A Renormalization Group Approach for Radiative Corrections in Nuclear Effective Field Theory

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CIPANP
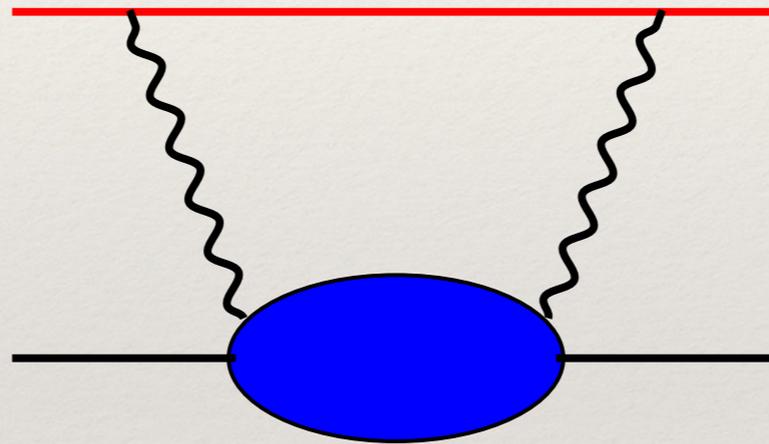June 11, 2025

In collaboration with Immo C. Reis
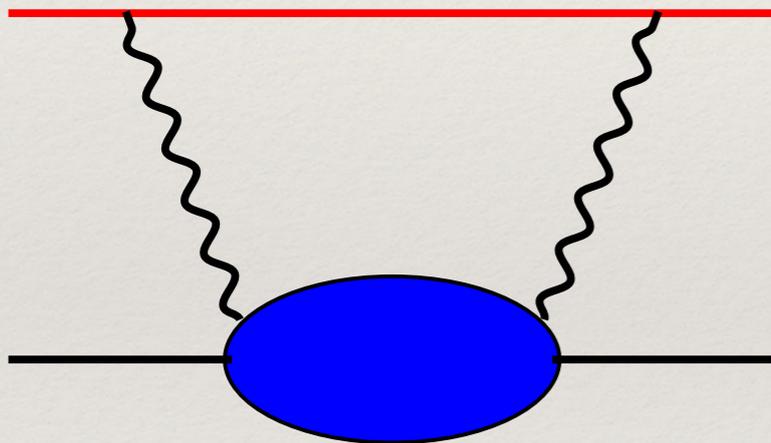
Radiative Corrections in Nuclear Physics

Spectroscopy

Lepton Scattering

Beta Decay

Radiative Corrections in Nuclear Physics

Li Muli, TRR, Bacca PRL 134

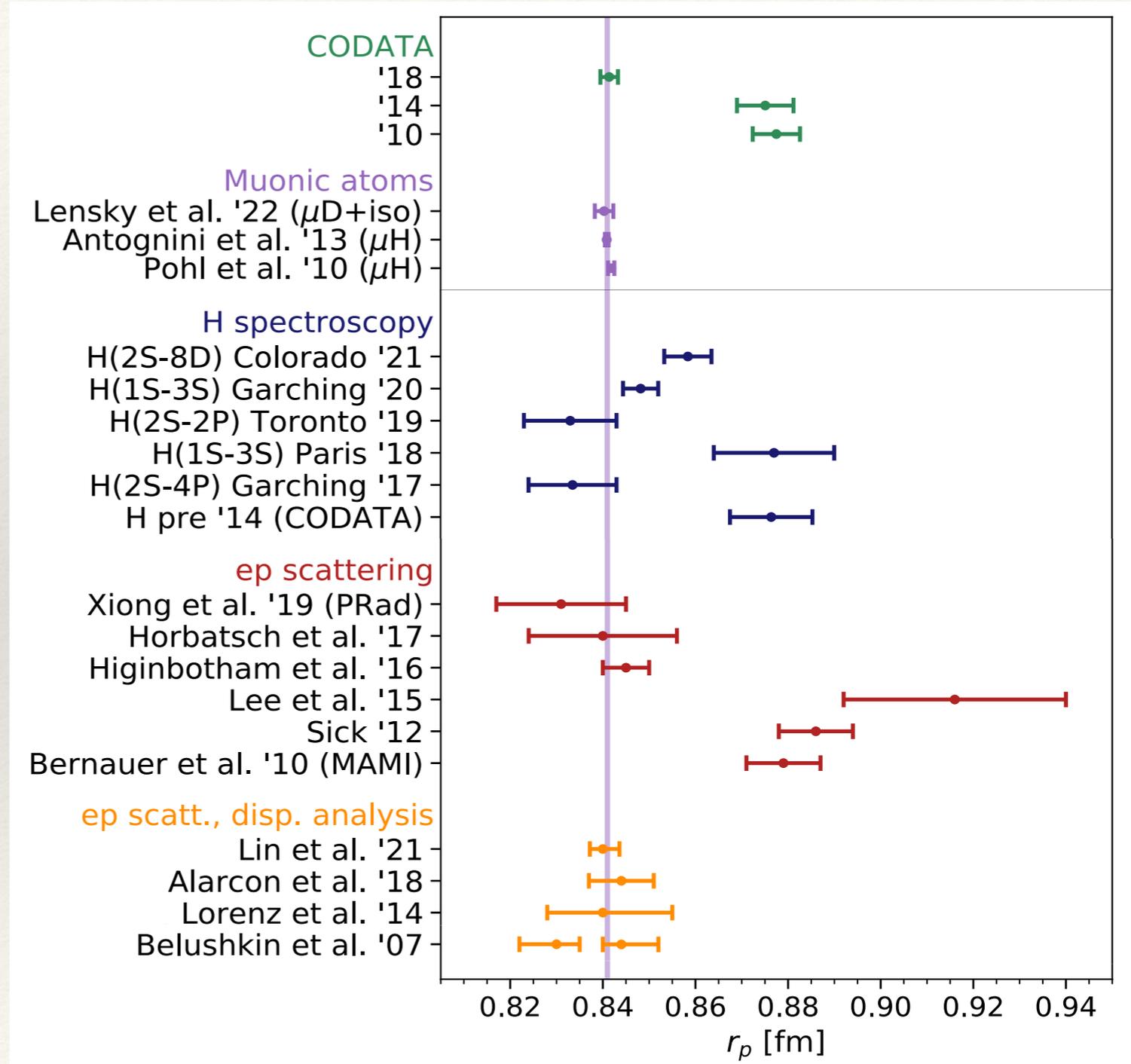# Radiative Corrections in Nuclear Physics

Lepton Scattering

Antognini et al. ARNPS 72

Radiative Corrections in Nuclear Physics

Beta Decay

Cirigliano et al. PLB 838

$K \to \pi l \nu$

$K \to \mu\nu/\pi \to \mu\nu$

neutron

$0^+ \to 0^+$

Effective Field Theory

$$\mathcal{L}_{\text{eff}} = \sum_n \left(\frac{p}{\Lambda}\right)^n c_{\mathcal{O}} \mathcal{O}_n$$

?

TeV

Standard Model

M_W

LEFT

Quarks, gluons, leptons, photons

RG evolution

1 GeV

χPT

Lattice input

Mesons, nucleons, photons

m_π

EFT$_{\not\pi}$

Few- and many-body methods

$1/a$

Nucleons, photons

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EFT Requirements

1. All particles that are near the mass-shell are included

2. Homogeneous power counting governed by a single ratio of scales

3. Renormalization group invariance up to the order we are working

4. Symmetries (gauge, discrete, internal) are preserved

Pionless Effective Field Theory

- Valid for $p \ll m_\pi$

- Fine-tuned $a^{-1} \ll m_\pi$

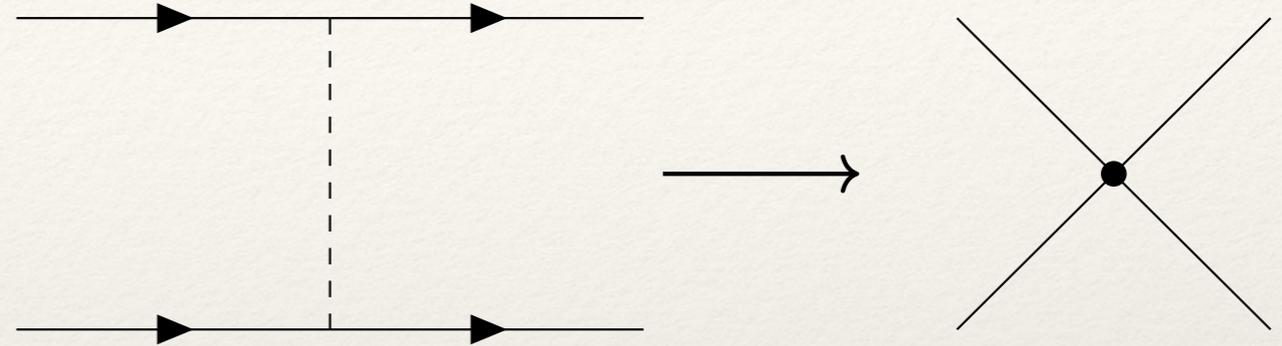$$\mathcal{L}_{NN} = N^\dagger \left(iD_0 + \frac{1}{2m_N} D^2 \right) N - C_0 \left(N^T P N \right)^\dagger \left(N^T P N \right) + \cdots$$

- Generate a shallow bound state through explicit resummation

$$B_d \approx \frac{1}{M_N a^2} \sim 2 \text{ MeV}$$

Kaplan, Savage, and Wise, 1998; van Kolck, 1999

Nonrelativistic EFT with Virtual Photons

❖ Bound state properties are easier with nonrelativistic theories

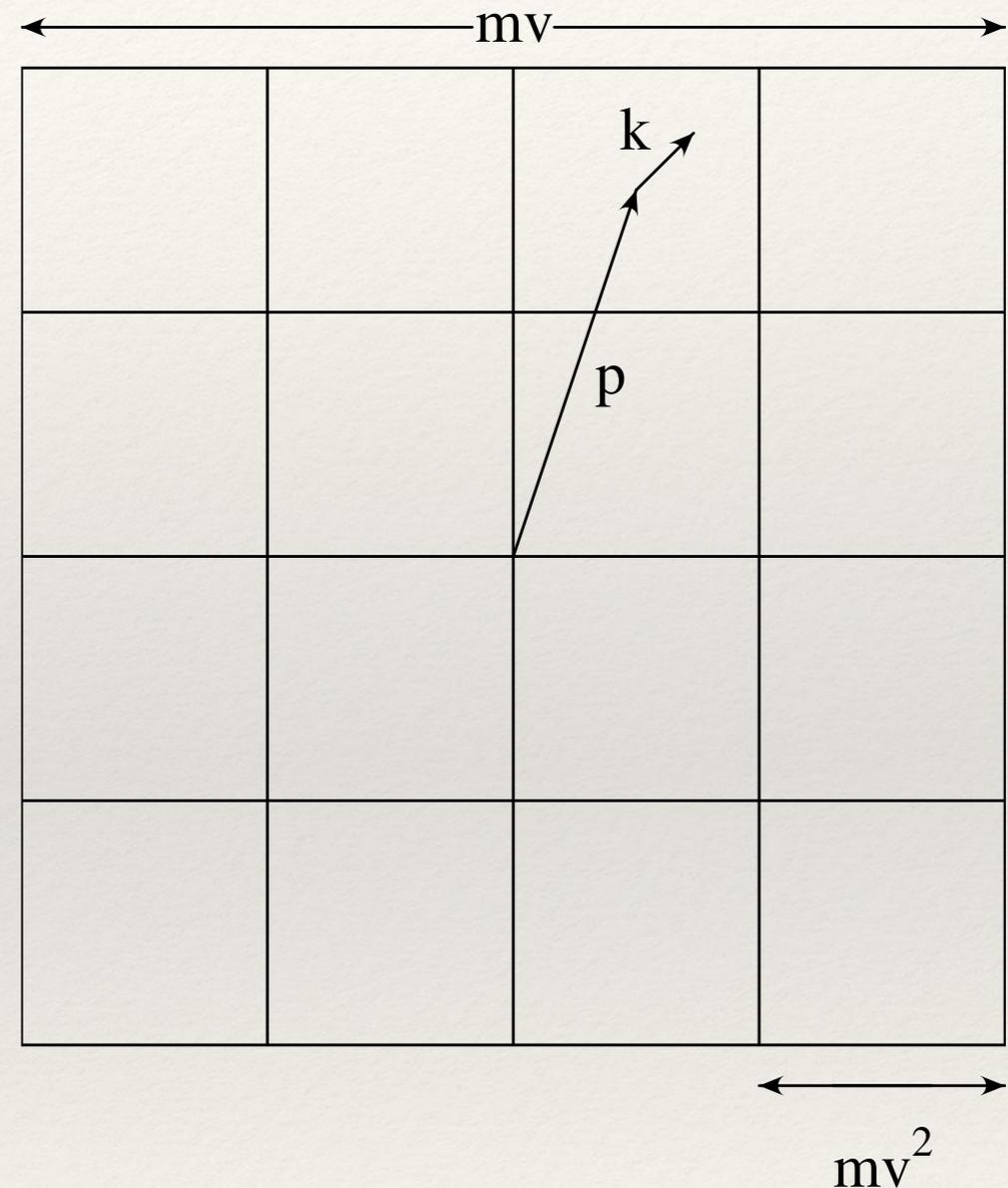❖ Energy and momenta are different but correlated scales

$$v \sim p/M = \sqrt{E/M}$$

❖ Homogeneous power counting requires mode separation

$$\text{potential} \sim (Mv^2, Mv) : N_{\mathbf{p}}$$
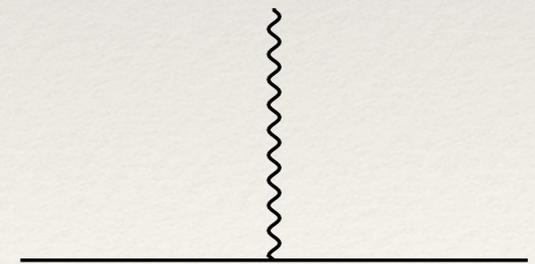$$\text{soft} \sim (Mv, Mv) : A_p$$
$$\text{ultrasoft} \sim (Mv^2, Mv^2) : A$$

Caswell, Lepage, Labelle, Luke, Manohar, Rothstein, Savage, Grießhammer, Grinstein, Stewart, Hoang, Pineda, Soto, Brambilla, Vairo

Velocity EFT Lagrangian

$$\mathcal{L} = \sum_p N_p^\dagger \left(iD_0 - \frac{(p - iD)^2}{2M_N} \right) N_p - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$$+ \sum_p \left| p^\mu A_p^\nu - p^\nu A_p^\mu \right|^2 - \sum_{p',p} V(p', p)$$

$$- \frac{4\pi\alpha}{2M_N} \sum_{q,q',p,p'} A_{q'} \cdot A_q N_{p'}^\dagger Q N_p$$

$$+ \frac{e}{2M_N} \epsilon^{ijk} \left(\nabla^j A^k \right) \sum_p N_p^\dagger \sigma^i \left[\kappa_0 + \kappa_1 \tau^3 \right] N_p \,,$$

TRR and Reis, FBS 65

Neutron-Proton Potential in Pionless EFT

$$V_{pn} = \sum_{v=-1} \sum_{p',p} V_{abcd}^{(v)}(p',p) p_{p',a}^{\dagger} p_{p,b} n_{-p',c}^{\dagger} n_{-p,d}$$

$$V_{abcd}^{(-1)} = C_{0,pn}^{(S=1)} P_{ab,cd}^{(1)} + C_{0,pn}^{(S=0)} P_{ab,cd}^{(0)}$$
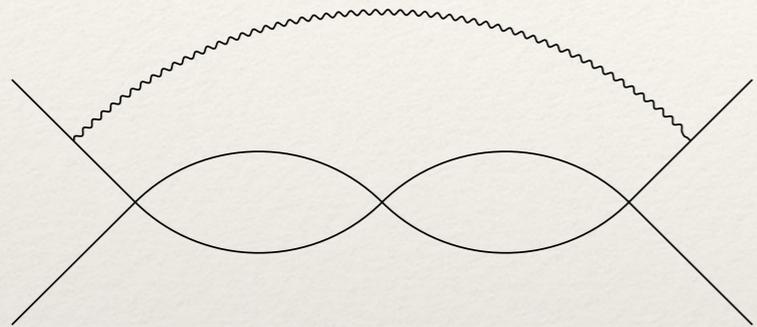
$$V_{abcd}^{(0)} = \frac{1}{2} \left(p'^2 + p^2 \right) \left[C_{2,pn}^{(S=1)} P_{ab,cd}^{(1)} + C_{2,pn}^{(S=0)} P_{ab,cd}^{(0)} \right]$$

$$V_{abcd}^{(1)} = \frac{1}{4} \left(p'^2 + p^2 \right)^2 \left[C_{4,pn}^{(S=1)} P_{ab,cd}^{(1)} + C_{4,pn}^{(S=0)} P_{ab,cd}^{(0)} \right]$$

Kaplan, Savage, Wise NPB 478, PLB 424, NPB 534
van Kolck NPA 645

Velocity Renormalization Group

❖ Perturbation theory generates two (possibly) large logarithms

$$p^2 C_0^3 \left[\lambda_S \log \frac{\mu}{p} + \lambda_{US} \log \frac{\mu}{E} \right]$$

❖ One μ will not minimize logs $\Longrightarrow$ Introduce two correlated scales in dimensional regularization

$$\mu_S = M_N \nu \qquad\qquad \mu_U = M_N \nu^2$$

❖ Run in ν—sum soft and ultrasoft logarithms simultaneously

$$\nu \frac{dV}{d\nu} = \gamma_S + 2\gamma_U \qquad \mu_U \frac{dV}{d\mu_U} = \gamma_U \qquad \mu_S \frac{dV}{d\mu_S} = \gamma_S$$

Luke et al. PRD 61

Strategy for Bound State Calculations

1. Obtain anomalous dimensions for the couplings in perturbation theory

2. Integrate the RG equations from matching scale to typical bound state velocity

3. Calculate bound state matrix elements

$$L \sim \log(m_L/m_H)$$

LO	1		
NLO	$\alpha\, L$	α	
NNLO	$\alpha^2\, L^2$	$\alpha^2\, L$	α^2
$\cdots$	$\cdots$	$\cdots$	$\cdots$
N^kLO	$\alpha^k\, L^k$	$\alpha^k\, L^{k-1}$	$\alpha^k\, L^{k-2}$

Cohen arXiv:1903.03622

Velocity RG Solution

$$C_2(\nu) = C_2\left(\frac{m_\pi}{M_N}\right) - \frac{27}{8}\left(\frac{M_N}{4\pi}\right)^2 C_0^3 \log\left(\frac{\alpha(M_N\nu^2)}{\alpha(m_\pi^2/M_N)}\right),$$

$$C_4(\nu) = C_4\left(\frac{m_\pi}{M_N}\right) + \frac{15}{4}\left(\frac{M_N}{4\pi}\right)^4 C_0^5 \log\left(\frac{\alpha(M_N\nu^2)}{\alpha(m_\pi^2/M_N)}\right)$$

TRR and Reis, FBS 65

Fixing the couplings

❖ The LECs need to be matched at $\nu = m_\pi/M_N$ at $O(\alpha^0)$

$$C_0^{(s)} = \frac{4\pi a_s}{M_N}$$

$$C_2^{(s)}\left(\frac{m_\pi}{M_N}\right) = \frac{2\pi a_s^2 r_s}{M_N}$$

$$C_4^{(s)}\left(\frac{m_\pi}{M_N}\right) = \frac{\pi}{M_N}a_s^3 r_s^2$$

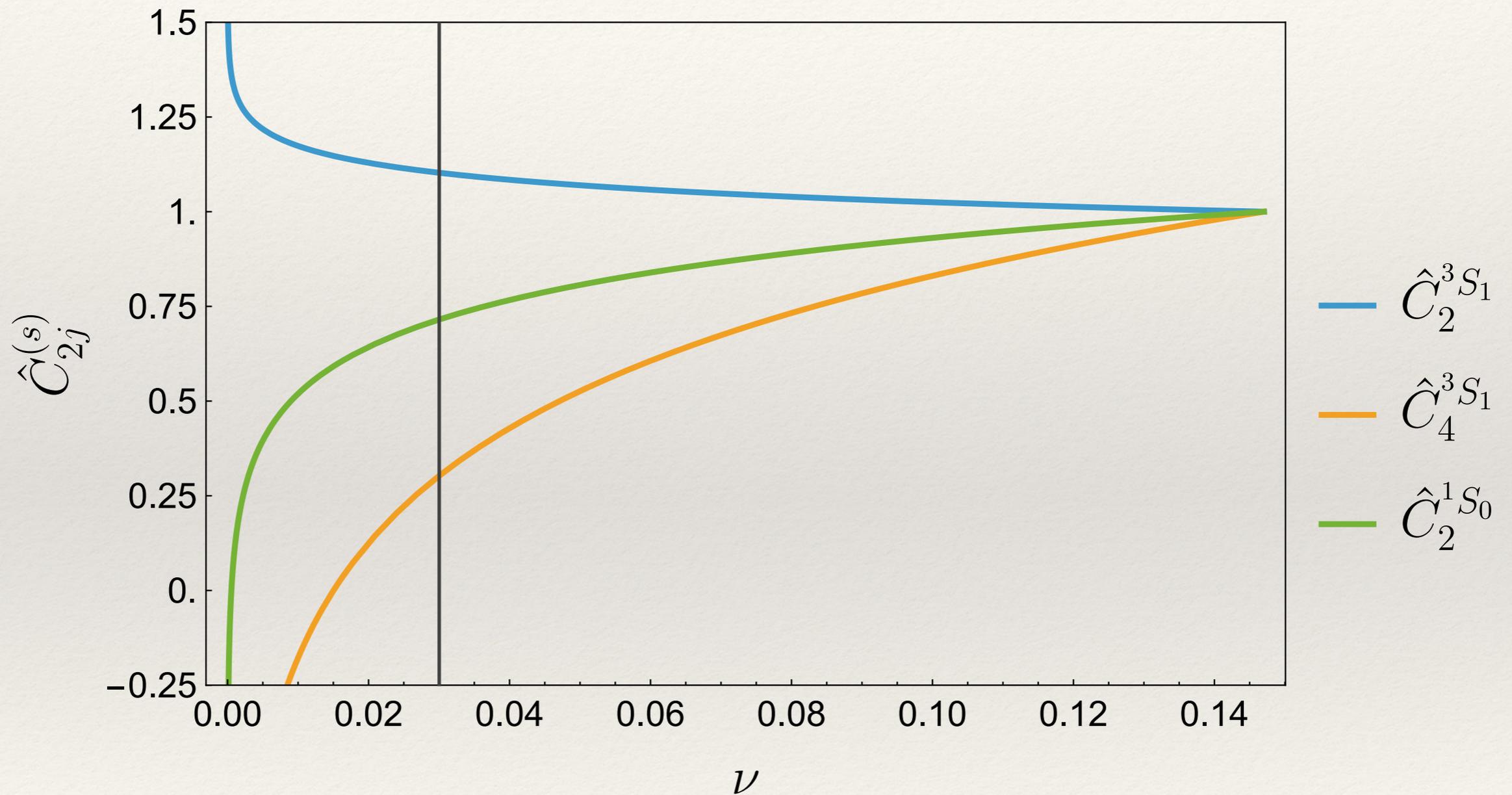❖ Use Argonne v18 as proxy for lattice QCD   Wirigna et al. PRC 51

$$a_0 = -23.084\,\text{fm} \qquad\qquad r_0 = 2.703\,\text{fm}$$
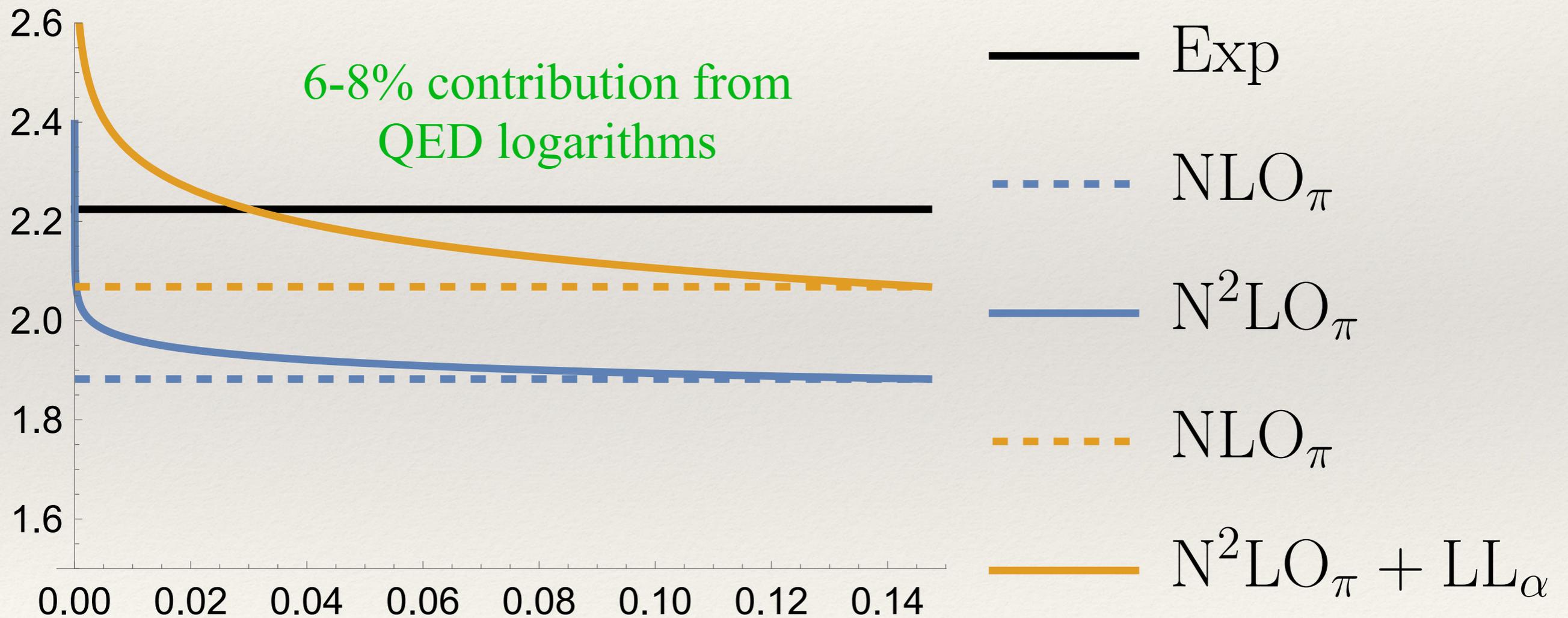
$$a_1 = 5.402\,\text{fm} \qquad\qquad r_1 = 1.752\,\text{fm}$$
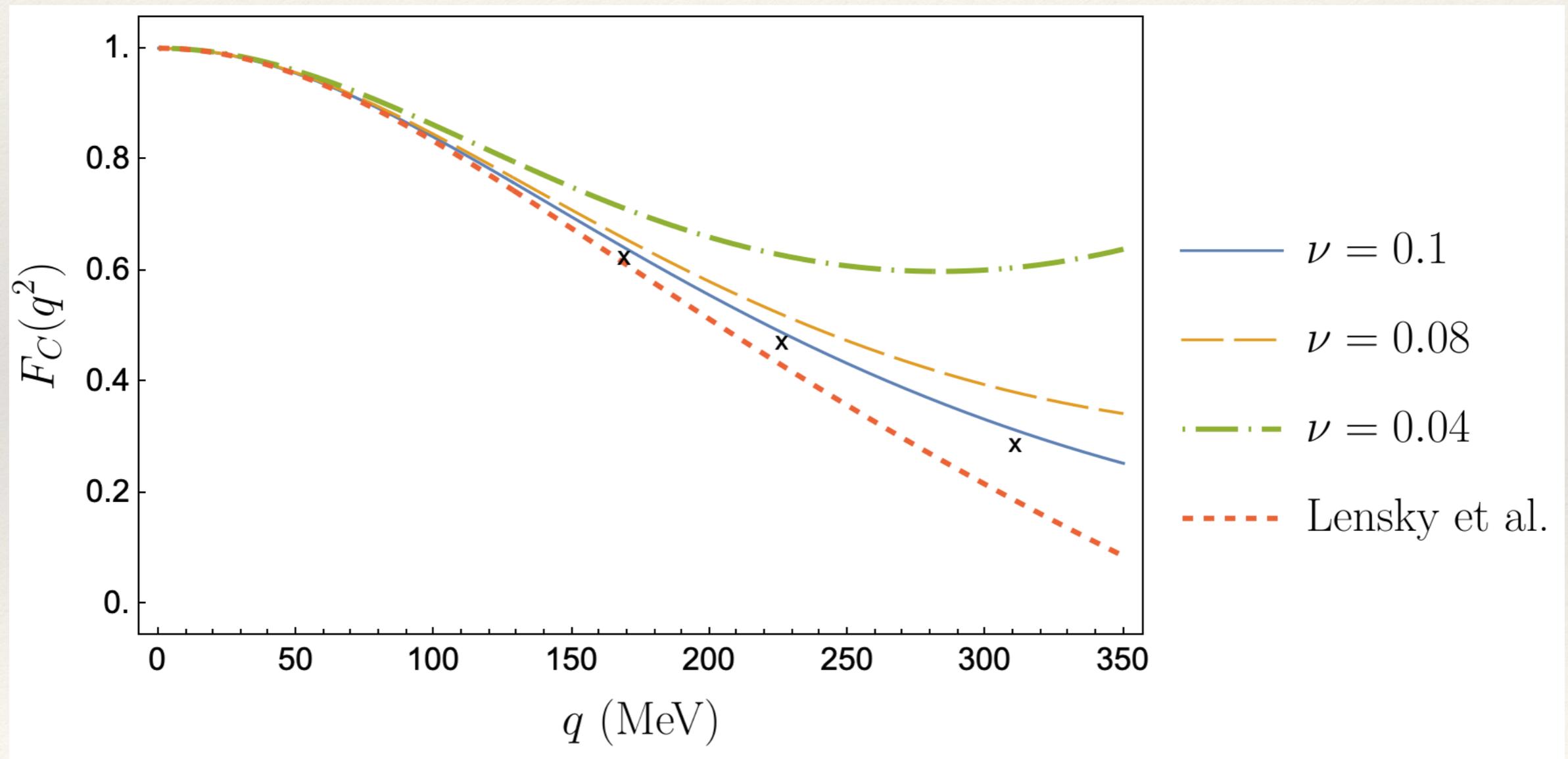
Running potentials

TRR and Reis, FBS 65

Deuteron Binding Energy

$$B = \frac{1}{M_N}\left(\frac{4\pi}{M_N C_0}\right)^2 + \frac{1}{2\pi}C_2\left(\frac{4\pi}{M_N C_0}\right)^5 + \frac{7}{16\pi^2}M_N C_2^2\left(\frac{4\pi}{M_N C_0}\right)^8 - \frac{1}{2\pi}C_4\left(\frac{4\pi}{M_N C_0}\right)^7$$

6-8% contribution from QED logarithms

Legend:
- Exp
- NLO_π
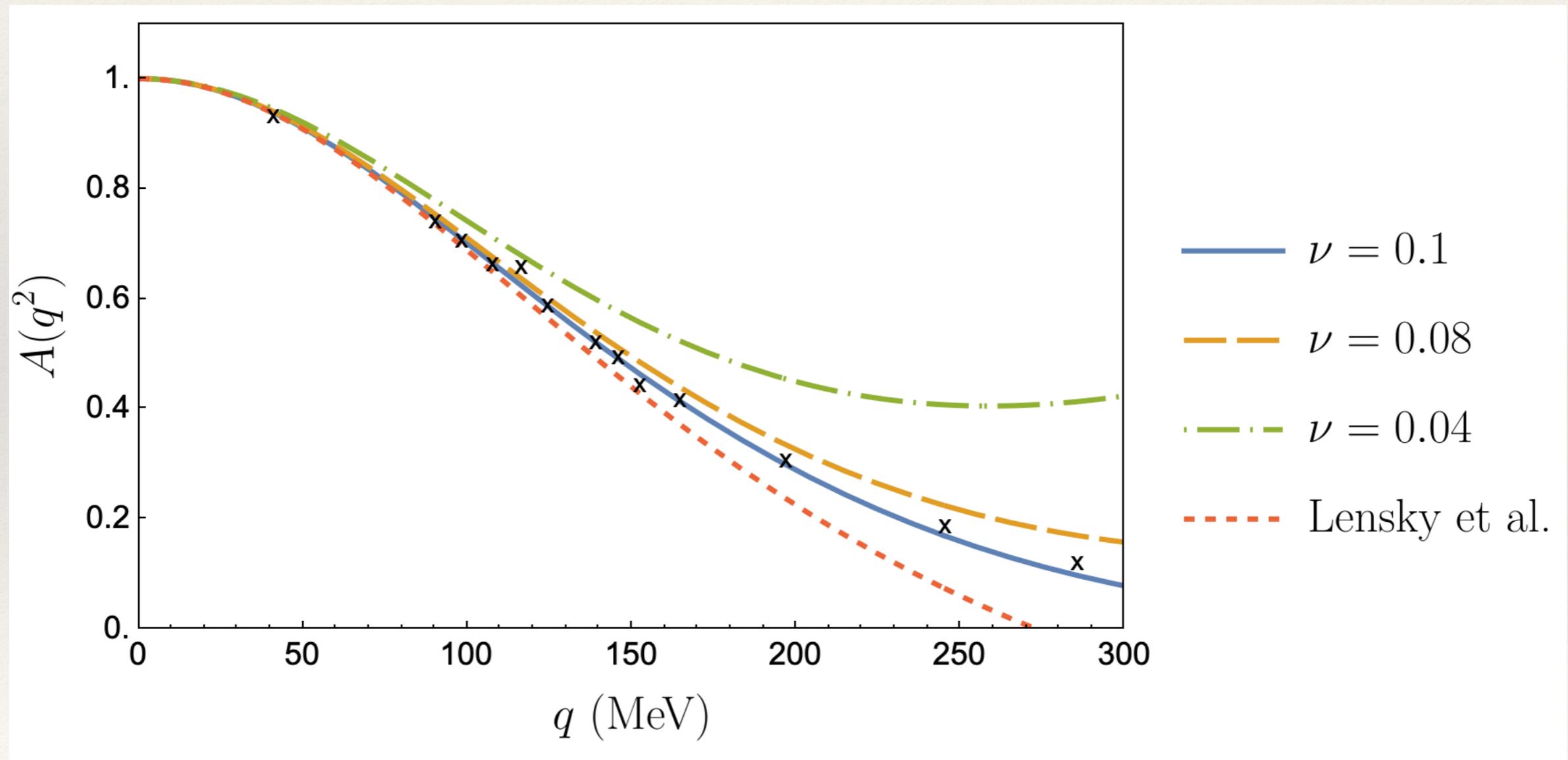- $\mathrm{N^2LO}_\pi$
- NLO_π
- $\mathrm{N^2LO}_\pi + \mathrm{LL}_\alpha$

Charge Form Factor

$$\langle \mathbf{p}', I | J_{\mathrm{EM}}^0 | \mathbf{p}, J \rangle \supset F_C(q^2)\delta^{IJ}$$

Abbot et al. EPJ A 7, KSW PRC 59, Lensky et al. PRC 104

Electron Scattering Structure Function

$$A(q^2) = F_C^2(q^2) + \frac{2}{3}\eta F_M^2(q^2) + \frac{8}{9}\eta^2 F_Q^2(q^2)$$

Simon et al. NPA 364, KSW PRC 59, Lensky et al. PRC 104

Charge Radius

❖ Conventional definition of charge radius is not scale-independent at $O(\alpha)$ and higher

$$\langle r_d^2 \rangle_C = -6 \frac{dF_C(q^2)}{dq^2} \bigg|_{q^2=0}$$

$$r_{\mathrm{CREMA}} = 2.12562(13)_{\mathrm{exp}}(77)_{\mathrm{th}}$$

CREMA Science 353

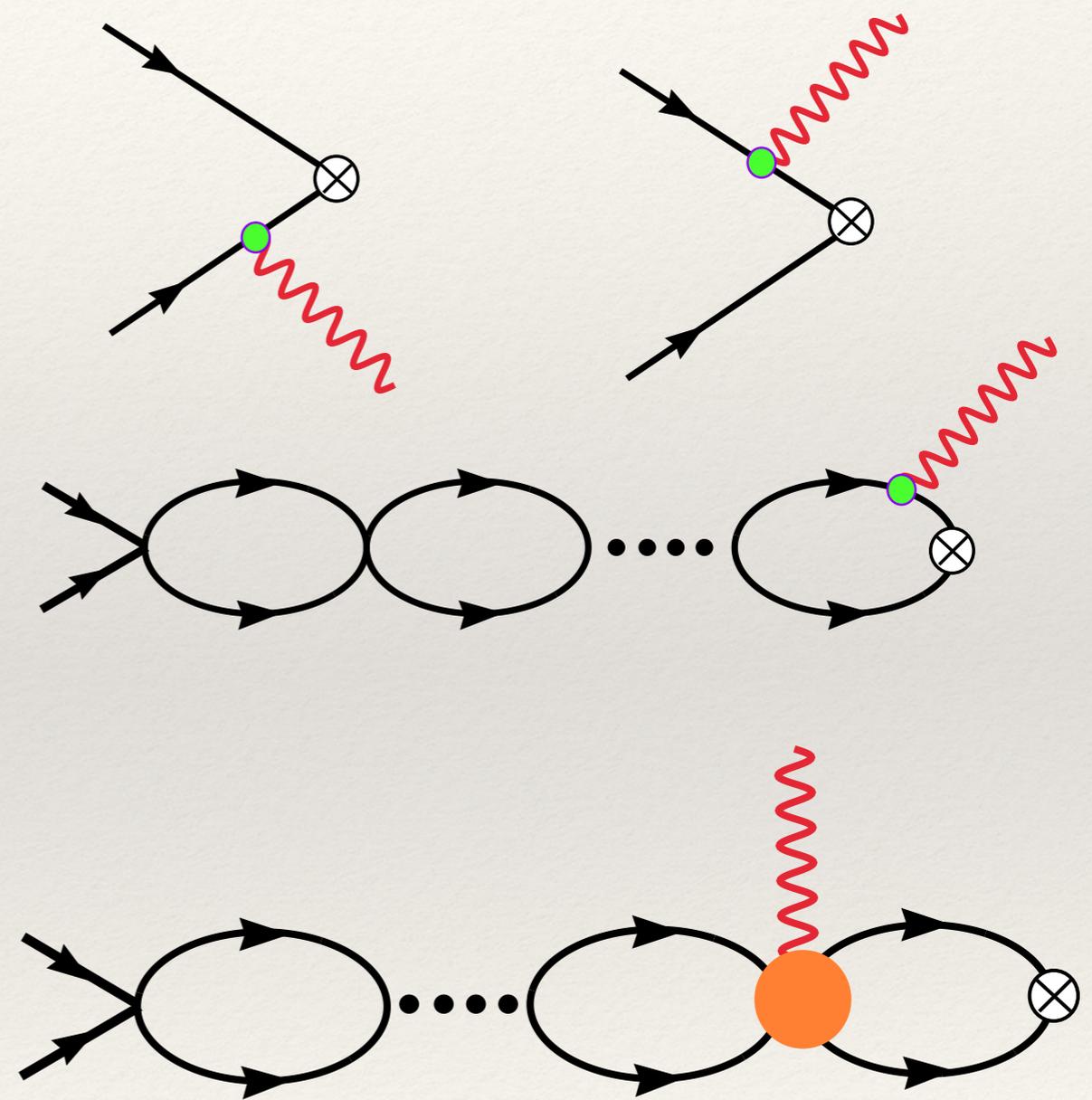|		$\nu = \frac{m_\pi}{M_N}$	$\nu = 0.1$	$\nu = 0.06$
r_d (fm)	no truncation error	2.154(6)	2.111(6)	2.049(7)
	with truncation error	2.15(6)	2.11(6)	2.05(6)

2% shift

Radiative Neutron Capture

- First step in Big Bang Nucleosynthesis network

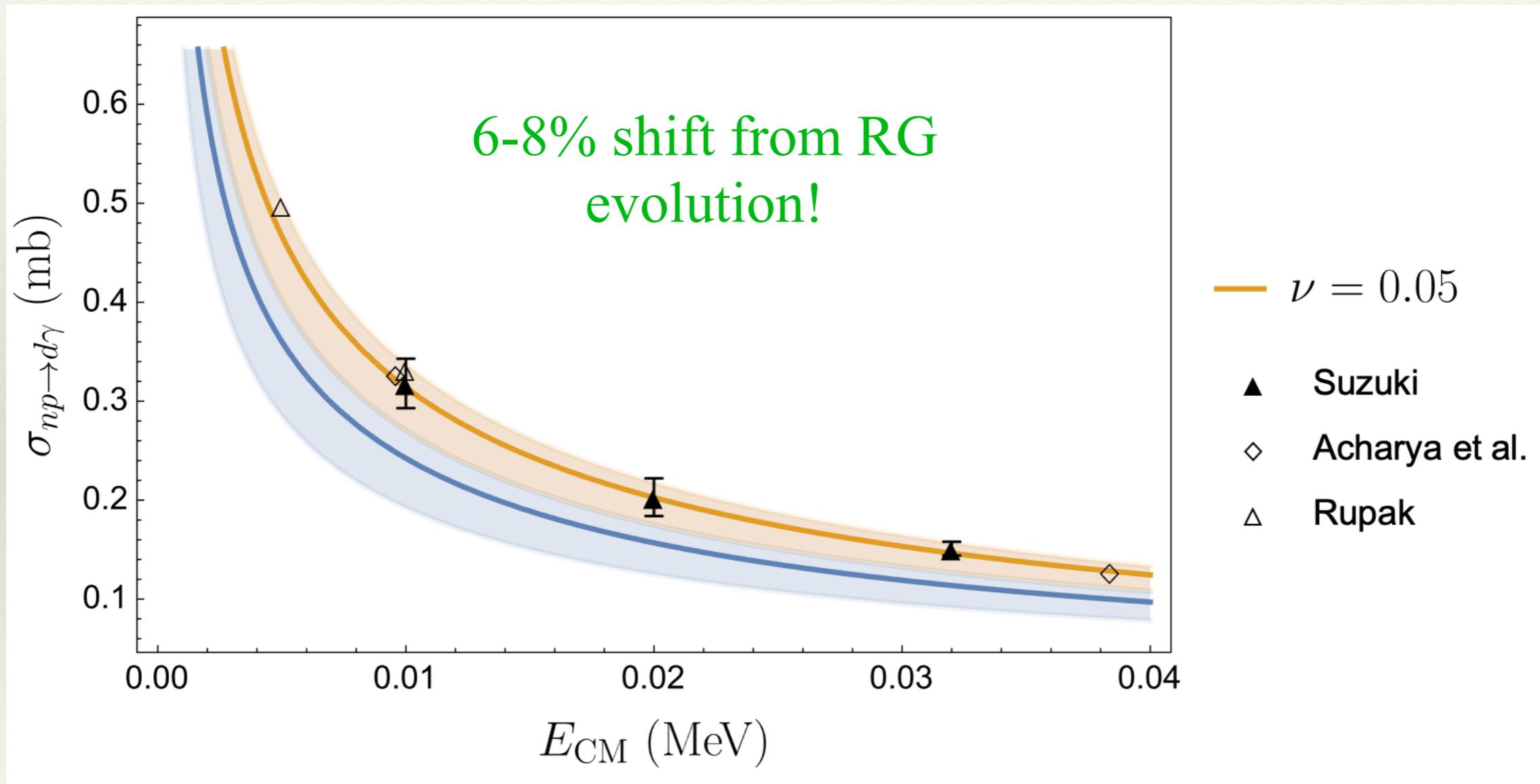 Wagoner, Fowler, Hoyle, Steigman, Iocco, Mangano, Miele, Pisanti, Serpico, Cyburt, Fields, Olive, Yeh

- Uncertainty in cross section sets scale for uncertainty in light element abundances

- Could be a probe of new physics with precise Standard Model predictions

- EFT analyses claim O(1%) uncertainty*

Chen et al. NPA, 653, Rupak NPA 678, Acharya and Bacca PLB 827

*with LECs fit to experimental data

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Radiative Neutron Capture

Cox et al. NP 74, Suzuki et al. 1995, Nagai et al. PRC 56, Tudoric-Ghemo, NPA 92, Bosman et al. PLB 82, Steiler et al. 1986, Michel et al. JPG 1989
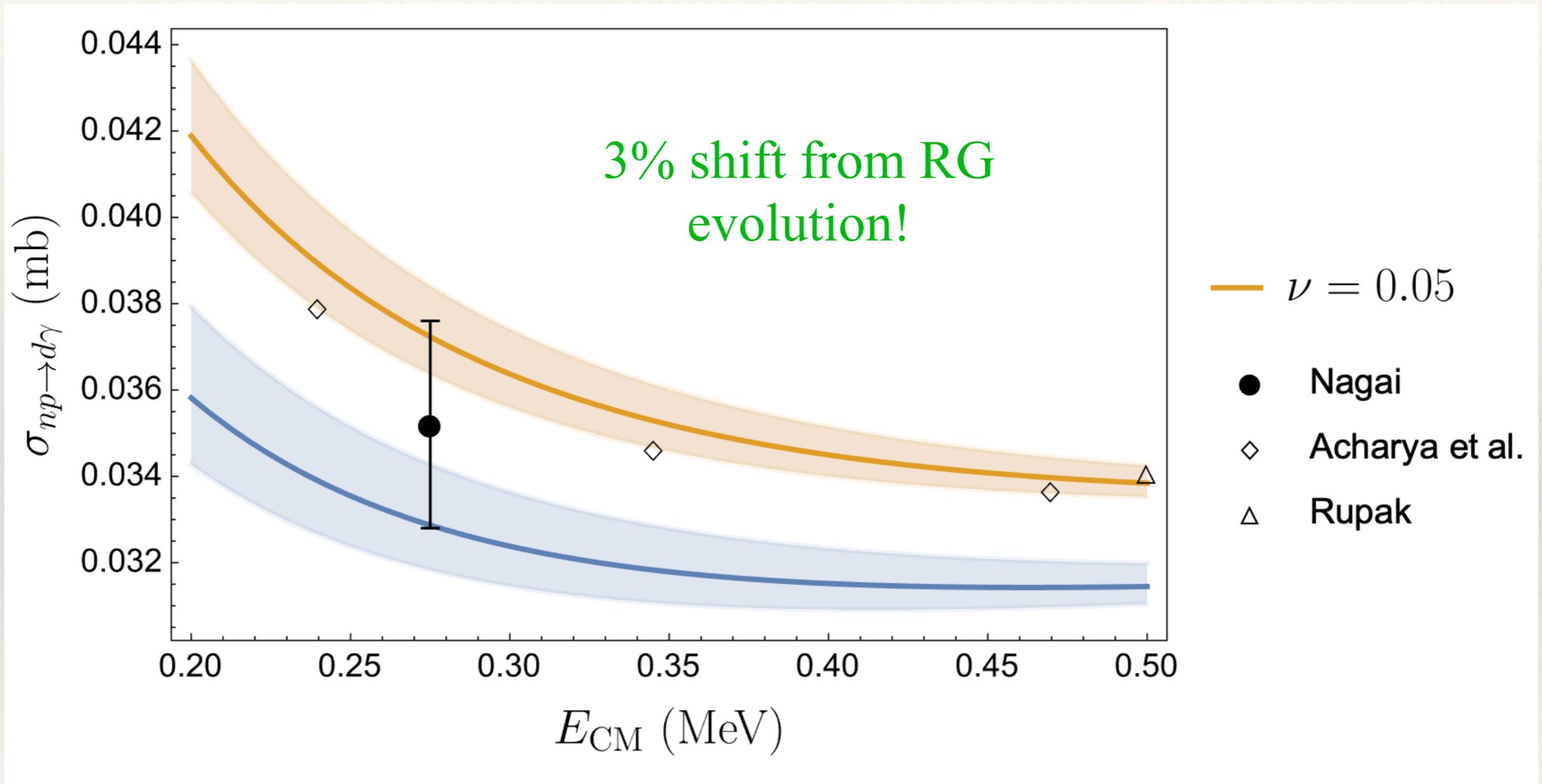
Radiative Neutron Capture

Cox et al. NP 74, Suzuki et al. 1995, Nagai et al. PRC 56, Tudoric-Ghemo, NPA 92, Bosman et al. PLB 82, Steiler et al. 1986, Michel et al. JPG 1989

Summary

❖ Nonrelativistic theories with photons couple different scales together—velocity formalism cures this

❖ RG improvement leads to few percent corrections in

    1. Deuteron binding energy (7-8%)

    2. Charge form factor and radius (2%)

    3. $np \rightarrow d\gamma$ cross section relevant for BBN (3-8%)

❖ Older EFT calculations used parameters fit to experimental data without accounting for radiative corrections

    ➡ How robust is the uncertainty quantification?

    ➡QED logarithms seem to be more important than previously expected

❖ Next steps: pp-fusion, include pions, next-to-leading-log, more nucleons