On-shell approach to NLO multi-parton processes at hadron colliders.

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Based on publications: **JHEP 0511:027,2005; Phys.Rev.D78:036003,2008**, **Phys.Rev.D78:036003,2008**; **arXiv:0902.2760**.

In collaboration with:

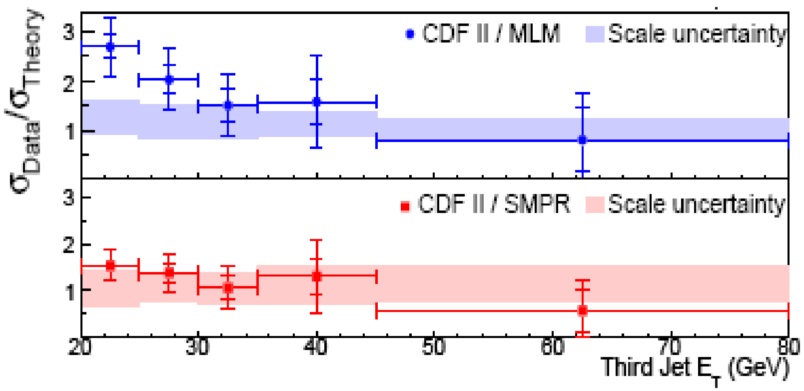
Bjerrum-Bohr, Dunbar; Carola Berger, Zvi Bern, Fernando Febres Cordero, Lance Dixon, Darren Forde, Daniel Maitre, David Kosower; Tanju Gleisberg;

Content:

- Brief discussion of recent progress.
- Complexity of W+3jet amplitudes.
- Color sampling and timing.
- Structuring on-shell methods using analytic properties:
  - Integral coefficients.
  - Recursive techniques.
- Stability.

First Precise Predictions for W + 3 Jet Production at Hadron Colliders

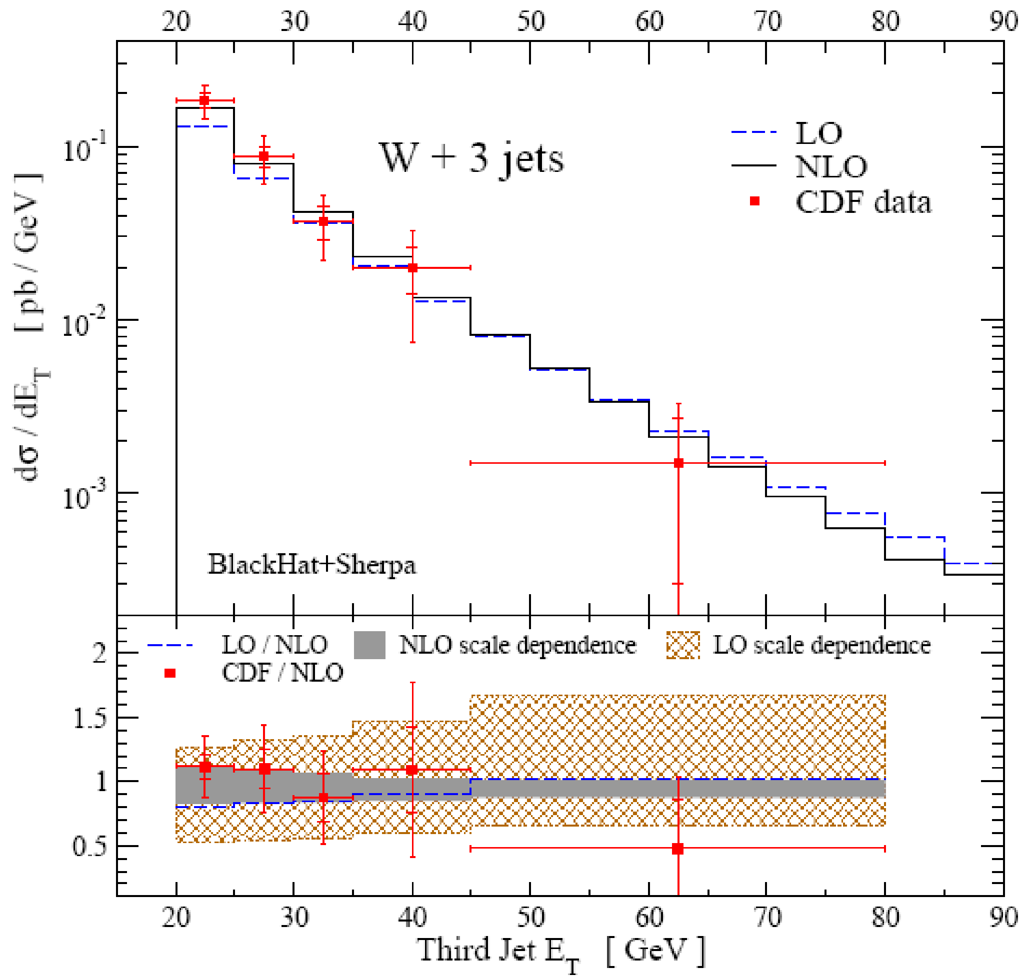
T. Aaltonen et al. [CDF Collaboration], arXiv:0711.4044 data from $320pb^{-1}$



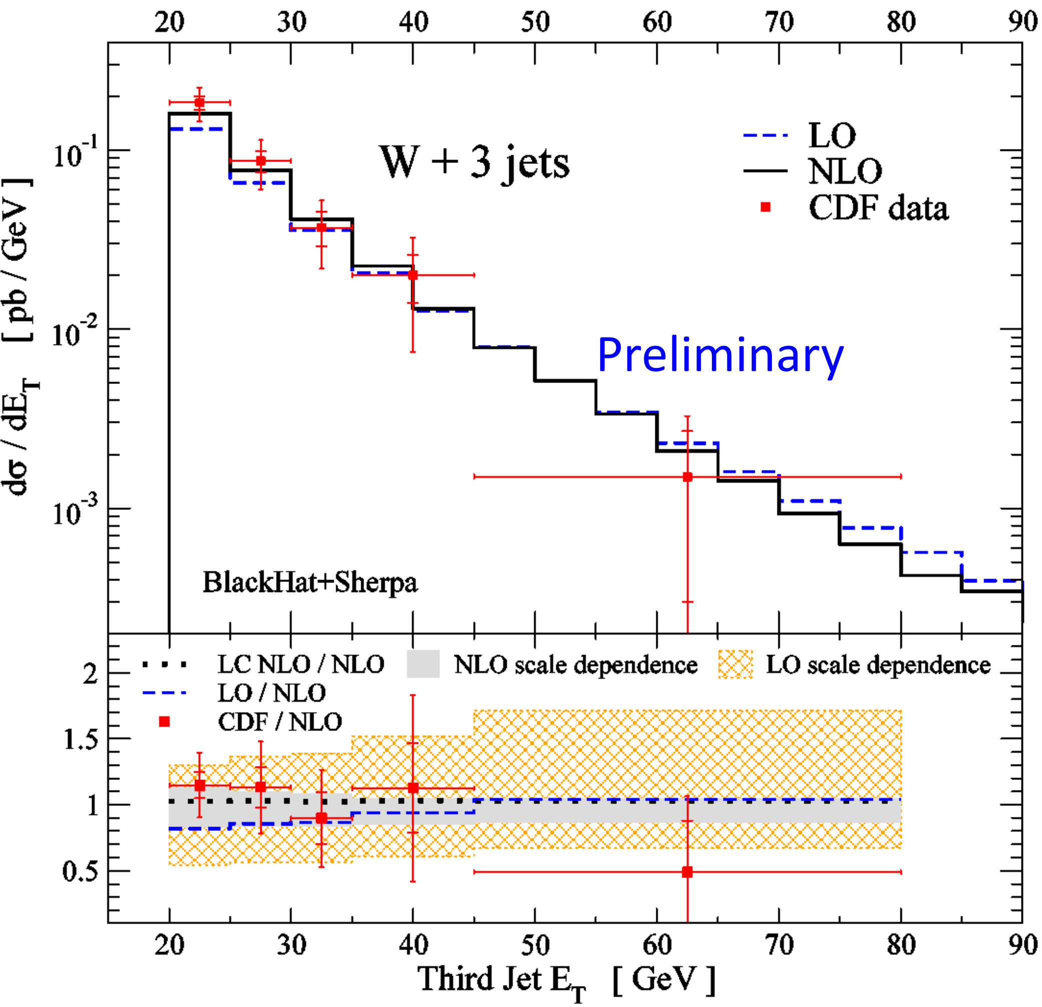
Comparison: LO with different matching Schemes.

•BlackHat+Sherpa.
•Leptonic decay of Ws.
•Leading color approximation.
•Scale uncertainty reduced significantly from 30% to 10%.

Berger, Bern, Dixon, Febres Cordero, Forde, H.I., Maitre, Kosower; Gleisberg; **arXiv:0902.2760**.



New: complete W+3jets



- Full result.
- Quantify our earlier leading color approx. to be good to within 3%.

$E_T^e > 20$ GeV $\quad E_T^{\text{jets}} > 20$ GeV

$|\eta^e| < 1.1 \quad \not{E}_T > 30$ GeV

$|\eta^{\text{jets}}| < 2 \quad M_T^W > 20$ GeV

$$\mu_0 \equiv \mu_F = \mu_R = \sqrt{M_W^2 + p_T^2(W)}$$

$$\mu_0/2 < \mu < 2\mu_0$$

Jet alg.: SISCone (Salam, Soyez, '07);

CTEQ6M, CTEQ6L1 PDF sets

Further remarks:

- All processes + leptonic decays (partial on-shell W study: Ellis, Melnikov, Zanderighi ‘09):

$$\mathrm{TeV}: \quad p\bar{p} \to W^{\pm} + 3 \text{ jets} \to \ell^{\pm}\nu_{\ell} + 3 \text{ jets}$$

- Partonic loop amplitudes (7-point): (Berger, Bern, Dixon, Febres Cordero, Forde, H.I., Maitre, Kosower ‘08; completed by Ellis, Giele, Kunszt, Melnikov, Zanderighi.)

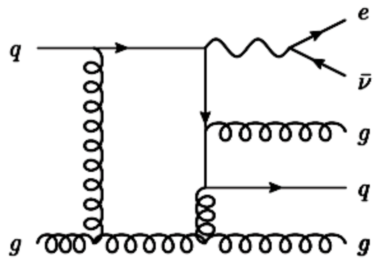
$$q\bar{q}ggg \to W^{\pm} \to e^{\pm}\nu_e\,, \qquad q\bar{q}Q\bar{Q}g \to W^{\pm} \to e^{\pm}\nu_e$$

- 5 light flavors used in *massless approximation*.
- We do not include contributions from a single top-loop.

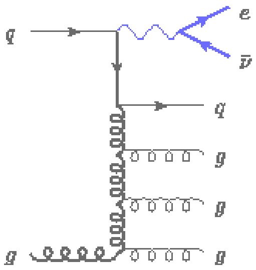
Complete NLO with: BlackHat+Sherpa.

$$\sigma^{NLO} = \int_{m+1} \left[d^{(4)}\sigma^{R} - d^{(4)}\sigma^{CS} \right] + \int_{m} \left[\int_{loop} d^{(D)}\sigma^{virt} - \int_{1} d^{(D)}\sigma^{CS} \right]_{\varepsilon=0} .$$

- BlackHat: (Berger, Bern, Dixon, Febres Cordero, Forde, H.I., Maitre, Kosower)
  - ONE-LOOP matrix elements



- SHERPA event generator (including AMEGIC++) (Gleisberg, Hoeche, Krauss, Schoenherr, Schumann, Siegert, Winter,...)
  - REAL radiative corrections
  - Phase space INTEGRATION
  - Automated DIPOLES (S.Catani, M.H.Seymour '97; Gleisberg, Krauss '07)
  - Framework for ANALYSIS



Pointer.

- Many details of the BlackHat matrix element extraction presented in Darren Forde's talk.
- Physics see Fernando Febres Cordero's talk later today.
- HERE: discuss some highlights of techniques used for W+3jets computation.

Color “sampling” and timing.

Color Expansion.

ME squared organized by group theory parameters:

$$\begin{aligned}\sigma^{virt}(N_c, n_f; p_i) \;=\; & \sigma^{virt}_{\text{leading}}(N_c, n_f; p_i) + \\ & + \frac{n_f}{N_c}\, \sigma^{virt}_{\text{n}_\text{f}}(N_c, n_f; p_i) + \\ & + \frac{1}{N_c^2}\, \sigma^{virt}_{\text{subleading}}(N_c, n_f; p_i)\,.\end{aligned}$$

N_c .... number of colors

n_f .... number of active flavors.

NOTICE:

- Some freedom in color expansion.
- Full real part and dipoles.
- Full singular terms $1/\epsilon,\ 1/\epsilon^2$
- Only finite part divided into leading & subleading terms.

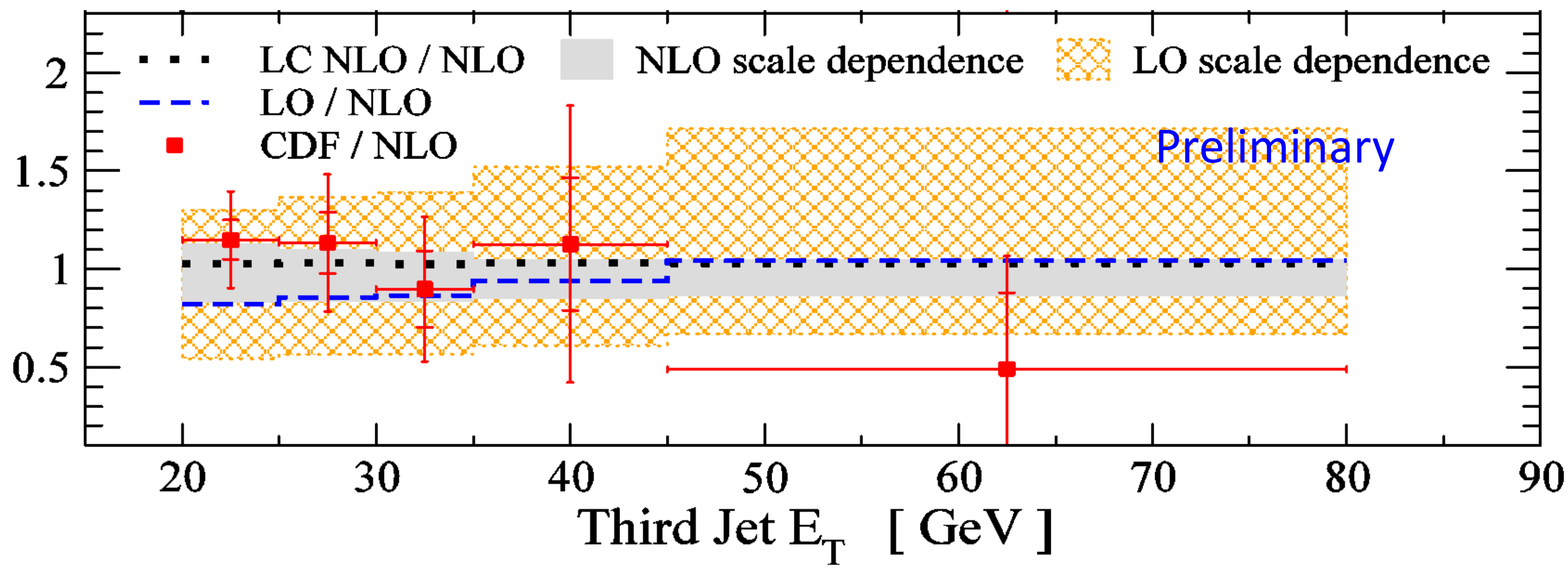
Subleading color quantitatively very *small*, but *statistically relevant*:

number of jets	CDF	LC NLO	NLO
1	53.5 ± 5.6	$58.3^{+4.6}_{-4.6}$	$57.8^{+4.4}_{-4.0}$
2	6.8 ± 1.1	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.908^{+0.044}_{-0.142}$	$0.882(5)^{+0.057}_{-0.138}$

Preliminary

BlackHat+Sherpa target

for statistical err.	0.5%.
Subleading color	3%.
Pdf uncertainty	5-10%.
Scale uncertainty	11%.



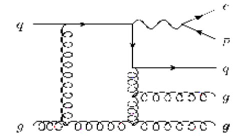
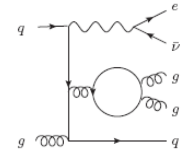
Color “Sampling”.

BlackHat computes & assembles primitive amplitudes:

partial amplitudes

sum of primitive amplitudes

$$\mathcal{A}^{\text{loop}} = \begin{cases} (T^{c_1}T^{c_2}T^{c_3})_{i\bar{j}}\, A_1^{\text{loop}} \\ \ldots \\ \text{tr}(T^{c_1}T^{c_2})(T^{c_3})_{i\bar{j}}\, A_3^{\text{loop}} \\ \ldots \end{cases}$$

$$A_1^{\text{loop}} = \Big\{$$  $$+ \frac{n_f}{N_c}$$  $$+ \mathcal{O}\left(\frac{1}{N_c^2}\right)$$

“primitive” = basic color-ordered amplitudes.

- Count of primitive amplitudes: subleading color vs. leading color: factor 20 but only 3% effect.
- To achieve precision target of 0.5% less than 1/10 of PS-points necessary.
- Effectively adding sub-leading color increases computation time by a factor of two.

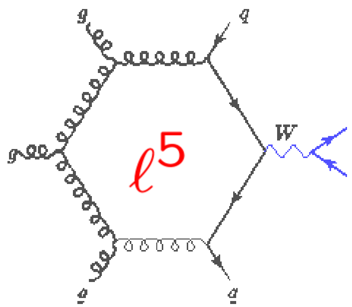
Complexity of one-loop computation.

Computational Complexity: W+3jets.

$$\int d\ell^D \frac{\ell_{\mu_1}\ell_{\mu_2}\cdots\ell_{\mu_{\text{rank}}}}{\left[(\ell-k_1)^2(\ell-k_2)^2\cdots(\ell-k_6)^2\right]}\,.$$

- Rank 5 tensor integrals with

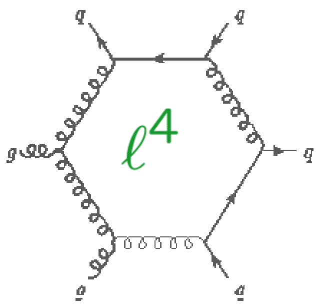
on-shell + unitarity methods.



- State of the art rank 4 tensor integrals

with Feynman-diagrammatic approach

in $pp \to t\bar{t}b\bar{b}$. (Bredenstein, Denner, Dittmaier, Pozzorini '09)



Confident with shown scaling with multiplicity that more can be done, e.g. W+4jets. (Berger, Bern, Febres Cordero, Dixon, Forde, H.I., Maitre, Kosower '08, see Forde's talk)

Structuring on-shell methods using analytic properties.

Reminder: one-loop basis.

See Bern, Dixon, Dunbar, Kosower, hep-ph/9212308.

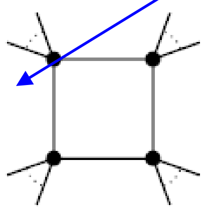
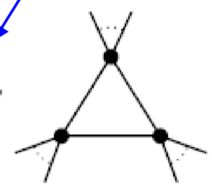
All external momenta in *D=4*, loop momenta in *D=4-2ε* (dimensional regularization).

Rational part

Cut part

Process dependent D=4 rational integral coefficients

$$A = R + C$$

$$C = \sum_i b_i$$  $$+ \sum_i c_i$$  $$+ \sum_i d_i$$ 

HERE we discuss:

- Cut Part from unitarity cuts in 4 dimensions.
- Rational part from on-shell recurrence relations.

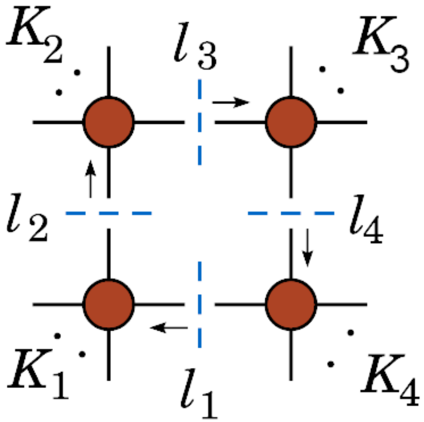
Speed & Precision from Analytic Structure:

- Generalized unitarity:
  - *Spinor variables* in loop momentum parameterizations. (Forde '07; Berger, Bern, Dixon, Febres Cordero, Forde, H.I., Maitre, Kosower '08)
  - Reduced tensor degree for certain *helicity structures*. (Bern, Carrasco,Forde, H.I., Johansson '08)
  - …
- On-Shell recursions:
  - Recursions for integral coefficients. (Bern, Bjerrum-Bohr, Dunbar, H.I.)
  - Tree-like speed for rational terms for *split-helicity configurations*. (used for W+3jets in BlackHat.)
  - …

Unitarity Method.

- Unitarity Approach:
  - Bern, Dixon, Dunbar, Kosower, hep-ph/9403226, hep-th/9409265.
- Generalized Unitarity:
  - Bern, Dixon, Kosower, hep-ph/9708239, hep-ph/0001001.
  - Britto, Cachazo, Feng, hep-th/0412103.
- Recent advances for numerical implementation:
  - del Aguila and Pittau, hep-ph/0404120; Ossola, Papadopoulos and Pittau, hep-ph/0609007.
  - Forde, 0704.1835; Badger, 0806.4600, 0807.1245.
  - Giele, Kunszt and Melnikov, 0801.2237; Ellis, Giele, Kunszt, Melnikov, 0806.3467;

Boxes: Precision from spinors.



Quadruple cut: (Britto, Cachazo, Feng '04)

$$d_i = \frac{1}{2}\sum_{\sigma=\pm} d_i^\sigma ,$$

$$d_i^\sigma = A_{(1)}^{\text{tree}} A_{(2)}^{\text{tree}} A_{(3)}^{\text{tree}} A_{(4)}^{\text{tree}} \Big|_{l_i = l_i^{(\sigma)}}$$

Expectation: $d_i \sim \left(\frac{1}{\Delta_4}\right)^4$

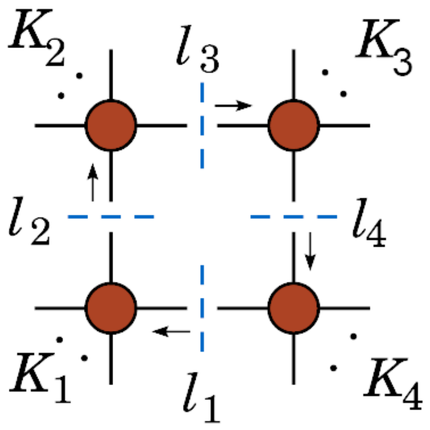
$$(l_1^{(\pm)})^\mu = \frac{\langle 1^\mp | \not{K}_2 \not{K}_3 \not{K}_4 \gamma^\mu | 1^\pm \rangle}{2 \langle 1^\mp | \not{K}_2 \not{K}_4 | 1^\pm \rangle} ,$$

Berger, Bern, Dixon, Febres Cordero, Forde, H.I., Kosower, Maitre '08; Risager '08.

$$\Delta_4 = -2 \langle 1^- | \not{K}_2 \not{K}_4 | 1^+ \rangle \langle 1^+ | \not{K}_2 \not{K}_4 | 1^- \rangle$$

RESULT: Reduced power of spurious factor from Gram determinant. Improved PRECISION!

Example (a): box-coefficient.

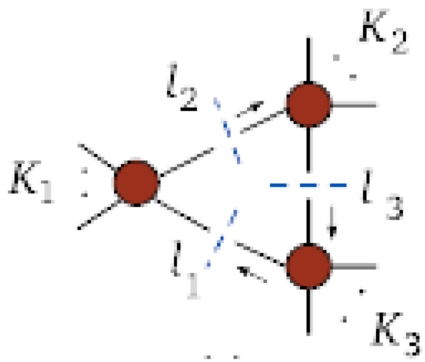


e.g.: A(1−, 2+, 3−, 4+, 5−, 6+)

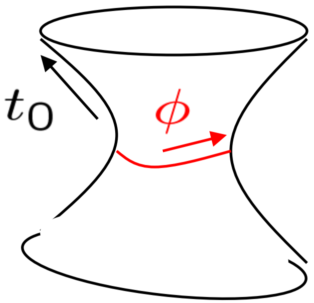
(Britto, Feng, Mastrolia '06)

$$c^{2m\ h}_{4:2;2} = \frac{2[1\ 2]\langle 5\ 6\rangle\langle 5|P_{123}|1]^2\langle 6|P_{123}|2]^2 P_{123}^2}{[2\ 3]\langle 4\ 5\rangle\langle 4|P_{123}|1]\langle 6|P_{123}|3]\langle 6|P_{123}|1]^4}$$

Triangle coefficients:



del Aguila, Pittau '04, Ossola, Papadopoulos, Pittau '06
Forde '07; Berger, Bern, Dixon, Febres Cordero,
Forde, H.I., Kosower, Maitre '08.



Three on-shell conditions:

$$t = t_0 * e^{i\phi}$$

$$l_1^\mu(t) \;=\; \tilde{K}_1^\mu + \tilde{K}_3^\mu + \frac{t}{2}\langle \tilde{K}_1^-|\gamma^\mu|\tilde{K}_3^-\rangle + \frac{1}{2t}\langle \tilde{K}_3^-|\gamma^\mu|\tilde{K}_1^-\rangle$$

Coefficient from analysing triple cut:

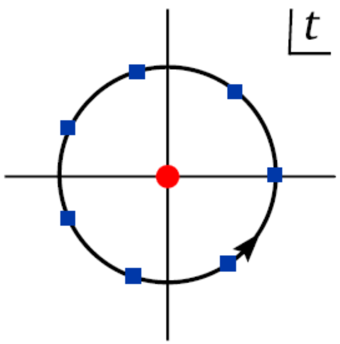
$$C_3(t) \equiv A_{(1)}^{\text{tree}} A_{(2)}^{\text{tree}} A_{(3)}^{\text{tree}}\Big|_{l_i = l_i(t)} .$$

Simple analytic dependence on t!

Example (a): general case.

3-mass triangle:

Forde '07; Berger, Bern, Dixon, Febres Cordero, Forde, H.I., Kosower, Maitre '08.



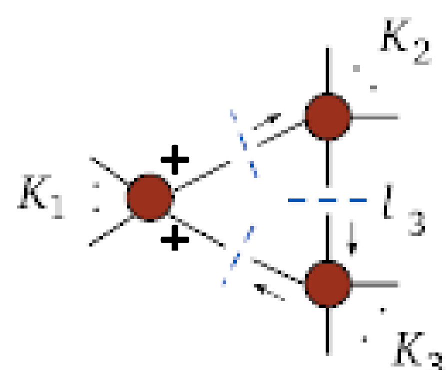
$$C_3(t) \equiv A^{\text{tree}}_{(1)} A^{\text{tree}}_{(2)} A^{\text{tree}}_{(3)} \Big|_{l_i = l_i(t)} .$$

$$C_3(t) = C_{-3} t^{-3} + C_{-2} t^{-2} + C_{-1} t^{-1} + C_0 + C_1 t^1 + C_2 t^2 + C_3 t^3 + \text{boxes} .$$

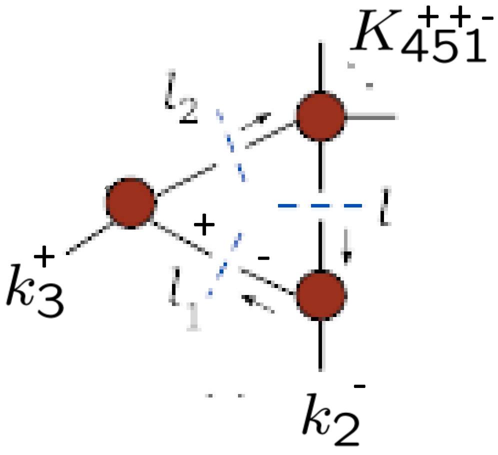
•Simple dependence on parameter t allows to extract triangle coefficient with discrete Fourier analysis.

•ZEROS for specific helicities (SUSY):

$$C_3(t) = \text{boxes} .$$



Example (b): 1-mass triangles.



$$l = \left(-\lambda_2 + \frac{t}{2}\lambda_3\right)\tilde{\lambda}_2\,,$$
$$l_1 = \frac{t}{2}\lambda_3\,\tilde{\lambda}_2\,,$$
$$l_2 = \lambda_3\left(\tilde{\lambda}_3 + \frac{t}{2}\tilde{\lambda}_2\right).$$

$$C_3(t) \;=\; C_0 + C_1 t^1 + C_2 t^2 + C_3 t^3 + \text{boxes}\,.$$

Many ZEROS! Only positive powers of t.

Triangle with scalar loop:

$$c_{4,5,1;2;3}(t) \sim \frac{\left(\frac{t}{2}\right)^2 \langle 1\,3\rangle^2\,[3\,2]\,\left(\langle 1\,2\rangle + \frac{t}{2}\,\langle 1\,3\rangle\right)}{\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}\,.$$

$$c_{4,5,1;2;3}(l) \sim \frac{\langle 2|l|3]^2\,\langle 1\,3\rangle^2\,[3\,2]\,\left(\langle 1\,2\rangle + \frac{\langle 1\,3\rangle\langle 2|l|3]}{s_{23}}\right)}{s_{23}^2\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 5\,1\rangle}\,.$$

On-shell recursions.

- Loop-level on-shell recursion.
  - Bern, Dixon, Kosower, hep-th/0501240,hep-ph/0505055,
  - Berger, Del Duca, Dixon, hep-ph/0608180,
  - Forde, Kosower, hep-ph/0509358, Berger, Bern, Dixon, Forde, Kosower, hep-ph/0607014
  - Bern, Bjerrum-Bohr, Dunbar, H.I., hep-ph/0507019.
  - Berger, Bern, Febres Cordero, Dixon, Forde, Maitre, Kosower arXiv:080341.80;

Recursions for Loops?

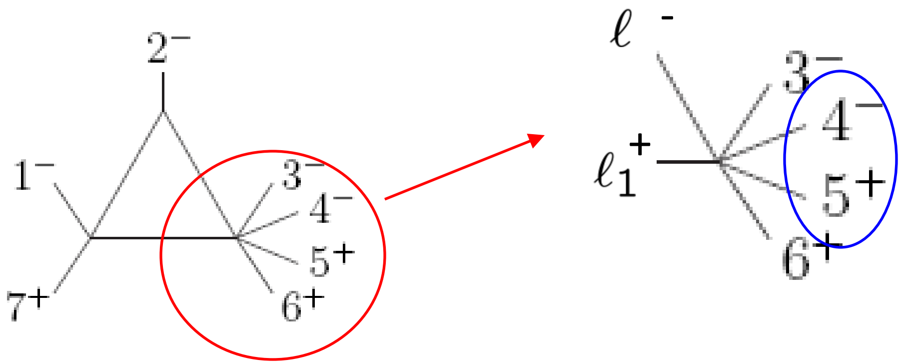
- BCFW recursion for trees rely on universal information:
  - Trees rational expressions in loop momenta.
  - Universal factorization & pole structure *in complex momenta*.
  - Good scaling behaviour for large complex momenta.

- All this known for rational terms R giving rational recursions. (At work in BlackHat. See Forde's talk.)
- All this known for certain integral coefficients and we find on-shell recursions for integral coefficients.

Fast integral coefficients.

- Simple BCFW-like recursion for coefficients with split helicity corners. (Bern, Bjerrum-Bohr, Dunbar, H.I. '06)

Example: scalar loop, external gluons:



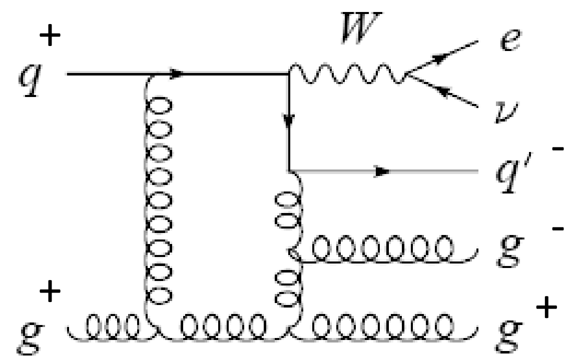
- Recursion using momenta 4 & 5.
- Tree-like BCFW recursion relation.

$$c_n(0) = \sum_{\alpha,h} A^h_{n-m_\alpha+1}(z_\alpha)\, \frac{i}{K_\alpha^2}\, c^{-h}_{m_\alpha+1}(z_\alpha)$$

- On-shell recursions for integral coefficients have tree-like speed & precision!

Fast rational terms.

- Particularly effective for split helicity amplitudes which have only split coefficients.

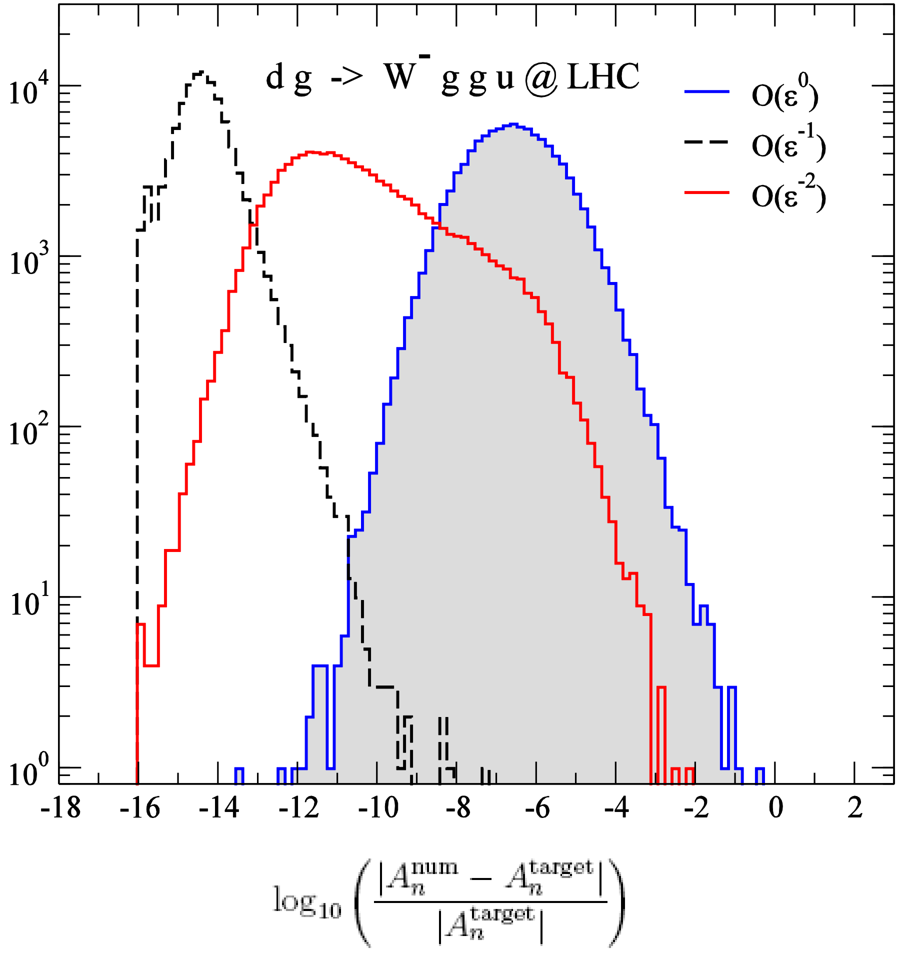


- Rational terms inherited speed & stability from integral coefficients.
- Usually 80% of computation time spent on a given rational. For fast rational terms down to a few percent.
- Out of 8 helicity structures of this primitive amplitudes, 3 fall into this class.

Numerical stability.

W+3jets: Stability Study

100,000 PS points produced by Sherpa for this single subprocess integrated at the LHC.



Precision tests for different parts of one-loop amplitude.

Rescue strategy: locally recomputed with higher precision.

The difference in
the total cross section about 3 orders of magnitude smaller than
the statistical error from the numerical integration.

Conclusions

- Presented full NLO results for W+3jets at the Tevatron from BlackHat+Sherpa.

- More physics see Fernando's talk later today.

- Discussed a combination of modern techniques: on-shell methods, unitarity together with color "sampling", analytic methods some of which are already used in BlackHat.

Many improvements are in sight and a variety of multi-parton processes seems within reach.

EXTRA SLIDES.