

Local-equilibrium theory of neutrino oscillations in supernovae and mergers

Luke Johns



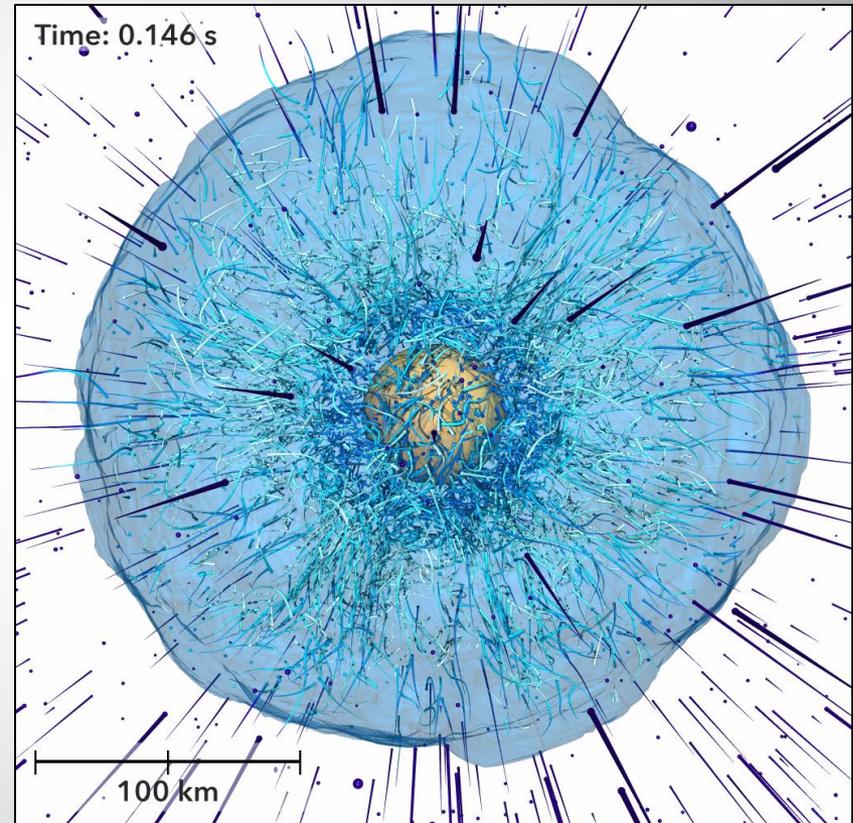
Core-collapse supernovae & neutron star mergers

Astrophysical simulations have now entered *the multidimensional era* — and neutrino oscillations are perhaps the most glaring omission.

Volpe, Rev Mod Phys (2024)

Johns, Richers, & Wu, accepted Annual Reviews (2025)

CCSN simulation



Burrows & Vartanyan, Nature (2021)

Neutrino quantum kinetics is computationally intractable

Quantum kinetic equation for density matrix $\rho(t, \mathbf{r}, \mathbf{p})$:

$$i \underbrace{(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}})}_{\text{Particle advection}} \rho = \underbrace{[H, \rho]}_{\text{Flavor mixing}} + \underbrace{iC}_{\text{Collisions}}$$

Hamiltonian H depends on ρ because neutrinos are coupled through neutrino-neutrino forward scattering.

Flavor instabilities imply significant effects from oscillations

Slow instabilities

Kostelecký & Samuel, PLB (1993)

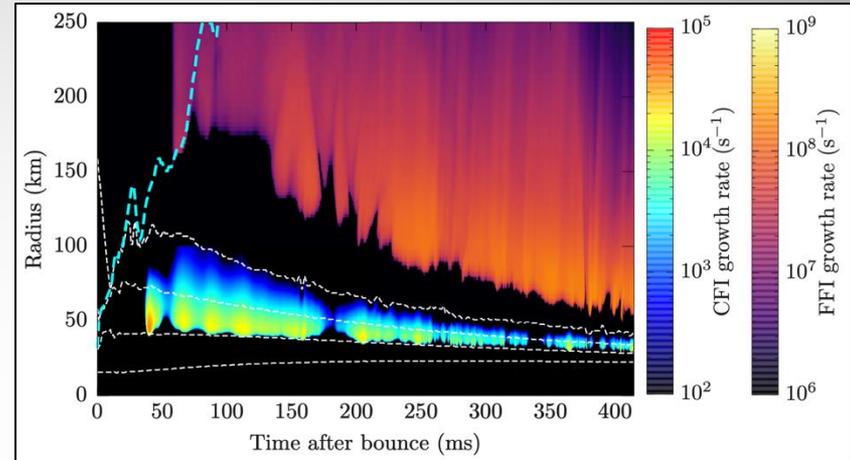
Fast instabilities

Sawyer, PRL (2016)

Collisional instabilities

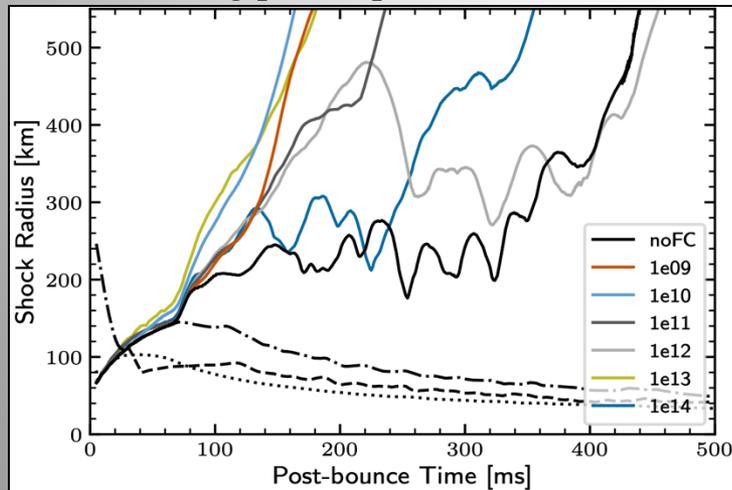
Johns, PRL (2023)

Instability prevalence in CCSN simulation



Akaho et al., PRD (2025)

CCSN dynamics with different flavor-mixing prescriptions



Ehring et al., PRL (2023)

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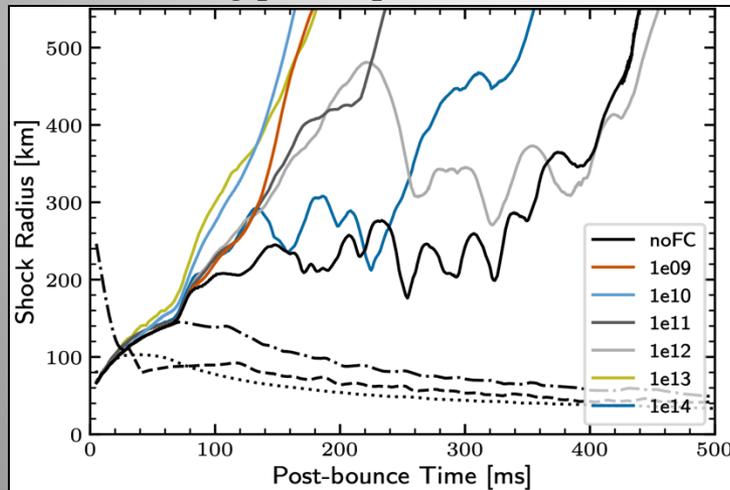
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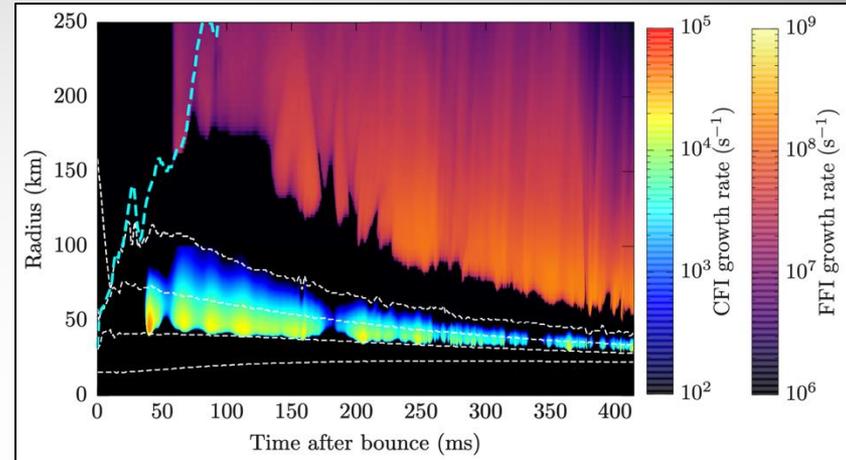
Johns, PRL (2023)

CCSN dynamics with different
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Ehring et al., PRL (2023)

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Akaho et al., PRD (2025)

The ways oscillations are being incorporated
into simulations are simple but imperfect.

Li & Siegel, PRL (2021)

Nagakura, Johns, & Zaizen, PRD (2024)

Xiong et al., PRL (2025)

Proposed solution:

A coarse-grained transport theory based on local mixing equilibrium.

Johns, 2306.14982 (*Thermodynamics of oscillating neutrinos*)

Johns, 2401.15247 (*Subgrid modeling of neutrino oscillations in astrophysics*)

Johns & Kost, 2506.03271 (*Local-equilibrium theory of neutrino oscillations*)

$$i \left(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}} \right) \rho = [H, \rho] + iC$$

Particle advection

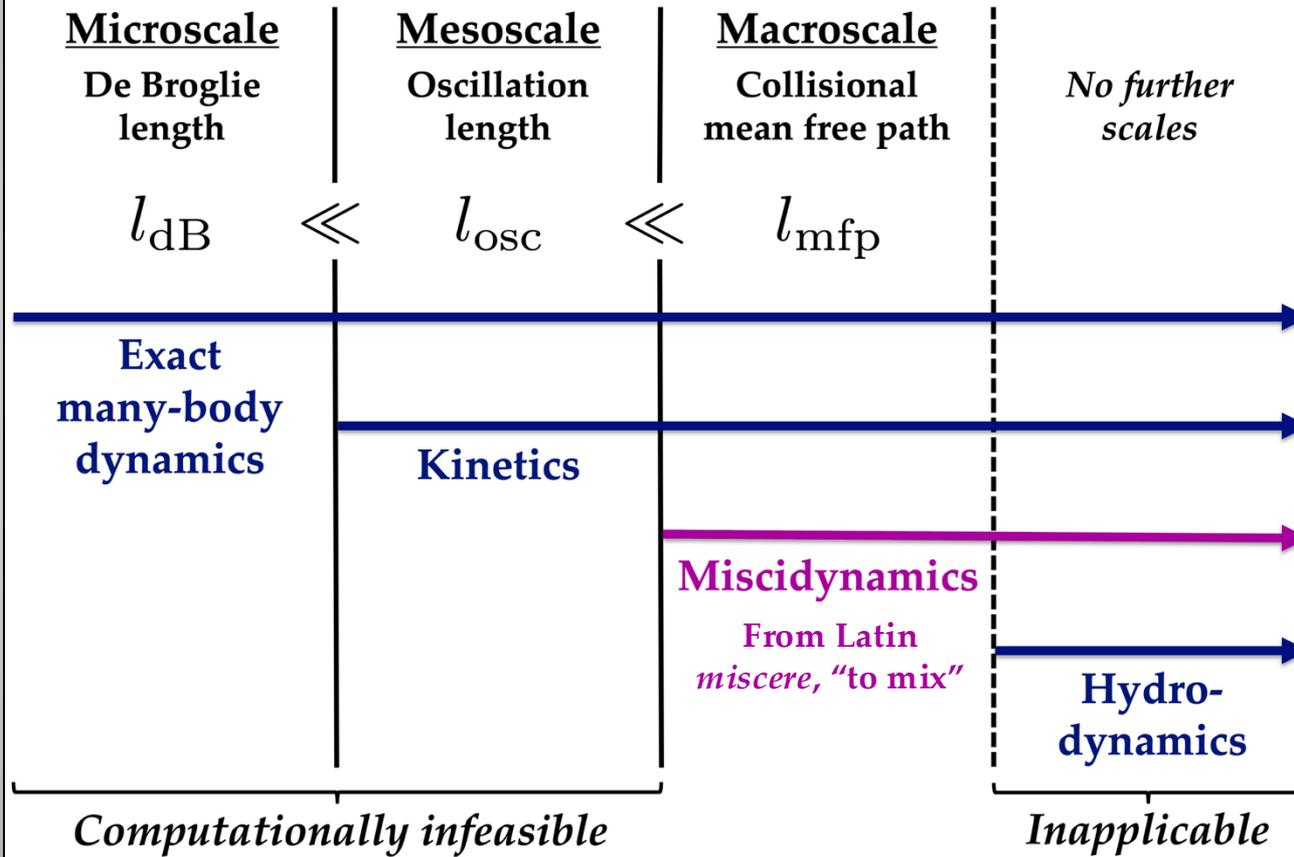
Flavor
mixing

Collisions

Vanishes in
mixing eq

Vanishes in
collisional eq

Length scales, coarse-grainings, & transport theories



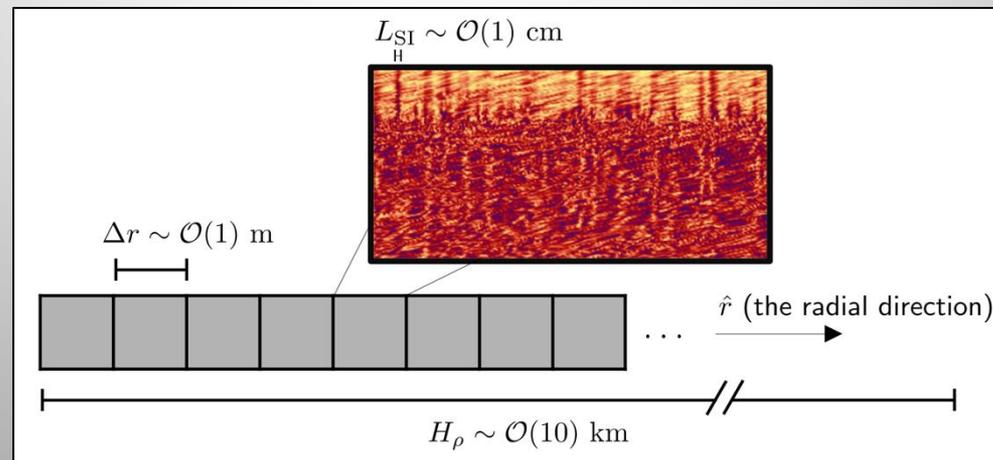
Miscidynamic equation in the local equilibrium frame:

Flavor
polarization
vector

Collisional
depolarization
rate

$$(\partial_t + \hat{\mathbf{p}} \cdot \partial_{\mathbf{r}}) \mathbf{P}^{\text{eq}} = -(\gamma + \Gamma) \mathbf{P}^{\text{eq}}$$

Turbulent
flavor-wave
viscosity



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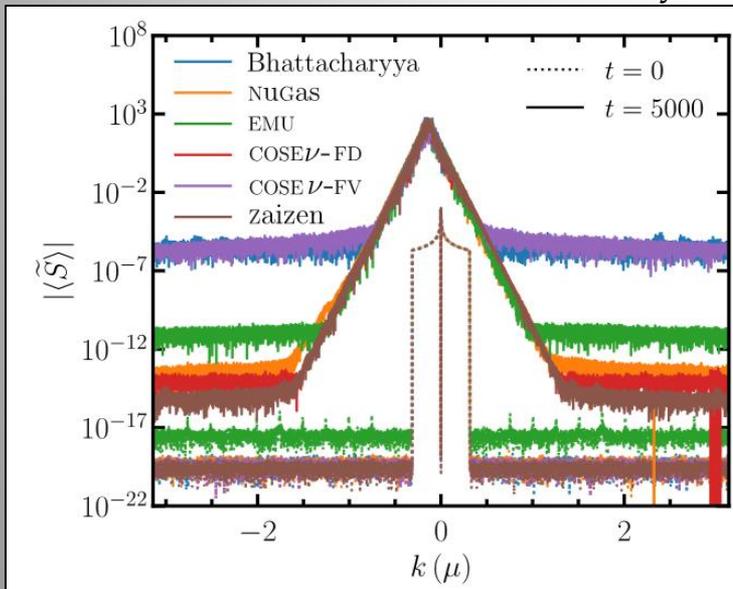
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**Turbulent
flavor-wave
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(compare *turbulent eddy
viscosity* in a fluid)

Flavor-wave turbulence after instability



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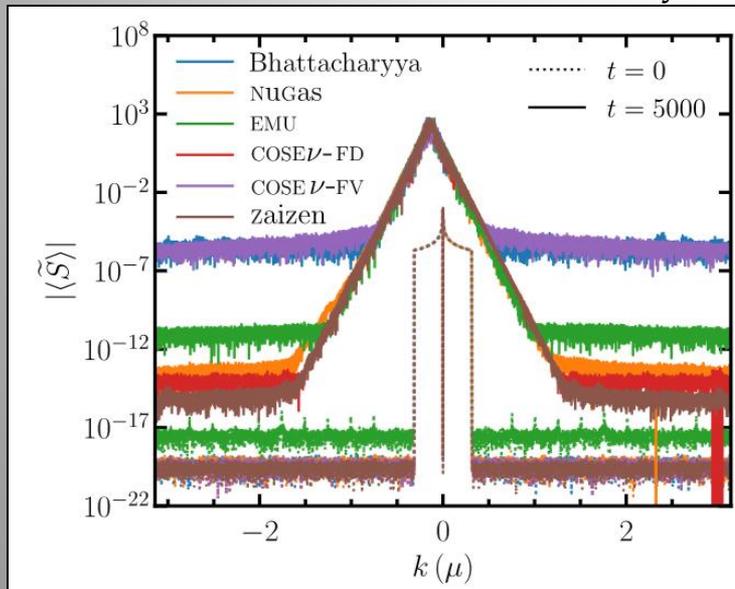
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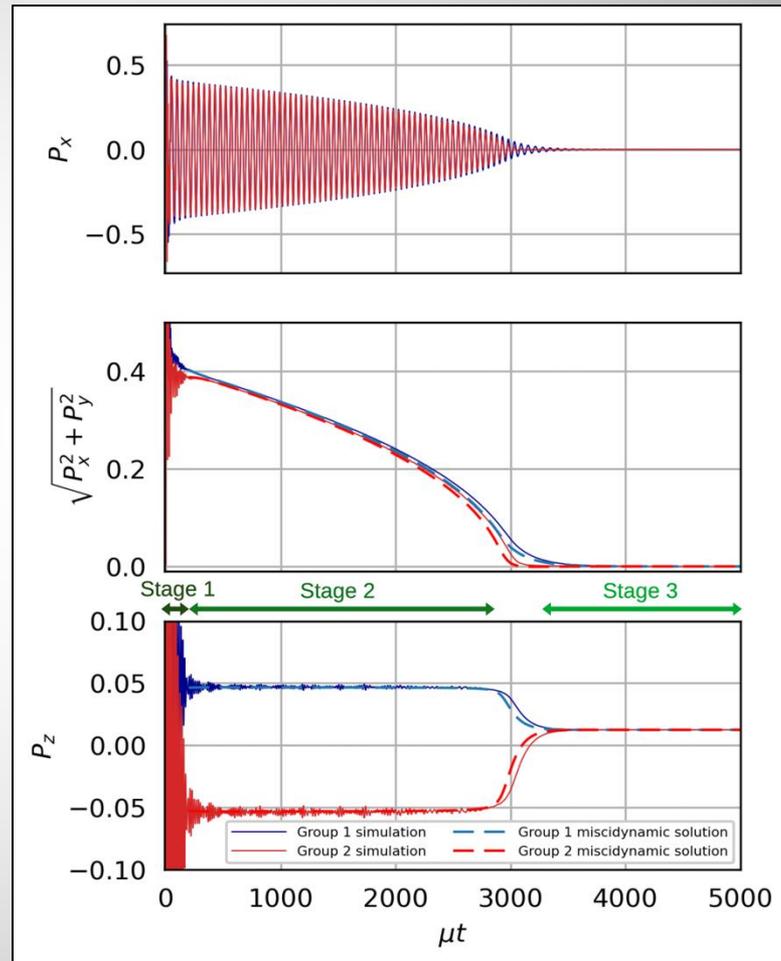


As in hydrodynamics, there's a closure problem: $\gamma \sim \langle \mathbf{P} \mathbf{P} \rangle$, which couples to $\langle \mathbf{P} \mathbf{P} \mathbf{P} \rangle$, ...

- Adiabatic closure ($\gamma = 0$)
- Kinetic theory of flavor waves ($\gamma \neq 0$)

Adiabatic closure ($\gamma = 0$)

Collision-driven adiabatic flavor relaxation shows the numerical success of miscidynamics:



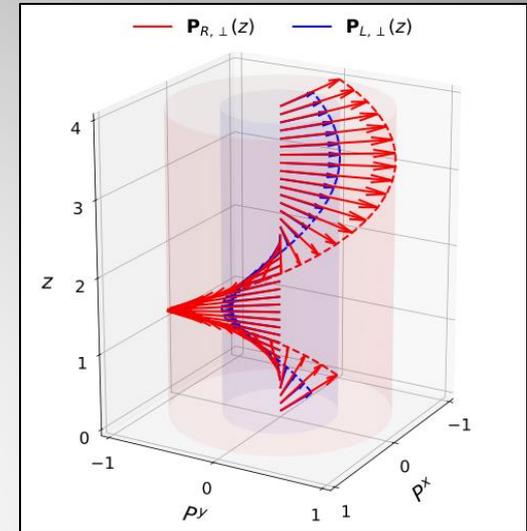
Kost, **Johns**, & Duan, in preparation
See also Kost, **Johns**, & Duan, PRD (2024)

Kinetic theory of flavor waves ($\gamma \neq 0$)

Idea: Approximate flavor waves as weakly coupled quasiparticles (compare phonons, magnons, etc.).

For a closely related approach, see Fiorillo & Raffelt, PRL (2025)

Illustration of a flavor wave



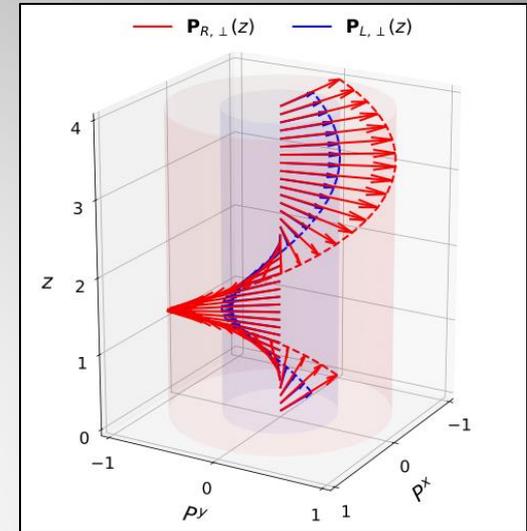
Liu, Johns, & Nagakura, in prep

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Liu, Johns, & Nagakura, in prep

Kinetic equation for flavor-wave amplitude:

$$(\partial_t + \mathbf{v}_i^g \cdot \partial_{\mathbf{r}} + \mathbf{F}_i \cdot \partial_{\mathbf{k}}) |A_i| = \Omega_i^I |A_i| + \mathcal{N}_i^I$$

Propagation along rays
like in geometric optics

Neutrino decay
to flavor waves

Three-wave
interactions

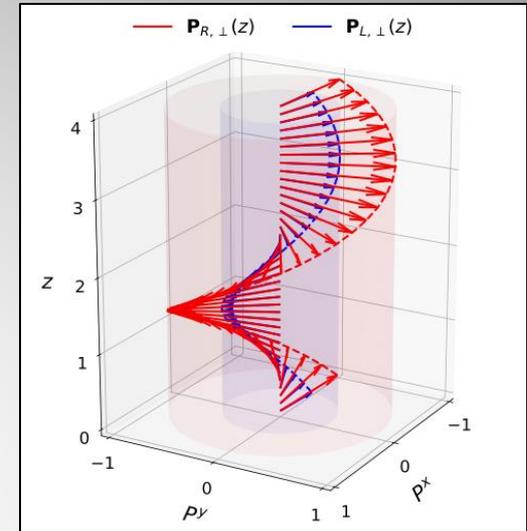
(similar to Boltzmann collision
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**Propagation along rays
like in geometric optics**

**Neutrino decay
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**Three-wave
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(similar to Boltzmann collision integrals for particle interactions)

This evolution feeds into the viscosity γ . We're now working on testing the $\gamma \neq 0$ theory numerically.

Summary

- Neutrino oscillations are estimated to have **significant effects** on core-collapse supernovae & neutron star mergers.
- Miscodynamics is a **local-equilibrium theory** that will hopefully enable more reliable simulations.
- There are new & interesting avenues to explore regarding the **thermodynamics** of neutrino oscillations.

e.g., see Sec. 9 of Fiorillo & Raffelt, 2505.20389, for a proof of the **oscillation *H*-theorem** conjectured in the first paper below.

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