Generalized unitarity
&
W + 3 jets

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Based on work done with Keith Ellis, Walter Giele, Zoltan Kunszt, Kirill Melnikov

LoopFest, Madison, May 2009

This talk

I won't explain the method in detail, only remind of the main ideas.
I will concentrate on practical aspects:
numerical implementation, efficiency, performance,
applications & new results

References:

- Ellis, Giele, Kunszt '07 *[Unitarity in D=4]*
- Giele, Kunszt, Melnikov '08 *[Unitarity in D≠4]*
- Giele & GZ '08 *[All one-loop N-gluon amplitudes]*
- Ellis, Giele, Melnikov, Kunszt '08 *[Massive fermions, ttggg amplitudes]*
- Ellis, Giele, Melnikov, Kunszt, GZ '08 *[W+5p one-loop amplitudes]*
- Ellis, Melnikov, GZ '09 *[W+3 jets]*

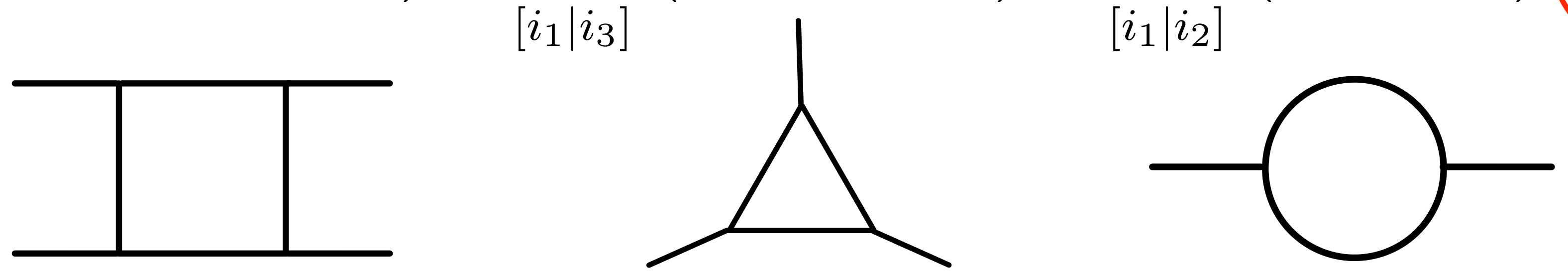
These papers heavily rely on previous work

- Bern, Dixon, Kosower '94 *[Unitarity, oneloop from trees]*
- Ossola, Pittau, Papadopoulos '06 *[OPP]*
- Britto, Cachazo, Feng '04 *[Generalized cuts]*
- *[....]*

One-loop virtual amplitudes

Cut constructable part can be obtained by taking residues in D=4

$$\mathcal{A}_N = \sum_{[i_1|i_4]} \left(d_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} \right) + \sum_{[i_1|i_3]} \left(c_{i_1 i_2 i_3} \; I^{(D)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b_{i_1 i_2} \; I^{(D)}_{i_1 i_2} \right) + \mathcal{R}$$



Rational part: can be obtained with D ≠ 4

Generic D dependence

Two sources of D dependence

dimensionality of loop momentum D

# of spin eigenstates/ polarization states D_s

Keep D and D_s distinct

$$\mathcal{A}^{D} \Rightarrow \mathcal{A}^{(D,D_s)}$$

Two key observations

1. External particles in D=4 ⇒ no preferred direction in the extra space

$$\mathcal{N}(l) = \mathcal{N}(l_4, \tilde{l}^2) \qquad \tilde{l}^2 = -\sum_{i=5}^{D} l_i^2$$

$\mathcal{N}$: numerator function

☛ in arbitrary D up to 5 constraints ⇒ get up to pentagon integrals

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2. Dependence of $\mathcal{N}$ on D_s is linear (or almost)

$$\mathcal{N}^{D_s}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

☛ evaluate at any D_{s1}, D_{s2} ⇒ get $\mathcal{N}_0$ and $\mathcal{N}_1$, i.e. , full $\mathcal{N}$

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Choose D_{s1}, D_{s2} integer ⇒ suitable for numerical implementation

[D_s = 4 - 2ϵ 't-Hooft-Veltman scheme, D_s = 4 FDH scheme]

In practice

- Start from

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\overline{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\overline{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{[i_1|i_3]} \frac{\overline{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\overline{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\overline{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

- Use unitarity constraints to determine the coefficients, computed as products of tree-level amplitudes with complex momenta in higher dimensions

- Berends-Giele recursion relations are natural candidates to compute tree level amplitudes: they are very fast for large N and very general (spin, masses, complex momenta)

Berends, Giele '88

Final result

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1 i_2 i_3 i_4 i_5}\; I^{(D)}_{i_1 i_2 i_3 i_4 i_5}$$
$$+ \sum_{[i_1|i_4]} \left(d^{(0)}_{i_1 i_2 i_3 i_4}\; I^{(D)}_{i_1 i_2 i_3 i_4} - \frac{D-4}{2}\, d^{(2)}_{i_1 i_2 i_3 i_4}\; I^{(D+2)}_{i_1 i_2 i_3 i_4} + \frac{(D-4)(D-2)}{4}\, d^{(4)}_{i_1 i_2 i_3 i_4}\; I^{(D+4)}_{i_1 i_2 i_3 i_4} \right)$$
$$+ \sum_{[i_1|i_3]} \left(c^{(0)}_{i_1 i_2 i_3}\; I^{(D)}_{i_1 i_2 i_3} - \frac{D-4}{2} c^{(9)}_{i_1 i_2 i_3}\; I^{(D+2)}_{i_1 i_2 i_3} \right) + \sum_{[i_1|i_2]} \left(b^{(0)}_{i_1 i_2}\; I^{(D)}_{i_1 i_2} - \frac{D-4}{2} b^{(9)}_{i_1 i_2}\; I^{(D+2)}_{i_1 i_2} \right)$$

Cut-constructable part:

$$\mathcal{A}_N^{CC} = \sum_{[i_1|i_4]} d^{(0)}_{i_1 i_2 i_3 i_4}\; I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3}\; I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2}\; I^{(4-2\epsilon)}_{i_1 i_2}$$

Rational part:

$$R_N = -\sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{6} + \sum_{[i_1|i_3]} \frac{c^{(9)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m^2_{i_1} + m^2_{i_2}}{2} \right) b^{(9)}_{i_1 i_2}$$

Vanishing contributions: $\mathcal{A} = \mathcal{O}(\epsilon)$

The F90 Rocket program

Rocket science!

Eruca sativa =Rocket=roquette=arugula=rucola
Recursive unitarity calculation of one-loop amplitudes

So far computed one-loop amplitudes:

- ✓ N-gluons
- ✓ qq + N-gluons
- ✓ qq + W + N-gluons
- ✓ qq + QQ + W
- ✓ tt + N-gluons
- ✓ tt + qq + N-gluons [Schulze]

Issues of automated one-loop

- checks of the results
  - ☛ poles, ward identities, independence of choice of D_1 and D_2, independence of the choice of the solution of the unitarity constraints, independence from choice of auxiliary vectors (gauge)
- numerical instabilities at special points
  - ☛ efficient procedure for identification of special points, than run in quadruple precision
- numerical efficiency
  - ☛ polynomial scaling for any NLO amplitude (N^9 for gluons)
- practicality: computation of realistic LHC processes
  - ☛ *first application: W + 3 jets*

First application: W + 3 jets

1. W + 3 jets measured at the Tevaton, but LO varies by more than a factor 2 for reasonable changes in scales

	$W^{\pm}$, TeV	W^{+}, LHC	W^{-}, LHC
σ [pb], $\mu = 40$ GeV	74.0 ± 0.2	783.1 ± 2.7	481.6 ± 1.4
σ [pb], $\mu = 80$ GeV	45.5 ± 0.1	515.1 ± 1.1	316.7 ± 0.7
σ [pb], $\mu = 160$ GeV	29.5 ± 0.1	353.5 ± 0.8	217.5 ± 0.5

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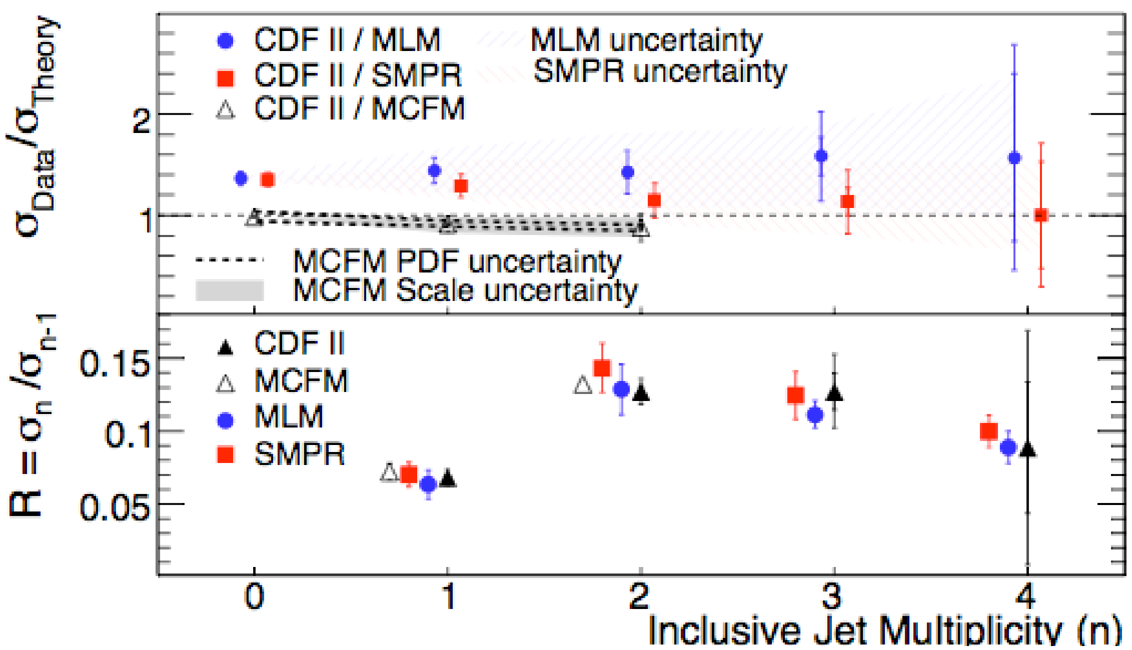
II. Measurements at the Tevaton: for W + n jets with n=1,2 data is described well by NLO QCD
$\Rightarrow$ verify this for 3 and more jets



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III. W + 3 jets of interest at the LHC, as one of the backgrounds to model-independent new physics searches using jets + MET

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IV. Calculation highly non-trivial optimal testing ground

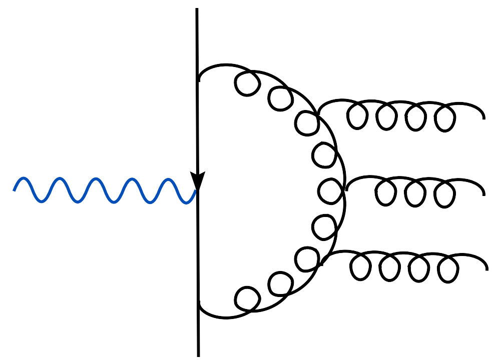
$0 \rightarrow \bar{u}\, d\, g\, g\, g\, W^+$ ⇒ 1203 +104 Feynman diagrams

$0 \rightarrow \bar{u}\, d\, \bar{Q}\, Q\, g\, W^+$ ⇒ 258 +18 Feynman diagrams

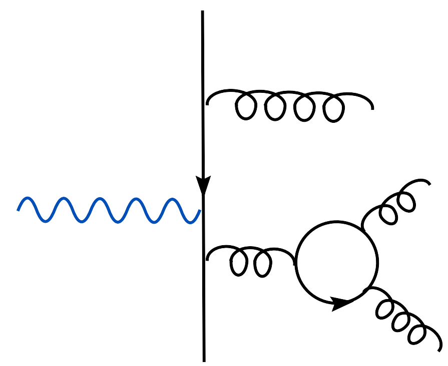
Primitive amplitudes: color structures

Leading color | Fermion loops | Subleading color

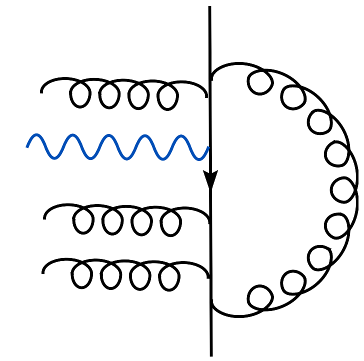
2-quark 3-gluon



$$\mathrm{LC} \equiv (N_c^2 - 1) N_c^3$$



$$\mathrm{LC} \cdot \frac{n_f}{N_c}$$



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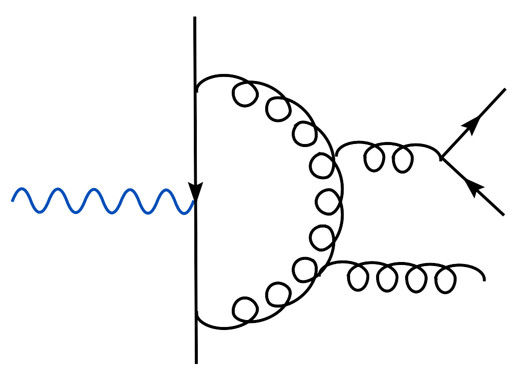
4-quark 1-gluon



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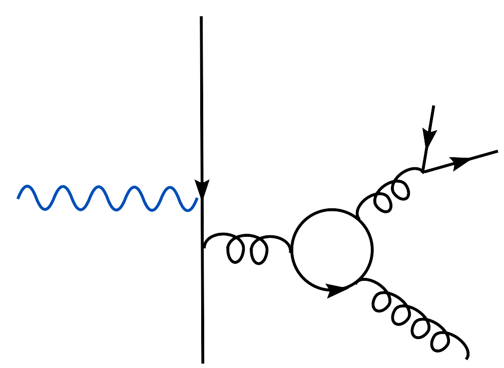

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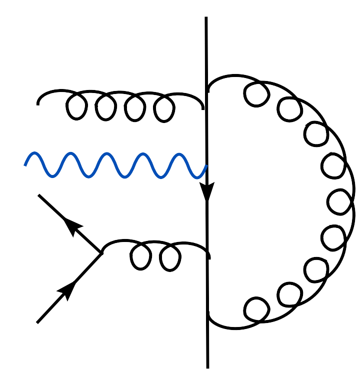

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Rules of the game

Procedure:

- order all SU(3) particles & allow all orderings of colorless particles

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Explicitly for W + 3 jets:

❶ ❷ ❸ ❹ ❺

u_1 g_2 g_3 g_4 d_5 + W

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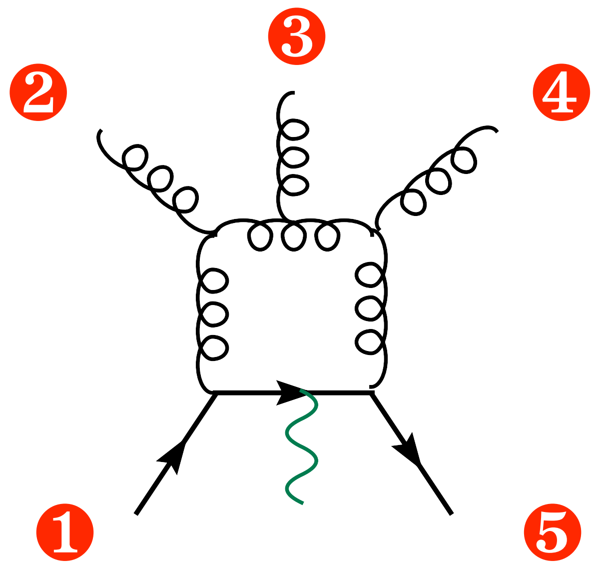

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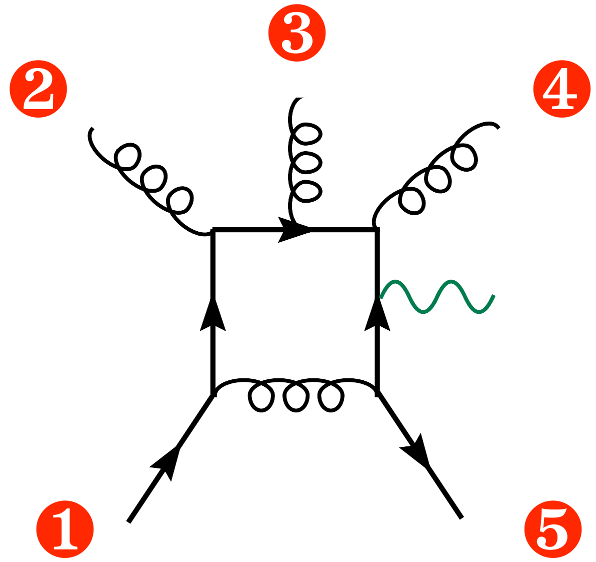

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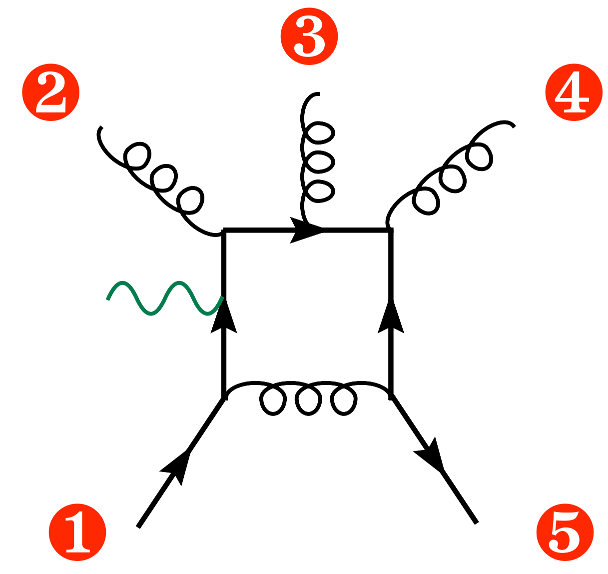

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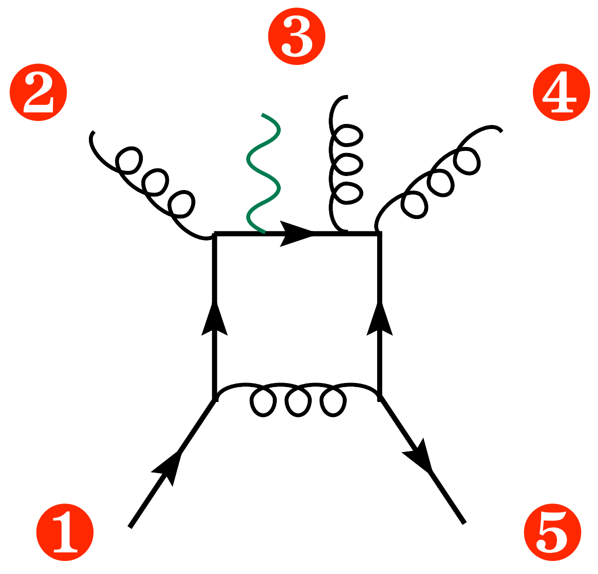

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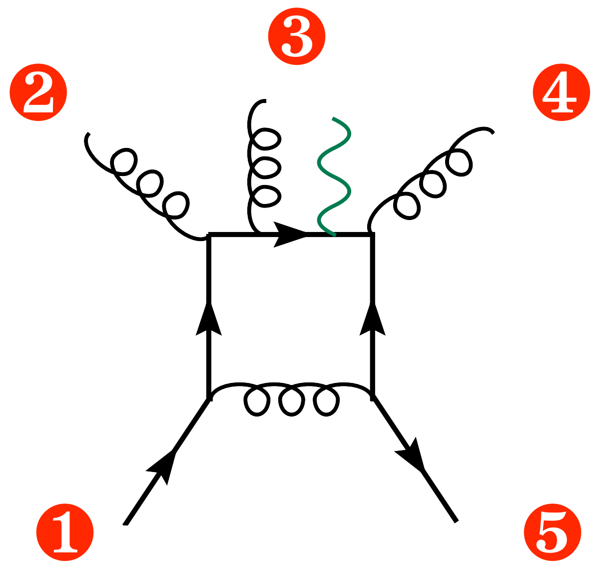

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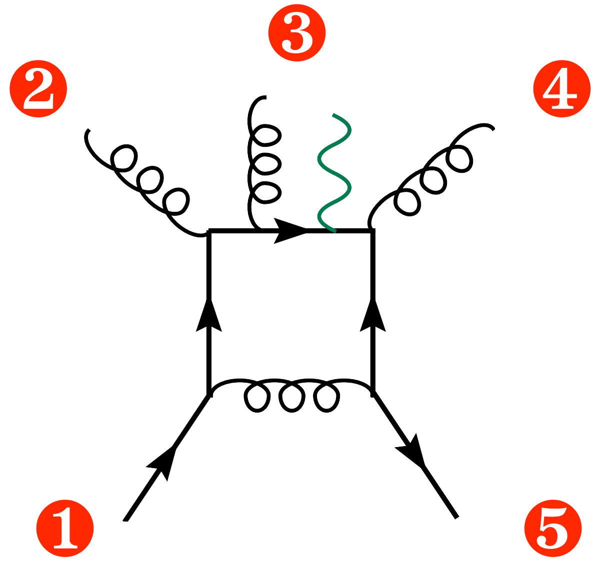

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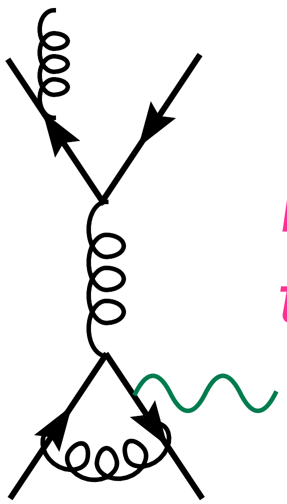

How does this work?

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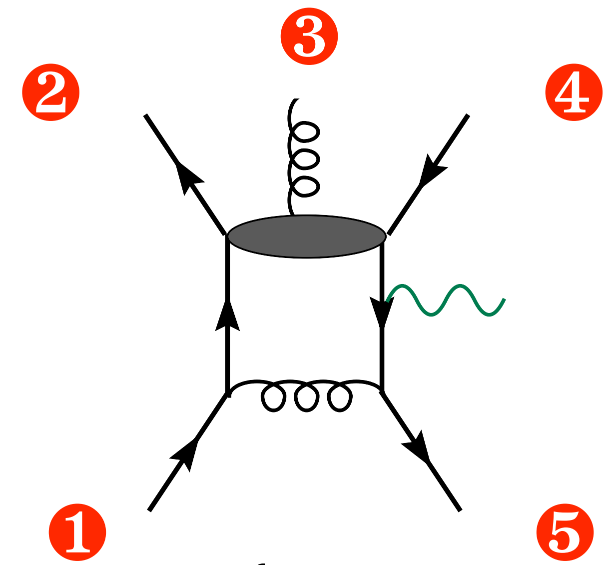
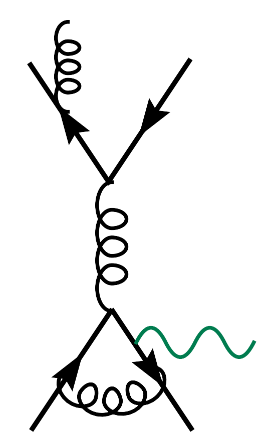
$u_1 \quad q_2 \quad g_3 \quad q_4 \quad d_5 + W$



Refers e.g. to:



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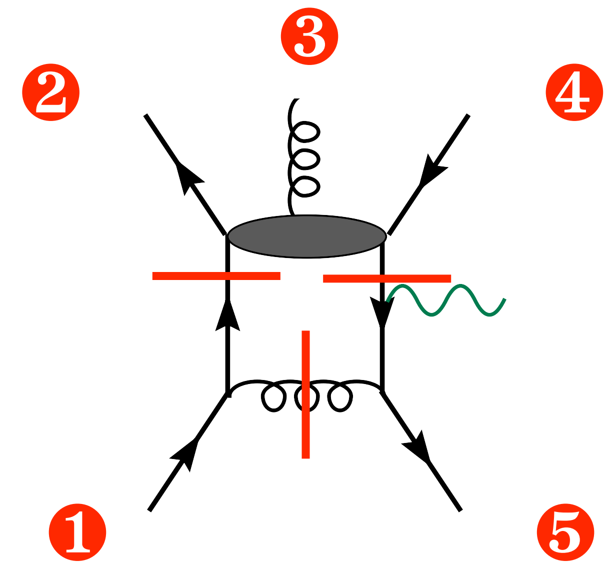

✓ *accept*

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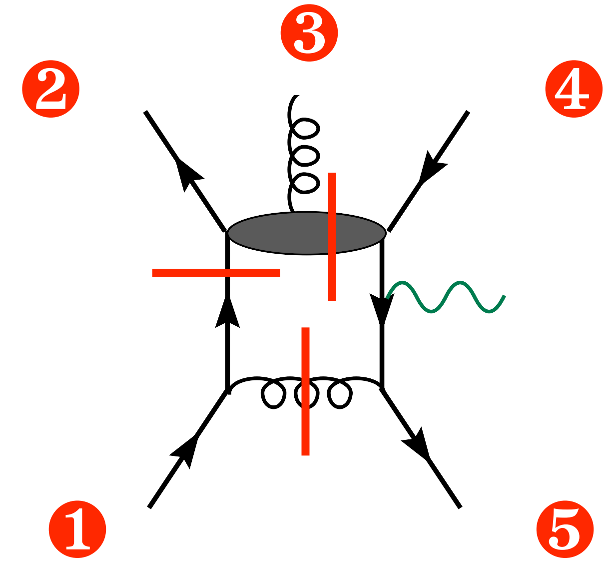

✗ *reject*

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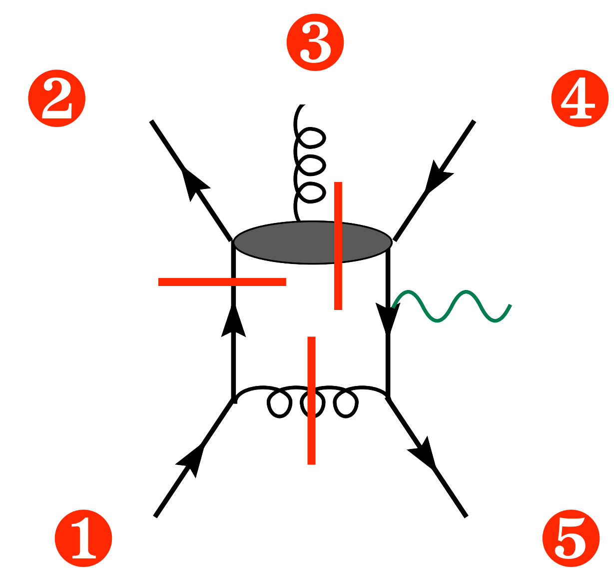

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- consider all cuts and throw away those involving dummy lines
- process each cut use standard D-dimensional unitarity
- tree-level amplitudes are computed via color stripped Feynman rules

Bern, Dixon, Kosower '94

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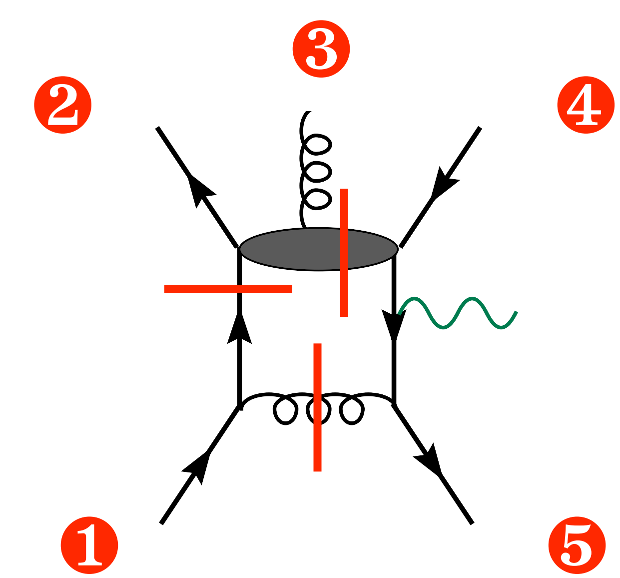


✗ *reject*

Sample results

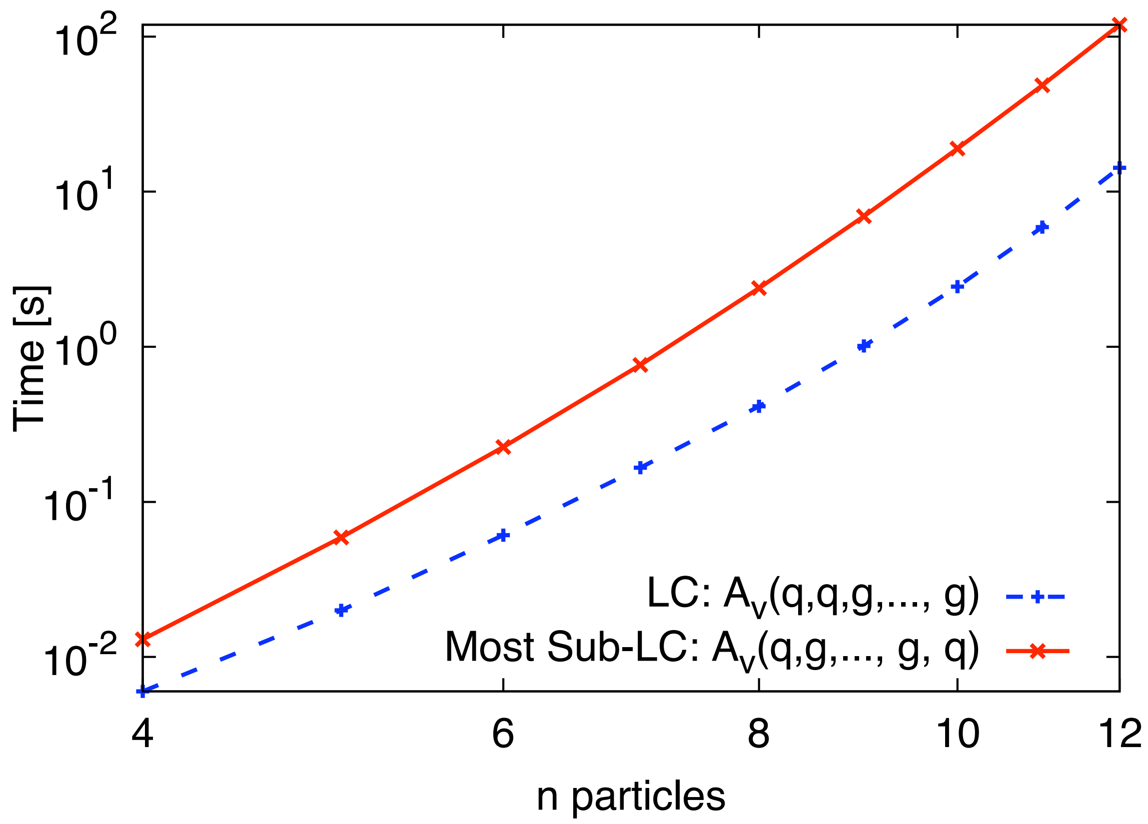
Helicity	$1/\epsilon^2$	$1/\epsilon$	ϵ^0
$A^{\rm tree}(1_{\bar q}^+ 2_q^- 3_g^+ 4_g^+ 5_g^+ 6_{\bar l}^+ 7_l^-)$			$-0.006873 + i\,0.011728$
$r_L^{[1]}(1_{\bar q}^+ 2_q^- 3_g^+ 4_g^+ 5_g^+ 6_{\bar l}^+ 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$5.993700 - i\,19.646278$
$A^{\rm tree}(1_{\bar q}^+ 2_q^- 3_g^+ 4_g^+ 5_g^- 6_{\bar l}^+ 7_l^-)$			$0.010248 - i\,0.007726$
$r_L^{[1]}(1_{\bar q}^+ 2_q^- 3_g^+ 4_g^+ 5_g^- 6_{\bar l}^+ 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-14.377555 - i\,37.219716$
$A^{\rm tree}(1_{\bar q}^+ 2_q^- 3_g^- 4_g^+ 5_g^+ 6_{\bar l}^+ 7_l^-)$			$0.495774 - i\,1.274796$
$r_L^{[1]}(1_{\bar q}^+ 2_q^- 3_g^- 4_g^+ 5_g^+ 6_{\bar l}^+ 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-1.039489 - i\,30.210418$
$A^{\rm tree}(1_{\bar q}^+ 2_q^- 3_g^- 4_g^+ 5_g^- 6_{\bar l}^+ 7_l^-)$			$-0.294256 - i\,0.223277$
$r_L^{[1]}(1_{\bar q}^+ 2_q^- 3_g^- 4_g^+ 5_g^- 6_{\bar l}^+ 7_l^-)$	-4.00000	$-10.439578 - i\,9.424778$	$-1.444709 - i\,26.101951$

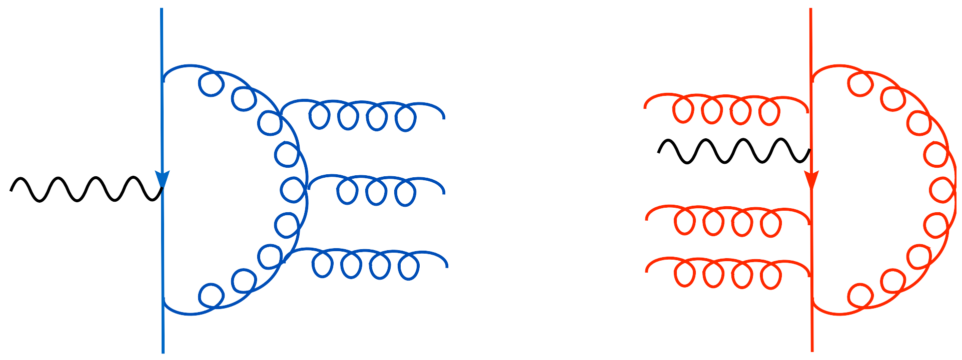
$$r_L^{[j]}(1,2,3,4,5,6,7) = \frac{1}{c_\Gamma}\frac{A_L^{[j]}(1,2,3,4,5,6,7)}{A^{\rm tree}(1,2,3,4,5,6,7)}, \quad c_\Gamma = \frac{\Gamma(1+\epsilon)\Gamma(1-\epsilon)^2}{(4\pi)^{2-\epsilon}\Gamma(1-2\epsilon)},$$

Leading color amplitudes in 0808.0941
[Berger, Bern, Cordero, Dixon, Forde, Ita, Kosower, Maitre]

All amplitudes in 0810.2542
[Ellis, Giele, Kunszt, Melnikov, GZ]

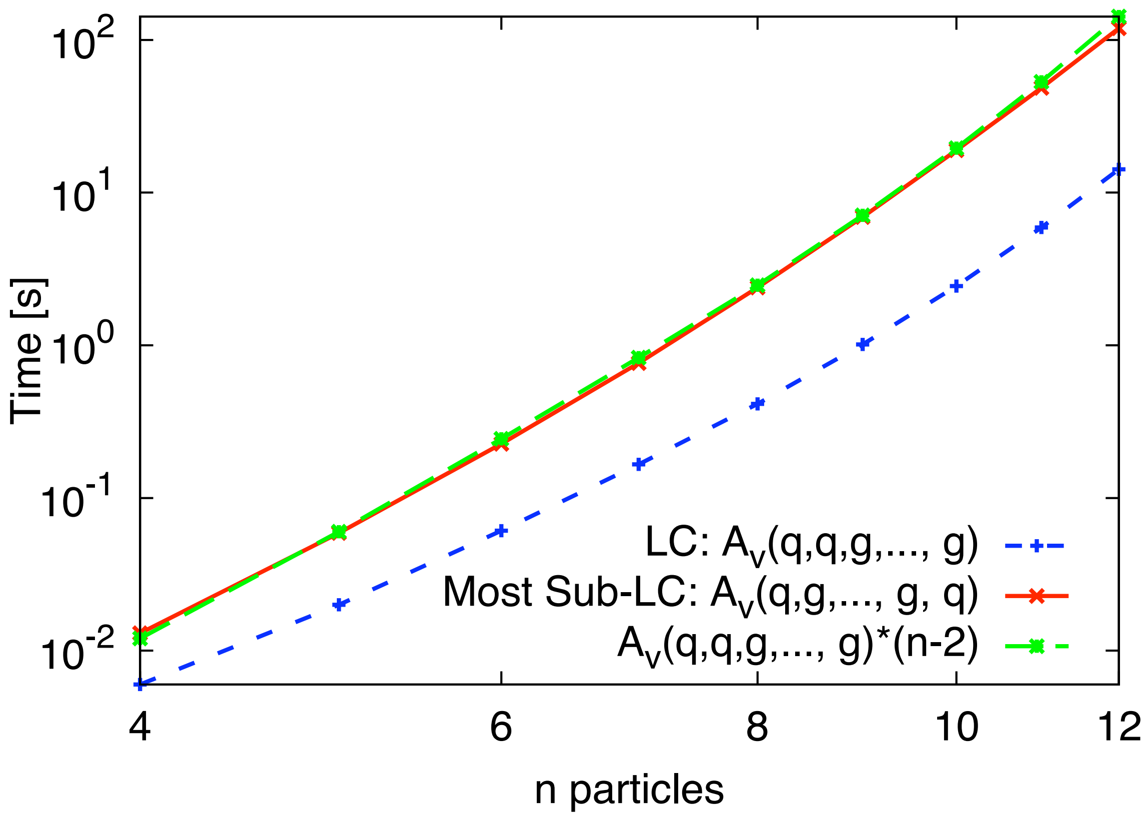
Time dependence of qq + W + n gluons

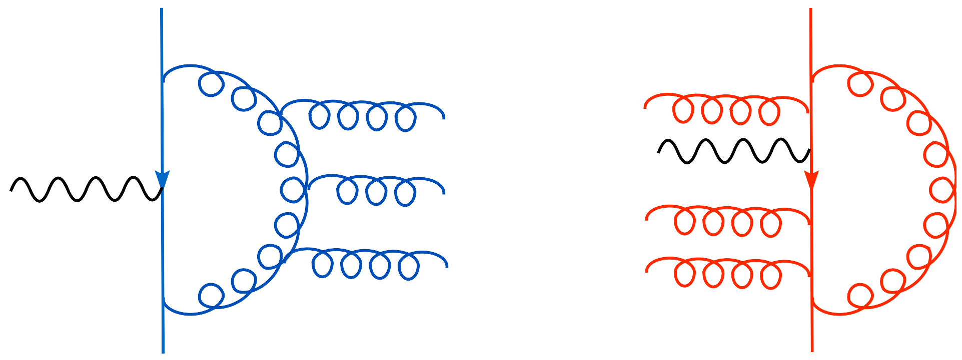




# of cuts: $N_{\rm cuts}$ $N_{\rm cuts} \cdot (n-2)$

Time dependence of qq + W + n gluons





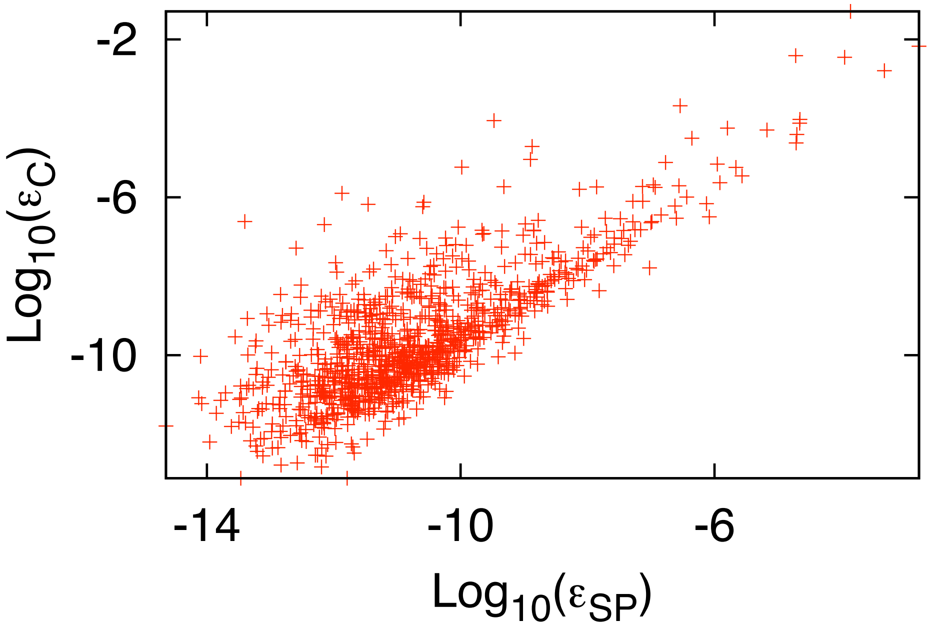
# of cuts: $N_{\rm cuts}$ $N_{\rm cuts} \cdot (n-2)$

Similar plots for qq + n gluons

Finding instabilities

1. Correlation in the accuracy of single pole and constant part

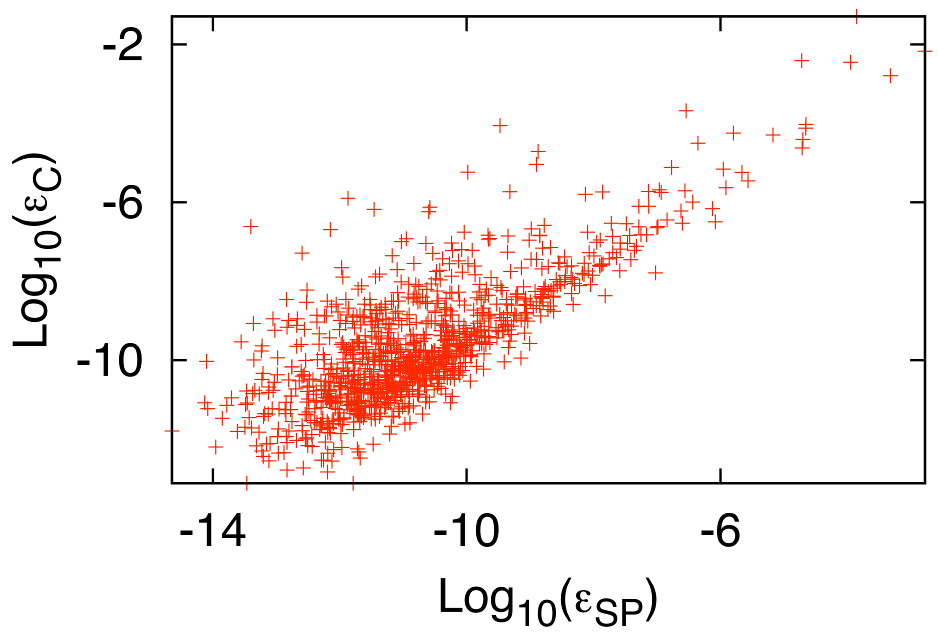
$\Rightarrow$ if the accuracy on the poles is worse than X use higher precision
This does not check the rational part



Finding instabilities

1. Correlation in the accuracy of single pole and constant part

⇒ if the accuracy on the poles is worse than X use higher precision
This does not check the rational part



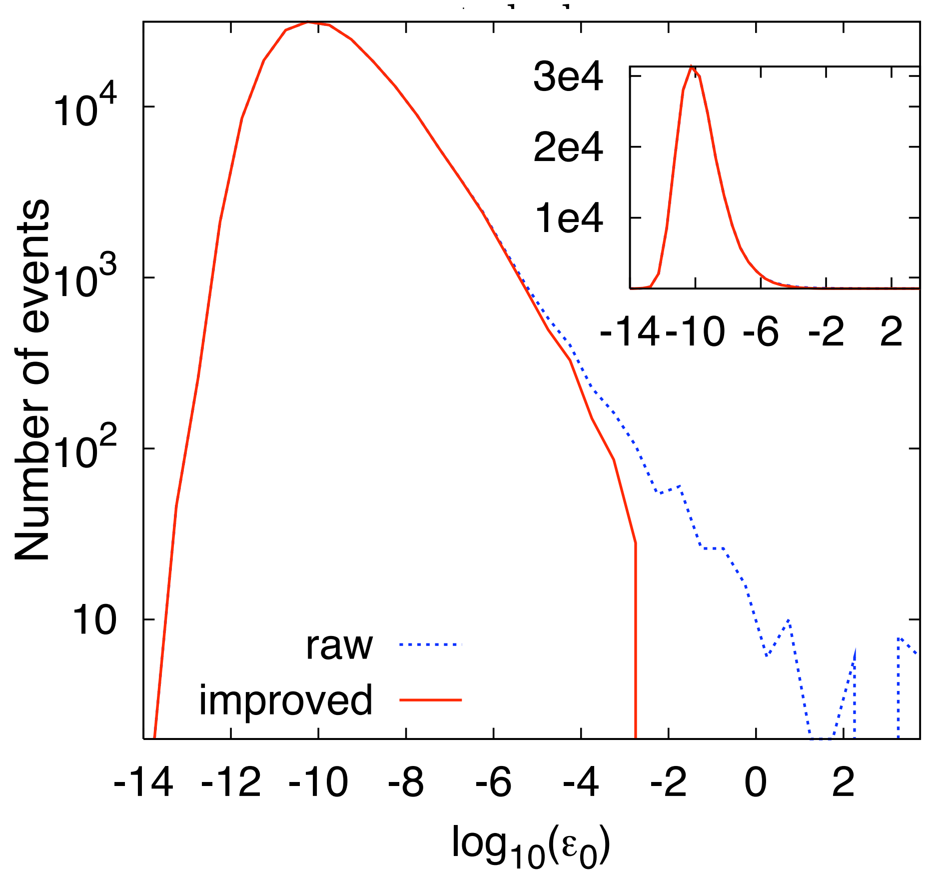
2. How good is the system of equations solved ?

Look at how well residues are reconstructed using the coefficients
In practice: choose a random loop momentum and for a given cut
- compute the residue as linear combination of coefficients
- compute the residue directly

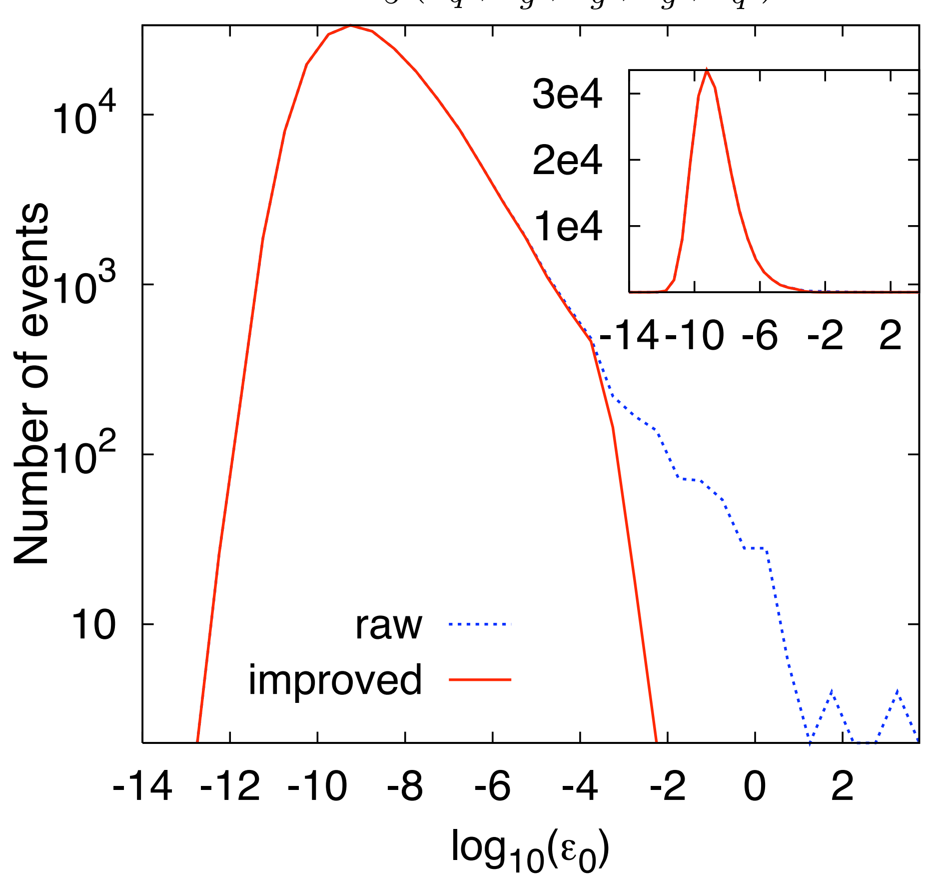
⇒ if the results differ more than X use higher precision

Instabilities and accuracy

$A_5^L(1_{\bar{q}}^+, 2_q^-, 3_g^-, 4_g^+, 5_g^-)$



$A_5^L(1_{\bar{q}}^+, 3_g^-, 4_g^+, 5_g^-, 2_q^-)$



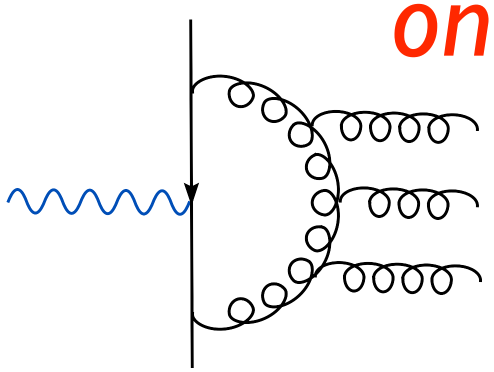
⇒ *All instabilities detected and cured with quadruple precision*

Primitive amplitudes: color structures

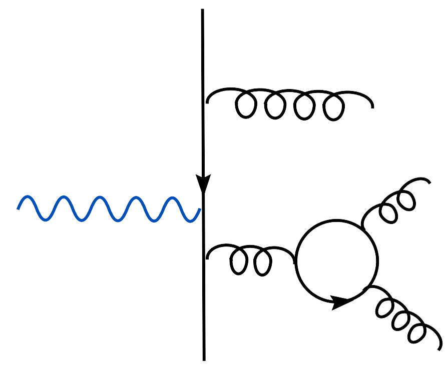
Leading color | Fermion loops | Subleading color

2-quark 3-gluon

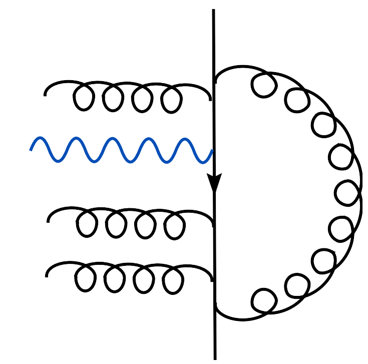
only



$$\mathrm{LC} \equiv (N_c^2 - 1)N_c^3$$

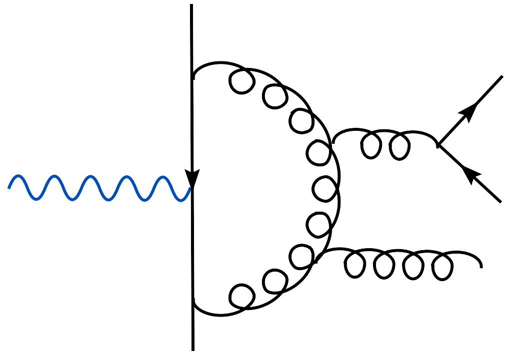


$$\mathrm{LC} \cdot \frac{n_f}{N_c}$$

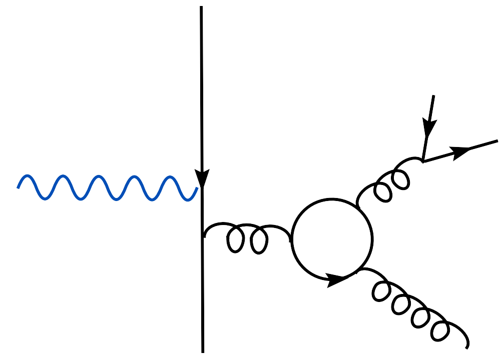


$\ldots$

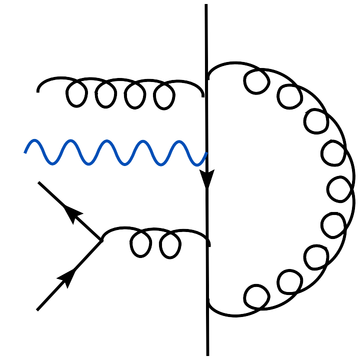
4-quark 1-gluon



$$\mathrm{LC} \cdot \frac{n_f}{N_c}$$



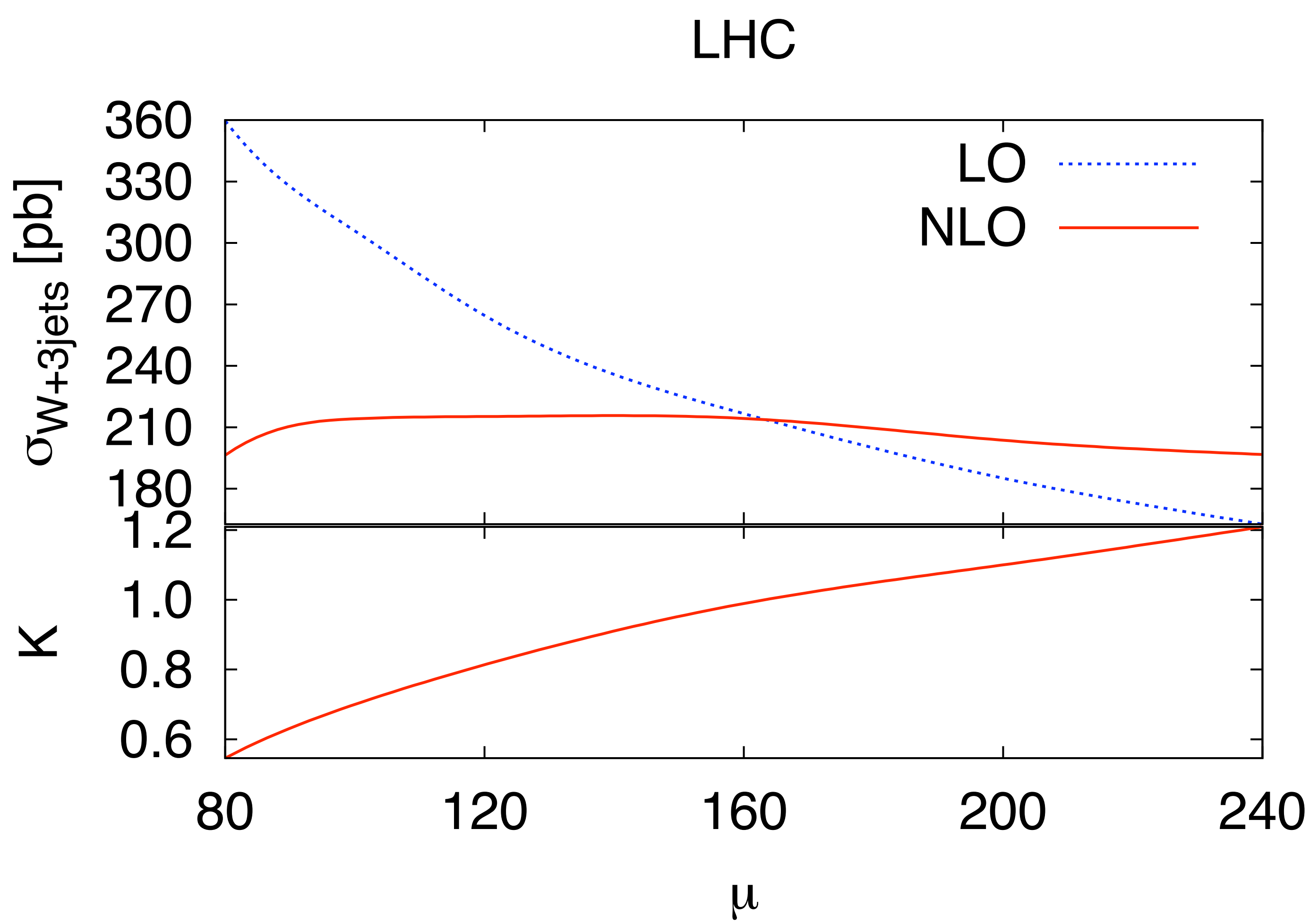
$\ldots$



$\ldots$

At tree level: leading color works up to O(10%), 4-quark processes O(30%)

Scale variation: W^+ + 3 jets



[Cuts and input defined in Ellis, Melnikov, GZ '09]

- very strong dependence at LO, remarkable independence at NLO
- LO = NLO at scales ~ 160 GeV
- W + 3 jets similar to W + 2 jets, however the price to pay for an infelicitous choice of scales is higher now
- similar results at the Tevatron

Second W + 3 jet calculation

More recently, similar calculation for W + 3 jets done in Blackhat+Sherpa

C. F. Berger, Z. Bern, L. J. Dixon, F. Febres Cordero, D. Forde, T. Gleisberg, H. Ita, D.A. Kosower, D. Maitre [0902.2760]

In the above paper: still leading color approximation in virtual (not real), all subprocesses included (but no fermion loops)

Next step: inclusion of all subprocesses and comparison with Berger et al.

CDF cuts

$$p_{\perp,j} > 20\text{GeV} \qquad p_{\perp,e} > 20\text{GeV} \qquad E_{\perp,\text{miss}} > 30\text{GeV}$$

$$|\eta_e| < 1.1 \qquad M_{\perp,W} > 20\text{GeV}$$

$$\mu_0 = \sqrt{p_{\perp,W}^2 + M_W^2} \qquad \mu = \mu_R = \mu_F = [\mu_0/2, 2\mu_0]$$

- CDF uses JETCLU with R = 0.4, but this is not infrared safe, use SIScone with the same R
  Difference O(1-2%) in inclusive cross-section [more in distributions]
  SIScone ⇒ Salam & Soyez '06

- CDF applies lepton-isolation cuts. This is a O(10%) effect. No lepton isolation in order to compare with Berger et al.
  Lepton-isolation and detector acceptance cuts are believe to cancel out

- PDFs: cteq6l1 and cteq6m, all other input as in 0902.2760
  NB: diagonal CKM O(1-2%) effect relative to Cabibbo rotated one

Cross-section at the Tevatron

$$\sigma_{W+3j}(p_{\perp,j} > 25\,\mathrm{GeV}) = (0.84 \pm 0.24)\,\mathrm{pb}$$

CDF

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CDF

LO^{LC}					
$0.89^{+0.55}_{-0.31}$					

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$\mathrm{LO}^{\mathrm{LC}}$	$\mathrm{LO}^{\mathrm{FC}}$				
$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$				

Cross-section at the Tevatron

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CDF

LO^{LC}	LO^{FC}	$r = \frac{\mathrm{LO}^{\mathrm{FC}}}{\mathrm{LO}^{\mathrm{LC}}}$			
$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$	0.91			

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$\mathrm{LO}^{\mathrm{LC}}$	$\mathrm{LO}^{\mathrm{FC}}$	$r = \frac{\mathrm{LO}^{\mathrm{FC}}}{\mathrm{LO}^{\mathrm{LC}}}$	$\mathrm{NLO}^{\mathrm{LC}}$		
$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$	0.91	$0.89^{+0.03}_{-0.11}$		

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'Our best shot'

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$0.89^{+0.55}_{-0.31}$	$0.81^{+0.50}_{-0.28}$	0.91	$0.89^{+0.03}_{-0.11}$	$0.81^{+0.03}_{-0.10}$	$0.908^{+0.044}_{-0.142}\ (v3)$

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'Our best shot'

NB: errors are standard scale variation errors, statistical errors smaller

12% discrepancy with Berger et al.
Disagreement ?

Cross-section at the Tevatron

Differences with respect to Berger et al.

- Fermion loops
  O(1%) for 2q subprocesses, up to O(-15%) for 4q ones [as in W+2j]
  ☛ up to 5% in the right direction

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- Leading color approximation
  ☛ another 3% in the right direction

number of jets	CDF	LC NLO	NLO
1	53.5 ± 5.6	$58.3^{+4.6}_{-4.6}$	$57.8^{+4.4}_{-4.0}$
2	6.8 ± 1.1	$7.81^{+0.54}_{-0.91}$	$7.62^{+0.62}_{-0.86}$
3	0.84 ± 0.24	$0.908^{+0.044}_{-0.142}$	—

(LC approximation good to about 3%)

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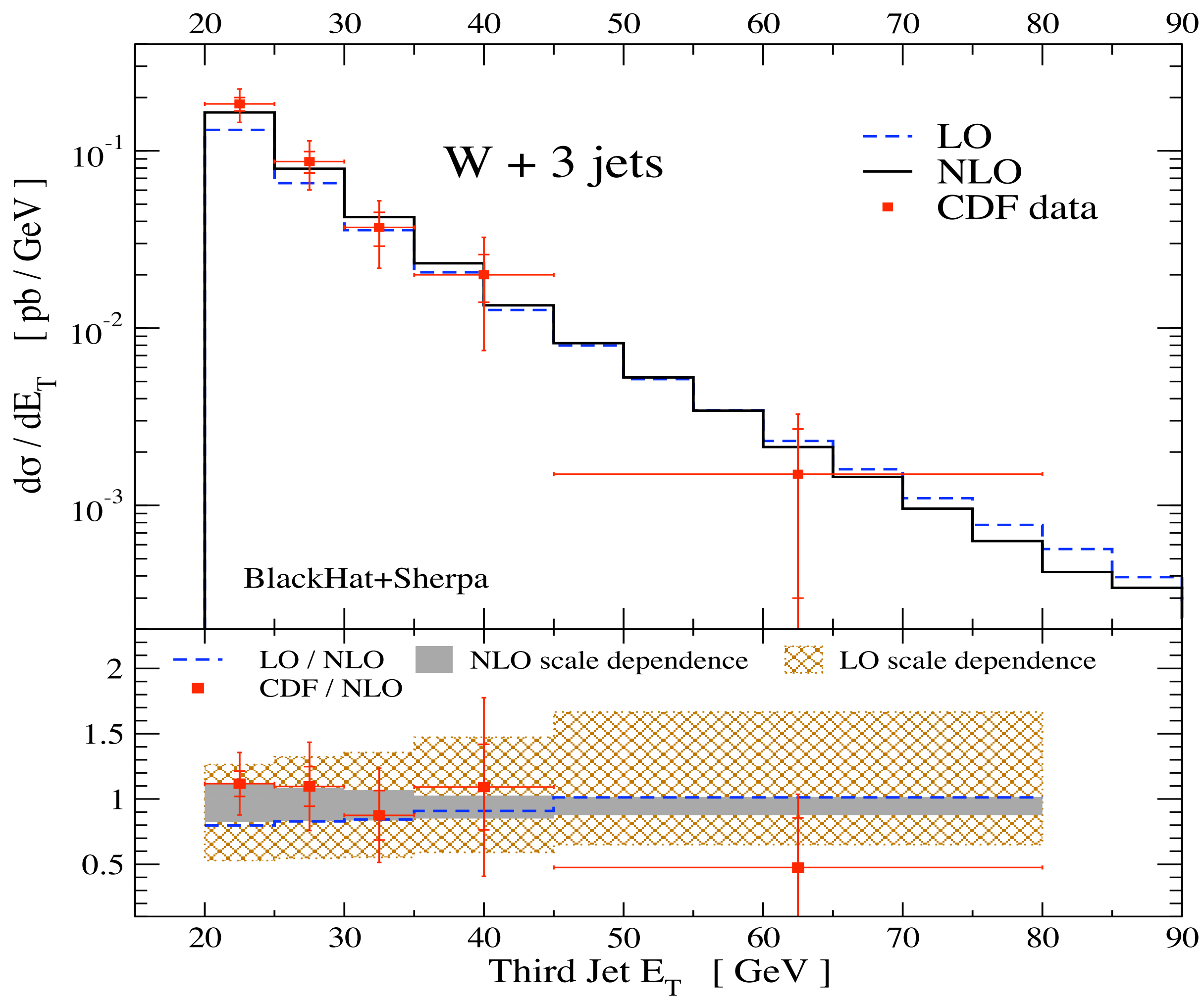
(LC approximation good to about 3%)

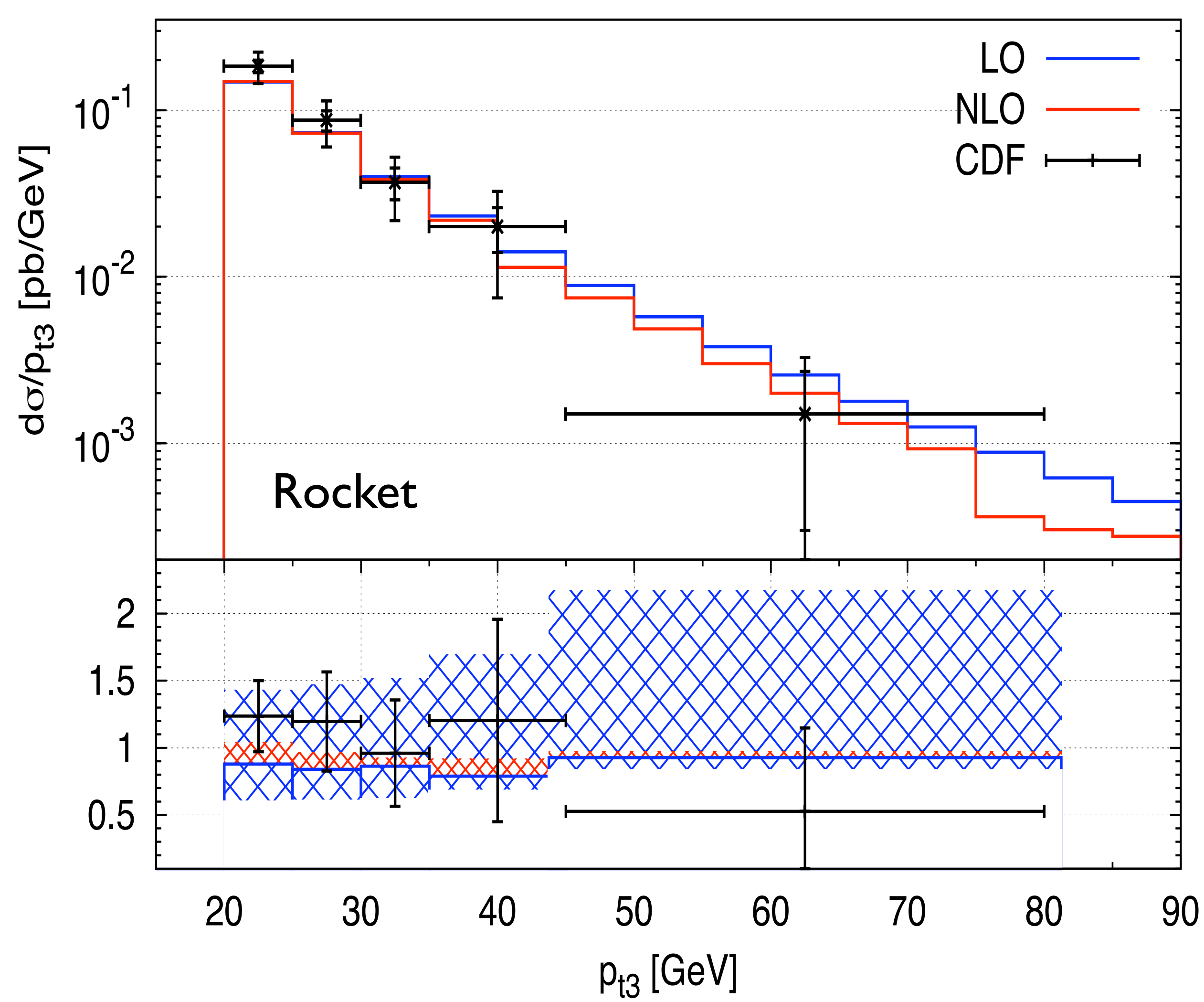
Remaining discrepancy acceptable, more detailed comparison not possible, however tension stronger with full NLO calculation presented yesterday: Leading color *only 3% effect* $\Rightarrow$ $\sigma^{FC}_{\rm NLO} = 0.882^{+0.057}_{-0.138}$

☛see Forde's talk

Sample distribution: $p_{t,j3}$



Berger et al ’09



Ellis, Melnikov, GZ preliminary

LHC cuts

$$E_{\text{CM}} = 10\,\text{TeV} \qquad E_{\perp,\text{jet}} = 30\,\text{GeV} \qquad E_{\perp,e} = 20\,\text{GeV}$$

$$E_{\perp,\text{miss}} = 15\,\text{GeV} \quad M_{\perp,W} = 30\,\text{GeV} \quad |\eta_e| < 2.4 \quad |\eta_{\text{jet}}| < 3$$

$$\mu_0 = \sqrt{p_{\perp,W}^2 + M_W^2} \qquad \mu = \mu_R = \mu_F = [\mu_0/2, 2\mu_0]$$

- Jet definition: SIScone with R = 0.5
- PDFs: cteq6l1 and cteq6m
- Other input parameters as before

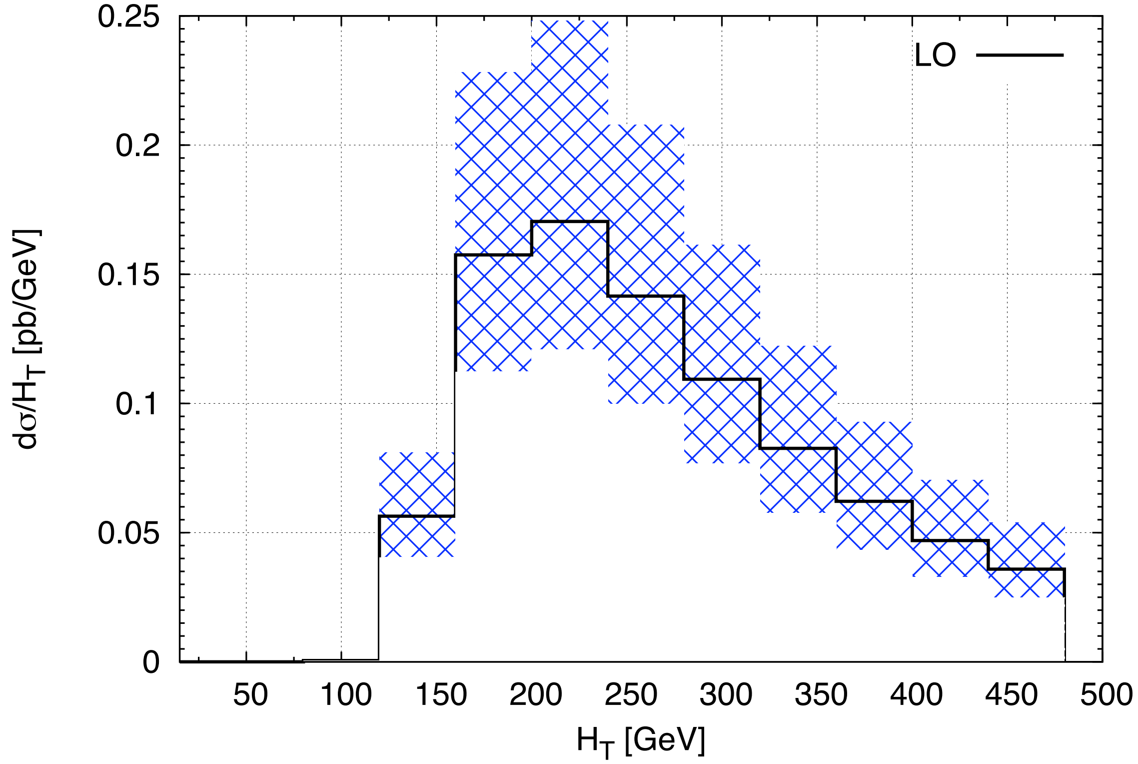
LHC: W^+ +3 jet cross-section

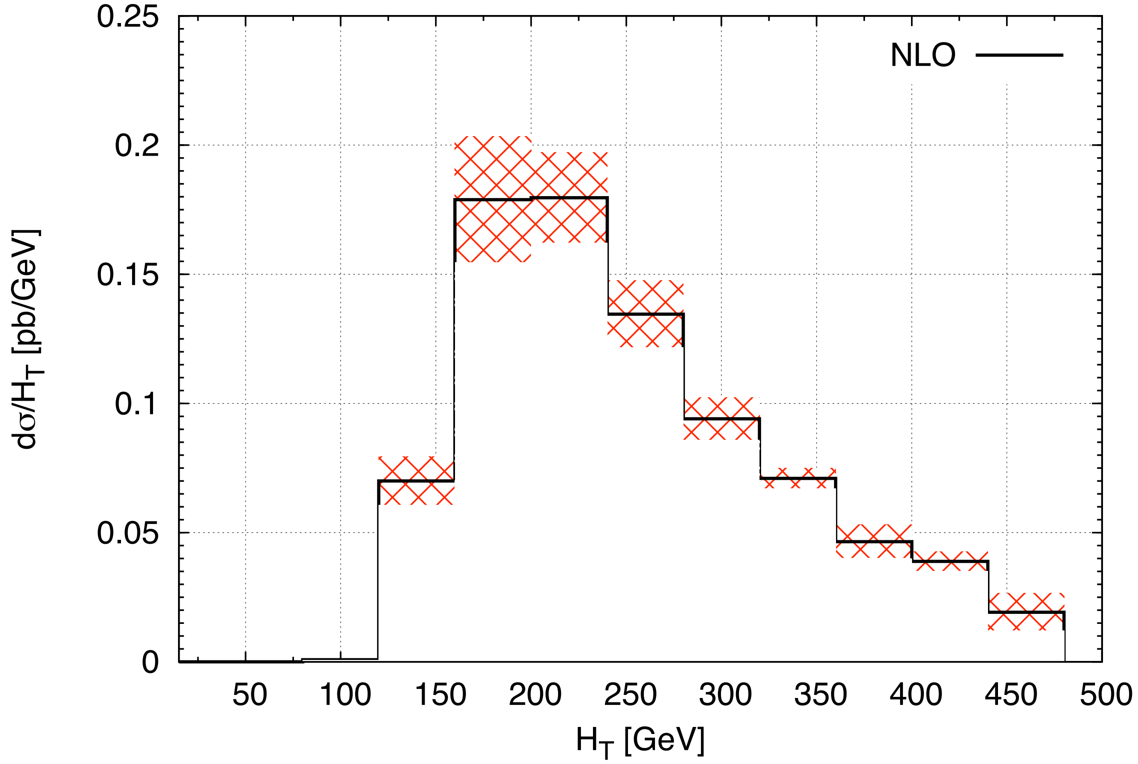
	$\sigma(\mu/2)$ [pb]	$\sigma(\mu)$ [pb]	$\sigma(2\,\mu)$ [pb]
LO	59.1	40.1	28.1
NLO [excl]	23.1	28.7	28.9
K [excl]	0.39	0.72	1.03
NLO [incl]	36.1	36.5	33.9
K [incl]	0.61	0.91	1.21

Ellis, Melnikov, GZ preliminary

- scale dependence considerably reduced at NLO
- NLO tends to reduce cross-section
- because of very large scale dependence of LO, quoting a K-factor not very meaningful

LHC: W^+ +3 jet sample distribution





Ellis, Melnikov, GZ preliminary

$$H_T = \sum_{j=1,2,3} p_{\perp,j} + p_{\perp,e}$$

Final remarks

<u>Generalized D-dimensional unitarity</u>

- ✗ general Berends-Giele recursion for tree level amplitudes: numerically efficient (large N), general (D, spins, masses)

- ✗ simple method, suitable for automation

- ✗ universal method (general masses, spins) and unified approach, no 'special' cases, no exceptions

- ✗ speed: numerical performance as expected (polynomial)

- ✗ transparent: full control on all parts

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- ✗ speed: numerical performance as expected (polynomial)
- ✗ transparent: full control on all parts

Maturity reached for cross-sections calculations?
Demonstrated by first explicit calculation of W + 3 jets
(but still room for further improvements)