Introduction to entanglement of neutrinos in collective oscillations

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Joint N3AS-iTHEMS MEETING ON QIS IN MULTIMESSENGER ASTROPHYSICS

N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
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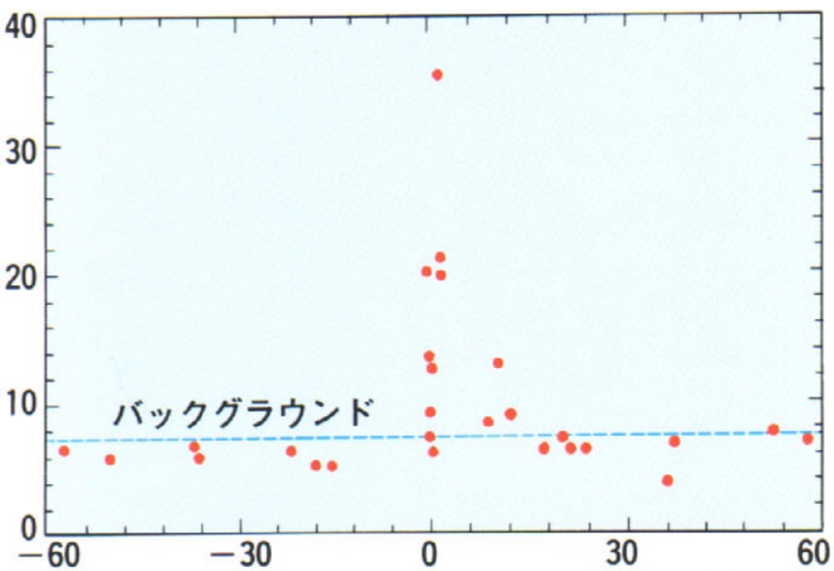
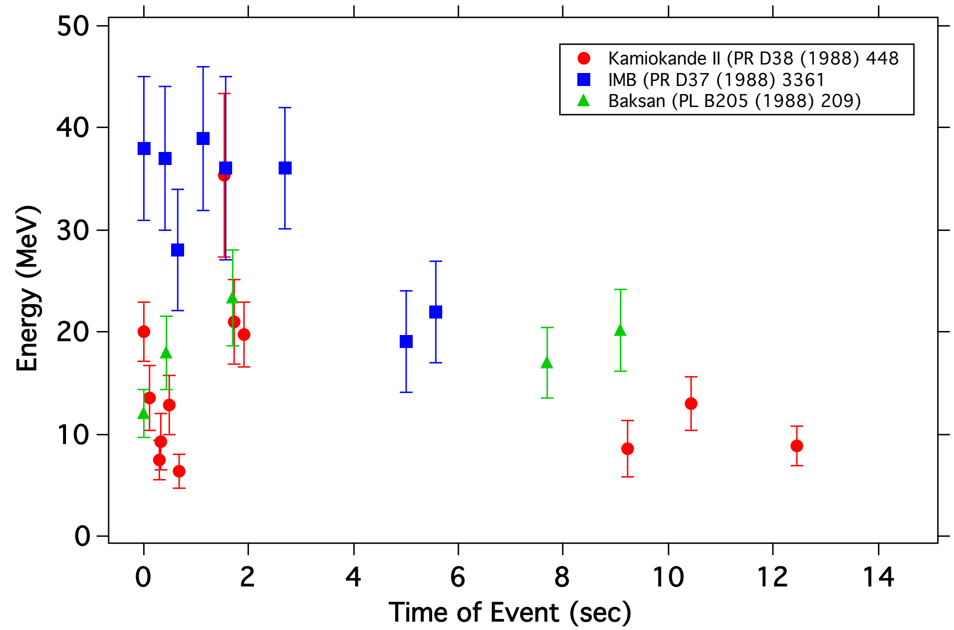
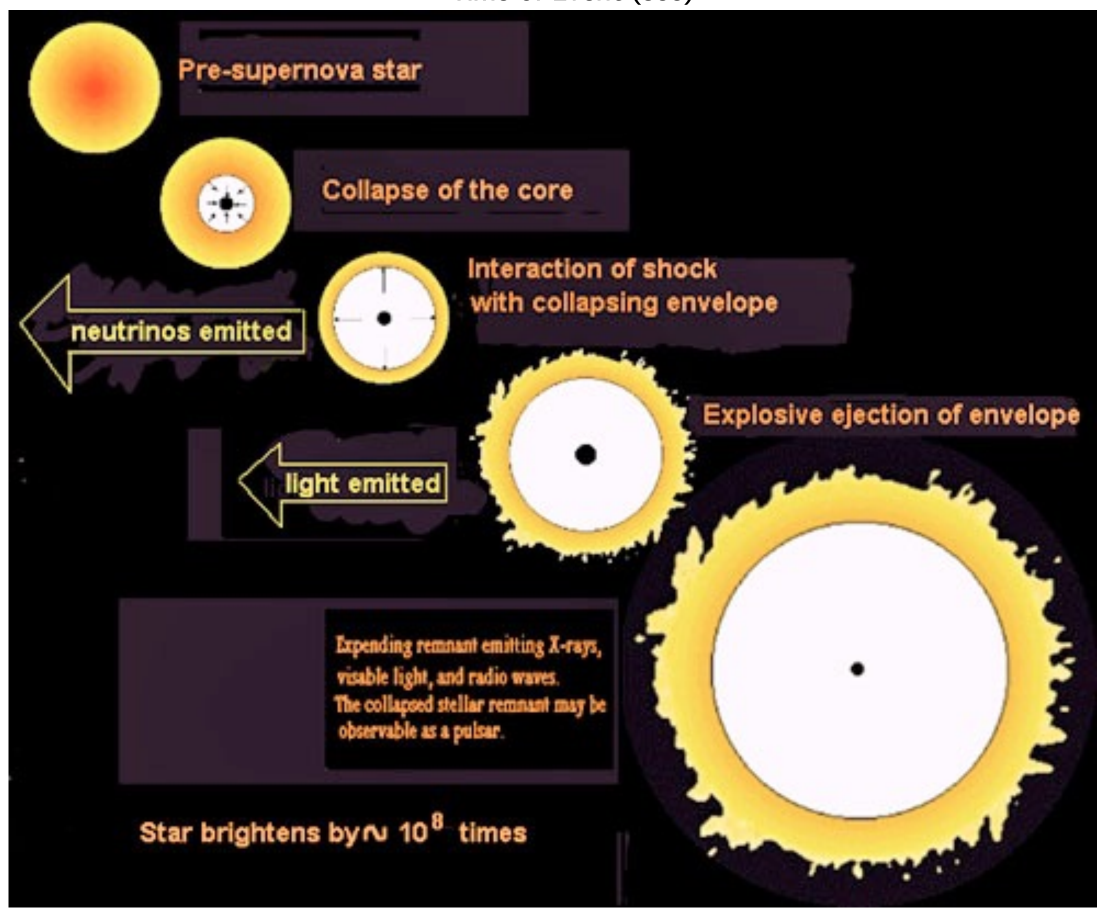
Neutrinos from core-collapse supernovae 1987A



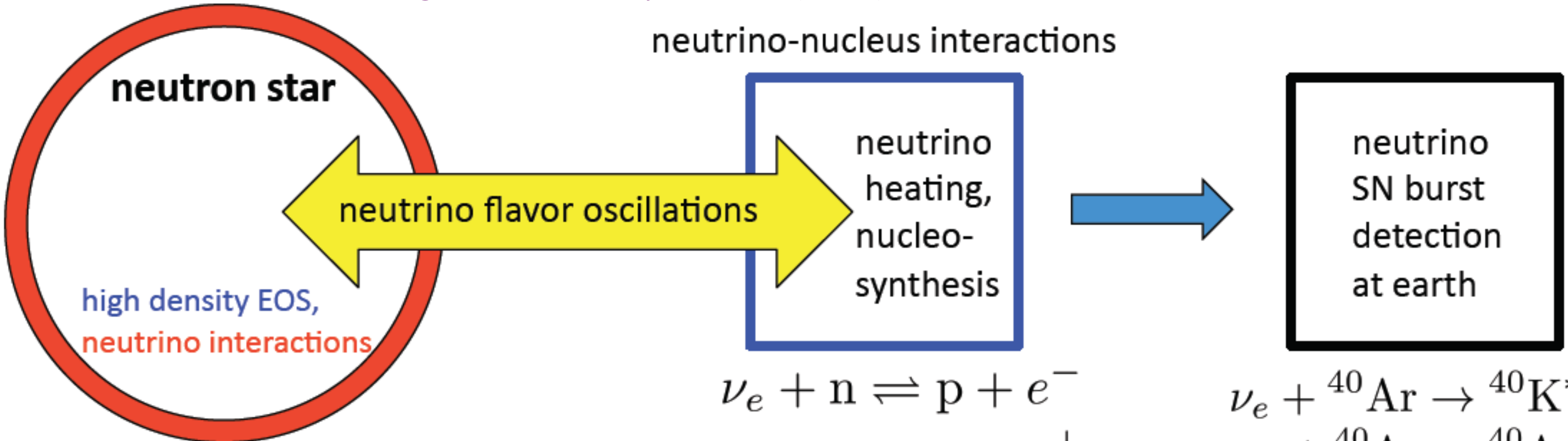




- $M_{prog} \gtrsim 8\ M_{sun} \Rightarrow \Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
- 99% of the energy is carried away by neutrinos and antineutrinos with $10 \lesssim E_\nu \lesssim 30$ MeV $\Rightarrow 10^{58}$ neutrinos

Understanding a core-collapse supernova requires answers to a variety of questions some of which need to be answered, both theoretically and experimentally.

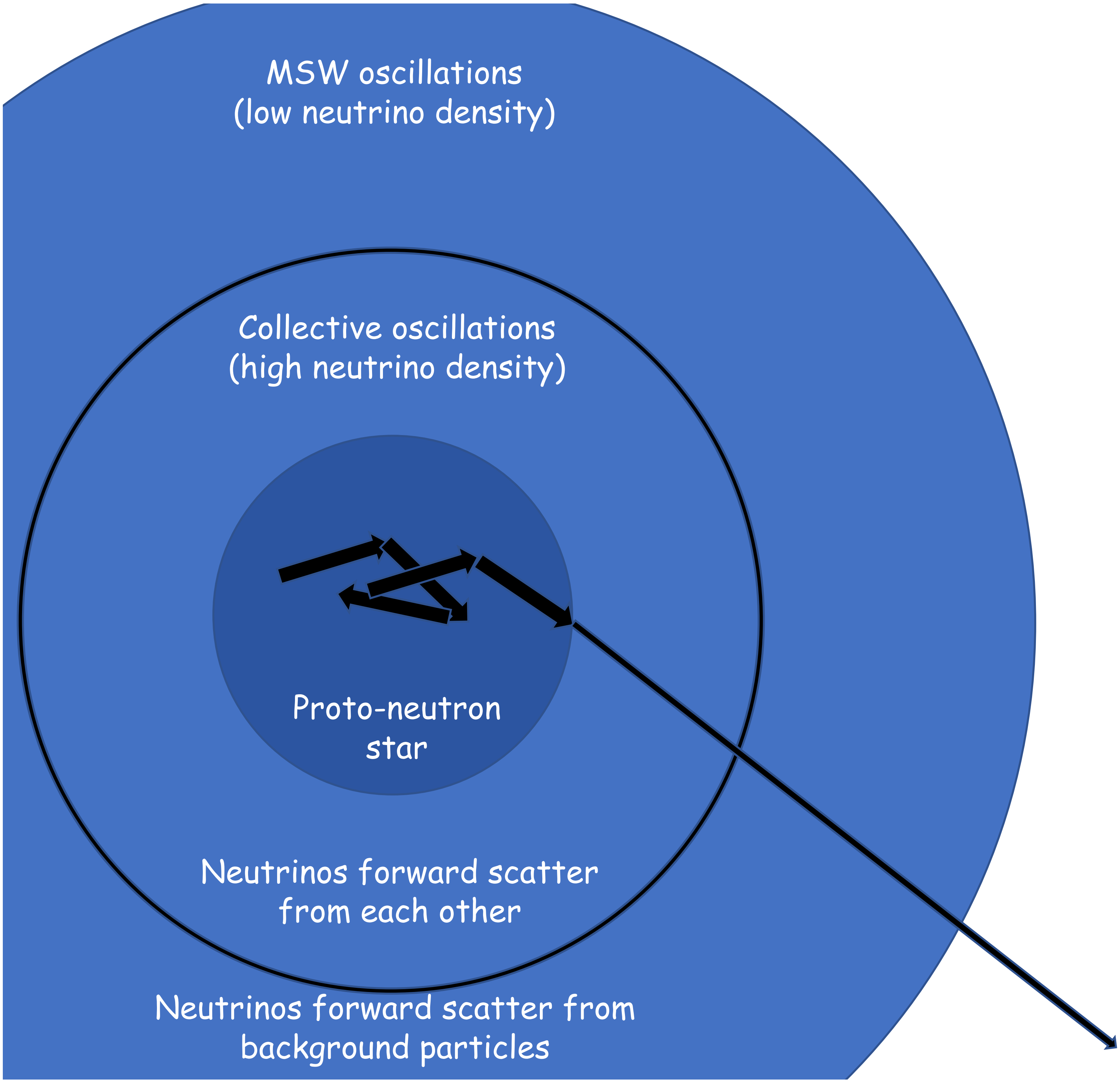
Balantekin and Fuller, Prog. Part. Nucl. Phys. **71** 162 (2013)

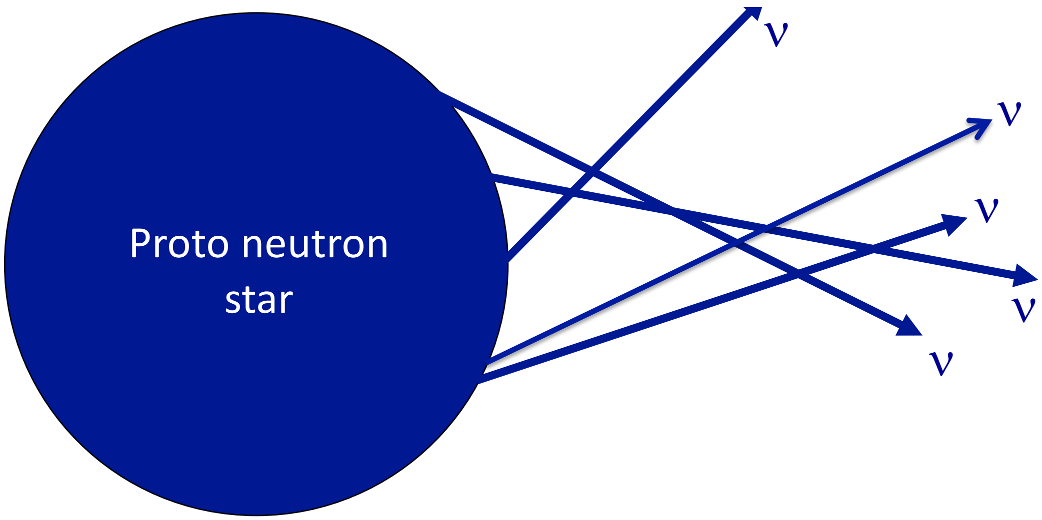


$$\nu_e + \mathrm{n} \rightleftharpoons \mathrm{p} + e^-$$
$$\bar{\nu}_e + \mathrm{p} \rightleftharpoons \mathrm{n} + e^+$$

$$\nu_e + {}^{40}\mathrm{Ar} \rightarrow {}^{40}\mathrm{K}^* + e^-$$
$$\nu_x + {}^{40}\mathrm{Ar} \rightarrow {}^{40}\mathrm{Ar}^* + \nu_x$$

Neutron-to-proton ratio depends on relative intensities of electron neutrinos and electron antineutrinos, which in turn depend on neutrino oscillations


MSW oscillations
(low neutrino density)
Collective oscillations
(high neutrino density)
Proto-neutron
star
Neutrinos forward scatter
from each other
Neutrinos forward scatter from
background particles



Energy released in a core-collapse SN: $\Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
99% of this energy is carried away by neutrinos and antineutrinos!
~ 10^{58} Neutrinos!
This necessitates including the effects of νν interactions ("collective neutrino oscillations")!

$$H = \underbrace{\sum a^\dagger a}_{\text{ν oscillations, MSW effect}} + \underbrace{\sum (1 - \cos\varphi) a^\dagger a^\dagger a a}_{\text{neutrino-neutrino interactions}}$$

ν oscillations
MSW effect

neutrino-neutrino interactions

The second term makes the physics of a neutrino gas in a core-collapse supernova a very interesting many-body problem, driven by weak interactions.

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral "swaps" or "splits").

A system of N particles each of which can occupy k states (k = number of flavors)

Exact Solution → **Mean-field approximation**

Entangled and unentangled states → Only unentangled states

Dimension of Hilbert space: k^N

Dimension of the diagonalizing space: kN

von Neumann entropy $S = -\mathrm{Tr}(\rho \log \rho)$

	Pure State	Mixed State
Density matrix	$\rho^2 = \rho$	$\rho^2 \neq \rho$
Entropy	$S = 0$	$S \neq 0$

Mean Field: Each neutrino moves independently interacting with a mean-field created by other neutrinos. Momenta of these neutrinos do not change.

Many-body: Momentum changing interactions are possible, but they vanish in the mean-filed limit. For simplicity in this talk we will only consider forward scattering.

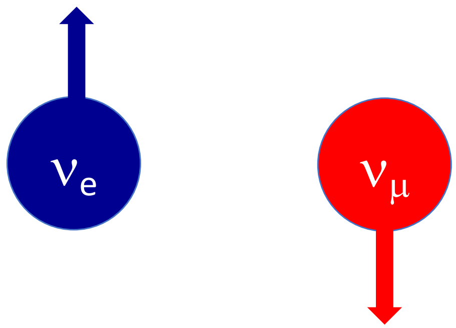
For momentum changing many-body calculations, see the talk by Yukari Yamauchi

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + C(\rho)$$

H = neutrino mixing
+ forward scattering of neutrinos off other background particles (MSW)
+ forward scattering of neutrinos off each other

C = collisions

Neutrino flavor isospin



$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

Free neutrinos (only mixing)

$$\hat{H} = \frac{m_1^2}{2E} a_1^\dagger a_1 + \frac{m_2^2}{2E} a_2^\dagger a_2 + (\cdot\cdot)\hat{1}$$

$$= \frac{\delta m^2}{4E}\cos 2\theta\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)'\,\hat{1}$$

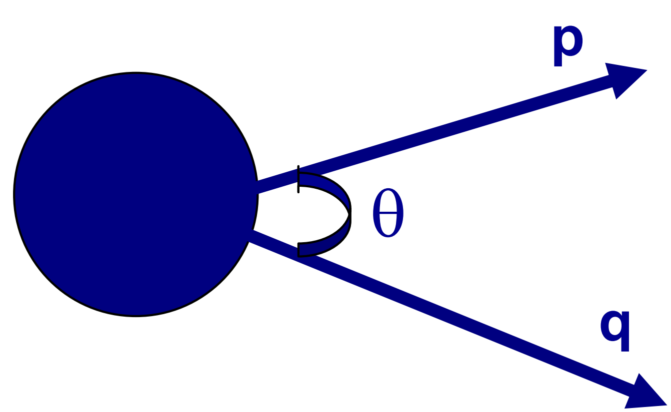
Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)''\,\hat{1}$$

Neutrino-Neutrino Interactions

Smirnov, Fuller, Qian, Pantaleone, Sawyer, McKellar, Friedland, Lunardini, Raffelt, Duan, Balantekin, Volpe, Kajino, Pehlivan …

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\, dq \left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q$$



This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

$$\hat{H} = \int dp \left(\frac{\delta m^2}{2E}\vec{\mathbf{B}}\cdot\vec{\mathbf{J}}_p - \sqrt{2}G_F N_e \mathbf{J}_p^0 \right) + \frac{\sqrt{2}G_F}{V}\int dp\, dq \left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p \cdot \vec{\mathbf{J}}_q$$

$$\vec{\mathbf{B}} = \left(\sin 2\theta,\ 0, -\cos 2\theta\right)$$

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral “swaps” or “splits”).

This problem is "exactly solvable" in the single-angle approximation

$$H = \sum_p \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}}) \, \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

$$H = \sum_p \omega_p \vec{B} \cdot \vec{J}_p + \mu(r) \vec{J} \cdot \vec{J}$$

Note that this Hamiltonian commutes with $\vec{B} \cdot \sum_p J_p$.

Hence $\mathrm{Tr}\left(\rho \vec{B} \cdot \sum_p J_p\right)$ is a constant of motion.

In the mass basis this is equal to $\mathrm{Tr}(\rho J_3)$.

Two of the adiabatic eigenstates of this equation are easy to find in the single-angle approximation:

$$H = \sum_p \omega_p \vec{B} \cdot \vec{J}_p + \mu(r) \vec{J} \cdot \vec{J}$$

$$|j, +j\rangle = |N/2, N/2\rangle = |\nu_1, \ldots, \nu_1\rangle$$

$$|j, -j\rangle = |N/2, -N/2\rangle = |\nu_2, \ldots, \nu_2\rangle$$

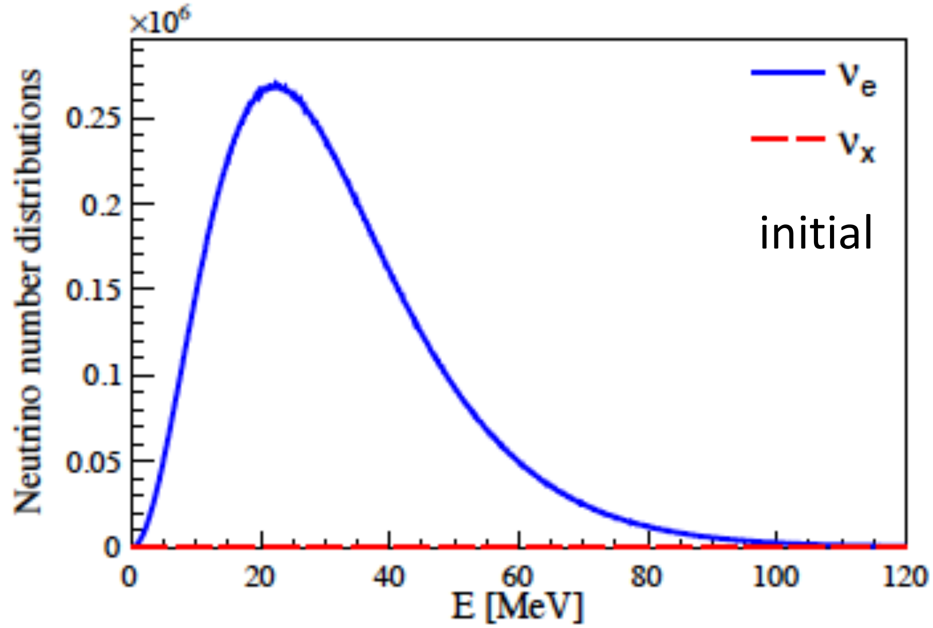
$$E_{\pm N/2} = \mp \sum_p \omega_p \frac{N_p}{2} + \mu \frac{N}{2} \left(\frac{N}{2} + 1 \right)$$

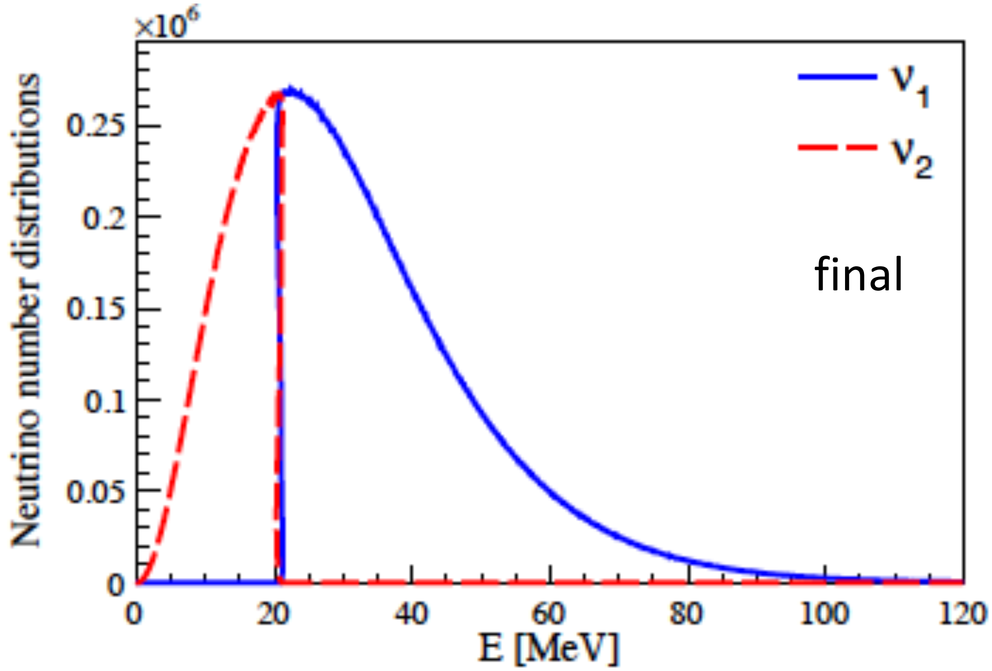
To find the others will take a lot more work

Away from the mean-field: Adiabatic solution of the *exact* many-body Hamiltonian for extremal states

Adiabatic evolution of an initial thermal distribution (T = 10 MeV) of electron neutrinos. 10^8 neutrinos distributed over 1200 energy bins with solar neutrino parameters and normal hierarchy.

Birol, Pehlivan, Balantekin, Kajino
arXiv:1805.11767
PRD98 (2018) 083002





BETHE ANSATZ

Single-angle approximation Hamiltonian:

$$H = \sum_p \frac{\delta m^2}{2p} J_p^0 + 2\mu \sum_{\substack{p,\,q \\ p \neq q}} \mathbf{J}_p \bullet \mathbf{J}_q$$

Eigenstates:

$$|x_i\rangle = \prod_{i=1}^{N} \sum_k \frac{J_k^\dagger}{\left(\delta m^2 / 2k\right) - x_i} |0\rangle$$

$$-\frac{1}{2\mu} - \sum_k \frac{j_k}{\left(\delta m^2 / 2k\right) - x_i} = \sum_{j \neq i} \frac{1}{x_i - x_j}$$

Bethe ansatz equations

$$\mu = \frac{G_F}{\sqrt{2}V} \langle 1 - \cos\Theta \rangle$$

Invariants:

$$h_p = J_p^0 + 2\mu \sum_{\substack{p,\,q \\ p \neq q}} \frac{\mathbf{J}_p \bullet \mathbf{J}_q}{\delta m^2 \left(\frac{1}{p} - \frac{1}{q} \right)}$$

Pehlivan, ABB, Kajino, & Yoshida
Phys. Rev. D 84, 065008 (2011)

- Bethe ansatz method has numerical instabilities for larger values of N. However, it is very valuable since it leads to the identification of conserved quantities.

  Patwardhan *et. al.*, PRD **99**, 123013 (2019); Cervia *et al.*, PRD **100**, 083001 (2019)

- Runge Kutta method (RK4)

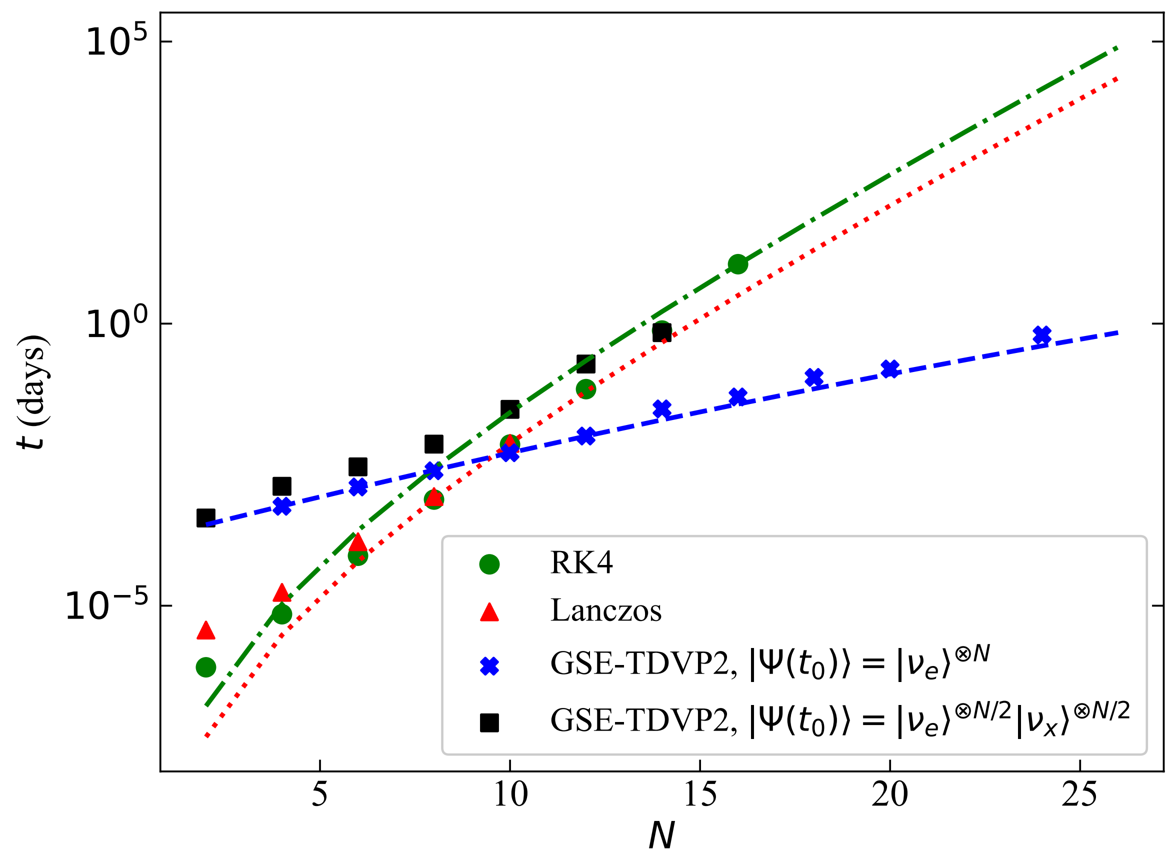
  Patwardhan *et. al.*, PRD **104**, 123035 (2021), Siwach *et. al.* PRD **107**, 023019 (2023)

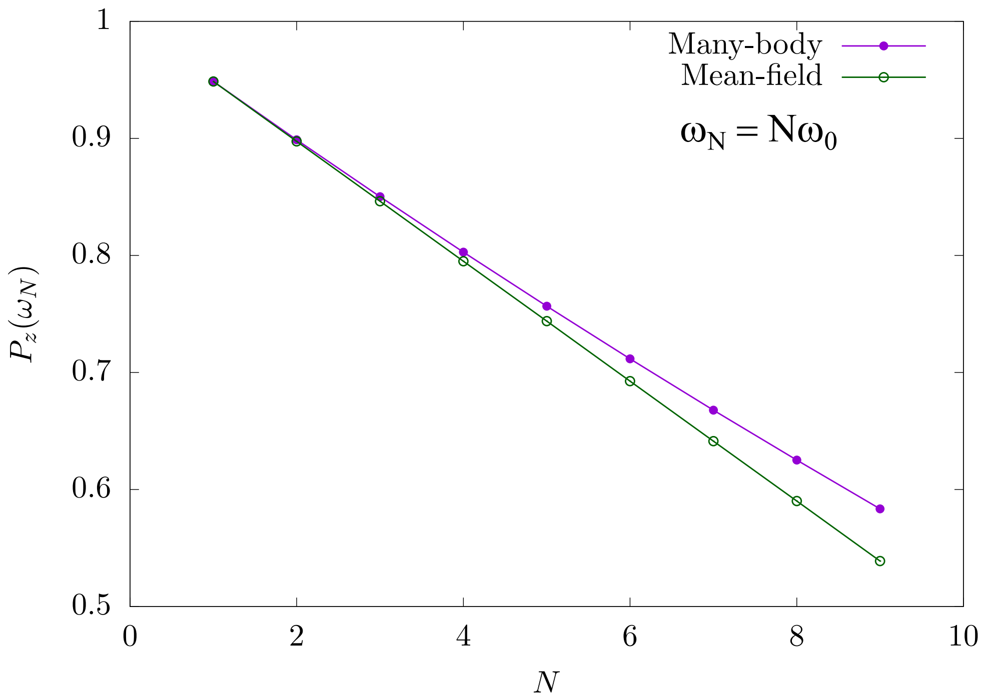
- Tensor network techniques

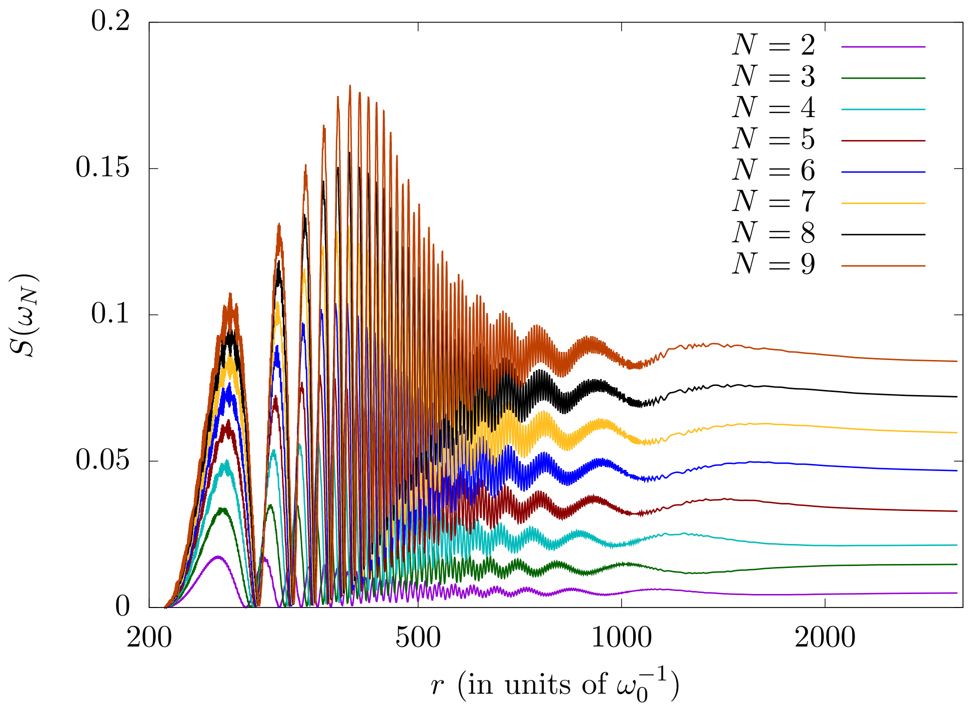
  Cervia *et al.*, PRD **105**, 123025 (2022)

- Noisy quantum computers

  Siwach *et. al.*, PRD **108**, 083039 (2023)





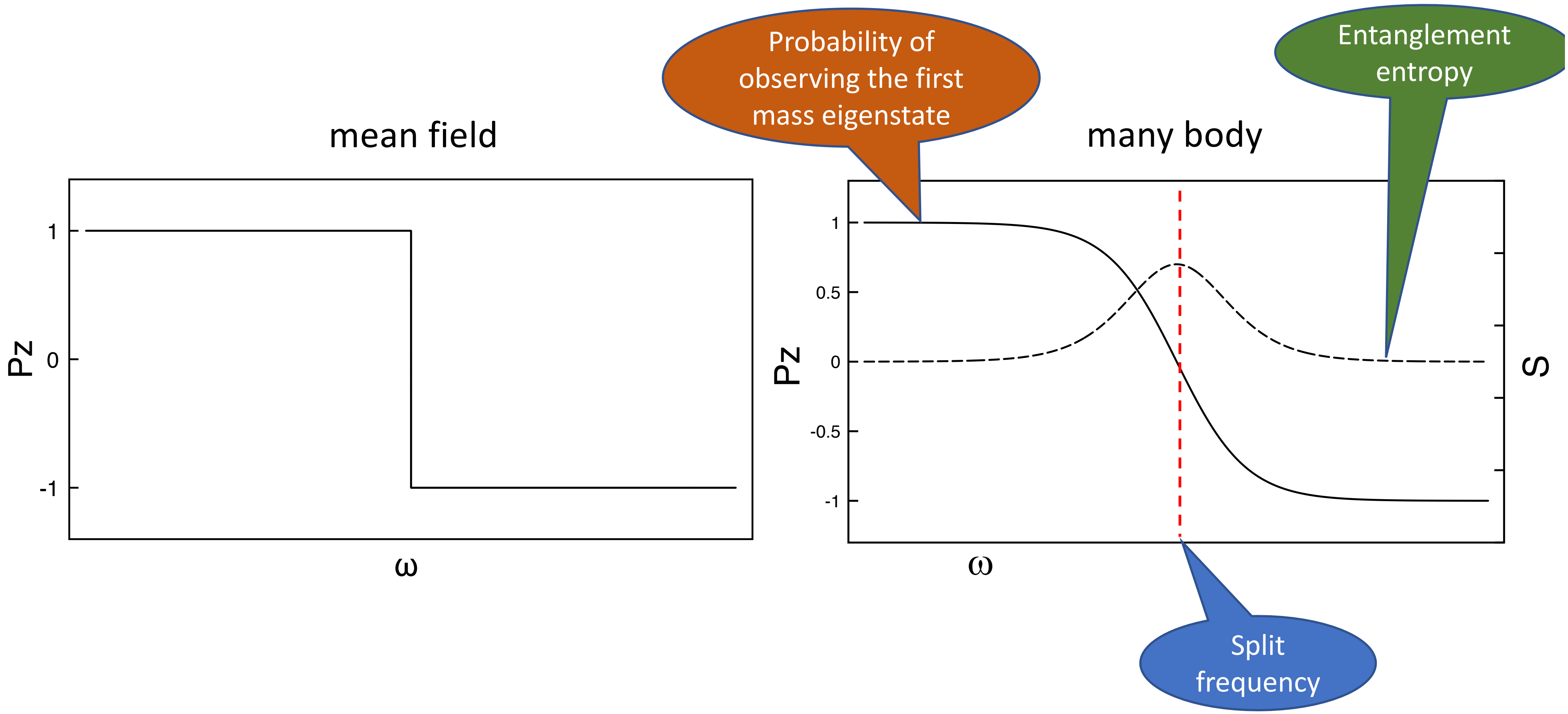


Initial state:
all electron neutrinos

Note: S = 0 for mean-field approximation

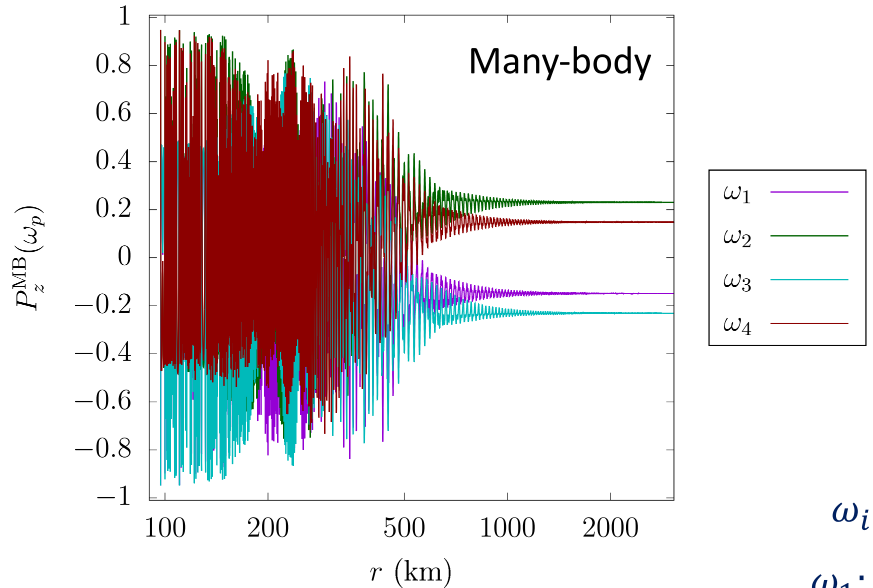
Cervia, Patwardhan, Balantekin, Coppersmith, Johnson,
arXiv:1908.03511
PRD, **100**, 083001 (2019)

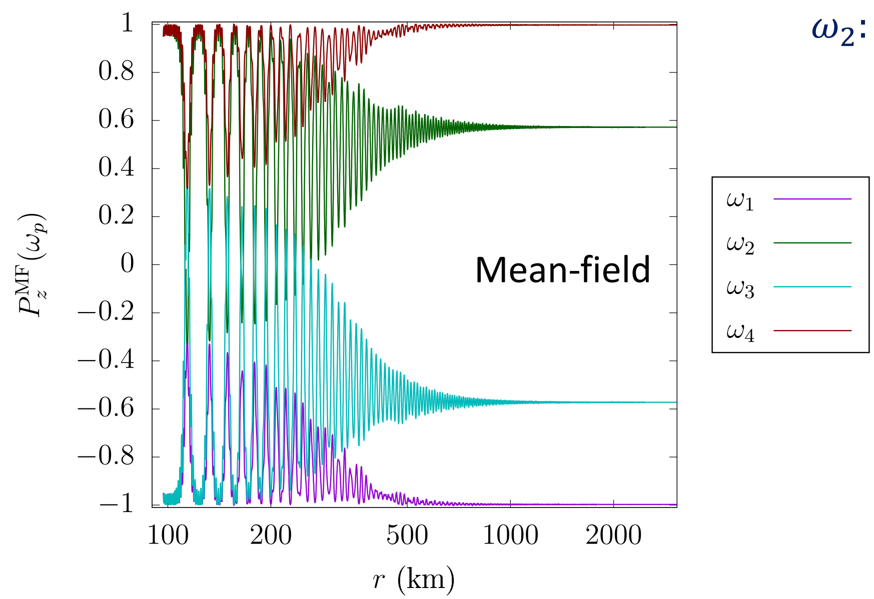
We find that the presence of spectral splits is a good proxy for deviations from the mean-field results



For the behavior with three flavors (qubits $\rightarrow$ qutrits) see the talk by Anna Suliga

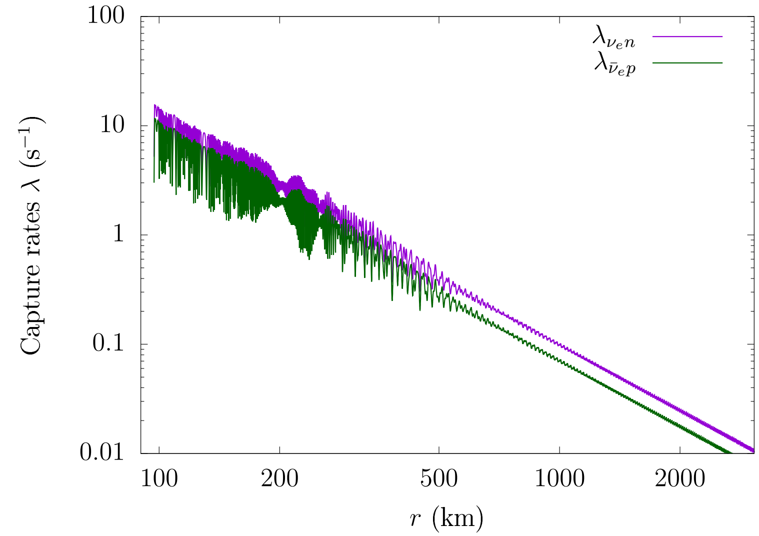
The impact of two different treatments of collective neutrino oscillations (with and without entanglement)

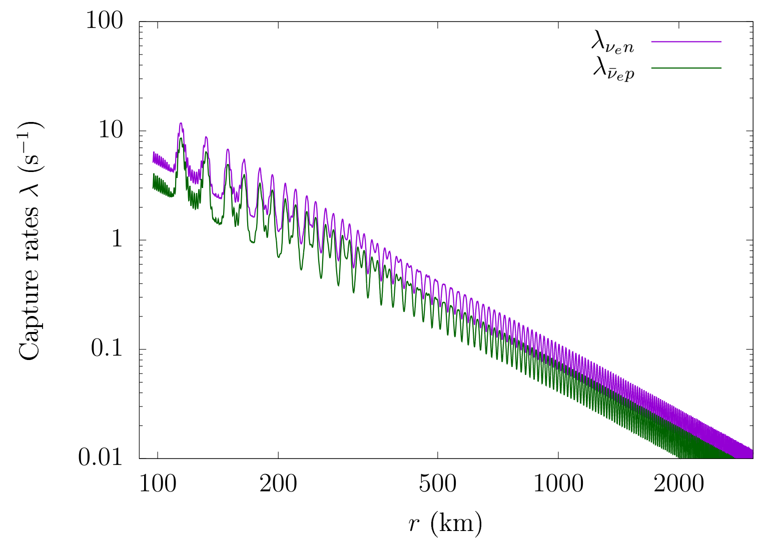




$$\omega_i = \frac{\delta m^2}{2E_i}$$

ω_1: $\bar{\nu}_e$,
ω_2: $\bar{\nu}_x$, ω_3 : ν_x, ω_4: ν_e





Considerations of collective effects unveiled a new kind of nucleosynthesis: "The νi process".

Balantekin, Cervia, Patwardhan, Surman, Wang; 2311.02562 [astro-ph.HE], Astrophys. J. **967**, 2 146 (2024)


どうもありがとうございました