QUANTINUUM

Simulating Floquet scrambling circuits on trapped-ion quantum computers

Presented by:

Yuta Kikuchi (Quantinuum, RIKEN iTHEMS)
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with Kazuhiro Seki (RIKEN), Tomoya Hayata (Keio U., RIKEN), Seiji Yunoki (RIKEN)

arXiv:2405.07613

Introduction

Quantum advantage for artificial tasks

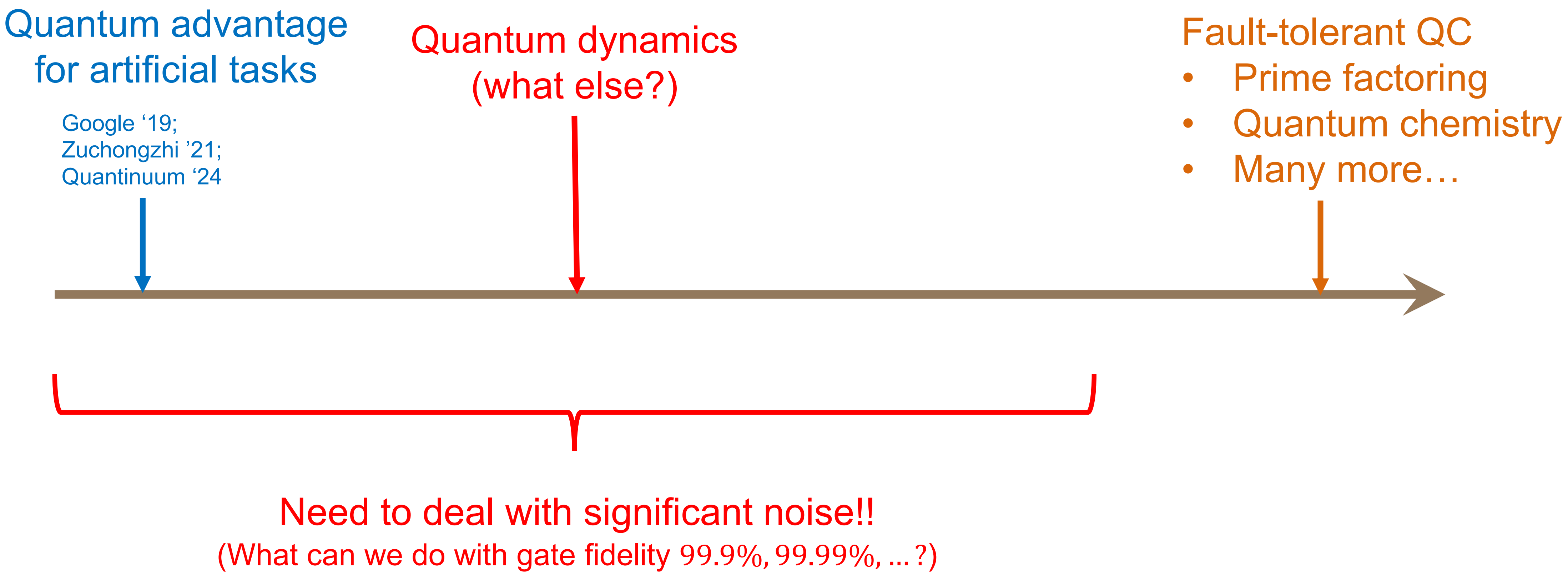
Google '19;
Zuchongzhi '21;
Quantinuum '24

Quantum dynamics (what else?)

Fault-tolerant QC

- Prime factoring
- Quantum chemistry
- Many more…

Introduction



Digital quantum simulation on noisy hardware

Setup

- Measure a local observable.
- Circuit has geometrically local connectivity.

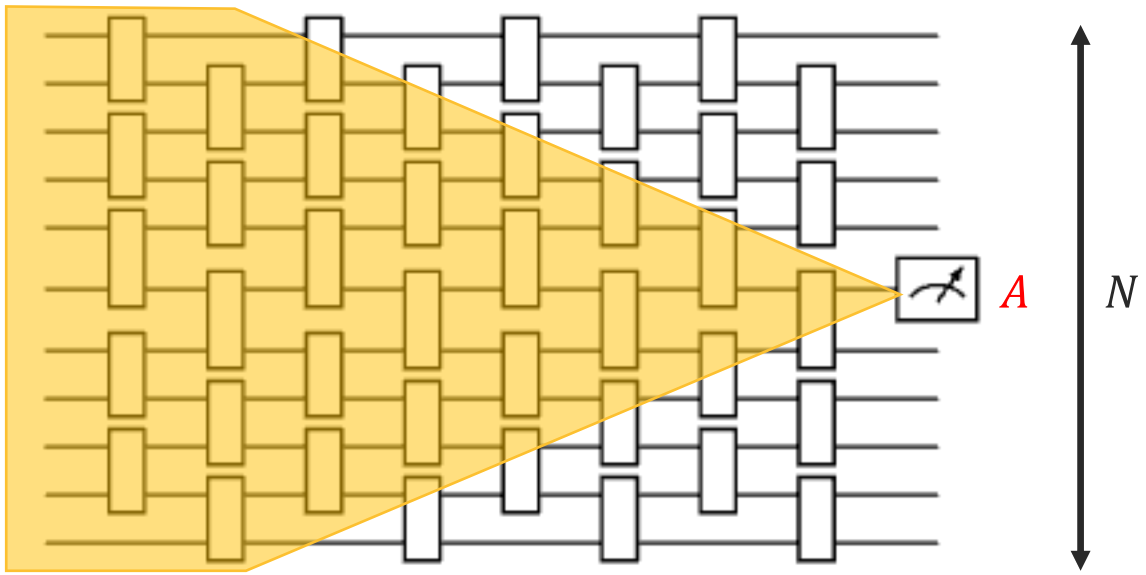
Crude estimate of error

- Dominant source of error is 2-qubit gates.
- Yellow contains N_{2Q} 2-qubit gates.
- Model the noisy circuit by the reduced density matrix

$$\rho_A^{\text{noisy}} = f\rho_A^{\text{ideal}} + (1-f)\frac{I^{|A|}}{2^{|A|}}, \qquad f = \left(1 - p_{2Q}\right)^{N_{2Q}}$$

- Expectation value of traceless operator O is

$$\langle O \rangle^{\text{noisy}} = f\langle O \rangle^{\text{ideal}}$$



Digital quantum simulation on noisy hardware

Hamiltonian simulation by Trotterization

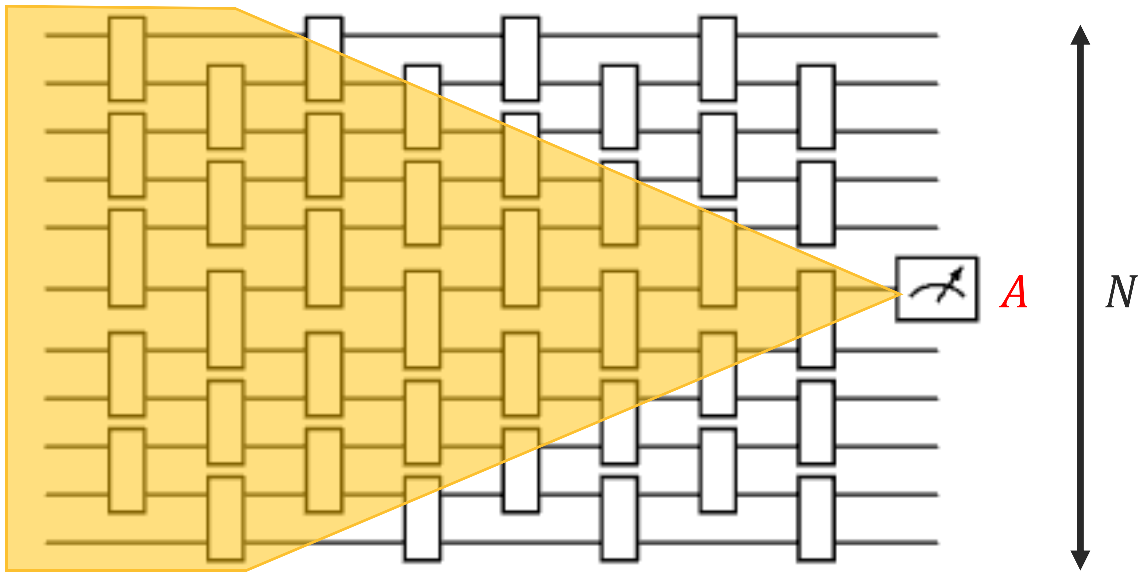
$$H = H_1 + H_2$$

$$e^{-iHt} \approx \left(e^{-iH_1T} e^{-iH_2T}\right)^{t/T}$$

Crude estimate of error

- Parameters: $t = 5,\ T = 0.1,\ N = 20.$
- # of 2Q gates: $N_{2Q} \approx N\frac{t}{T} = 1000.$
- 2Q gate error: $p_{2Q} = 2\times10^{-3}$
- Expectation value of traceless operator O is

$$\langle O\rangle^{\text{noisy}} = f\langle O\rangle^{\text{ideal}}, \qquad f = \left(1 - p_{2Q}\right)^{N_{2Q}} = 0.135$$



Floquet dynamics

Floquet dynamics:

- Periodically driven dynamics described by Hamiltonian $H(t+T) = H(t)$.
- Floquet systems eventually heat up to infinite temperature by acquiring energy from the driving force (Floquet heating).

Lanzarides, Das & Moessner '14;
D'Alessio & Rigol '14;
Abanin, Roeck & Huveneers '15;
Mori, Kuwahara & Saito '16

Trotterization

- Trotter dynamics for time mT, $\left(\mathrm{e}^{-\mathrm{i}H_1 T/2}\mathrm{e}^{-\mathrm{i}H_2 T/2}\right)^{2m}$, is a Floquet dynamics of an m cycles.
- Qualitative change happens as the period T is increased (Trotter transition)

→ Floquet heating (thermalization) happens during Trotter dynamics

Heyl, Hauke & Zoller '18;
Varnier, Bertini, Giudici & Piroli '23

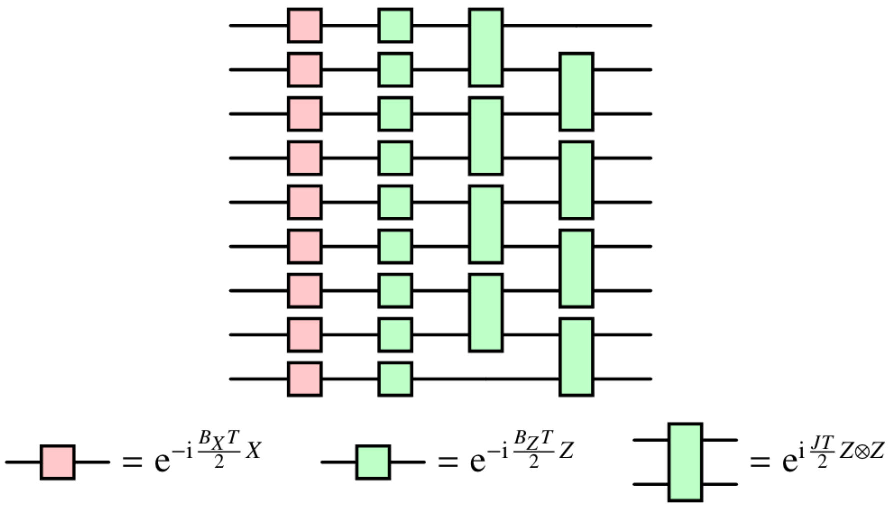
Information scrambling by Floquet circuits

Our target model:

Prosen '02, '07

Kicked-Ising model / BKP model / Floquet Ising model / Trotterized Ising spin chain

- $U_{\mathrm{F}} = \mathrm{e}^{-\mathrm{i}H_Z\frac{T}{2}}\mathrm{e}^{-\mathrm{i}H_X\frac{T}{2}}$,
- $H_Z = -J\sum_i Z_i Z_{i+1} + B_Z \sum_i Z_i, \quad H_X = B_X \sum_i X_i$
- Maximally chaotic (self-dual) point at $|JT| = |B_X T| = \frac{\pi}{2}$



Information scrambling:

- A process of the lost information spreading across the system.
- The more scrambling the dynamics is the harder it is to recover the initial information.

→ Diagnose the complexity of dynamics

Experiments on trapped-ion quantum computers

GOAL:

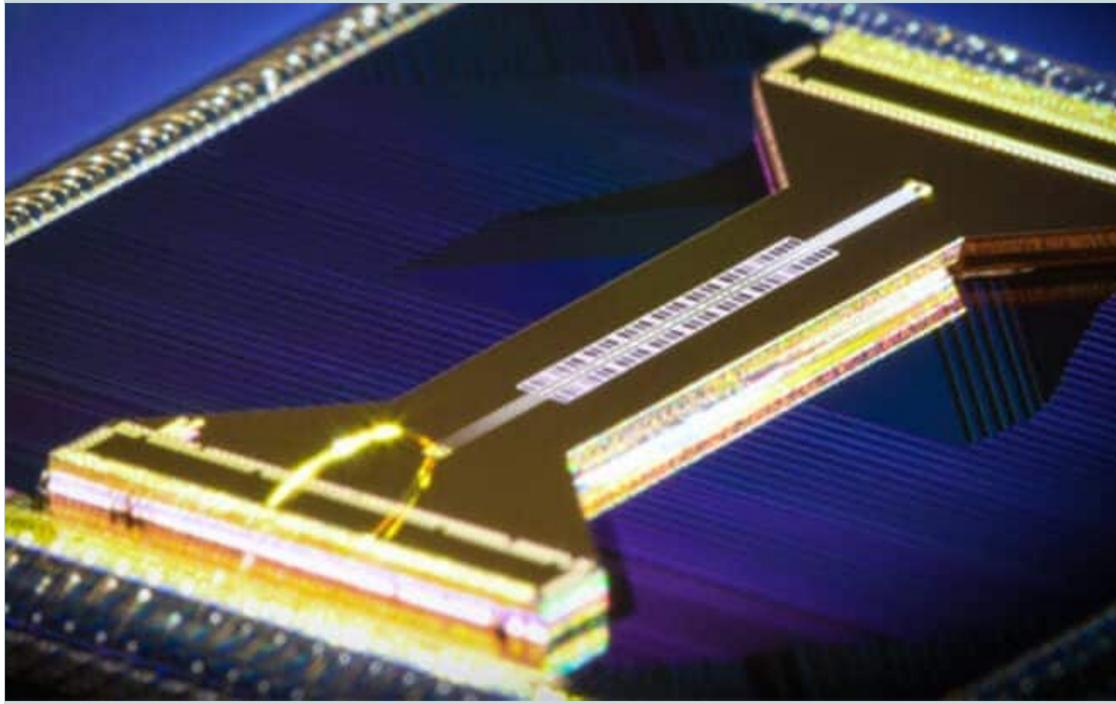
Access the feasibility of scrambling simulation on the current hardware

Seki, YK, Hayata, Yunoki '24

1. Hayden-Preskill recovery protocol
2. Interferometric protocol for out-of-time-ordered correlators
3. Thermal expectation value with microcanonical TPQ states

System Model H1

Available on various platforms



99.91% Two-qubit gate fidelity (arbitrary angle)

20 qubits

99.998% Single-qubit gate fidelity

Measurement cross talk error < 0.01%

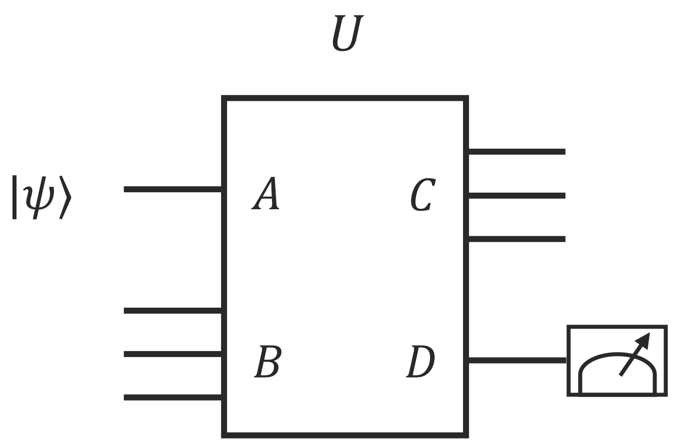
All-to-all-connectivity

SPAM fidelity > 99.7%

Hayden-Preskill recovery (HPR) protocol [Theory]

Information scrambling

Hayden & Preskill '07;
Sekino & Susskind '08

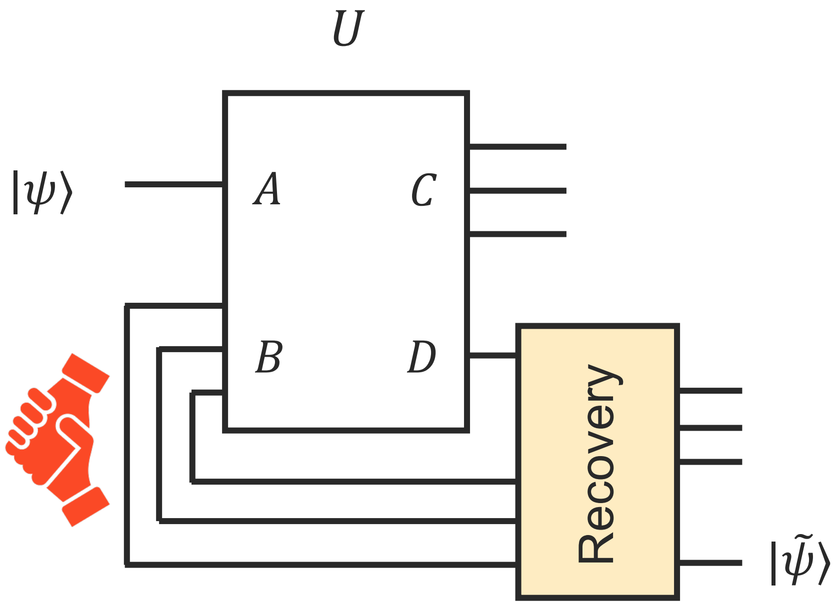


- Cannot learn about $|\psi\rangle$ from any local measurement if U scrambles information.

Hayden-Preskill recovery (HPR) protocol [Theory]

Information recovery

Hayden & Preskill '07

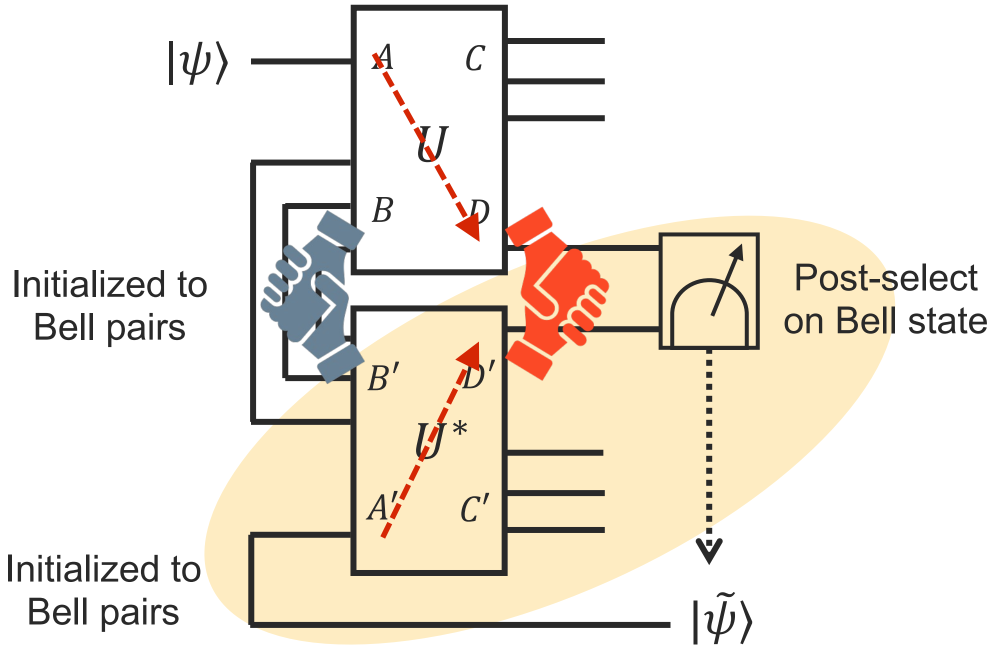


- Entanglement resource helps the recovery of the input state.
- $F_{\mathrm{EPR}} = |\langle\psi|\tilde{\psi}\rangle| \approx 1$ at late times if U is scrambling
- What is the recovery protocol?

Hayden-Preskill recovery (HPR) protocol [Theory]

Recovery protocol

Yoshida & Kitaev '17



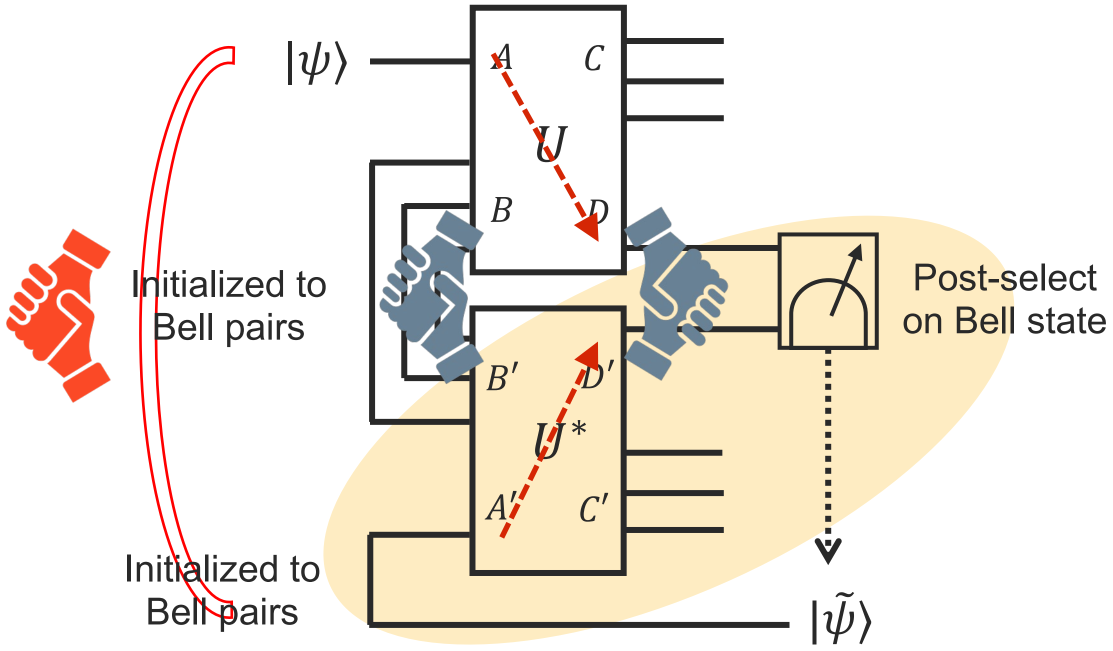
- States on B and B' are maximally entangled
- Information propagates from A to D
- Information propagates from A' to D'
- States on D and D' are projected on a Bell state

  → States on A and A' are maximally entangled

Hayden-Preskill recovery (HPR) protocol [Theory]

Recovery protocol

Yoshida & Kitaev '17



- States on B and B' are maximally entangled
- Information propagates from A to D
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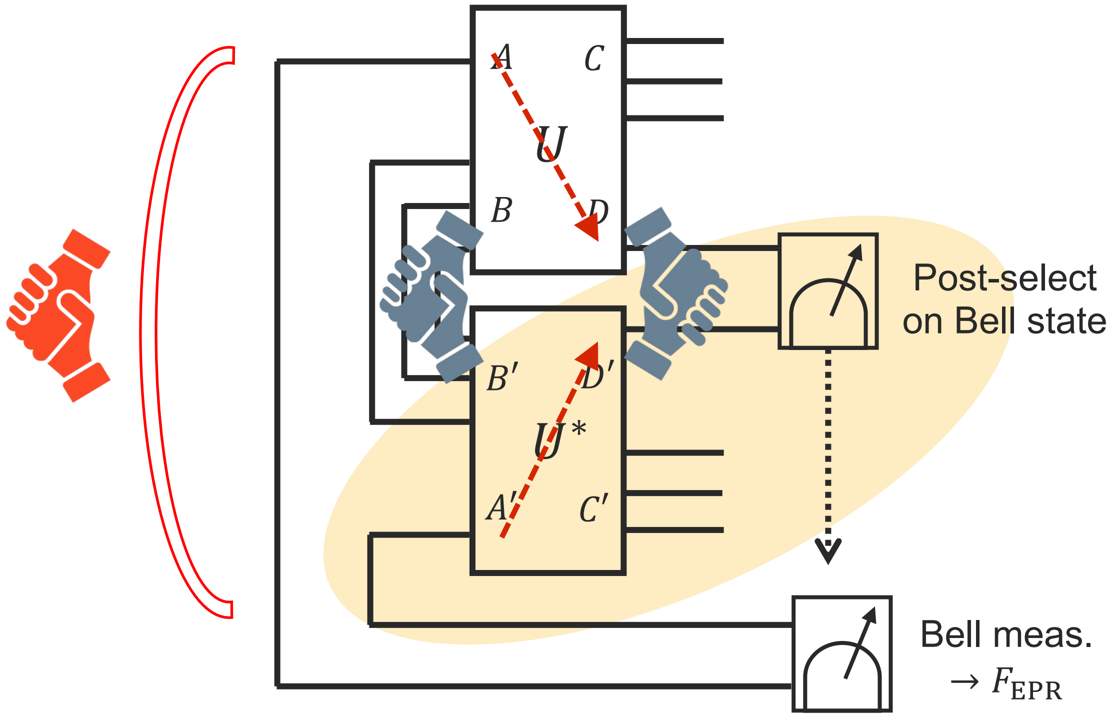
→ States on A and A' are maximally entangled

Hayden-Preskill recovery (HPR) protocol [Theory]

Recovery protocol

Yoshida & Kitaev '17



- States on B and B' are maximally entangled
- Information propagates from A to D
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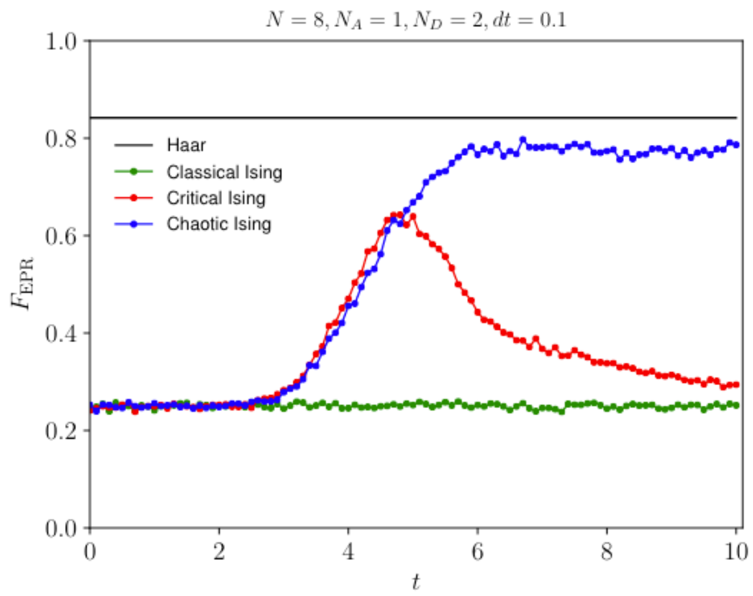
- States on D and D' are projected on a Bell state

  → States on A and A' are maximally entangled



Hayata, Hidaka, YK '21

$$H = -J\sum_i Z_i Z_{i+1} + B_Z \sum_i Z_i + B_X \sum_i X_i$$

Hayden-Preskill recovery (HPR) protocol [Experiment]

Seki, YK, Hayata, Yunoki '24

Setup

- 2 copies of 9-qubit spin chain + 2 ancilla qubits
- $U = \left(e^{-iH_Z\frac{T}{2}}e^{-iH_X\frac{T}{2}}\right)^m$
- Mitigation is done assuming the global depolarizing channel on U:

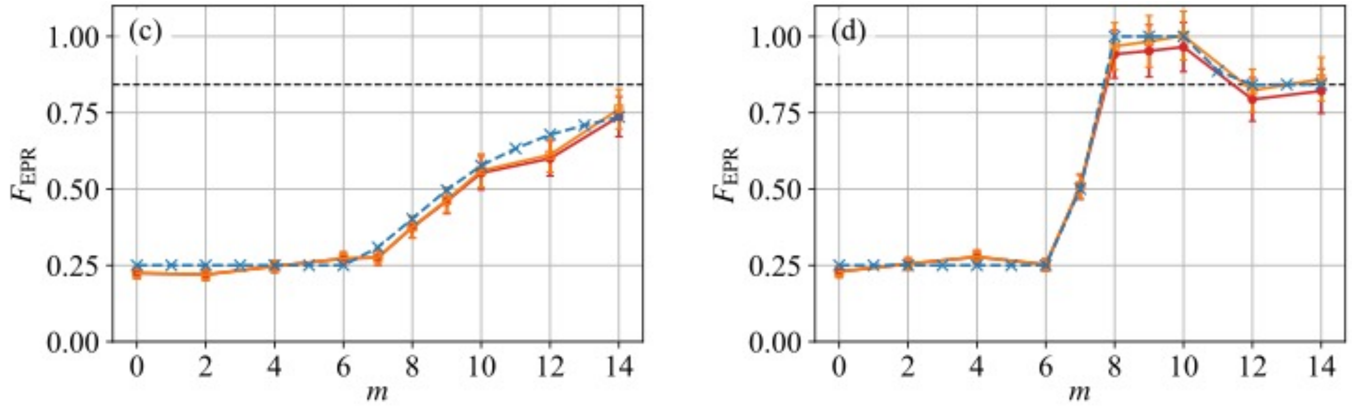
$$\rho^{\text{noisy}} = fU^\dagger\rho_{\text{input}}U + (1-f)\frac{I}{2^N}$$

$$f = \left(1-p_{2Q}\right)^{N_{2Q}}$$

$JT = 1.3$

Maximally chaotic

$JT = \frac{\pi}{2}$



dashed: exact, red: raw, orange: mitigated

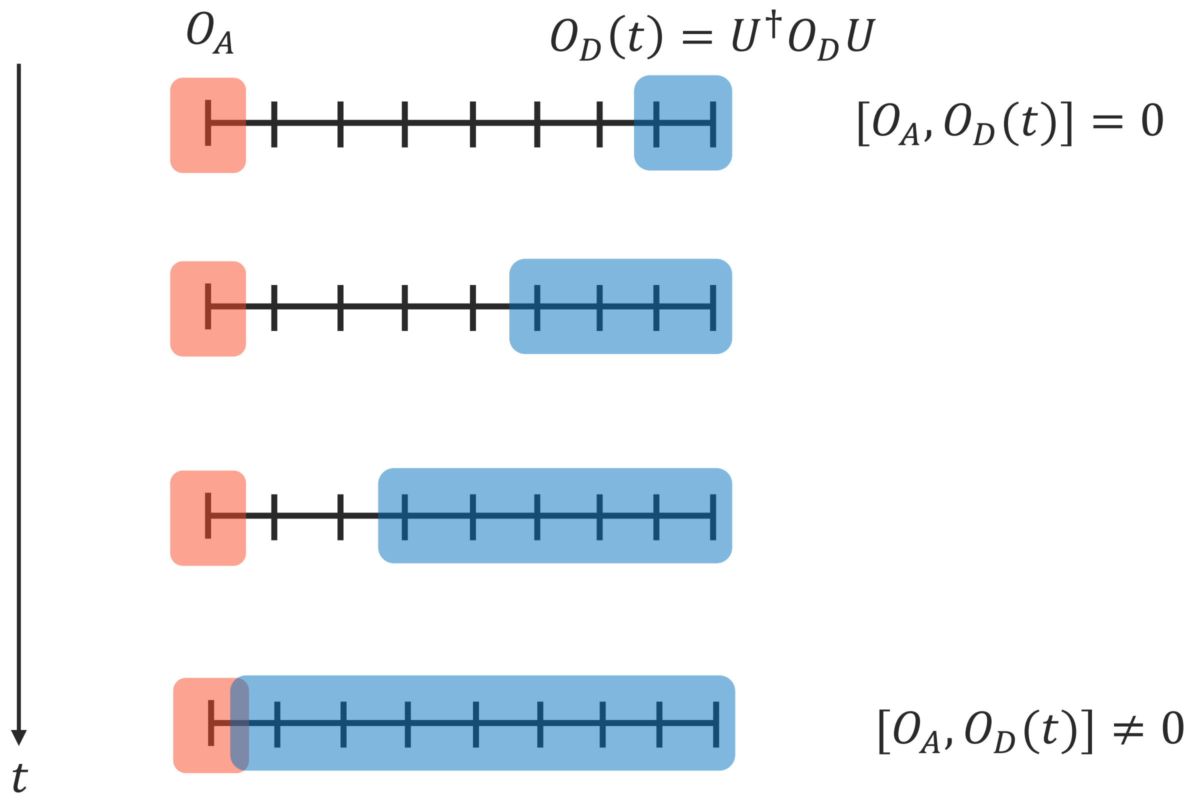
Observations

- A sharp increase once the information reaches the measured register in the chaotic model.
- Consistent with Haar random value (dashed lines) at late times.
- Depolarizing channel models the chaotic model better.

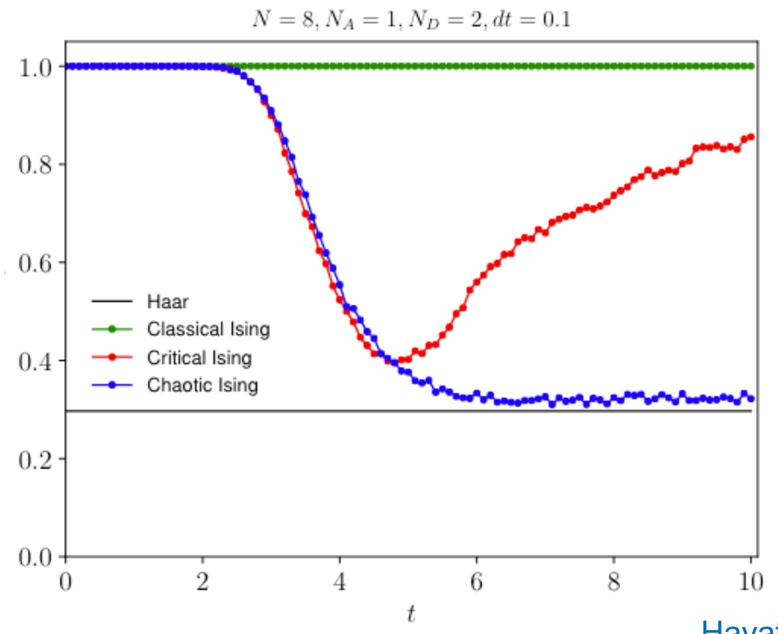
Out-of-time-ordered correlators (OTOCs) [Theory]

Larkin & Ovchinikov 1969;
Shenker & Stanford 2014

Operator growth



$$\text{OTOC} = \langle O_A O_D(t) O_A^\dagger \; O_D(t)^\dagger \rangle \approx \frac{1}{d_A F_{\text{EPR}}}$$



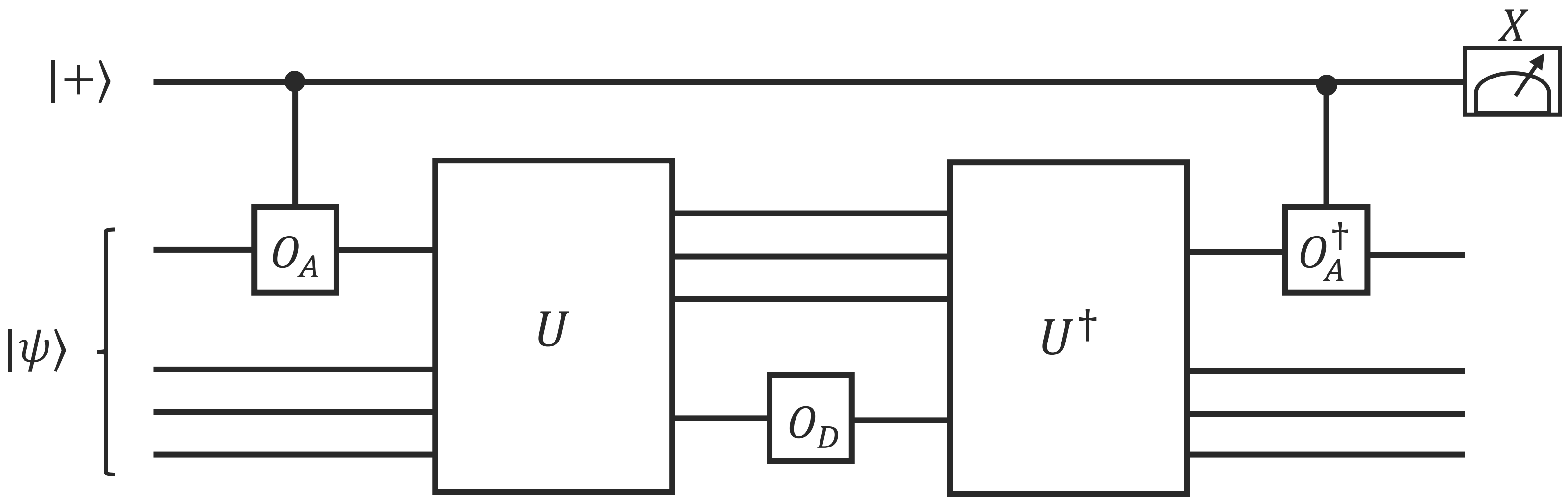
Hayata, Hidaka, YK '21

$$H = -J \sum_i Z_i Z_{i+1} + B_Z \sum_i Z_i + B_X \sum_i X_i$$

Out-of-time-ordered correlators (OTOCs) [Theory]

Interferometric protocol

$$\text{OTOC} = \langle O_A O_D(t) O_A^\dagger \; O_D(t)^\dagger \rangle$$



Swingle, Bentsen, Schleier-Smith & Hayden '16
Swingle & Halpern '18
Mi et.al. '21

Out-of-time-ordered correlators (OTOCs) [Experiment]

Interferometric protocol

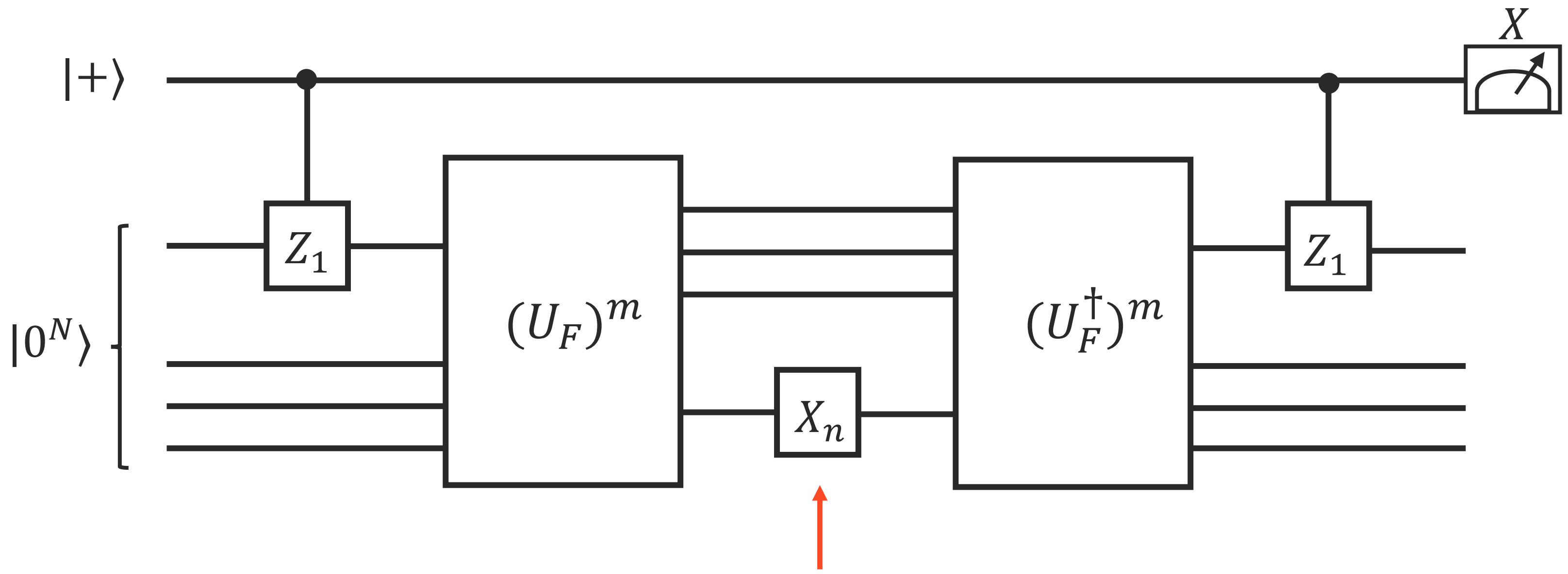
$\text{OTOC} = \langle Z_1\, X_n(t) Z_1\, X_n(t)\rangle$

$\text{OTOC'} = \frac{\langle Z_1\, X_n(t) Z_1\, X_n(t)\rangle}{\langle Z_1\, I_n(t) Z_1\, I_n(t)\rangle}$: mitigates incoherent errors

$\langle Z_1\, I_n(t) Z_1\, I_n(t)\rangle = 1$ without noise

Setup

- 19-qubit spin chain + 1 ancilla qubit
- $U_F = e^{-iH_Z \frac{T}{2}} e^{-iH_X \frac{T}{2}}$
- 367 two-qubit gates inside the causal cone at maximum

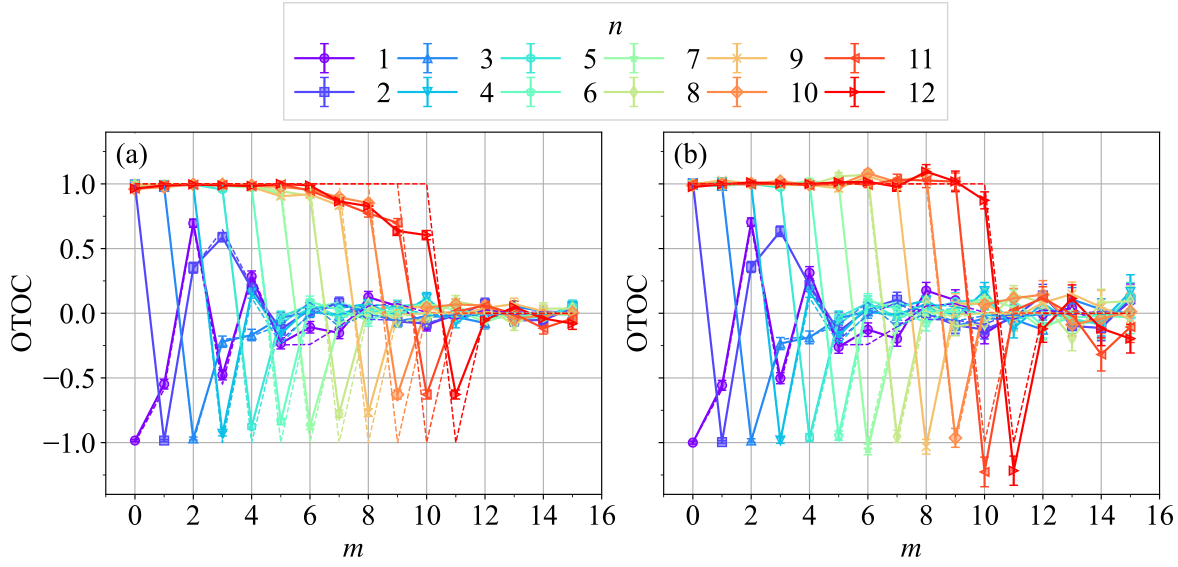


Replaced by I_n for mitigation

Swingle, Bentsen, Schleier-Smith & Hayden '16
Swingle & Halpern '18
Mi et.al. '21

Out-of-time-ordered correlators (OTOCs) [Experiment]

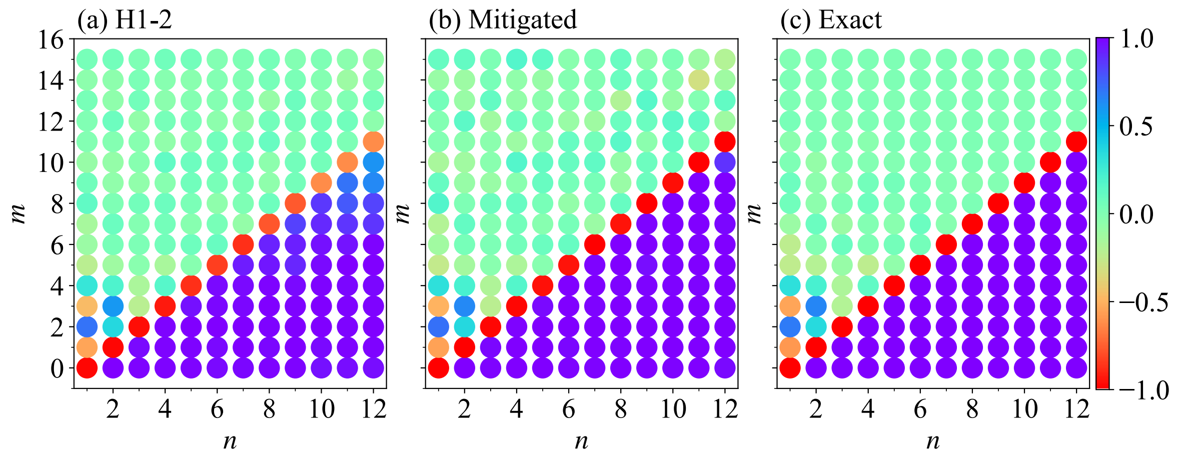
Seki, YK, Hayata, Yunoki '24



Dashed: exact

Left: $\text{OTOC} = \langle Z_1\, X_n(t) Z_1\, X_n(t) \rangle$

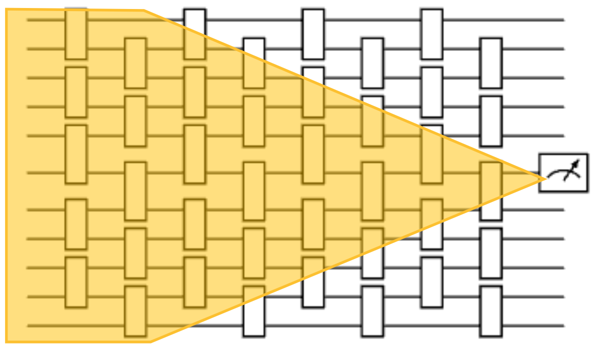
Right: $\text{OTOC'} = \dfrac{\langle Z_1\, X_n(t) Z_1\, X_n(t) \rangle}{\langle Z_1\, I_n(t) Z_1\, I_n(t) \rangle}$



Observations

- Sharp drop once the operators overlap.
- Normalized OTOCs suffer less from noise at early times, but amplify the statistical errors at late times.
- Agree with Haar random unitary at late times within error bars.
- Ballistic operator growths are visualized below.

Discussion



Circuit fidelity is crudely (sometimes accurately) estimated by

$$f = (1 - p_{2Q})^{N_{2Q}}$$

Geometrically local models requires $\mathrm{poly}(N)$ gates to scramble.

- 1D geometrically circuit requires $t \approx O(N)$ for the entire system to get involved: $N_{\mathrm{gate}} \sim O(N^2)$
- E.g. we used 400 2Q gates for the 20-qubit 1D system
  $\Longrightarrow$ a 40-qubit system requires 4 times larger gate counts ~ 1600 2Q gates : $f = 0.999^{1600} \approx 0.2$

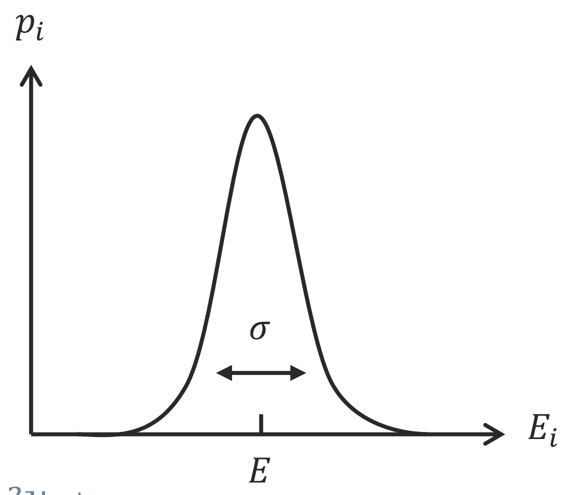
What if hardware results deviate from the estimate?

- Memory error (on idling qubits, during ion shuttling)
- SPAM error (bias between $0 \to 1$ and $1 \to 0$)
- Wrong gate counting (relevant # of 2Q gates)

Backup

Thermal expectation values [Theory]

Microcanonical ensemble = isolated thermal state



- Mixed state $\rho_\sigma(E) = \frac{\exp\left[-\frac{(\mathcal{H}-E)^2}{2\sigma^2}\right]}{\mathrm{Tr}\left[\exp\left[-\frac{(\mathcal{H}-E)^2}{2\sigma^2}\right]\right]} = \sum_i p_i |E_i\rangle\langle E_i|$
- Thermal expectation value $\mathrm{Tr}[\rho_\sigma(E) O] = \frac{\mathrm{Tr}\left[O \exp\left[-\frac{(\mathcal{H}-E)^2}{2\sigma^2}\right]\right]}{\mathrm{Tr}\left[\exp\left[-\frac{(\mathcal{H}-E)^2}{2\sigma^2}\right]\right]} \approx \frac{\left\langle\psi\middle|O \exp\left[-\frac{(\mathcal{H}-E)^2}{2\sigma^2}\right]\middle|\psi\right\rangle}{\left\langle\psi\middle|\exp\left[-\frac{(\mathcal{H}-E)^2}{2\sigma^2}\right]\middle|\psi\right\rangle}$ for $|\psi\rangle$ sampled from a 2-design

Jin et.al. (2021)
Coopmans, Y.K. & Benedetti (2022);
Seki & Yunoki (2022)

- $\left\langle\psi\middle|\exp\left[-\frac{(\mathcal{H}-E)^2}{2\sigma^2}\right]\middle|\psi\right\rangle = \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} \mathrm{e}^{-\sigma^2 t^2/2} \mathrm{e}^{\mathrm{i}Et} \langle\psi|\mathrm{e}^{-\mathrm{i}\mathcal{H}t}|\psi\rangle$
- Calculate Loschmidt amplitude $\mathcal{L}(\mathrm{t}) \coloneqq \langle\psi|\mathrm{e}^{-\mathrm{i}\mathcal{H}t}|\psi\rangle$

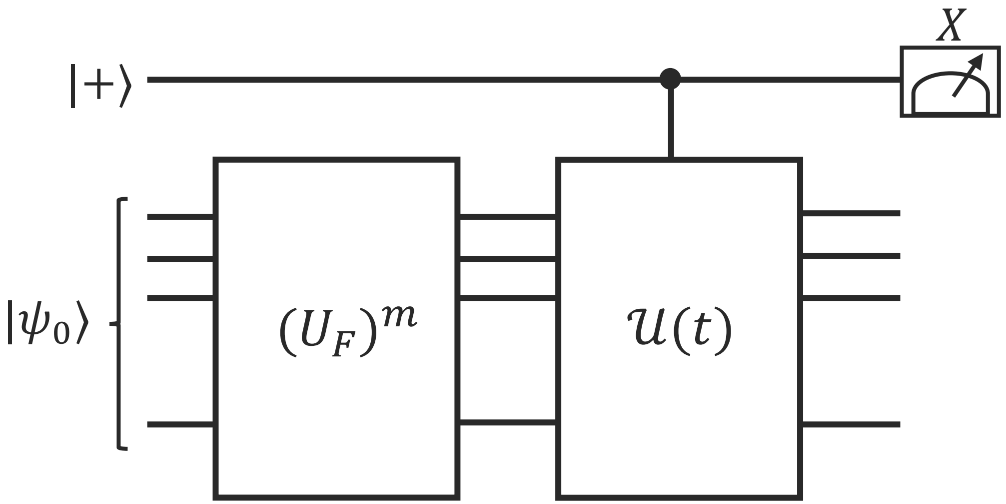
Lu, Banuls & Cirac (2021);
Yang, Cirac & Banuls (2022);
Schuckert, Bohrdt, Crane & Knap (2023);
Ghanem, Schukert & Dreyer (2023)

- Use Floquet scrambling circuits instead of unitary 2-design.

Thermal expectation value [Experiment]

Related experiments:
Summer, Chiaracane, Mitchison & Goold (2024);
Hemery et.al. (2023)

Calculate $\langle Z_1 Z_2 \rangle \coloneqq \mathrm{Tr}[\rho_\sigma(E) Z_1 Z_2]$ for the Heisenberg model $\mathcal{H} = \frac{\mathcal{J}}{2} \sum_i (X_i X_{i+1} + Y_i Y_{i+1} + Z_i Z_{i+1})$



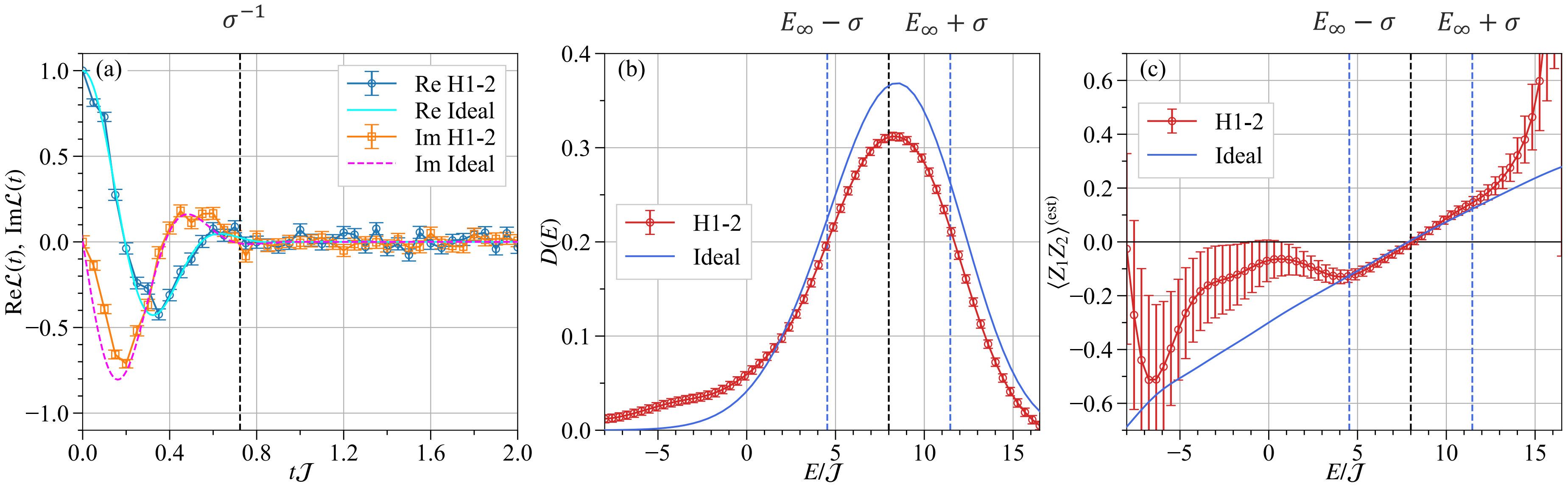
$\mathcal{L}(\mathrm{t}) = \langle \psi | \mathrm{e}^{-\mathrm{i}\mathcal{H}t} | \psi \rangle$

Setup

- Run on H1-2
- 16-qubit spin chain (PBC) + 1 ancilla qubit
- $|\psi_0\rangle$: a randomly chosen product state
- $U_F = \mathrm{e}^{-\mathrm{i} H_Z \frac{T}{2}} \mathrm{e}^{-\mathrm{i} H_X \frac{T}{2}}$
- $\mathcal{U}(t)$: time-evolution operator of Heisenberg model

Thermal expectation value [Experiment]

Related experiments:
Summer, Chiaracane, Mitchison & Goold (2024);
Hemery et.al. (2023)



Observations

- $\mathcal{L}(\mathrm{t}) \coloneqq \langle\psi|\mathrm{e}^{-\mathrm{i}\mathcal{H}t}|\psi\rangle$: Loschmidt amplitude.
- $D(E) \coloneqq \left\langle\psi\left|\exp\left[-\frac{(\mathcal{H}-E)^2}{2\sigma^2}\right]\right|\psi\right\rangle$: Density of states concentrates around $E_\infty - \sigma \lesssim E \lesssim E_\infty + \sigma$.
- Obtained $\langle Z_1 Z_2\rangle$ agrees with exact data in the range where $D(E)$ is large.

TPQ error $\propto \mathrm{Tr}[\rho_\sigma(E)^2]$, shot noise $\propto D(E)^{-1}$