# Quantum Computation & Quantum Field Theory

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(本多正純)

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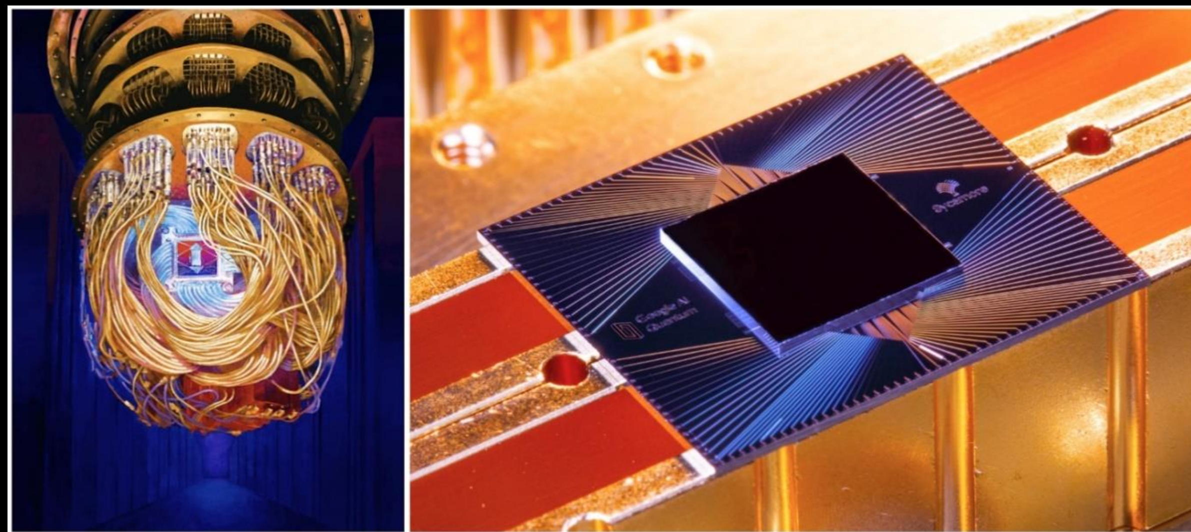
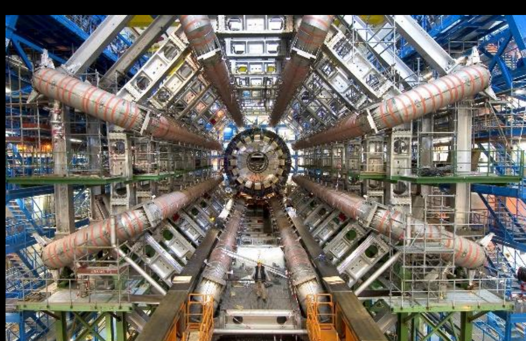
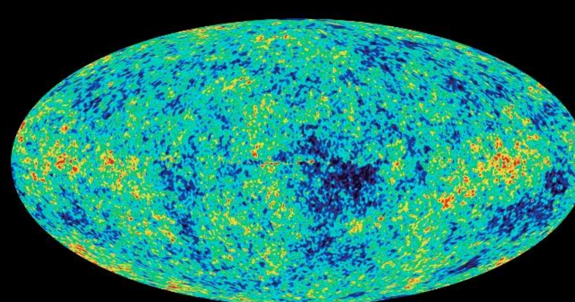
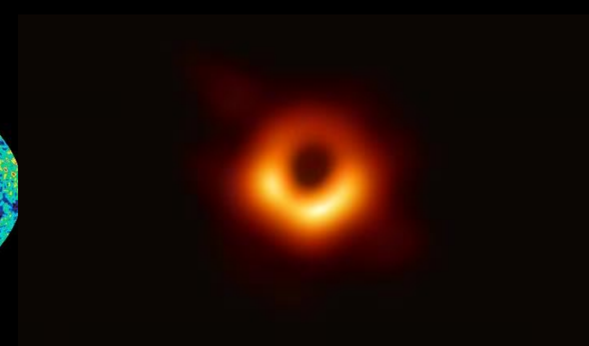
JOINT N3AS - iTHEMS MEETING ON QUANTUM INFORMATION SCIENCE IN MULTI-MESSENGER ASTROPHYSICS

# Long term goal









etc...

# What can we do by quantum computer?: (details later)

Numerical simulation in situations where conventional approach has a problem

*Monte Carlo method* *sign problem*

## Physical situations w/ sign problems (typically)

- systems w/ topological terms (ex. Chern-Simons, theta)
- fermionic systems w/ chemical potentials
  - —— that's why QCD phase diagram is still incomplete
- real time

# Challenges

- needs real device w/ certain specification

  (qubit #, fidelity, etc...)

- pioneers problems efficiently solved by QC

To do : (except waiting for development of hardware)

- bench mark, estimation of required computational resource
- inventing/improving methods → reduces required resource
  - —— developing lattice field theory in operator formalism
  - —— especially finding nice regularization of gauge theory
- Quantum field theory → Quantum computation?
  - —— proposing new error correcting codes etc...

# Contents

# Sign problem in Monte Carlo simulation

## Conventional approach to simulate QFT:

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi \ \mathcal{O}(\phi) e^{-S[\phi]} \quad \Rightarrow \quad \int d\phi \ \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

②

# Sign problem in Monte Carlo simulation

## Conventional approach to simulate QFT:

① Discretize Euclidean spacetime by lattice:

& make path integral finite dimensional:

$$\int D\phi \; \mathcal{O}(\phi) e^{-S[\phi]} \quad \Longrightarrow \quad \int d\phi \; \mathcal{O}(\phi) \underbrace{e^{-S(\phi)}}_{\text{probability}}$$

② Numerically Evaluate it by (Markov Chain) Monte Carlo method regarding the Boltzmann factor as a probability:

$$\langle \mathcal{O}(\phi) \rangle \simeq \frac{1}{\sharp(\text{samples})} \sum_{i \in \text{samples}} \mathcal{O}(\phi_i)$$

# Sign problem in Monte Carlo simulation (Cont'd)

Markov Chain Monte Carlo:

$$\int d\phi \ \mathcal{O}(\phi) e^{-S(\phi)}$$

probability

problematic when Boltzmann factor isn't $\mathbf{R}_{\geqq 0}$ & is highly oscillating

Examples w/ sign problem:

- topological term —— complex action
- chemical potential —— indefinite sign of fermion determinant
- real time —— " $e^{iS(\phi)}$ " much worse

In operator formalism,

sign problem is absent from the beginning

($\exists$ various approaches within framework of path integral formalism but I'll skip it )

# Cost of operator formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

*regularization needed!*

Technically, computers have to

memorize huge vector & multiply huge matrices

Quantum computers do this job?

# "Regularization" of Hilbert space

Hilbert space of QFT is typically $\infty$ dimensional

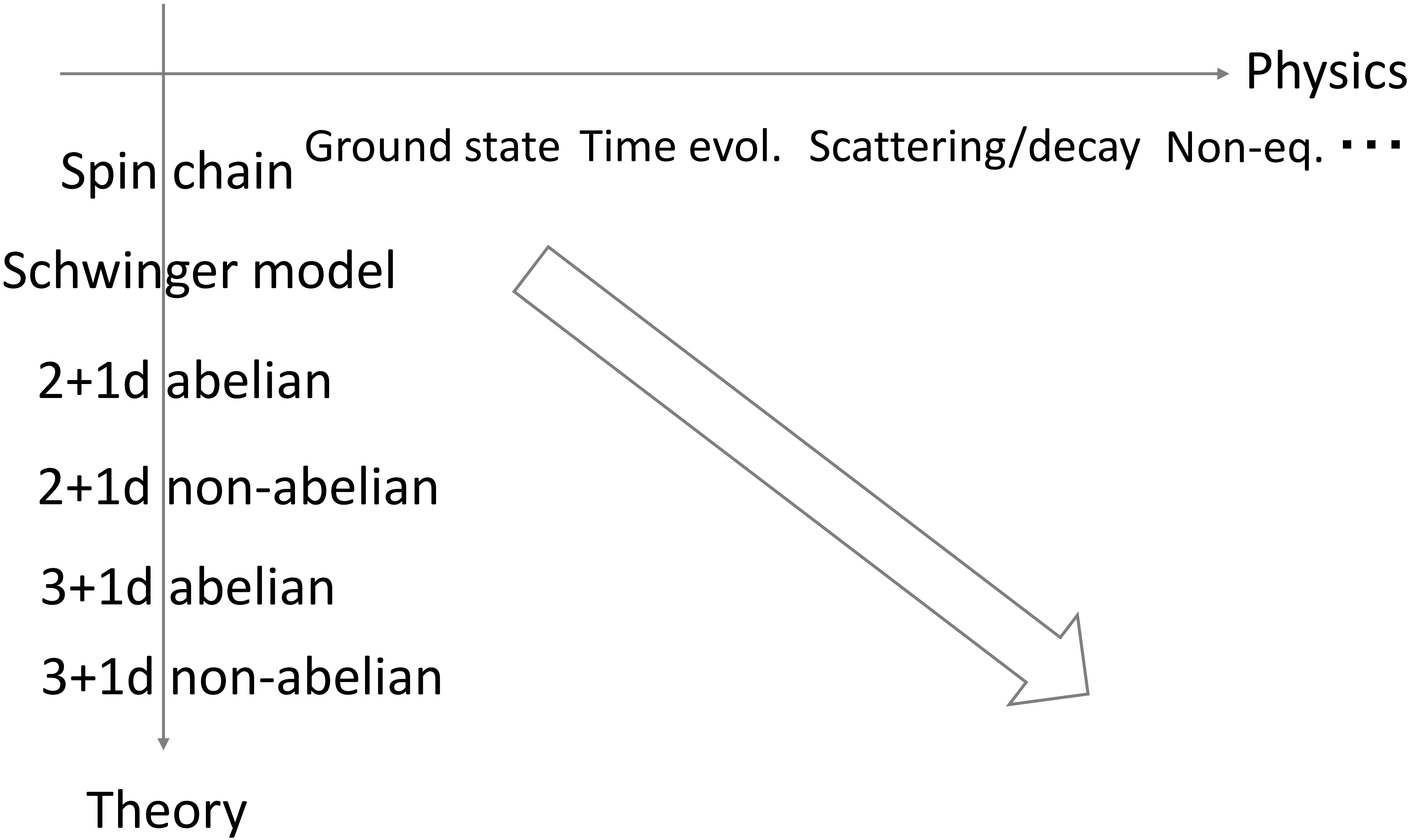
$\longrightarrow$ Make it finite dimensional!

- Fermion is easiest (up to doubling problem)
  — Putting on spatial lattice, Hilbert sp. is finite dimensional
- scalar
  — Hilbert sp. at each site is $\infty$ dimensional
  (need truncation or additional regularization)
- gauge field (w/ kinetic term)
  — no physical d.o.f. in 0+1D/1+1D (w/ open bdy. condition)
  — $\infty$ dimensional Hilbert sp. in higher dimensions

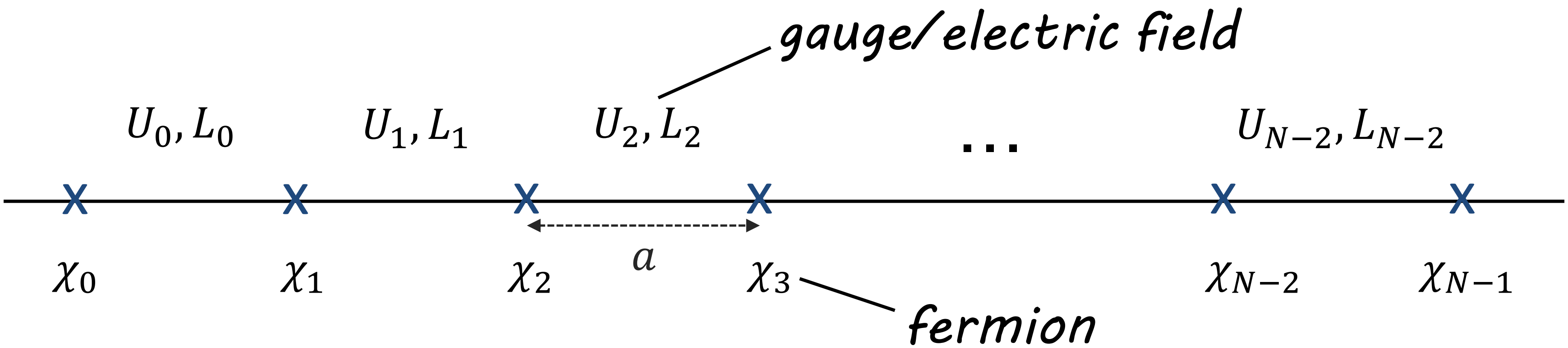
# Citation history of “Hamiltonian Formulation of Wilson's Lattice Gauge Theories” by Kogut-Susskind

(totally 2330 at this moment)

# Flow of researches in the field (?)



# Charge-$q$ Schwinger model




$$H = J \sum_{n=0}^{N-2} \left( L_n + \frac{\theta_0}{2\pi} \right)^2 - \mathrm{i}w \sum_{n=0}^{N-2} \left[ \chi_n^\dagger (U_n)^q \chi_{n+1} - \mathrm{h.c.} \right] + m \sum_{n=0}^{N-1} (-1)^n \chi_n^\dagger \chi_n$$

$$[L_n, U_m] = U_m \delta_{nm}, \quad \{\chi_n, \chi_m^\dagger\} = \delta_{nm}$$

Physical states are subject to Gauss law:

$$(L_n - L_{n-1})|\mathrm{phys}\rangle = q \left[ \chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2} \right] |\mathrm{phys}\rangle$$

"$\nabla \cdot \vec{E}\,(x)$" "$\rho(x)$ "

# Schwinger model as qubits

1. Take open b.c. & solve Gauss law:

$$L_n = L_{-1} + q \sum_{j=1}^{n} \left( \chi_j^\dagger \chi_j - \frac{1-(-1)^j}{2} \right) \qquad \text{w/ } L_{-1} = 0$$

2. Take the gauge $U_n = 1$

3. Map to spin system: $\chi_n = \frac{X_n - \mathrm{i}Y_n}{2} \left( \prod_{i=1}^{n-1} -\mathrm{i}Z_i \right)$ ($X_n, Y_n, Z_n$: $\sigma_{1,2,3}$ at site $n$)

*"Jordan-Wigner transformation"*

[Jordan-Wigner'28]

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[Jordan-Wigner'28]

⇓

$$H = J \sum_{n=0}^{N-2} \left[ q \sum_{i=0}^{n} \frac{Z_i + (-1)^i}{2} + \frac{\vartheta_n}{2\pi} \right]^2 + \frac{w}{2} \sum_{n=0}^{N-2} \left[ X_n X_{n+1} + Y_n Y_{n+1} \right] + \frac{m}{2} \sum_{n=0}^{N-1} (-1)^n Z_n$$

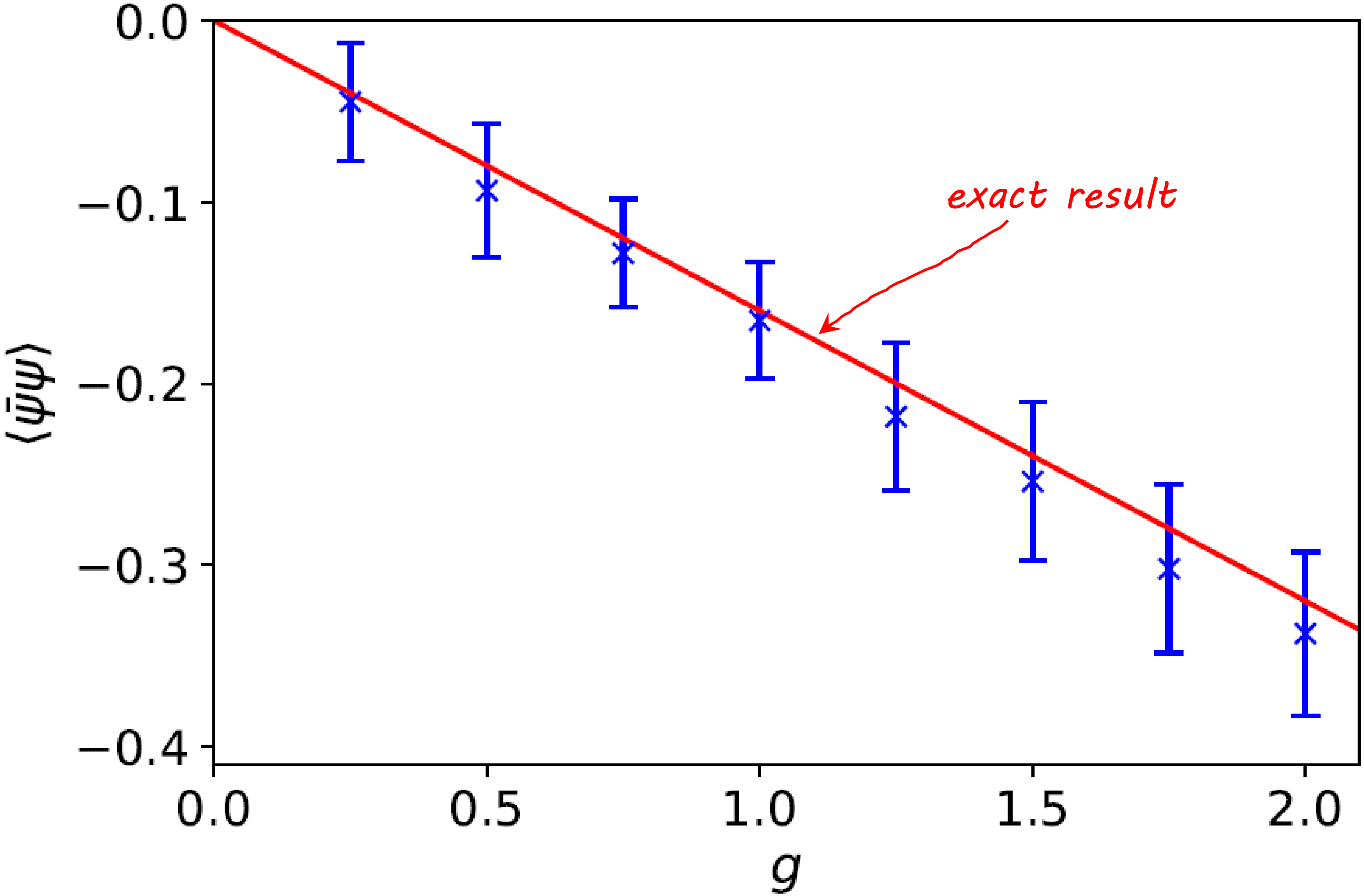
*Qubit description of the Schwinger model !!*

# Ground state expectation value in massless case

$T = 100, \delta t = 0.1, N_{\mathrm{max}} = 16, 1M$ shots

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

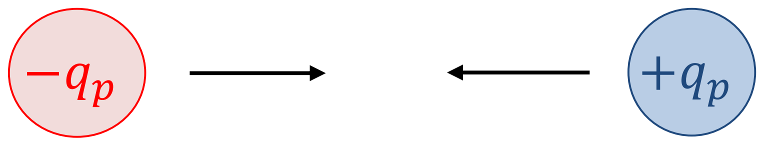
(after continuum limit)



# Screening versus Confinement

Let's consider

potential between 2 heavy charged particles




Classical picture:

$$V(x) = \frac{q_p^2\, g^2}{2} x \;?$$

*Coulomb law in 1+1d*
*||*
*confinement*

too naive in the presence of dynamical fermions

# Expectations from previous analyzes

Potential between probe charges $\pm q_p$ has been analytically computed

[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95 ]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}(1 - e^{-q\mu x})$$ *screening*

- massive case:

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[Iso-Murayama '88, Gross-Klebanov-Matytsin-Smilga '95 ]

- massless case:

$\mu \equiv g/\sqrt{\pi}$

$$V(x) = \frac{q_p^2\, g^2}{2\mu}\left(1 - e^{-q\mu x}\right) \quad \textit{screening}$$

- massive case: [cf. Misumi-Tanizaki-Unsal '19 ]

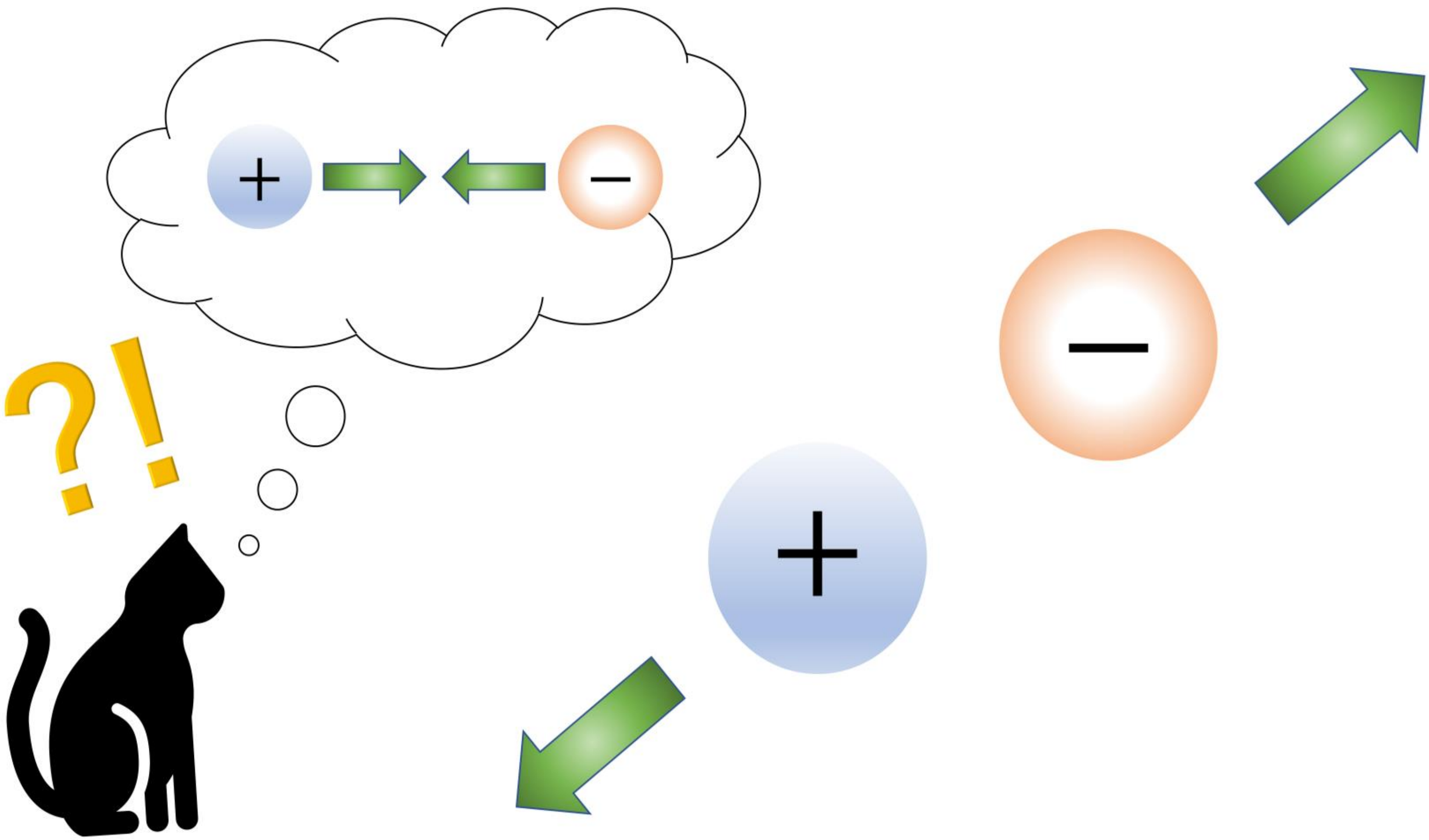
$\Sigma \equiv g e^{\gamma}/2\pi^{3/2}$

$$V(x) \sim mq\Sigma\left(\cos\left(\frac{\theta + 2\pi q_p}{q}\right) - \cos\left(\frac{\theta}{q}\right)\right) x \qquad (\mathrm{m} \ll g,\ |x| \gg 1/g\ )$$

$$\begin{cases} = \mathrm{Const.} & \text{for } \mathrm{q_p/q} = \mathbf{Z} \quad \textit{screening} \\ \propto x & \text{for } \mathrm{q_p/q} \neq \boldsymbol{Z} \quad \textit{confinement?} \end{cases}$$

*but sometimes **negative** slope!*

# That is, as changing the parameters…



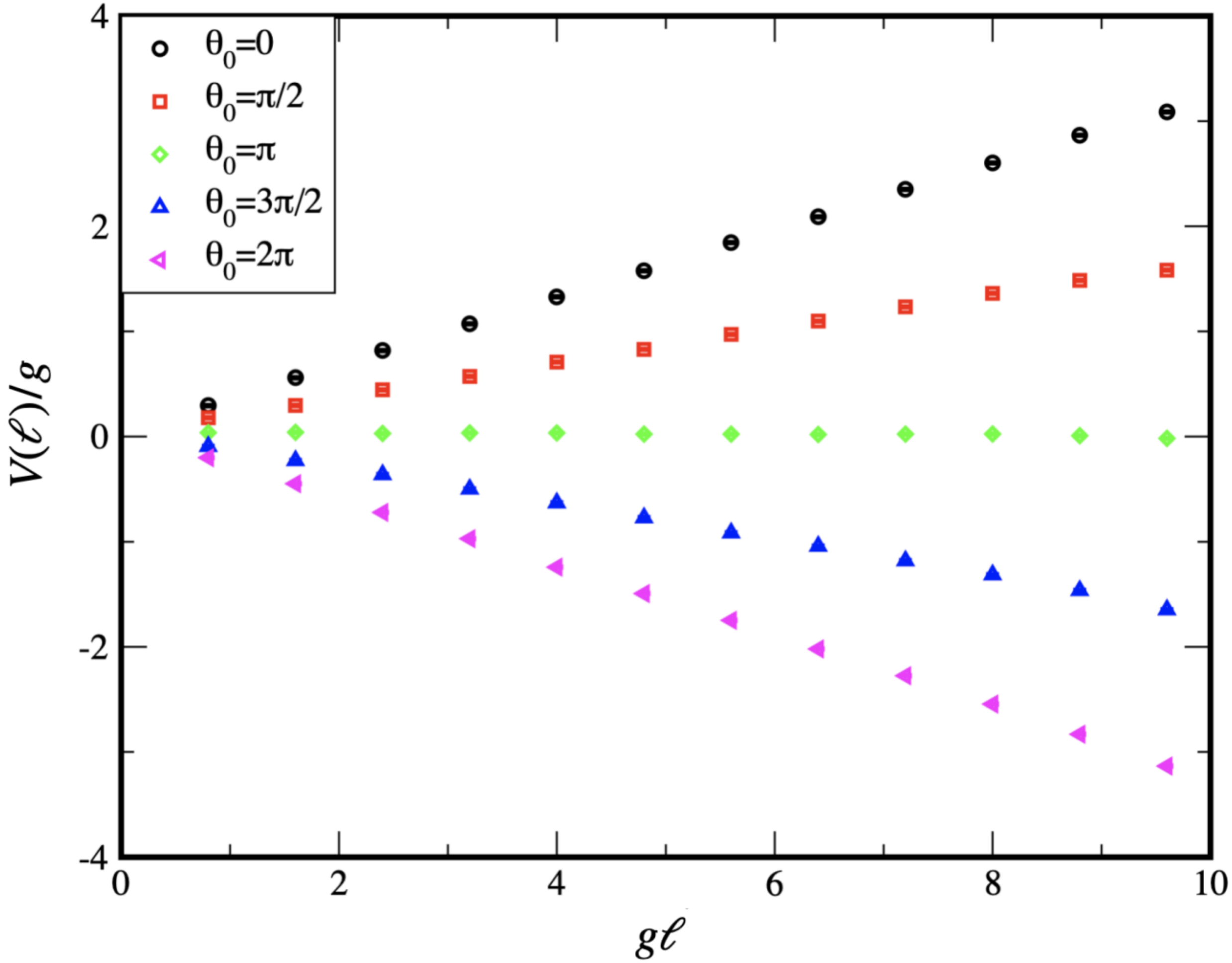

Let’s explore this aspect by quantum simulation!

# Positive / negative string tension

[MH-Itou-Kikuchi-Tanizaki ’21]

[cf. MH-Itou-Kikuchi-Nagano-Okuda ’21]

Parameters: $g = 1, a = 0.4, N = 25, T = 99, q_p/q = -1/3, m = 0.15$

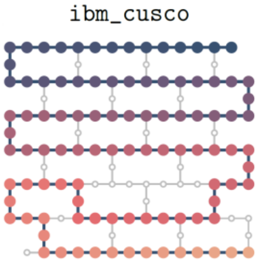



Sign(tension) changes as changing $\theta$-angle!!

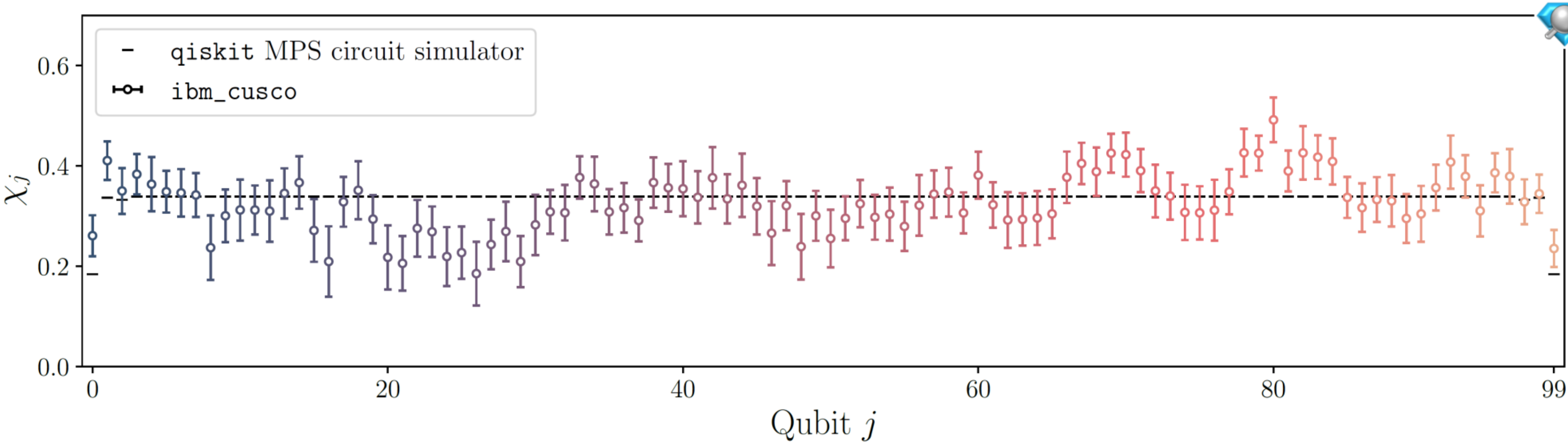
# 100 qubit simulation of Schwinger model

(127-qubit device: **ibm_cusco** w/ error mitigation)

[Farrel-Illa-Ciavarella-Savage '23]




## Ground state exp. of local chiral condensate :



# Other simulations of Schwinger model

- decay of massive vacuum under time evolution

  [cf. Martinez etal. **Nature** 534 (2016) 516-519]

- quenched dynamics of $\theta$ [Nagano-Bapat-Bauer '23]

- Schwinger model in open quantum system

  [De Jong-Metcalf-Mulligan-Ploskon-Ringer-Yao '20, de Jong-Lee-Mulligan-Ploskon-Ringer-Yao '21, Lee-Mulligan-Ringer-Yao '23]

- 112 qubit simulation of meson propagation

  [Farrell-Illa-Ciavarella-Savage '24]

- finding energy spectrum [MH-Ghim, work in progress]

- finite temperature [Itou-Sun-Pedersen-Yunoki '23]

etc...

# Scattering in Thirring model

## Thirring model on lattice:

[Chai-Crippa-Jansen-Kuhn-Pascuzzi-Tacchino-Tavernelli '23]

$$H = \sum_{n=0}^{N-1} \left( \frac{i}{2a} \left( \xi_{n+1}^\dagger \xi_n - \xi_n^\dagger \xi_{n+1} \right) + (-1)^n m \, \xi_n^\dagger \xi_n \right) + \sum_{n=0}^{N-1} \frac{g(\lambda)}{a} \xi_n^\dagger \xi_n \xi_{n+1}^\dagger \xi_{n+1},$$

# Scattering in Thirring model

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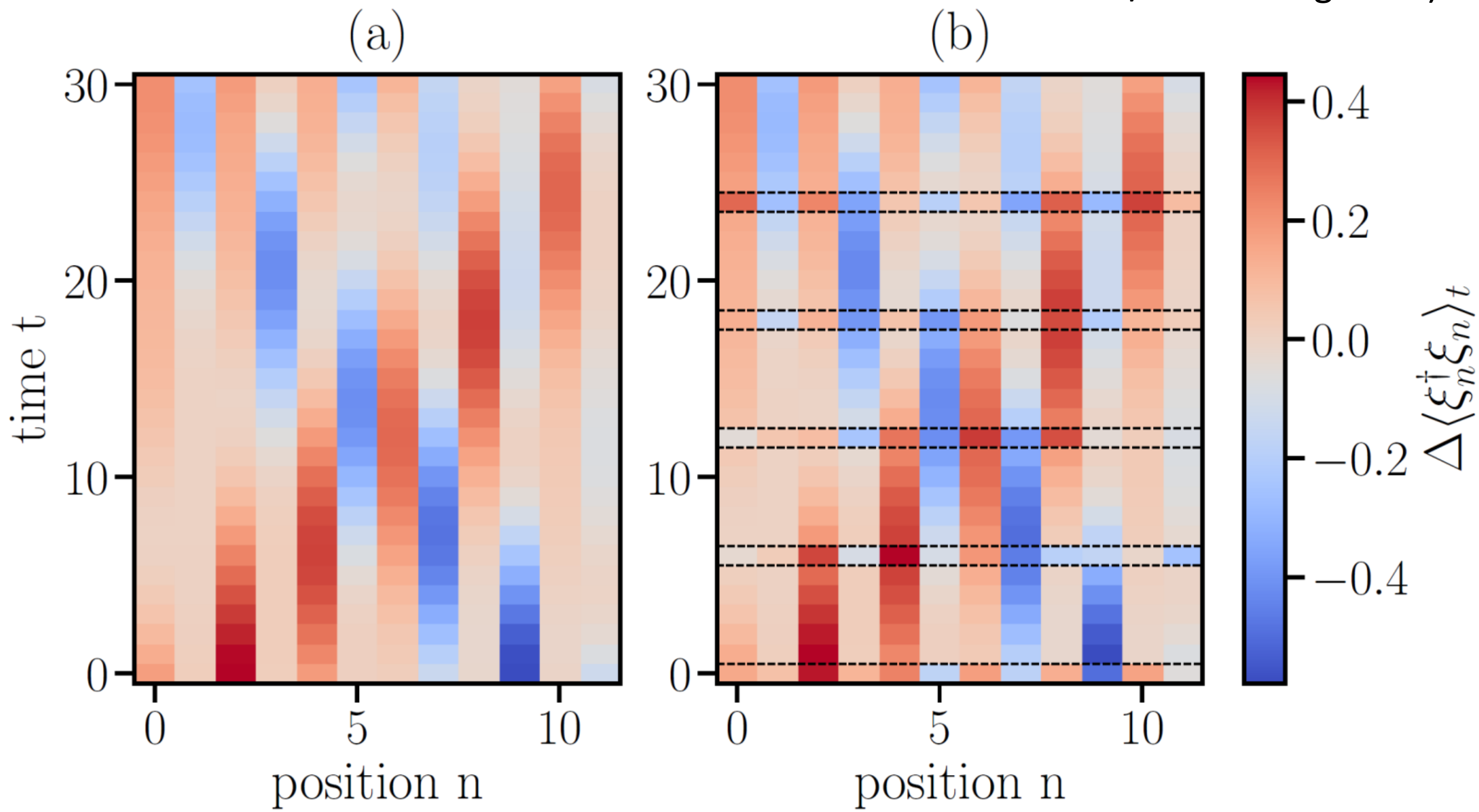
## Particle density of two wave packets:

(12-qubit device: **ibm_peekskill** w/ error mitigation )



# On higher dimensional fermion

Go to higher dimensions!

[MH, work in progress]

⇩

1st step: find a nice way to map 2d fermion to spins

Problem in naïve approach:

$$\chi_n = \frac{X_n - \mathrm{i}Y_n}{2}\left(\prod_{i=1}^{n-1} -\mathrm{i}Z_i\right)$$

- 1d

$$\chi_{n+1}^{\dagger}\chi_n \xrightarrow{\text{Jordan-Wigner}} {}^{\exists}X_{n+1}X_n, Y_{n+1}Y_n, X_{n+1}Y_n, Y_{n+1}X_n$$

*local*

- 2d

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*local*

- 2d ($N \times N$ square lattice)

Relabeling site $(i,j)$ like 1d label (say $n = i + Nj$),

$$\chi_{(i,j+1)}^{\dagger}\chi_{(i,j)} = \chi_{I+N}^{\dagger}\chi_I \xrightarrow{\text{JW}} {}^{\exists}X_{I+N}X_I \prod_{i=I+1}^{I+N-1} Z_i \text{ , etc...}$$

(cf. $\mathcal{O}(\log N)$ for Bravyi-Kitaev trans.) *non-local*

# Application of a new map to field theory

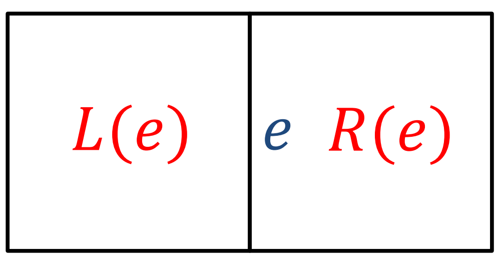
[Chen-Kapustin-Radicevic '17]

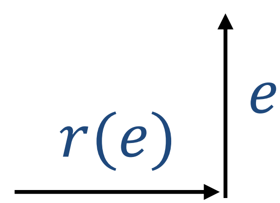
2 Majorana **fermions** on **face** ⟺ **Spin** op. on **edge**

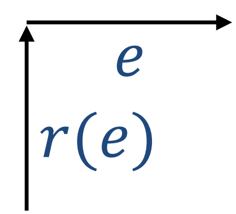
$$(-1)^{F_f} = -i\gamma_f\gamma'_f \longleftrightarrow W_f. \qquad S_e = i\gamma_{L(e)}\gamma'_{R(e)} \longleftrightarrow U_e$$

where $$W_f = \prod_{e\subset f} Z_e. \qquad U_e = X_e Z_{r(e)}.$$

"Gauss law" constraint at site $v$: $$W_{\mathrm{NE}(v)} \prod_{e\supset v} X_e = 1.$$




ex.) $$H = t\sum_e (c^\dagger_{L(e)}c_{R(e)} + c^\dagger_{R(e)}c_{L(e)}) + \mu\sum_f c^\dagger_f c_f.$$

$$\Longrightarrow \quad H = \frac{t}{2}\sum_e X_e Z_{r(e)}(1 - W_{L(e)}W_{R(e)}) + \frac{\mu}{2}\sum_f (1 - W_f)$$

*local*

# Some other applications

- Efficient simulation of (2+1)d U(1) gauge th. [Kane-Grabowska-Nachman-Bauer ’22]
- Inflation (scalar in curved spacetime) [Liu-Li ’20]
- Chiral fermion [Hayata-Nakayama-Yamamoto ’23]
- Quantum group approach to Non-abelian gauge th. [Zache-Gonzalez-Cuadra-Zoller ’23, Hayata-Hidaka ’23]
- Conformal bootstrap [Bao-Liu ’18]
- Dark sector showers [Chigusa-Yamazaki ’22, Bauer-Chigusa-Yamazaki ’23]
- Measurement-based quantum computation [Okuda-Sukeno ’22]
- quantum machine learning [Nagano-Miessen-Onodera-Tavernelli-Tacchino-Terashi ’23, etc…]
- String/M-theory [Gharibyan-Hanada-MH-Liu ’20]

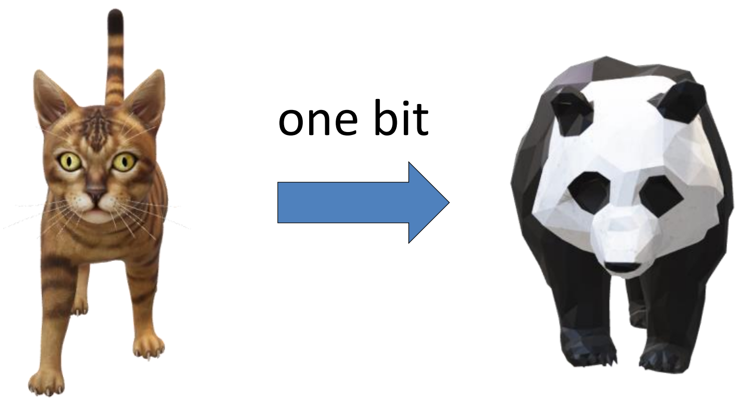
etc…

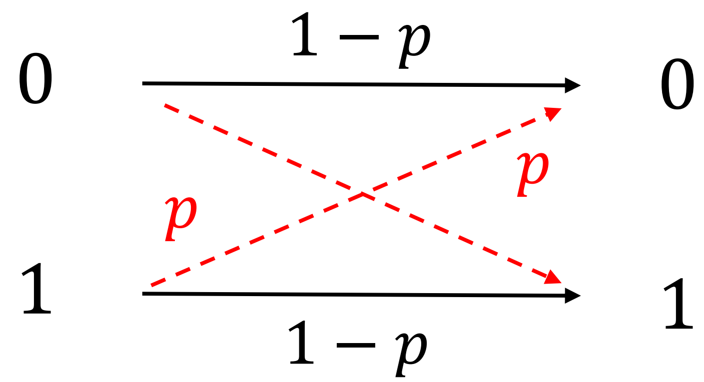
# Contents

1. Introduction
2. QC for QFT
3. QFT for QC (briefly)
4. Summary

# Errors in classical computers

Computer interacts w/ environment ⇒ error/noise




Suppose we send a bit but have “error” in probability $p$

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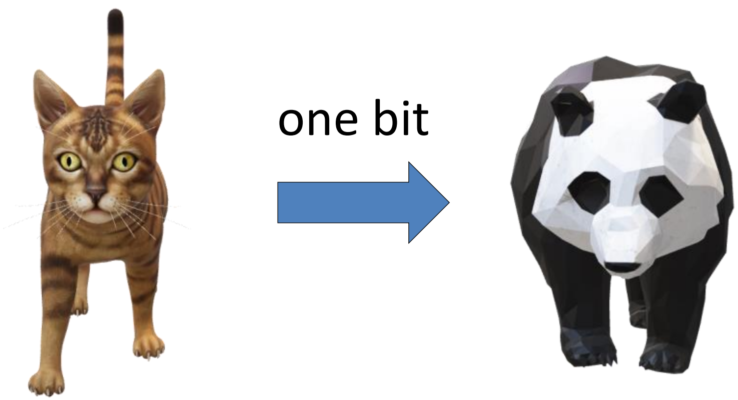


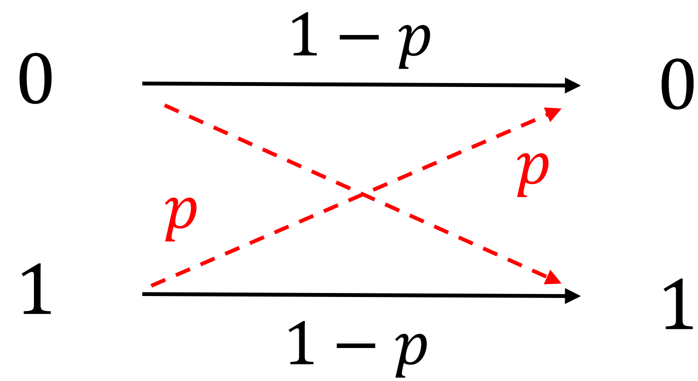


Suppose we send a bit but have “error” in probability $p$

A simple way to correct errors:

① Duplicate the bit (encoding): $0 \rightarrow 000, \quad 1 \rightarrow 111$

② Error detection & correction by “majority voting”:

$$001 \rightarrow 000, \quad 011 \rightarrow 111, \quad \text{etc...}$$

⇒ $P_{\text{failed}} = 3p^2(1-p) + p^3$ (improved if $p < 1/2$)

# Quantum Error Correction

## 1.Encoding

$$|\psi\rangle \in \mathcal{H} \longrightarrow |\psi_E\rangle \in \mathcal{H}_E \quad (\mathcal{H} \subset \mathcal{H}_E)$$

## 2. Error detection

Take set of operators $\{O_1, \cdots\}$ s.t.

$$O_i|\psi_E\rangle = |\psi_E\rangle, \quad O_i(\mathrm{error})|\psi_E\rangle \neq (\mathrm{error})|\psi_E\rangle$$

Then find eigenvalues of $O_i$'s using ancillary qubits

## 3. Error recovery

Act "inverse of error" based on the eigenvalues

# relations between QEC & gauge theory

## Motivations

[Spirit may be similar to Rajput-Roggaro-Wiebe ’21, Gustafson-Lamm ’23, etc...]

1. ∃ explicit examples

   ex.) Toric code = $\boldsymbol{Z_2}$ lattice gauge theory [Kitaev ’97]

2.

3.

4.

# relations between QEC & gauge theory

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   QEC = redundant description of logical qubits

   Gauge theory = redundant description of physical states

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3. Nature = Gauge theory & Nature = Quantum computer

   ⟹ Gauge theory may know something on QEC?

4.

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4. ∃ proposals on relations among QEC & concepts in HEP

   ex.) Holography, Black hole, CFT, Renormalization group

   [Almheiri-Dong-Harlow ’14, Hayden-Preskill ’07, Dymarsky-Shapere ’20, Kawabata-Nishioka-Okuda ’22, Furuya-Lashkari-Moosa ’21, etc...]

# What I'm doing...

[MH, work in progress]

to make dictionary for classes of codes/gauge theories:

| QEC | Gauge theory |
| --- | --- |
| errors | unphysical op. (& excitation) |
| logical qubits | physical states (w/ low energy) |
| "no error conditions" (stabilizer) | Gauss law (& min[energy]) |
| logical op. | gauge invariant op. |
| ancilla for recovery | additional matter |
| ⋮ | ⋮ |

# QFT as a generator of error correcting code?

## Toric code

- Lattice model interpreted as QEC
- Low energy effective theory = QFT (BF theory)

QFT ↔ Lattice model ↔ QEC

Idea : if we get something new in one of them,
then try to fill the other parts

ex.) "Dipolar" generalization of Toric code [Pace-Wen '22]

⇩

corresponds to a "layer" of BF theory w/ some rule

[Ebisu-MH-Nakanishi '23]

# 新たな種類のエニオンを系統的に作る方法を発見―量子コンピュータへの新たな応用の可能性―

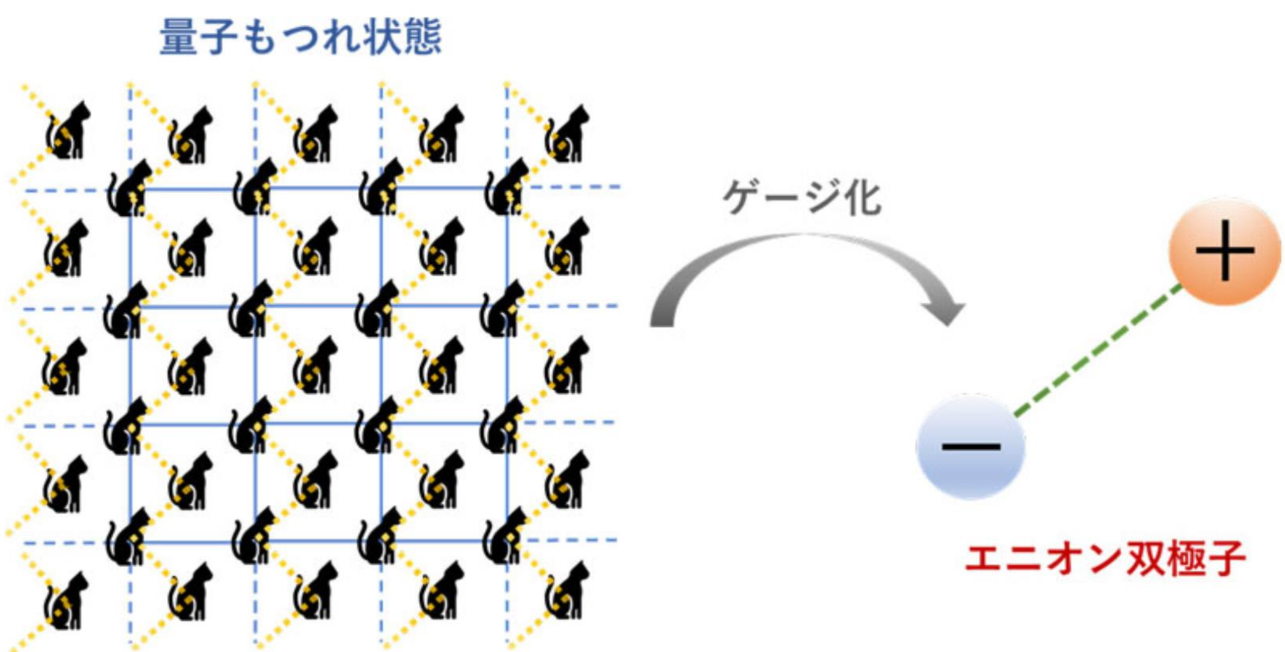
(Press release: systematic construction of new anyons)

person 企業・研究者の方

公開日： 2024年05月16日

液体・固体・気体など物質は状況に応じて異なる状態（相）を持つことがあり、相を理解することは物理学で重要な課題です。現代的な相の分類においては、エニオンと呼ばれる分数電荷を持つ準粒子が重要であり、量子コンピュータへの応用の観点からも研究されています。

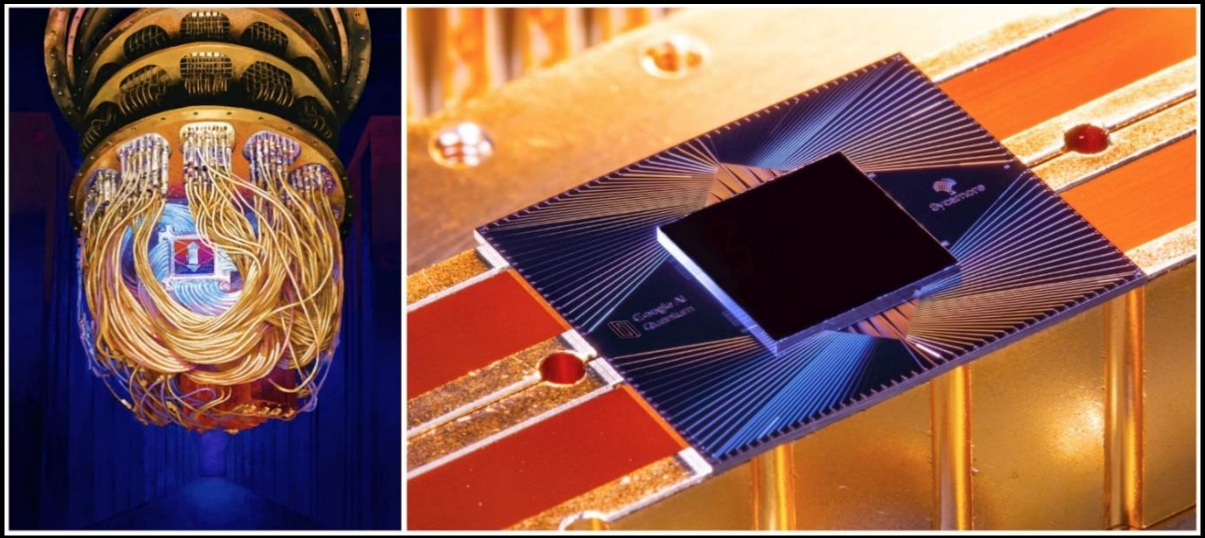
戎弘実 基礎物理学研究所研究員、中西泰一 同博士課程学生（兼：理化学研究所大学院生リサーチ・アソシエイト）、本多正純 理化学研究所上級研究員の共同研究グループは、動き方に制限がかかる新しい種類のエニオンを系統的に記述する理論的枠組みを発見しました。図のように層に沿って量子もつれを導入し、ゲージ化と呼ばれる操作により新奇な物質の相を構成しました。この相ではエニオンが対を形成する双極子の構造が存在し、動き方に制限が見られるなどの新奇な性質があることが分かりました。この結果はフラクトン・トポロジカル相と呼ばれる新しい物性の解明や、量子情報の保存などの量子コンピュータへの応用の問題にも貢献していくことが考えられます。

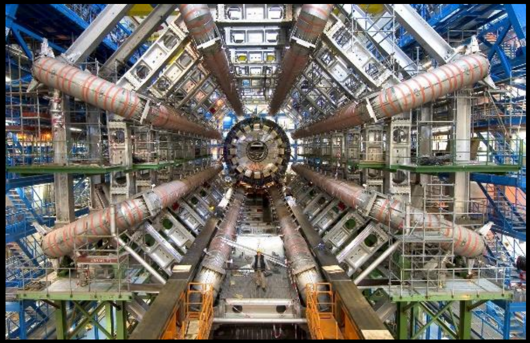


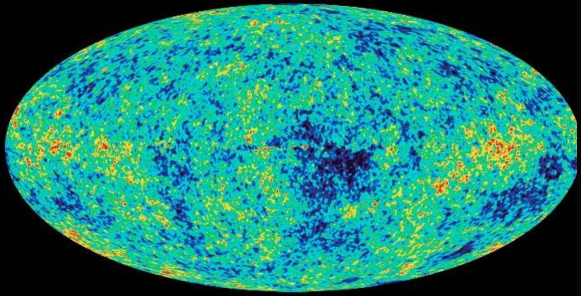

図：量子もつれの状態を導入し（左）ゲージ化を考えることで、双極子を持つエニオンの存在（右）を明らかにした。

# Summary

# Long term goal







etc...

# Challenges

- needs real device w/ certain specification
  (qubit #, fidelity, etc...)
- pioneers problems efficiently solved by QC

To do : (except waiting for development of hardware)

- bench mark, estimation of required computational resource
- inventing/improving methods → reduces required resource
  — developing lattice field theory in operator formalism
  — especially finding nice regularization of gauge theory
- Quantum field theory → Quantum computation?
  — proposing new error correcting codes etc...

Thanks!