# Entanglement in three-flavor collective neutrino oscillations

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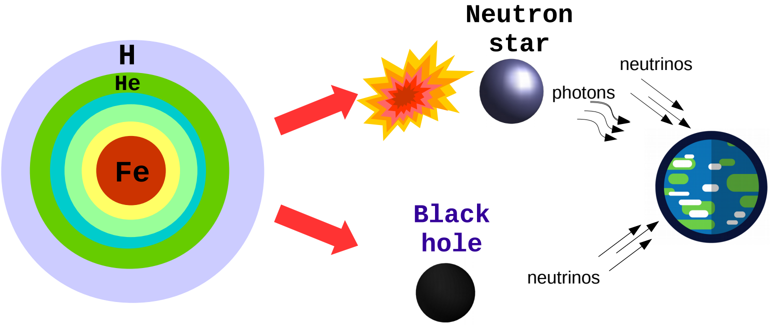
PHYSICS FRONTIER CENTER

# Why are neutrinos important for a core-collapse supernova?

**Neutrinos**:

- $\sim 10^{58}$ of them emitted from a single core collapse
- only they (+ GW) can reveal the deep interior conditions
- only they (+ GW) are emitted from the collapse to a black hole




Earth image: Kurzgesagt

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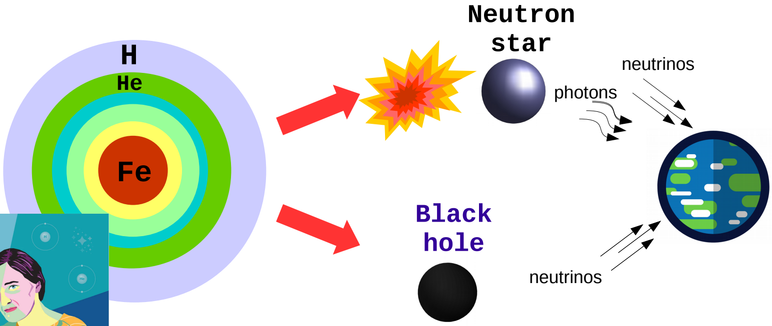




Earth image: Kurzgesagt

# Why core-collapse supernovae are good physics probes?

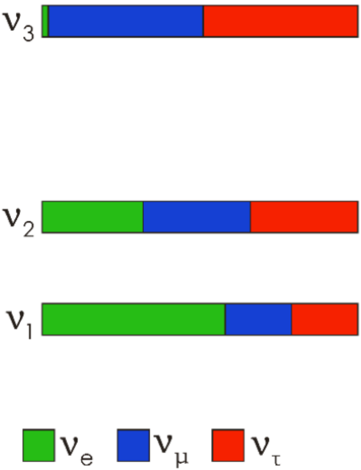
**Advantages**

- extreme physical conditions not accessible on Earth:
  very high densities, long baselines etc.
- within our reach to detect (SK, JUNO, XENON, PandaX...)

**What can we learn with a variety of detectors?**

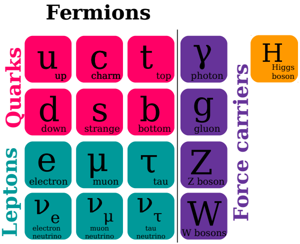
- explosion mechanism — Bethe & Wilson (1985), Fischer et al. (2011)...
- yields of heavy elements — Woosley et al. (1994), Surman & McLaughlin (2003)...
- compact object formation — Warren et al. (2019), Li, Beacom et al. (2020)...
- neutrino flavor evolution — Balantekin & Fuller (2013), Tamborra & Shalgar (2020)...
- non-standard physics — McLaughlin et al. (1999), de Gouvêa et al. (2019) ...

# Neutrino mass and flavor states




mass states $\neq$ flavor states

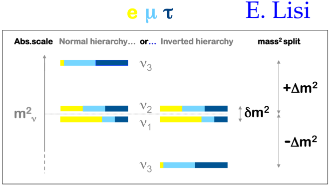
# Neutrino flavor and mass states




flavor basis mass basis

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = U \begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

E. Lisi




beam, atmospheric | beam, reactor | solar, reactor

$$U = \begin{bmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{bmatrix} \begin{bmatrix}c_{13} & 0 & s_{13}e^{-i\delta_{CP}}\\ 0 & 1 & 0\\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13}\end{bmatrix} \begin{bmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{bmatrix}$$

is $\nu_s$ $(\nu_4)$ missing?

Pontecorvo (1957), Maki, Nakagawa, Sakata (1962)

# The Neutrino Many-Body Interactions Hamiltonian

$$H = H_\nu + H_{\nu\nu}$$

Vaccum term

$$H_\nu = \sum_{\vec{p}} \sum_{i=1}^{3} \sum_{j(\neq i)} \frac{\Delta m_{ij}^2}{2E} T_{ii}(p, \vec{p}),$$

$$T_{ij}(p, \vec{p}) = a_i^\dagger(\vec{p}) a_j(\vec{p}) \ ,$$

Neutrino-neutrino interaction term

$$H_{\nu\nu} = \frac{G_F}{\sqrt{2}V} \sum_{i,j=1}^{3} \sum_{E,\vec{p}} \sum_{E',\vec{p'}} \left(1 - \cos\theta_{\vec{p}\vec{p'}}\right) \ \times T_{ij}(E, \vec{p}) T_{ji}(E', \vec{p'}) \ ,$$

Pehlivan, Balantekin, Kajino (2014)

# The Neutrino Many-Body Interactions Hamiltonian

$$T_{ij} = \sum_{i'} (\lambda_{i'})_{ji} Q_{i'} + \frac{1}{3}\delta_{ij} \sum_i a_i^\dagger a_i,$$

$$Q_{i'} = \frac{1}{2} \sum_{i,j=1}^{3} a_i^\dagger (\lambda_{i'})_{ij} a_j,$$

$$H = \sum_p \vec{B} \cdot \vec{Q_p} + \sum_{p,p'} \mu_{pp'} \vec{Q_p} \cdot \vec{Q_{p'}} \ ,$$

$$\mu(r) = \frac{G_F}{\sqrt{2}V} \left(1 - \sqrt{1 - \frac{R_\nu^2}{r^2}}\right)^2 \ ,$$

$$\vec{B} = \left(0, 0, \omega_p, 0, 0, 0, 0, \Omega_p\right) \ .$$

$\omega_p = -\frac{1}{2E}\delta m^2$ and $\Omega_p = -\frac{1}{2E}\Delta m^2$

# Density Matrix Evolution

$$\rho \equiv |\Psi\rangle\langle\Psi|$$

Reduced density matrix for a single neutrino

$$\rho_n = \mathrm{Tr}_n[\rho] = \frac{1}{3}\left[\mathbb{I} + \frac{3}{2}\sum_j \lambda_j P_j\right]$$

extracting the polarization vector components

$$P_j = \mathrm{Tr}[\rho_n \lambda_j]\ .$$

$$P_{\nu_1} = \tfrac{1}{3}\left(1 + \tfrac{3}{2}P_3 + \tfrac{\sqrt{3}}{2}P_8\right)\ ,$$
$$P_{\nu_2} = \tfrac{1}{3}\left(1 - \tfrac{3}{2}P_3 + \tfrac{\sqrt{3}}{2}P_8\right)\ ,$$
$$P_{\nu_3} = \tfrac{1}{3}\left(1 - \sqrt{3}P_8\right)\ .$$

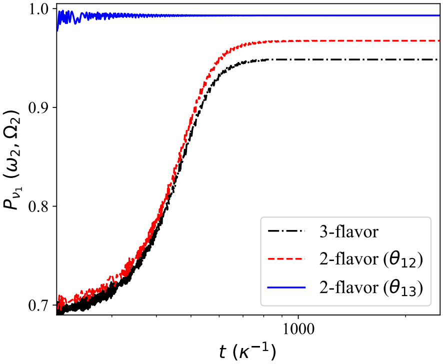
| Mass eigenstate | $P_3$ | $P_8$ |
|---|---|---|
| $\nu_1$ | 1 | $1/\sqrt{3}$ |
| $\nu_2$ | $-1$ | $1/\sqrt{3}$ |
| $\nu_3$ | 0 | $-2/\sqrt{3}$ |

Bipartite Entanglement Entropy

$$S_n = -\mathrm{Tr}\left[\rho_n \log \rho_n\right] \ ,$$

$$S_n = \log 3 - \frac{1}{3}\mathrm{Tr}\left[\left(\mathbb{I} + \frac{3}{2}\lambda_j P_j\right)\log\left(\mathbb{I} + \frac{3}{2}\lambda_j P_j\right)\right] \ ,$$

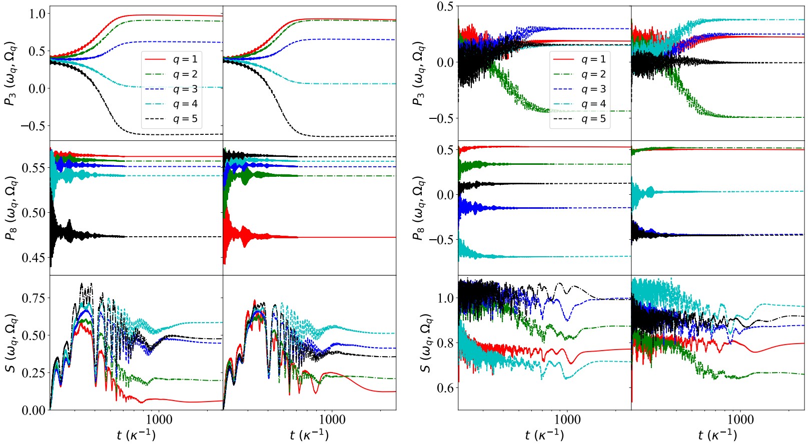
# Comparison between 2-flavor and 3-flavor approach




The results are for $N = 5$; initial state $|\psi\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_e\rangle$ in NO

Always more mixing in 3-flavor case

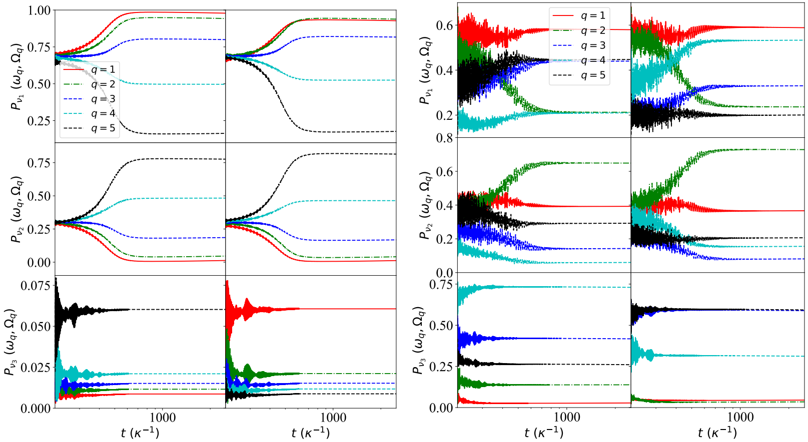
# The evolution of a neutrino systems




initial state $|\psi\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_e\rangle \rightarrow$ small entanglement entropy

initial state $|\psi\rangle = |\nu_e\nu_e\nu_\mu\nu_\mu\nu_\tau\rangle \rightarrow$ small entanglement entropy

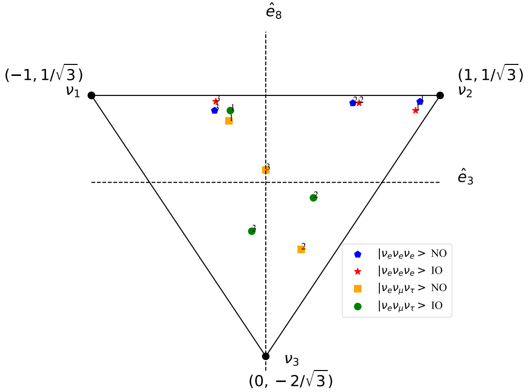
# The evolution of a neutrino systems




$$\text{initial state } |\psi\rangle = |\nu_e\nu_e\nu_e\nu_e\nu_e\rangle \rightarrow \text{small mixing}$$

$$\text{initial state } |\psi\rangle = |\nu_e\nu_e\nu_\mu\nu_\mu\nu_\tau\rangle \rightarrow \text{large mixing}$$

# Asymptotic values




initial state $|\psi\rangle = |\nu_e\nu_e\nu_e\rangle \to$ small mixing

initial state $|\psi\rangle = |\nu_e\nu_\mu\nu_\tau\rangle \to$ large mixing

# Aditional conditions arising in 3-flavor approach

A density matrix should satisfy four conditions:

1. It is Hermitian,
2. Its trace is one,
3. It is positive semi-definite
4. $\rho^2 \leq \rho$.

3. Elementary symmetric polynomials formed from eigenvalues of $\rho$ non-negative

# Aditional conditions arising in 3-flavor approach

The characteristic equation of any matrix $\mathcal{M}$ with eigenvalues $x_i$ can be written as

$$\prod_{i=1}^{N}(x - x_i) = \sum_{k=1}^{N}(-1)^k x^{N-k} e_k = 0$$

where $e_k$ are the elementary symmetric polynomials of the eigenvalues $x_i$. The elementary symmetric polynomials can in turn be expressed in terms of power sums, $p_k = \sum_i x_i^k$. via the equality

$$\sum_k e_k x^k = \exp\left(\sum_k \frac{(-1)^{k+1}}{k} p_k x^k\right) .$$

Note that $p_k = \mathrm{Tr}\,(\mathcal{M}^k)$.

# Aditional conditions arising in 3-flavor approach

$$e_1 = \mathrm{Tr}\,\rho = 1,$$

$$e_2 = \frac{1}{2} - \frac{1}{2}\mathrm{Tr}\,\rho^2 = \frac{N-1}{2N} - \frac{1}{N^2}|P|^2,$$

$$e_3 = \frac{1}{6} - \frac{1}{2}(\mathrm{Tr}\rho^2) + \frac{1}{3}(\mathrm{Tr}\rho^3) = \frac{(N-1)(N-2)}{6N^2} - \frac{N-2}{N^3}|P|^2 + \frac{2}{3N^3}Q\,,$$

That yields conditions $|P|^2 \leq 3$ and $\frac{2}{3}Q \geq |P|^2 - 1$ where
$Q = \mathrm{Tr}\{\lambda_j \lambda_i \lambda_k P_i P_j P_k\}/2 = d_{ijk} P_i P_j P_k$

Necessary but not sufficient conditions
One needs the roots of the characteristic polynomial

Deen, Kabir, Karl (1971)

# Aditional conditions arising in 3-flavor approach

The characteristic equation for the matrix $\mathcal{A} = \lambda_a P_a$

$$\mathrm{Tr}\,\mathcal{A} = 0$$

$$\mathrm{Tr}\,\mathcal{A}^2 = 2|P|^2$$

$$\mathrm{Tr}\,\mathcal{A}^3 = 2Q\,.$$

Hence the characteristic equation for the matrix $\mathcal{A}$ is

$$x^3 - |P|^2 x - \frac{2}{3}Q = 0\,,$$

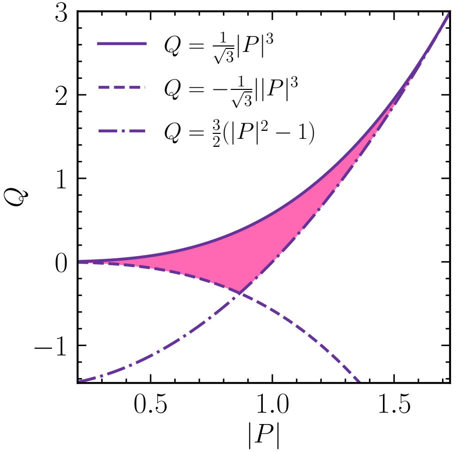
Positive semidefinite matrix $\rightarrow$ all roots real and take values so that $(1 + x_i)$ are positive.

All root real $\rightarrow$ discriminant is positive:

$$3\frac{Q^2}{|P|^6} < 1$$

# Aditional conditions arising in 3-flavor approach

In the 3-flavor, only certain solutions are allowed for $|P|$
reason: SU(3) and SO(8) are not isomorphic




$$Q = \mathrm{Tr}\{\lambda_j \lambda_i \lambda_k P_i P_j P_k\}/2 = d_{ijk} P_i P_j P_k$$

# Conclusions

- Entanglement can be underestimated in 2-flavor approach
- 3-flavor mixing depends on mass ordering
- Mixed initial states have more entanglement and mixing
- 3-flavor neutrino systems natural for qutrit-based quantum computers

**Thank you for the attention!**