Dominant NNLO corrections to four-fermion production at the WW threshold

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EFT for unstable particles by

Beneke, Chapovsky, Falgari, Kauer, Schwinn, Signer, Zanderighi

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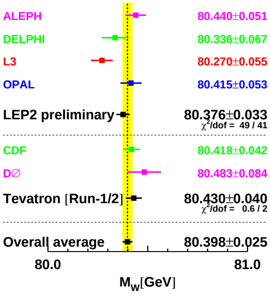
Outline

1. Why $e^- e^+ \to 4$ fermions at WW
2. EFT method for NNLO corrections
3. Numerical results
4. Conclusions

W mass at LEP2

$e^- e^+ \to 4$ fermions $\Rightarrow$ precise determination of the W mass



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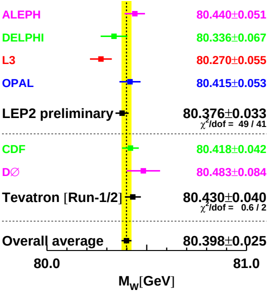

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- Total cross section at LEP2 from WW to 207 GeV with 1% accuracy $\Rightarrow$ W-mass uncertainty 33 MeV

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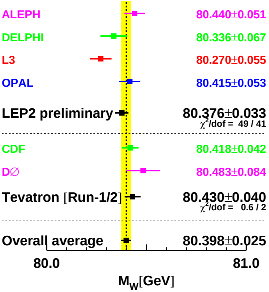

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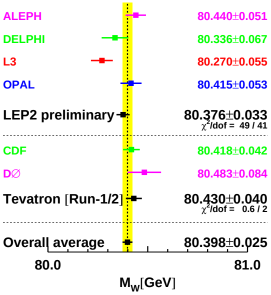

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- Combination of LEP2 / TEVATRON reduces to 25 MeV
- Foreseen improvement of 50% at the LHC

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W unstable $\Rightarrow$ effects due to the W width taken into account

NLO predictions for $e^- e^+ \to 4$ fermions

LEP2 analysis with YFSWW [Jadach-Placzek-Skrzypek-Ward-Was '00] and RacoonWW [Denner-Dittmaier-Roth-Wackeroth '00] (selected radiative corrections to LO); ILC accuracy $\Rightarrow$ full NLO computation needed

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	$\sigma(e^- e^+ \to \mu^- \bar{\nu}_\mu u \bar{d} X)$(fb)			
$\sqrt{s}$ [GeV]	Born	EFT	CMS	shift
161	107.06(4)	117.38(4) [+9.6%]	118.12(8) [+10.3%]	0.7 %
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EFT tailored to threshold region: simple computation, compact result, easy evaluation of dominant NNLO corrections at threshold

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- Calculation re-organized through a loop and kinematical expansion in the small parameters

$$\alpha_{ew} = \alpha / \sin^2\theta \qquad (s - 4M_W^2) / (4M_W^2) \qquad \Gamma_W / M_W$$

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Collectively denoted by δ, of the same order for power counting

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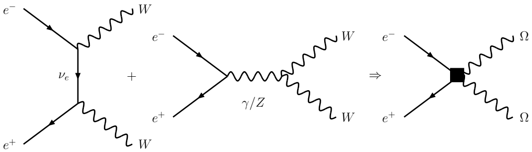
Integrate out hard modes introducing matching coefficients

$\Rightarrow$ Short-distance physics:

- ∗ non-resonant W's
- ∗ light with large k^2
- ∗ heavy particles



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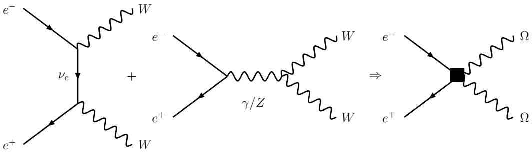
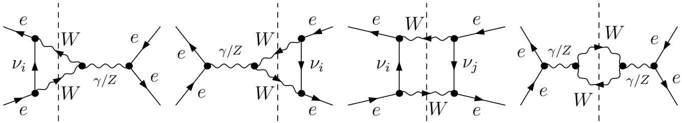


Potential, soft and collinear modes (long-distance physics) generate genuine radiative corrections in the EFT

Outline of the computation

1) Extract the total cross section for e^- $e^+ \to 4$ fermions from the cuts of the e^-e^+ forward-scattering amplitude



$$i\mathcal{A} = \underbrace{\sum_{k,l} \int d^4x < e^-e^+|T[i\mathcal{O}_p^{(k)\dagger}(0) i\mathcal{O}_p^{(l)}(x)|e^-e^+ >}_{\text{resonant (eff. operators)}} + \underbrace{\sum_k < e^-e^+|i\mathcal{O}_{4e}^{(k)}(0)|e^-e^+ >}_{\text{non resonant (eff. operators)}}$$

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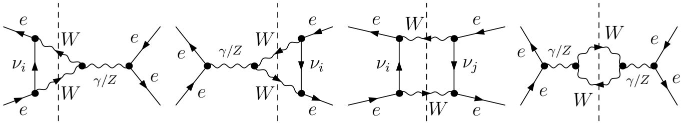

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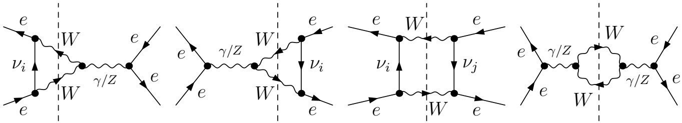

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3) Evaluate loop corrections to the EFT matrix element:

- resonant W's $\quad k^2 - M_W^2 \sim M_W \Gamma_W$
- Coulomb and soft photons $\quad k^2 \sim M_W \Gamma_W$ and $k^2 \sim \Gamma_W^2$
- high-energy external fermions $\quad k^2 = 0$

Dominant NNLO corrections

Re-organization of perturbative expansion $\Rightarrow$ identify the dominant NNLO corrections from the power of δ for each EFT diagram

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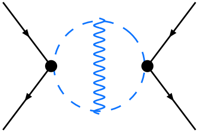

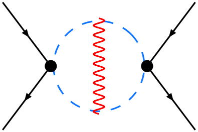

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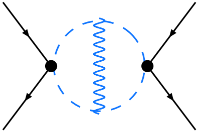

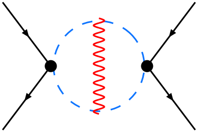

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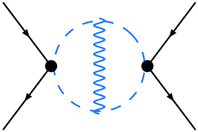

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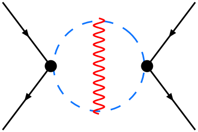

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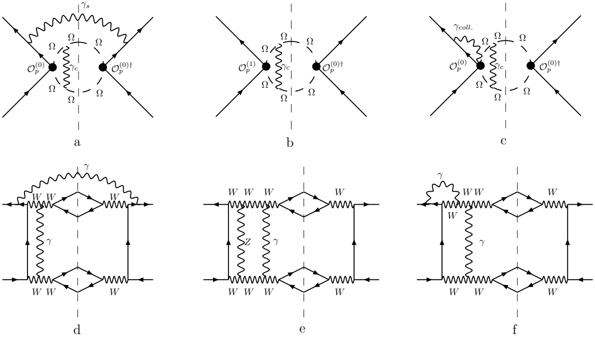
some corrections $\alpha_{ew}^2 \alpha\, \delta$ already included in the NLO computation [Denner-Dittmaier-Roth-Wieders '05]

Corrections to single-Coulomb-exchange diagrams

Single-Coulomb-exchange diagrams $\otimes$ soft- and collinear-photon emission and one-loop corrections to the matching coefficient of the production operator



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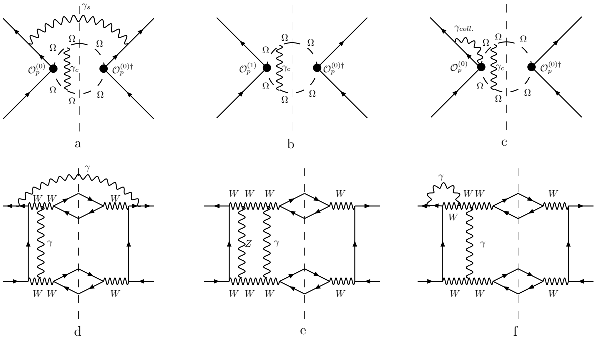

EFT: • point-like interaction between leptons and resonant W's

• bare W propagator replaced with effective re-summed one

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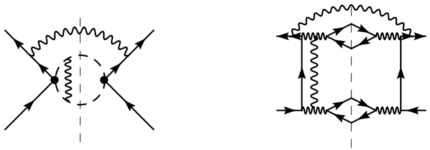

1) Sub-loop free from soft modes (EFT $\to$ non-covariant propagators)

- resonant W's $\to$ i $\left[k^0 - \vec{k}^2/(2M_W) + i\Gamma_W/2\right]^{-1}$ Coulomb photon $\to$ $i/\vec{k}^2$

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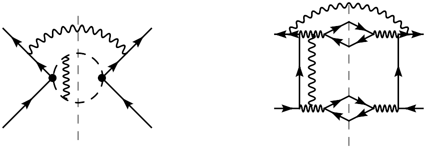

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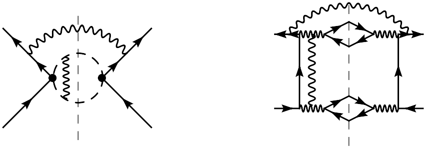

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3) Evaluate the convolution with the soft-photon integral using
Feynman parameters for non-covariant loop integrals $\Rightarrow$ tadpoles

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Two independent evaluations:

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$\mathcal{E}_W = \sqrt{s} - 2M_W + i\Gamma_W,\qquad c_p^{(1)} \to$ NLO matching coefficient of production operator

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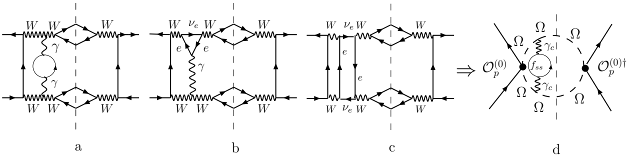
from [Beneke-Falgari-Schwinn-Signer-Zanderighi'07]

- full result sensitive to the electron mass $\Rightarrow \ln(2M_W/m_e)$
- large logarithms subtracted and re-absorbed in the ESFs $\Gamma(x)$ [Skrzypek '92, Beenakker et al. '96]

$$\sigma(s) = \int_0^1 dx_1 \int_0^1 dx_2\,\Gamma(x_1)\Gamma(x_2)\hat{\sigma}(x_1x_2s)$$

Corrections to the Coulomb potential

Fermion-loop corrections to the Coulomb force between resonant W's



Bubbles, triangles and boxes $\Rightarrow$ only bubbles dominant NNLO

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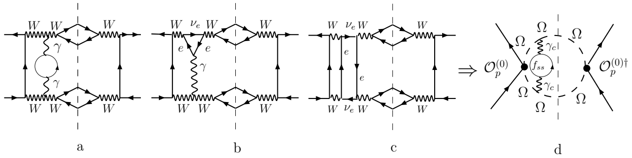

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$$\Delta\sigma_2|_{\alpha(M_Z)} = -\frac{\alpha_{ew}^2\alpha^2}{81s}\sum_f C_f Q_f^2 \left\{4\ln\left(\frac{2M_W}{M_Z}\right)\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}_W}{M_W}\right)\right] + \mathrm{Im}\left[\ln^2\left(-\frac{\mathcal{E}_W}{M_W}\right)\right]\right\}$$

$$\Delta\sigma_2|_{G_\mu} = \Delta\sigma_2|_{\alpha(M_Z)} - \frac{2\pi\alpha_{ew}^2\alpha}{27s}\,\delta_{\alpha(M_Z)\to G_\mu}\,\mathrm{Im}\left[\ln\left(-\frac{\mathcal{E}_W}{M_W}\right)\right]$$

large fermion-mass logarithms in the $\alpha(M_Z) \to G_\mu$ [Dittmaier-Krämer '01]

(as for NLO result [Beneke-Falgari-Schwinn-Signer-Zanderighi '07])

Corrections specific of the EFT

Decay corrections: after using unitarity, multiply the result by the leading order branching fraction product $\mathrm{Br}^{(0)}_{\mu^-\overline{\nu}_\mu} \cdot \mathrm{Br}^{(0)}_{u\overline{d}} = 1/27$ for selecting the flavour-specific case

$$\mathrm{Im}\mathcal{A} \Rightarrow \sigma = \frac{1}{27s}\mathrm{Im}\mathcal{A}$$

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NLO and NNLO: higher order flavour-specific corrections to the decay of the resonant W's to $\mu^-\overline{\nu}_\mu$ and $u\overline{d}$

$$\Delta\sigma_3 = -\left(\frac{\Gamma^{NLO}_{\mu^-\overline{\nu}_\mu}\Gamma^{NLO}_{u\overline{d}}}{\Gamma^2_{NLO}}\right)\underbrace{\frac{2\pi\alpha_{ew}^2\alpha}{27s}\ln\left(-\frac{\mathcal{E}_W}{M_W}\right)}_{\text{single Coulomb}}$$

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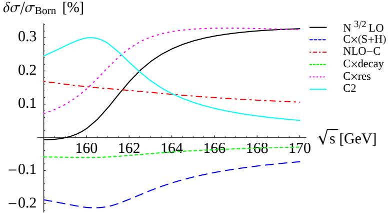
Residue corrections: matching coefficient for the production operator dressed with WFR factors

$$\Delta\sigma_4 = \frac{4\pi\alpha^2_{ew}\alpha}{27s}\,\frac{\Gamma_W}{M_W}\ln\left(\frac{2|\mathcal{E}_W|(\mathrm{Re}\mathcal{E}_W + |\mathcal{E}_W|)}{\Gamma^2_W}\right)$$

Results for the total cross section



	σ(fb)		
$\sqrt{s}$ [GeV]	Born	+ NLO	NNLO only
158	45.64(2)	49.19(2)	0.000 [+0.00%]
161	108.60(4)	117.81(5)	0.087 [+0.06%]
164	219.7(1)	234.9(1)	0.544 [+0.18%]
167	310.2(1)	328.2(1)	0.936 [+0.23%]
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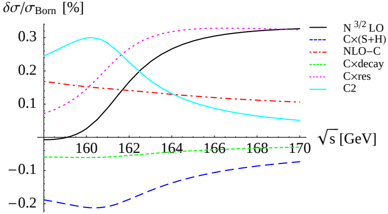

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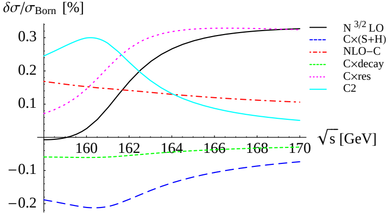

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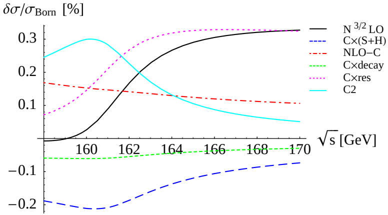

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- Screening effect $\Rightarrow$ combined effect below the permille level
- Conversion of shift on cross section $\to$ shift on W mass $\Rightarrow$ effect of NNLO corrections below 6 MeV accuracy expected at the ILC ($\delta\sigma \sim 1\ \% \Rightarrow \delta M_W \sim 15$ MeV)

Effect of experimental cuts

For practical applications, experimental cuts are needed; complicated in the EFT because they introduce new scales

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Estimate of the effect of cuts using WHIZARD [Kilian-Ohl-Reuter '07] for the SM Born result and the L3 cuts

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–	154.18(5)	
$\lvert \vec{p}_\mu \rvert > 20$ GeV	153.71(5)	99.69(5) %
$M_{\mu\nu} > 55$ GeV, 40 GeV $< M_{jj} < 120$ GeV	150.61(5)	97.68(5) %
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- implemented introducing $\theta(\Lambda^2 \pm p_W^2 \mp M_W^2)$
- $\Lambda \sim M_W >> \delta$ (here) effect irrelevant

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- angular cuts more complicated; Coulomb photons $\Rightarrow$ angular distributions not modified respect to the LO $\Rightarrow$ effect negligible

Ambiguities in higher order ISR

Different implementations of ISR are formally equivalent at the leading logarithmic level [Beneke-Falgari-Schwinn-Signer-Zanderighi '07]

$$\sigma(s) = \int_0^1 dx_1 \int_0^1 dx_2\, \Gamma(x_1)\Gamma(x_2)\, \underbrace{\hat{\sigma}(x_1 x_2 s)}_{\text{HO corrections}}$$

$$\sigma(s) = \int_0^1 dx_1 \int_0^1 dx_2\, \Gamma(x_1)\Gamma(x_2)\hat{\sigma}_{\text{Born}}(x_1 x_2 s) + \hat{\sigma}_{\text{NLO}}(s) + \hat{\sigma}_{\text{NNLO}}(s)$$

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	σ(fb)				
$\sqrt{s}$ [GeV]	Born	NLO (ISR)	NLO	NNLO (ISR)	NNLO
161	108.60	$+8.50\%$	$+10.50\%$	$+0.06\%$	$+0.1\%$
164	219.7	$+6.92\%$	$+7.78\%$	$+0.18\%$	$+0.27\%$
170	378.4	$+5.18\%$	$+5.26\%$	$+0.25\%$	$+0.33\%$

Different treatments lead to a $\sim 2\%$ difference at threshold $\Rightarrow$ residual uncertainty ~ 30 MeV on M_W

Conclusions

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- ▷ Using EFT methods for unstable particles it has been possible to evaluate all parametrically dominant NNLO corrections to four-fermion production at the WW threshold

- ▷ Even if finite-width effects are handled in a different way in the EFT respect to the full result in the CMS, the NNLO subset of corrections can be implemented in both schemes without modifications

- ▷ Each diagram contributes between -0.2% and $+0.3\%$ at threshold, but large cancellations occur, giving a $+0.06\%$ (ISR) [$+0.1\%$ no ISR]. The impact of the result on the W-mass determination is well below the 6 MeV goal at the ILC