

# NLO EW+QCD corrections to $W$ +jet production at hadron colliders for decaying (off-shell) $W$ bosons

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- 3 Numerical results for the LHC
- 4 Conclusions



# 1 Introduction

Importance of  $W$ +jet production:  $pp \rightarrow l\nu_l + \text{jet} + X$

- large SM cross section ( $\sim 1\text{nb}$  after basic cuts)  $\rightarrow$  **standard candle**
- single- $W$  production often shows additional jet activity  
 $\hookrightarrow$  relevance for  **$W$ -mass determination** at the LHC
- offers precision test for jet dynamics in QCD
- dominant channel for high- $p_T$  leptons  
 $\hookrightarrow$  important **background** for various new-physics searches

Theoretical status:

- NLO QCD DYRAD [Giele et al. '93]; MCFM [Campbell/R.K.Ellis '02]; Melnikov/Petriello '06; Catani et al. '09
- NLO EW for stable  $W$  bosons  
Kühn, Kulesza, Pozzorini, Schulze '07; Hollik, Kasprzik, Kniehl '07

Motivation for this work:

**Inclusion of full off-shell effects** in NLO QCD+EW predictions

$\hookrightarrow$  e.g. essential for  $W$ -mass determination

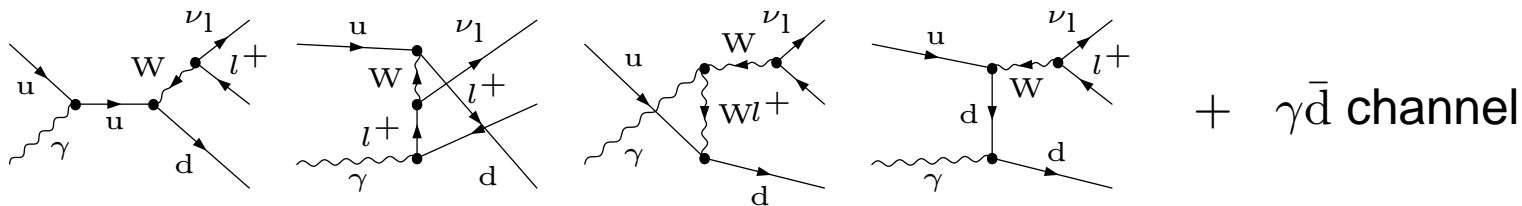
## 2 Calculation of NLO corrections

### 2.1 Lowest-order prediction

LO diagrams:



Contributions from photon-induced processes



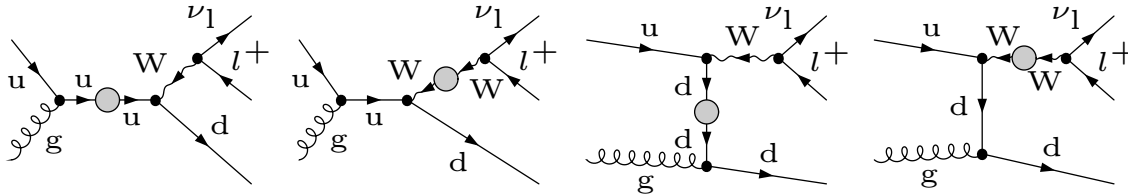
Features of the LO cross section:

- IR safety requires at least lower cut on  $p_{T,\text{jet}}$ 
  - ↪ apply jet algorithm for NLO cross section before cut on  $p_{T,\text{jet}}$
- contributions from photon-induced processes generically small ( $\sim 1\%$ )
  - ↪ inclusion of LO and NLO QCD as corrections

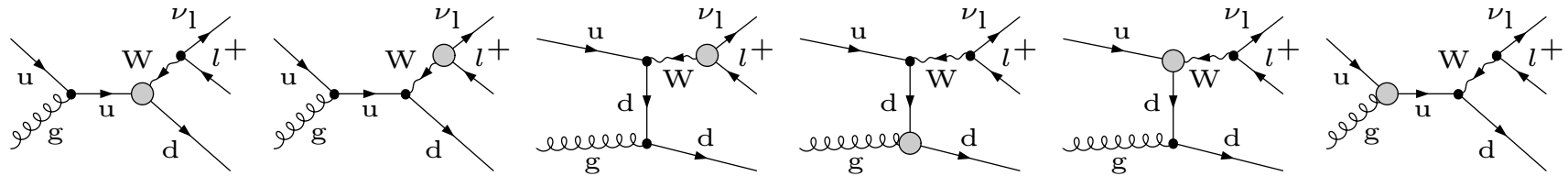
## 2.2 Survey of NLO EW+QCD corrections and technical details

1PI loop insertions in virtual corrections:  $\mathcal{O}(100)$  EW loop diagrams per channel

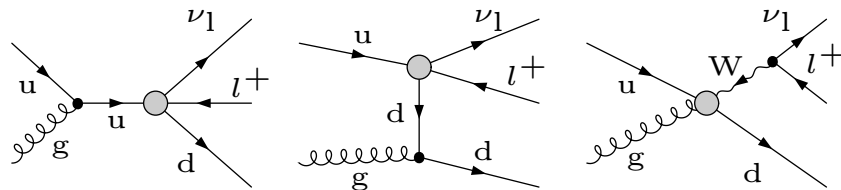
Self-energy insertions:



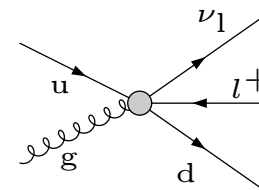
Vertex corrections:



Box corrections:

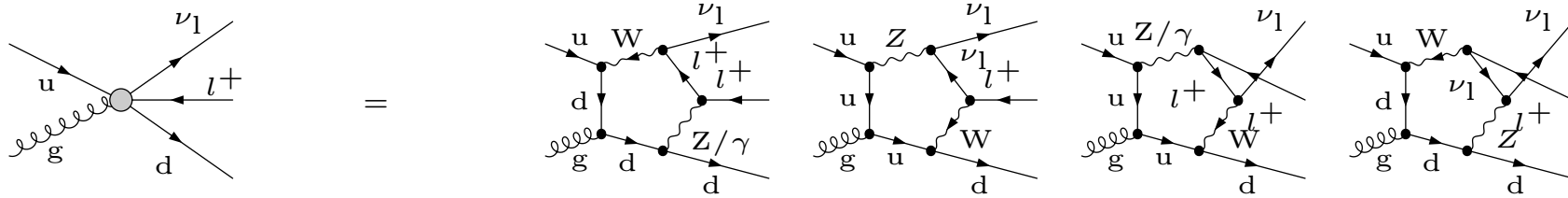


Pentagon graphs:



## Most complicated parts of the loop calculation:

- pentagon graphs:



↪ stable reduction to box integrals without inverse Gram determinants

Denner, S.D. '02,'05 (similar to Binoth et al. '05)

- numerical instabilities in Passarino–Veltman reduction of tensor integrals

↪ expansion about exceptional points

Denner, S.D. '05 (similar to Giele et al. '04; R.K.Ellis et al. '05)

- gauge-invariant treatment of  $W$  resonances

↪ “complex-mass scheme”

Denner, S.D., Roth, Wieders '05

# The complex-mass scheme at NLO Denner, S.D., Roth, Wieders '05

**Basic idea:**  $\text{mass}^2 = \text{location of propagator pole in complex } p^2 \text{ plane}$

$\hookrightarrow$  **consistent use of complex masses everywhere !**

**Application to gauge-boson resonances:**

• replace  $M_W^2 \rightarrow \mu_W^2 = M_W^2 - iM_W\Gamma_W$ ,  $M_Z^2 \rightarrow \mu_Z^2 = M_Z^2 - iM_Z\Gamma_Z$

and define (complex) weak mixing angle via  $c_W^2 = 1 - s_W^2 = \frac{\mu_W^2}{\mu_Z^2}$

• **virtues:**

◇ gauge-invariant result (Slavnov–Taylor identities, gauge-parameter independence)

$\hookrightarrow$  unitarity cancellations respected !

◇ perturbative calculations as usual (loops and counterterms)

◇ no double counting of contributions (bare Lagrangian unchanged !)

• **drawbacks:**

◇ unitarity-violating spurious terms of  $\mathcal{O}(\alpha^2)$   $\rightarrow$  but beyond NLO accuracy !

(from  $t$ -channel/off-shell propagators and complex mixing angle)

◇ complex gauge-boson masses also in loop integrals

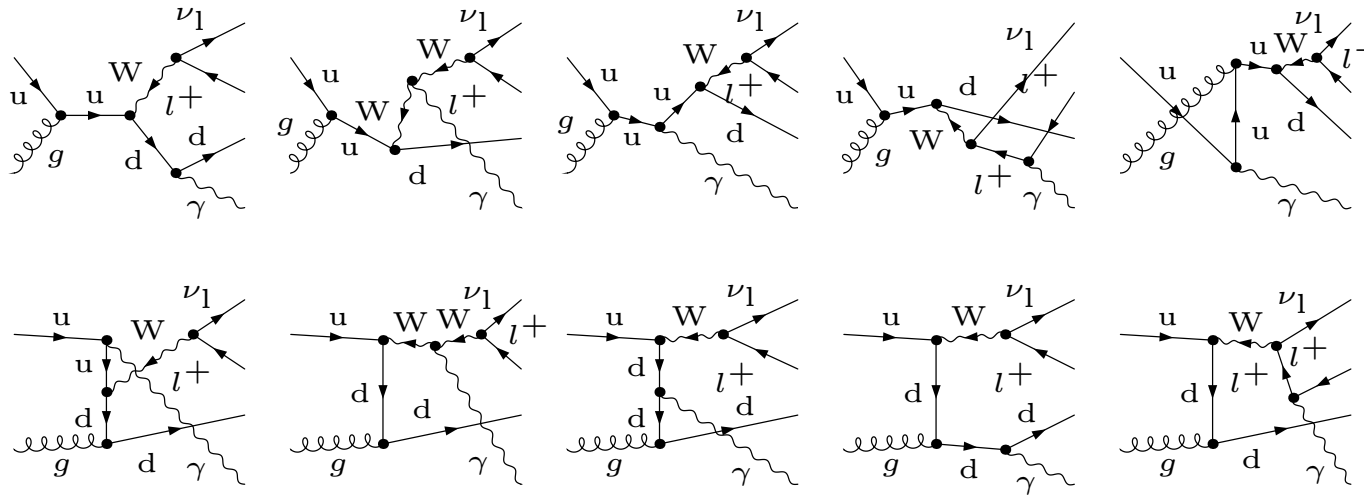
# Real emission corrections

- NLO QCD:**

- ◇ gluon bremsstrahlung:  $u\bar{d} \rightarrow l^+ \nu_l gg$ ,  $ug \rightarrow l^+ \nu_l dg$ ,  $\bar{d}g \rightarrow l^+ \nu_l \bar{u}g$
- ◇ gg fusion:  $gg \rightarrow l^+ \nu_l \bar{u}d$
- ◇ gluon splitting  $g^* \rightarrow q\bar{q}$ :  $u\bar{d} \rightarrow l^+ \nu_l q\bar{q}$  + crossed variants

- NLO EW:**

- ◇ photon bremsstrahlung:  $u\bar{d} \rightarrow l^+ \nu_l g\gamma$ ,  $ug \rightarrow l^+ \nu_l d\gamma$ ,  $\bar{d}g \rightarrow l^+ \nu_l \bar{u}\gamma$



- Note:**  $u\bar{d} \rightarrow l^+ \nu_l g\gamma$  contributes to both
- NLO **EW** corrections to  $u\bar{d} \rightarrow l^+ \nu_l g$  and
  - NLO **QCD** corrections to  $u\bar{d} \rightarrow l^+ \nu_l \gamma$



## Further technical details

- Generic features

- ◇ two completely independent calculations of all ingredients  
↳ results in mutual agreement
- ◇ dipole subtraction for QCD and photonic IR (soft and collinear) singularities  
Catani, Seymour '96; S.D. '99; S.D., Kabelschacht, Kasprzik '08  
↳ checked against phase-space slicing

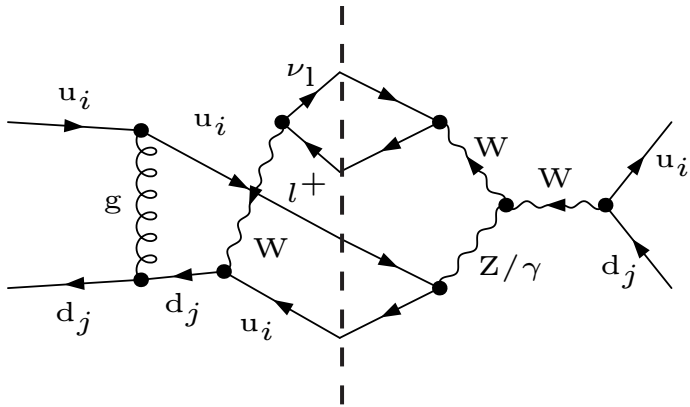
- MPI/FR

- ◇ loop amplitude generation with FEYNARTS 1 Böhm, Denner, Küblbeck '90  
↳ algebraic reduction with inhouse MATHEMATICA routines
- ◇ tree amplitudes evaluated analytically via spinor formalism
- ◇ phase-space integration via VEGAS

- PSI

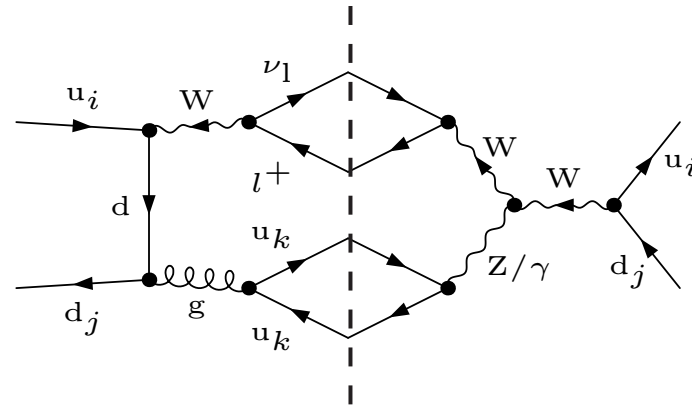
- ◇ loop amplitude generation with FEYNARTS 3 Hahn '00  
↳ algebraic reduction with FORMCALC Hahn '98-'09 and POLE Meier '05; Mück
- ◇ real corrections generated with program POLE  
↳ automatic generation of tree amplitudes via spinor formalism,  
dipole subtraction terms, and multi-channel phase-space integration

# Non-trivial interferences between real NLO QCD and EW corrections



$$\mathcal{M}_{\text{QCD}}(\mathcal{M}_{\text{EW}})^* \propto \text{Tr}[T^a T^a]$$

→ interference non zero



$$\mathcal{M}_{\text{QCD}}(\mathcal{M}_{\text{EW}})^* \propto (\text{Tr}[T^a])^2$$

→ interference vanishes

↪ Non-singular contributions of  $\mathcal{O}(\alpha^3 \alpha_s) =$  same order as NLO EW

## Interferences included in our calculation

↪ effect phenomenologically negligible

(%-effects only for  $p_{\text{T,jet}} \gtrsim 1 \text{ TeV}$ , otherwise  $\ll 1\%$ )

### 3 Numerical results for the LHC

#### Setup and definition of observables

- **PDF set MRST2004QED** which includes NLO QCD+EW corrections
  - ↪ initial-state collinear QCD and photonic singularities removed via factorization
  - ↪ **PDF set includes photon density**
- **two scale choices:**  $\mu = \mu_{\text{ren}} = \mu_{\text{fact}} = M_W$  (**fixed**) or  $\mu = \sqrt{M_W^2 + (p_T^{\text{had}})^2}$  (**variable**)
- **$k_T$ -algorithm** for jet definition
- **basic cuts:**  $p_{T,\text{jet1}/l/\text{miss}} > 25 \text{ GeV}$ ,  $|y_{\text{jet1}/l}| < 2.5$ ,  $R_{l,\text{jet}} > 0.5$
- **jet veto** optionally applied to 2nd hard jet if  $p_{T,\text{jet2}} > p_{T,\text{jet1}}/2$
- **two lepton identifications:** **“bare leptons”** ( $l = \mu^+$ )
  - ↪ large corrections  $\propto \alpha \ln(m_l^2/Q^2)$
  - or **photon–lepton recombination** if  $R_{\gamma,l} < 0.1$  ( $l = e^+$ )
    - ↪ cancellation of all  $\alpha \ln(m_l^2/Q^2)$  terms a la KLN
- **photon fragmentation function** for photon–jet separation **Glover, Morgan '94**

# Photon–jet separation via photon fragmentation function $D_{q \rightarrow \gamma}$

## Why?

- collinear quarks and photons have to be recombined  $\rightarrow$  quasiparticle otherwise corrections  $\propto \ln(m_q^2/Q^2) \rightarrow$  perturbative “IR instability”
  - quark and gluon jets cannot be distinguished event by event  
 $\hookrightarrow$  common recombination required for quarks/gluons with photons
- $\Rightarrow$   $\underbrace{(\mathbf{g}_{\text{hard}} + \mathbf{\gamma}_{\text{soft}})}_{\text{EW corr. to } \mathbf{W+jet}}$  and  $\underbrace{(\mathbf{g}_{\text{soft}} + \mathbf{\gamma}_{\text{hard}})}_{\text{QCD corr. to } \mathbf{W+\gamma}}$  both appear as 1 jet

## Solution:

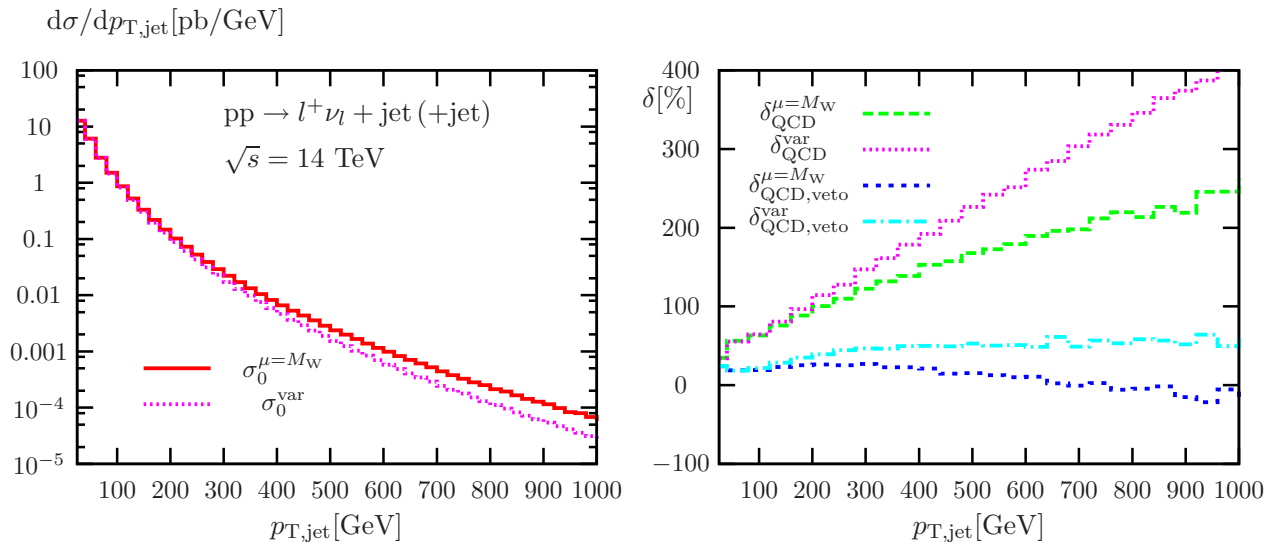
- exclude events with photon energy fraction  $z_\gamma = \frac{E_\gamma}{E_{\text{jet}} + E_\gamma} > z_0$  for (jet +  $\gamma$ ) quasiparticles (chosen value  $z_0 = 0.7$ )
- subtract convolution of LO cross section with

$$D_{q \rightarrow \gamma}^{\overline{\text{MS}}}(z_\gamma, \mu_{\text{fact}}) \Big|_{\text{mass.reg.}} = \frac{\alpha Q_q^2}{2\pi} P_{q \rightarrow \gamma}(z_\gamma) \left[ \ln \frac{m_q^2}{\mu_{\text{fact}}^2} + 2 \ln z_\gamma + 1 \right] \leftarrow \text{cancels coll. singularities}$$
$$+ D_{q \rightarrow \gamma}^{\text{ALEPH}}(z_\gamma, \mu_{\text{fact}}) \leftarrow \text{non-perturbative part fitted to ALEPH data}$$

where  $P_{q \rightarrow \gamma}(z_\gamma) = \frac{1+(1-z_\gamma)^2}{z_\gamma} =$  quark-to-photon splitting function

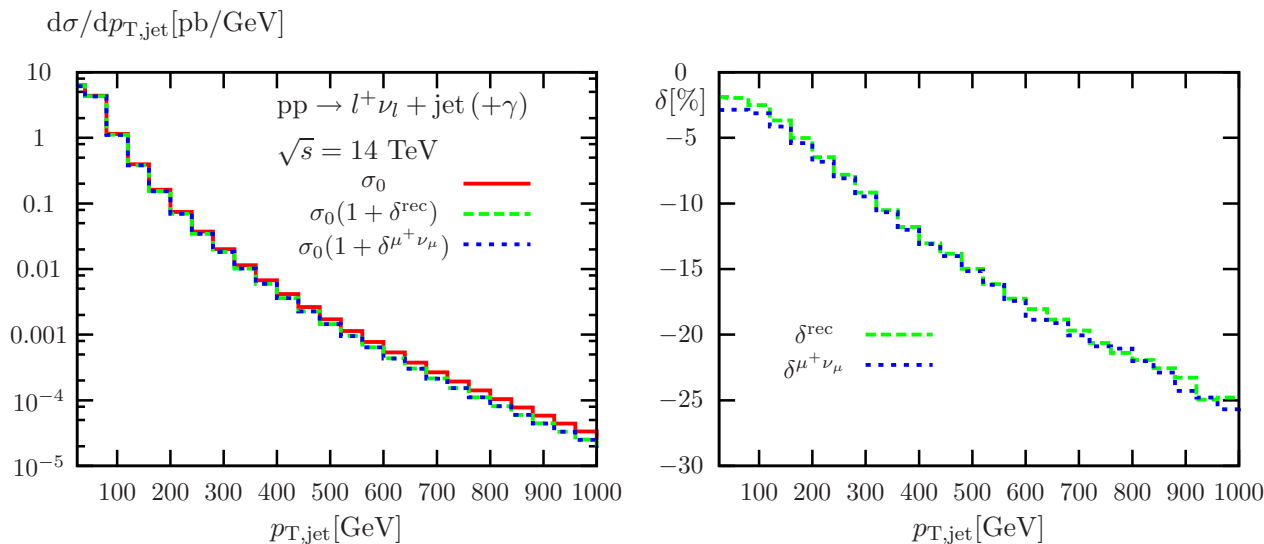
# Transverse-momentum distribution of the hardest jet

## QCD corrections:



Large positive corrections from W+2jets (mainly back-to-back jets)  
 $\hookrightarrow$  significant reduction of corrections via jet veto

## EW corrections:



Large neg. corrections due to EW Sudakov logs  
 $\hookrightarrow$  qualitative agreement with previous results for on-shell Ws [Kühn et al. '07](#) [Hollik et al. '07](#)

# Integrated cross section for various cuts on $p_{T,jet}$

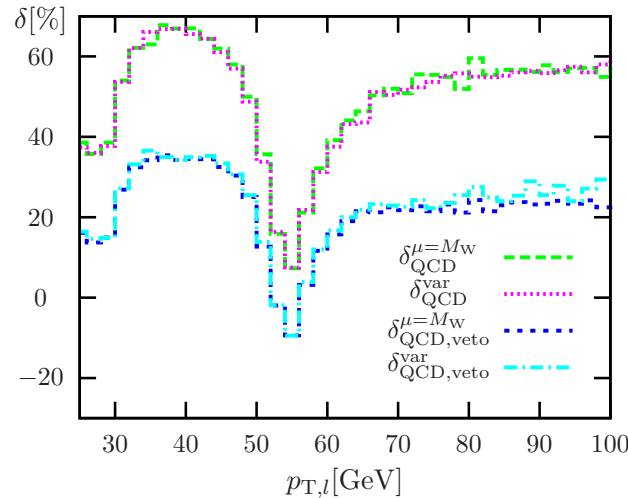
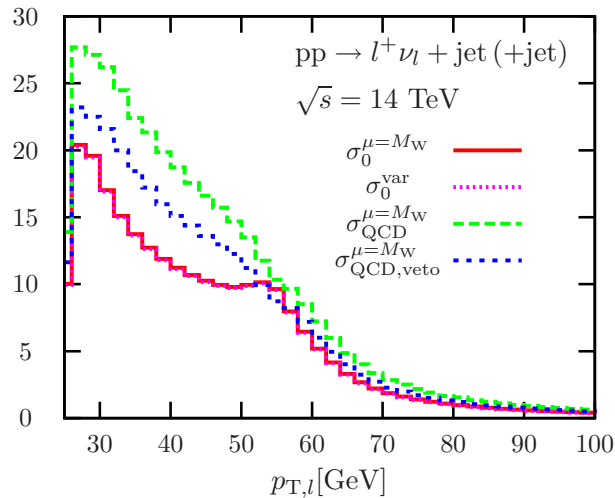
$$pp \rightarrow l^+ \nu_l \text{ jet} + X \text{ at } \sqrt{s} = 14 \text{ TeV}$$

$p_{T,jet} / \text{GeV}$	25 – $\infty$	50 – $\infty$	100 – $\infty$	200 – $\infty$	500 – $\infty$	1000 – $\infty$	
$\sigma_{\text{Born}}^{\mu=M_W} / \text{pb}$	509.45(5)	182.74(2)	49.777(5)	8.1086(9)	0.3156(1)	0.0117(1)	
$\sigma_{\text{Born}}^{\text{var}} / \text{pb}$	502.66(5)	176.39(1)	45.382(4)	6.4990(6)	0.1850(1)	0.0048(1)	
$\delta_{\text{EW}}^{\mu^+ \nu_\mu, \text{var}} / \%$	-3.07(6)	-3.35(1)	-4.64(1)	-8.50(1)	-18.0(1)	-28.0(1)	EW Sudakov
$\delta_{\text{EW}}^{\text{rec}, \text{var}} / \%$	-2.07(1)	-2.55(2)	-4.18(1)	-8.37(3)	-17.8(1)	-27.9(1)	logs
$\delta_{\text{QCD}}^{\mu=M_W} / \%$	48.0(1)	64.8(1)	80.7(1)	115	188	270(1)	
$\delta_{\text{QCD}}^{\text{var}} / \%$	47.9(2)	65.4(1)	85.8(1)	135	270	494(1)	
$\delta_{\text{QCD, veto}}^{\mu=M_W} / \%$	21.5(1)	18.2(1)	22.5(2)	24.6(1)	5.3(1)	-26.4(2)	jet veto
$\delta_{\text{QCD, veto}}^{\text{var}} / \%$	22.3(1)	20.8(1)	29.8(2)	43.3(2)	52.4(1)	58.7(1)	
$\delta_{\gamma, \text{NLO}}^{\text{var}} / \%$	0.38	0.70	1.18	1.86	3.26	5.18(1)	$\gamma$ -induced processes
$\delta_{\gamma, \text{NLO, veto}}^{\text{var}} / \%$	0.35	0.64	1.10	1.76	3.03	4.73(1)	
$\delta_{\text{IF}}^{\text{var}} / \%$	0.05	0.13	0.51	1.88	11.50	49.95	QCD-EW interferences
$\delta_{\text{IF, veto}}^{\text{var}} / \%$	0.01(1)	0.03	0.12	0.40	1.63	4.72	

# Transverse-momentum distribution of the charged lepton

## QCD corrections:

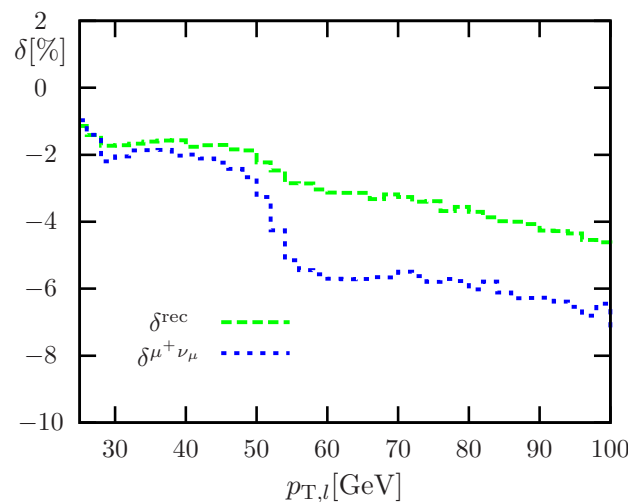
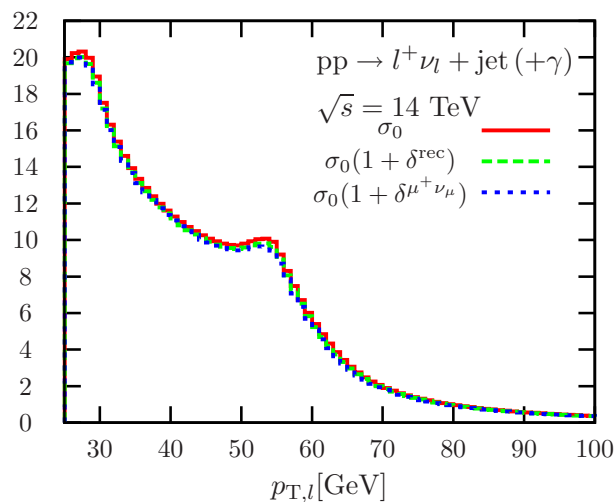
$d\sigma/dp_{T,l}[\text{pb}/\text{GeV}]$



- large corrections  
distorting the shape
- corrections reduced  
by jet veto

## EW corrections:

$d\sigma/dp_{T,l}[\text{pb}/\text{GeV}]$

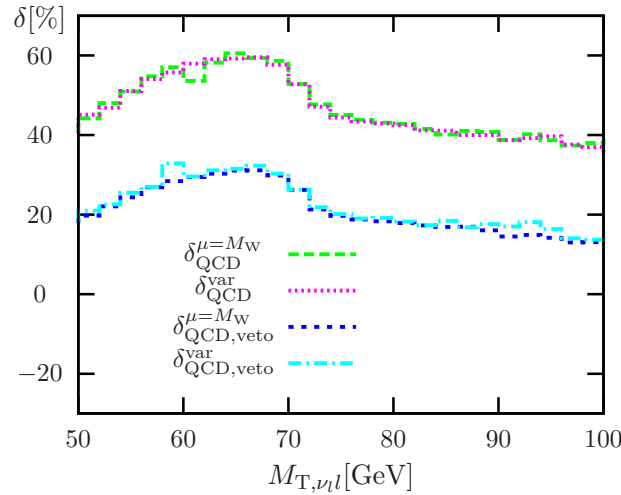
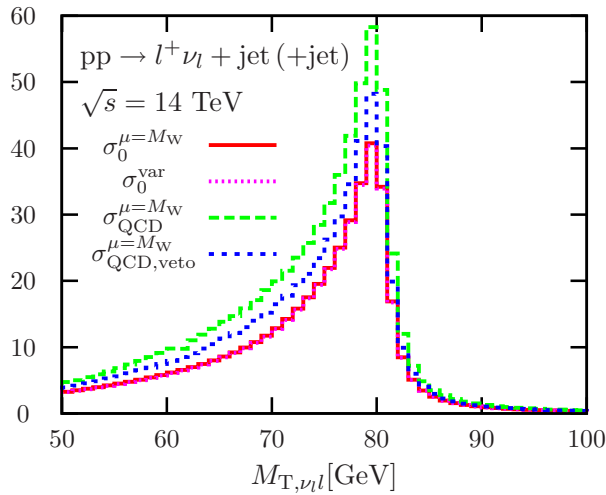


- moderate corrections  
of  $\mathcal{O}(5\%)$
- Jacobian peaks  
slightly distorted

# Transverse-mass distribution of the W boson

## QCD corrections:

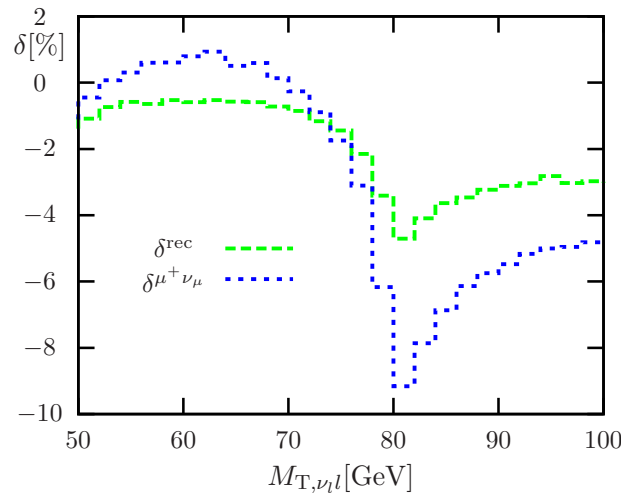
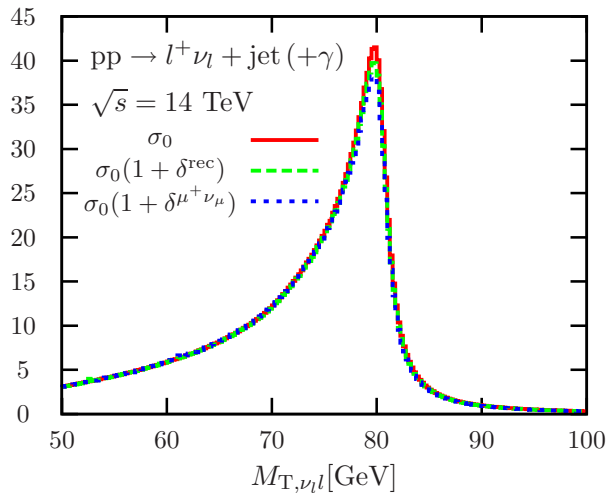
$d\sigma/dM_{T,\nu l}[\text{pb/GeV}]$



- corrections smooth near Jacobian peak
- corrections reduced by jet veto

## EW corrections:

$d\sigma/dM_{T,\nu l}[\text{pb/GeV}]$



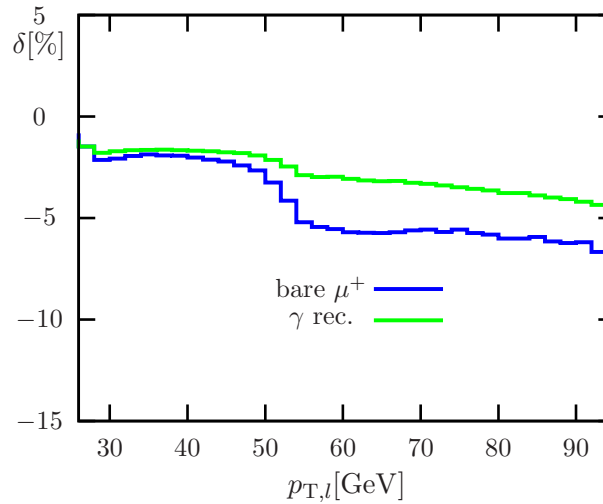
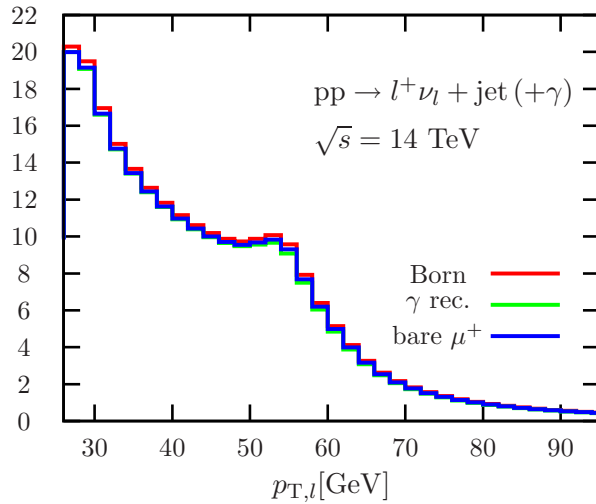
- corrections of  $\mathcal{O}(5-10\%)$
- Jacobian peak significantly distorted



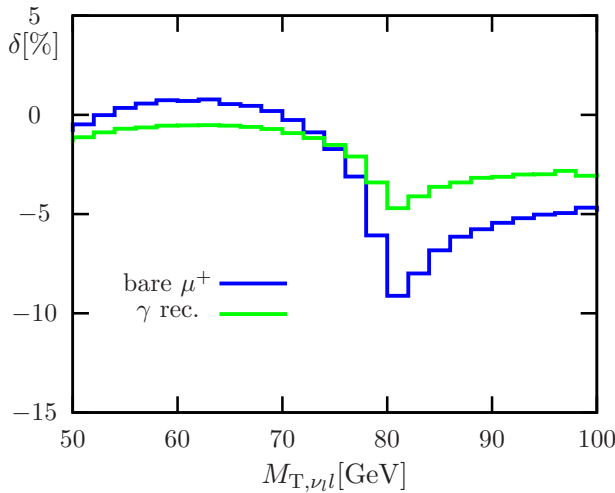
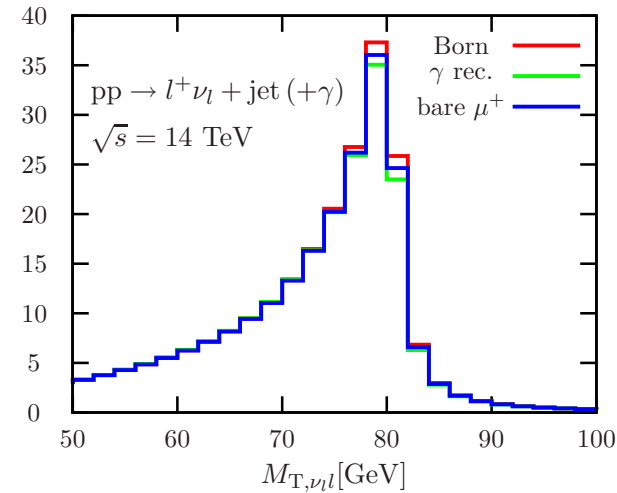
# Comparison of EW corrections to W+jet and single (jet-inclusive) W production

↪ interesting for W-mass determination via single-W production

$d\sigma/dp_{T,l}[\text{pb/GeV}]$



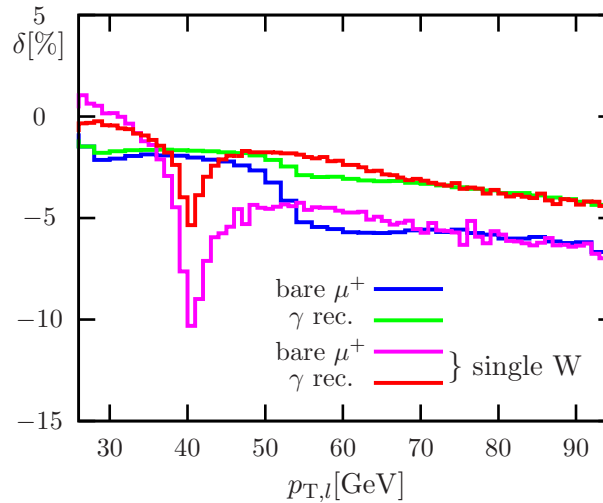
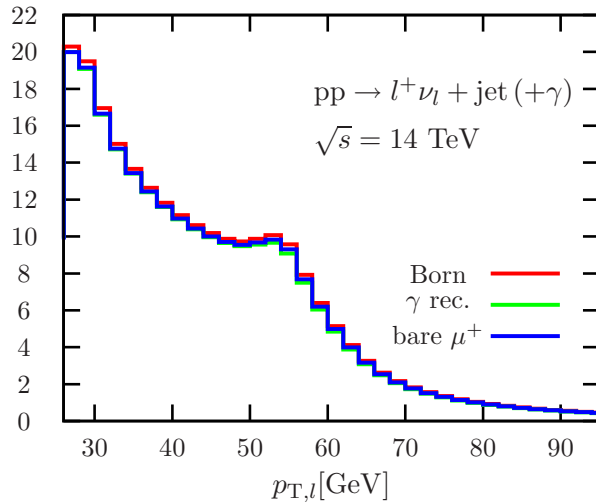
$d\sigma/dM_{T,\nu l}[\text{pb/GeV}]$



# Comparison of EW corrections to W+jet and single (jet-inclusive) W production

↪ interesting for W-mass determination via single-W production

$d\sigma/dp_{T,l}[\text{pb/GeV}]$

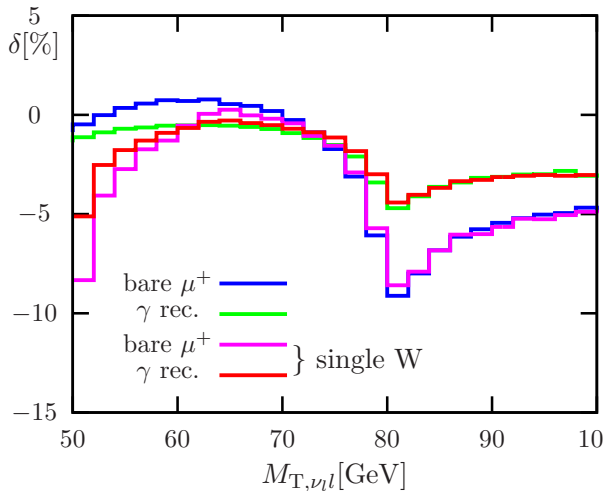
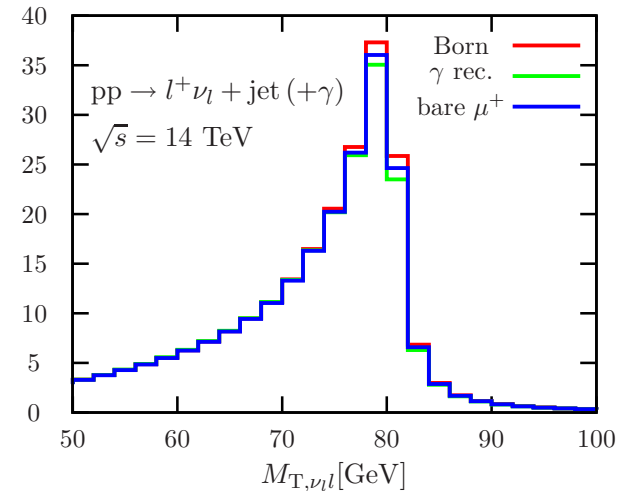


relative EW corrections  
 completely different

For single-W in NLO EW, see

- Baur et al. '98/'04
- S.D./Krämer '01
- Arbuzov et al. '06
- Carloni Calame et al. '06

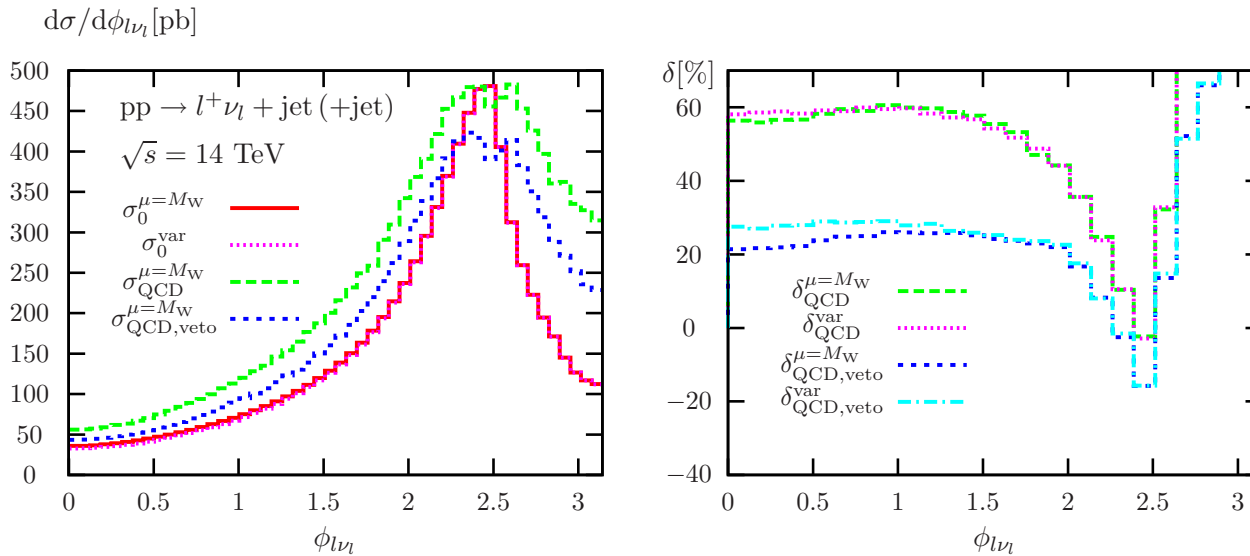
$d\sigma/dM_{T,\nu_l}[\text{pb/GeV}]$



relative EW corrections  
 practically identical  
 near Jacobian peak

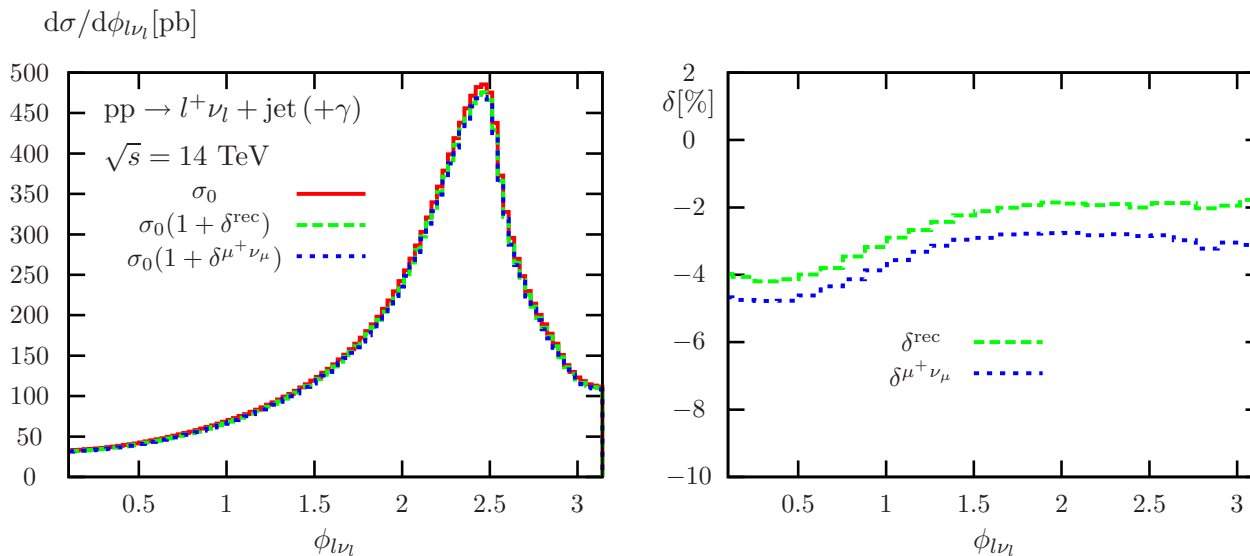
# Distribution in the lepton–neutrino ( $\phi_T$ ) azimuthal angle in the transverse plane

## QCD corrections:



- significant distortion of shape
- corrections reduced by jet veto

## EW corrections:



- corrections small and relatively smooth

## 4 Conclusions

W+jet production is a very important process at Tevatron and LHC.  
(standard candle, W mass, background, etc.)

### Our calculation provides

- recalculation of NLO QCD corrections
- first calculation of **NLO EW corrections including full off-shell effects**  
(e.g. relevant for W-mass determination)
  - ↪ building block for NNLO corrections of  $\mathcal{O}(\alpha\alpha_s)$  for single-W production

### Size of corrections:

- **EW corrections particularly large at high  $p_T$**  of leptons and jets
- photon-induced processes and EW–QCD interferences  
phenomenologically unimportant

### Outlook:

- low- $p_T$  range should be further improved by **soft-gluon resummation**  
or by merging with parton showers
- off-shell **Z+jet production** straightforward with the same approach