NLO QCD corrections to $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ at the LHC

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based on

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LoopFest VIII

Madison, May 9, 2009

Outline of the talk

(1) **Introduction** - NLO corrections to multi-leg processes, $t\bar{t}b\bar{b}$ production

(2) **Virtual corrections** - Feynman diagrams, tensor reduction, rational terms

(3) **Real corrections** - Dipole subtraction

(4) **Numerical results** - LHC cross section, CPU performance

(1) Introduction

Importance of **multi-leg processes** at the LHC

- huge W, Z and top-quark production rates + multiple jet emission
- multi-particle signatures with leptons and missing E
- serious backgrounds to Higgs and new-physics signals
  (often not fully accessible to measurements)

Importance of **NLO QCD corrections** at the LHC

- reduce scale uncertainties (high powers of α_S!); improve description of jets
- systematics better than Tevatron; very high statistics

Technical problems for $2 \to 3, 4, \ldots$ processes

- numerical instability of virtual corrections (Gram determinants)
- number and complexity of diagrams grow very fast

Challenges for NLO programs

- reliable predictions: **numerical stability**
- **sufficient speed**: distributions require >1 event/sec!

Feynman diagrams have provided hadronic NLO cross sections for several nontrivial $2 \to 3$ processes

- $\mathrm{pp} \to \mathrm{t\bar{t}H},\ \mathrm{b\bar{b}H}$ Beenakker/Dittmaier/Krämer/Plümper/Spira/Zerwas; Dawson/Reina/Wackeroth/Orr/Jackson; Peng/Wen-Gan/Hong-Shen/Ren-You/Yi
- $\mathrm{pp} \to \mathrm{HHH}$ Plehn/Rauch; Binoth/Karg/Kauer/Rückl
- $\mathrm{pp} \to \mathrm{Hjj}$ Del Duca/ Kilgore/Oleari/Schmidt/Zeppenfeld; Campbell/Ellis/Zanderighi Ciccolini/Denner/Dittmaier
- $\mathrm{pp} \to \mathrm{jjj}$ Bern/Dixon/Kosower; Kunszt/Signer/Trocsanyi; Giele/Kilgore/Nagy
- $\mathrm{pp} \to V\mathrm{jj}$ Bern/Dixon/Kosower; Ellis/Veseli; Campbell/Ellis;
- $\mathrm{pp} \to VV\mathrm{j}$ Dittmaier/Kallweit/Uwer; Campbell/Ellis/Zanderighi;
- $\mathrm{pp} \to VVV$ Lazopoulos/Melnikov/Petriello; Hankele/Zeppenfeld; Binoth/Ossola/Papadopoulos/Pittau
- $\mathrm{pp} \to V + \mathrm{b\bar{b}}$ Ferbes Cordero/Reina/Wackeroth
- $\mathrm{pp} \to \mathrm{t\bar{t}j}$ Dittmaier/Uwer/Weinzierl
- $\mathrm{pp} \to \mathrm{t\bar{t}Z}$ Lazopoulus/McElmurry/Melnikov/Petriello

...and a few pioneering $2 \to 4$ results for e^+e^- and $\gamma\gamma$ colliders

- $e^+e^- \longrightarrow 4f$ (EW) Denner/Dittmaier/Roth/Wieders '05
- $e^+e^- \longrightarrow HH\nu\bar{\nu}$ (EW) Boudjema/Fujimoto/Ishikawa/Kaneko/Kurihara/Shimizu/Kato/Yasui '05
- $\gamma\gamma \longrightarrow t\bar{t}b\bar{b}$ (QCD) Lei/Wen-Gan/Liang/Ren-You/Yi '07

Questions

- Can one repeat this success at hadron colliders?
- Should one switch to alternative methods like unitarity cuts?

Les Houches '05/'07 prioritized wishlist

	Reaction	background for	existing NLO cross sections
1.	VVj	$t\bar{t}$H, new physics	WWj: Dittmaier/Kallweit/Uwer '07; WWj: Campbell/Ellis/Zanderighi '07 WWj/ZZj: Binoth/Guillet/Karg/Kauer/Sanguinetti (in progress)
2.	$t\bar{t}b\bar{b}$	$t\bar{t}$H	**complete NLO:** Bredenstein/Denner/Dittmaier/P. '08-'09
3.	$t\bar{t}$jj	$t\bar{t}$H	$\emptyset$
4.	$VVb\bar{b}$	VBF→ H → VV, $t\bar{t}$, NP	$\emptyset$
5.	VVjj	VBF→ H → VV	**VBF:** Jäger/Oleari/Zeppenfeld '06 + Bozzi '07
6.	Vjjj	new physics	**leading colour approximation:** Ellis/Giele/Kunszt/Melnikov/Zanderighi '08; Ellis/Melnikov/Zanderighi '09; Berger/Bern/Dixon/Ferbes Cordero/Forde/Gleisberg/Ita/ Kosower/Maitre '09
7.	VVV	new physics	ZZZ: Lazopoulos/Melnikov/Petriello '07; WWZ: Hankele/Zeppenfeld '07; VVV: Binoth/Ossola/Papadopoulos/Pittau '07
8.	$b\bar{b}b\bar{b}$	Higgs and new physics	**qq channel (virtual):** Binoth/Guffanti/Guillet/Heinrich/Karg/ Kauer/Mertsch/Reiter/Reuter/Sanguinetti

virtual amplitudes for several 2→4 processes van Hameren/Papadopoulos/Pittau '09

Technical motivation for $t\bar{t}b\bar{b}$

Validate NLO algorithms by solving a non-trivial problem

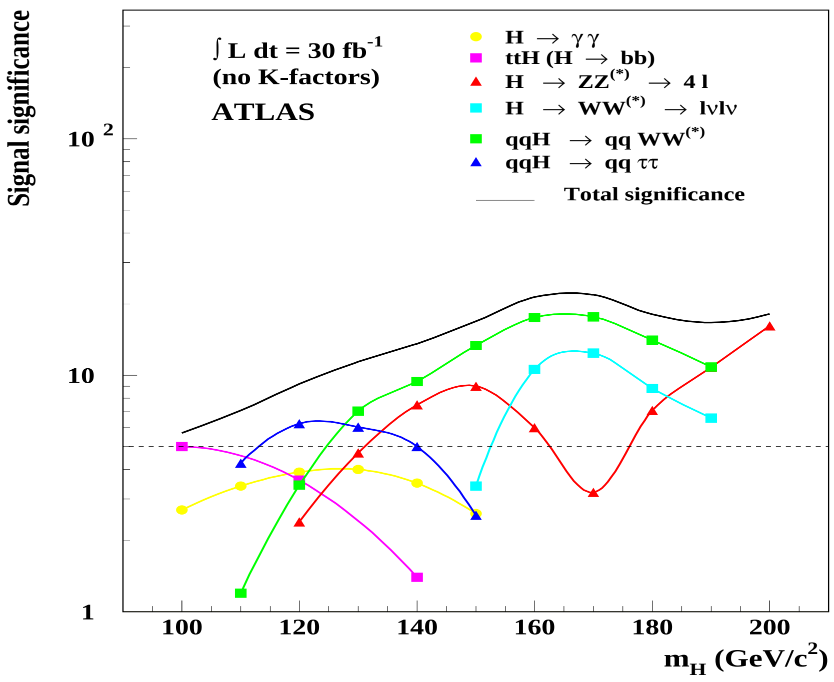
- $2 \to 4$ process with $\mathcal{O}(10^3)$ diagrams involving hexagons up to rank 4
- 6 coloured legs, massless and massive quarks, gluons

Phenomenological motivation for $t\bar{t}b\bar{b}$

Associated $t\bar{t}H(H \to b\bar{b})$ production

- can be observed in $H \to b\bar{b}$ channel
- exploits large $BR(H \to b\bar{b})$ for light H
- measurement of top Yukawa coupling

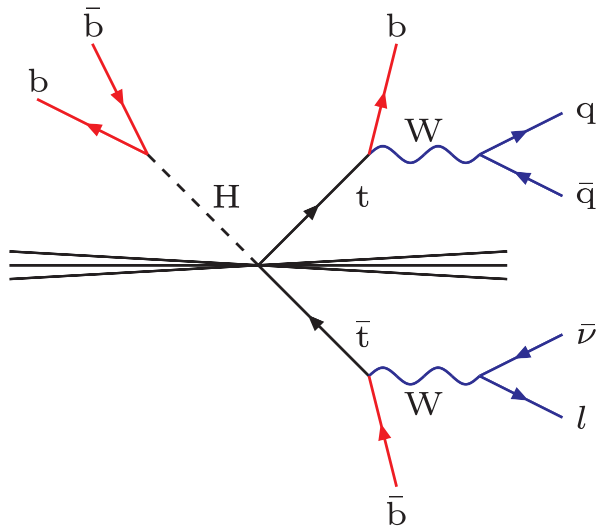
Early ATLAS studies of Higgs discovery potential (ATLAS '03)



Proposed analysis (ATLAS TDR)

- select final state $\mathrm{b\bar{b}b\bar{b}jj}l\nu$ (4 b-quarks!)
- reconstruct $\mathrm{t\bar{t}b\bar{b}}$
- select region $|m_{\mathrm{b\bar{b}}} - M_{\mathrm{H}}| < 30\,\mathrm{GeV}$

Richter-Was and Sapinski, ATL-PHYS-98-132



Backgrounds

- irreducible: $\mathrm{t\bar{t}b\bar{b}}$ (QCD+EW)
- reducible: $\mathrm{t\bar{t}jj}$

events and stat. significance (30 fb^{-1})

M_{H}(GeV)	110	120	130
$\mathrm{t\bar{t}H}$	60.9	41.9	25.5
$\mathrm{t\bar{t}b\bar{b}}$(QCD)	167.3	145.8	128.7
$\mathrm{t\bar{t}jj}$	66.2	54.6	41.6
$\mathrm{t\bar{t}b\bar{b}}$(EW)	21.8	18.4	15.2
$S/\sqrt{B}$	3.8	2.8	1.9
S/B	0.24	0.19	0.14

Cammin and Schumacher, ATL-PHYS-2003-024

Systematic uncertainty

Large $t\bar{t}b\bar{b}$ and $t\bar{t}jj$ background

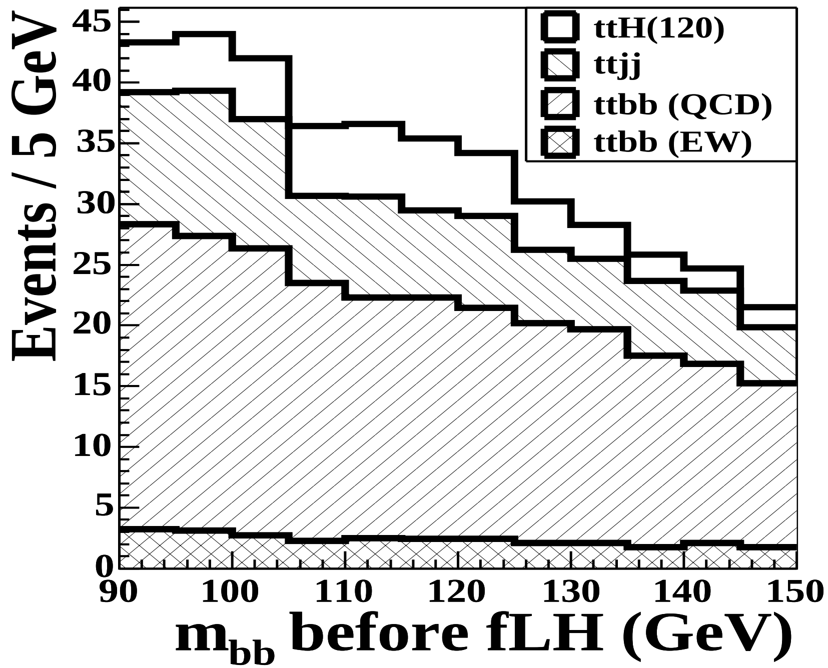
- $S/B = \mathcal{O}(1/10)$
- $\mathcal{O}(10\%)$ B uncertainty kills measurement!

Systematic uncertainty of background

- only statistics in TDR: $1/\sqrt{B} < 10\%$
- but error dominated by systematics!

Strategy for precise determination of B

- normalization from data outside signal region
- interpolate from signal-free to signal-rich region using precise shape predictions

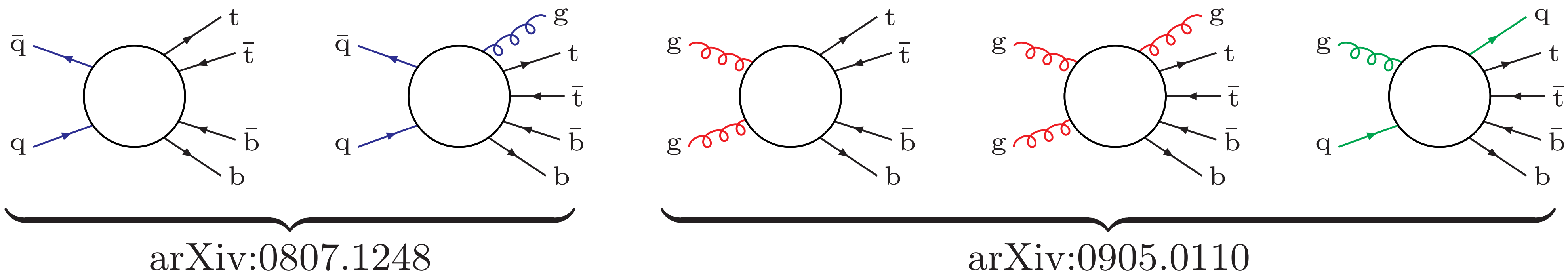


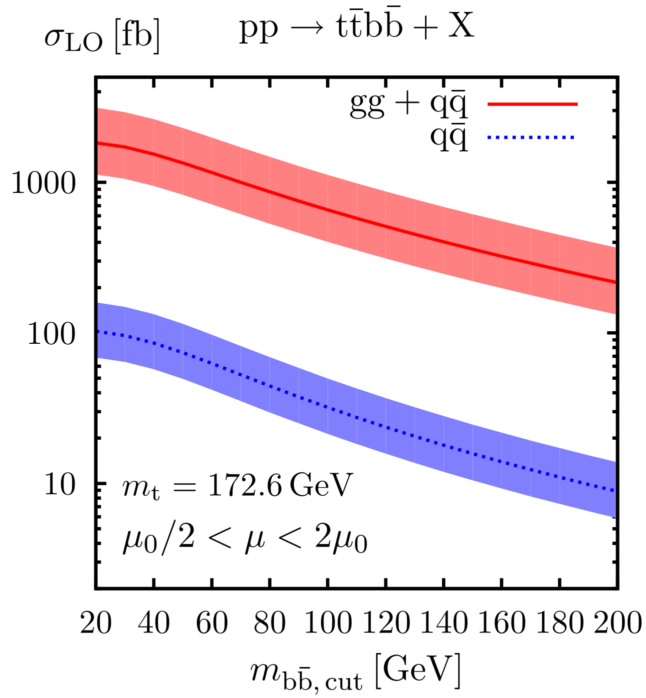
Cammin and Schumacher, ATL-PHYS-2003-024

$t\bar{t}H$ **measurement impossible without** $t\bar{t}b\bar{b}$ **and** $t\bar{t}jj$ **at NLO!**

$2 \to 4$ and $2 \to 5$ Feynman diagrams

Quark-antiquark and gluon induced processes

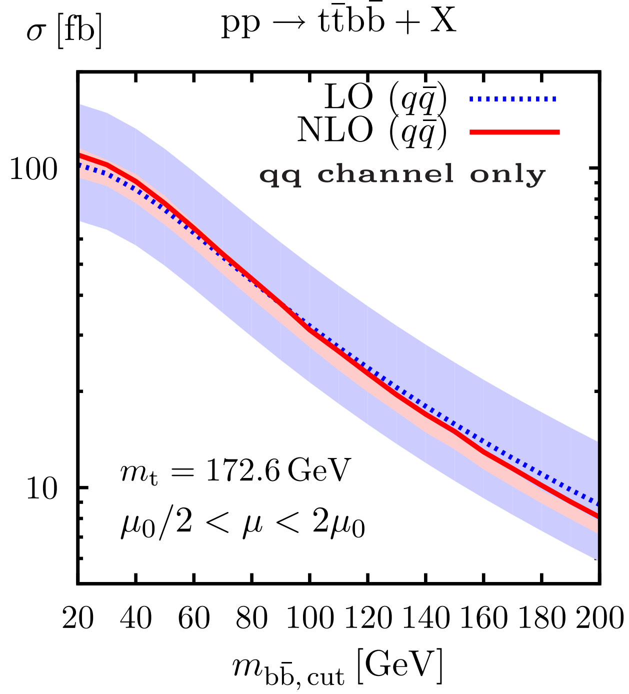




 # diagrams and impact on σ_{tot}

	$q\bar{q}$	gg	qg
LO	7	36	
virtual	188	1003	
real	64	341	64
$(\sigma/\sigma_{\mathrm{tot}})_{\mathrm{LO}}$	5%	95%	
$(\sigma/\sigma_{\mathrm{tot}})_{\mathrm{NLO}}$	3%	92%	5%

Step 1: quark anti-quark channel



Motivation

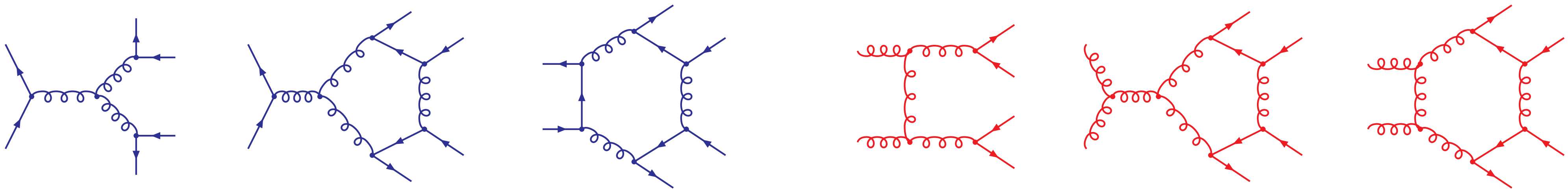
- 6-fermion process related to $e^+e^- \longrightarrow 4f$
- proof of principle of feasibility of calculation

Main results [arXiv:0807.1248]

- small NLO correction (K=1.03)
- strong reduction of scale dependence
- very high CPU performance of 13 ms/PS-point

(2) Tree and one-loop contributions to $q\bar{q}/gg \to t\bar{t}b\bar{b}$

Tree and one-loop sample diagrams in the $q\bar{q}$ and gg channels



7 trees | 24 pentagons | 8 hexagons | 36 trees | 114 pentagons | 40 hexagons

Two independent calculations

- diagrams generated with `FeynArts 1.0 / 3.2` [Külbeck/Böhm/Denner '90; Hahn '01]
- one calculation uses `FormCalc 5.2` [Hahn '06] for preliminary algebraic manipualations (Dirac algebra, covariant decomposition)
- bulk of reduction with two in-house `MATHEMATICA` programs
- numerics with two independent `Fortran77` codes
  (two libraries for tensor integrals)

Top quarks massive and bottom quarks massless

Structure of the calculation

Standard matrix elements and colour structures for **individual diagrams**

$$\mathcal{D} = \underbrace{\left[\sum_k a_k \mathcal{C}_k(\{c\})\right]}_{\text{factorized colour structure}} \sum_i \mathcal{F}_i \underbrace{\mathcal{S}_i(\{p\},\{\lambda\})}_{\text{standard matrix elements}}$$

Form factors $\mathcal{F}_i$ in terms of tensor integrals

$$\mathcal{F}_i = \sum_{j_1\dots j_R} \mathcal{K}_{i,j_1\dots j_R} \underbrace{T_{j_1\dots j_R}(\{p\})}_{\text{tensor loop coefficients}}$$

computed numerically diagram by diagram (no analytic reduction to scalar integrals)

Main goals

- reduction to small set of standard matrix elements $\mathcal{S}_i$
- fast and stable numerical evaluation of tensor integrals $T_{j_1\dots j_R}$

Colour structure

- colour basis in the $q\bar{q}$ and gg channels

$$\underbrace{\delta_{i_1 i_2}\delta_{i_3 i_4}\delta_{i_5 i_6}, \quad \delta_{i_1 i_2} T^a_{i_3 i_4} T^a_{i_5 i_6}, \quad \ldots}_{6 \text{ elements}} \qquad \underbrace{\delta^{a_1 a_2}\delta_{i_3 i_4}\delta_{i_5 i_6}, \quad T^{a_1}_{i_3 i_4} T^{a_2}_{i_5 i_6}, \quad \ldots}_{14 \text{ elements}}$$

- colour factorization for individual diagrams ⇒ CPU time doesn't scale with the number of colour structures

Rational terms originating from **UV** $1/(D-4)$ poles of tensor loop integrals

$$\mathcal{K}_{j_1\ldots j_R}(D) \quad \underbrace{T_{j_1\ldots j_R}}_{\frac{R_{j_1\ldots j_R}}{(D-4)} + \text{UV}-\text{finite part}} \quad = \mathcal{K}_{j_1\ldots j_R}(4)\, T_{j_1\ldots j_R} + \mathcal{K}'_{j_1\ldots j_R}(4)\, R_{j_1\ldots j_R} + \mathcal{O}(D-4)$$

- residues $R_{j_1\ldots j_R}$ of UV poles of tensor integrals explicitly available
- after $(D-4)$-expansion continue calculation in $D = 4$
- **rational terms from IR poles** appear in intermediate expressions but **cancel at amplitude level and can thus be ignored** (proven for any scattering amplitude in App. A of JHEP **0808**, 108 (2008) [arXiv:0807.1248])

Cancellation of rational terms originating from IR poles

Explicit calculations in terms of tensor and/or scalar integrals contain "rational" terms arising from double and single **IR poles** of tensor loop integrals

$$\mathcal{K}_{j_1\dots j_R}(D) \quad \underbrace{T_{j_1\dots j_R}}_{\frac{R_{2;j_1\dots j_R}}{(D-4)^2}+\frac{R_{1;j_1\dots j_R}}{(D-4)}+\mathrm{IR-finite\,part}} \quad \Rightarrow \mathcal{K}'_{j_1\dots j_R}(4)\, R_{1,j_1\dots j_R} + \frac{1}{2}\mathcal{K}''_{j_1\dots j_R}(4)\, R_{2,j_1\dots j_R} + \dots$$

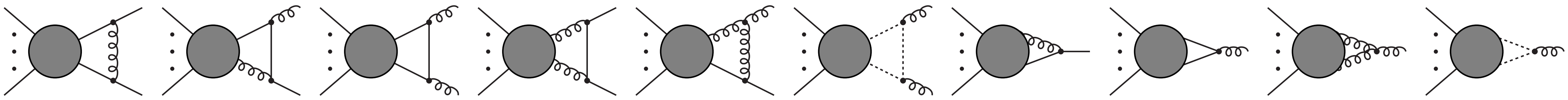
- can require lot of algebraic work (second-order expansion)
- cancel completely in truncated (!) one-loop amplitudes (observed in various calculations)
- explicit proof based on the following two observations

(1) **Tensor-reduction identities are free from rational terms of IR type**

- Only $g^{\mu\nu}$-type components of tensor integrals ($T_{00\dots}$) receive D-dependent factors in reduction identities
- while soft and collinear regions ($q^\mu \to xp^\mu$) yield only $p_i^\mu p_j^\mu \dots$ tensor structures

(B) IR-divergent part of (individual) loop diagrams can be expressed in terms of integrals with D-independent prefactors

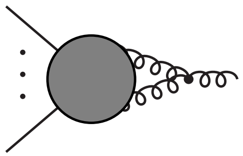
(B.1) consider all diagrams involving IR-divergent integrals



(B.2) look for trace-like contractions, $g_{\nu\lambda}\Gamma^{\nu\lambda}$, which can produce D-dep. coefficients:

$$g_{\nu\lambda}\, g^{\nu\lambda} = D, \qquad g_{\nu\lambda}\, \gamma^{\nu} \not{p} \gamma^{\lambda} = (2-D)\not{p}, \ldots \tag{1}$$

(B.3) Manipulating the IR-divergent part of the integrand in D dim (using appropriate loop parametrizations and taking into account double-counting subtleties)



$$= \int \mathrm{d}^D q \; \epsilon^{\mu *} \, \frac{(q+2p)_\lambda \, g_{\mu\nu} - (2q+p)_\mu \, g_{\nu\lambda} + (q-p)_\nu \, g_{\lambda\mu}}{q^2 (q+p)^2} \, \Gamma^{\nu\lambda}(q)$$

we find that the IR-singular part is free from trace-like contractions (due to the collinear limit $\epsilon^{\mu *}(2q+p)_\mu \to 0$ in the above example)

Practical consequence

- "rational" terms from IR poles can be systematically neglected
- subtlety related to scaleless 2-point functions

$$B_0(0,0,0) \;=\; \left(\frac{2}{D-4}\right)_{\mathrm{IR}} - \left(\frac{2}{D-4}\right)_{\mathrm{UV}} = 0$$

$$(D-4)B_0(0,0,0) \;\Rightarrow\; (D-4)\left(\frac{-2}{D-4}\right)_{\mathrm{UV}} = -2$$

Generality and applicability

- cancellation applies to any **truncated** QCD amplitude and is independent from reduction algorithm (OPP, unitarity, PV,...)
- proven for Feynman t'Hooft or similar gauges (power-1 propagators)
- δZ **factors contain rational terms of IR origin**

Reduction of Standard Matrix Elements (SMEs): strategy/simple examples

(A) q$\bar{\mathrm{q}}$ **channel**: after standard D-dim manipulations, $\mathcal{O}(10^3)$ Dirac struct. of type

$$\underbrace{\left[\bar{v}(p_1)\dots\gamma^\mu\gamma^\nu \not{p}_3\dots u(p_2)\right]}_{\mathrm{q\bar{q}\ chain}} \underbrace{\left[\bar{v}(p_3)\dots\gamma^\mu\gamma^\nu\gamma^\rho \not{p}_6\dots u(p_4)\right]}_{\mathrm{t\bar{t}\ chain}} \underbrace{\left[\bar{v}(p_5)\dots\gamma^\rho \not{p}_2\not{p}_3\dots u(p_6)\right]}_{\mathrm{b\bar{b}\ chain}}$$

Needs additional identities in order to eliminate $\not{p}$-terms and γ-contractions

- without introducing new denominators that might spoil numerical stability

After cancellation of $1/(D-4)$ poles we work in 4 dimensions and introduce

$$\omega_\pm = \frac{1}{2}(1 \pm \gamma^5) \qquad\qquad \bar{v}(p_i)\Gamma u(p_j) \Rightarrow \sum_{\lambda=\pm} \bar{v}(p_i)\Gamma\omega_\lambda u(p_j)$$

This permits to use the **Chisholm identity**

$$\mathrm{i}\varepsilon^{\alpha\beta\gamma\delta}\gamma_\delta\gamma^5 = \gamma^\alpha\gamma^\beta\gamma^\gamma - g^{\alpha\beta}\gamma^\gamma + g^{\alpha\gamma}\gamma^\beta + g^{\beta\gamma}\gamma^\alpha$$

Contracting with $\otimes\gamma_\gamma\gamma^5$ and symmetrizing $\Rightarrow$ eliminates the ε-tensor and yields

$$\mathrm{i}\varepsilon^{\alpha\beta\gamma\delta}\left[\gamma_\delta\gamma^5 \otimes \gamma_\gamma\gamma^5 + (\delta \leftrightarrow \gamma)\right] = 0 \quad \Rightarrow \quad \gamma^\mu\gamma^\alpha\gamma^\beta\omega_\pm \otimes \gamma_\mu\omega_\mp = \gamma^\mu\omega_\pm \otimes \gamma^\alpha\gamma^\beta\gamma_\mu\omega_\mp$$

and similar identities that permit to exchange $\gamma^\alpha\gamma^\beta$ between γ-contracted chains

Other identies involving doubly γ-contraced chains

$$\gamma^\mu \gamma^\alpha \gamma^\nu \omega_\pm \otimes \gamma_\mu \gamma^\beta \gamma_\nu \omega_\mp = 4\gamma^\beta \omega_\pm \otimes \gamma^\alpha \omega_\mp \qquad \text{etc.}$$

can be used (for instance)

- to reduce number of γ-contractions
- or (in the inverse direction) to shift $\not{p}$-terms and then exploit Dirac equation

$$4\Big[\bar{v}(p_1)\ldots \not{p}_4 \omega_\pm\, u(p_2)\Big]\Big[\bar{v}(p_3)\ldots \not{p}_2 \omega_\mp u(p_4)\Big] = \Big[\bar{v}(p_1)\ldots \gamma^\mu \not{p}_2 \gamma^\nu \omega_\pm\, u(p_2)\Big]\Big[\bar{v}(p_3)\ldots \gamma_\mu \not{p}_4 \gamma_\nu \omega_\mp u(p_4)\Big]$$
$$= 4(p_2 p_4)\Big[\bar{v}(p_1)\ldots \gamma^\mu \omega_\pm\, u(p_2)\Big]\Big[\bar{v}(p_3)\ldots \gamma_\mu \omega_\mp u(p_4)\Big] + \text{mass-terms}$$

In practice

- Chisolm identity + Dirac equation + momentum conservation (+ a lot of patience)
- yields several useful relations
- construct a sophisticated algorithm to reduce # of γ-contractions and $\not{p}$-terms

At the end of the day **25 types** of standard matrix elements

- 10 of "massless" type: one Dirac matrix per chain

$$\left[\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\right]\;\left[\bar{v}(p_3)\gamma^\mu\omega_\beta u(p_4)\right]\;\left[\bar{v}(p_5)\gamma^\mu\omega_\rho u(p_6)\right]$$
$$\left[\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\right]\;\left[\bar{v}(p_3)\not{p}_j\omega_\beta u(p_4)\right]\;\left[\bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)\right]$$

- 15 of "massive" type: 2/0 Dirac matrices inside the $t\bar{t}$ chain

$$\left[\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\right]\;\left[\bar{v}(p_3)\not{p}_j\gamma^\mu\omega_\beta u(p_4)\right]\;\left[\bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)\right]$$
$$\left[\bar{v}(p_1)\gamma^\mu\omega_\alpha u(p_2)\right]\;\left[\bar{v}(p_3)\not{p}_j\not{p}_j\omega_\beta u(p_4)\right]\;\left[\bar{v}(p_5)\gamma^\mu\omega_\rho u(p_6)\right]$$
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$$\left[\bar{v}(p_1)\not{p}_i\omega_\alpha u(p_2)\right]\;\left[\bar{v}(p_3)\omega_\beta u(p_4)\right]\;\left[\bar{v}(p_5)\not{p}_k\omega_\rho u(p_6)\right]$$

25×8 chiral structures $(\omega_\alpha \otimes \omega_\beta \otimes \omega_\rho) \Rightarrow$ **200 SMEs** for the $q\bar{q}$ channel

(B) Reduction strategy for gg channel

Standard D-dim manipulations for quark chains and gluon-helicity vectors ($\epsilon_1 p_1 = \epsilon_2 p_2 = \epsilon_1 p_2 = \epsilon_2 p_1 = 0$) yield structures of type

$$\underbrace{\left\{\epsilon_1^\mu \epsilon_2^\nu,\ (\epsilon_1\epsilon_2) p_2^\mu p_4^\nu,\ (\epsilon_1 p_4)(\epsilon_2 p_3) g^{\mu\nu}, \ldots\right\}}_{\text{gluon polarization vectors}} \underbrace{\left[\bar{v}(p_3)\ldots\gamma_\mu\gamma_\rho \not{p}_6 \ldots u(p_4)\right]}_{\text{t}\bar{\text{t}}\text{ chain}} \underbrace{\left[\bar{v}(p_5)\ldots\gamma^\rho\gamma_\nu \not{p}_2 \not{p}_3 \ldots u(p_6)\right]}_{\text{b}\bar{\text{b}}\text{ chain}}$$

Strategy B.1: employ q$\bar{\text{q}}$-channel method (γ^5 and Chisolm-based identities) to reduce two fermion chains $\Rightarrow$ **502 SMEs**

Strategy B.2: less-sophisticated 4-dim reduction based only on a relation of type

$$\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} = g^{\mu_1\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} + \ldots + g^{\mu_1\mu_2} g^{\mu_3\mu_4}\gamma^{\mu_5} + \ldots$$

- can be derived from Chisolm identitiy but does not involve γ^5
- permits to eliminate all γ-products with more than 3 terms $\Rightarrow$ **970 SMEs**
- factor 2 more SMEs but completely process-independent reduction

Surprising result: speed of codes based on B.1 and B.2 reduction almost identical.
CPU efficiency not due to highly sophisticated process-dependent manipulations!

Stable reduction of tensor loop integrals (developed for $e^+e^- \to 4f$) [Denner/Dittmaier '05]

6- and 5-point tensor integrals (exploit linear dependence of $4+n$ momenta)

- reduction to lower-point integrals [Melrose '65; Denner/Dittmaier '02]
- simultaneous rank-reduction[Binoth/Guillet/Heinrich/Pilon/Schubert '05; Denner/Dittmaier '05]
- no Gram determinants $\Rightarrow$ no numerical problems in practice

4- and 3-point tensor integrals

- Passarino–Veltman reduction [Passarino/Veltman '79]
- alternative methods for small Gram determinants [Denner/Dittmaier '05]
  (analogies with techniques proposed by Ferroglia/Passera/Passarino/Uccirati '03; Binoth/Guillet/Heinrich/Pilon/Schubert '05; Ellis/Giele/Zanderighi '06)

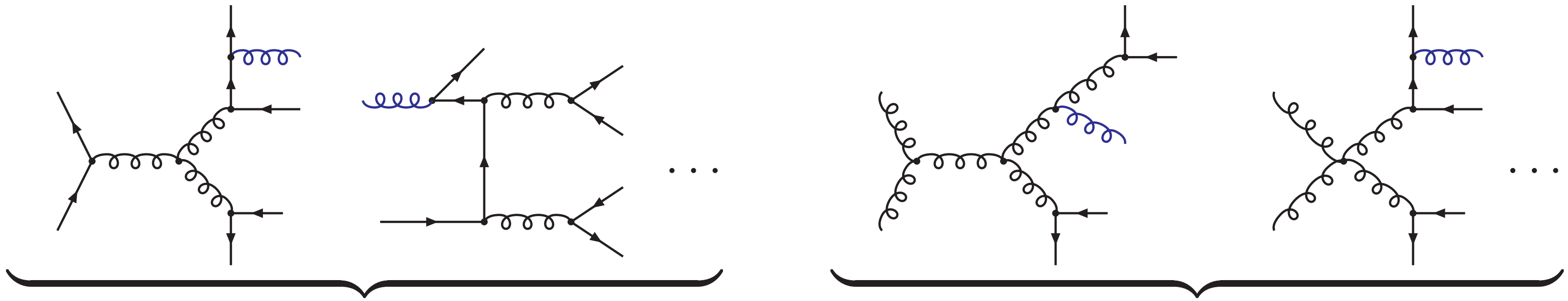
2- and 1-point tensor integrals

- numerically stable analytic representations [Passarino/Veltman '79; Denner/Dittmaier '05]

(3) Real corrections (qq/gg/qg channels)

- Also for real corrections: 2 independent calculations

Two types of matrix elements



$\bar{q}q \to t\bar{t}b\bar{b}g$ and $qg \to t\bar{t}b\bar{b}q$ (64 diagrams) $\quad$ $gg \to t\bar{t}b\bar{b}g$ (341 diagrams)

- Madgraph 4.1.33 [Alwall/Demin/deVisscher/Frederix/Herquet/Maltoni/Plehn/Rainwater/Stelzer'07] for all channels
- analytical calculation with Weyl–van der Waerden spinors [Dittmaier '98] for qq/qg channels
- in-house numerical algortihm based on off-shell recursions [Berends/Giele '88; Caravaglios/Moretti '95; Draggiotis/Kleiss/Papadopoulos '98] for gg channel

Treatment of soft and collinear singularities with dipole subtraction

Catani/Seymour '96; Dittmaier '99; Catani/Dittmaier/Seymour/Trócsányi '02

$$\int \mathrm{d}\sigma_{2\to 5} = \int \left[\mathrm{d}\sigma_{2\to 5} - \sum_{\substack{i,j=1\\ i\neq j}}^{6} \mathrm{d}\sigma_{2\to 5}^{\mathrm{dipole},ij}\right] + \sum_{\substack{i,j=1\\ i\neq j}}^{6} \mathcal{F}_{ij} \otimes \mathrm{d}\sigma_{2\to 4}$$

- numerically stable/efficient but non-trivial: 30 qq/gg (10 qg) subtraction terms
- in-house dipoles checked against `MadDipole`[Frederix/Gehrmann/Greiner '08] (gg/qg) and PS slicing[Giele/Glover '92; Giele et al. '93; Keller/Laenen '98; Harris/Owens '01] (qq)
- initial-state collinear singularities cancelled by $\overline{\mathrm{MS}}$-redefinition of PDFs

Phase-space integration

- adaptive multi-channel Monte Carlo [Berends/Kleiss/Pittau '94; Kleiss/Pittau '94] as in `RACOONWW`[Denner/Dittmaier/Roth/Wackeroth'99]/`PROFECY4f`[Bredenstein/Denner/Dittmaier/Weber'06]
- $\mathcal{O}(1400)$ channels to map all peaks from propagators (300) and dipoles (1100)

11-dimensional phase space, many channels and dipoles $\Rightarrow$ CPU-time! (see later)

Numerical checks

(A) LO checked against SHERPA [Gleisberg/Hoche/Krauss/Schalicke/Schumann/Winter '03]

(B) Precision checks for individual NLO components in single PS points
(typical precision: 10 to 14 digits)

Virtual corrections

- UV, soft and collinear cancellations
- agreement between 2 independent implementations

Real emission

- agreement of $2 \to 5$ matrix elements
- agreement between two dipole implementations
- cancellations in soft and collinear regions

(C) Integrated NLO cross section

- qq channel: agreement between 2 dipole implementations and PS slicing
- gg/qg channels: two independent calculations based on dipole subtraction agree at 1-2 sigma level with statistical accuracy at $2 \times 10^{-3} \times \sigma_{\mathrm{NLO}}$ level

(4) NLO results for the LHC

Parton masses

- $m_{\mathrm{t}} = 172.6\,\mathrm{GeV}$
- g, q and b massless $\Rightarrow$ recombination!

k_{T}-Jet-Algorithm [Run II Jet physics group: Blazey et al. hep-ex/0005012]

- Select partons with $|\eta| < 5$
- Reconstruct jets with $\sqrt{\Delta\phi^2 + \Delta y^2} > D = 0.8$

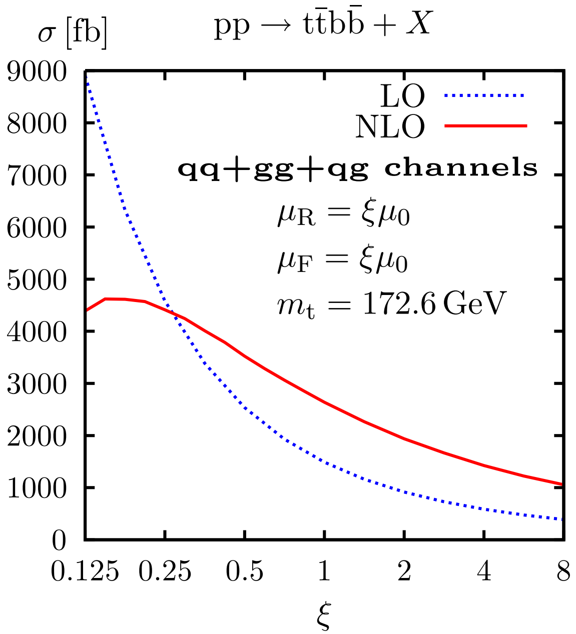
Cuts for b-jets

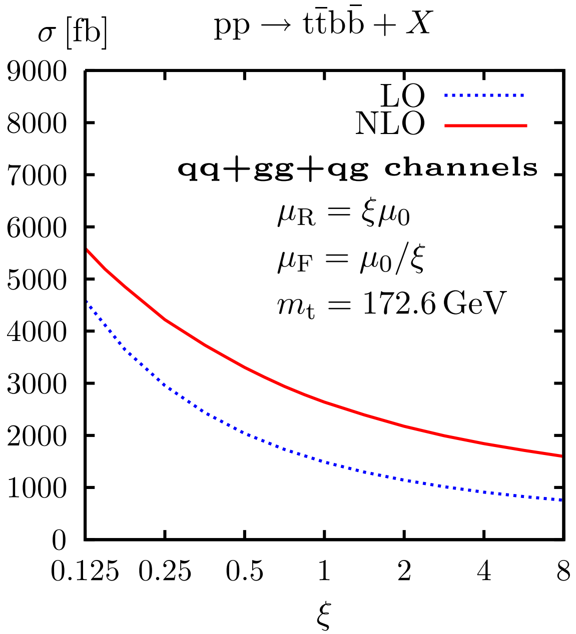
- require two b-jets with $p_{\mathrm{T},j} > 20\,\mathrm{GeV}$ and $y_j < 2.5$
- $\mathrm{b\bar{b}}$ invariant mass: $m_{\mathrm{b\bar{b}}} > m_{\mathrm{b\bar{b},cut}}$

Strong coupling, PDFs and central scale*

- CTEQ6M with $\alpha_{\mathrm{S}}(M_{\mathrm{Z}}) = 0.118$
- 2-loop running to $\mu = \mu_{\mathrm{R}} = \mu_{\mathrm{F}}$
- central scale $\mu_0 = m_{\mathrm{t}} + m_{\mathrm{b\bar{b},cut}}/2$

* LO obtained with LO α_{S}, LO PDFs and 1-loop running





LO and NLO scale dependence (qq+gg+qg channels)

Reduction of scale dependence for σ_{tot}

- rescaling $\mu_{\mathrm{R,F}} = m_{\mathrm{t}}$ by common factor $\xi \in [0.5, 2]$
- 70% dependence at LO
- 34% dependence at NLO

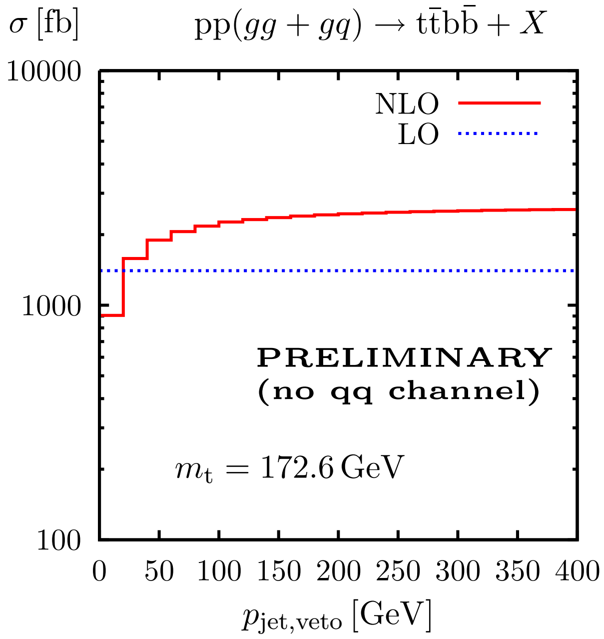
Rescaling μ_{F} by $1/\xi$ (lower plot)

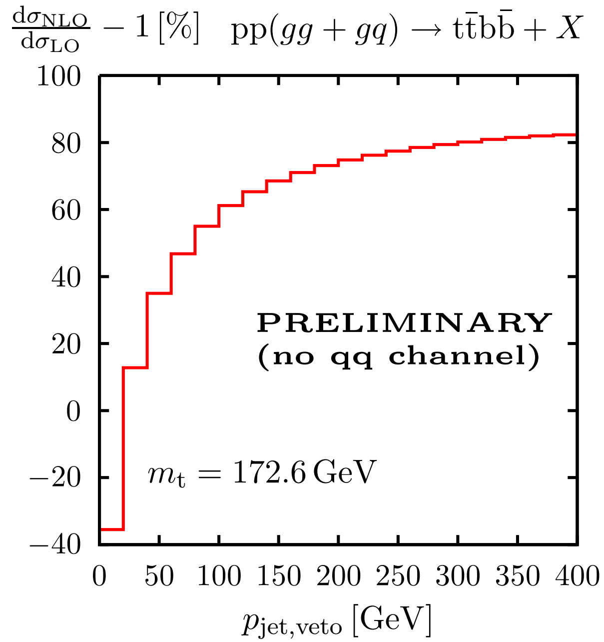
- qualitatively similar behaviour
- dominant dependence from $\alpha_{\mathrm{S}}(\mu_{\mathrm{R}})^4$

Very large NLO correction

- LO and NLO curves do not cross around $\xi = 1$
- $K = 1.77$ at central scale
- completely different wrt $\mathrm{q\bar{q}}$ channel ($K = 1.03$)
- bad news: strong $\mathrm{t\bar{t}H}$-background enhancement!

How to reduce large NLO contribution?





Idea

- large K-factor due to hard jet emission
- underestimated by PDF evolution (collinear approximation)

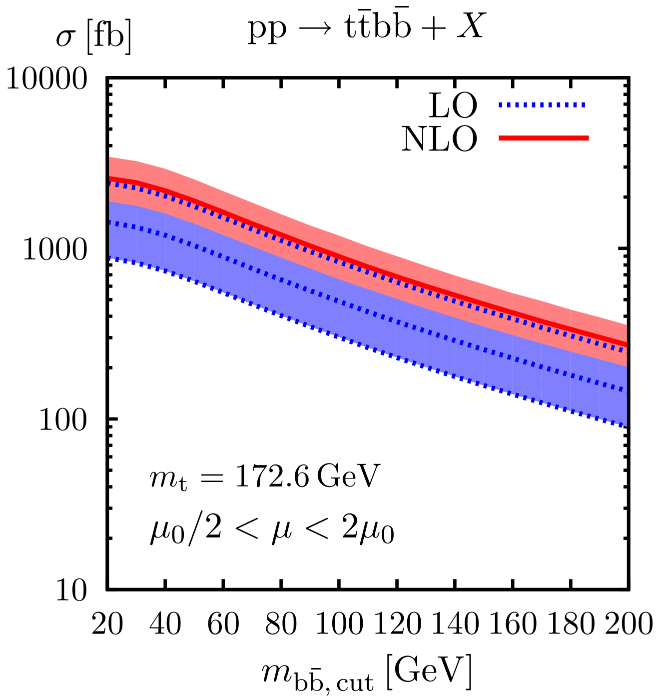
Introduce a veto against jets with $p_{\mathrm{T}}^{\mathrm{jet}} \geq p_{\mathrm{T}}^{\mathrm{veto}}$

- realistic choice $p_{\mathrm{T}}^{\mathrm{veto}} \sim 50 - 100\,\mathrm{GeV}$
- $\sigma_{\mathrm{NLO}}^{\mathrm{tot}}$ quite sensitive to $p_{\mathrm{T}}^{\mathrm{veto}}$ in this region
- can reduce K-factor up to roughly 1.2

Important issues remain to be studied

- impact of $p_{\mathrm{T}}^{\mathrm{veto}}$ on scale dependence?
- resulting systematic uncertainties?
- impact on distributions?

$m_{b\bar{b}}$-dependence of σ_{LO} and σ_{NLO}



Plotted curve

- $m_{b\bar{b}}$-distribution integrated over $m_{b\bar{b}} > m_{b\bar{b},\mathrm{cut}}$
- no jet veto
- LO and NLO with uncertainty bands around $\mu_0 = m_t + m_{b\bar{b},\mathrm{cut}}/2$

Similar behaviour as for σ_{tot}

- scale dependence reduced by factor 2
- bands overlap but NLO central value slightly above LO band (large K-factor)

K-factor quite insensitive to $m_{b\bar{b}}$

Statistical precision and speed of the calculation

Single 3GHz Intel Xeon processor

	$\sigma/\sigma_{\rm LO}$	# events (after cuts)	$(\Delta\sigma)_{\rm stat}/\sigma$	runtime	time/event
NLOtree (gg)	85%	5.8×10^6	0.4×10^{-3}	2h	1.4ms
virtual (gg)	10%	0.46×10^6	0.7×10^{-3}	20h	160ms
real + dipoles (gg/qg)	87%	16.5×10^6	2.6×10^{-3}	47h	10ms

- couple of days on single CPU $\Rightarrow \mathcal{O}(10^7)$ events and $\mathcal{O}(10^{-3})$ stat. accuracy
- for same precision $(\Delta\sigma)_{\rm stat}$ **virtual corrections require less CPU-time than real corrections** (scale-dependent statement!)
- **speed of virtual corrections is remarkably high**: 160 ms/event (including colour and polarization sums!)

Some (process-dependent) remarks about CPU efficiency

- Speed of diagrammatic method **in striking contrast to pessimistic expectations** based on generic arguments (factorial complexity of Feynman diagrams)
- **Is it possible to speed up the virtual corrections beyond 160ms/event?**

⇒ Comparison with unitarity-based algorithms would be very instructive

⇒ Looking at method-independent (and simple) ingredient:

$$\text{CPU-time for master integrals} \sim 10 \text{ ms/event}$$

suggests that there is not much room for further dramatic improvement

Conclusions

NLO QCD calculation for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ **at the LHC**

- very important for $\mathrm{t\bar{t}H}$ measurement
- $2 \to 4$ reaction with highest priority in the 2005 Les Houches wish list

Phenomenological result

- strong enhancement of $\mathrm{t\bar{t}H}$-background ($K \simeq 1.8$)
- might be reduced with a jet veto (to be studied)

Technical considerations

- First complete $2 \to 4$ NLO calculation at hadron colliders
- Excellent testing ground for NLO multi-leg methods
- Remarkably high CPU efficiency of diagrammatic tensor-reduction approach