Quantum vs. Classical Matched Filtering in Gravitational Wave Detection

(Work in progress)

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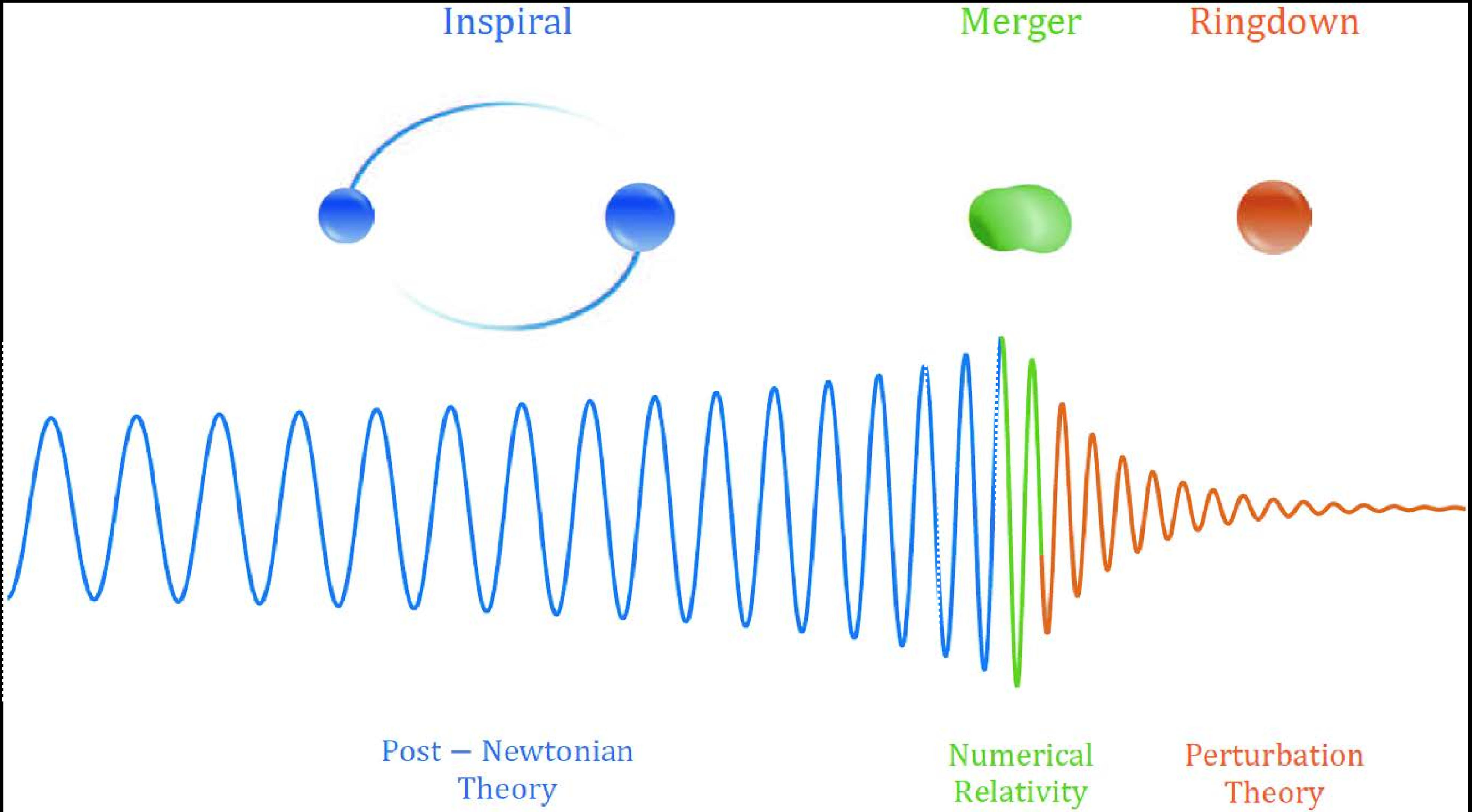
Ref.

- Quantum vs. Classical Matched Filtering in Gravitational Wave Detection. P. Karmakar, K. Cannon, M. Honda (draft in progress)
- K. Cannon et al., Astrophys. J. 748, 136 (2012),
- S. Gao, F. Hayes, S. Croke, C. Messenger, and J. Veitch, Phys. Rev. Res. 4, 023006 (2022),
- K. Miyamoto, G. Morr ás, T. S. Yamamoto, S. Kuroyanagi, and S. Nesseris, Phys. Rev. Res. 4, 033150 (2022).

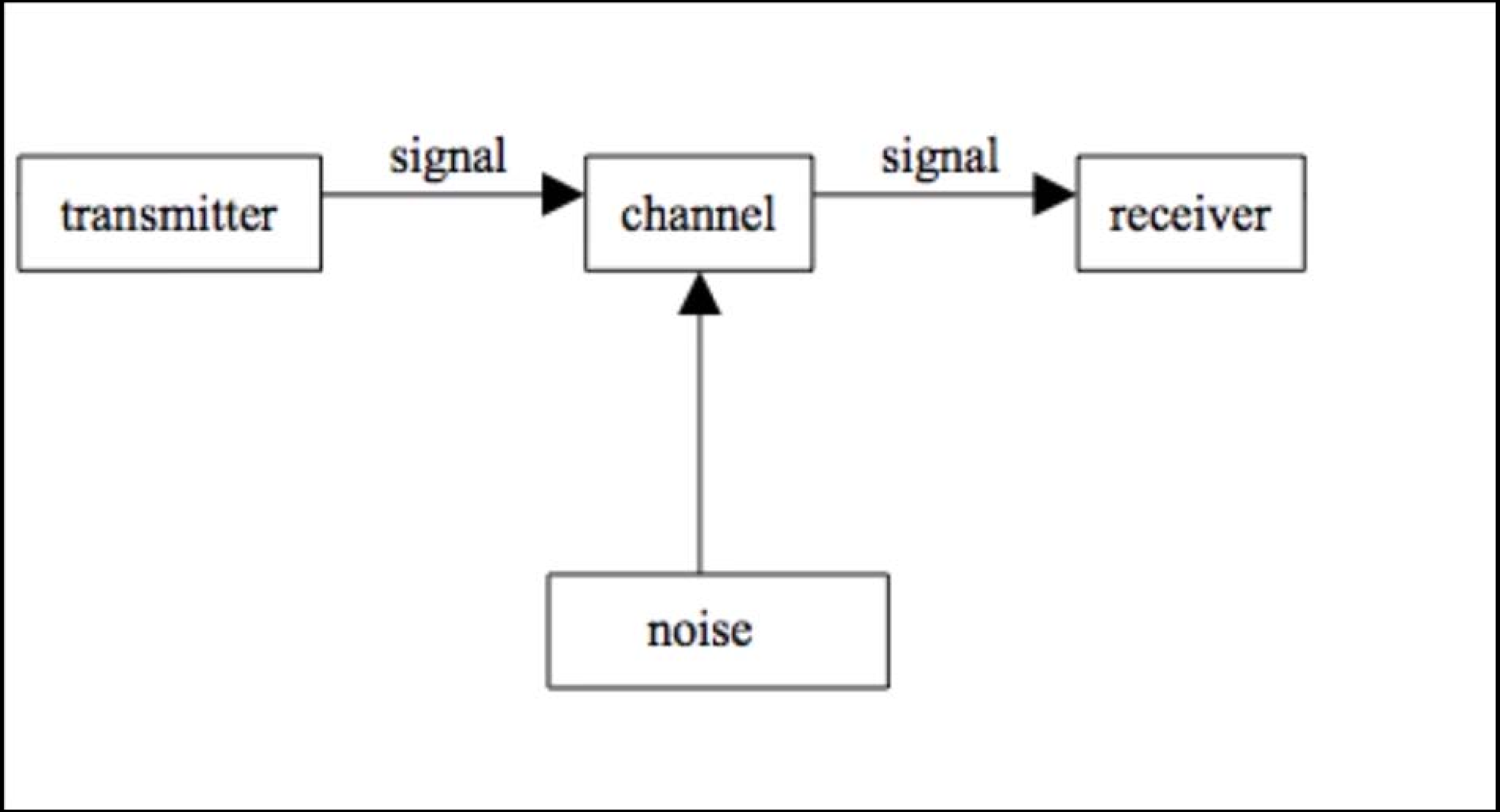


Low frequency region (band?)
Most of the waveforms are similar in the first region of inspiral phase (mostly fn of chirp-mass)

High Frequency region (band?)
Last part of inspiral differ a bit for different mergers

source

Matched Filtering

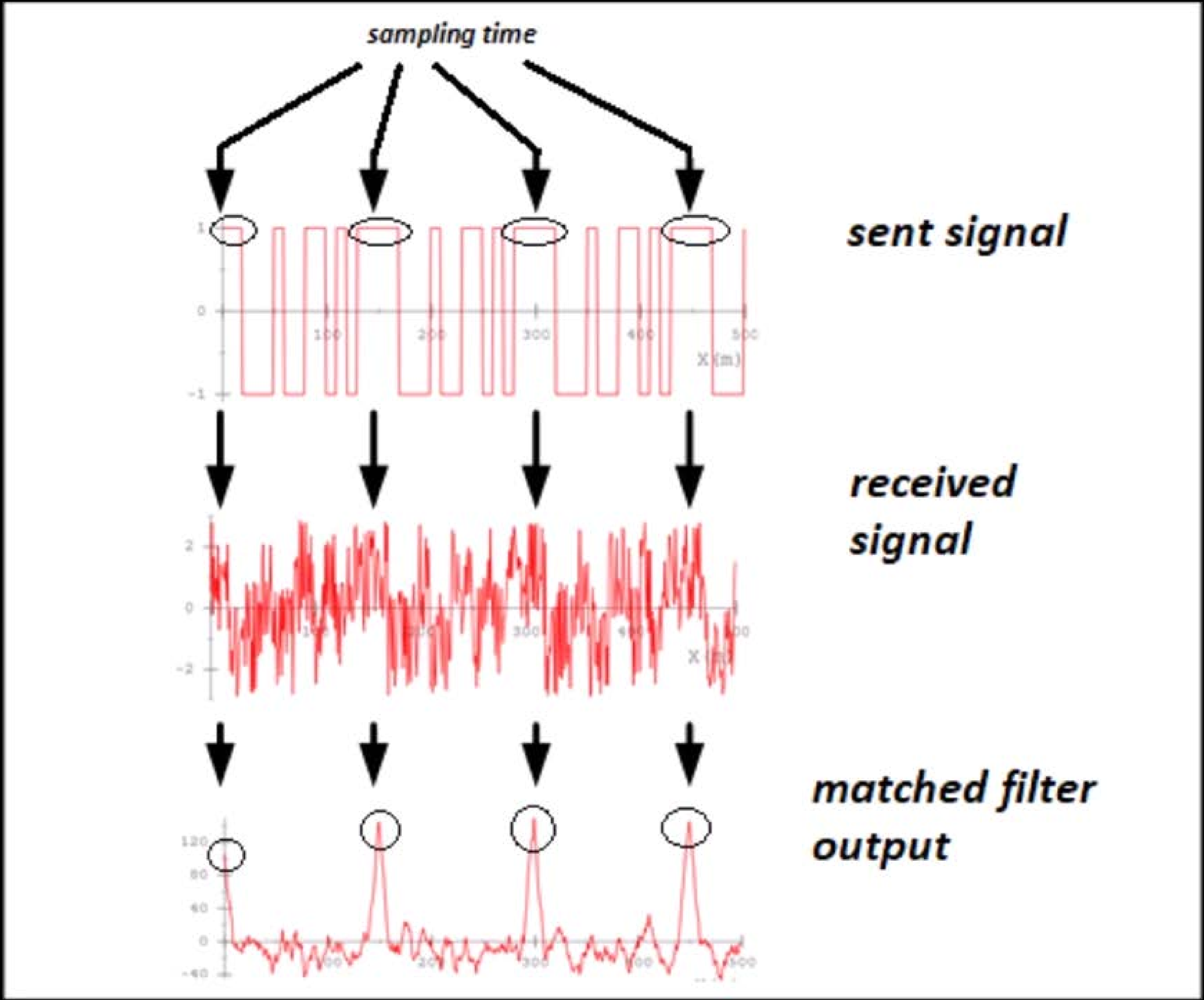


Solution: Filters - systems or processes used to remove unwanted components from a signal.

Convolution Operation

$$
\begin{aligned}
y(t) &= s(t) * h(t) \\
&= \int_{-\infty}^{\infty} s(\tau) \cdot h(t-\tau) d\tau
\end{aligned}
$$

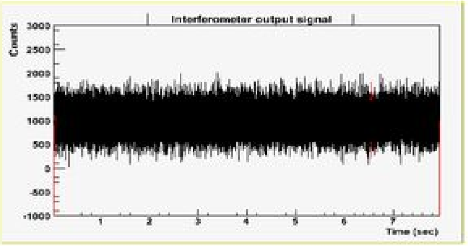
- Output of the matched filter is the convolution of its input signal (s(t)) with its impulse response (h(t)).
- **Convolution** is the integral of the product of two signals, one flipped and shifted.
- → the calculation of the overlap of the input signal S(T) with the reverse and pulse response H(T)



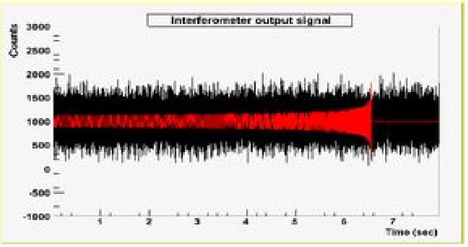
sampling time
sent signal
received
signal
matched filter
output

Matched filtering

Detector output (d)

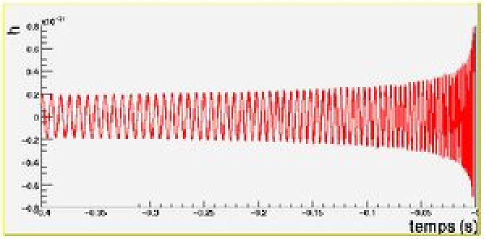


or



$$S(t) = 4\int_{f_{low}}^{f_{final}} \frac{\tilde{d}(f)\ \tilde{T}^*(f)}{S_n(f)}\ e^{2\pi i f t}\ df$$

Model waveform (T)



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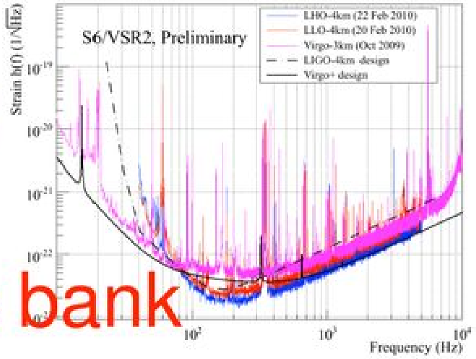
Detector noise sensitivity (S_n)



$T(m_1, m_2, \chi_{eff}) \sim 10^3 - 10^5$ template bank

$10^3 - 10^5$ matched filters are done at once

==> νεεδ χομπυτινγ ρεσουρχεσ

$\tilde{d}(f) = \tilde{n}(f)$ ==> νο ουτπυτ

$\tilde{d}(f) = \tilde{n}(f) + \tilde{s}(f)$ ==> στρονγ ουτπυτ

source

Time-Domain Method: SNR time series

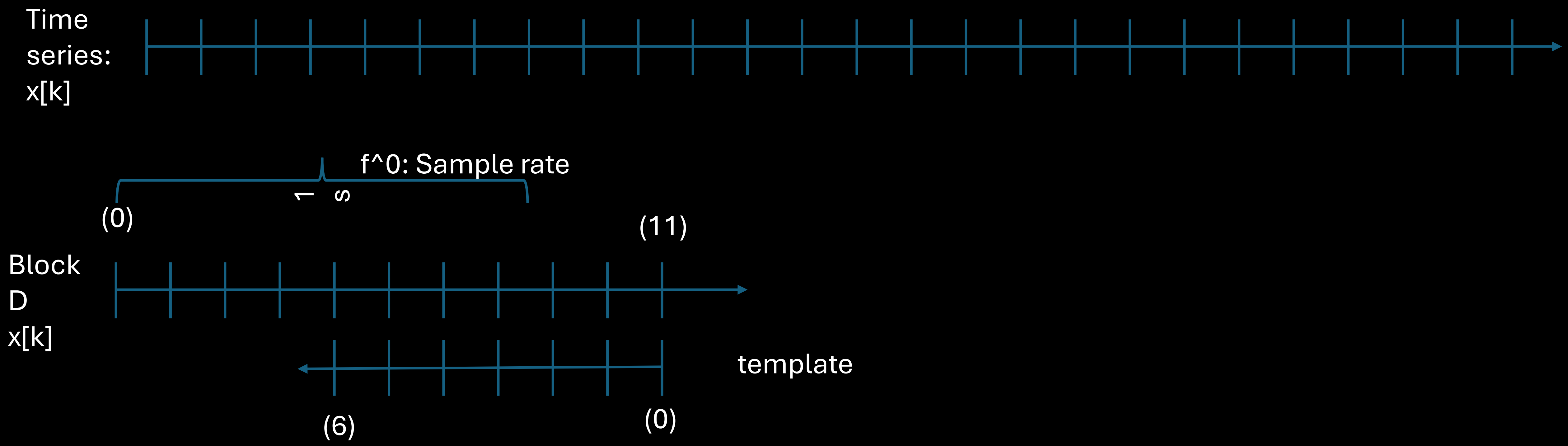
$$\rho_i[k] = \sum_{n=0}^{M-1} \hat{h}_i^*[n]\hat{x}[k-n],$$

The total cost per unit time of input data is

$$2NMf^0.$$

	Definition
N	Number of templates
M	Number of samples in each template
f^0	Sample rate for data and templates (*e.g.* in Hertz)
S	Number of template time slices
M^s	Number of samples in decimated time slice s
f^s	Sample rate for decimated template time slice s
L^s	Number of basis vectors spanning time slice s

Finite Block in Reality: FFT (example of overlapping)



Samples per template / time step / template length: M (=6)
Total Numbers of Templates: N
D > M
Overlap: M-1

Frequency-Domain Method:

The total cost per unit time of input data is

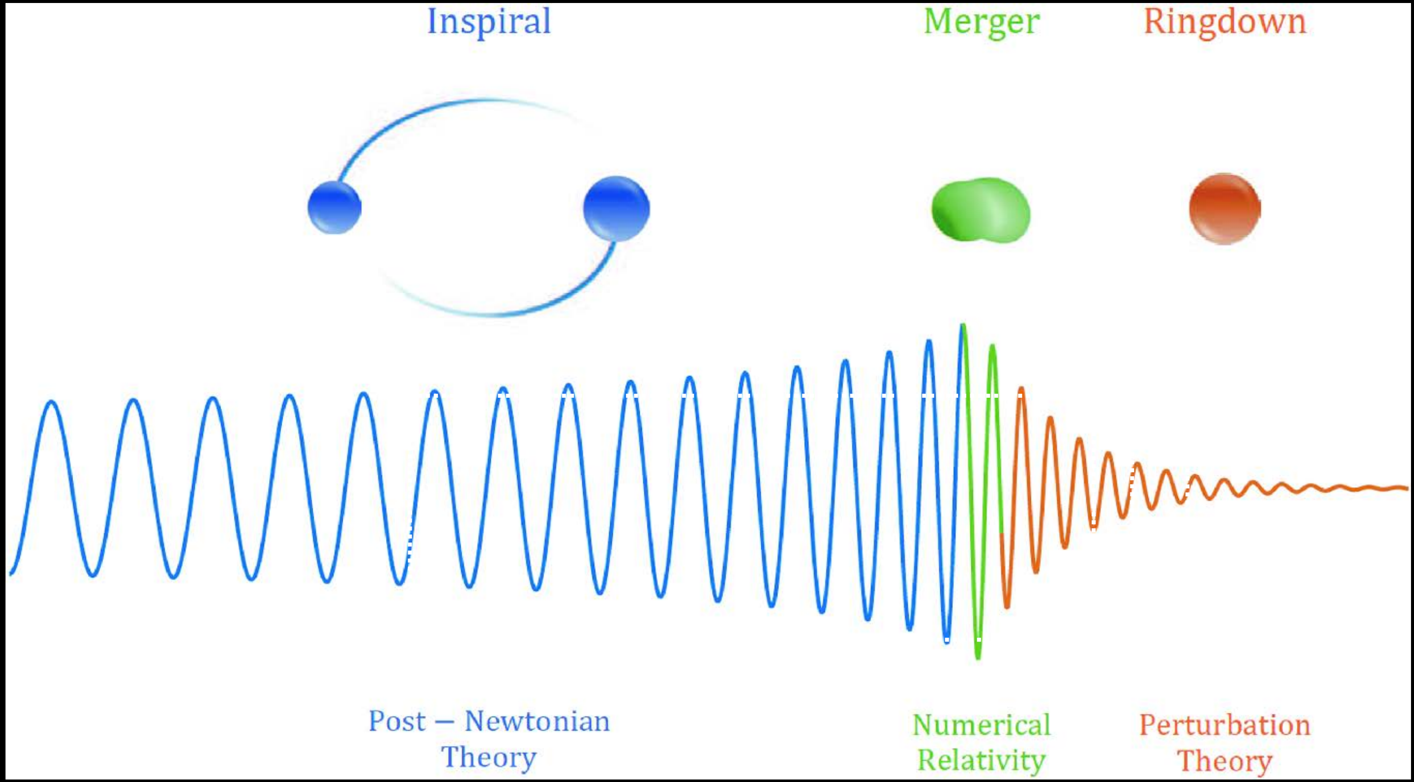
$$\frac{2(N+1)\log D + 2N}{1 - M/D} f^0.$$

Approx: For N>>1, D>>M

$$\approx 2N f^0 \log D,$$

	Definition
N	Number of templates
M	Number of samples in each template
f^0	Sample rate for data and templates (*e.g.* in Hertz)
S	Number of template time slices
M^s	Number of samples in decimated time slice s
f^s	Sample rate for decimated template time slice s
L^s	Number of basis vectors spanning time slice s

Modern search pipeline (Gst-LAL): Reduction



- Orthonormal basis
- Non-uniform frequency
- Uses high pass and low pass filters,
- Ordinary matched filtering SNR > 8.
- SNR > 3.5 → Ranking statistics (Reduce risk of loosing signals)

Low frequency region (band?)
Most of the waveforms are similar in the first region of inspiral phase (mostly fn of chirp-mass)

High Frequency region (band?)
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source

Gst-LAL (LLOID): Cost Reduction

- The number of arithmetic operations per unit time of input data:

$$2\sum_{s=0}^{S-1} f^s L^s \left(N + \begin{cases} \lg D^s \\ M^s \end{cases} \right).$$

w.r.t. Frequency Domain:

$$\frac{2\sum_{s=0}^{S-1} f^s L^s M^s}{2N f^0 \log D}.$$

~ 0.1

Grover’s Quantum Computing Algorithm

Original purpose was to solve the unstructured search problem faster than what can be done classically

References

- https://arxiv.org/abs/quant-ph/9605043
- https://arxiv.org/abs/quant-ph/9909040

Grover's Algorithm for Multiobject Search in Quantum Computing

G. Chen, S. A. Fulling, M. O. Scully

L. K. Grover's search algorithm in quantum computing gives an optimal, square-root speedup in the search for a single object in a large unsorted database. In this paper, we expound Grover's algorithm in a Hilbert-space framework that isolates its geometrical essence, and we generalize it to the case where more than one object satisfies the search criterion.

A fast quantum mechanical algorithm for database search

Lov K. Grover (Bell Labs, Murray Hill NJ)

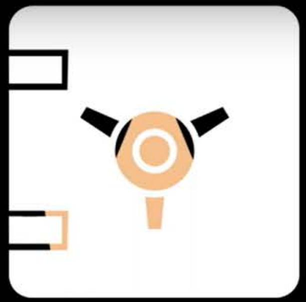
Imagine a phone directory containing N names arranged in completely random order. In order to find someone's phone number with a 50% probability, any classical algorithm (whether deterministic or probabilistic) will need to look at a minimum of N/2 names. Quantum mechanical systems can be in a superposition of states and simultaneously examine multiple names. By properly adjusting the phases of various operations, successful computations reinforce each other while others interfere randomly. As a result, the desired phone number can be obtained in only O(sqrt(N)) steps. The algorithm is within a small constant factor of the fastest possible quantum mechanical algorithm.

1. Unstructured Search

- Example: 1 lock, 8 identical keys (N), Only one Key opens the lock

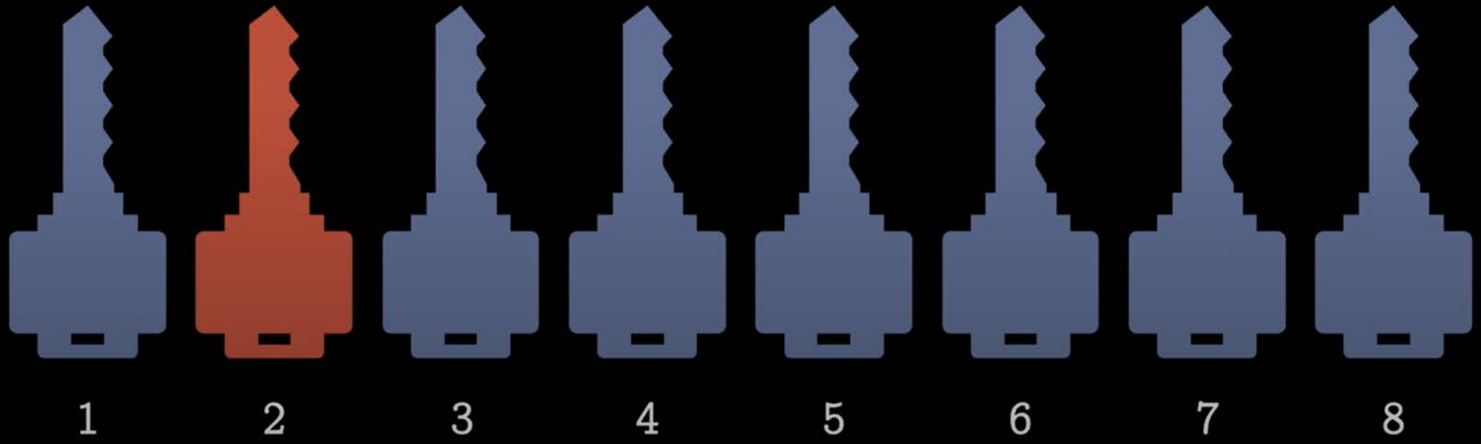
- Criteria: Stop search if matching happens

Best Case:
Opens in first try



Worst Case:
Opens in all 7 tries fails: 7 (=N-1)

- The upper bound on our computational complexity is $\mathcal{O}(N)$



2. Grover's algorithm: 3 Steps

Step 1: Probability initialization: Initialize Search of $N = 2^n$ items

- It is to initialize our quantum state vector to represent the unstructured search problem.

$$H^{\otimes n}|0^n\rangle = \frac{1}{\sqrt{N}} \sum_{x \in \{0,1\}^n} |x\rangle = |\phi\rangle$$

Initially, there is an equal probability that any of the keys will be the right key,
Our state vector is a superposition of all possibilities with uniform amplitude.

Grover's algorithm: 3 Steps

Step 2: Search Operator: Search and Reflect the desired state $|s\rangle$

- Functional form:

$$U_s|x\rangle = (-1)^{s(x)}|x\rangle \begin{cases} s(x) = 1 \text{ if } x = s \\ s(x) = 0 \text{ if } x \neq s \end{cases}$$

$$U_s = I - 2|s\rangle\langle s|$$

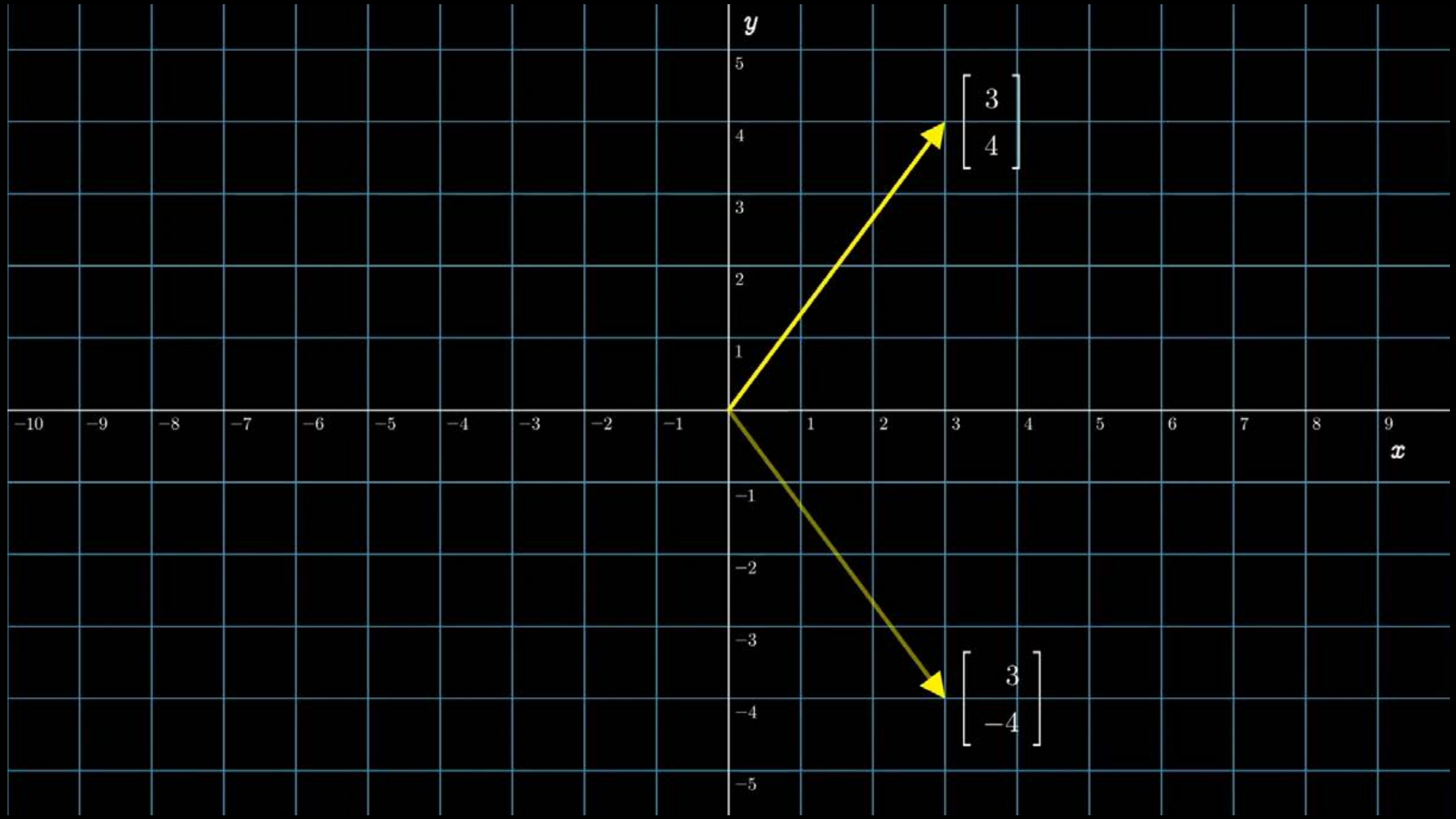
- Linear operator: Reflection operator

Step 3: Diffusion Operator:

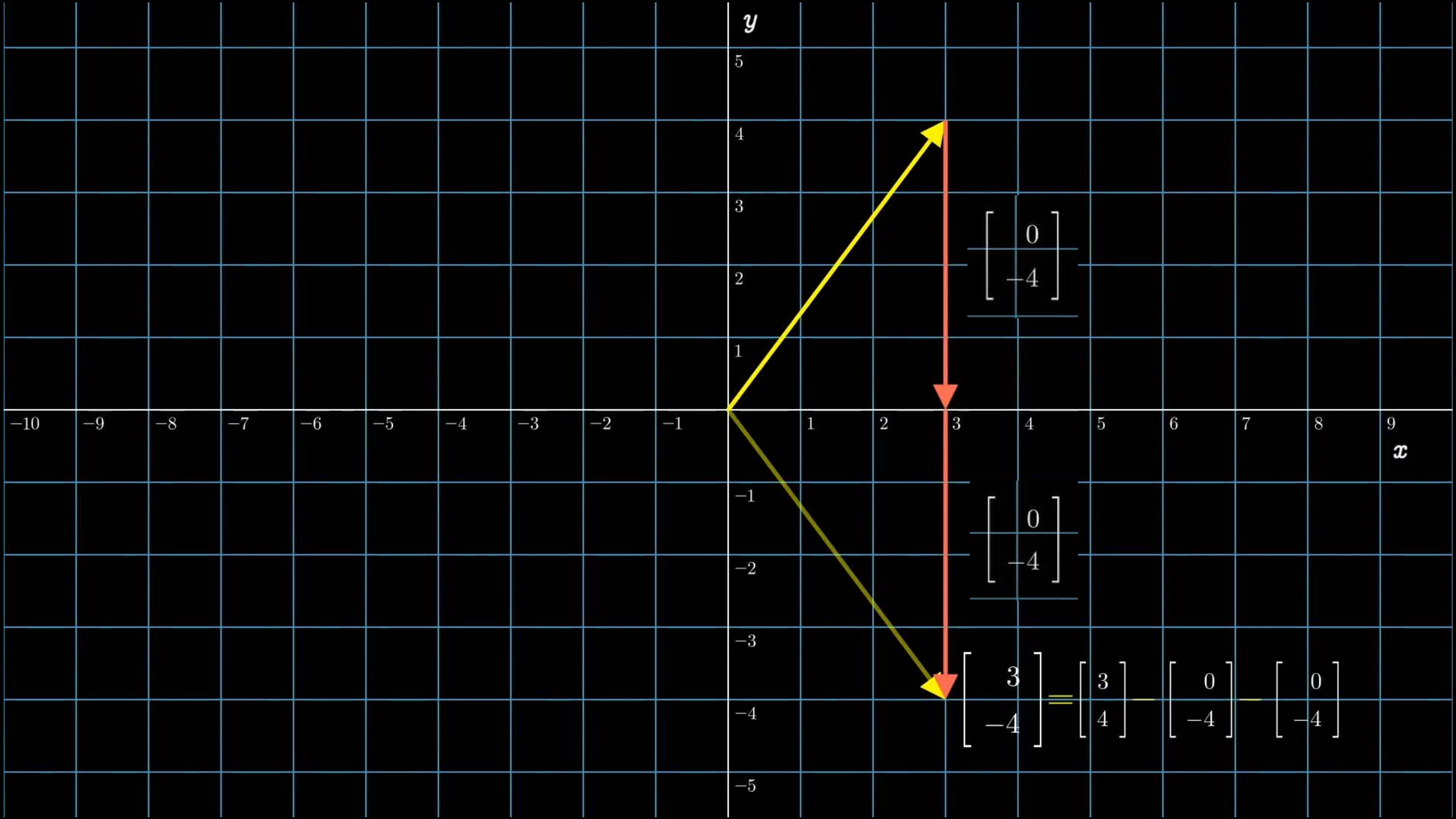
Reflection of our state vector about the mean amplitude

$$U_d = 2|\phi\rangle\langle\phi| - I$$

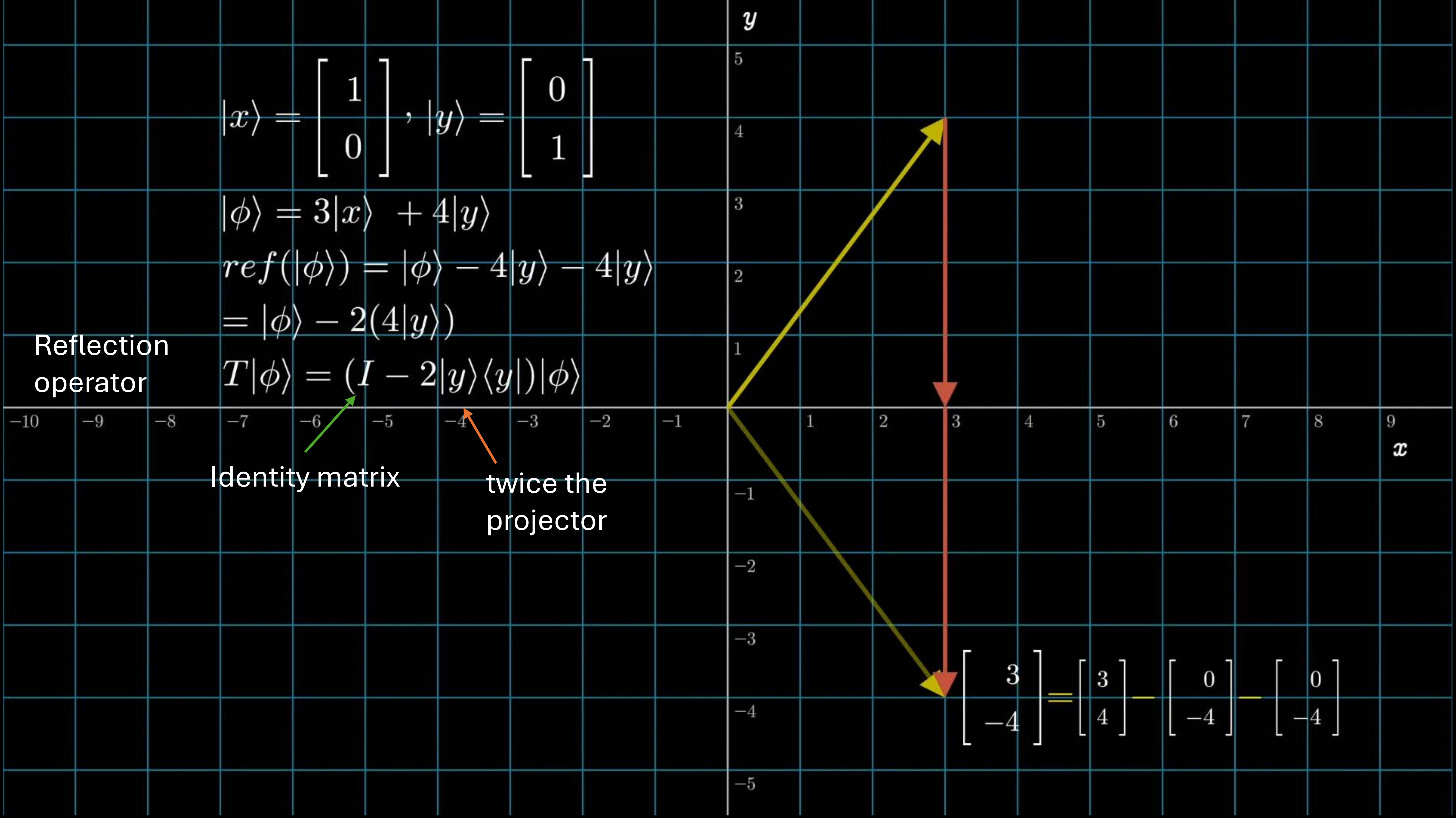
3. Reflections



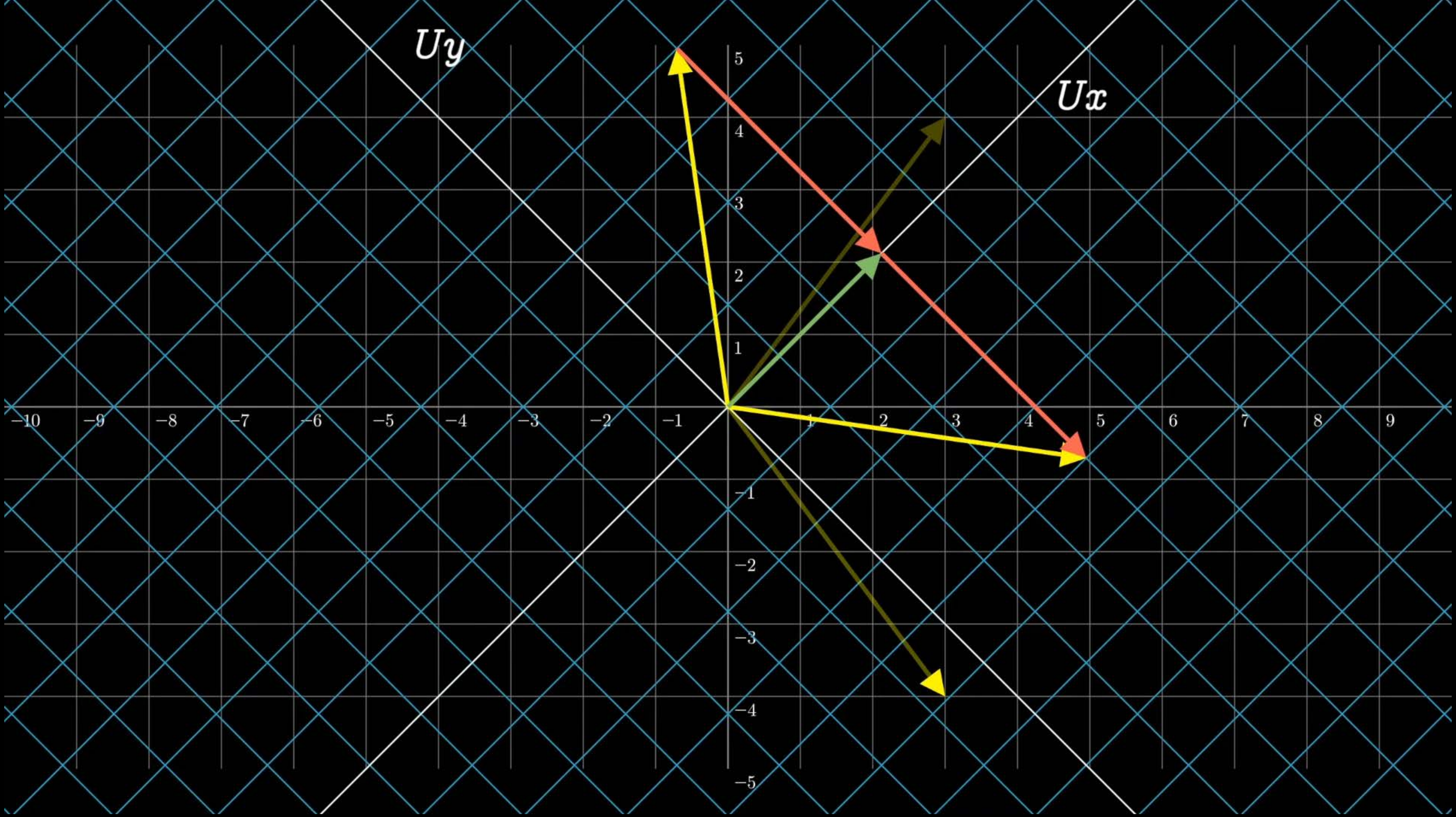
Next Q: How can we represent flipping with a more basic operation?
(e.g. subtraction or addition?)



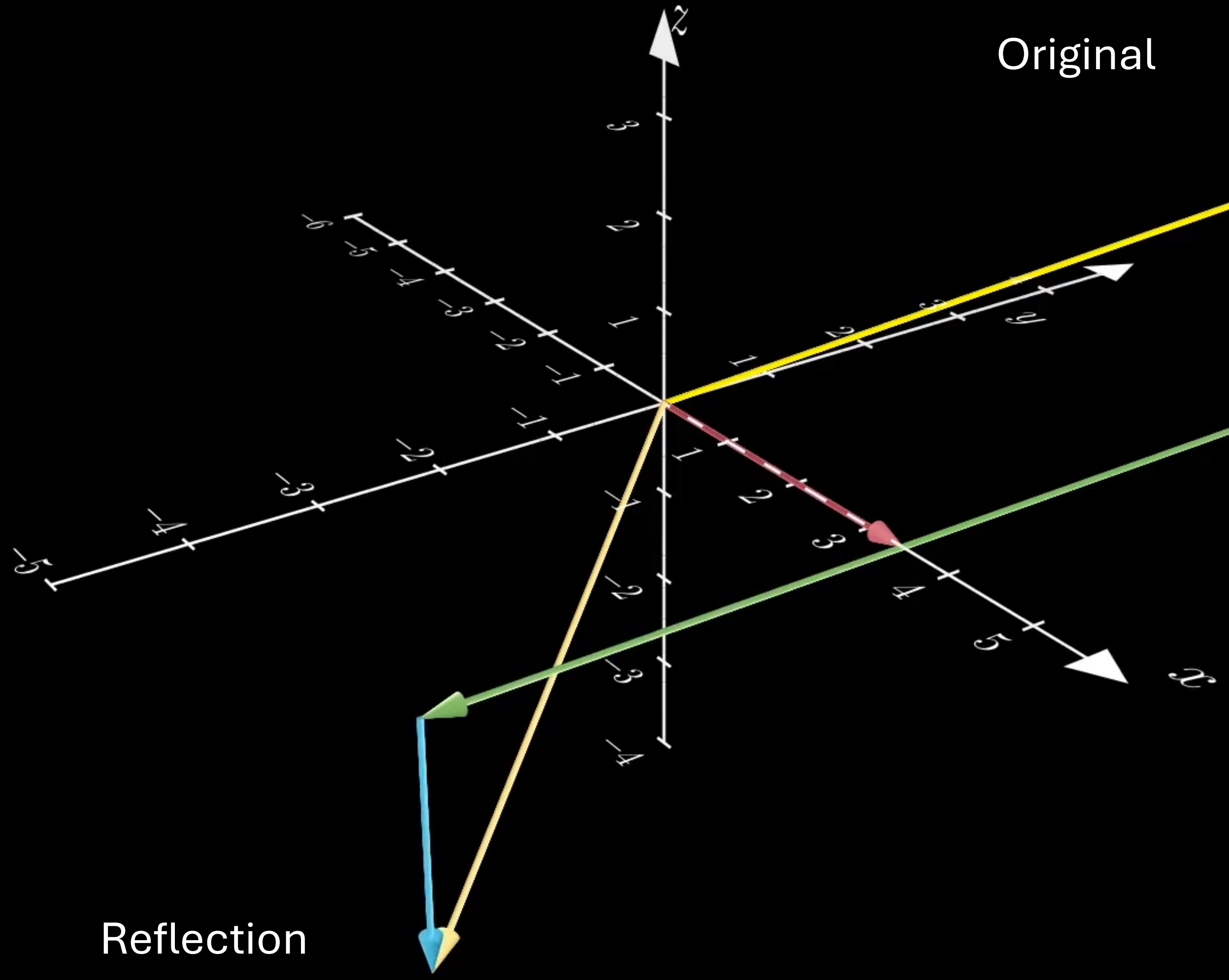
Next Q: How to we generalize this operation using a linear operator on our vector?



Next Q: Arbitrary alignment of x, y axis?



Next Q: Can we generalize this approach to a higher dimensions?



Same approach:
Subtract each components twice

Next Q: Instead of expressing what has changed, can we express what does not change?

Reflection: Linear Algebra

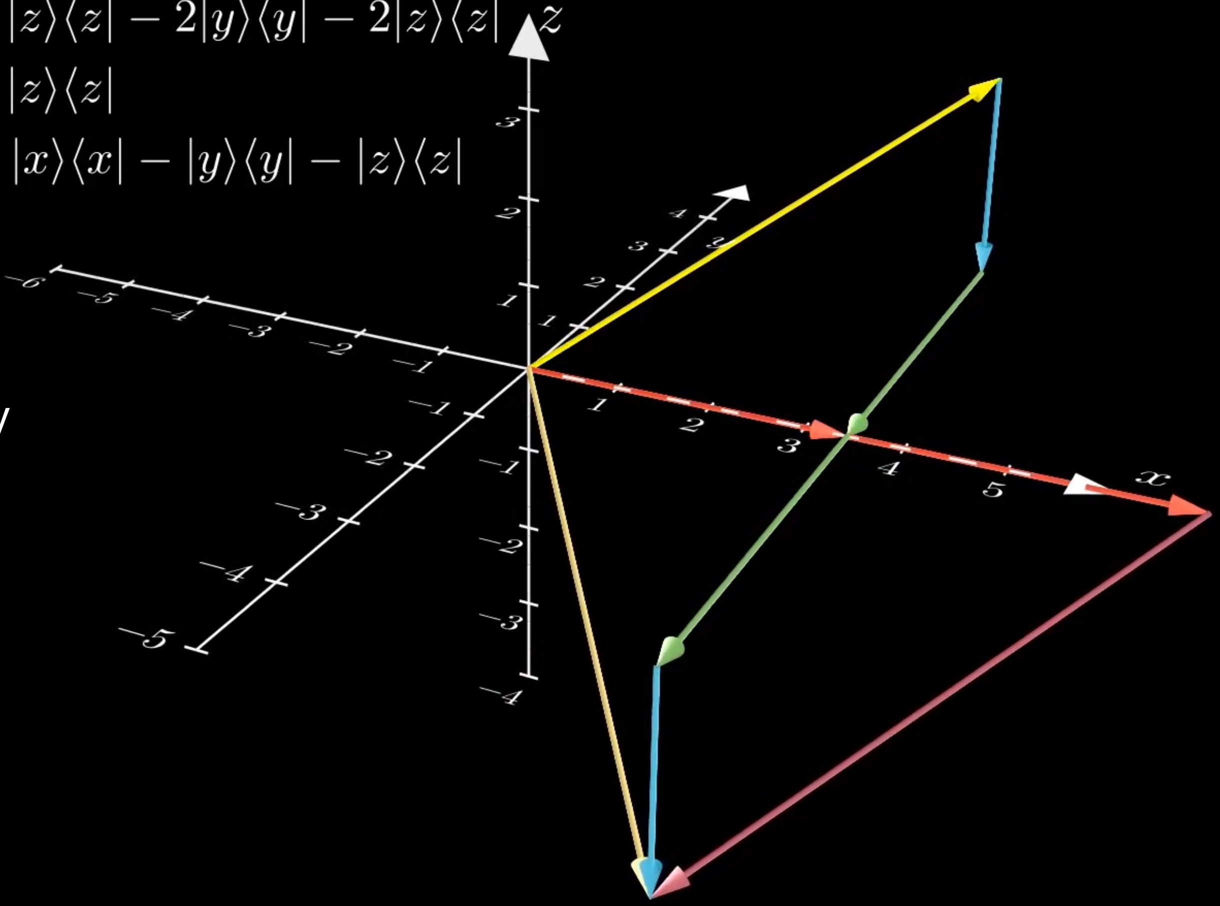
Subtracting all other components

Reflection operator about X

$$
\begin{aligned}
T &= I \boxed{- 2|y\rangle\langle y| - 2|z\rangle\langle z|} \\
&= |x\rangle\langle x| + |y\rangle\langle y| + |z\rangle\langle z| - 2|y\rangle\langle y| - 2|z\rangle\langle z| \\
&= |x\rangle\langle x| - |y\rangle\langle y| - |z\rangle\langle z| \\
&= |x\rangle\langle x| + |x\rangle\langle x| - |x\rangle\langle x| - |y\rangle\langle y| - |z\rangle\langle z| \\
&= 2|x\rangle\langle x| - I
\end{aligned}
$$

Twice the projector of x

Identity



Step 1: Preparing Initial State

Step 1: Probability initialization: **Initialize Search of $N = 2^n$ items**

- It is to initialize our quantum state vector to represent the unstructured search problem.

$$H^{\otimes n}|0^n\rangle = \frac{1}{\sqrt{N}} \sum_{x \in \{0,1\}^n} |x\rangle = |\phi\rangle$$

Hadamard gates

Puts them in a uniform superposition of all possible states

Qubits in 0-state

Initially, there is an equal probability that any of the keys will be the right key,
Our state vector is a superposition of all possibilities with mean uniform amplitude.

1 Qubit in 0-state

$$H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

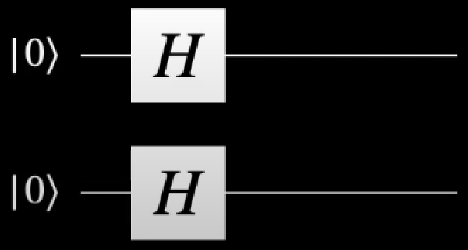
Equal probability/likelihood of measuring zero or one

2 Qubits in 0-state

$$(H|0\rangle)(H|0\rangle) = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)$$

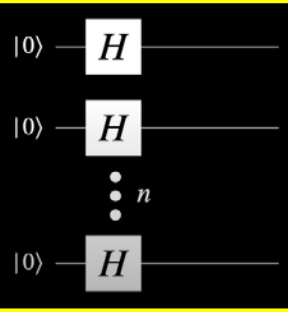
$$H^{\otimes 2}|00\rangle = \frac{1}{\sqrt{2^2}}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

four permutations of two qubits



n- Qubits in 0-state

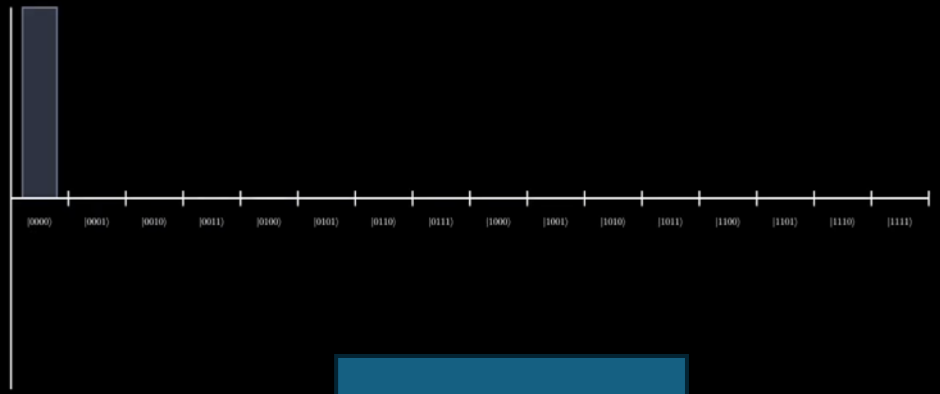
$$H^{\otimes n}|0^n\rangle = \frac{1}{\sqrt{2^n}} \sum_{x \in \{0,1\}^n} |x\rangle = |\phi\rangle$$



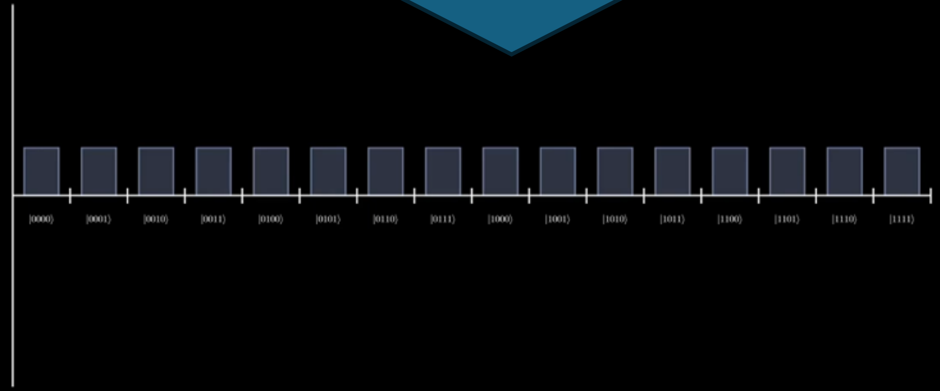
Possible permutations.
Each of these permutations represents one state in our computational basis for n qubits

possible permutation with replacement of 0 and 1 that forms a string of length n

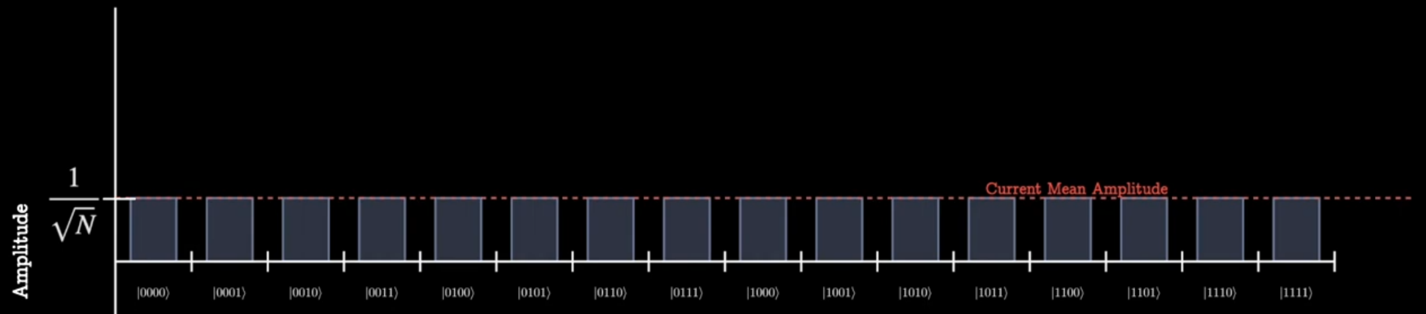
System: 4 Qubits: 16 possible states



Apply Hadamard gates

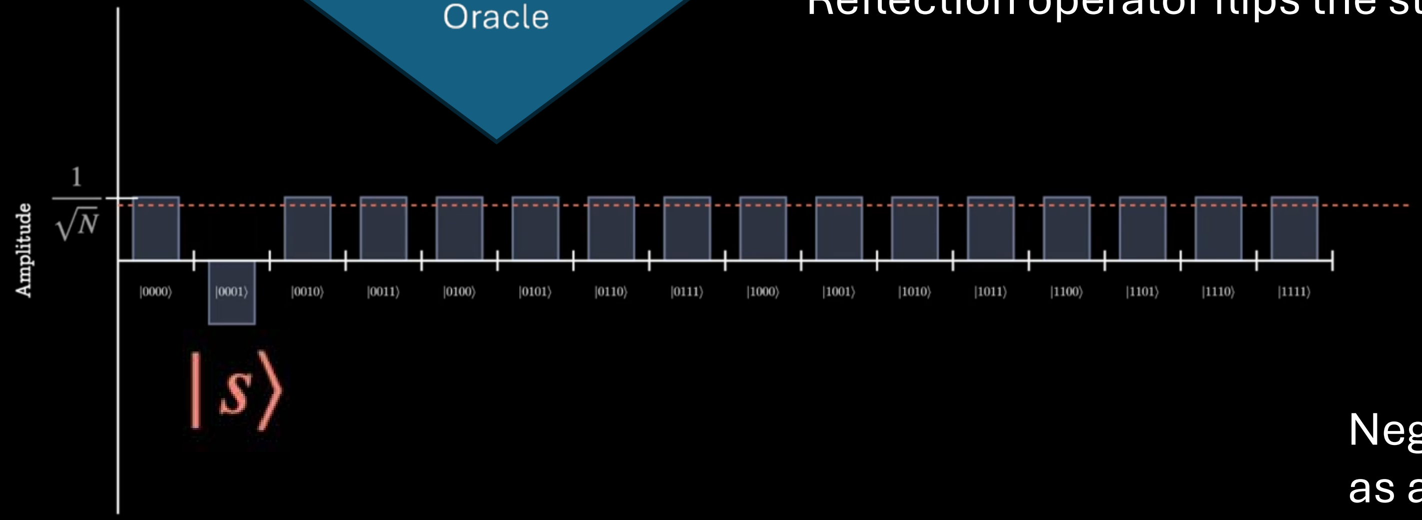


Uniform superposition state of $|\phi\rangle$

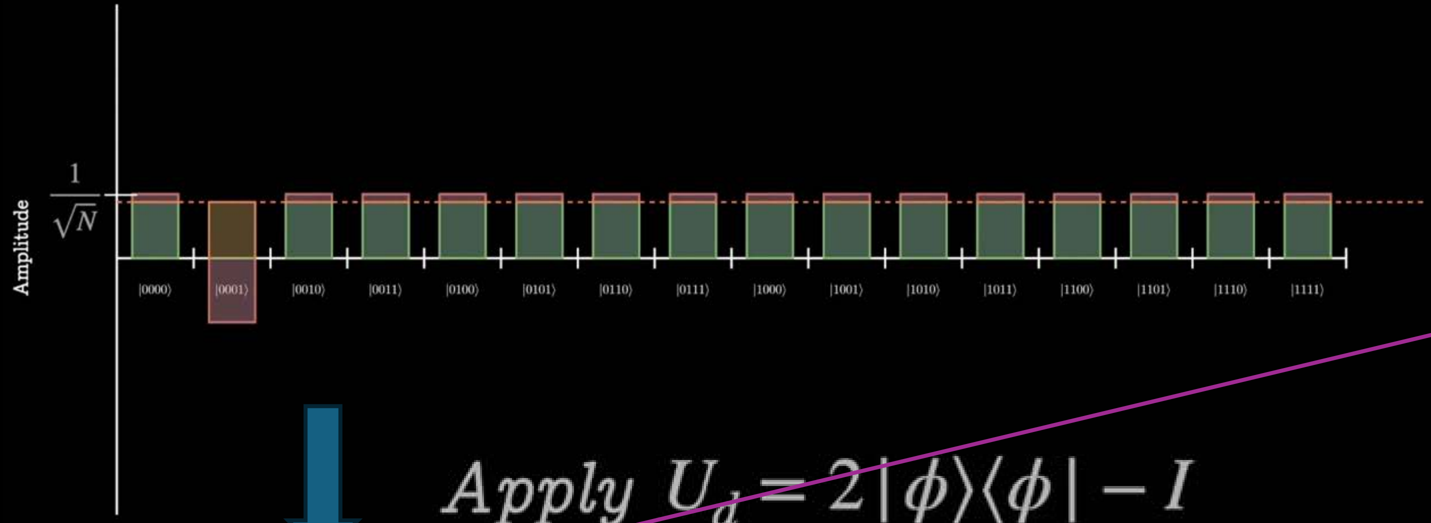


$$Apply\ U_s = I - 2|s\rangle\langle s|$$

Reflection operator flips the state we are searching for



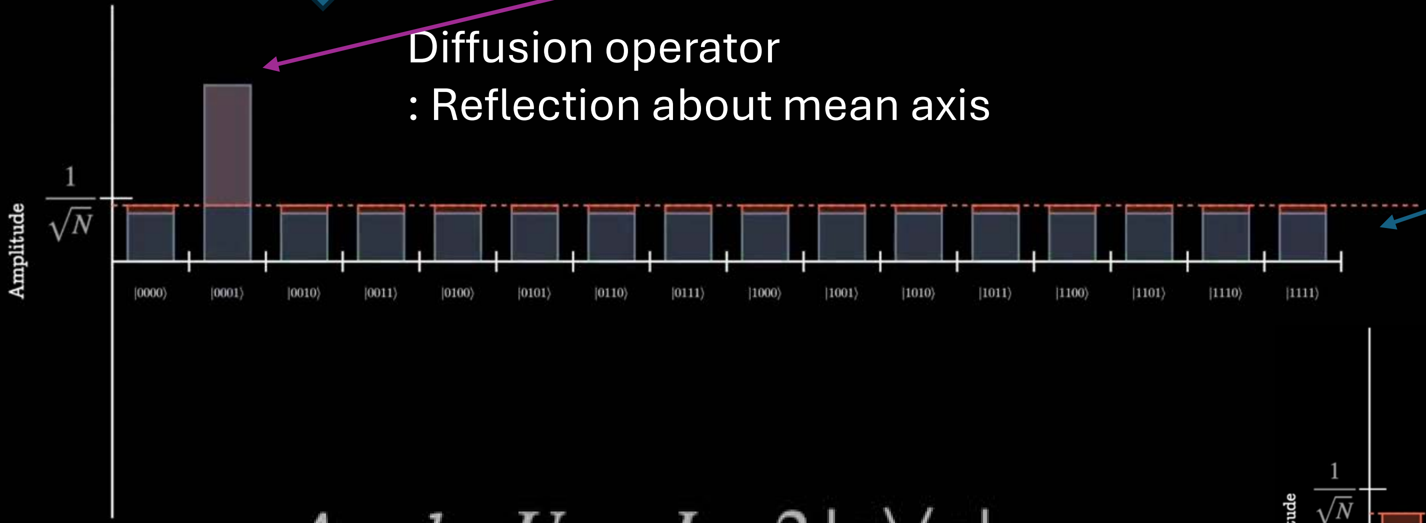
Negative amplitude still gives us the same probability as a positive amplitude.
Therefore, reflection alone does not help much.



When we flip across the mean, our good state amplitude increases more since there's a greater difference between its negative amplitude and the mean.

$$Apply\ U_d = 2|\phi\rangle\langle\phi| - I$$

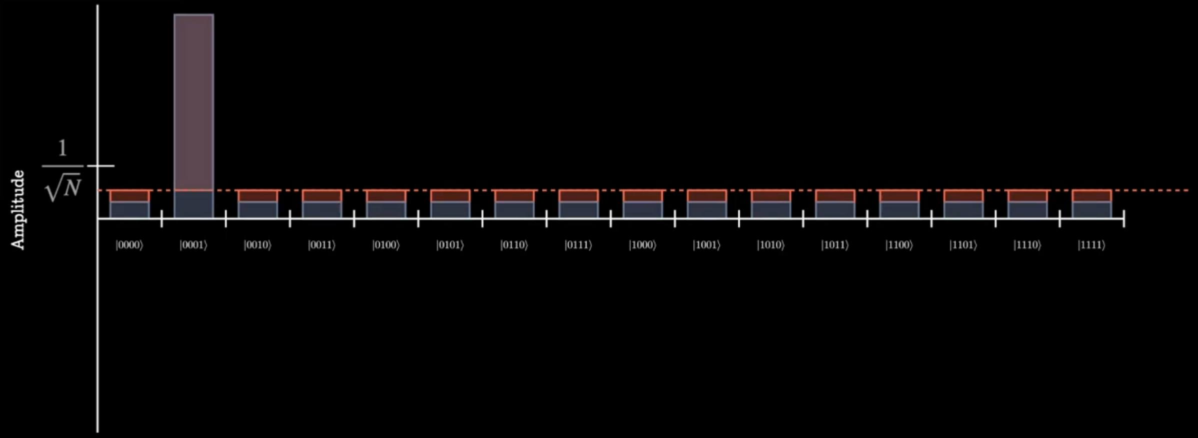
Diffusion operator
: Reflection about mean axis

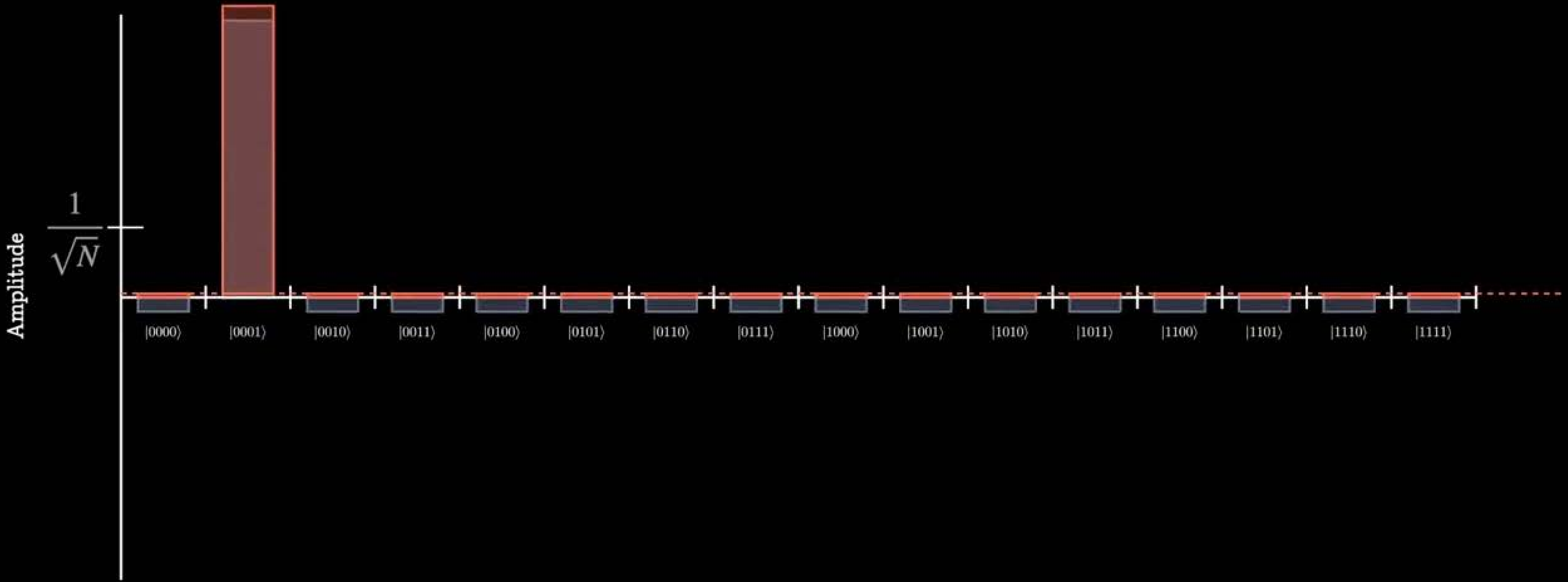


every other bad state decreases slightly

$$Apply\ U_s = I - 2|s\rangle\langle s|$$

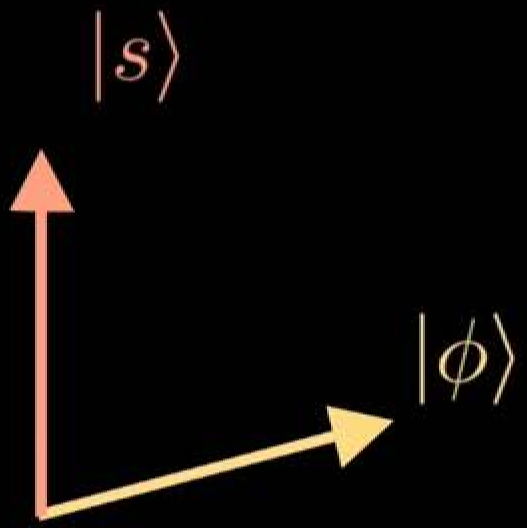
Then $$Apply\ U_d = 2|\phi\rangle\langle\phi| - I$$





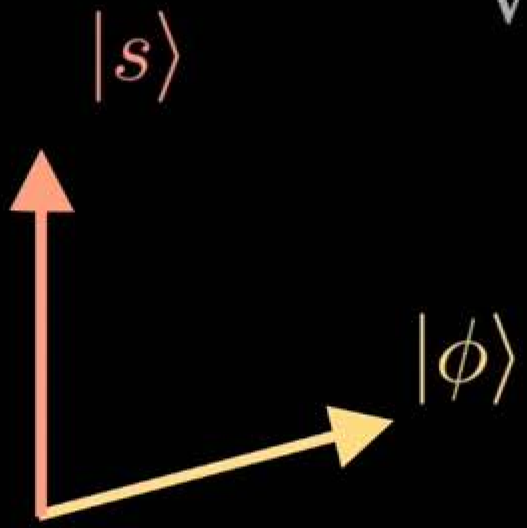
Q: How many times should we continue these operations?

If we amplify too much, our mean amplitude will become negative. This causes us to actually decrease the amplitude of our good states and we amplify the bad states again, which is quite interesting. Therefore, we have to figure out how many times we should run this Grover's operation before we stop to measure.



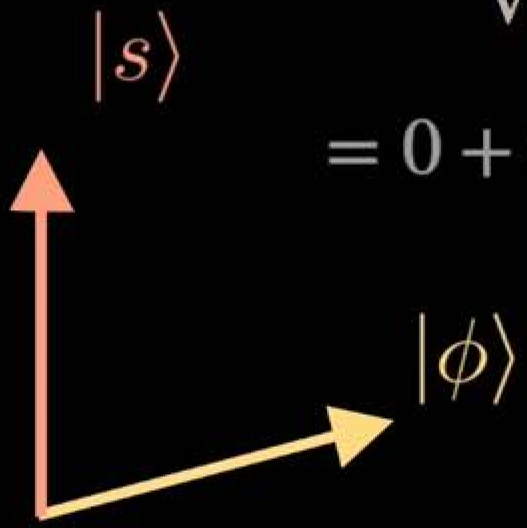
|Phi> contains a component of a good state |s>, i.e., not orthogonal

$$\langle s | \phi \rangle = \langle s | \frac{1}{\sqrt{N}} \sum_{x \in \{0,1\}^n} | x \rangle$$

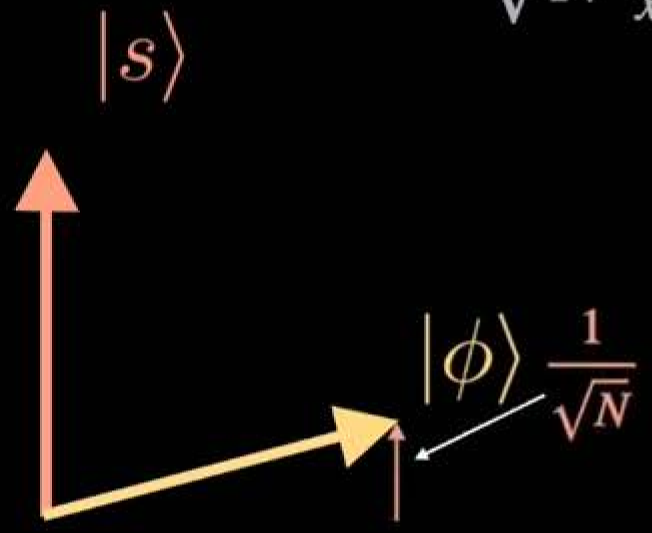


$$\langle s | \phi \rangle = \langle s | \frac{1}{\sqrt{N}} \sum_{x \in \{0,1\}^n} | x \rangle$$

$$= 0 + 0 + \ldots + \frac{1}{\sqrt{N}} + \ldots + 0$$

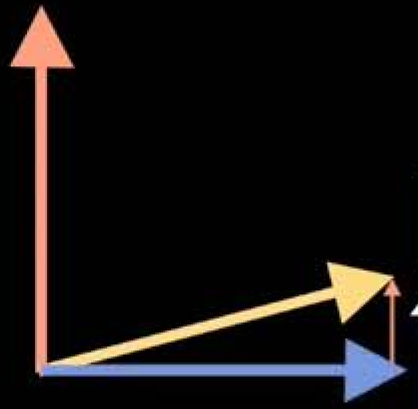


$$\langle s|\phi\rangle = \langle s|\frac{1}{\sqrt{N}}\sum_{x\in\{0,1\}^n}|x\rangle$$



$$\langle s|\phi\rangle = \langle s|\frac{1}{\sqrt{N}}\sum_{x\in\{0,1\}^n}|x\rangle$$

$|s\rangle$



$|\phi\rangle$ $\frac{1}{\sqrt{N}}$

$$|s^{\perp}\rangle = \frac{1}{\sqrt{N-1}}\left(\sum_{x\in\{0,1\}^n}|x\rangle - |s\rangle\right)$$

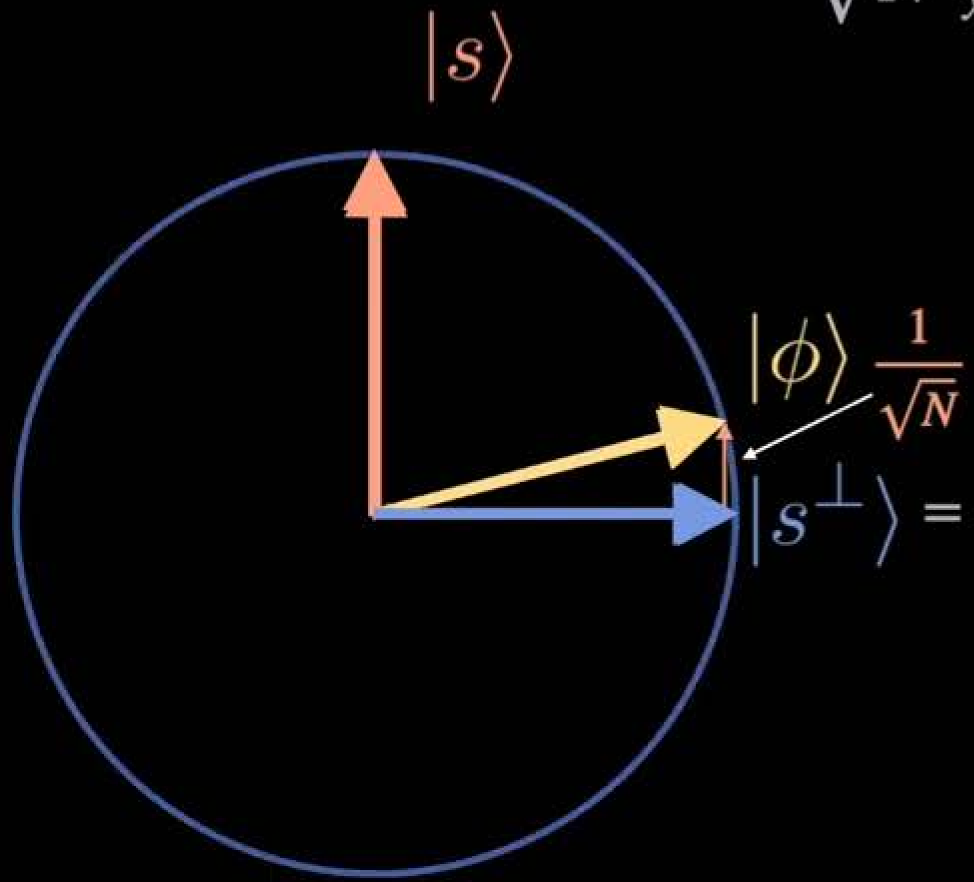
Orthogonal state

Normaliz ation

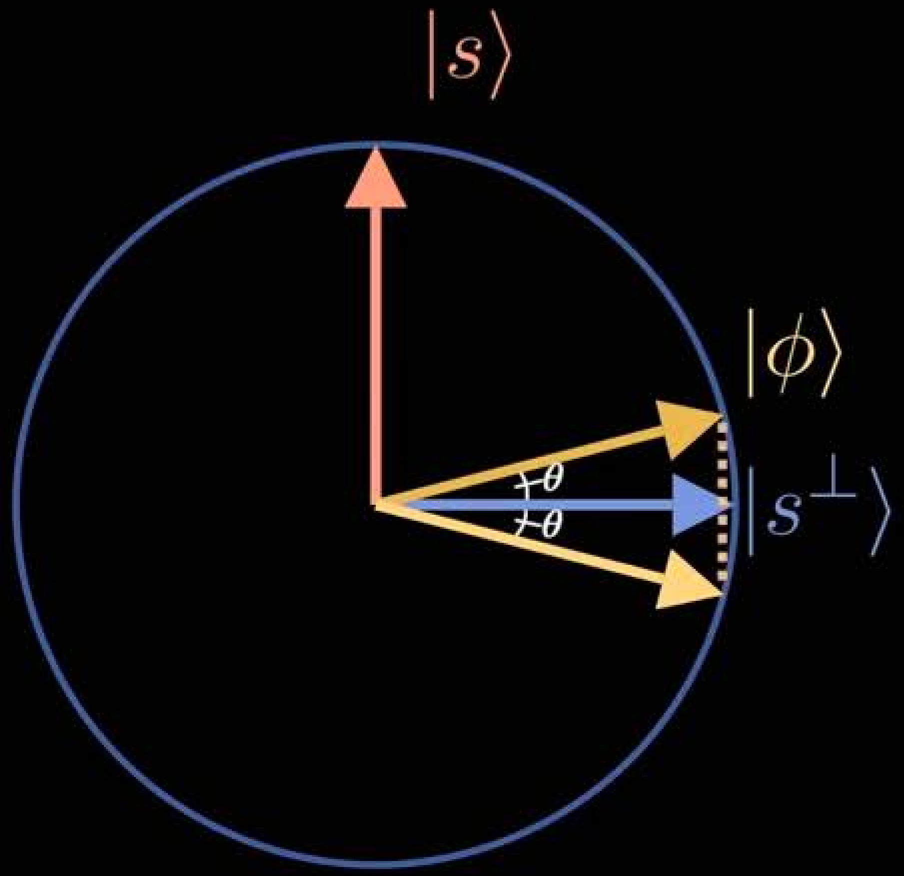
All states

Good state

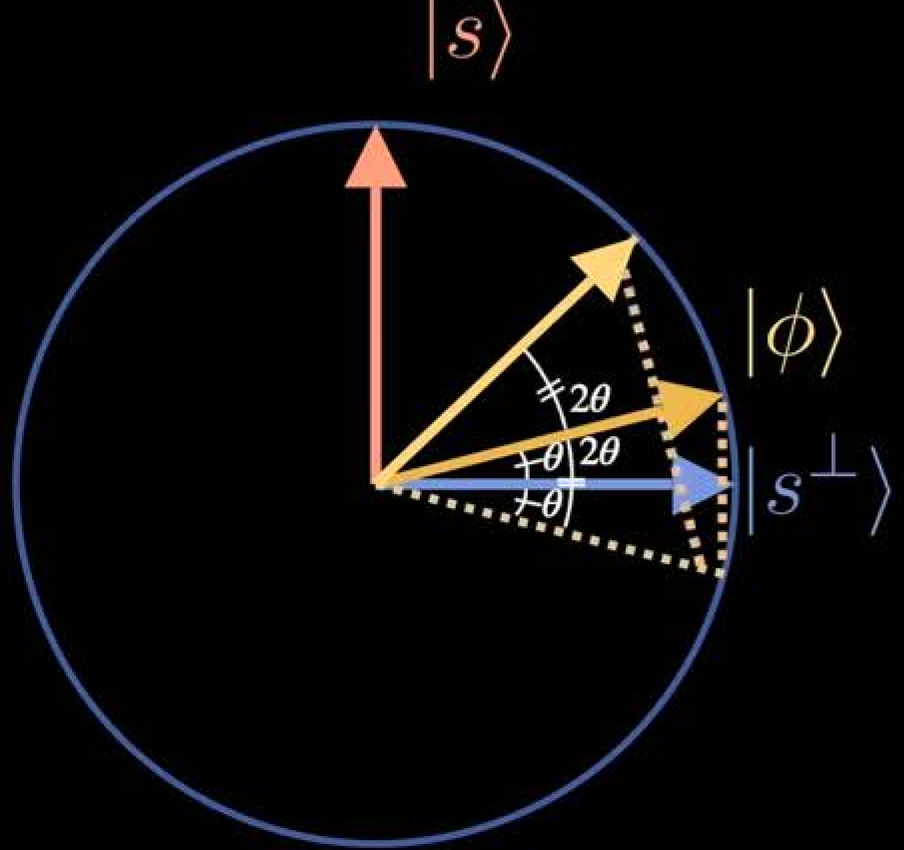
$$\langle s | \phi \rangle = \langle s | \frac{1}{\sqrt{N}} \sum_{x \in \{0,1\}^n} | x \rangle$$



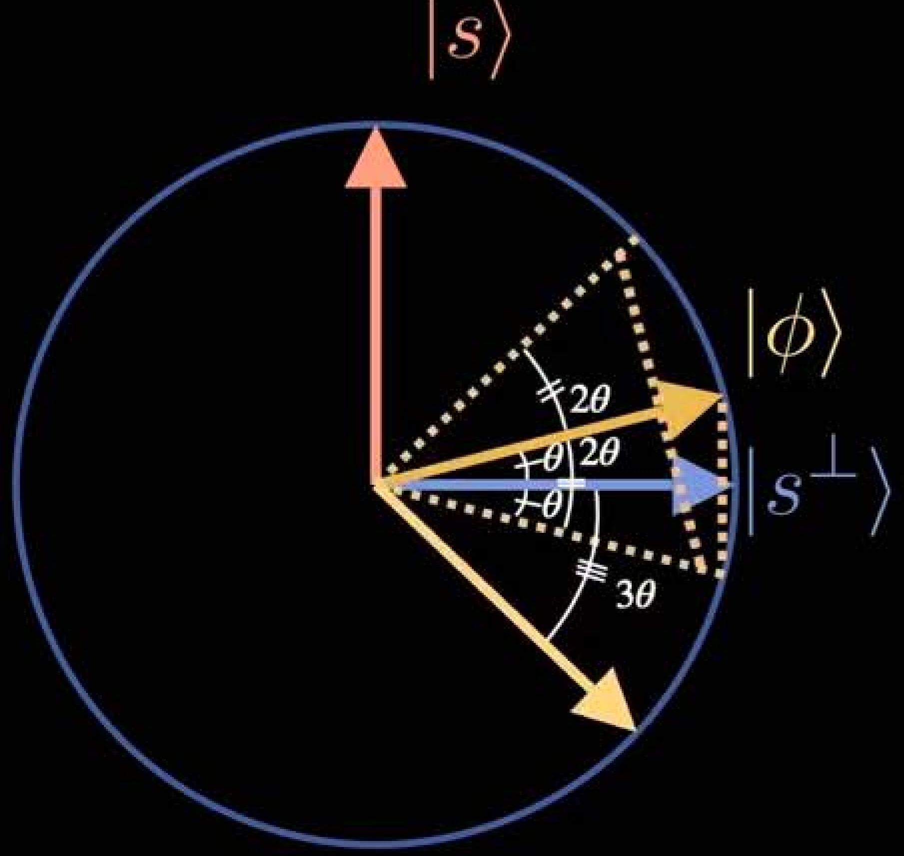
$$|s^{\perp}\rangle = \frac{1}{\sqrt{N-1}} \left(\sum_{x \in \{0,1\}^n} |x\rangle - |s\rangle \right)$$



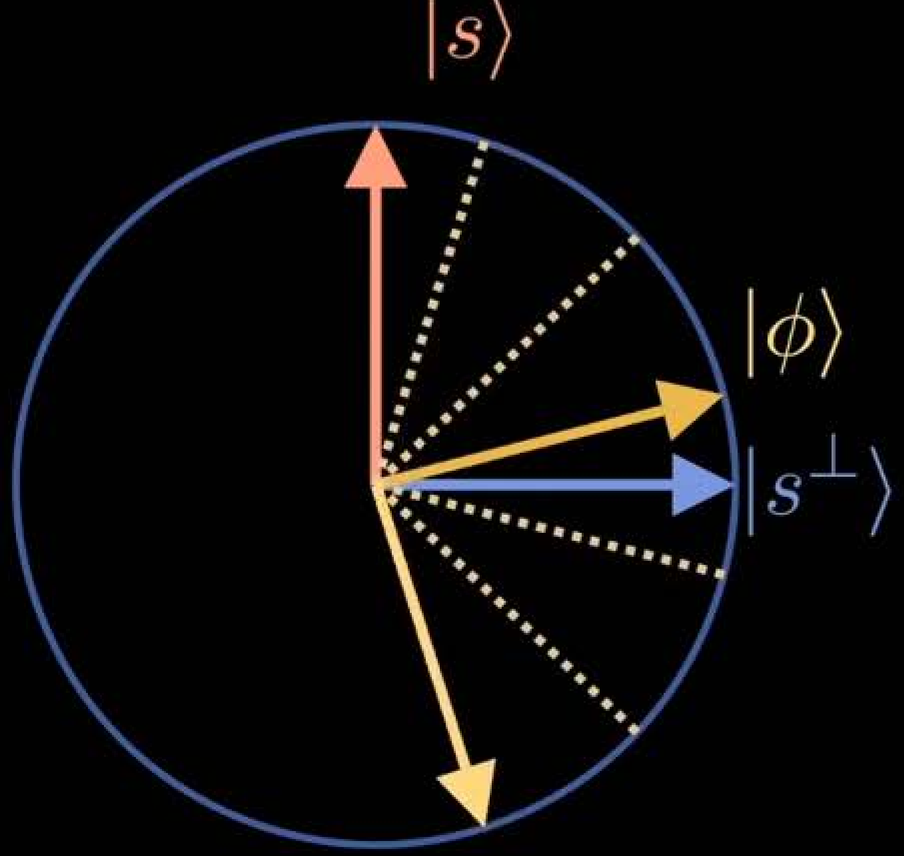
After Applying:
Search Operator



After Applying:
Diffusion operator



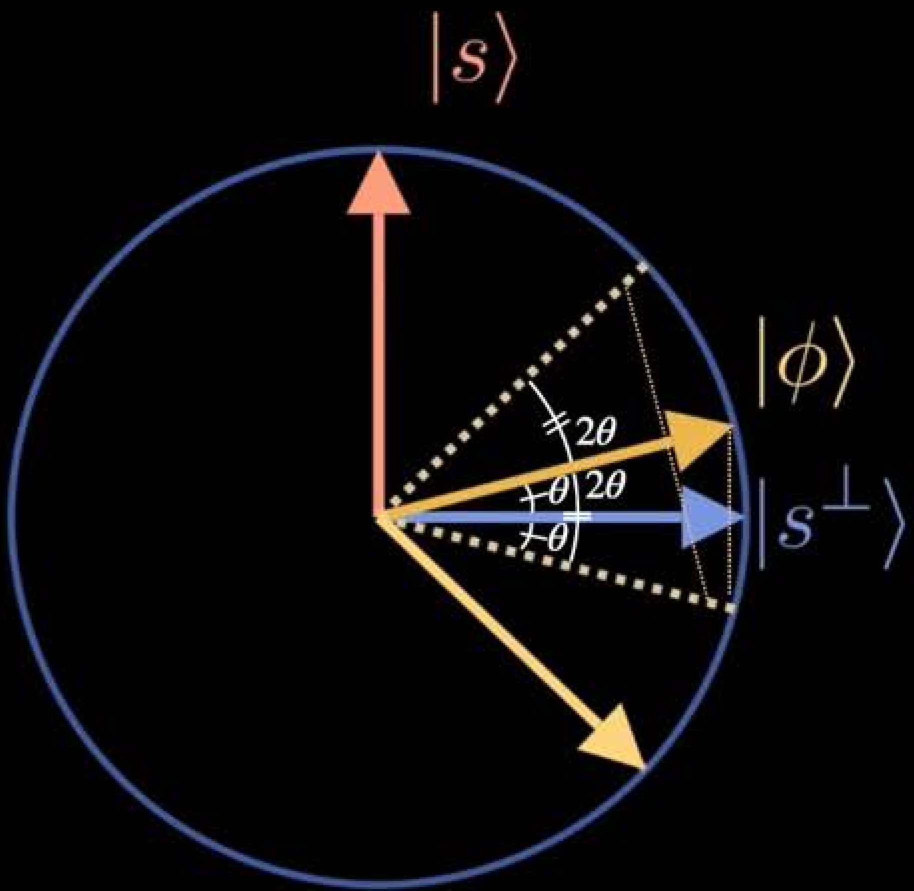
$|s\rangle$
$|\phi\rangle$
$|s^{\perp}\rangle$
2θ
2θ
θ
θ
3θ



$|s\rangle$
$|\phi\rangle$
$|s^{\perp}\rangle$

Q: How many thetas will get us the closest to pi over 2 when the amplitude of |s> will be the greatest? → Maximize the search probability.

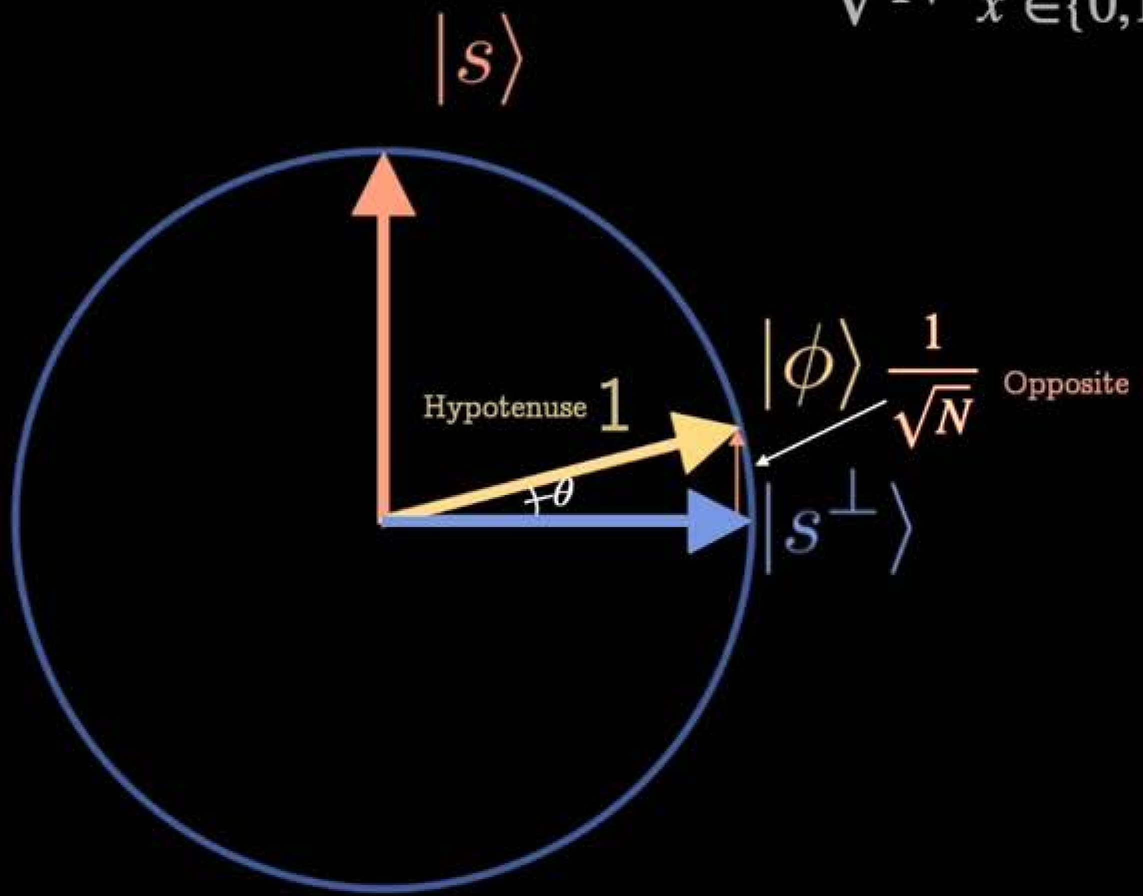
$$(2t+1)\theta \longrightarrow \frac{\pi}{2}$$



Q: what is theta?

$$\langle s | \phi \rangle = \langle s | \frac{1}{\sqrt{N}} \sum_{x \in \{0,1\}^n} |x\rangle$$

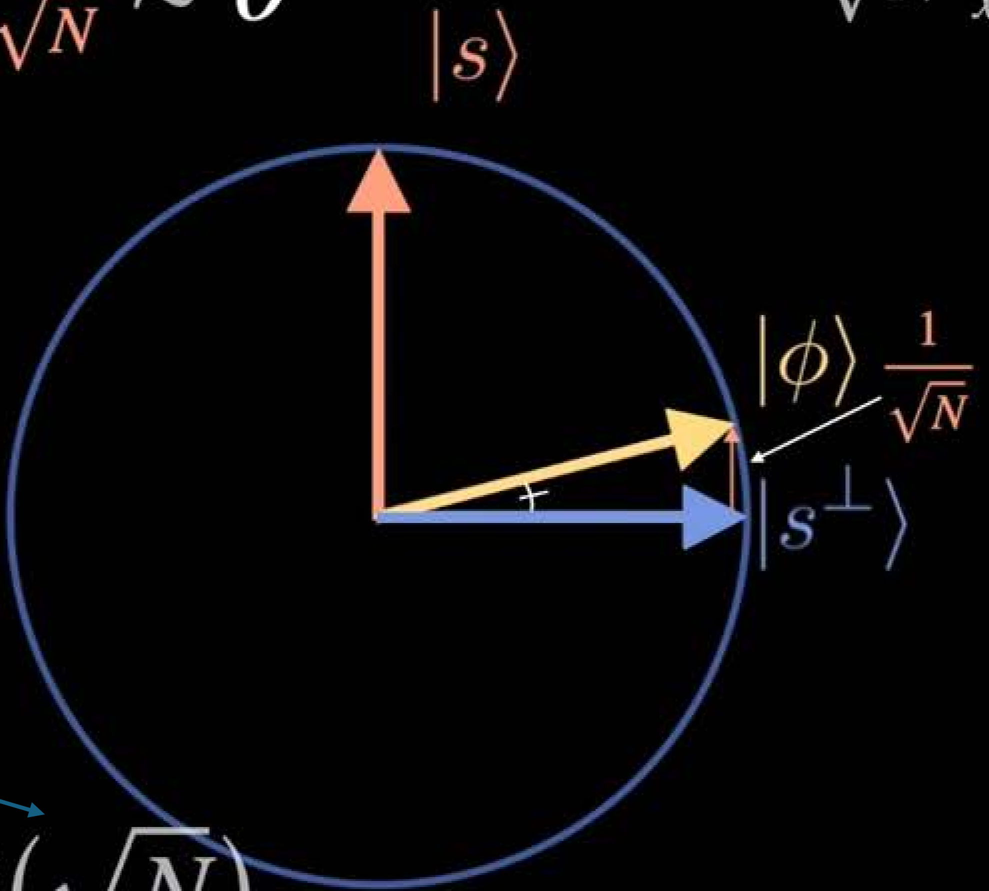
$$arcsin(\frac{\text{Opposite}}{\text{Hypotenuse}})$$



$arcsin(\frac{\text{Opposite}}{\text{Hypotenuse}})$

$$arcsin(\frac{1}{\sqrt{N}}) \approx \frac{1}{\sqrt{N}} \approx \theta$$

$$\langle s|\phi\rangle = \langle s|\frac{1}{\sqrt{N}}\sum_{x\in\{0,1\}^n}|x\rangle$$



$$(2t+1)\frac{1}{\sqrt{N}} \longrightarrow \frac{\pi}{2}$$

$$\frac{2t}{\sqrt{N}} = \frac{\pi}{2} - \frac{1}{\sqrt{N}}$$

$$t = \frac{\pi}{4}\sqrt{N} - \frac{1}{2}$$

Scaling matters,
and not coefficient

$$\text{Run } t \text{ times} \longrightarrow \mathcal{O}(\sqrt{N})$$

$$\mathcal{O}(N) \longrightarrow \mathcal{O}(\sqrt{N})$$

Classical Quantum

$$\mathcal{O}(N) \longrightarrow \mathcal{O}(\sqrt{N})$$

(Only for one block)

Old Classical → Quantum

- Modern Classical vs Quantum?
- Using custom built FFT Chips: Classical cost can be reduced further.
- Initializing state: sum over qubit, and sum over templates (to be fixed!)
- Compact-binary coalescence (CBC): Modern classical algorithm wins
- Continuous Waveform (CW): Days long: Quantum Computer might be necessary!

Open Questions:

- Finding the best way to compare the classical and quantum algorithm results:
  - Classical algorithm measures in the number of arithmetic operations per unit time of input data.
    Hence, Total cost should be evaluated by multiplying by the total observation time.
  - Quantum computer measures total cost
- Are there any cost reduction method in quantum algorithm (similar to the modern pipeline orthonormal basis reduction?
- Is this quantum algorithm applicable to any other research fields of the other speakers? OR could any of your methods be useful to improve GW detection?