Theoretical Advances in $g-2$

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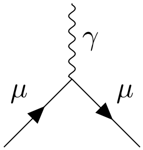
15th Conference on the Intersections of Particle and Nuclear Phyiscs (CIPANP) - 10th June 2025

$g-2$ Overview

- Magnetic moment of muon μ:

$$\vec{\mu}_\mu = g\frac{e}{2m_\mu}\vec{S}$$

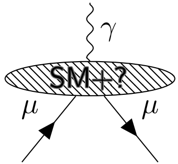


- Dirac eqn. (tree-level QFT):

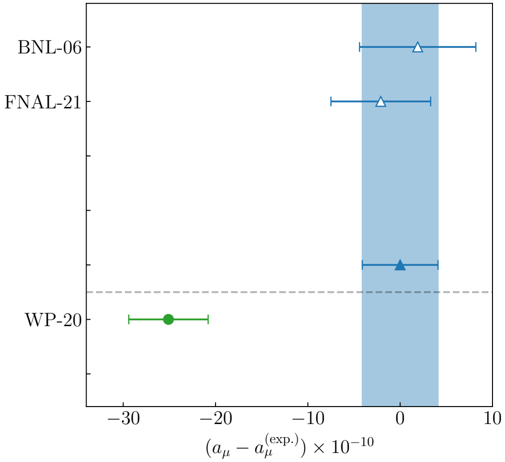
$$g = 2$$

- Anomalous moment from loop effects:

$$a_\mu = (g-2)/2$$



Muon $g-2$: A precision test of the Standard Model



- 4 years ago:
  - Exp. avg: 0.35 ppm.
  - WP20: 0.37 ppm (data-driven HVP).
  - Tension: $4.2\,\sigma$.

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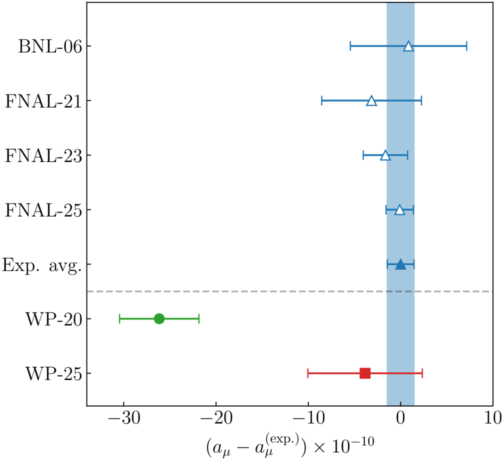

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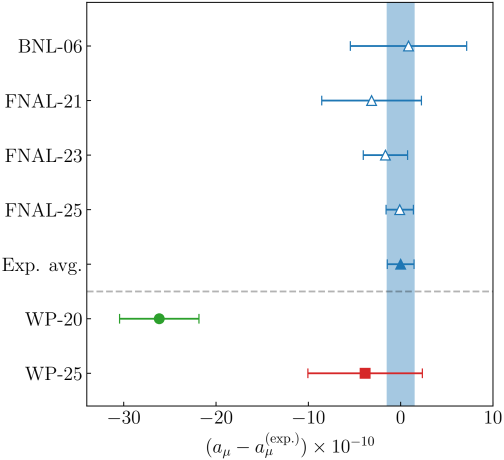
- Today:
  - Exp. avg: 0.12 ppm.
  - WP25: 0.53 ppm (lattice-QCD HVP).
  - Agreement: $0.4\,\sigma$.

Muon $g-2$: A precision test of the Standard Model



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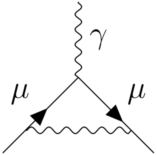
This talk: Developments from WP20 to WP25.

SM Theory Overview

- Computed perturbatively in power series of EM coupling α:

$$a_\mu^{\mathrm{SM}} = \sum_{i=1} a_\mu^{(i)}(\alpha^i)$$

- $a_\mu^{(1)}$ contribution:



$$\Rightarrow \quad a_\mu^{(1)} = \frac{\alpha}{2\pi} \approx 0.00116171491\ldots \quad \text{Schwinger, } 1948$$

- $a_\mu^{(n>1)}$ contributions: $\approx 10^{n-1}$ diagrams, mass dependence,

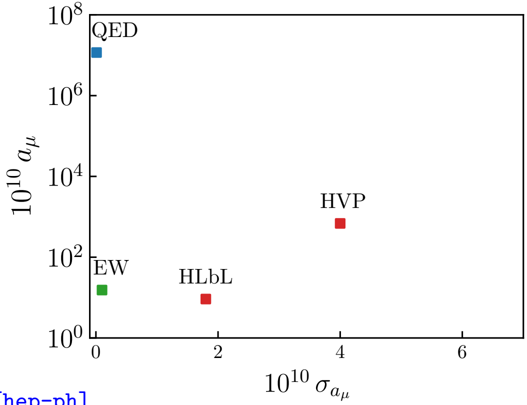
$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Had.}} + a_\mu^{\mathrm{EW}}$$

Theoretical result - a_μ^{SM} (WP20 & WP25)[1,2]

$$a_\mu^{\mathrm{SM}} = a_\mu^{\mathrm{QED}} + a_\mu^{\mathrm{Had.}} + a_\mu^{\mathrm{EW}}$$

$$a_\mu^{\mathrm{Had.}} = a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$

$$10^{10} a_\mu^{\mathrm{SM}}(\mathrm{WP20}) = 11659181.0(4.3)$$



[1] T. Aoyama et al., Phys. Rept. 887, 1–166 (2020).
[2] R. Aliberti et al. "" May 2025 arXiv: 2505.21476 [hep-ph].

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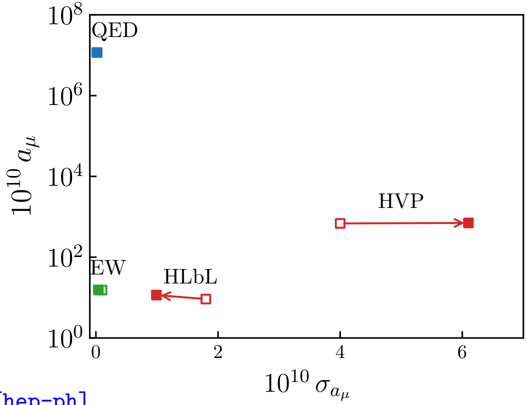
$$a_\mu^{\mathrm{Had.}} = a_\mu^{\mathrm{HVP}} + a_\mu^{\mathrm{HLbL}}$$

$$10^{10} a_\mu^{\mathrm{SM}}(\mathrm{WP20}) = 11659181.0(4.3)$$

$$10^{10} a_\mu^{\mathrm{SM}}(\mathrm{WP25}) = 11659203.3(6.2)$$

- HVP update : $+19.1$
  (data-driven $\rightarrow$ lattice QCD)
- HLbL update: $+2.5$



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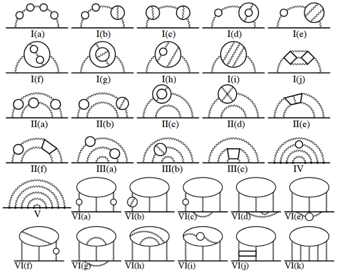
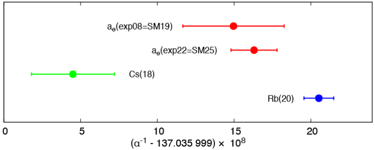
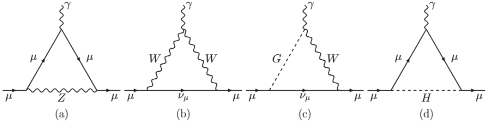




- WP20: Computed up to $\mathcal{O}(\alpha^5)$.
- All $\mathcal{O}(\alpha^5)$ diagrams (10k+) by one team, AHKN (numerically), subsets crosschecked by other groups (numerical and analytical).
- WP25:
  - Further crosschecks of $\mathcal{O}(\alpha^5)$ contributions.
  - Neglegible shift in central value.
  - Uncertainty doubled due to tensions in α determinations.

The EW contribution

- Contributions containing at least one EW boson (W, Z or higgs)



- Suppressed due to mass ratios

$$a_\mu^{\mathrm{EW}(1)} \propto \frac{\alpha}{4\pi s_{\mathrm{W}}^2}\frac{m_\mu^2}{M_W^2} \sim 10^{-9}$$

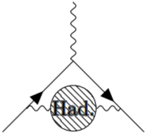
- Computed up to 3 loops

$$a_\mu^{\mathrm{EW}} = a_\mu^{\mathrm{EW}(1)} + a_{\mu;\mathrm{bos}}^{\mathrm{EW}(2)} + a_{\mu;\mathrm{ferm}}^{\mathrm{EW}(2)} + a_\mu^{\mathrm{EW}(\geq 3)}$$

- WP25: Uncertainty halved with improved M_H, refined calculation of hadronic contributions.

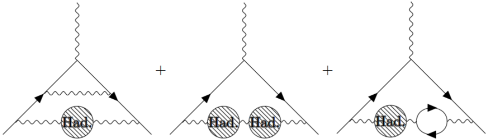
The Hadronic Vacuum Polarization

(LO) :

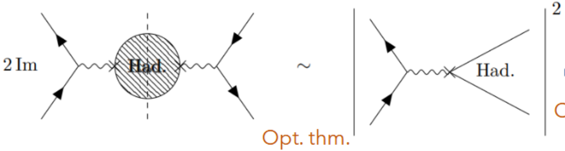


$$\Rightarrow a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty \mathrm{d}Q^2 K_E(Q^2)\widehat{\Pi}(Q^2).$$

(NLO):



- Enters at $\mathcal{O}(\alpha^2)$
- WP: LO, NLO (α^3) & NNLO (α^4)
- Two approaches:
  - **Data-driven dispersive**: $\sigma(e^+e^- \to \mathrm{Had.}\gamma)$
  - **Lattice QCD** ("ab-inito"): $(n_f + 1$ inputs to set physical scales)



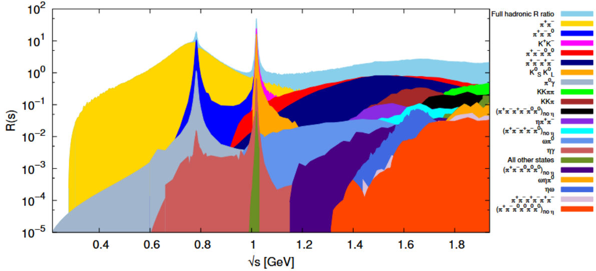
Cauchy. ⟹ *"R-ratio"*

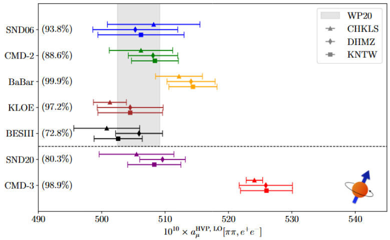
$$\hat{\Pi}\left(q^2\right) = \frac{4\pi\alpha^2 q^2}{9}\int_{4m_\pi^2}^{\infty}\frac{\sigma_{e^-e^+\to Had.}(s)}{s^2\left(s-q^2\right)}ds$$

$$a_\mu^{\mathrm{HVP,LO}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2\int_{s_{\mathrm{thr}}}^{\infty} ds\frac{\hat{K}(s)}{s^2}R_{\mathrm{Had.}}(s), \quad R_{\mathrm{Had.}}(s) = \frac{3s}{4\pi\alpha^2}\sigma\left[e^+e^- \to \text{ Hadrons } (+\gamma)\right]$$

$R(s)$ taken from e^+e^- collider data.

Data driven dispersive results





- R(s) results from 100+ datasets in 35+ channels summed to $\sqrt{s} \approx 2$ GeV.
- Dominant $\pi^+\pi^-$ channel approx 70% of total $a_\mu^{\mathrm{HVP,LO}}$.
- Discrepencies in $\pi\pi$ channel unresolved $\rightarrow$ WP25 data-driven HVP theory average not sensible.

Lattice QCD approach

$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt \tilde{K}(t)\, C(t), \quad C(t) = \frac{1}{3}\sum_i^3 \int d^3x \, \langle J_i(x) J_i(0) \rangle$$ (F.T. of HVP)

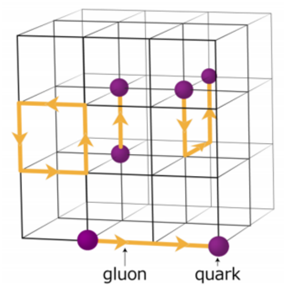
$$J_i(x) = \sum_f Q_{q_f} \bar{q}_f(x) \gamma_i q_f(x), \quad Q_u = +\frac{2}{3}, \; Q_d = -\frac{1}{3}, \; Q_s = -\frac{1}{3}, \ldots$$

$C(t)$ is computed on the lattice.

What is Lattice QCD

Lattice perscription: Wick rotate $(t \to it)$ and discretize $(a \neq 0)$ the path integral.

$$\langle O \rangle = \frac{1}{Z_{\mathrm{LQCD}}} \int \mathcal{D}\left[q_f, \bar{q}_f\right] \mathcal{D}[U] \mathrm{e}^{-S_F[q_f, \bar{q}_f, U] - S_G[U]} O\left[q_f, \bar{q}_f, U\right]$$



M. Savage,
2015 National Nuclear Physics Summer School

- Quarks on sites, gluons $(U_\mu(n) \equiv \exp\left(\mathrm{i}aG_\mu(n)\right))$ on links.
- Perform path integral:
  - Fermionic: Analytic $\to$ Wick contractions.
  - Bosonic: Markov Chain Monte Carlo (Importance Sampling).
- Observables: $\langle O \rangle(\{a, L, m_q\})$.
- Extrapolate, interpolate, or correct (EFT/pQCD) to continuum, infinite volume, physical point.

$a_\mu^{\text{HVP,LO}} = 4\alpha^2 \int_0^\infty dt \tilde{K}(t)\, C(t)$ calculation broken down into:

- Isospin-symmetric + broken pieces. $a_\mu(m_u = m_d = m_{\text{light}},\ Q_f = 0) + \Delta_{\text{I.B.}}$

$a_\mu^{\text{HVP,LO}}$:

Lattice HVP breakdown

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- Conn. + Disc. wick contractions. $C(t) =$

$a_\mu^{\text{HVP,LO}}$:

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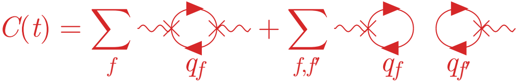
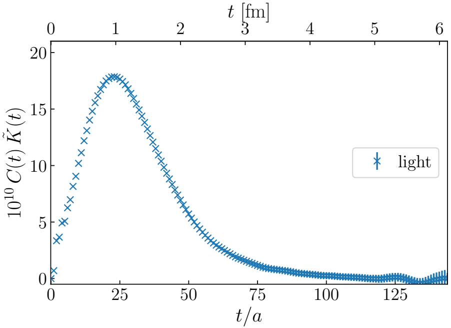


- $\sum$ quark flavor.

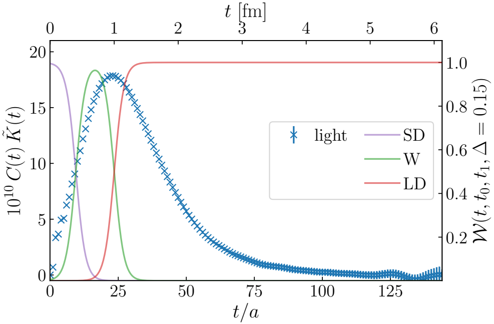
$a_\mu^{\mathrm{HVP,LO}}$:

C(t) & Euclidean windows[3]



$$a_\mu^{\mathrm{HVP,LO}} = 4\alpha^2 \int_0^\infty dt\, \tilde{K}(t)\, C(t)$$

[3] T. Blum et al., Phys. Rev. Lett. 121.2, 022003 (2018).

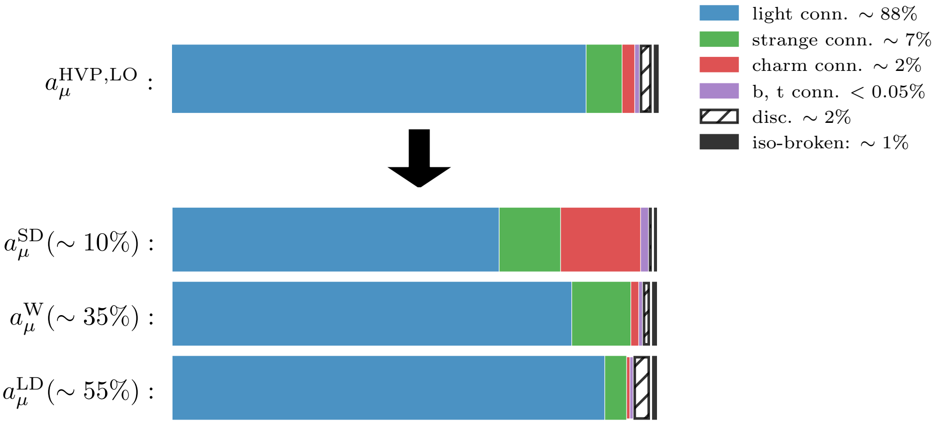


$$a_\mu^{\mathrm{W}} = 4\alpha^2 \int_0^\infty dt\, \tilde{K}(t)\, C(t) \mathcal{W}(\mathrm{W}, t)$$

- SD($\sim$ high energy (pQCD)):
  - sensitive to a (lattice spacing).
- LD($\sim$ low energy (χPT)):
  - sensitive to L (lattice length).
- W: *Goldilocks* window.
- $a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$

[3] T. Blum et al., Phys. Rev. Lett. 121.2, 022003 (2018).

Lattice HVP windows breakdown: $a_\mu^{\mathrm{HVP,LO}} = a_\mu^{\mathrm{SD}} + a_\mu^{\mathrm{W}} + a_\mu^{\mathrm{LD}}$



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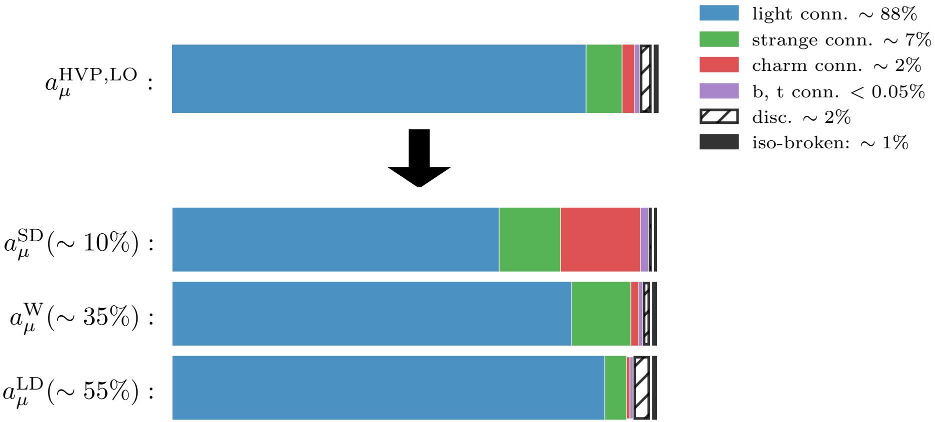


Each piece is a separate calculation!

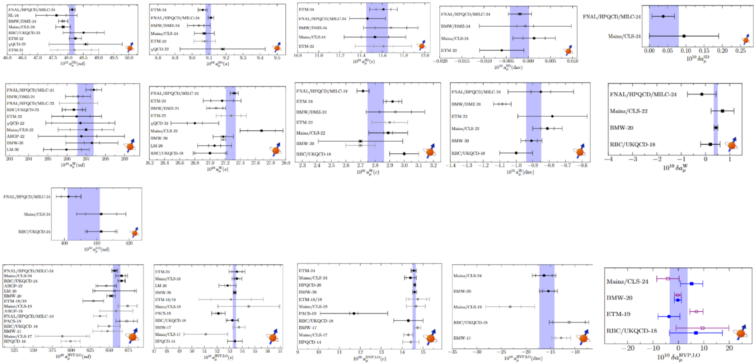
light strange charm disconnected δ IB

SD

W

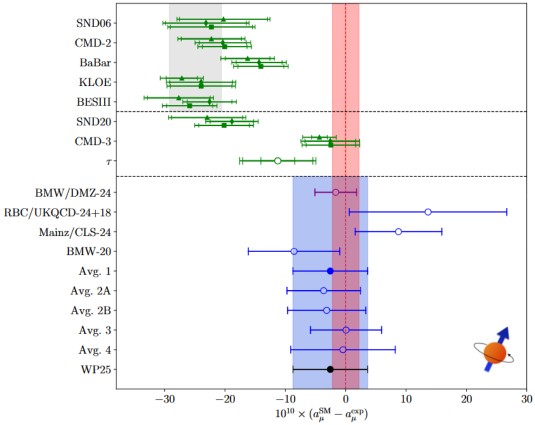
LD

Full



- WP20$\rightarrow$ WP25: 17 lattice papers from 8 lattice groups.
- Many ways to obtain full (SD+W+LD), complete (isosymmetric + isobroken) result.

WP25 HVP result

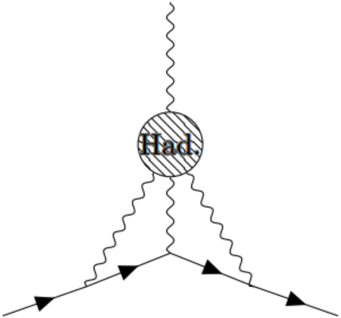


- Lattice (blue) average performed 5 ways (1, 2A, 2B, 3, 4).
- Data-driven dispersive (green)
- 3 lattice groups with (near) complete calculations (BMW20, RBC/UKQCD24, Mainz24)
- Hybrid evalutation: data-driven+lattice, maroon (BMW/DMZ24)

The Hadronic Light-by-Light Contribution

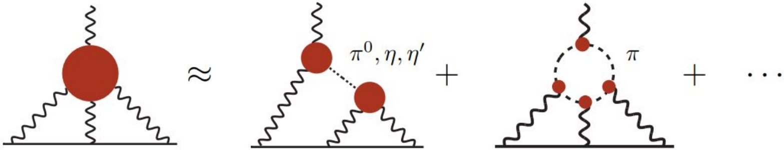
Hadronic light-by-light

(LO) :



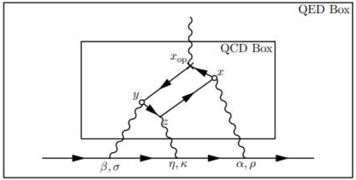
- Enters at $\mathcal{O}(\alpha^3)$
- WP20 & WP25: computed at LO and NLO
- Two approaches:
  - Dispersive (Optical Thrm: Multiple cuts $\rightarrow$ multiple process to reproduce)
  - Lattice QCD

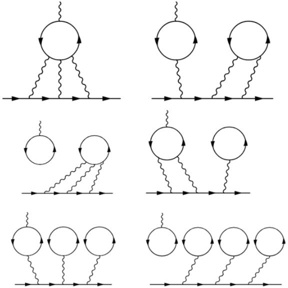
Dispersive HLbL Calculation details



- Dispersive framework developed late 2010s, just before WP20.
- Dominant contributions: single, light pseudo-scalar intermediate states π, η, η, ...
- Relies on input for Transition form-factors $PS \to \gamma\gamma$ (TFFS).
- Intermediate two-pion and two-kaon intermediate states also relevant.

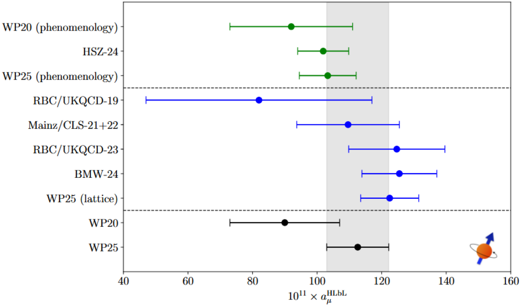
Lattice HLbL Calculation details





- QCD part on the lattice, QED part in the continuum and infinite volume limit.
- 6 Wick contractions (top 2 are dominant).
- Computed by three lattice groups with different dirac discretizations.

HLbL Results



- Dispersive (green) in good agreement with lattice (blue).
- WP25 from combined average of two approaches, $<10\%$ uncertainty.

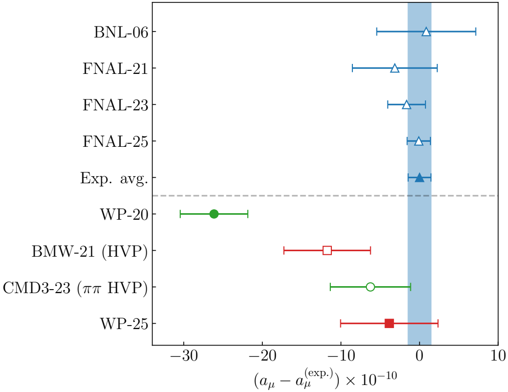
Plot from talk: Luchang Jin @ g-2 TI workshop, Bern 2023

Summary

- WP25 is a snapshot in time, posted before the final release of FNAL $g-2$ experimental results.
- Central value shift: WP20(data-driven HVP) $\rightarrow$ WP25(lattice-QCD HVP).
- HVP
  - Lattice: Consolidated average with all but a tiny contribution (0.06%) cross-checked by at least three (up to 8 for the benchmark windows) lattice-QCD collaborations.
  - Data-driven: CMD3 puzzle remains. Upcoming results from BaBar, KLOE, SND, CMD-3, BESIII, and Belle II, and improved theoretical tools (MC generators) will hopefully shed more light.
- HLbL
  - WP25 result is average of dispersive and lattice approaches ($<$10% precision goal achieved).
- Theory uncertainty is now 4 times larger than experimental, more work needed.

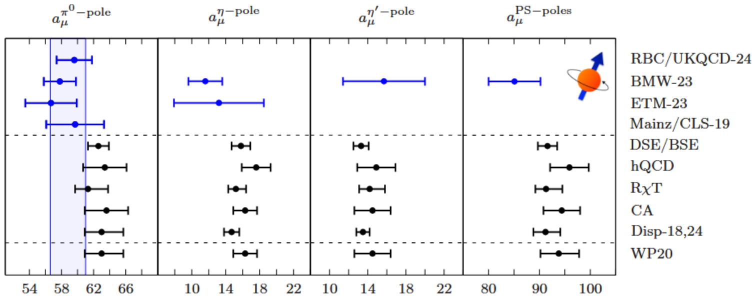
Thank you

Muon $g-2$: Recent History



1. BNL exp.: 0.54 ppm
2. WP20 (data-driven HVP), $3.7\,\sigma$ tension.
3. FNAL exp. run 1, Exp avg: 0.35 ppm $(4.2\,\sigma)$
4. BMW21 (lattice HVP) in between.
5. CMD3 (data-driven HVP) agrees with Exp avg.
6. FNAL experiment run 2+3, Exp avg: 0.19 ppm, $(5.1\,\sigma)$
7. A bunch of lattice results
8. WP25 (Lattice LO-HVP)
9. Final FNAL exp result, Exp avg: 0.13 ppm

Dispersive HLbL Results



- TFFs taken from experiment (Dispersive)
- Also computed using a number of approaches including lattice-QCD (blue)