Massachusetts Institute of Technology

Ab initio neutrinoless double beta decay matrix elements.

CIPANP 2025

Antoine Belley

12 June 2025

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DEPARTMENT OF ENERGY UNITED STATES OF AMERICA

NSERC CRSNG

$2\nu\beta\beta$ vs $0\nu\beta\beta$

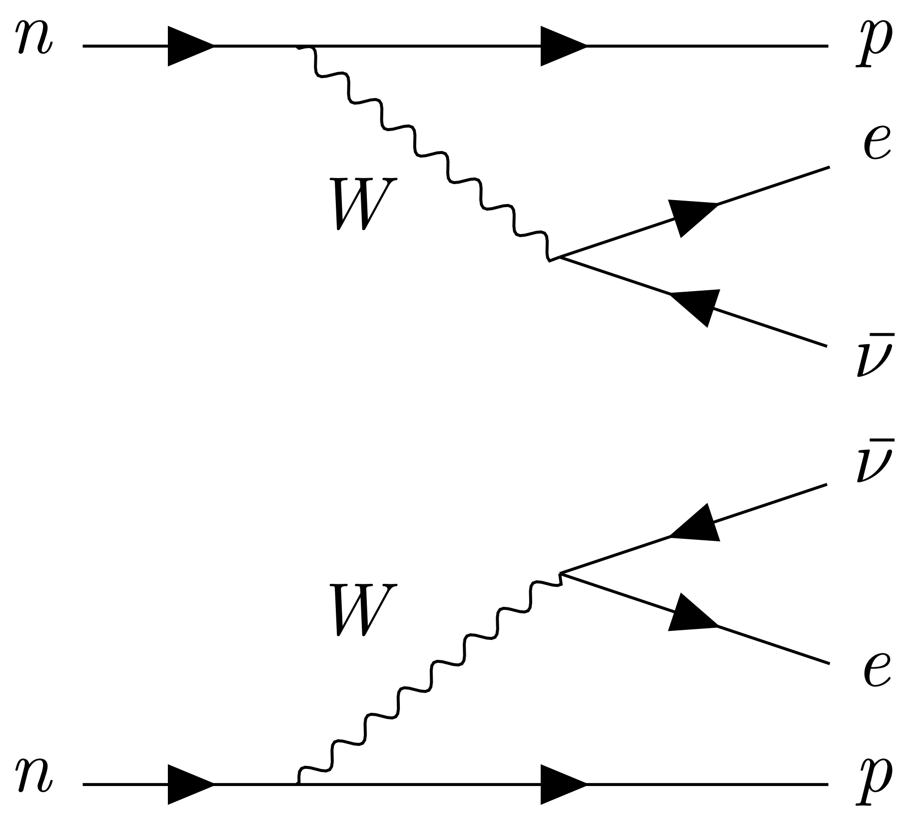
Decay	$2\nu\beta\beta$
Diagram	

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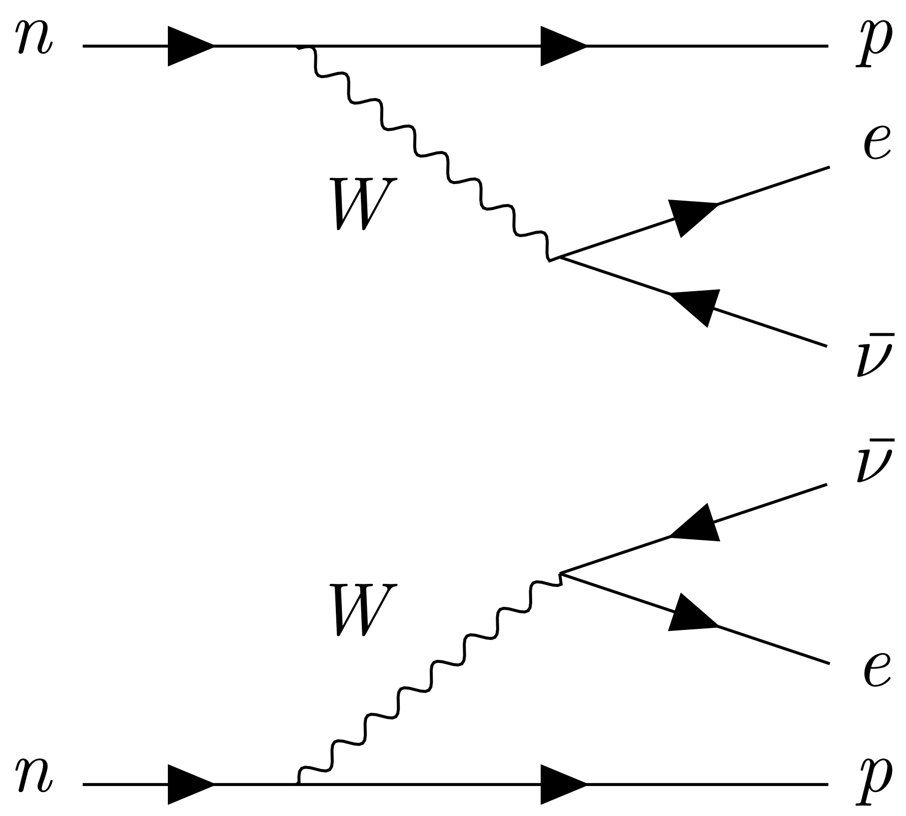
Diagram	
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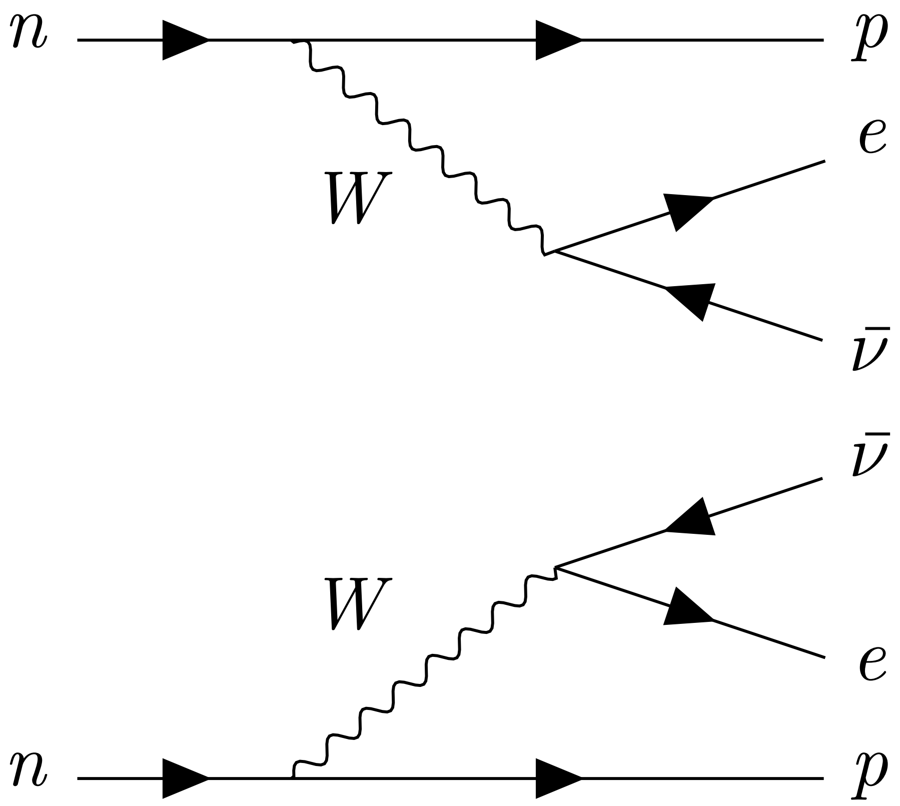
Diagram	
Half-life Formula	$[T^{2\nu}_{1/2}]^{-1} = g_A^4 G^{2\nu} \lvert M^{2\nu} \rvert^2$
NME Formula	$M^{2\nu} \approx M^{2\nu}_{GT}$

*NME : Nuclear matrix elements

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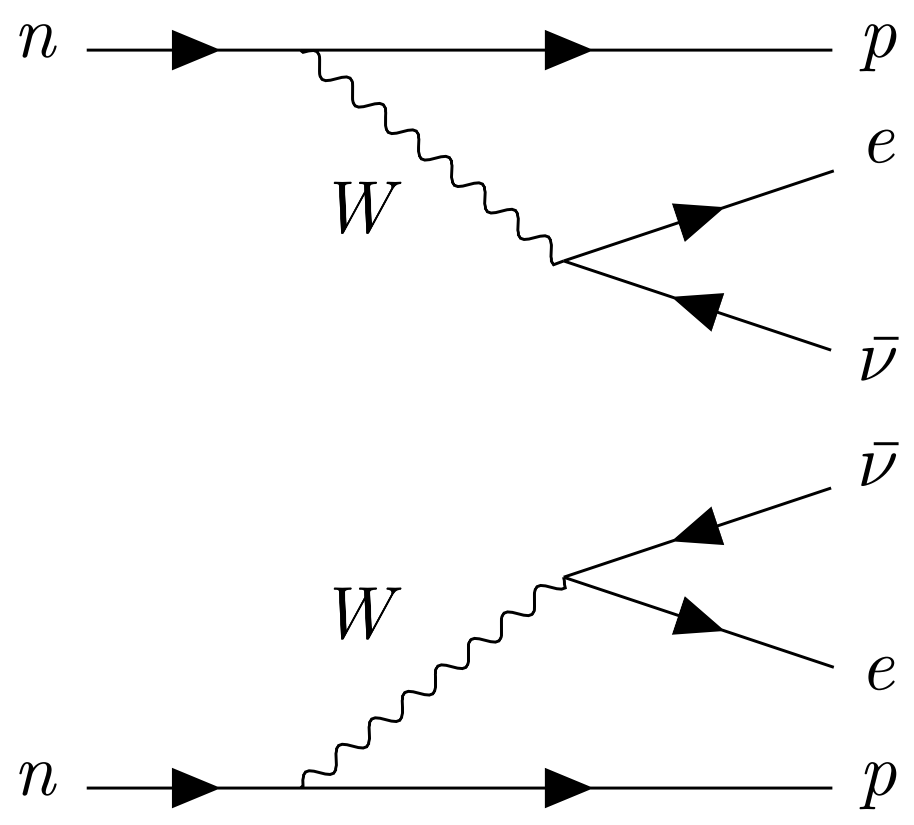
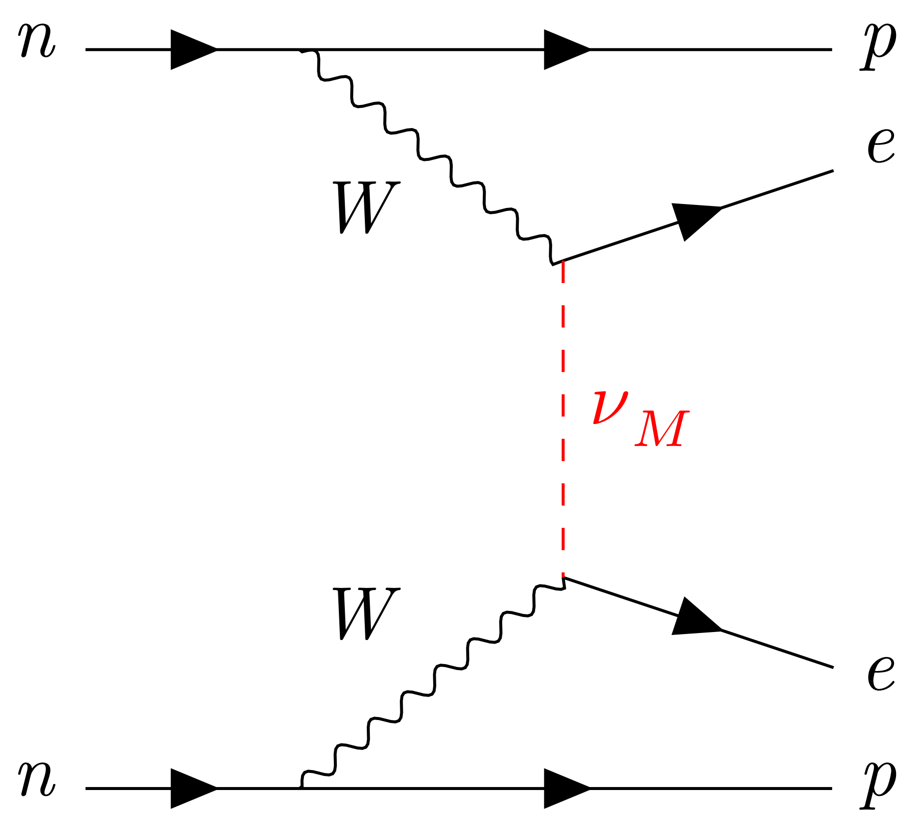
Diagram		
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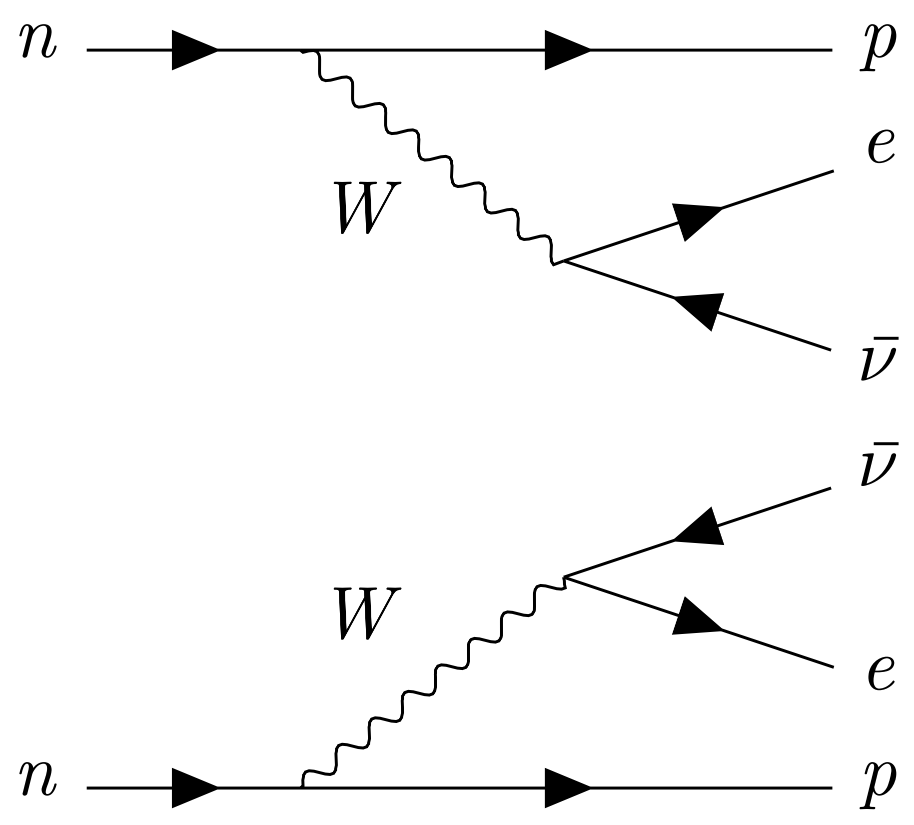
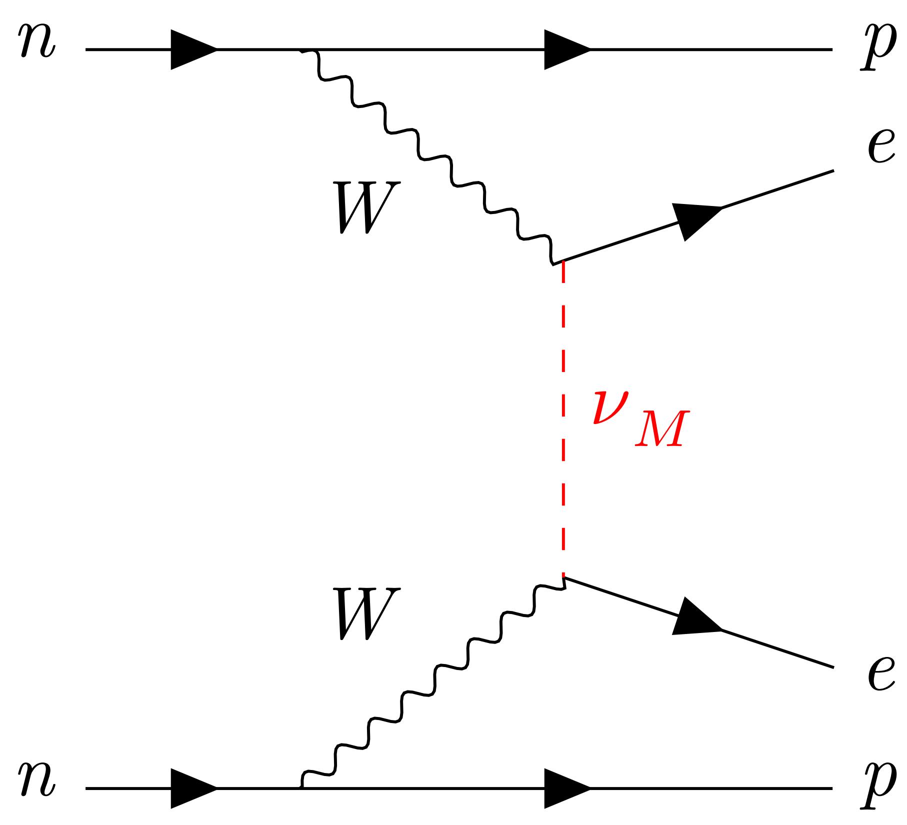
Diagram		
Half-life Formula	$[T^{2\nu}_{1/2}]^{-1} = g_A^4 G^{2\nu} \lvert M^{2\nu} \rvert^2$	Create matter without any antimatter!
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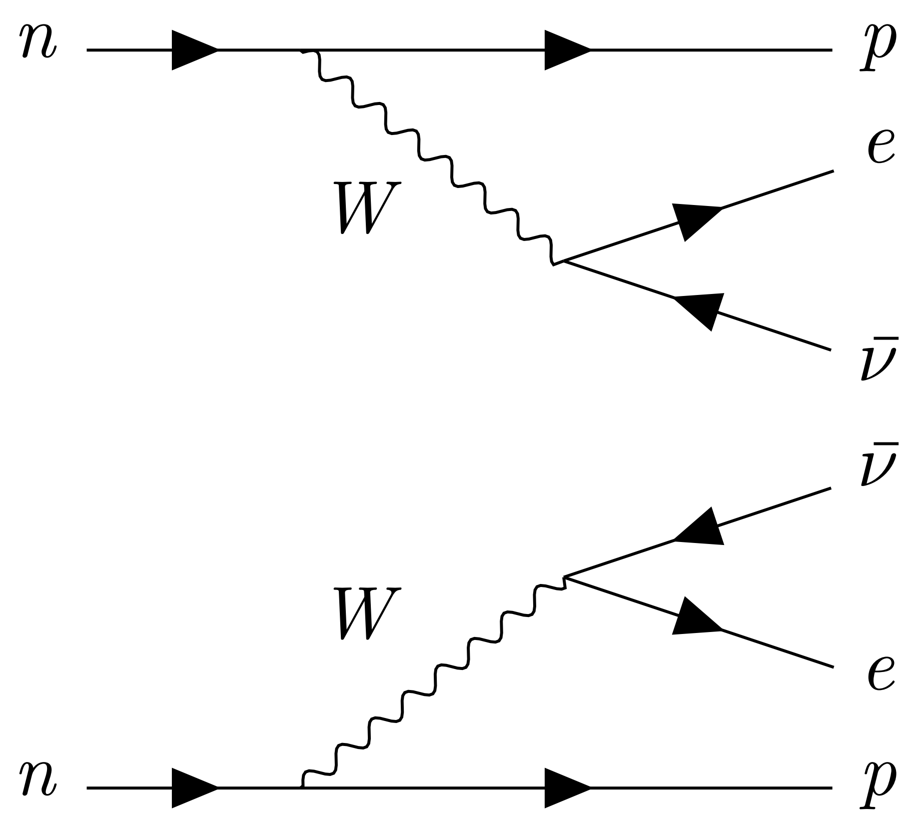
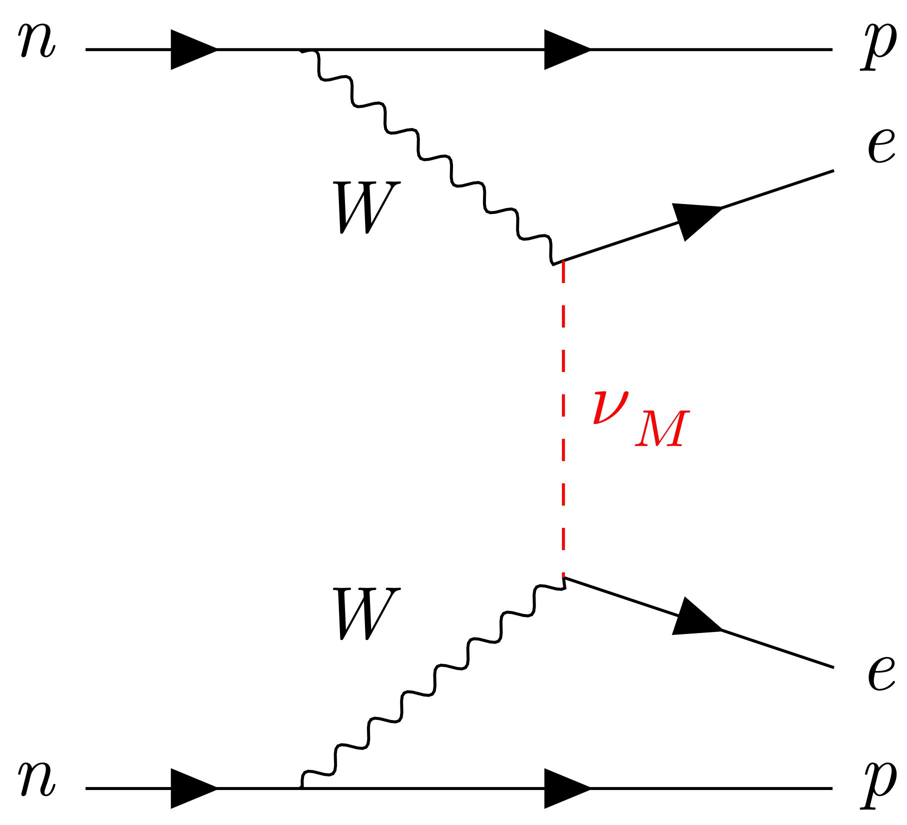
Diagram		
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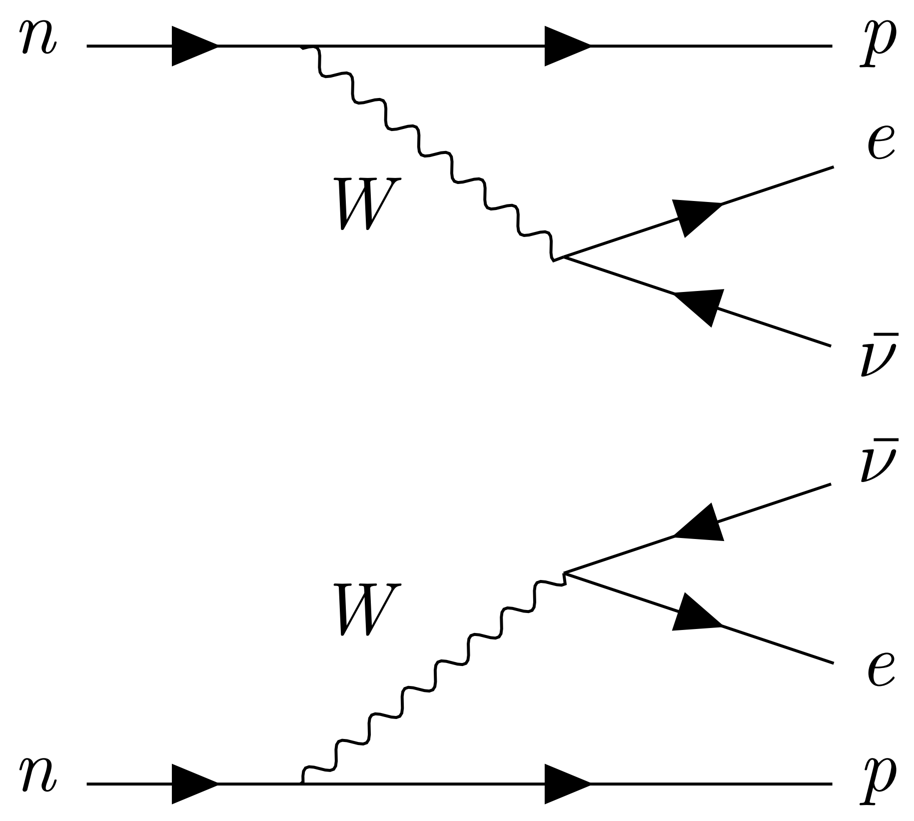
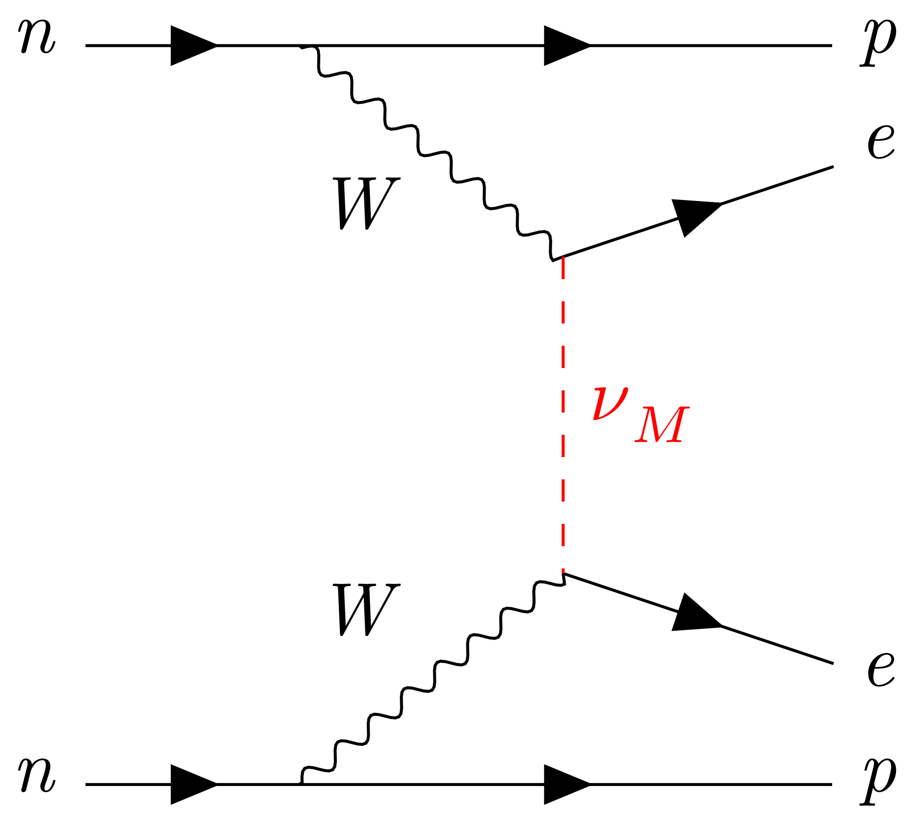
Diagram		
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NME Formula	$M^{2\nu} \approx M^{2\nu}_{GT}$	$\boxed{M^{0\nu} = M^{0\nu}_{GT} - (\frac{g_V}{g_A})^2 M^{0\nu}_F + M^{0\nu}_T - 2g_{\nu\nu}M^{0\nu}_{CT}}$

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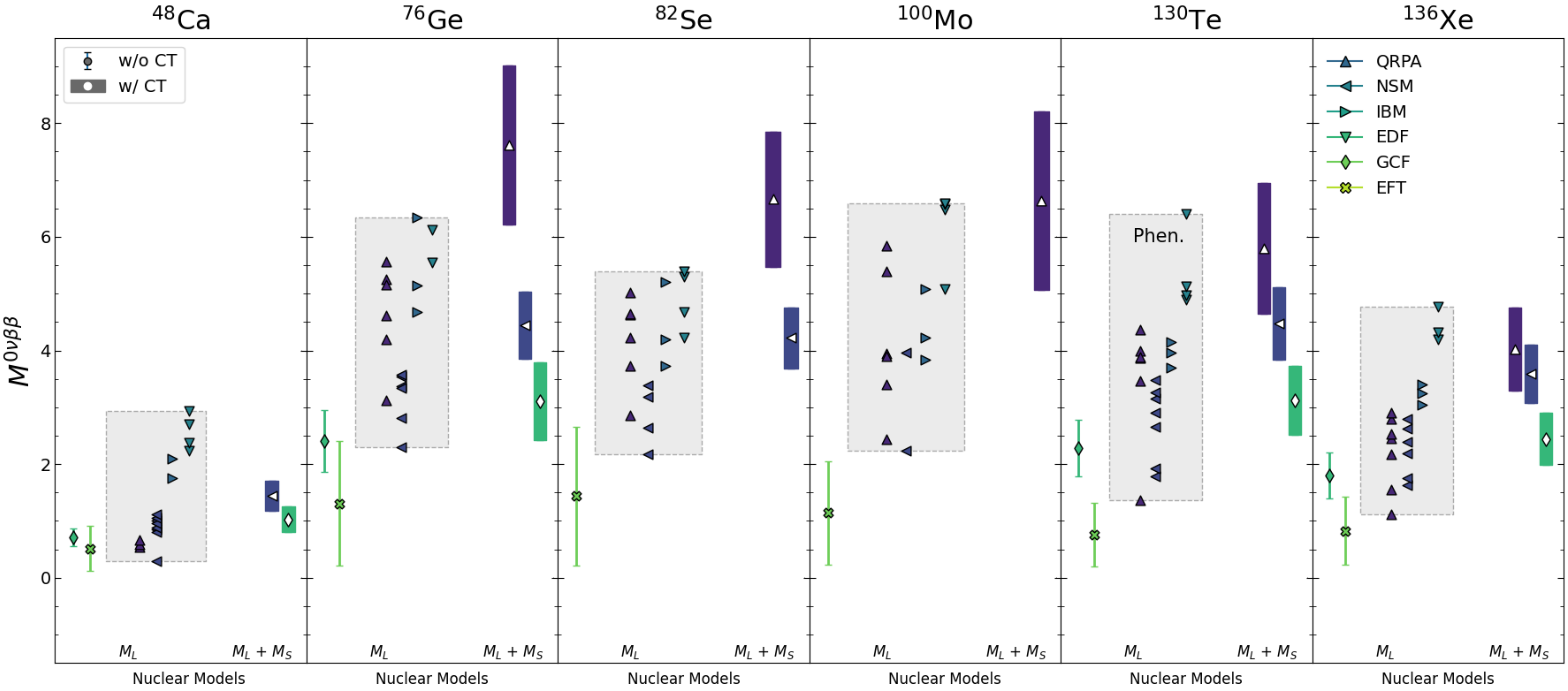
Status of $0\nu\beta\beta$-decay Matrix Elements



Values from Engel and Menéndez, Rep. Prog. Phys. **80** 046301 (2017); Yao, Sci. Bull. 10.1016 (2020); Brase et al, Phys. Rev. C **106**, 034309 (2021)

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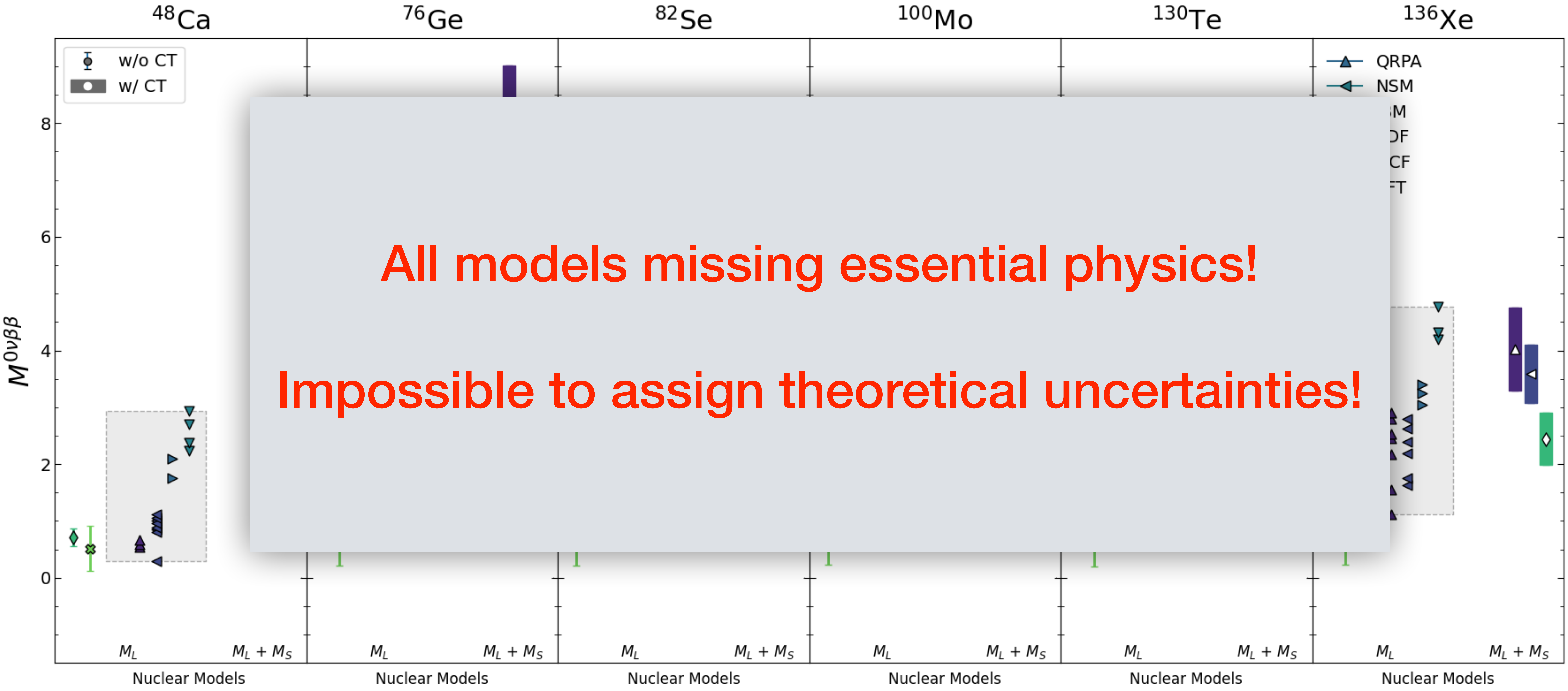

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Nuclear Theory Challenges

Understanding nuclear structure from microscopic physics

Figures courtesy of J. Muñoz

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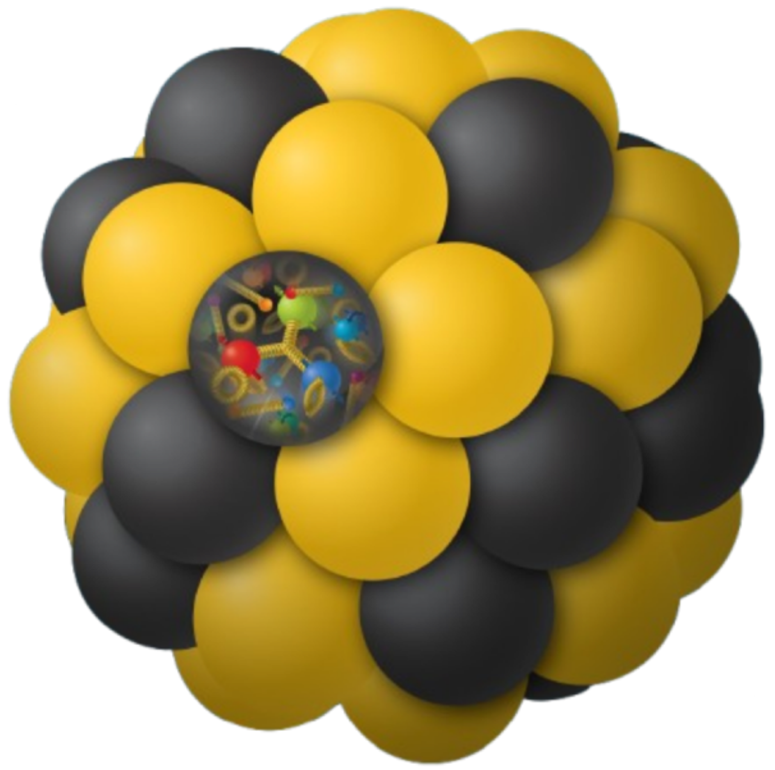
Nuclear Interactions



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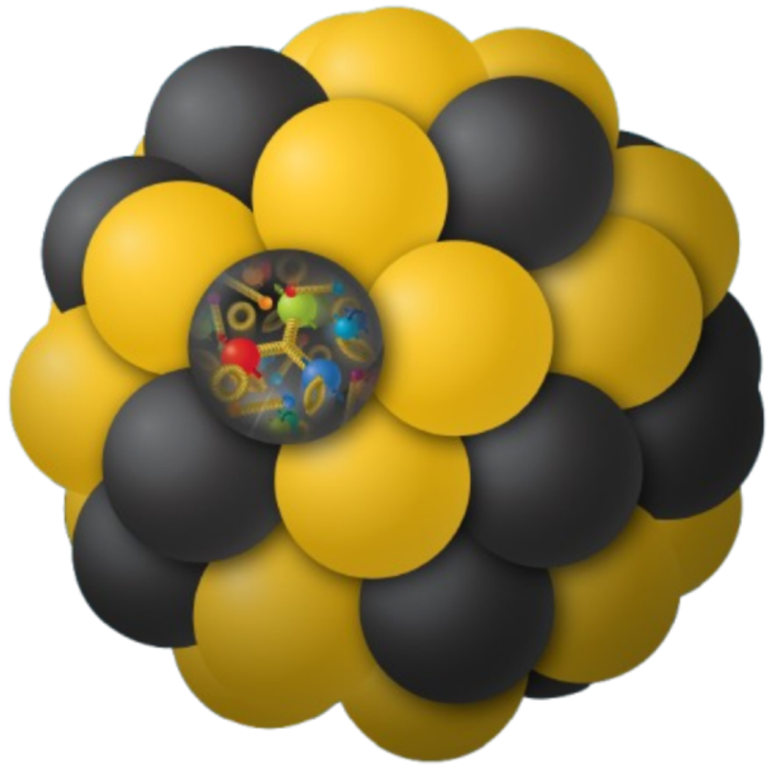
Nuclear Interactions

Wave functions



$$H|\Psi\rangle = E|\Psi\rangle$$

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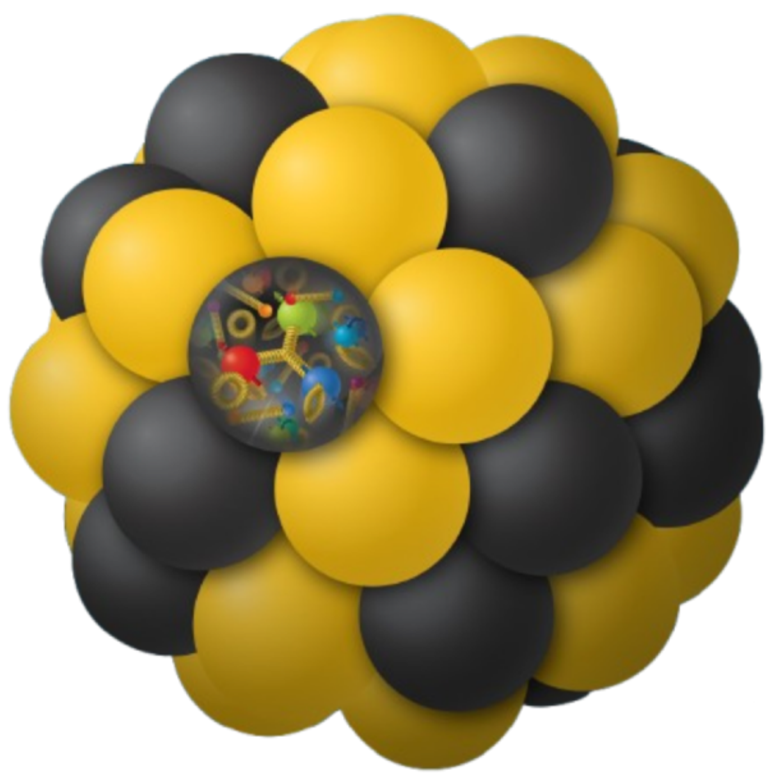

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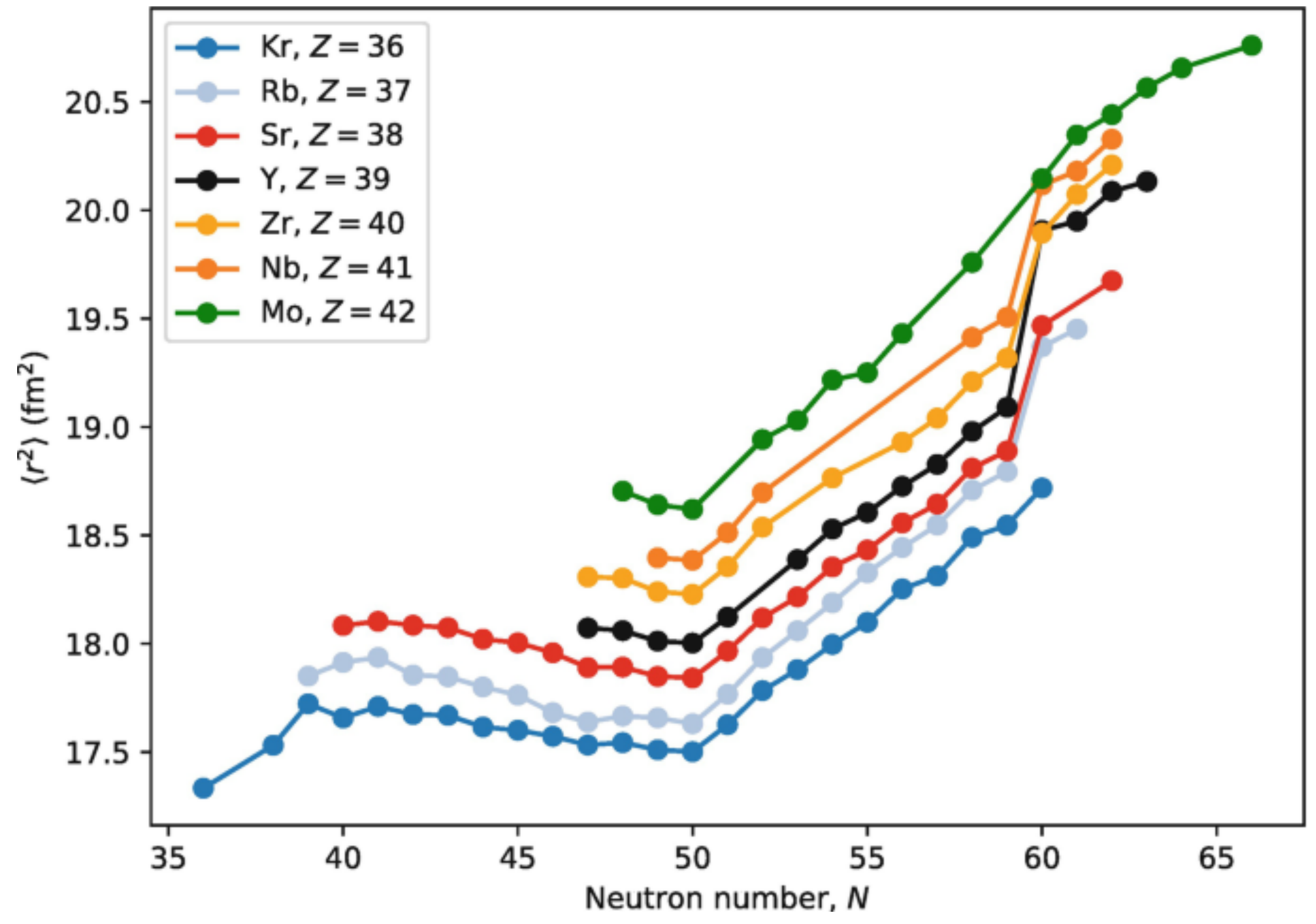

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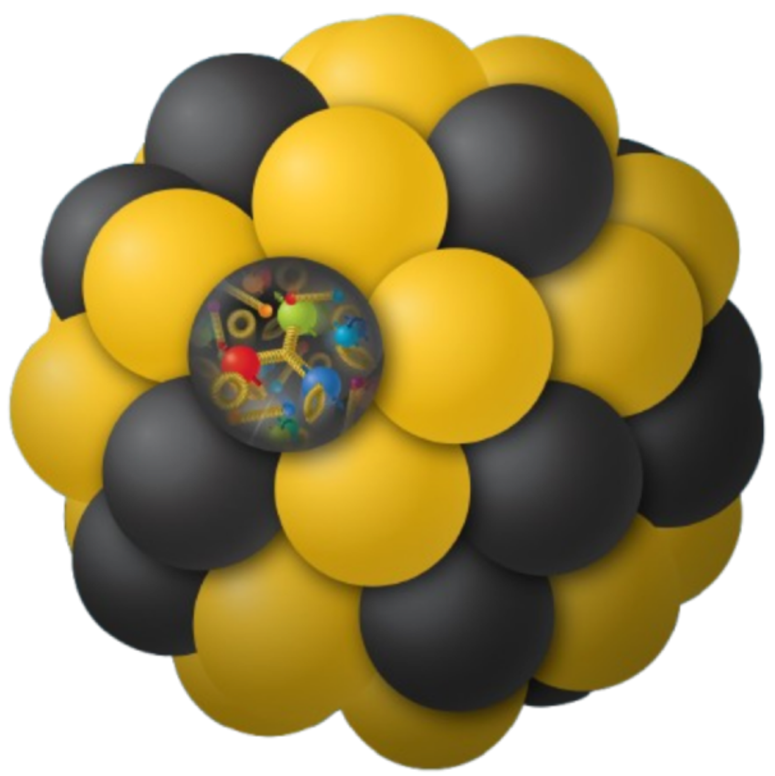

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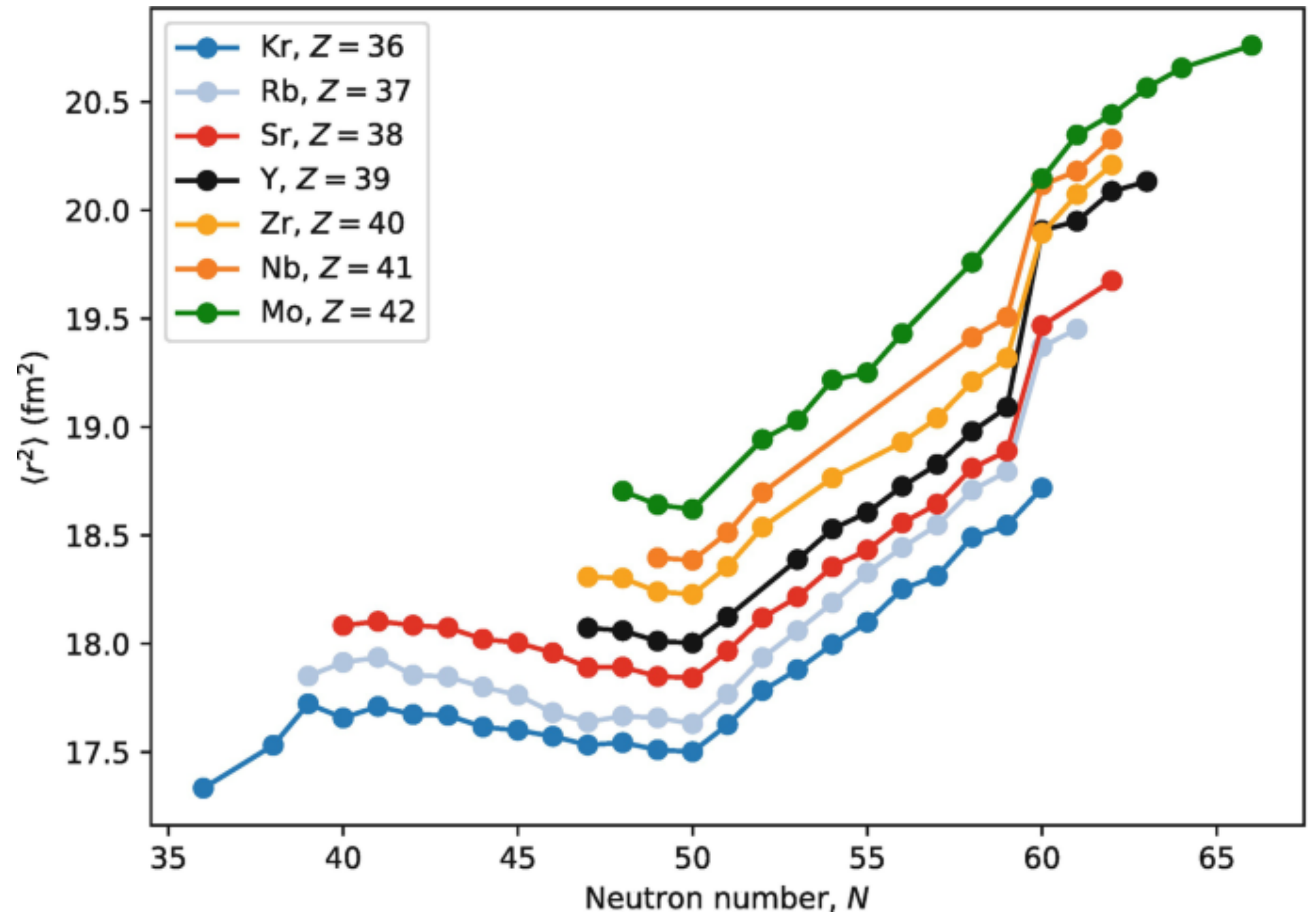

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List of Challenges

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- Obtaining a result:

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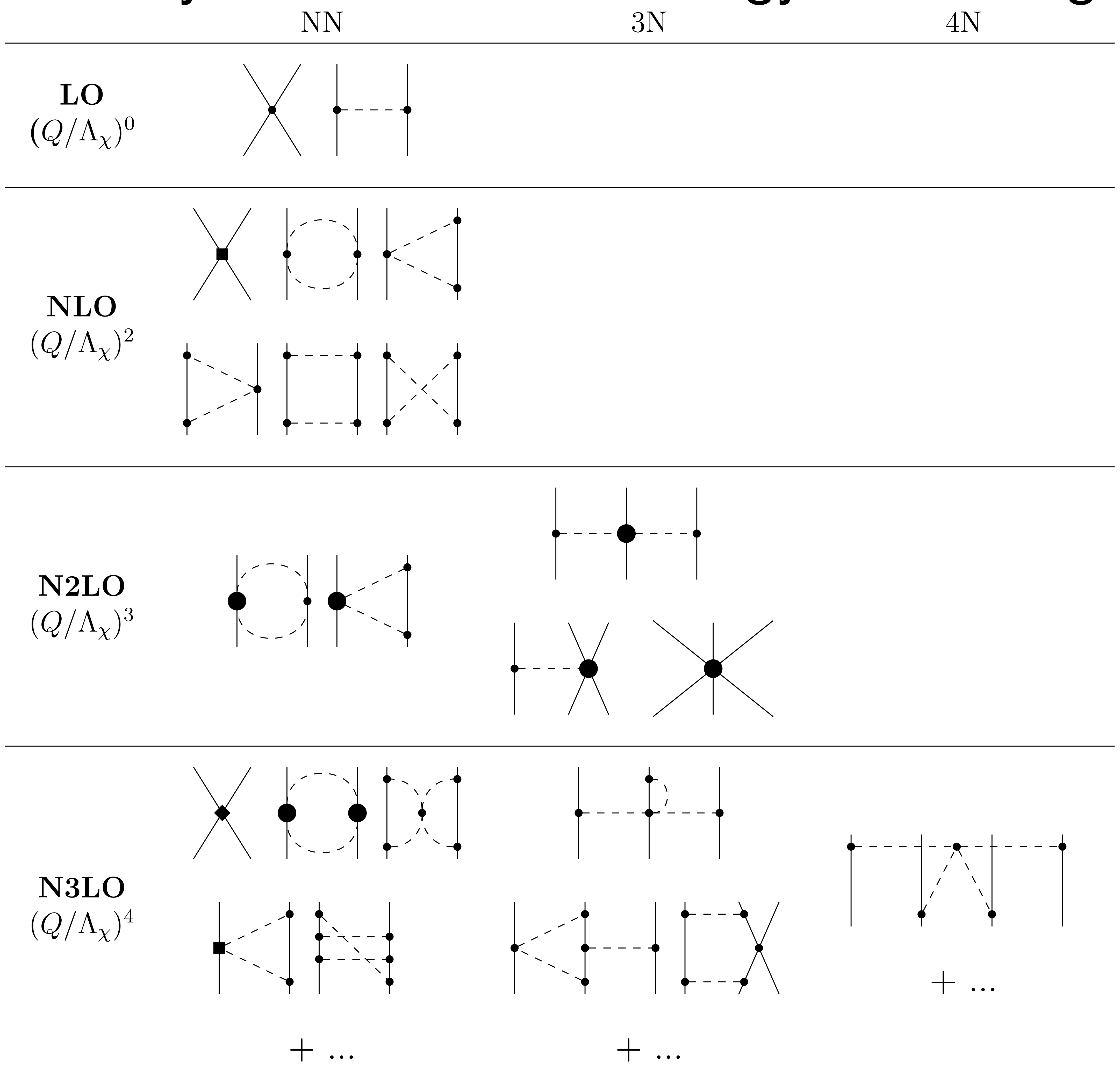
Expansion order by order of the nuclear forces

Reproduces symmetries of low-energy QCD using nucleons as fields and pions as force carriers.



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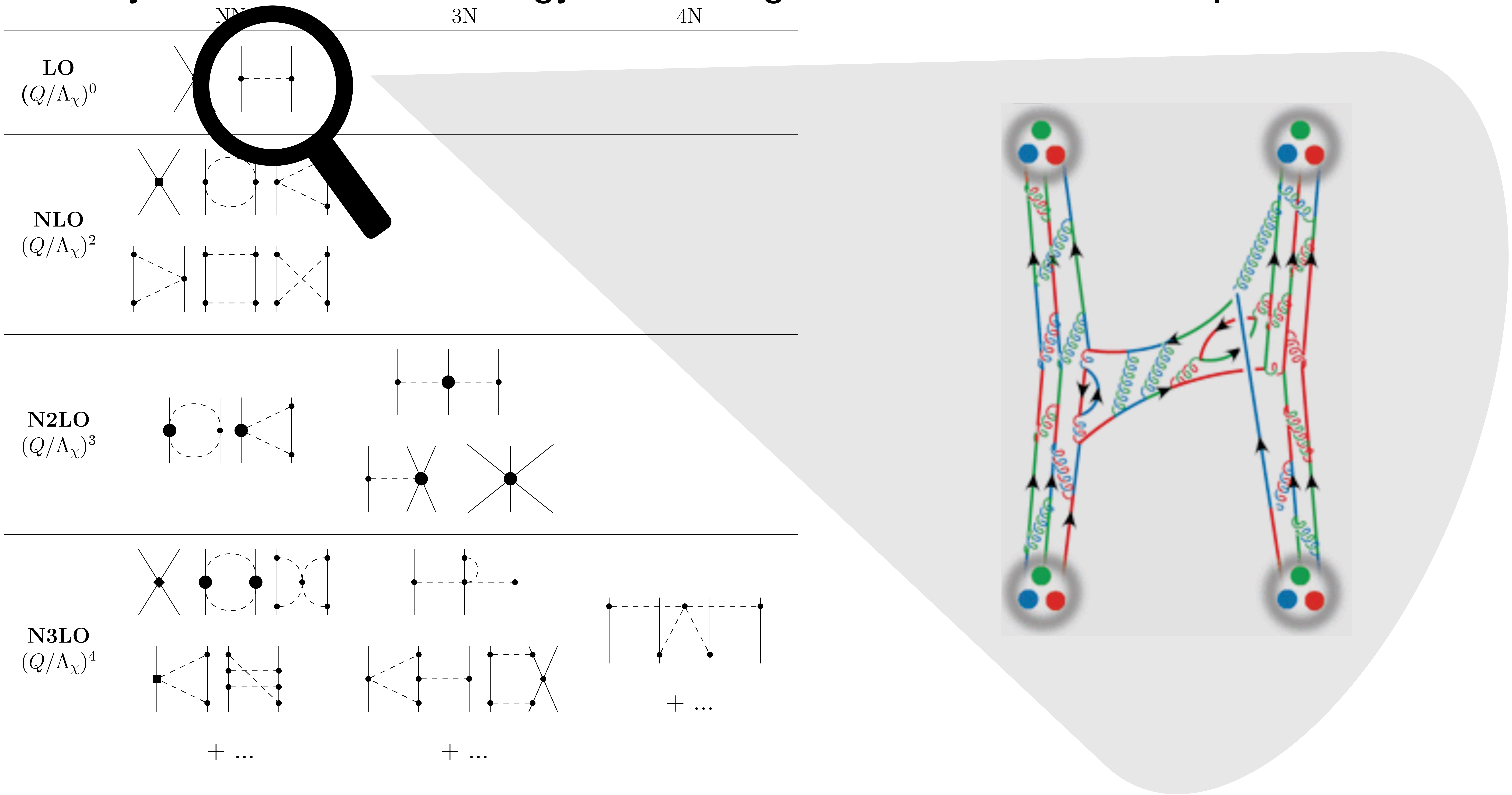

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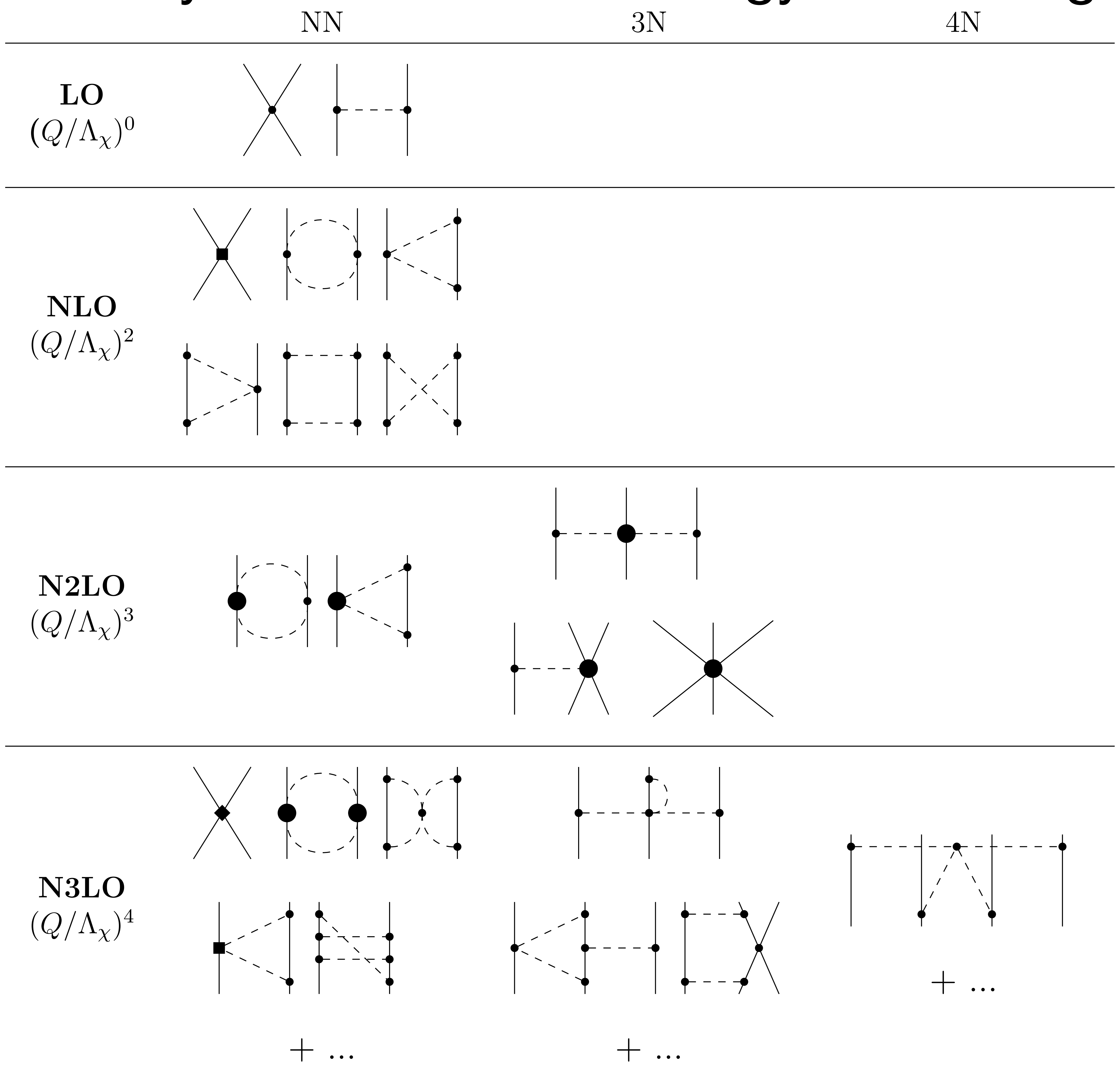

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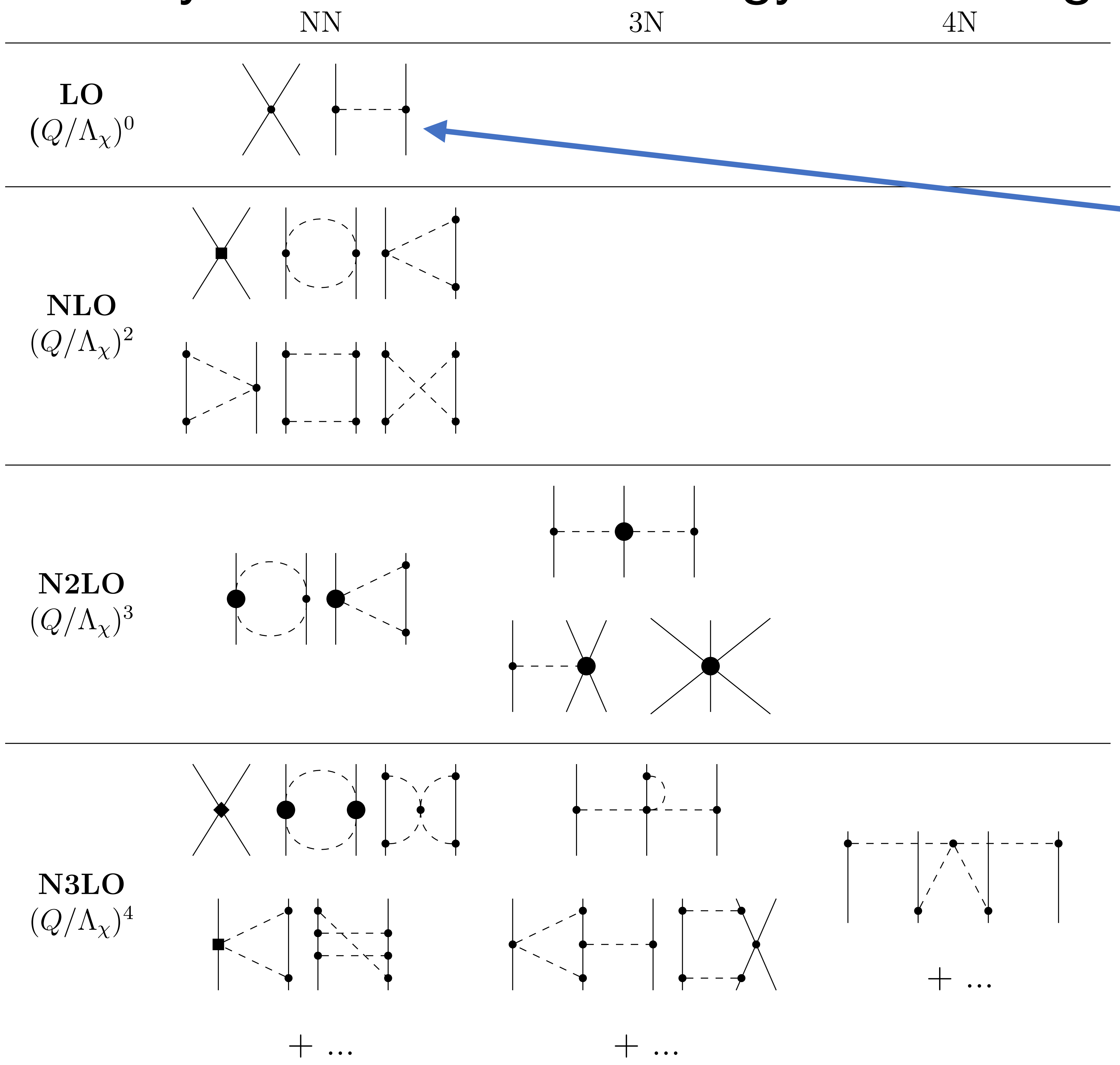

The different low energy coupling constants (LECs) are fitted to few-nucleon data to absorb the effect of higher order terms

χ-EFT

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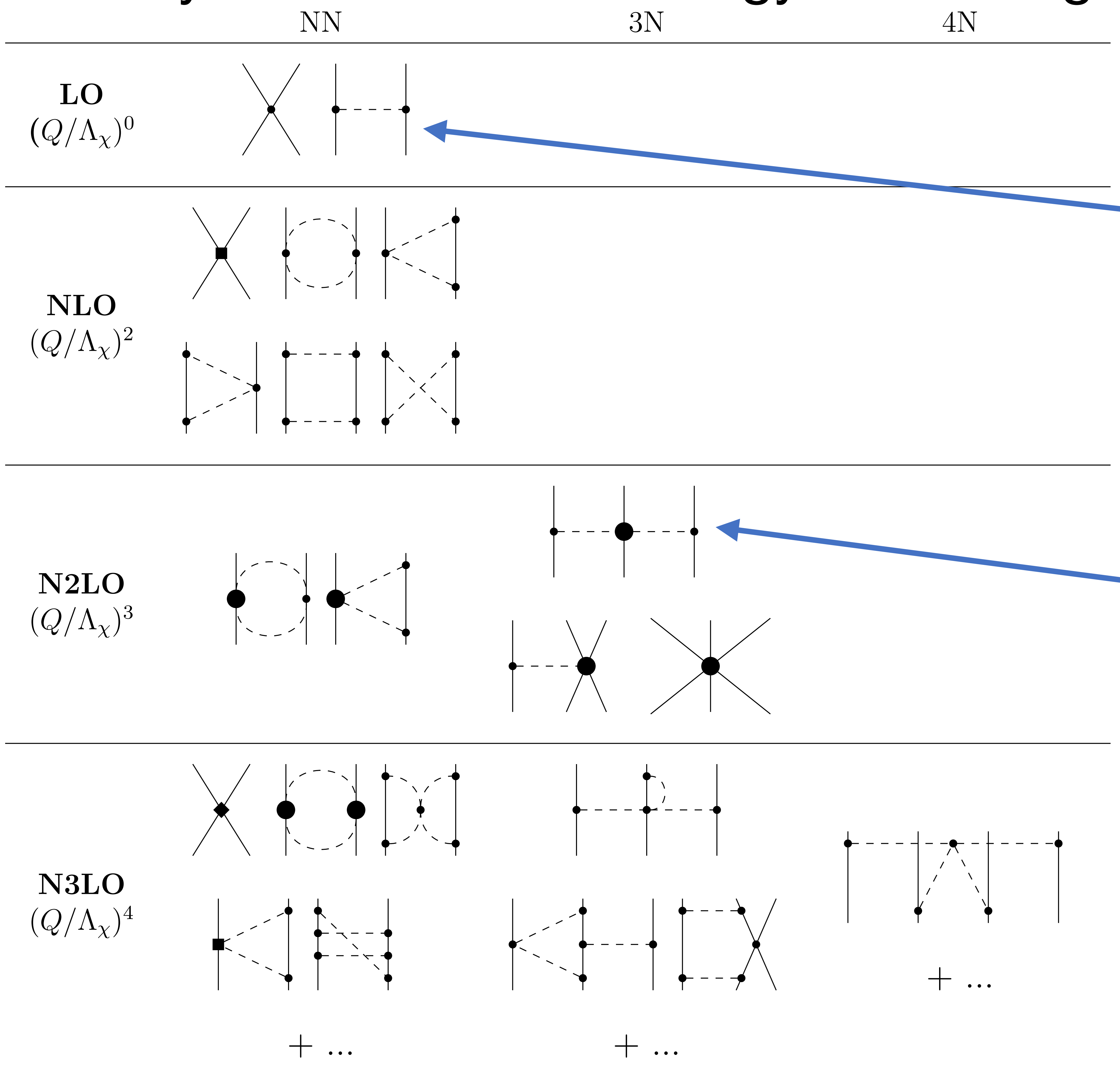

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Three- (and higher-)body forces needed

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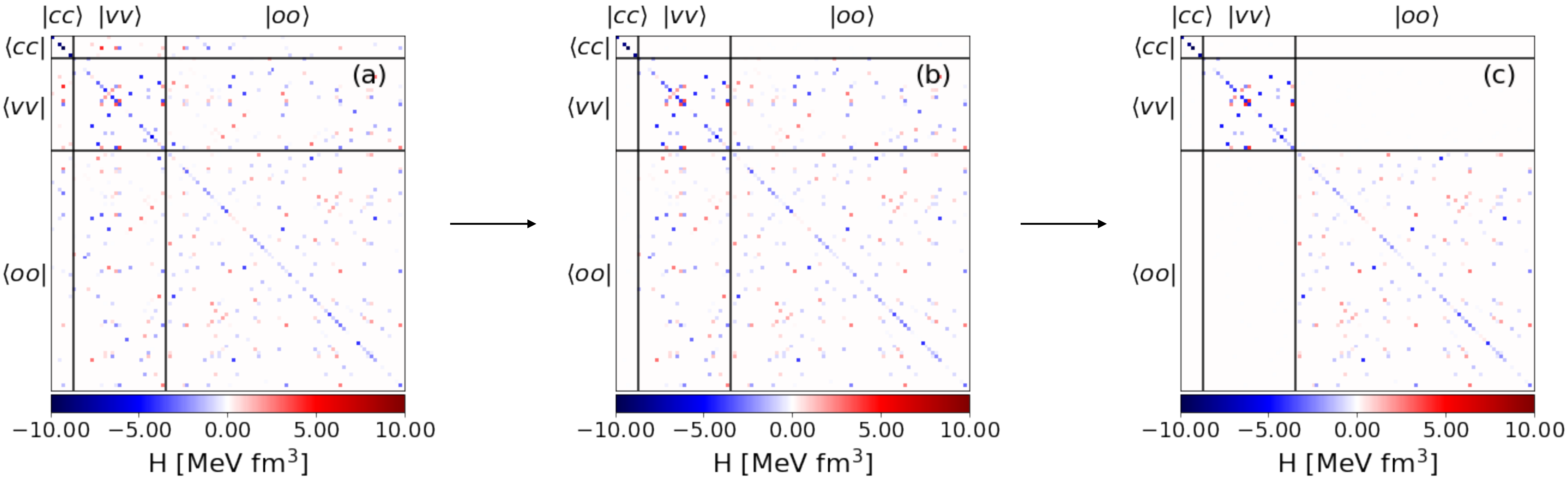
The VS-IMSRG

Valence-Space In Medium Similarity Renormalization Group



Bare Hamiltonian

$$\hat{H}(0)$$

Core is decoupled

$$\hat{H}(s) = e^{\Omega_c(s)}\hat{H}(0)e^{-\Omega_c(s)}$$

$$\hat{H}_c = e^{\Omega_c(\infty)}\hat{H}(0)e^{-\Omega_c(\infty)}$$

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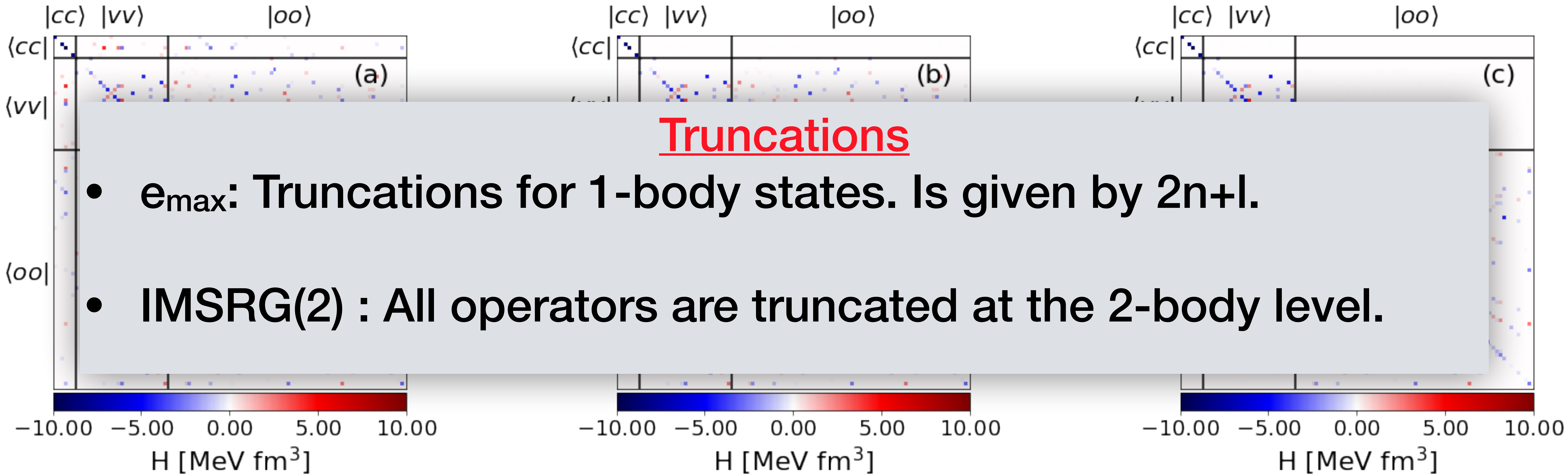

Truncations

- e_{max}: Truncations for 1-body states. Is given by 2n+l.
- IMSRG(2) : All operators are truncated at the 2-body level.

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EFT Corrections to the Operator

Complete approach based on EFT allows to find corrections to operators:

$$\boxed{[T_{1/2}^{0\nu}]^{-1} = g_A^4 G^{0\nu} \,|\, M_{LR}^{0\nu} + M_{SR}^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{\text{loops}}^{0\nu} \,|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2}$$

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)

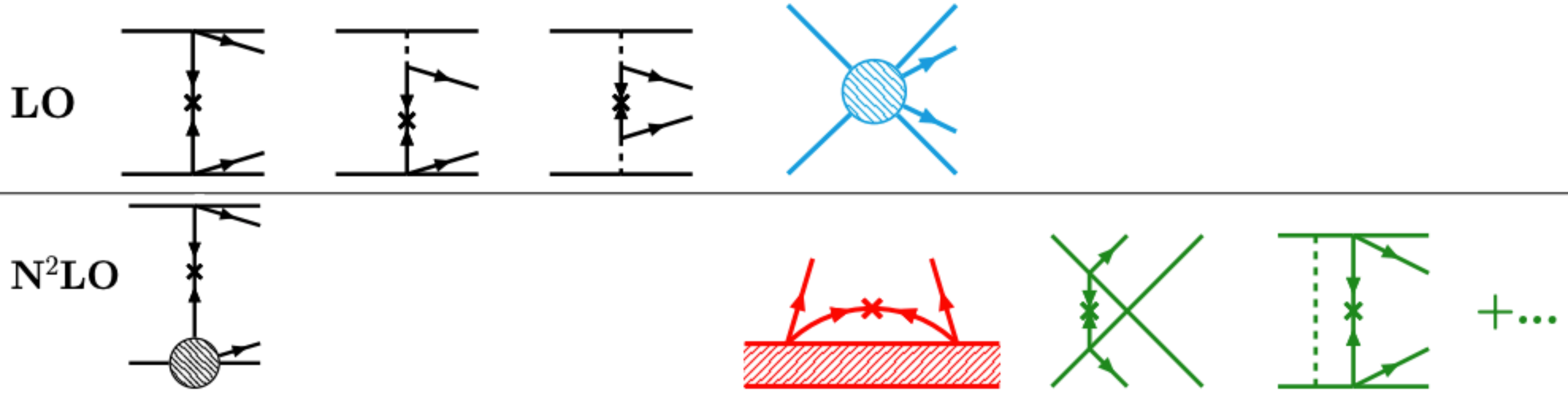
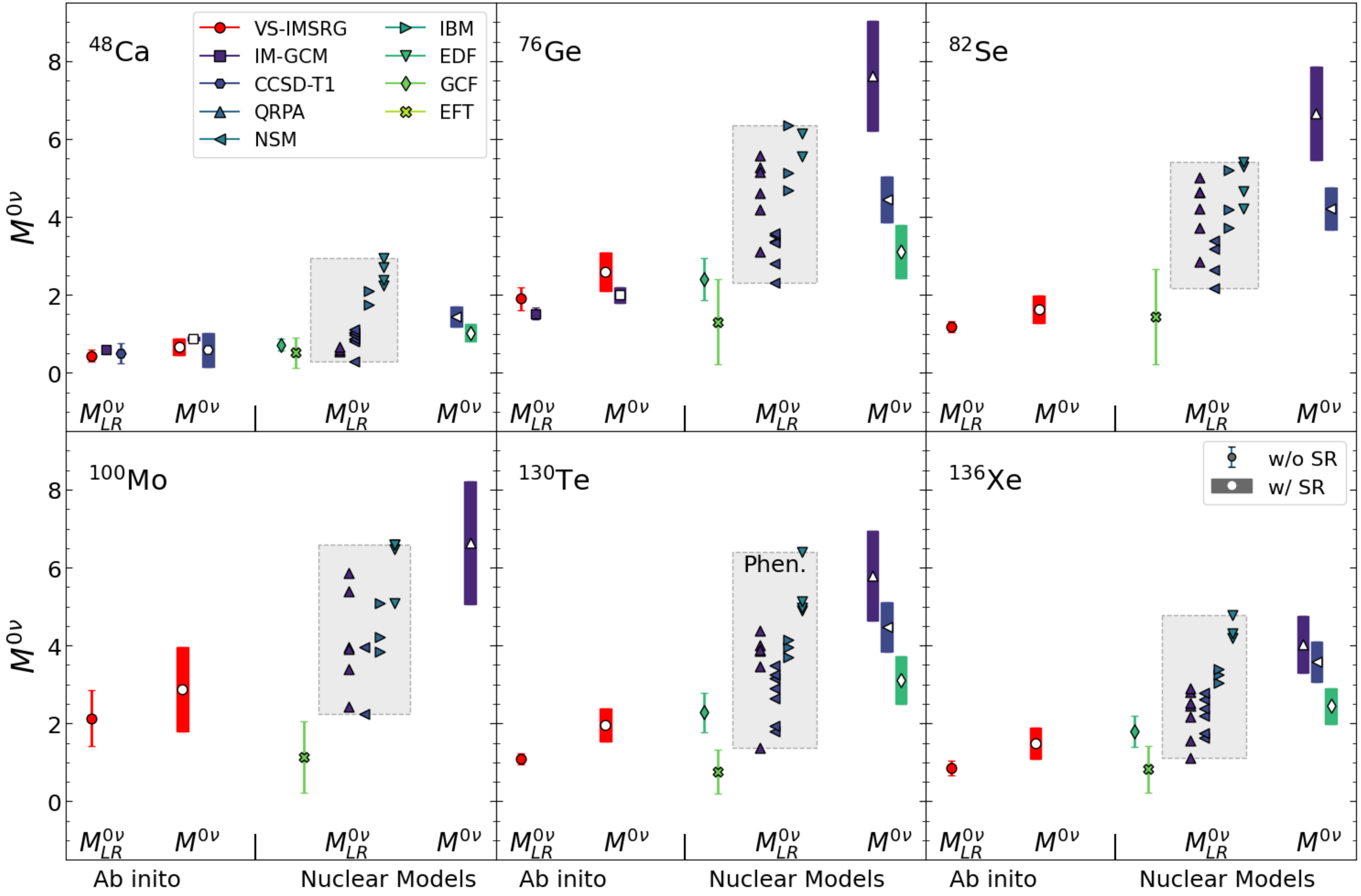


Figure courtesy of L. Jokiniemi

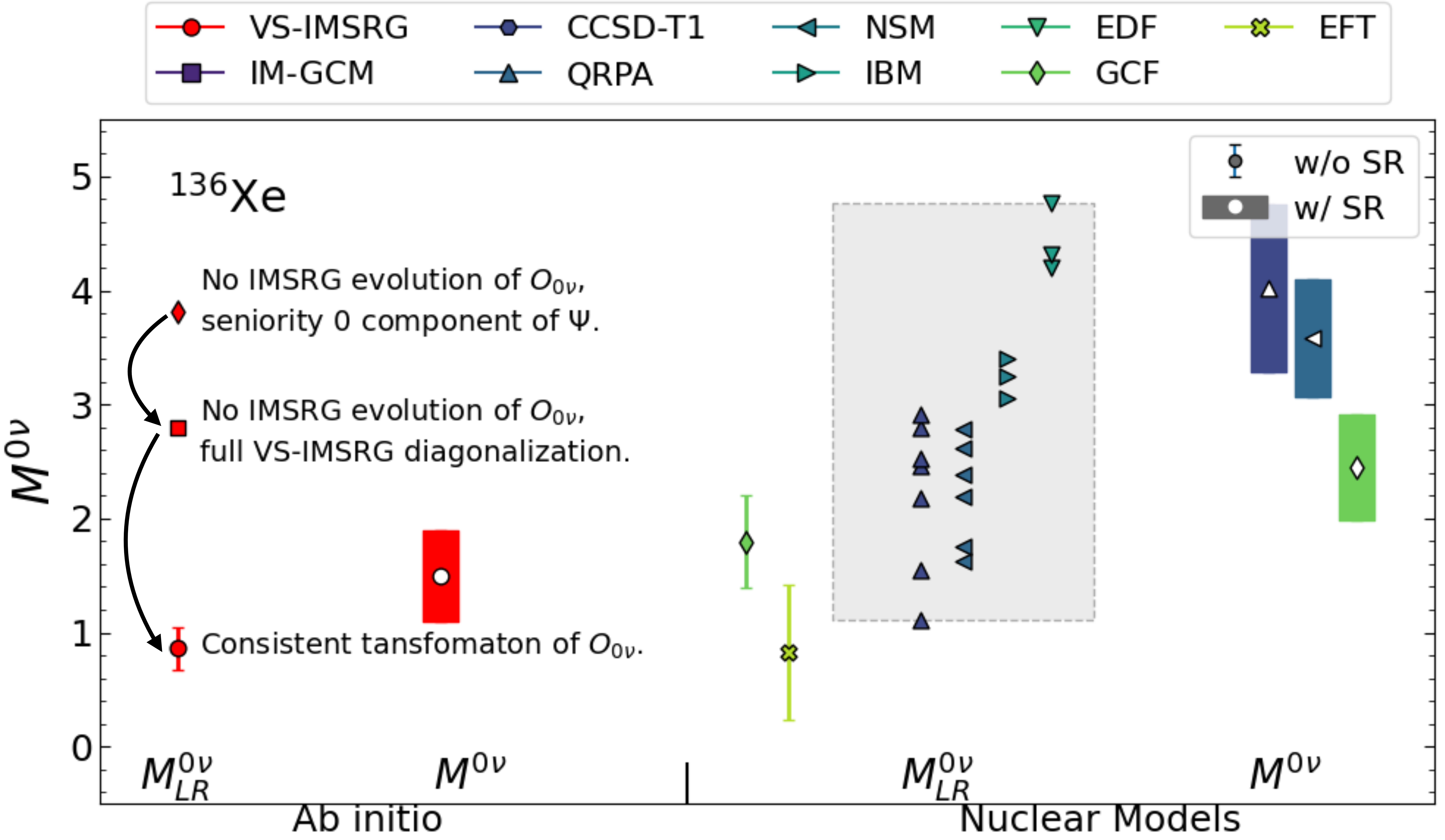
Getting a Result



Values from:

- Agostini et al., Rev. Mod. Phys. 95, 025002 (2023);
- Yao, Sci. Bull. 10.1016 (2020);
- **Belley** et al., Phys. Rev. Lett. 126, 042502 (2020);
- Brase et al., Phys. Rev. C 106, 034309 (2021);
- Weiss et al., Phys. Rev. C 106, 064401 (2022);
- **Belley** et al., arXiv:2307.15156 (2023);
- **Belley** et al., Phys. Rev. Lett. 132, 182502 (2024).

Comparison with Previous Results



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- Obtaining a **reliable** result:
  - Uncertainty Quantification

Uncertainty quantification

Propagating the LECs Error

Recall that the nuclear potential depends on a set of LECs α:

$$M^{0\nu\beta\beta}(\alpha) = \langle \psi_f(\alpha) \,|\, O \,|\, \psi_i(\alpha) \rangle$$

that are fitted to NN and few-nucleon data, i.e. each LEC has an uncertainty $\delta\alpha$ associated with it.

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Bayesian Statistics!

Bayesian Approach

We read $prob(A \mid B)$ as probability of A given B

$$prob(y \mid y_k, I) \propto prob(y_k \mid y, I) \times prob(y \mid I)$$

Bayesian Approach

The “true” value of the LECs for the nuclear interaction.

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$$prob(y \mid y_k, I) \propto \underbrace{prob(y_k \mid y, I)}_{\text{Likelihood}} \times \underbrace{prob(y \mid I)}_{\text{Prior}}$$

Likelihood
Probability that this sample gives a result that is representative of experimental values.

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Posterior distribution
Probability distribution for the final value given the data and our previous knowledge (what we want to obtain).

Likelihood
Probability that this sample gives a result that is representative of experimental values.

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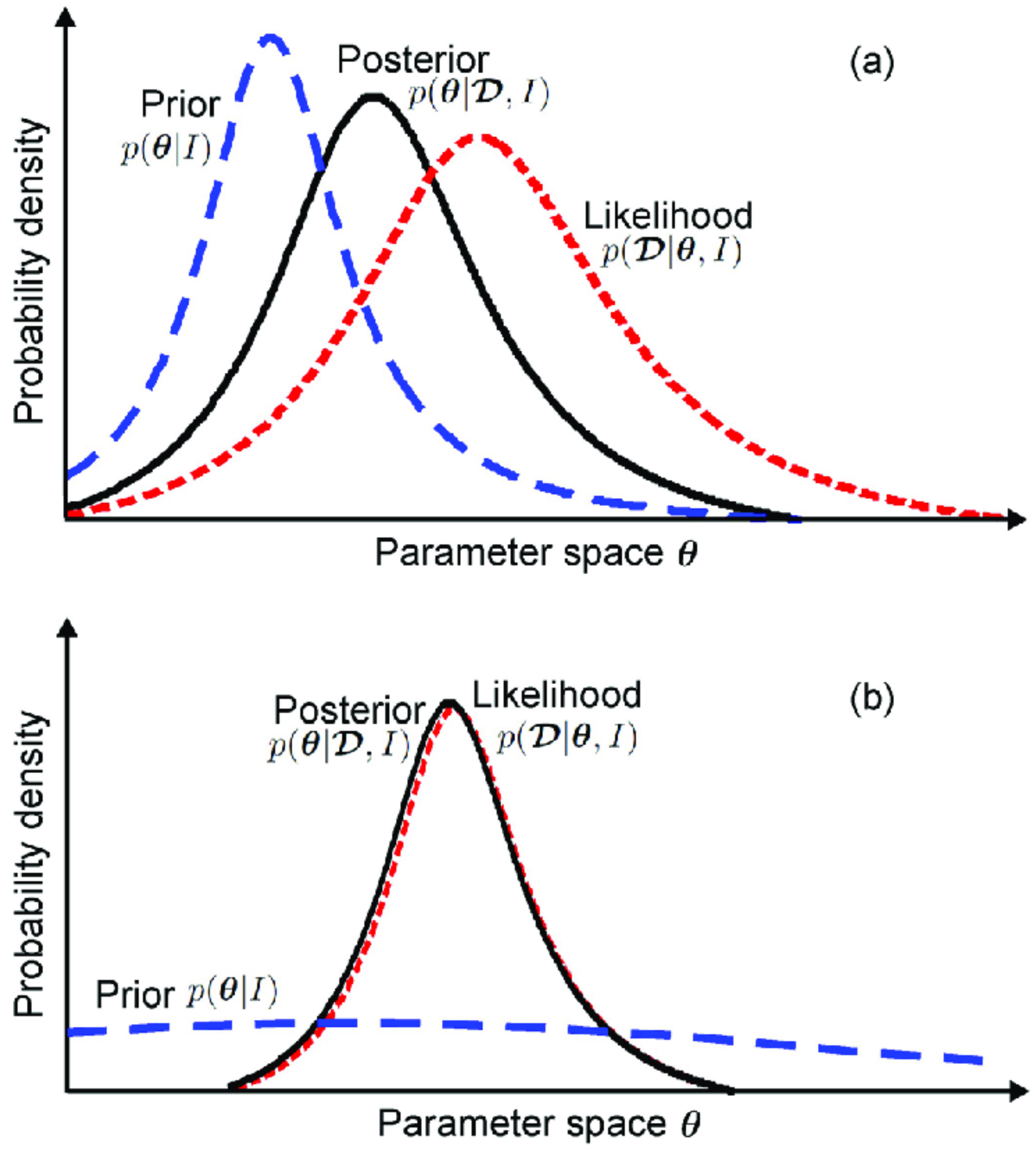
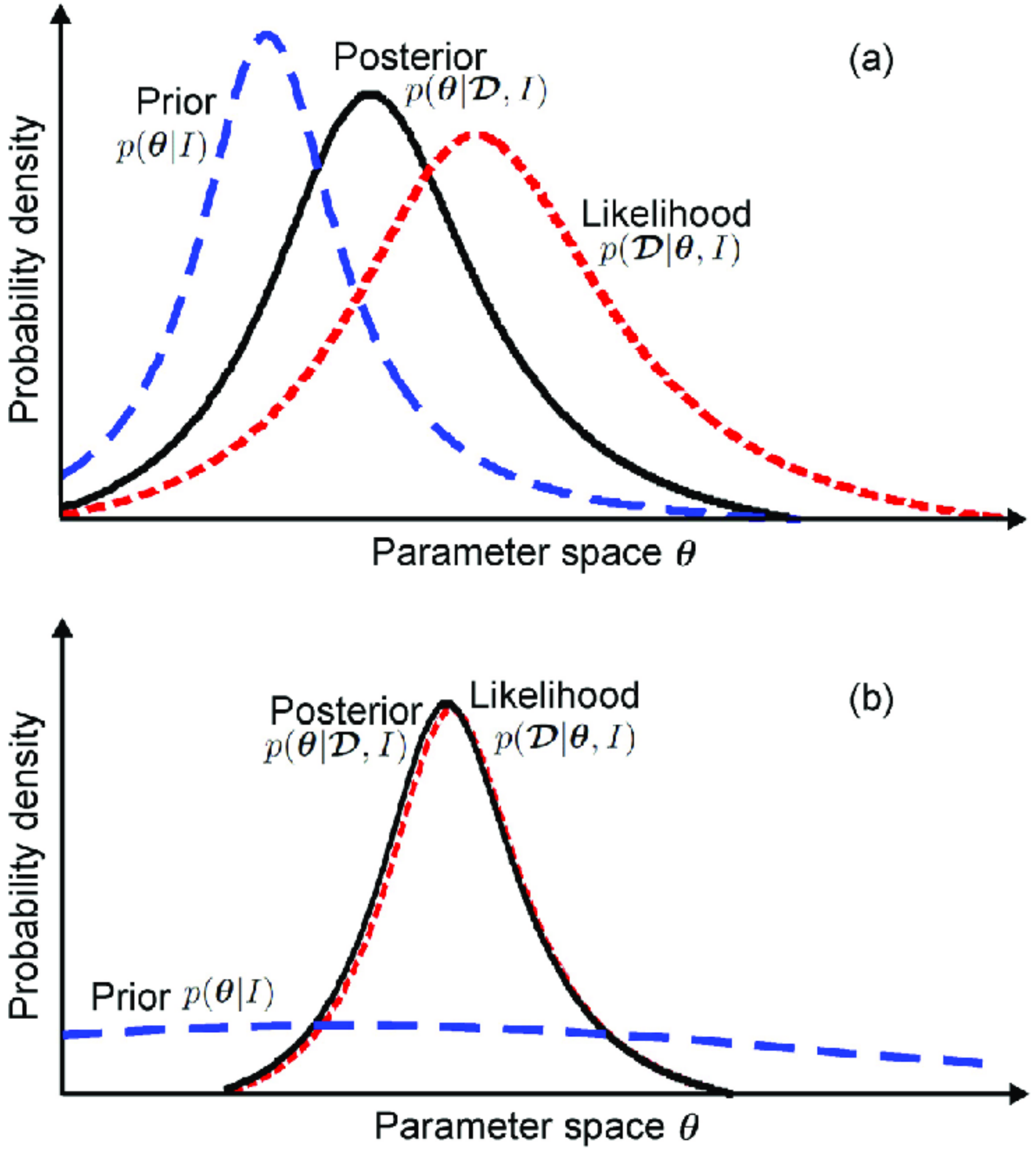
Procedure for UQ in the Bayesian Approach



Ning Xiang - https://doi.org/10.1121/10.0001731

Procedure for UQ in the Bayesian Approach



The catch

Need many samples.

Due to the large cost of many-body methods, for 1 isotope:

- Take ~**1 year** to compute all samples on HPC cluster.
- Cost > **$2 million!**
- Huge environmental impact (**220 tree-years** calculated using green-algorithms.org v3.0)

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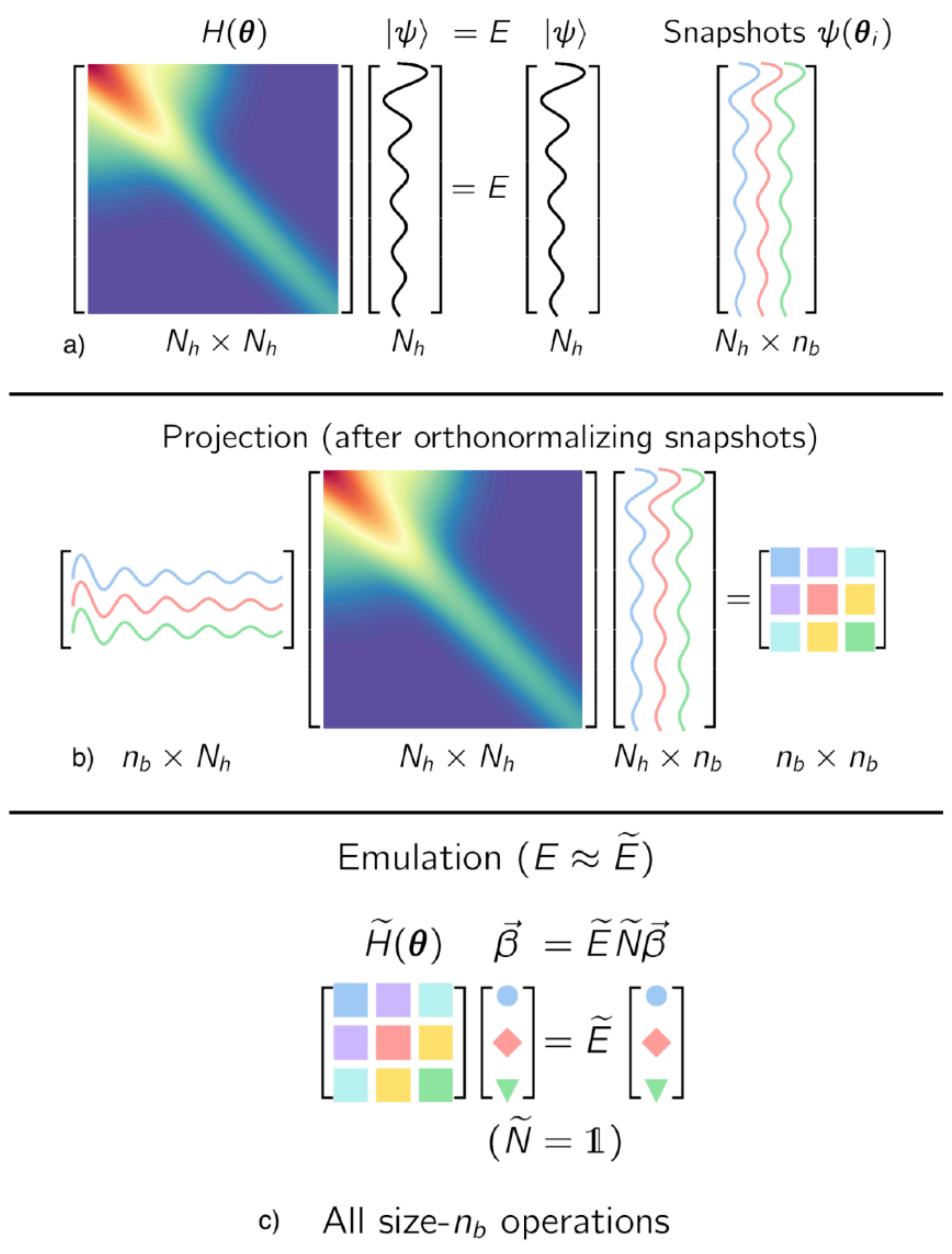

Duguet, et al., Rev. Mod. Phys. **96**, 031002 (2024)

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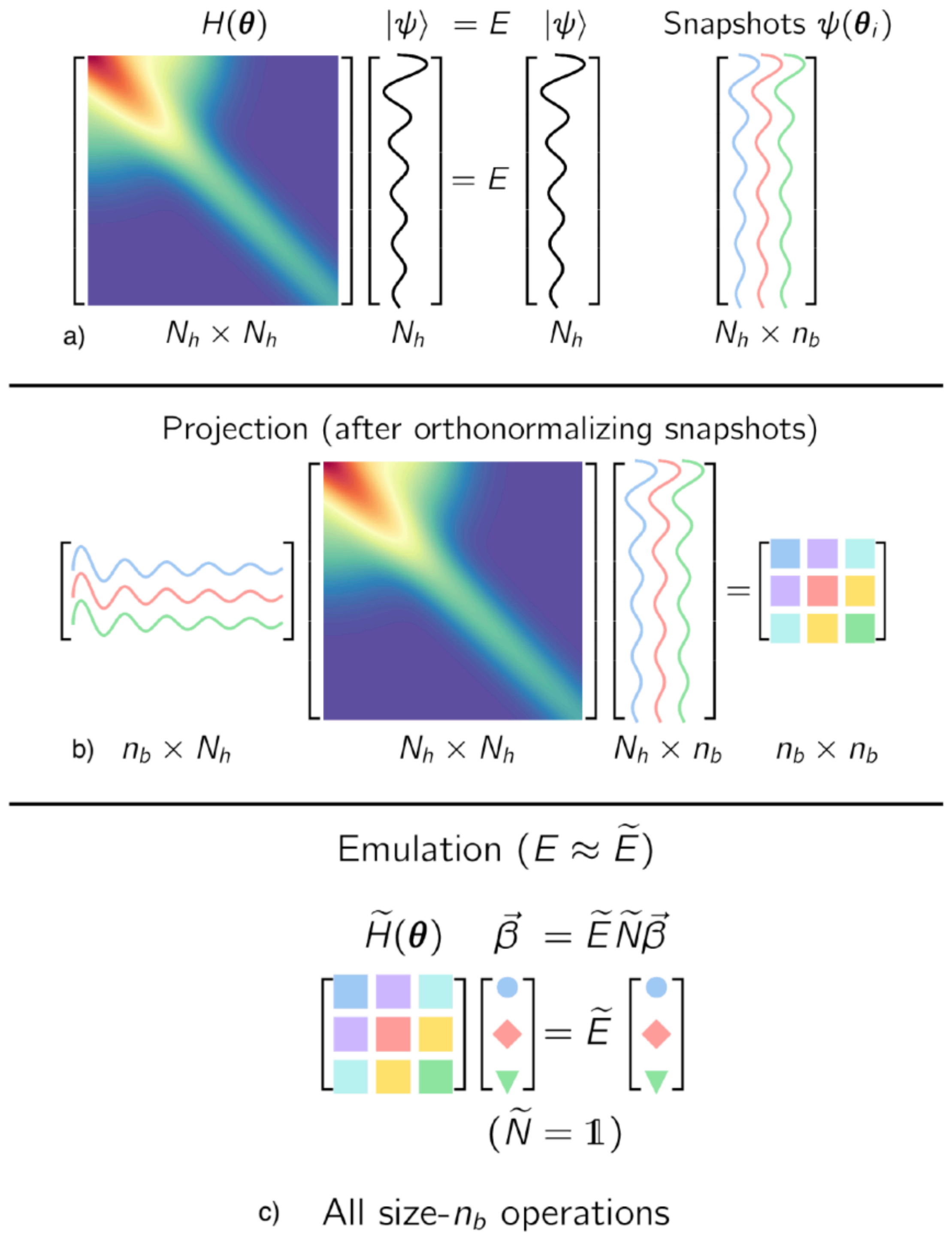
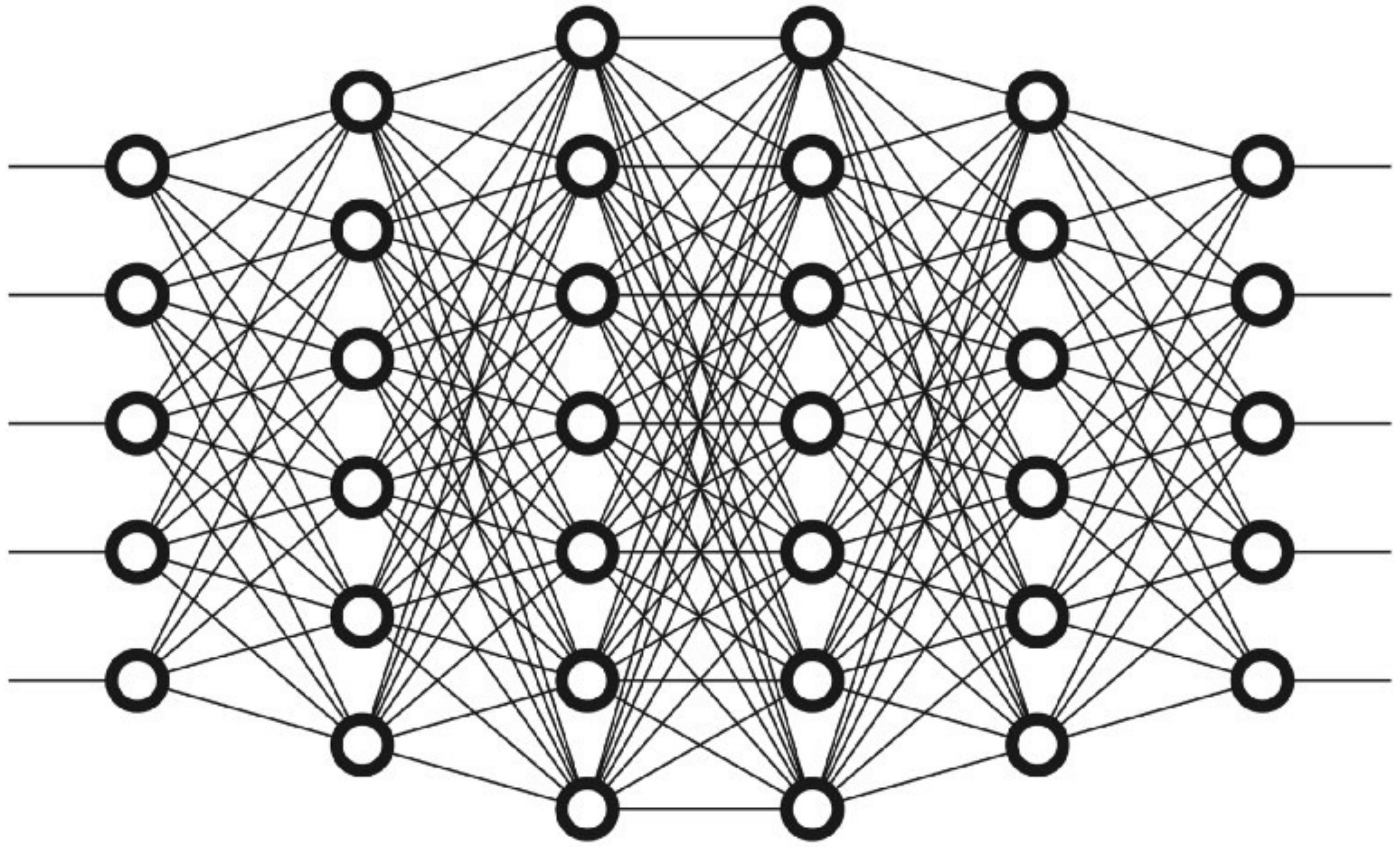
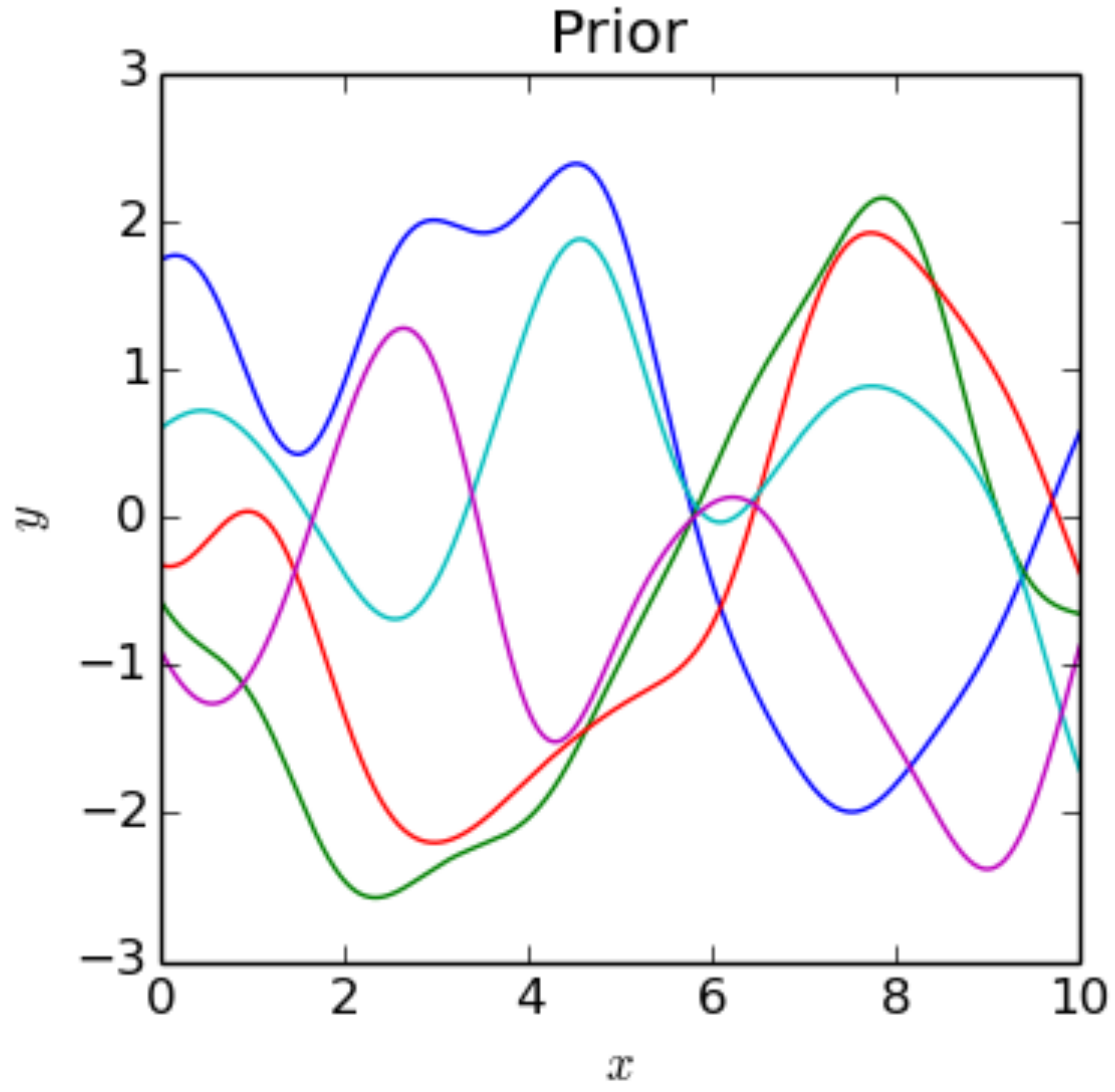


Duguet, et al., Rev. Mod. Phys. **96**, 031002 (2024)

2. Data driven

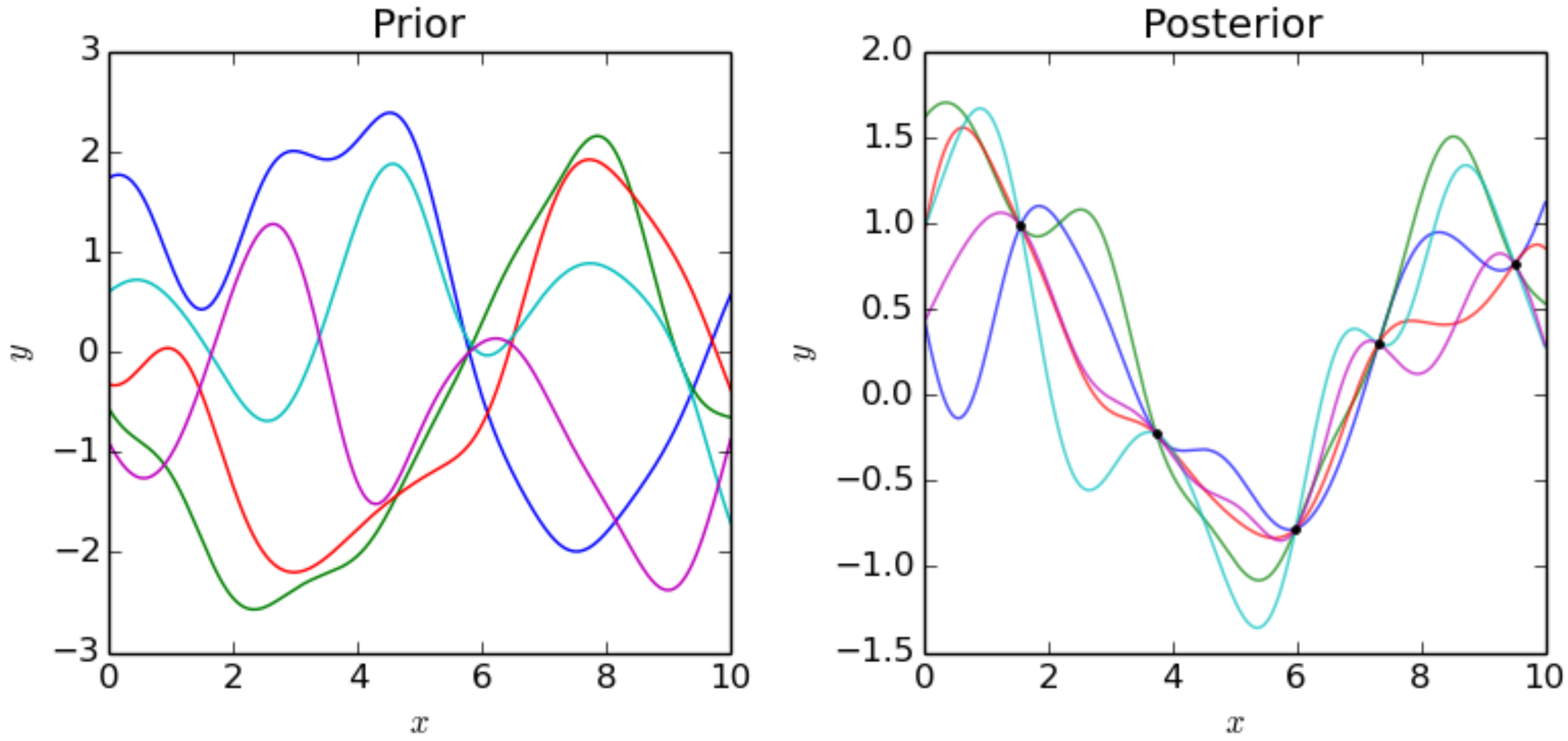


Using Gaussian Process as an Emulator



$$f(\mathbf{x}) = \mathcal{N}(\mu, K(\mathbf{x}, \mathbf{x}))$$

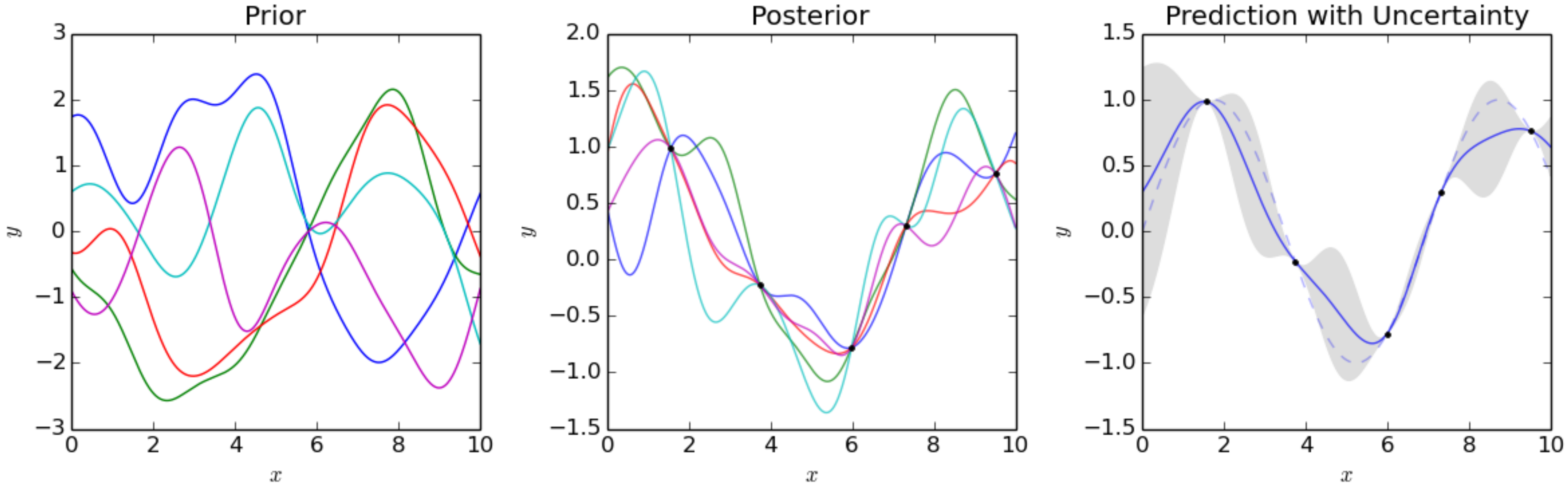
Using Gaussian Process as an Emulator



$$f(\mathbf{x}) = \mathcal{N}(\mu, K(\mathbf{x}, \mathbf{x}))$$

$$P_{Y*|Y} \sim \mathcal{N}\left(\Sigma_{X*X}\Sigma_{XX}^{-1}Y, \Sigma_{X*X*} - \Sigma_{X*X}\Sigma_{XX}^{-1}\Sigma_{XX*}\right)$$

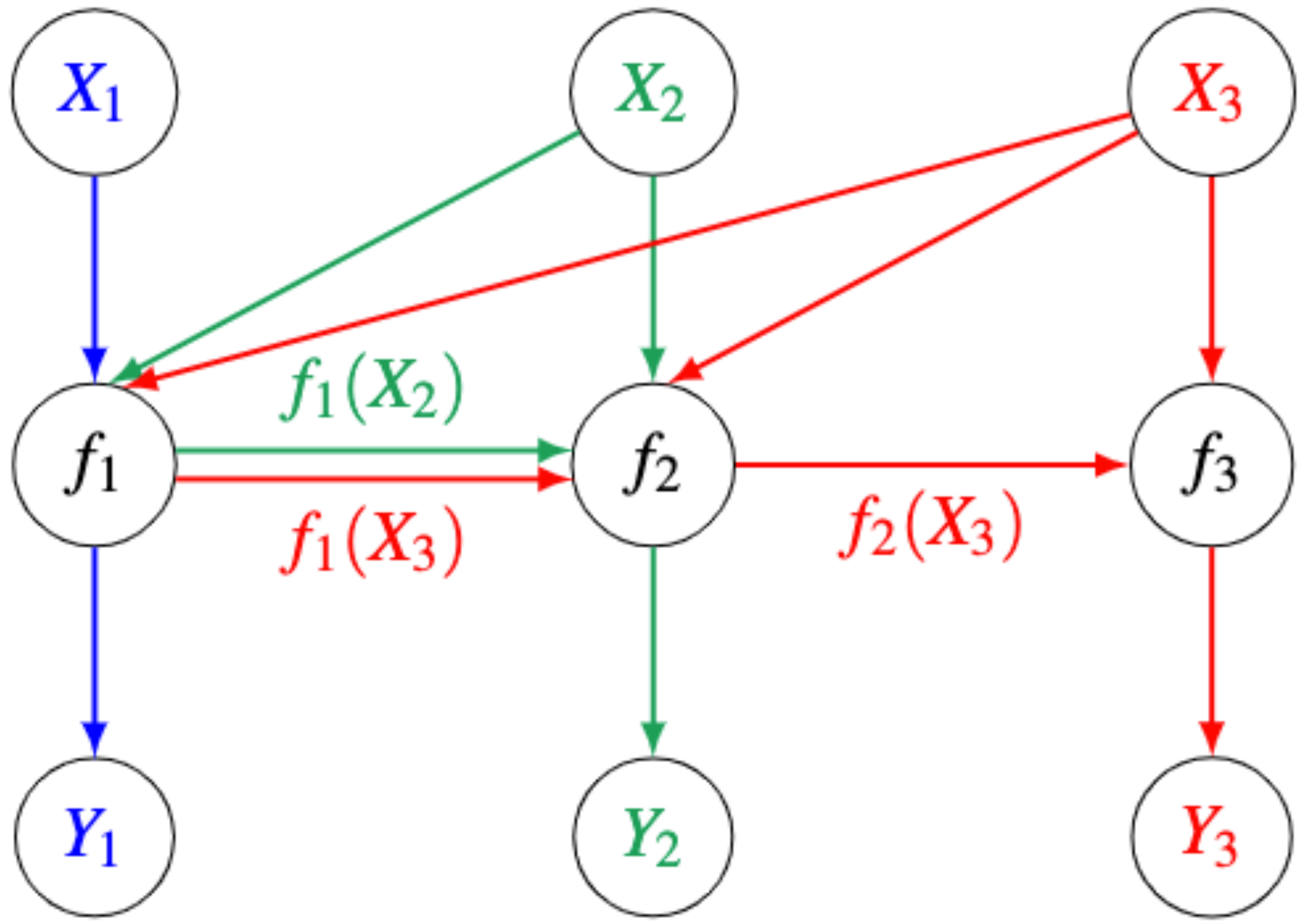
Using Gaussian Process as an Emulator



$$f(\mathbf{x}) = \mathcal{N}(\mu, K(\mathbf{x}, \mathbf{x})) \qquad P_{Y^*|Y} \sim \mathcal{N}\left(\Sigma_{X^*X}\Sigma_{XX}^{-1}Y, \Sigma_{X^*X^*} - \Sigma_{X^*X}\Sigma_{XX}^{-1}\Sigma_{XX^*}\right)$$

The MM-DGP Algorithm

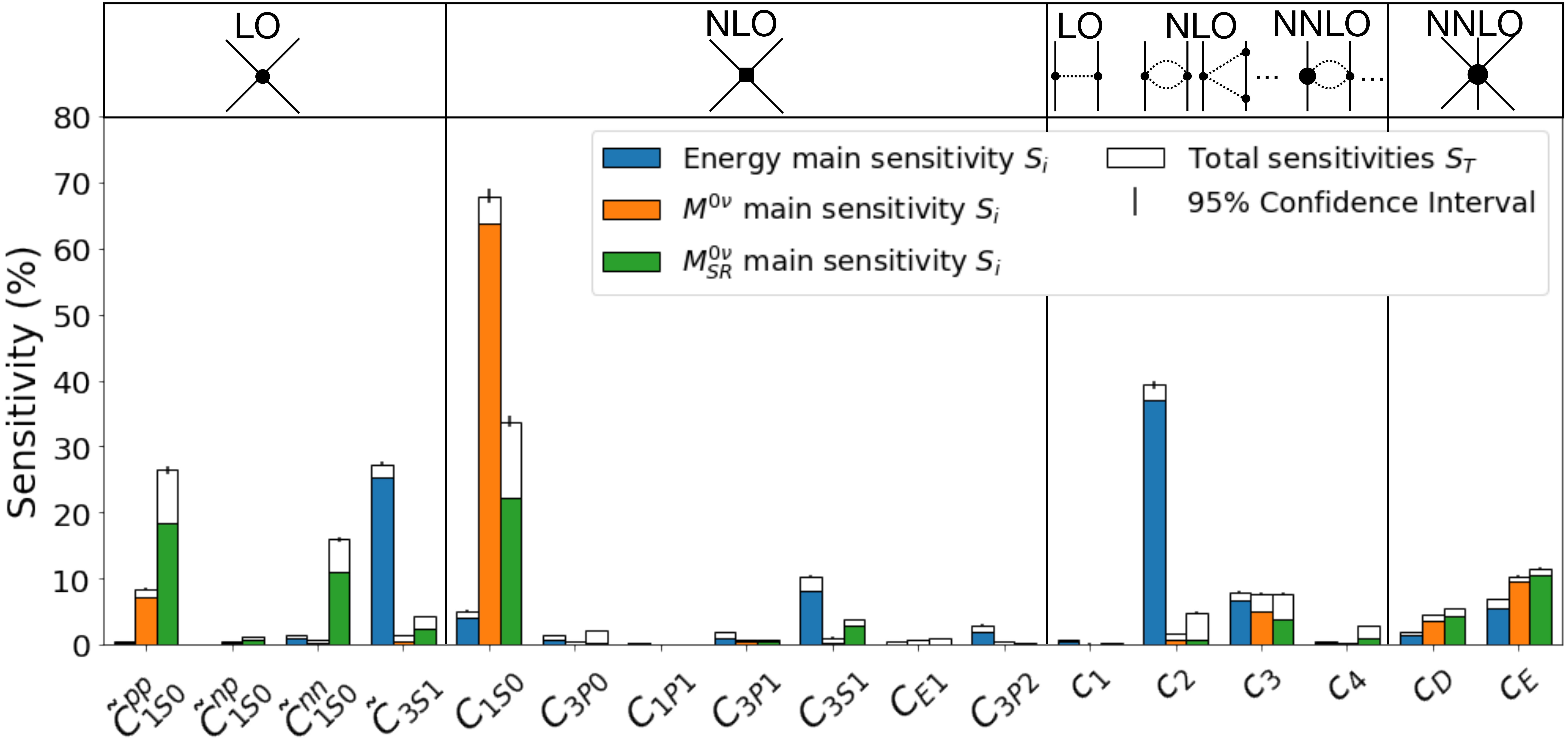
- **Deep Gaussian Processes [1]**: Stack multiple GPs in a neural network-like architecture for improved hierarchical learning.
- **Multi-Fidelity Modelling**: Model low-to-high fidelity differences by passing outputs from one fidelity as inputs to the next.
- **MM-DGP Extension**: Adapted to handle multiple outputs across fidelity levels, creating the Multi-output Multi-fidelity Deep Gaussian Process (MM-DGP).



[1] Kurt Cutajar, Mark Pullin, Andreas Damianou, Neil Lawrence, Javier González arXiv:1903.07320 (2021).

The MM-DGP Algorithm: GSA

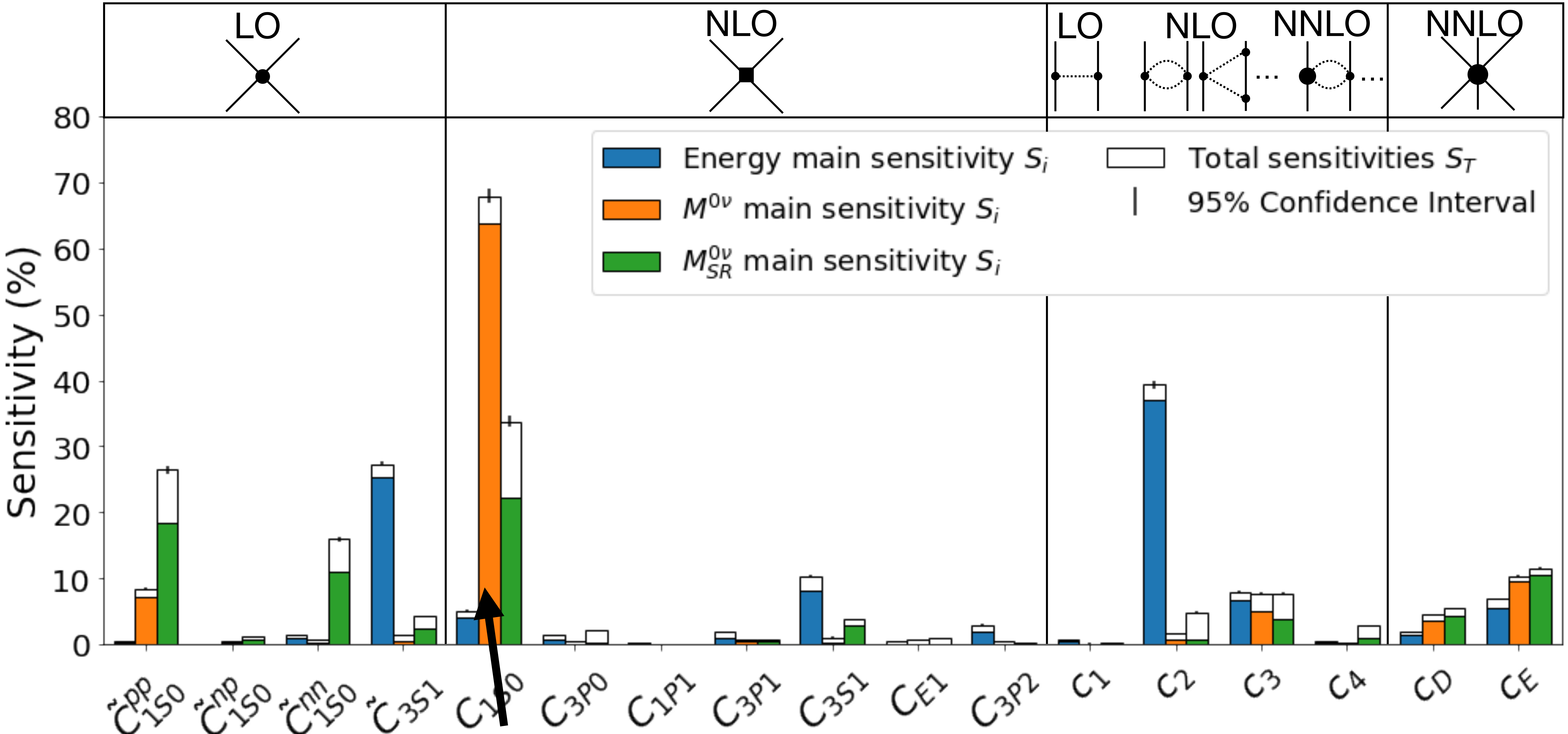
Belley, et al., arXiv:2408.02169 (2024)



Low-energy constants, i.e. parameters of the nuclear force

The MM-DGP Algorithm: GSA

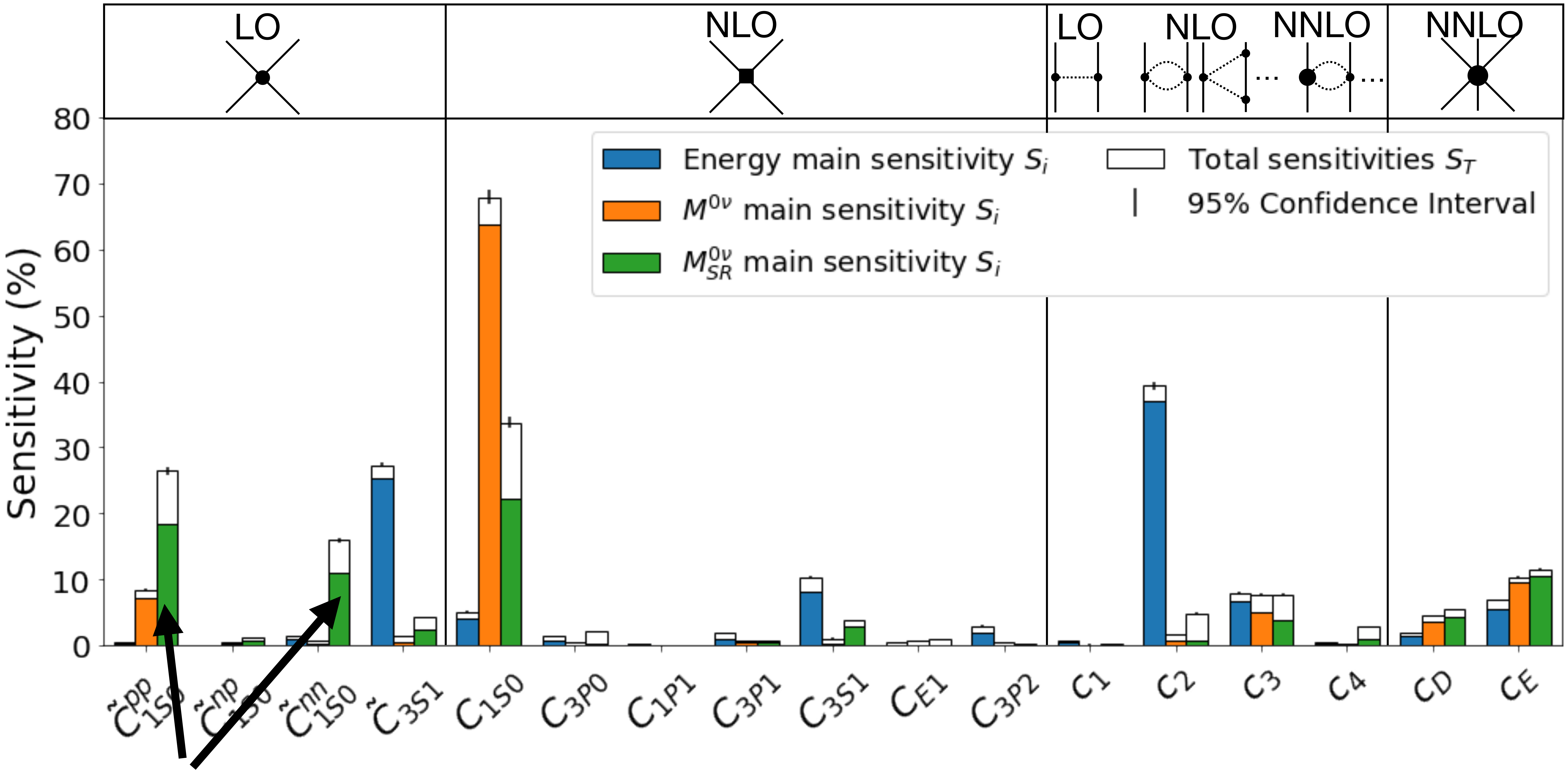
Belley, et al., arXiv:2408.02169 (2024)



The total matrix element mostly depends on one LEC!

The MM-DGP Algorithm: GSA

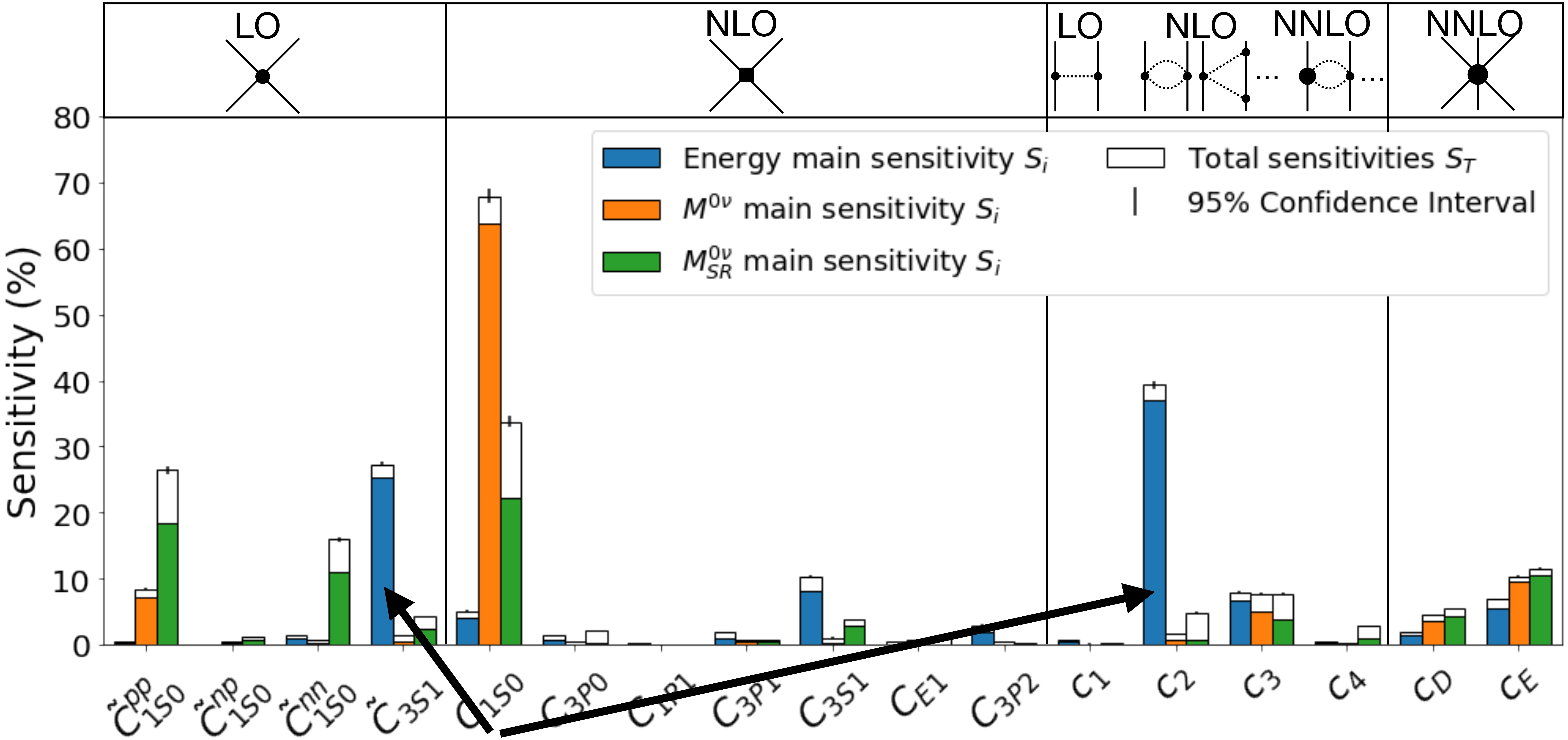
Belley, et al., arXiv:2408.02169 (2024)



The short-range matrix element however sees other contributions from LECs associated to the short-range nuclear interaction.

The MM-DGP Algorithm: GSA

Belley, et al., arXiv:2408.02169 (2024)

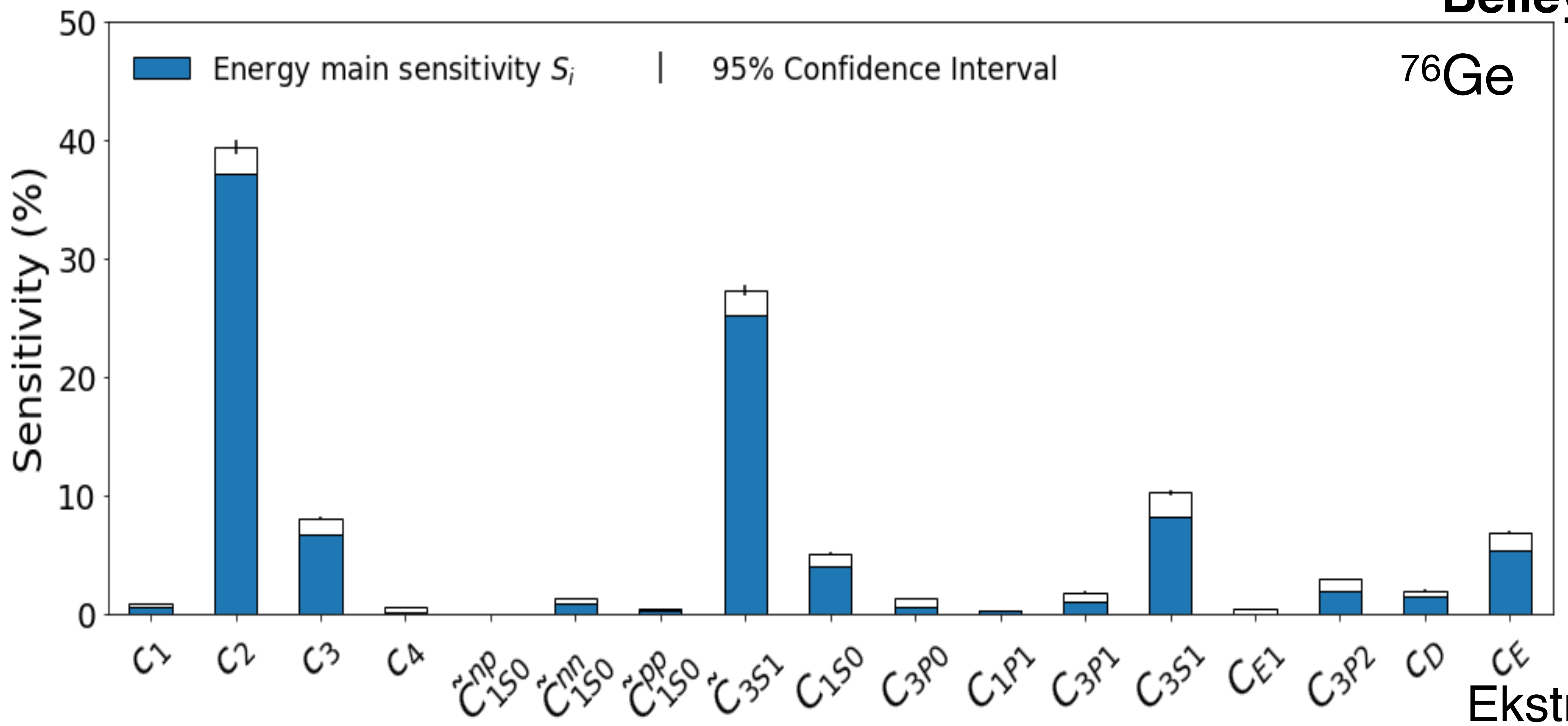


Results for energies are consistent with results of physics-based emulators of the coupled cluster method.

The MM-DGP Algorithm: GSA

Belley, et al., arXiv:2408.02169 (2024)

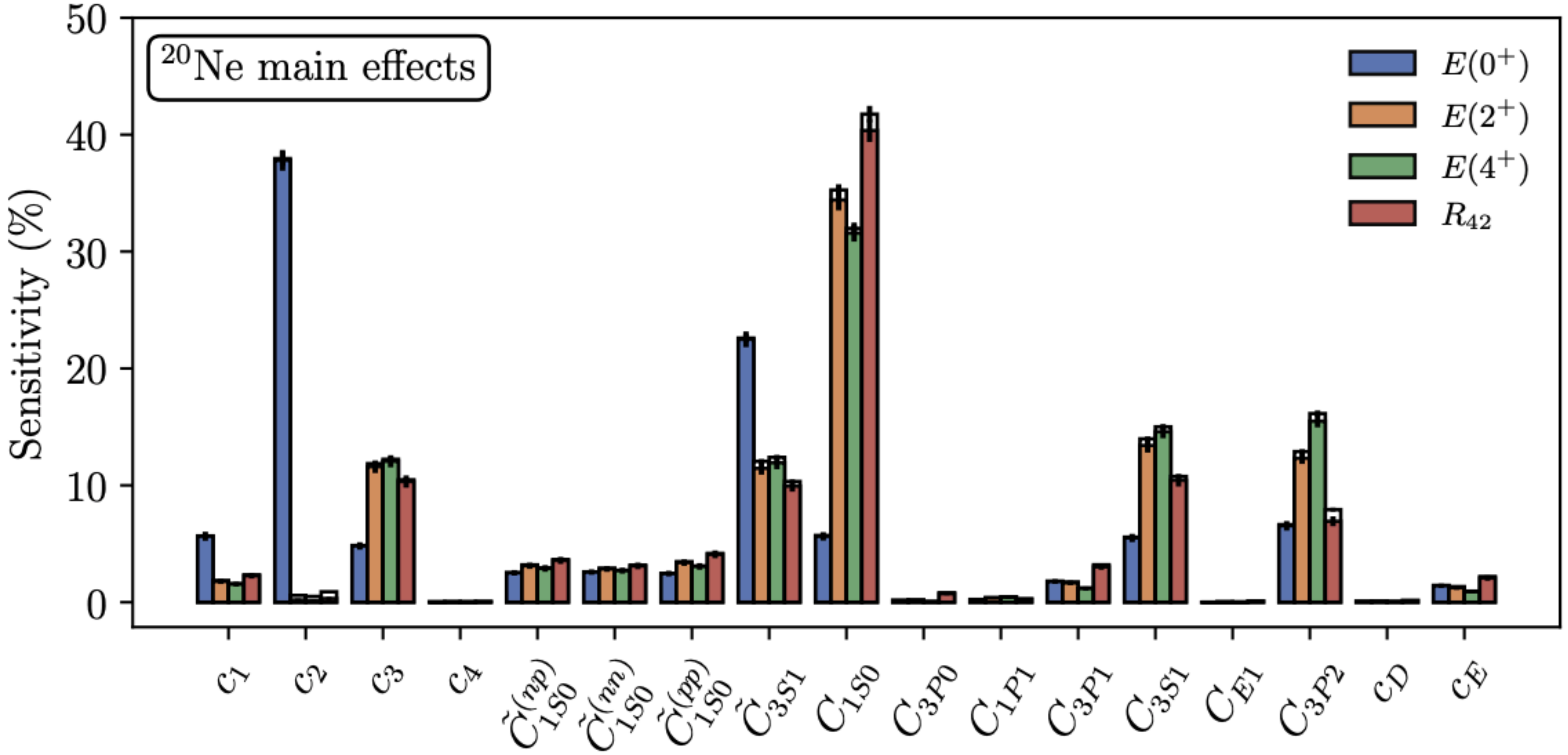
Data driven MM-DGP



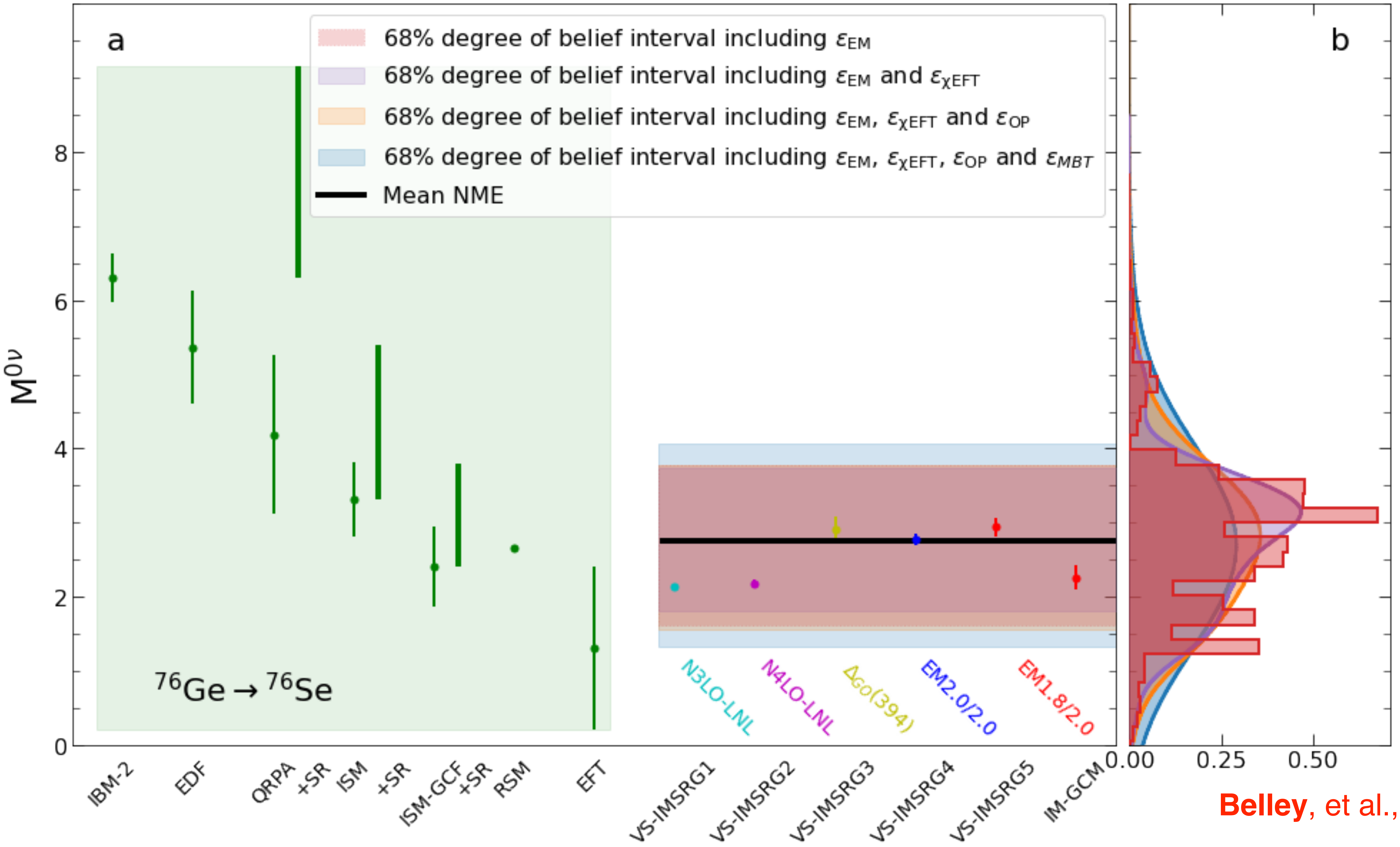
vs

Ekström, et al., arXiv:2305.06955 (2023)

Physics Driven



Combining All Sources of Uncertainty



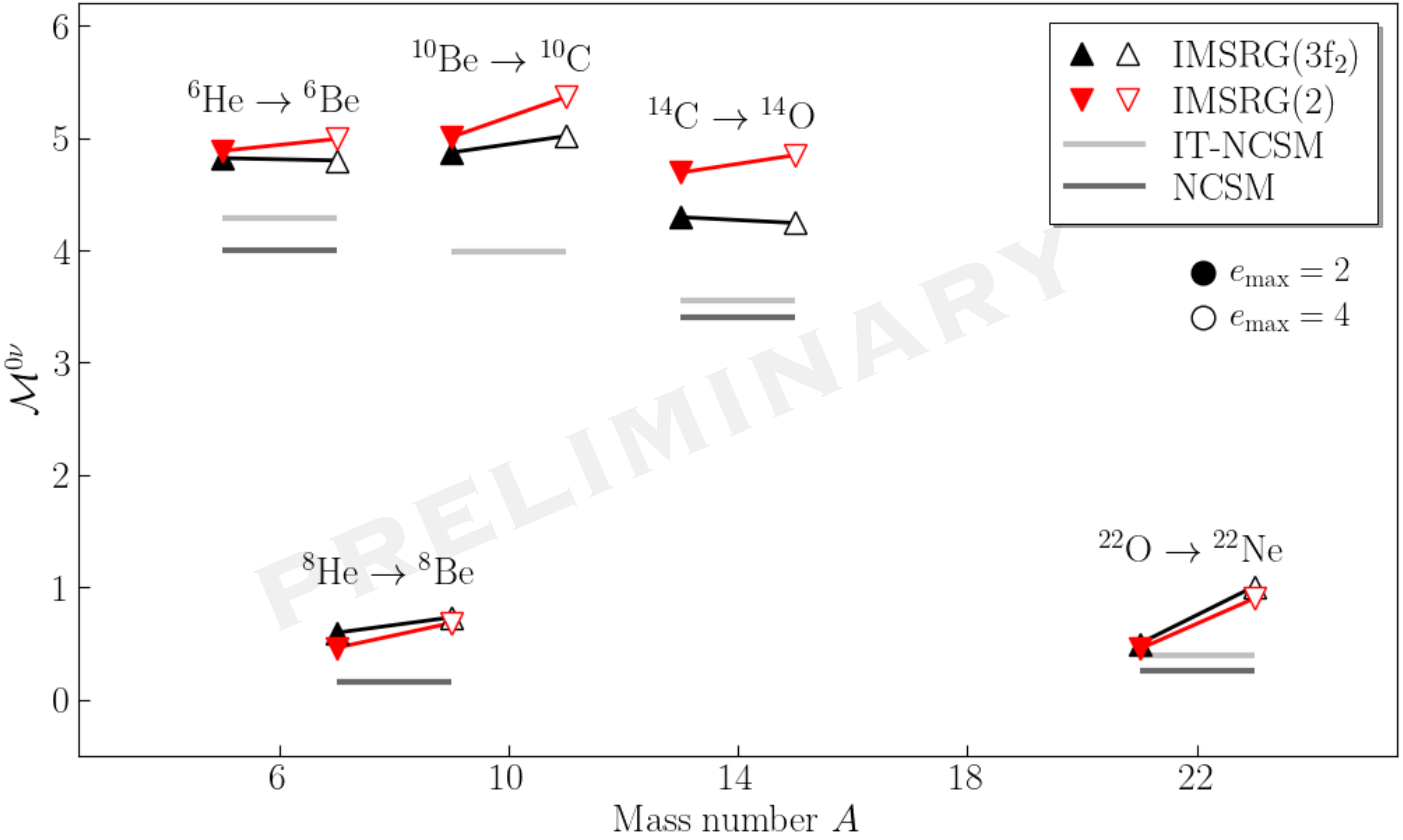
$$M^{0\nu\beta\beta} = 2.60^{+1.28}_{-1.36}$$

Belley, et al., Phys. Rev. Lett. 132, 182502

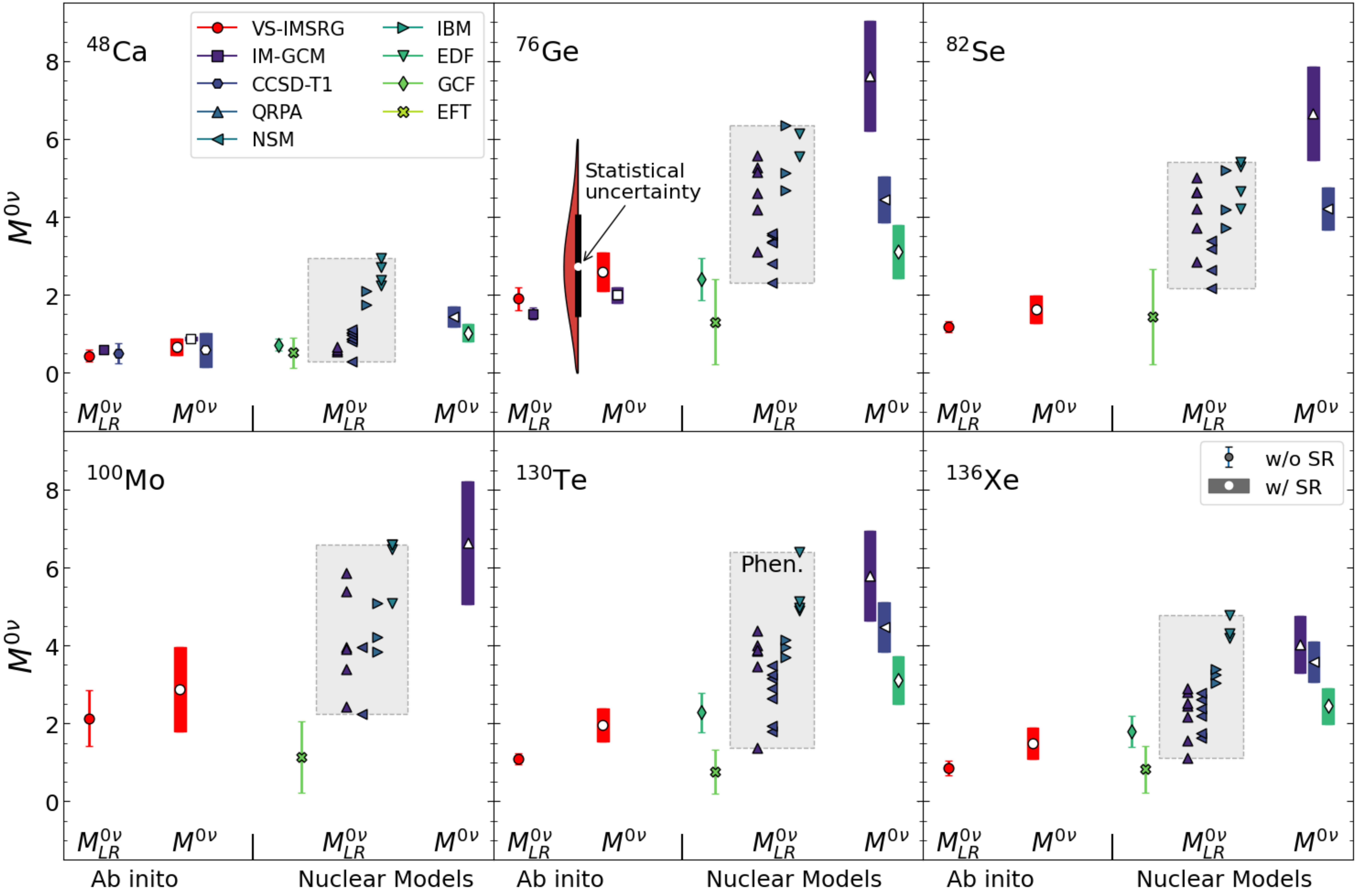
Reducing Uncertainties

Alex Todd

Preliminary results of IMSRG(3) show expected improvements.



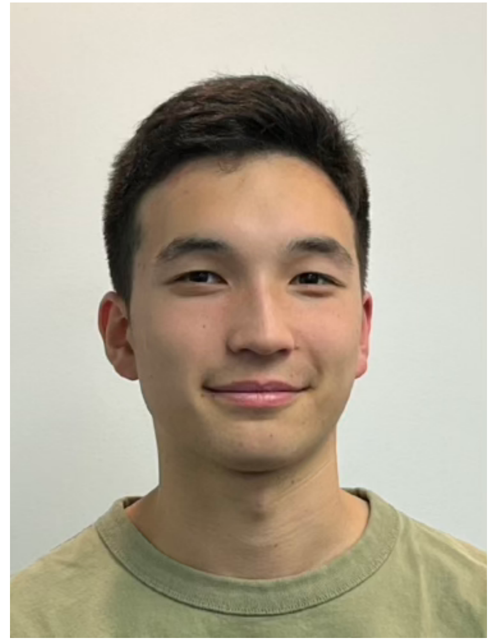
The Current Picture



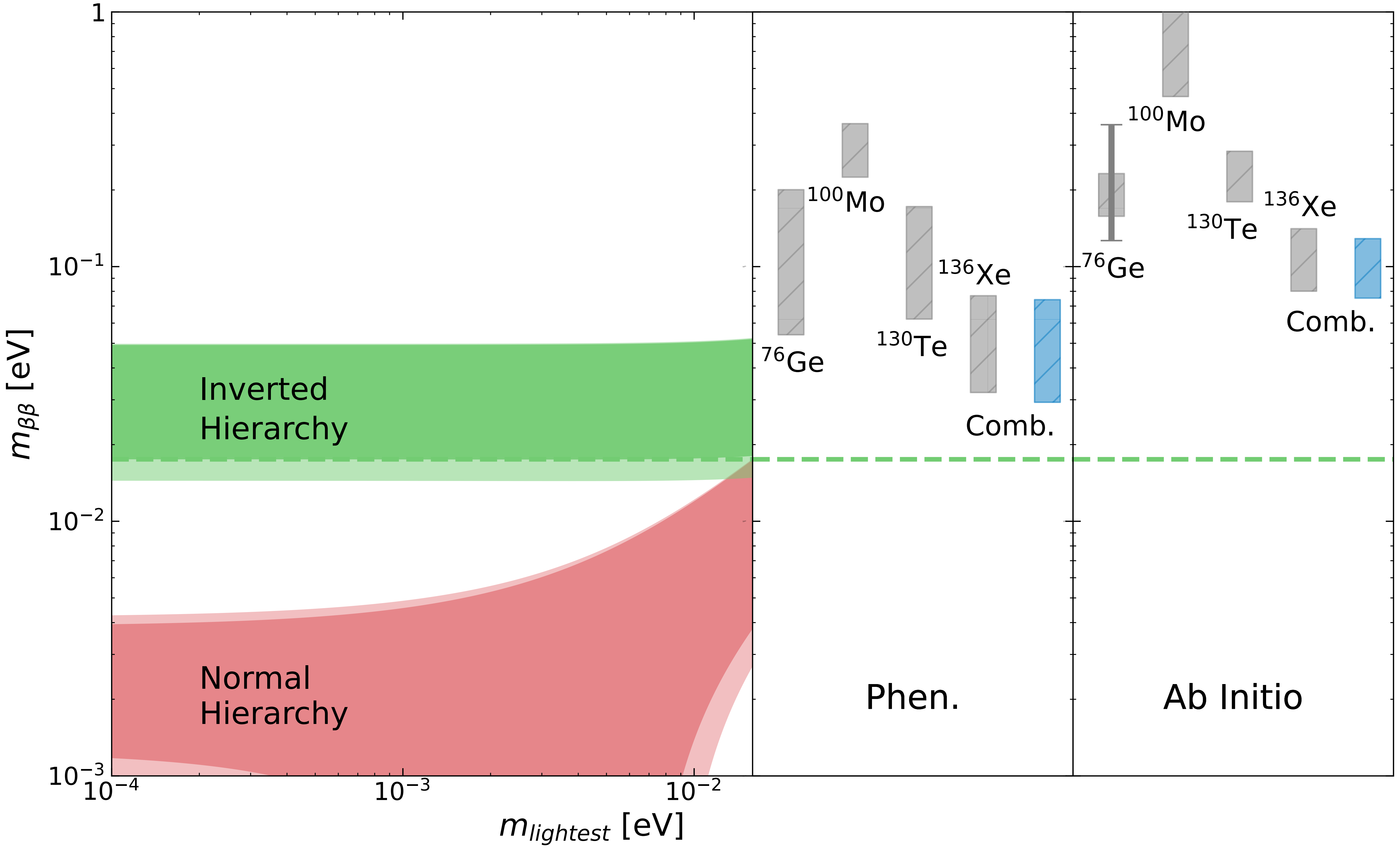
Values from:

- Agostini et al., Rev. Mod. Phys. 95, 025002 (2023);
- Yao, Sci. Bull. 10.1016 (2020);
- **Belley** et al., Phys. Rev. Lett. 126, 042502 (2020);
- Brase et al., Phys. Rev. C 106, 034309 (2021);
- Weiss et al., Phys. Rev. C 106, 064401 (2022);
- **Belley** et al., arXiv:2307.15156 (2023);
- **Belley** et al., Phys. Rev. Lett. 132, 182502 (2024).

Combining Limits of Different Isotopes



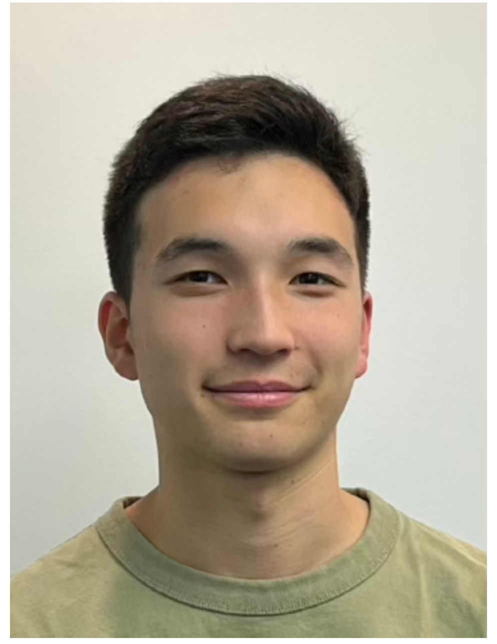
Taiki Shickele



Current

Experimental limits: **GERDA (^{76}Ge)** Phys. Rev. Lett. 125, 252502, **CUPID-Mo (^{100}100)** Eur. Phys. J. C 82 11, 1033, **CUORE(^{130}Te)** arXiv:2404.04453, **EXO(^{136}Xe)** Phys. Rev. Lett. 123, 161802 and **Kamland Zen (^{136}Xe)** arXiv:2406.11438

Combining Limits of Different Isotopes



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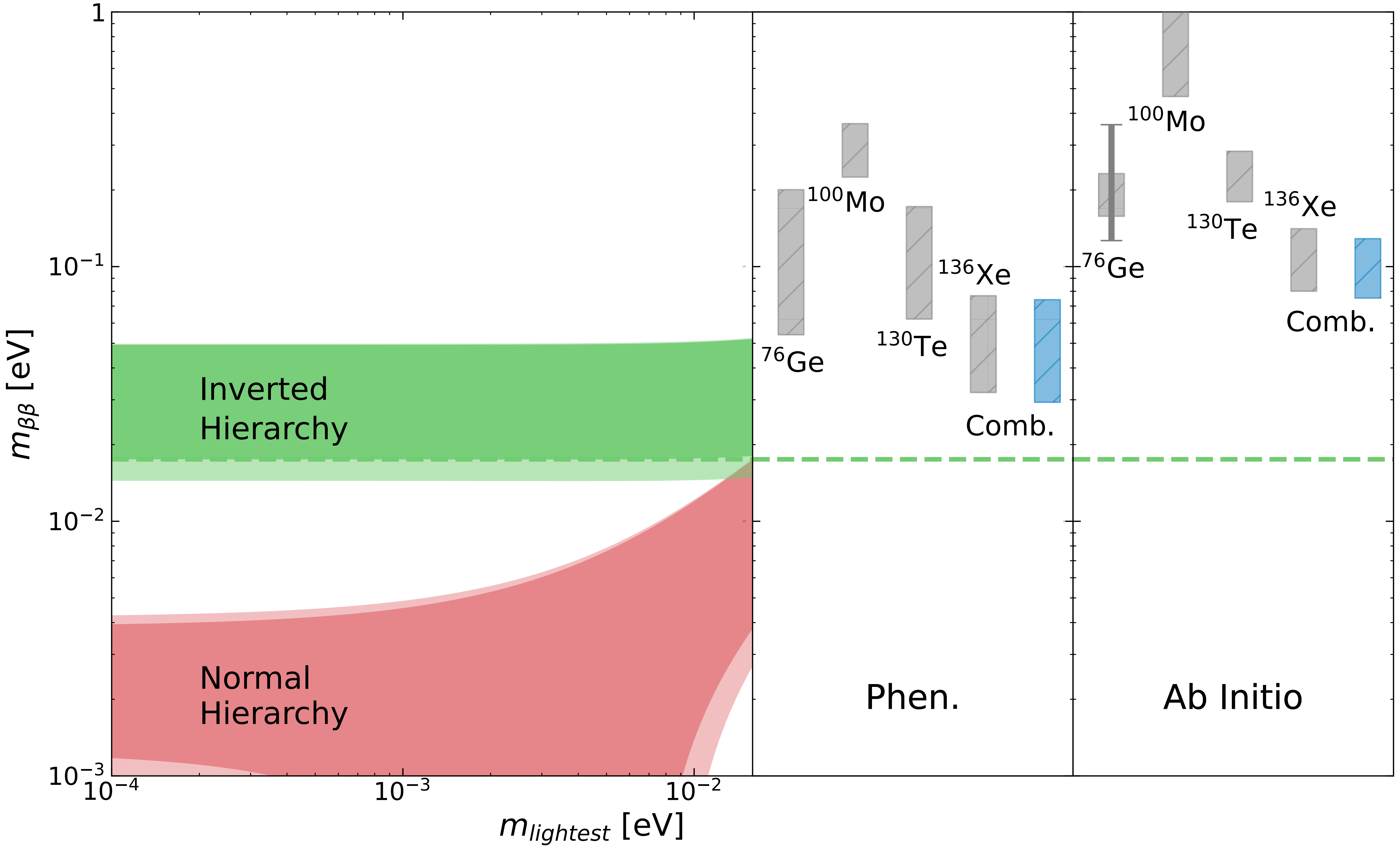
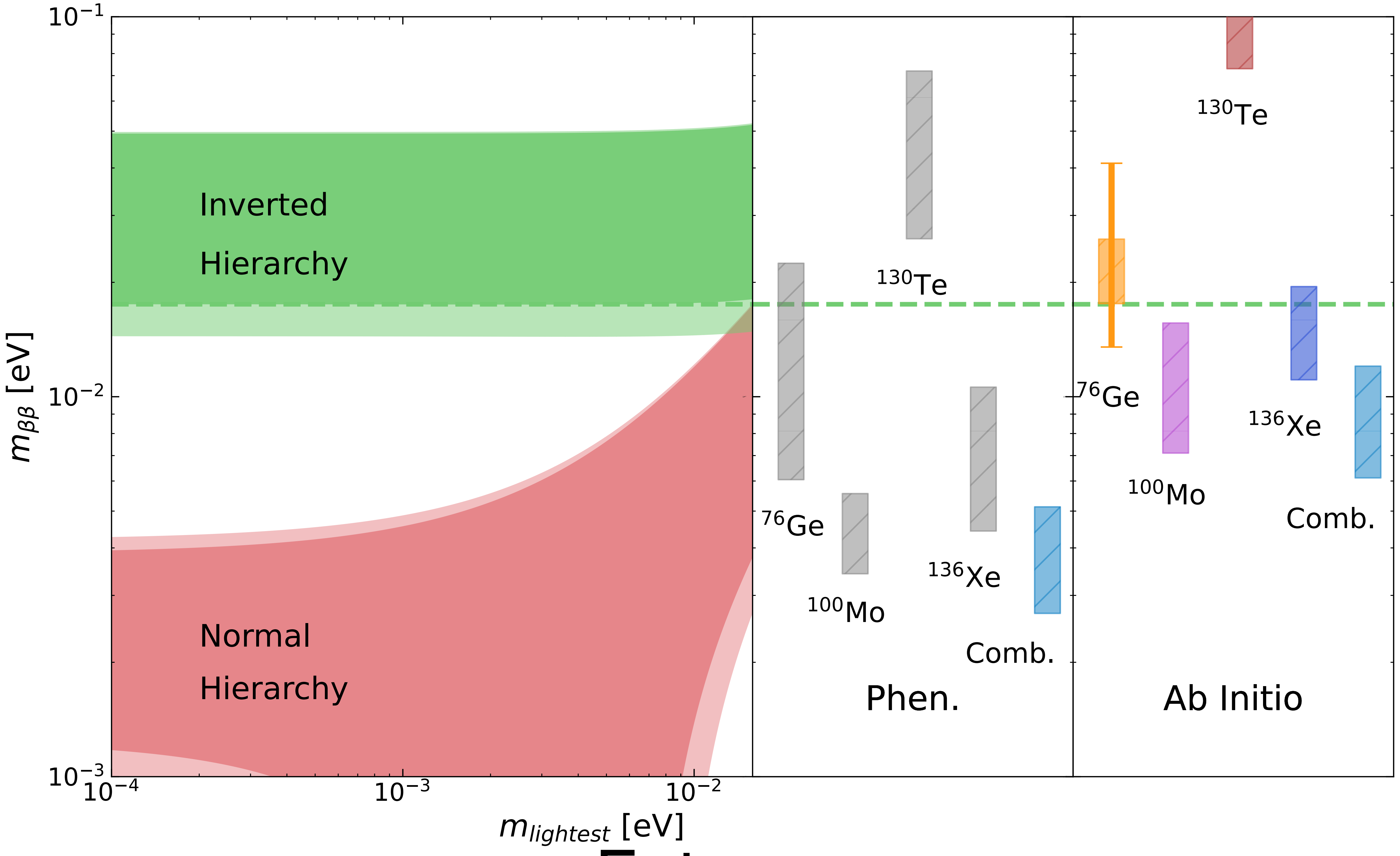


Current



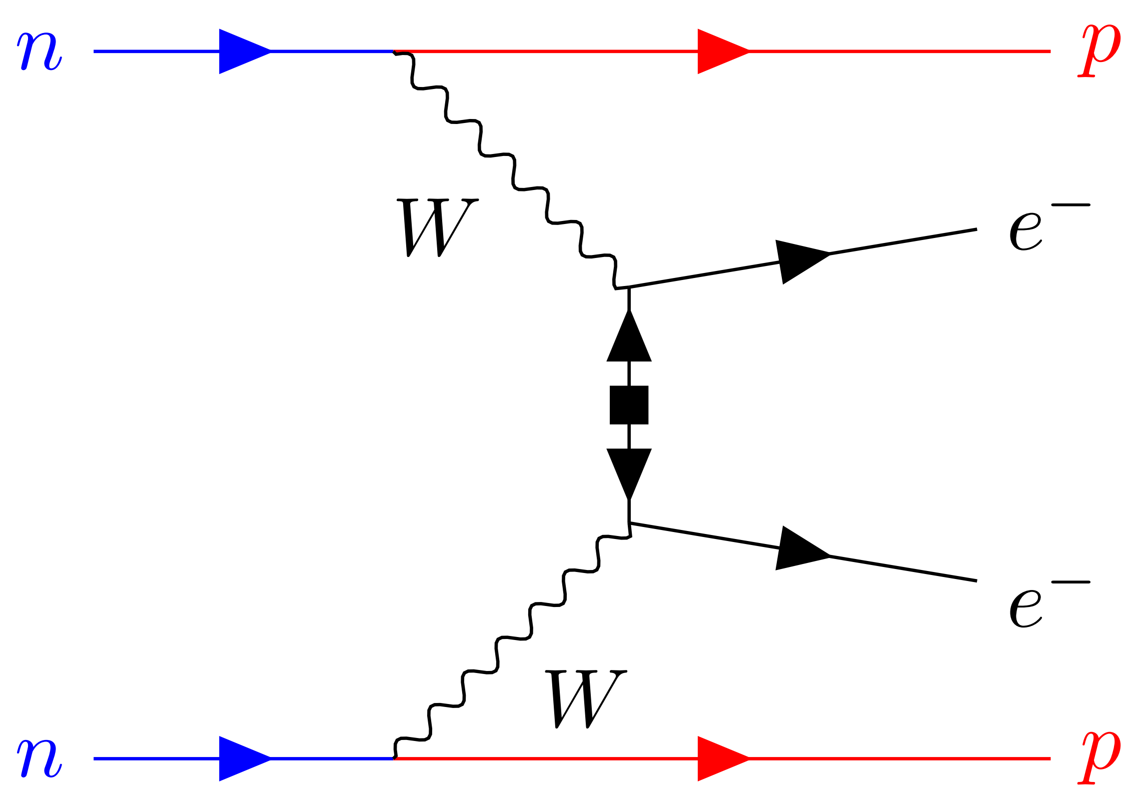
Future

Experimental limits: **GERDA (^{76}Ge)** Phys. Rev. Lett. 125, 252502, **CUPID-Mo (^{100}100)** Eur. Phys. J. C 82 11, 1033, **CUORE(^{130}Te)** arXiv:2404.04453, **EXO(^{136}Xe)** Phys. Rev. Lett. 123, 161802 and **Kamland Zen (^{136}Xe)** arXiv:2406.11438

Expected limits: **LEGEND (^{76}Ge)** arXiv:2107.11462, **CUPID-1T (^{100}100)** arXiv:2203.08386, **AMoRE (^{100}100)** arXiv:2406.09698, **SNO+(^{130}Te)** arXiv:2104.11687, **NEXT (^{136}Xe)** JHEP09(2023)190 and **nEXO (^{136}Xe)** J. Phys .G 49 1, 015104.

Going Past the Standard Mechanism

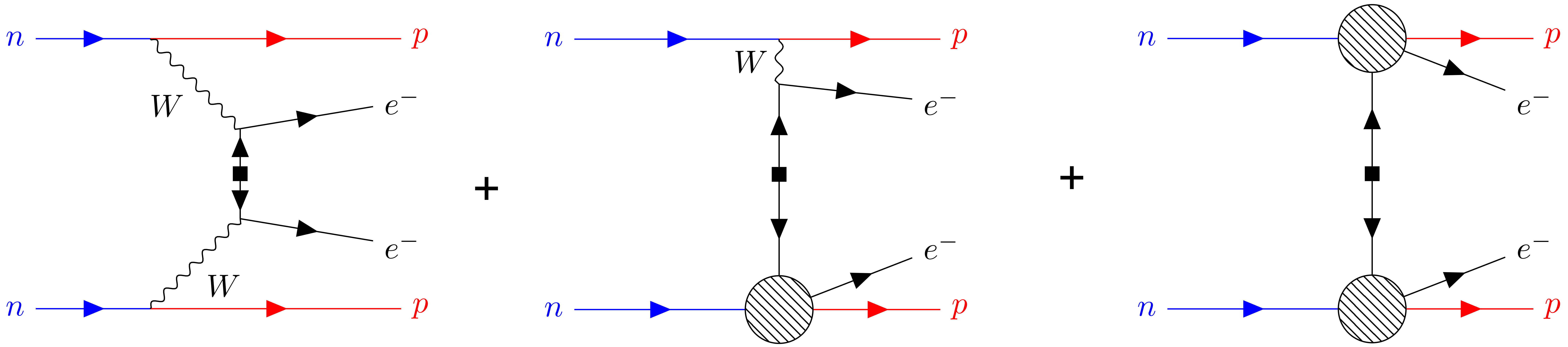
$$[T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu} M^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta} \rangle}{m_e} \right)^2$$



+

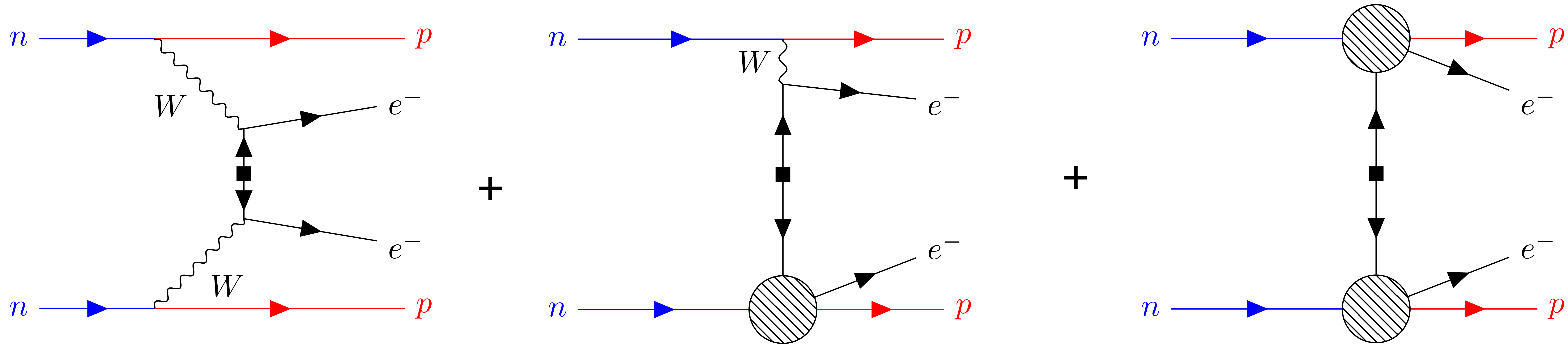
Going Past the Standard Mechanism

$$[T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu} M^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 \longrightarrow [T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu}|M^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 + \sum_i G_i |M_i|^2 \eta_i^2$$



Going Past the Standard Mechanism

$$[T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu} M^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 \longrightarrow [T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu}|M^{0\nu}|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2 + \sum_i G_i |M_i|^2 \eta_i^2$$



Simplest extension is to add heavy sterile neutrinos $\Rightarrow [T^{0\nu}_{1/2}]^{-1} = g_A^4 G^{0\nu} \left| M^{0\nu}\left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right) + M^{0N}\left(\frac{m_p}{m_N}\right)\right|^2$

Mass of heavy neutrino

Heavy Sterile Neutrino NMEs

Alex Todd

$$\boxed{M^{0N} = M_{GT}^{0N} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0N} + M_T^{0N}}$$

Alex Todd

Heavy Sterile Neutrino NMEs

$$\boxed{M^{0N} = M^{0N}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0N}_F + M^{0N}_T} \longrightarrow$$

All operators are short-range contact operators.

Heavy Sterile Neutrino NMEs

Alex Todd

$$\boxed{M^{0N} = M^{0N}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0N}_F + M^{0N}_T}$$

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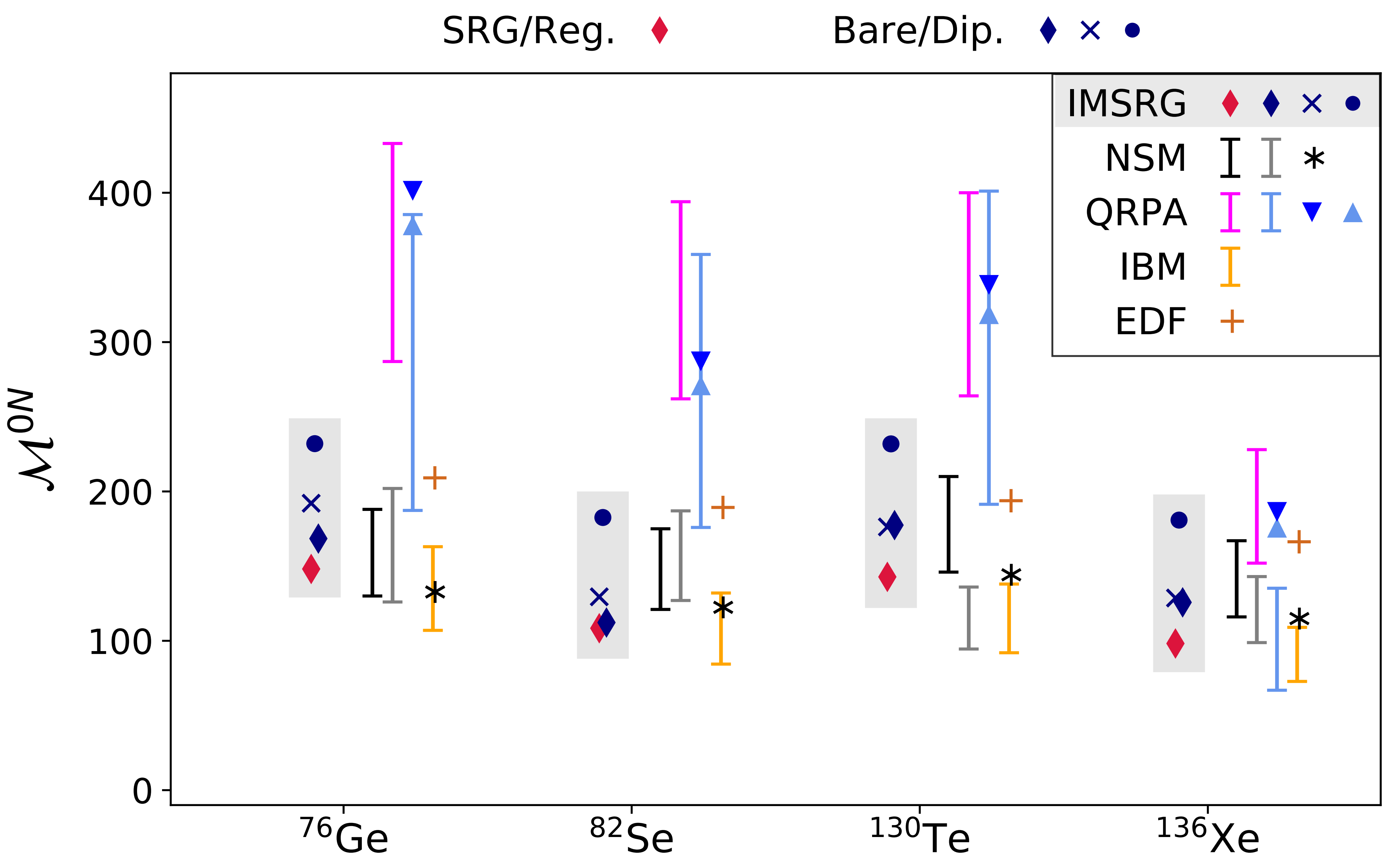
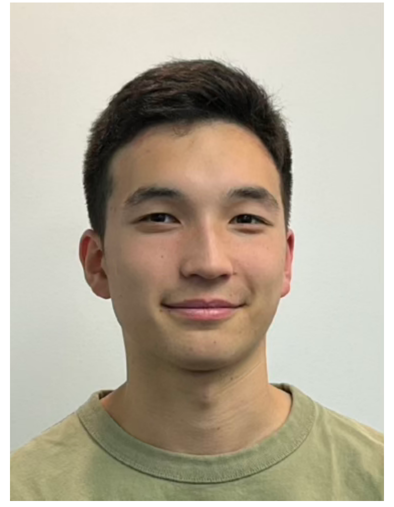


Heavy Sterile Neutrino NMEs




Taiki Shickele   Alex Todd

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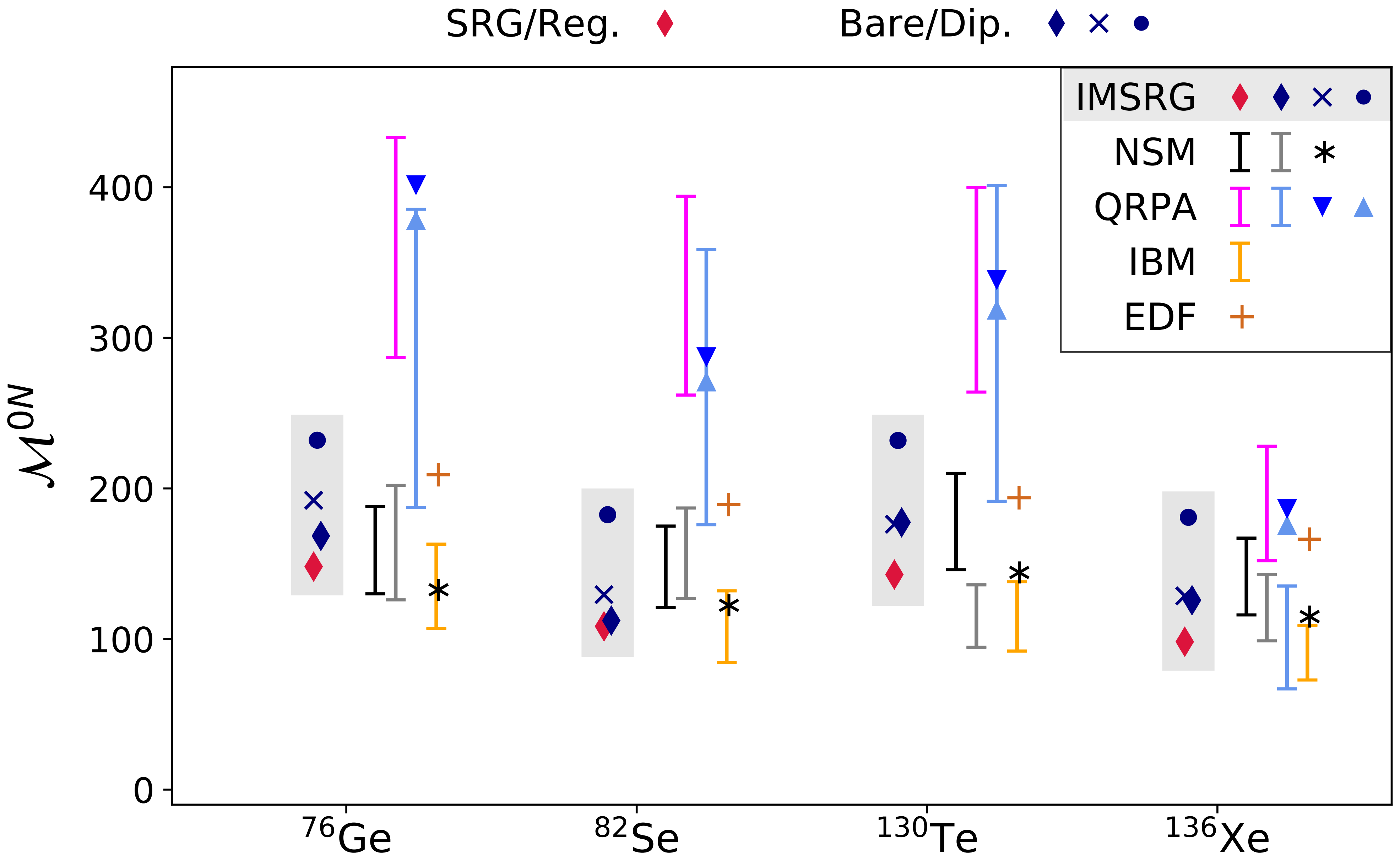
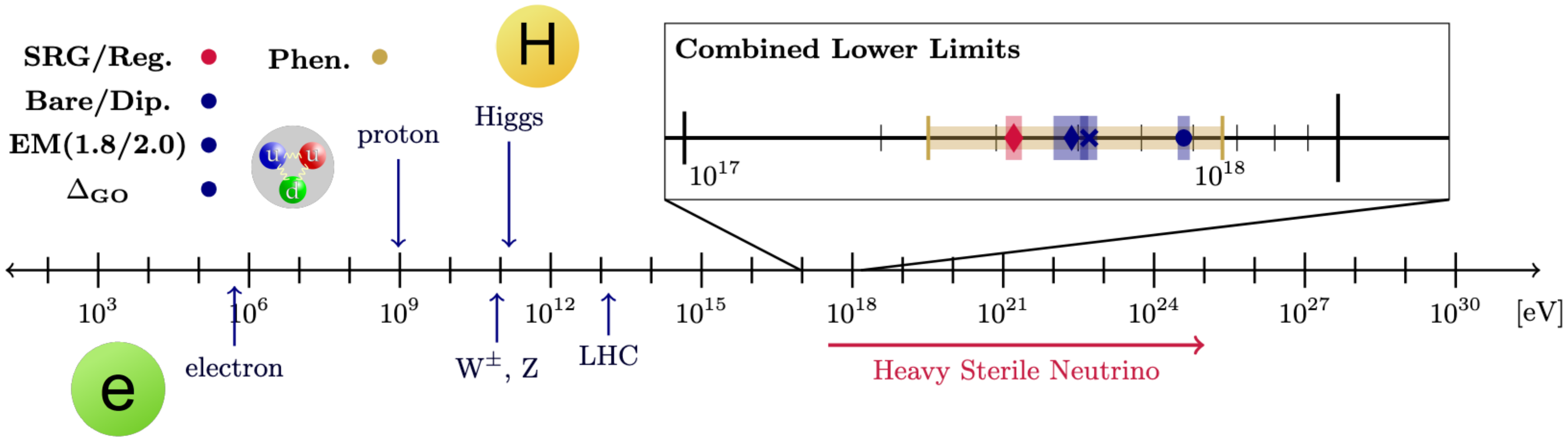
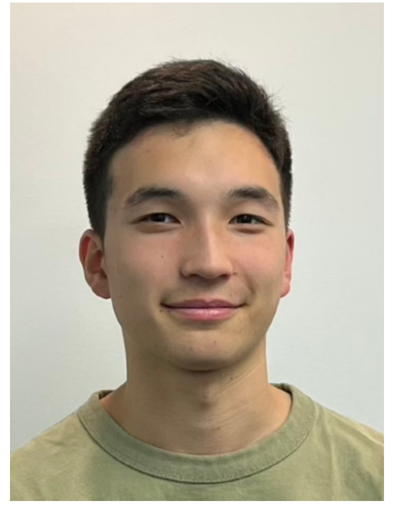




Heavy Sterile Neutrino NMEs




Taiki Shickele   Alex Todd

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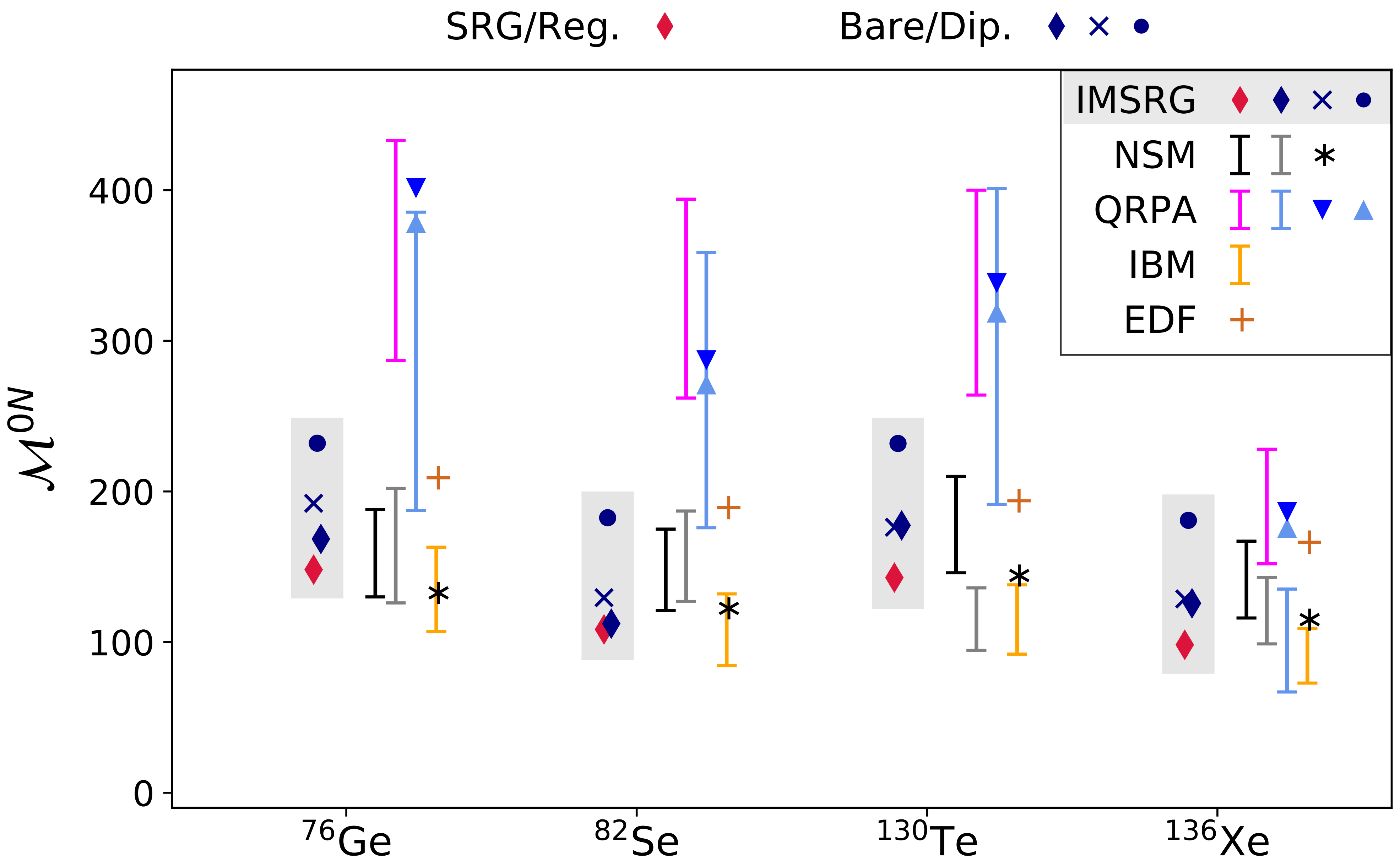
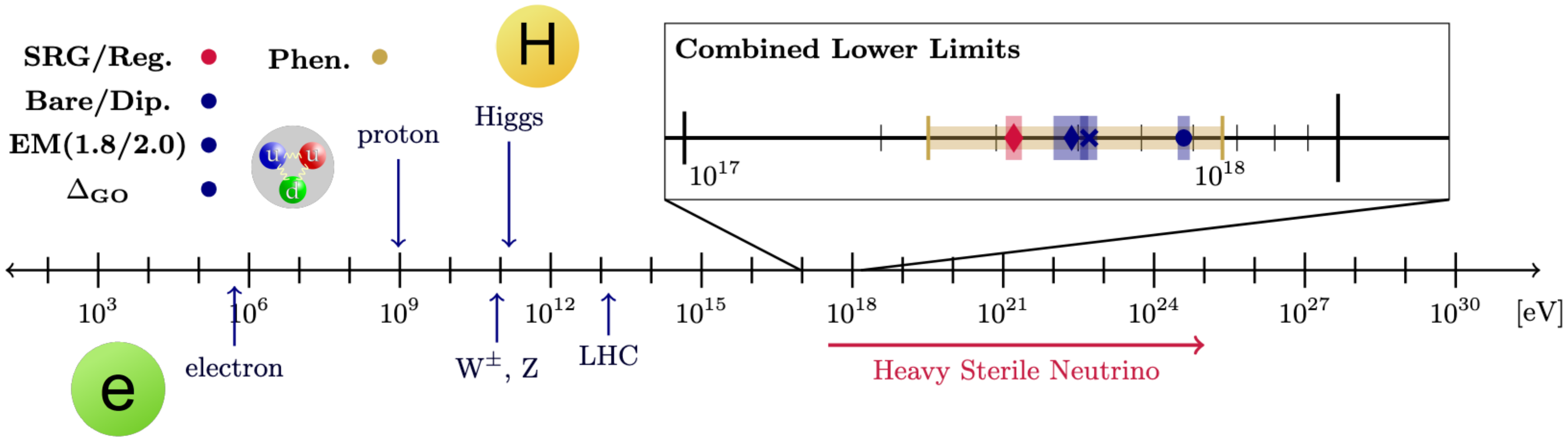
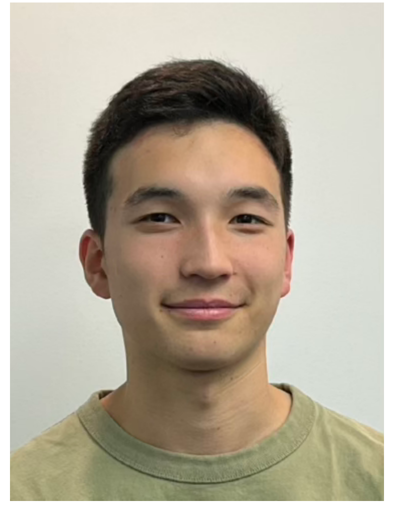


Can probe energy scales many orders of magnitude higher than particle accelerators!



Heavy Sterile Neutrino NMEs



Taiki Shickele   Alex Todd

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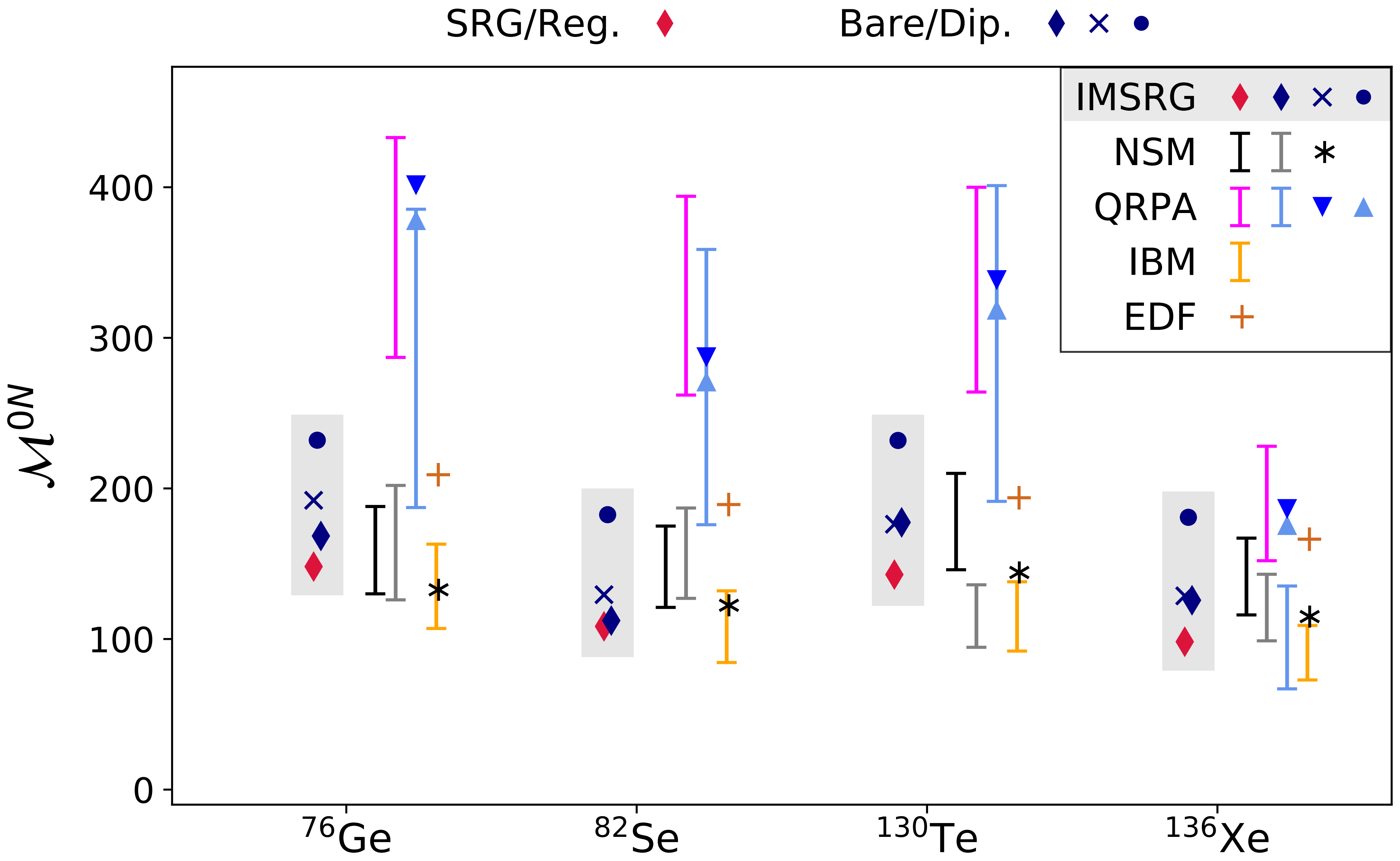

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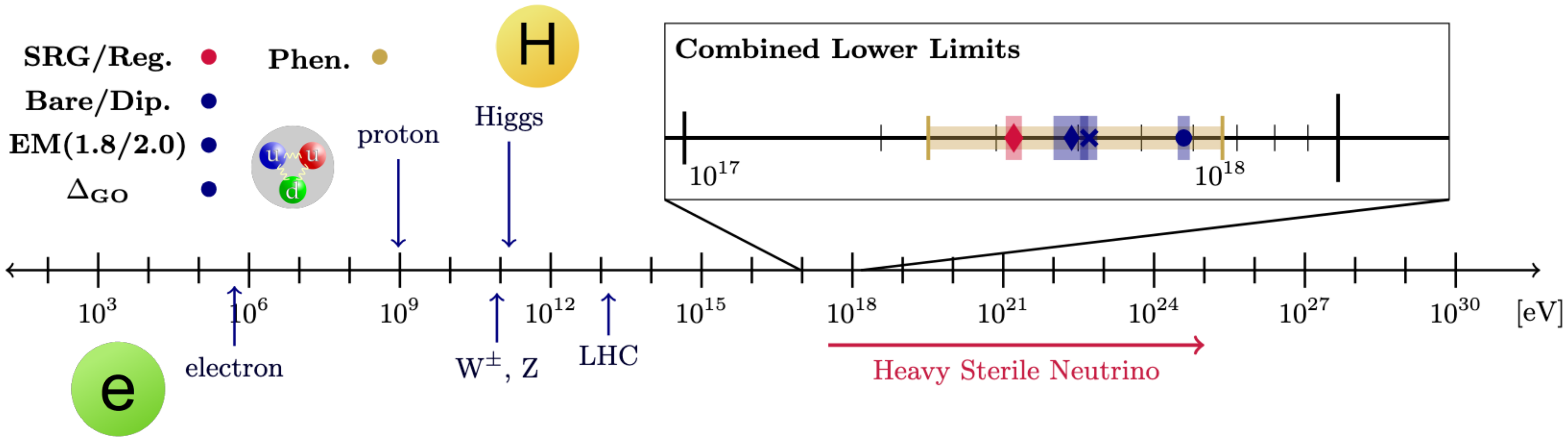


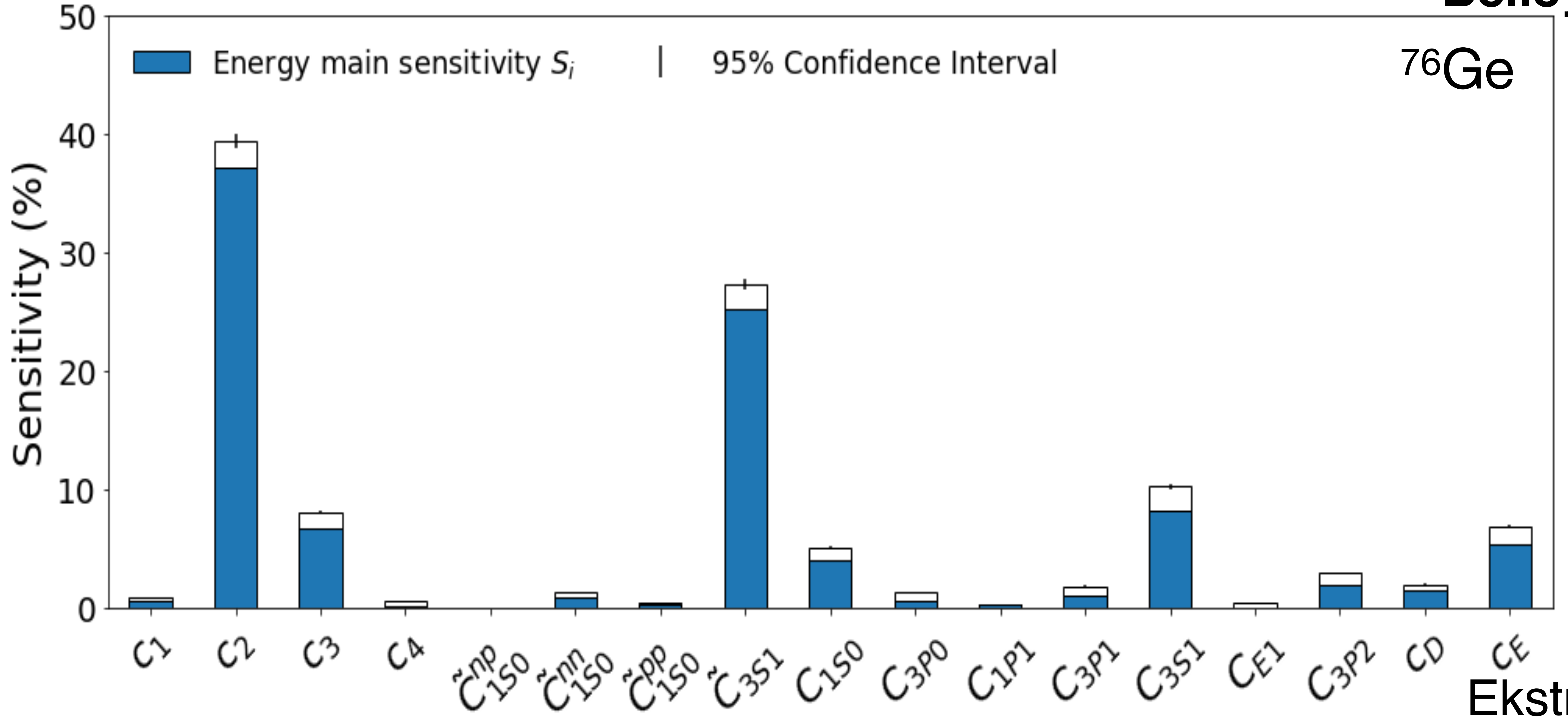
From V. Cirigliano, et al., JHEP12(2018)097, only 15 different nuclear matrix elements can contribute to mechanisms involved in $0\nu\beta\beta$. Observation in many isotopes is required to identify (or at least constrain) the mechanisms at play.

Global emulation for nuclear structure

The MM-DGP Algorithm: GSA

Data driven MM-DGP

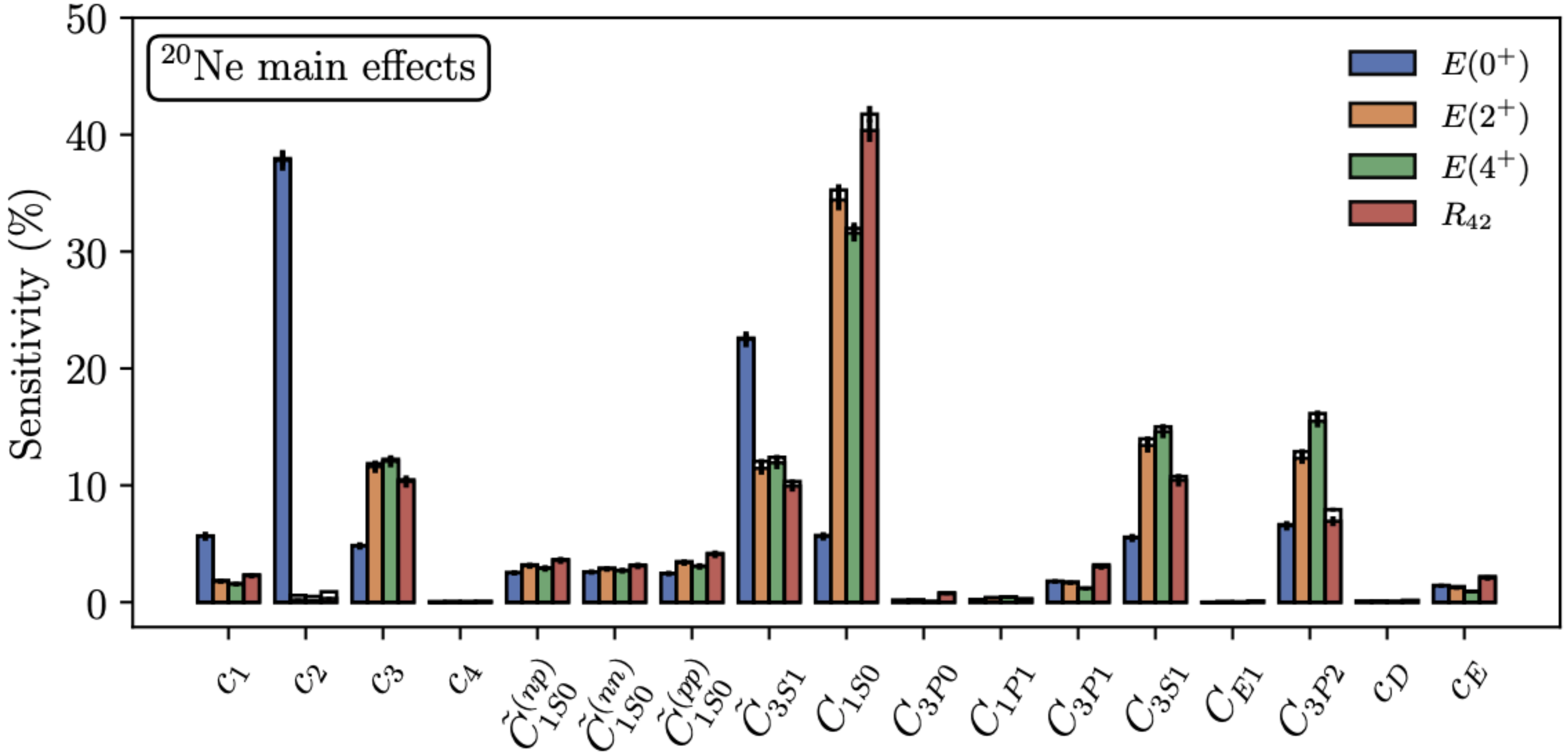
Belley, et al., arXiv:2408.02169 (2024)



vs

Physics Driven

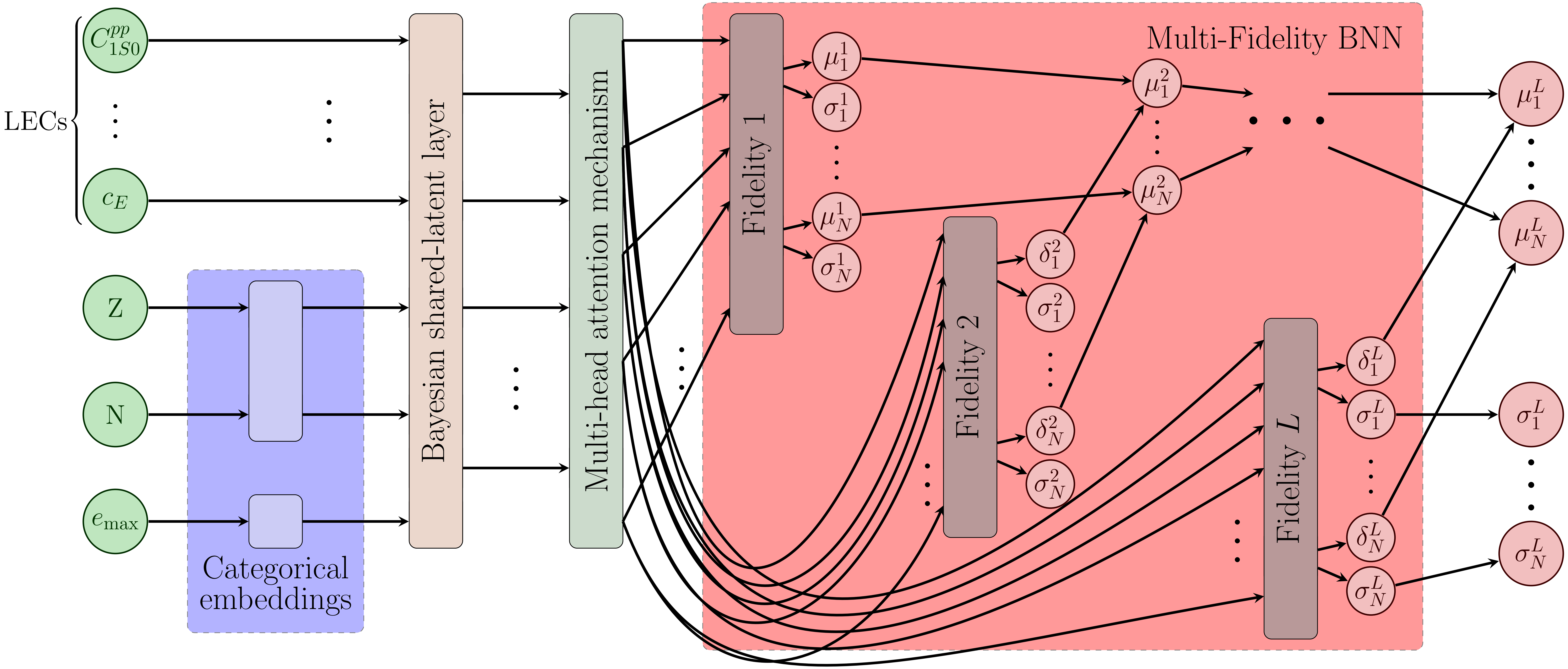
Ekström, et al., arXiv:2305.06955 (2023)



Jose Miguel Muñoz Arias

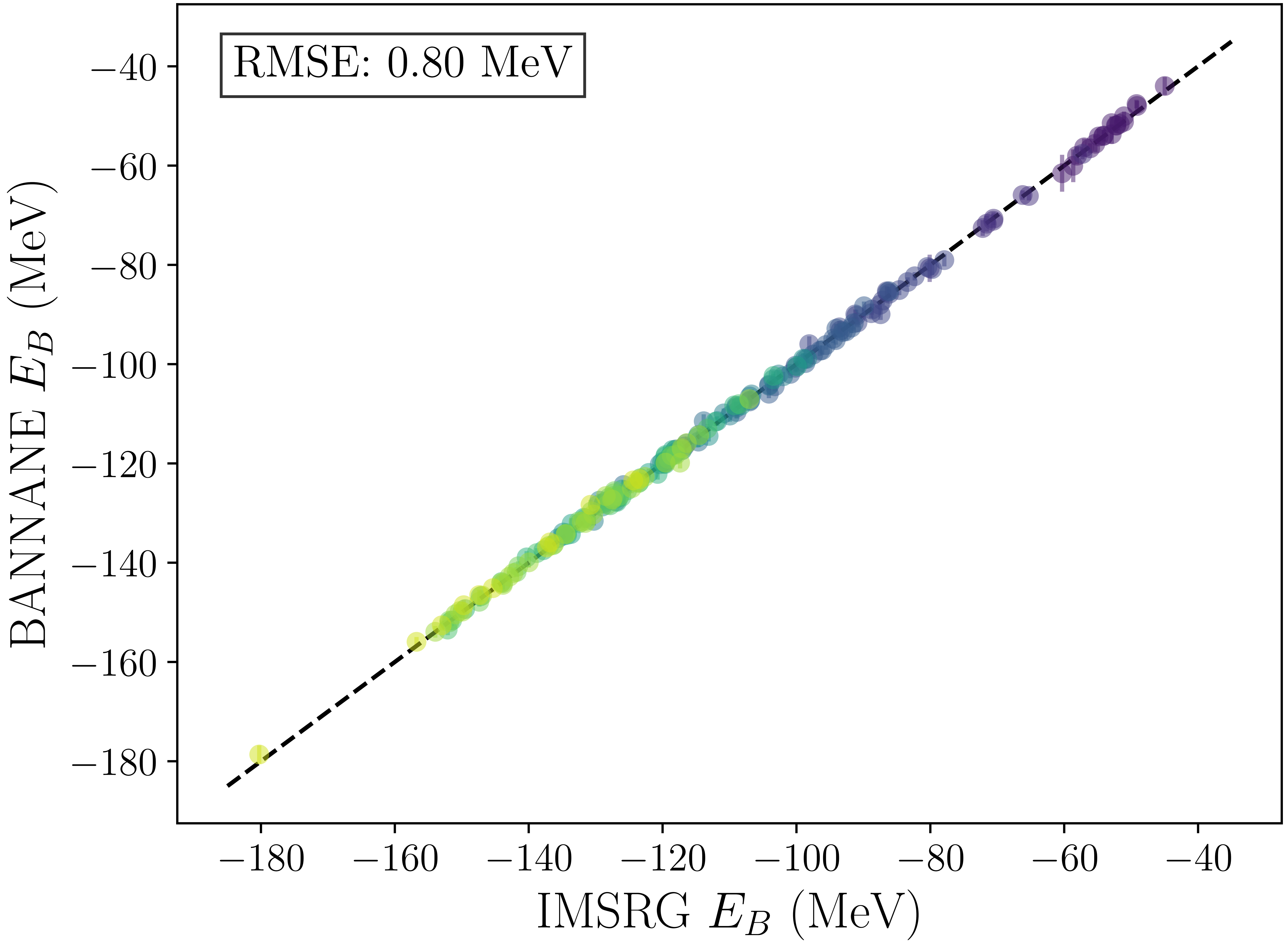
BANNANE

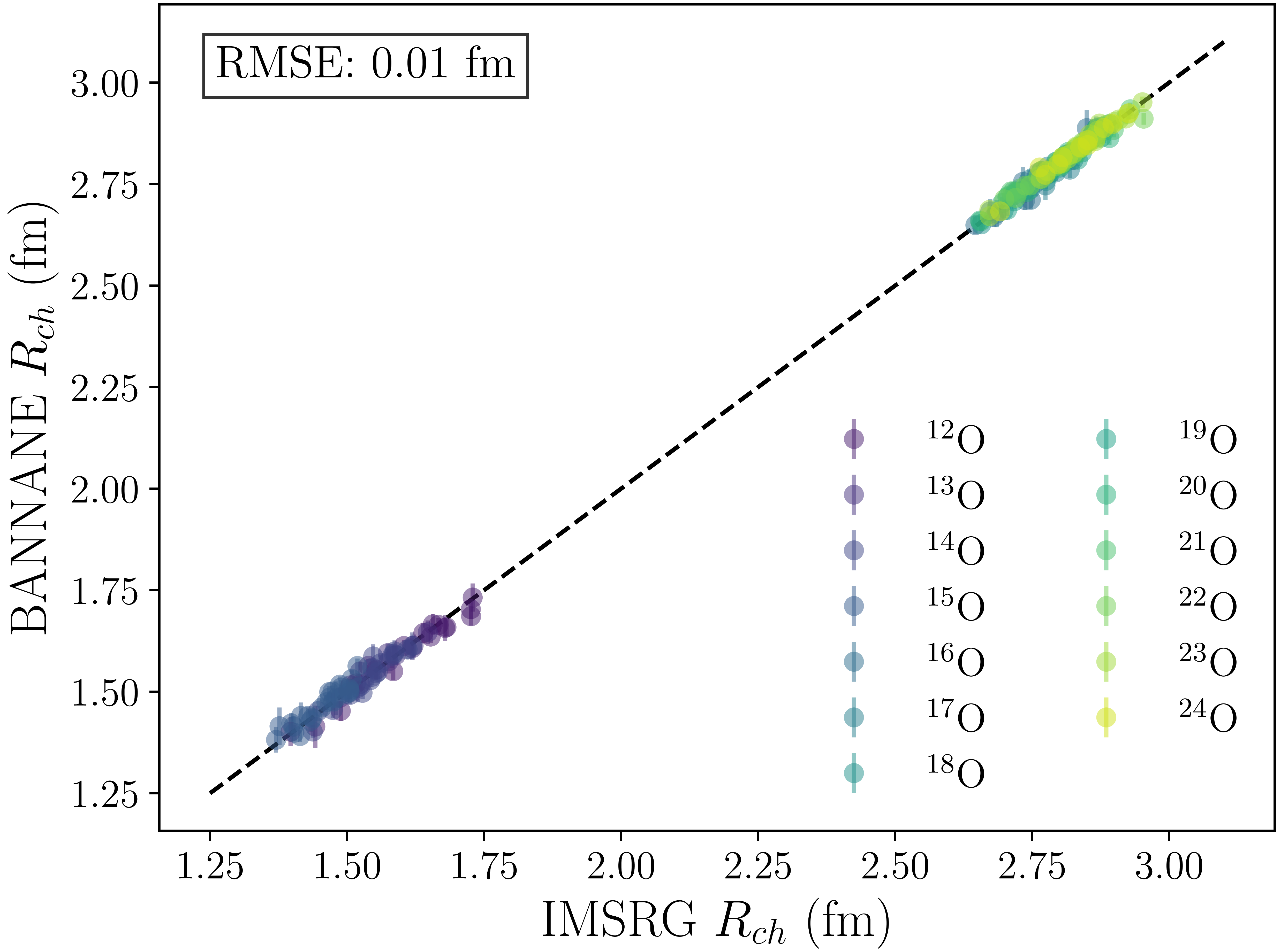
BAyesian Neural Network for Atomic Nuclei Emulation



Emulating Multiple Isotopes

Belley**, Munoz,** Gàrcia.,́ arxiv:2502.20363





BANNANE achieves state-of-the-art emulation, with smaller errors than both MM-DGP and EC while emulating over a full isotopic chain.

Summary …

- Emulators are required to obtain uncertainty quantification of nuclear theory observables required for searches of new physics.
- Emulator further allows the use of other statistical tools like global sensitivity analysis.
- Many-body uncertainty is the main source of uncertainty in current calculations.

… and Outlook

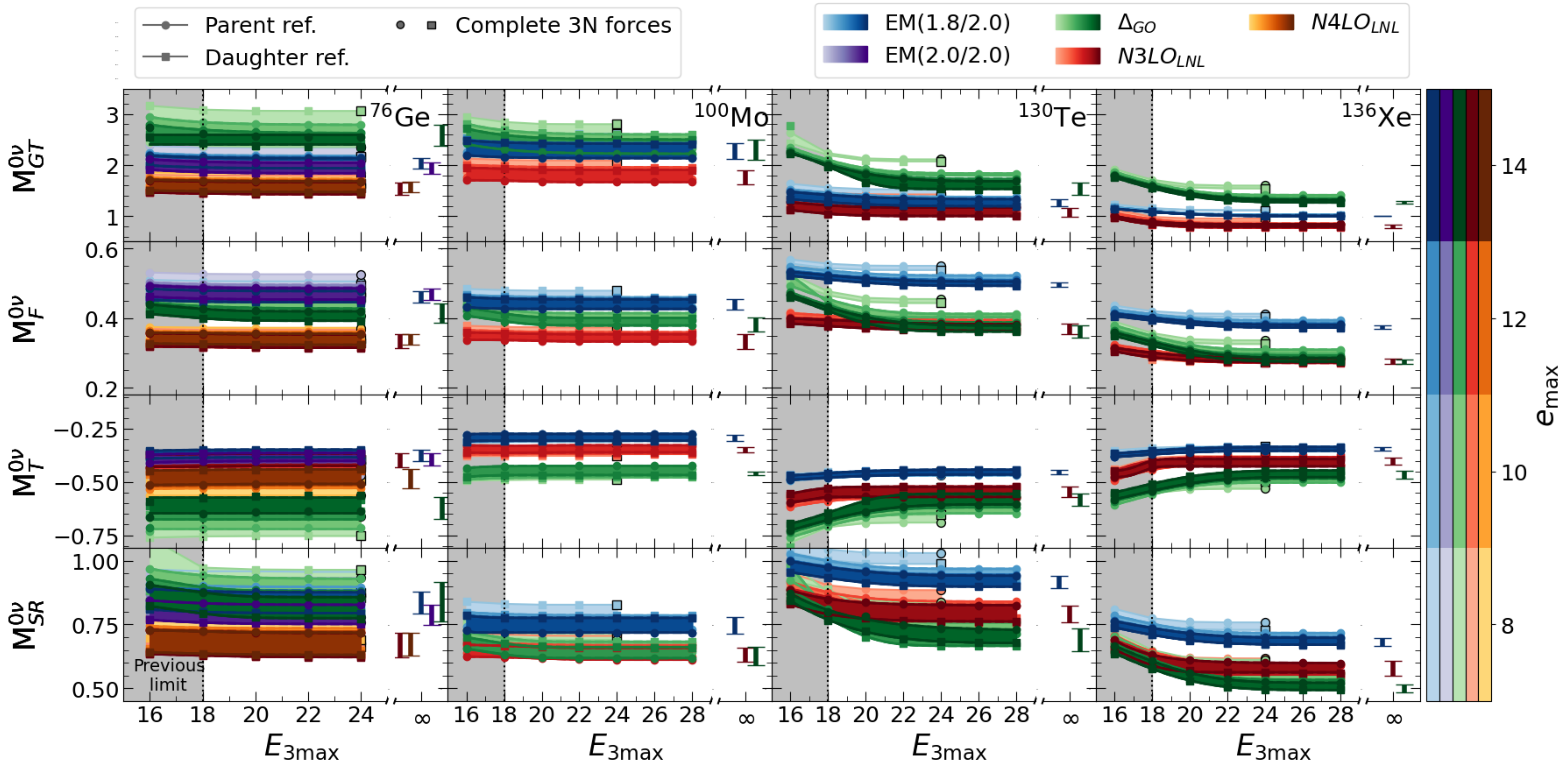
- Improving the emulator with other machine learning models.
- Reducing the many-body error using methods that probe the IMSRG(3).
- Doing a similar analysis for other nuclear processes.
- Computing other observables for BSM searches with uncertainties.

Thank you!

Questions?

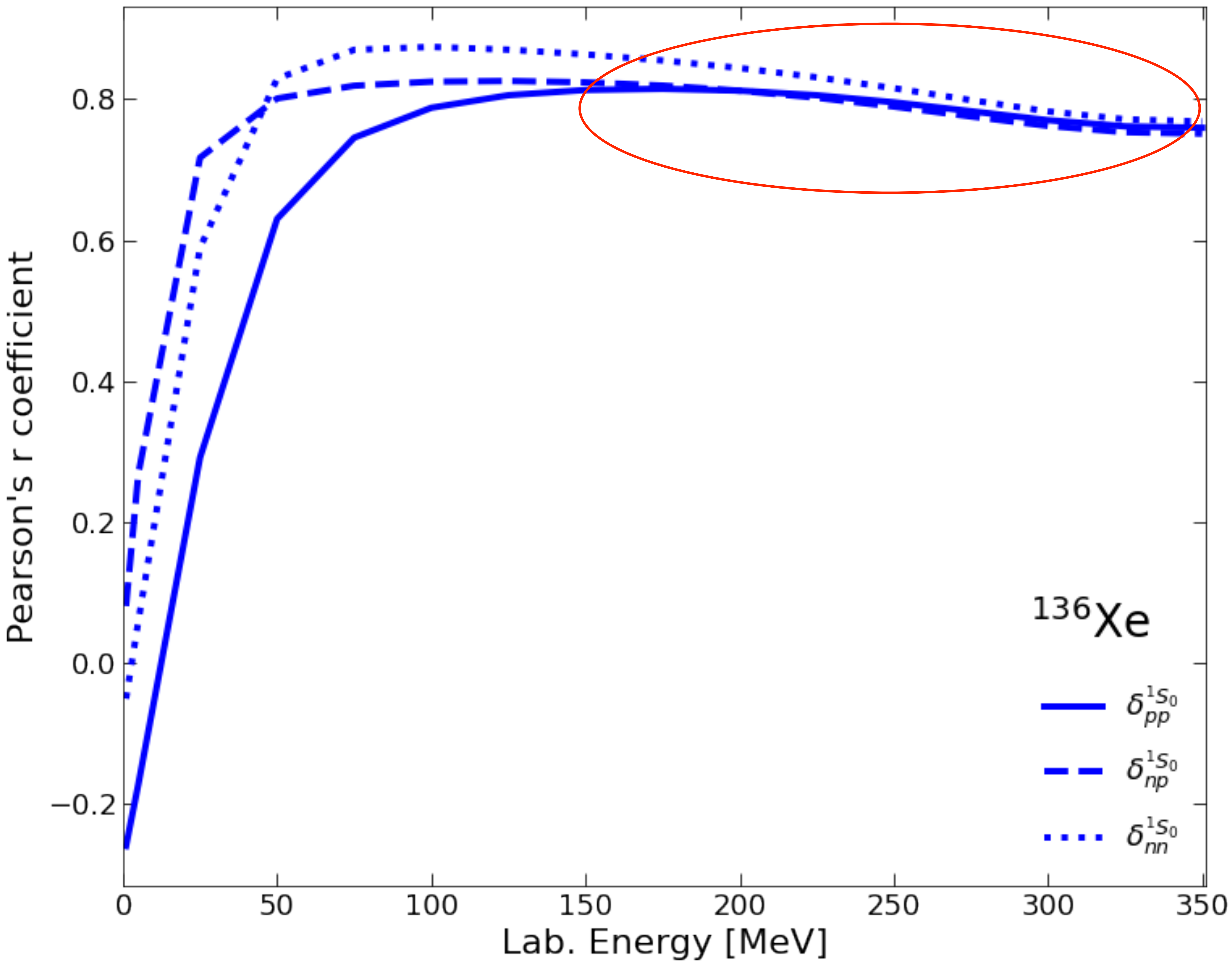
abelley@mit.edu

Results in Heavy Nuclei



Belley, et al., arXiv:2307.15156 (2023)

Correlation with Phase Shift



Strong correlation for energies > 50 MeV

⇒

The size of matrix elements is mostly constrained by the interaction between the two nucleons that undergo the decay, given they are close enough from each other.

Belley, et al., arXiv:2408.02169 (2024)

Posterior Distribution of the NMEs

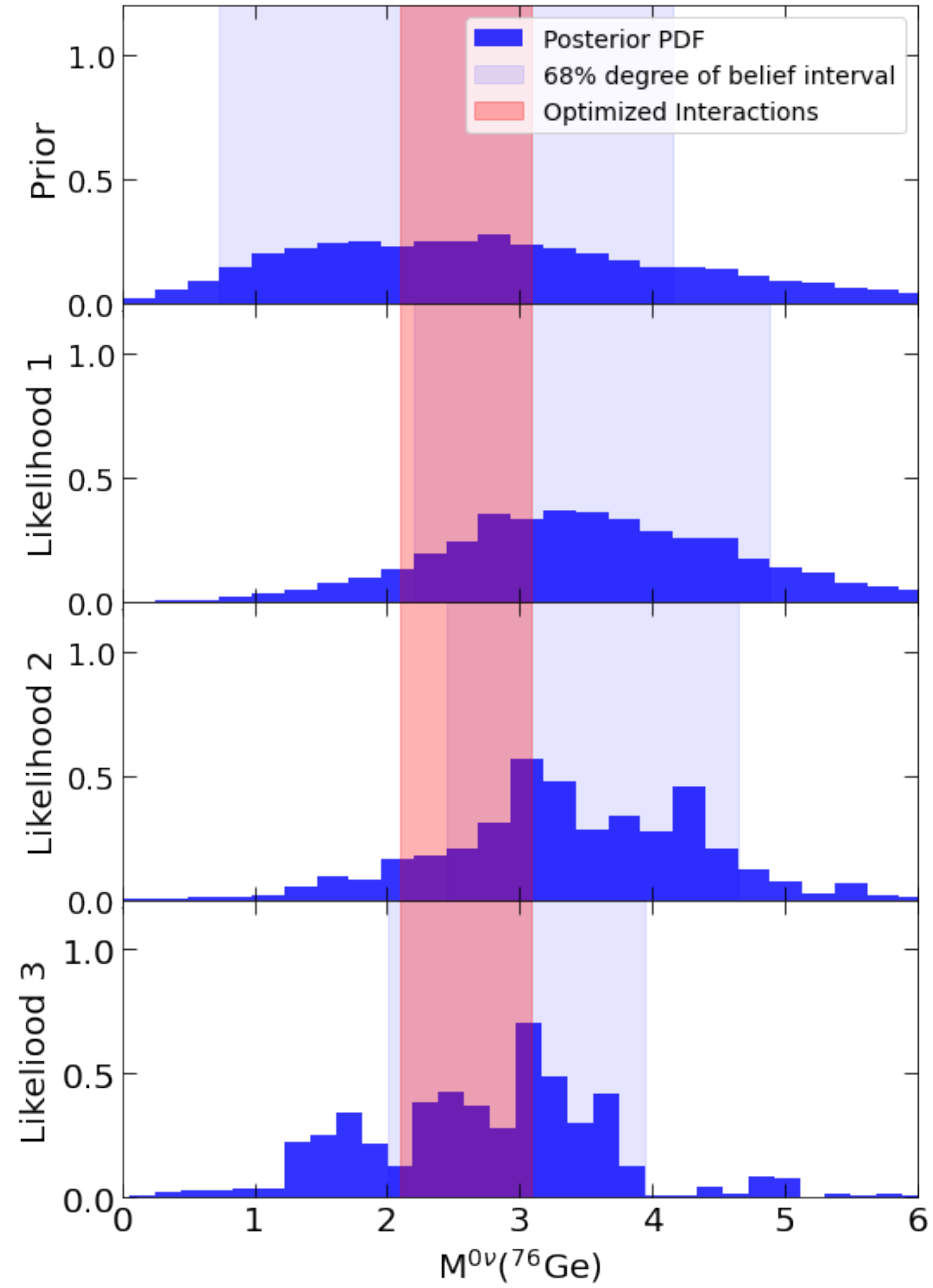
- Use 8188 “non-implausible” samples obtain by Jiang, W. G. et al. (Phys. Rev. C **109**, 064314) .
- Many-body problem is “solved” with the MM-DGP.
- Consider all sources of uncertainties by taking:

$$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

where the ϵ’s are the errors coming from different sources and are assumed to be normally distributed and independent.

- Interactions are weighted by the ${}^1S_0$ neutron-proton phase shifts at 50 MeV and observables for mass A=2-4,16.

Choosing a Likelood



Likelihood 1: Only contains $^1S_0$ neutron-proton phase shifts at 50 MeV.

Likelihood 2: Contains $^1S_0$ neutron-proton phase shifts at 50 MeV and observables for A=2-4.

Likelihood 3: Contains $^1S_0$ neutron-proton phase shifts at 50 MeV and observables for A=2-4,16.

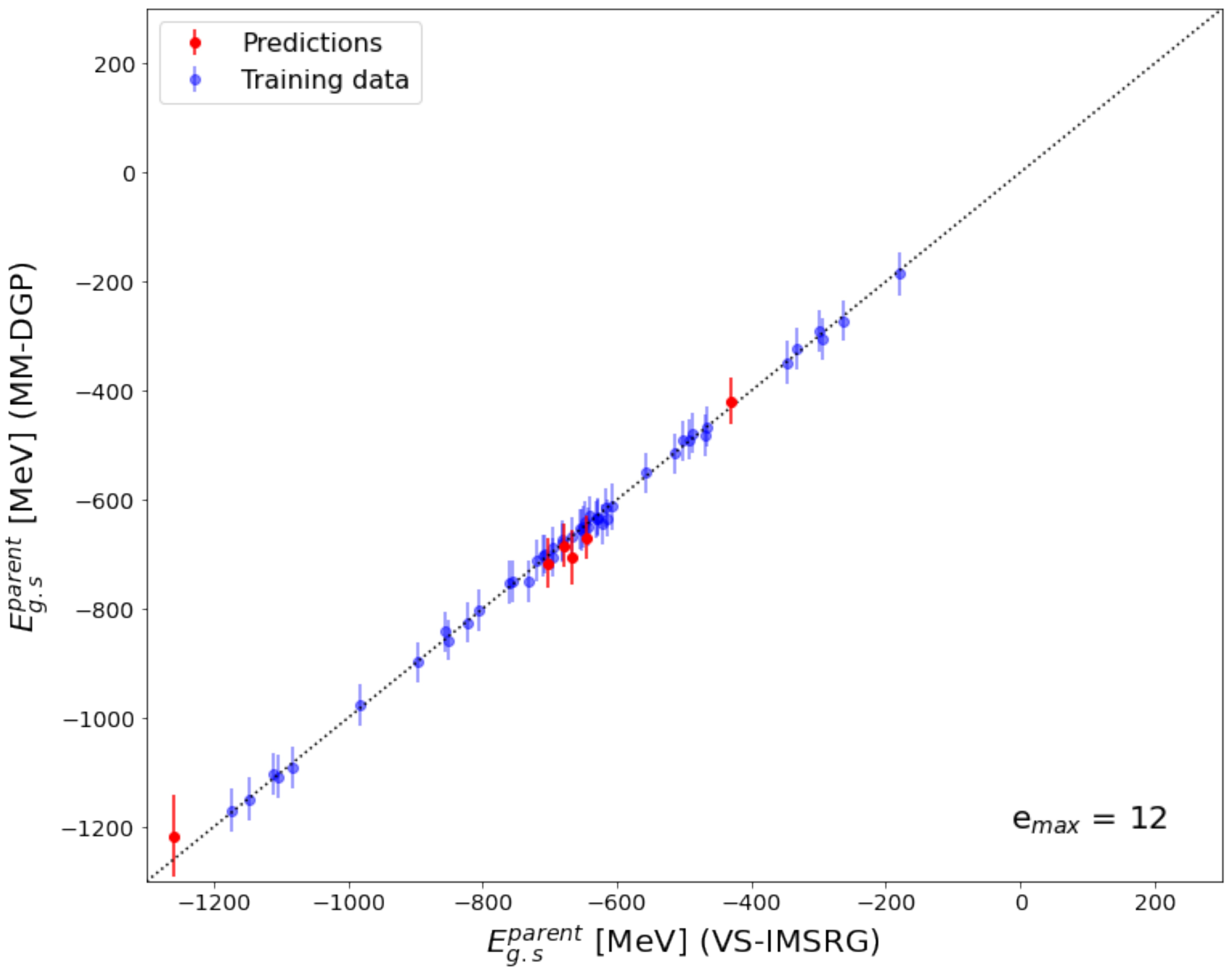
A2-4: $E(^2H)$, $r_p(^2H)$, $Q(^2H)$, $E(^3H)$, $E(^4He)$, $r_p(^4He)$

A16: $E(^{16}O)$, $r_p(^{16}O)$

Emulator error

$$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

This error is given directly by the Gaussian Process and depends on the LECs (i.e. each predicted point has its own error).



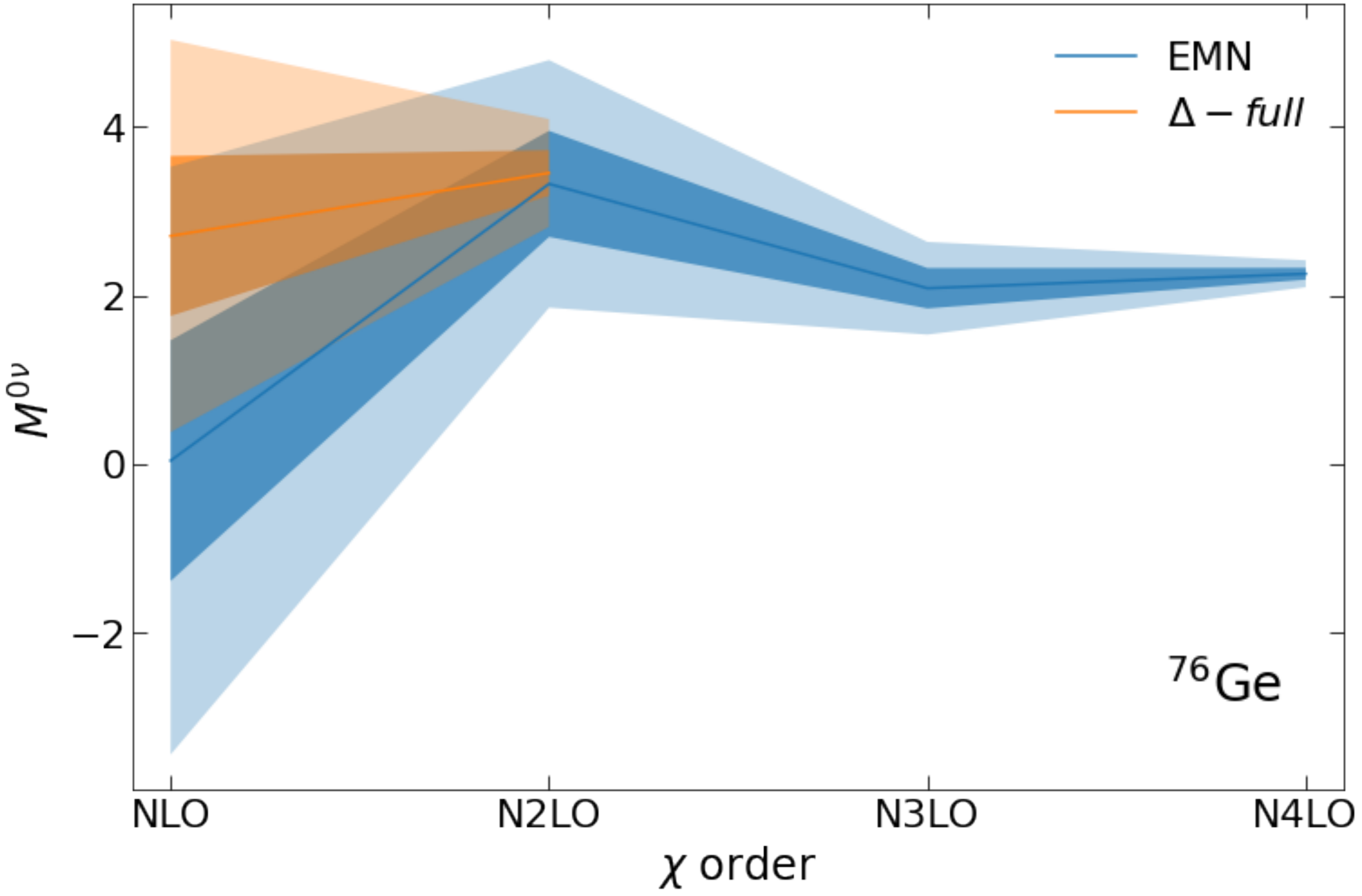
EFT Truncation error

$$y = y_{MM-DGP} + \epsilon_{emulator} + \boxed{\epsilon_{EFT}} + \epsilon_{many-body} + \epsilon_{operator}$$

Error due to the truncation of the nuclear interactions (the samples are truncated at N2LO, including delta excitations).

Use EMN interaction at NLO, N2LO, N3LO and N4LO, without delta excitations, to verify convergence of chiral expansion.

Using the Δ-full interaction of this work, only NLO and N2LO orders are available. Using expansion from BUQEYE collaboration, we get $\epsilon_{EFT} = 0.3$.

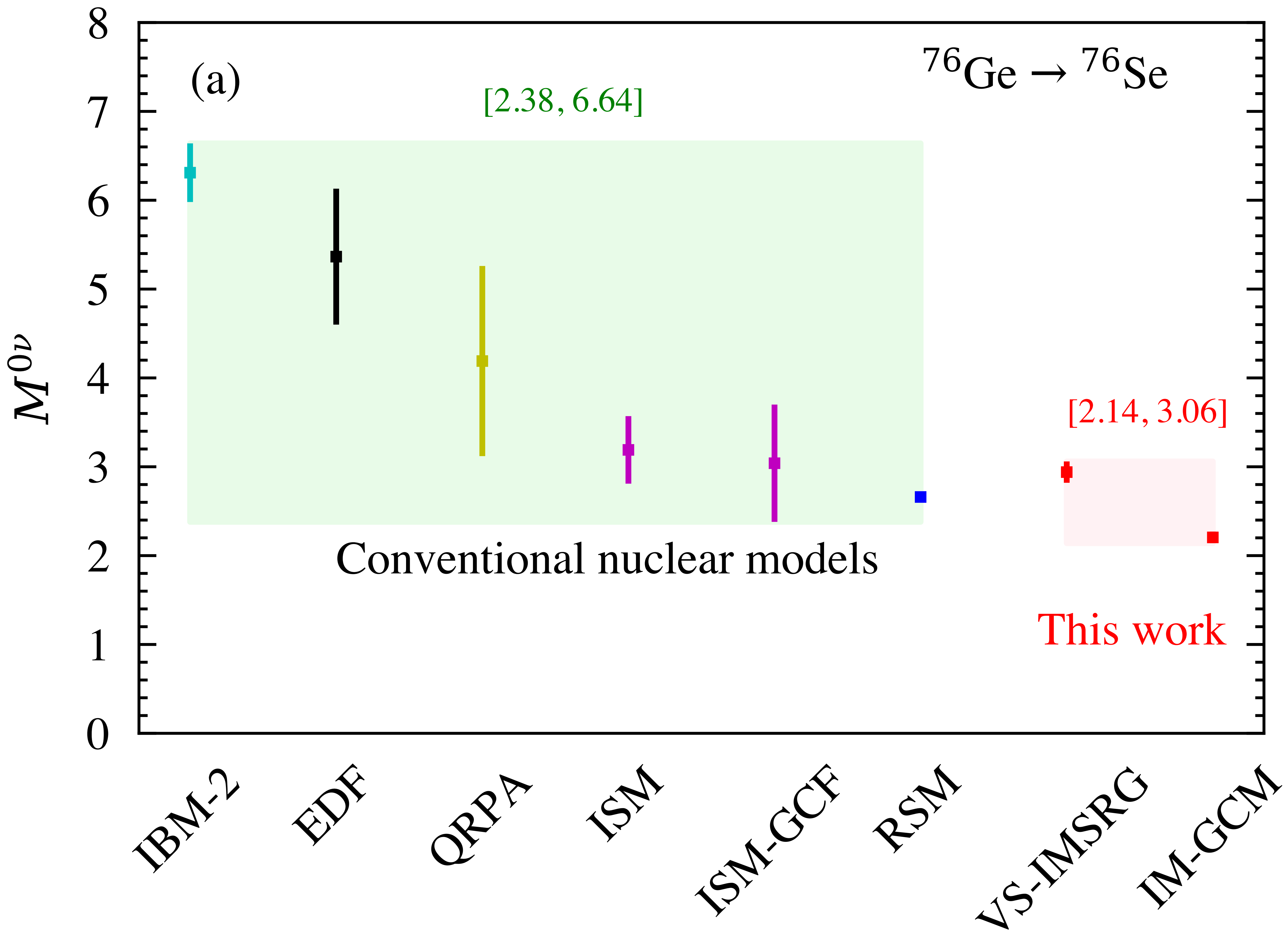


EFT Truncation error

$$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \epsilon_{operator}$$

Error due to the truncation of the many-body method. This is studied by comparing the results of the IM-GCM and VS-IMSRG using the magic interaction.

This error is surprisingly large as we find $\epsilon_{many-body}$ = 0.88.



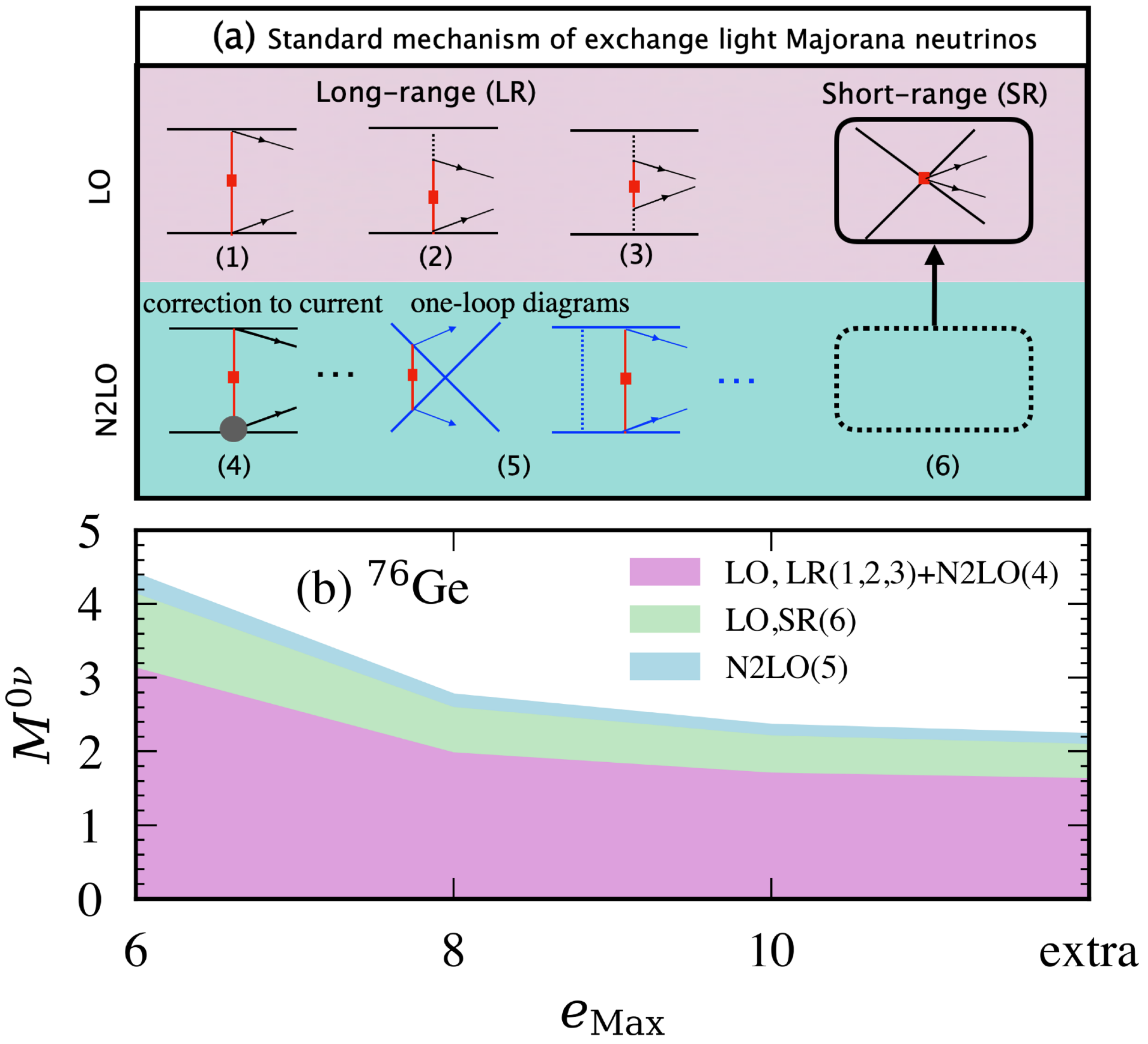
Operator error

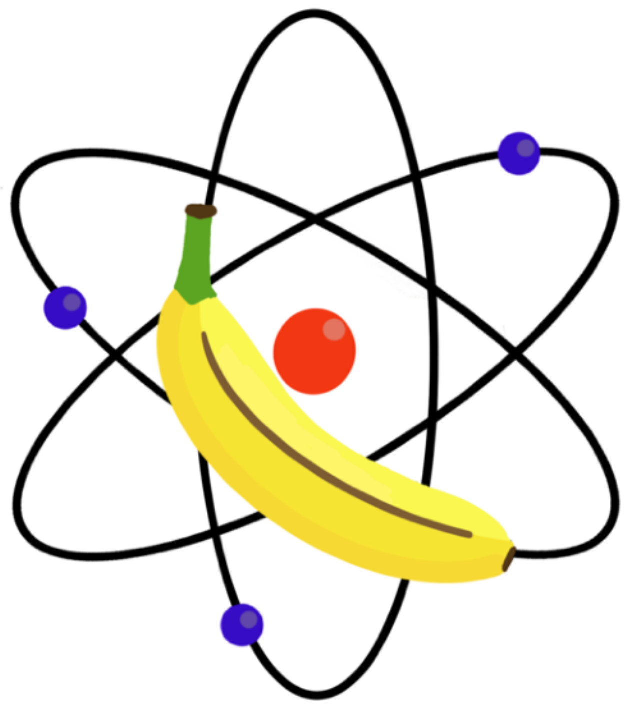
$y = y_{MM-DGP} + \epsilon_{emulator} + \epsilon_{EFT} + \epsilon_{many-body} + \boxed{\epsilon_{operator}}$

Error due to the truncation of the operator in chiral expansion + closure energy correction + value of the contact LEC.

Adding N2LO operators has very small contribution (< 0.2). Biggest contribution comes from determination of contact term.

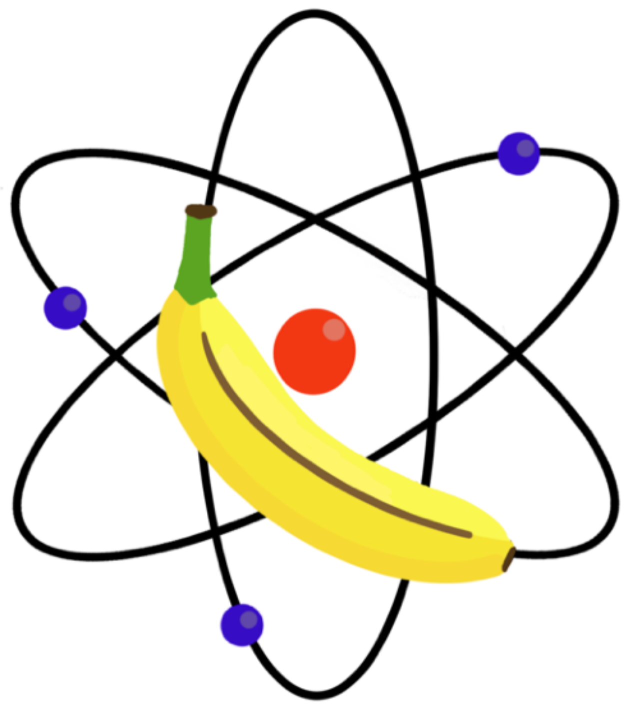
Total error amounts to $\epsilon_{operator} = 0.47$.





Attention!

- **Attention Mechanisms learns how the embeddings need to be adapted due to other inputs**
- **Responsible to for the improvements of large language models in recent years!**



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Attention Is All You Need

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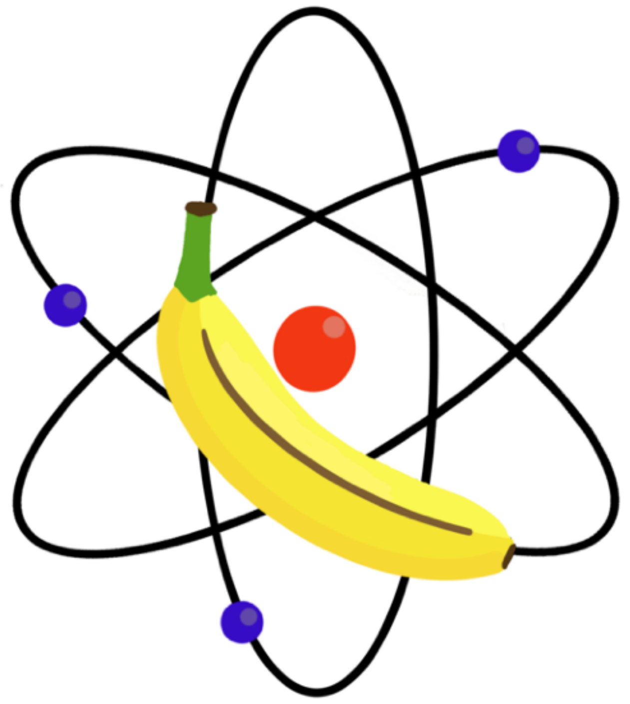
181, 436 citations



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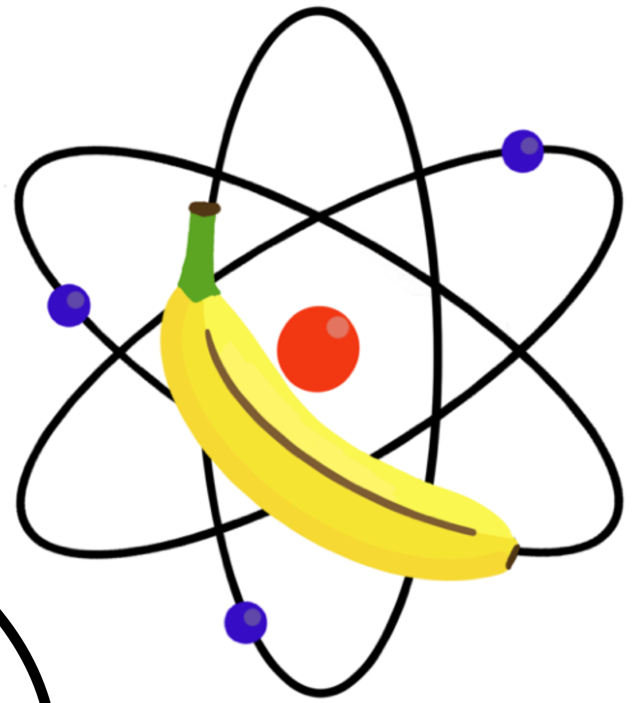
Illia Polosukhin* ‡ illia.polosukhin@gmail.com

181, 436 citations

Highly accurate protein structure prediction with AlphaFold

John Jumper, Richard Evans, Alexander Pritzel, Tim Green, Michael Figurnov, Olaf Ronneberger, Kathryn Tunyasuvunakool, Russ Bates, Augustin Žídek, Anna Potapenko, Alex Bridgland, Clemens Meyer, Simon A. A. Kohl, Andrew J. Ballard, Andrew Cowie, Bernardino Romera-Paredes, Stanislav Nikolov, Rishub Jain, Jonas Adler, Trevor Back, Stig Petersen, David Reiman, Ellen Clancy, Michal Zielinski, ... Demis Hassabis + Show authors

35, 675 citations



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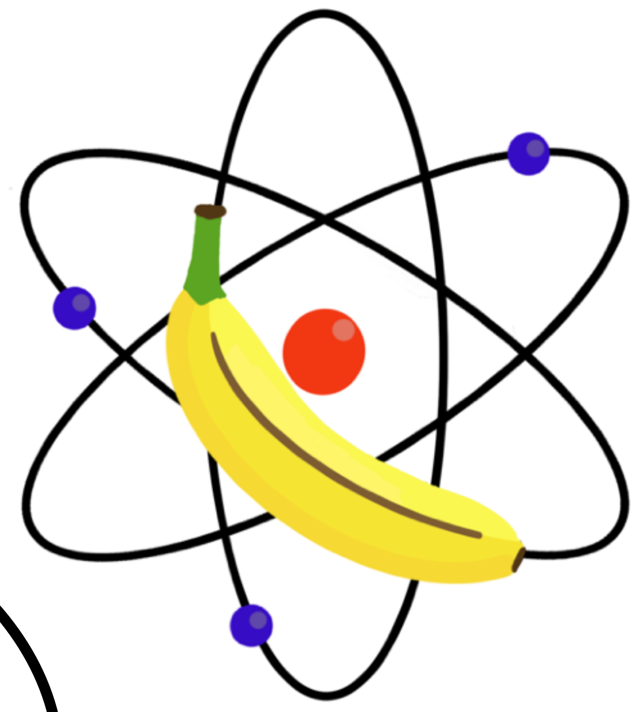

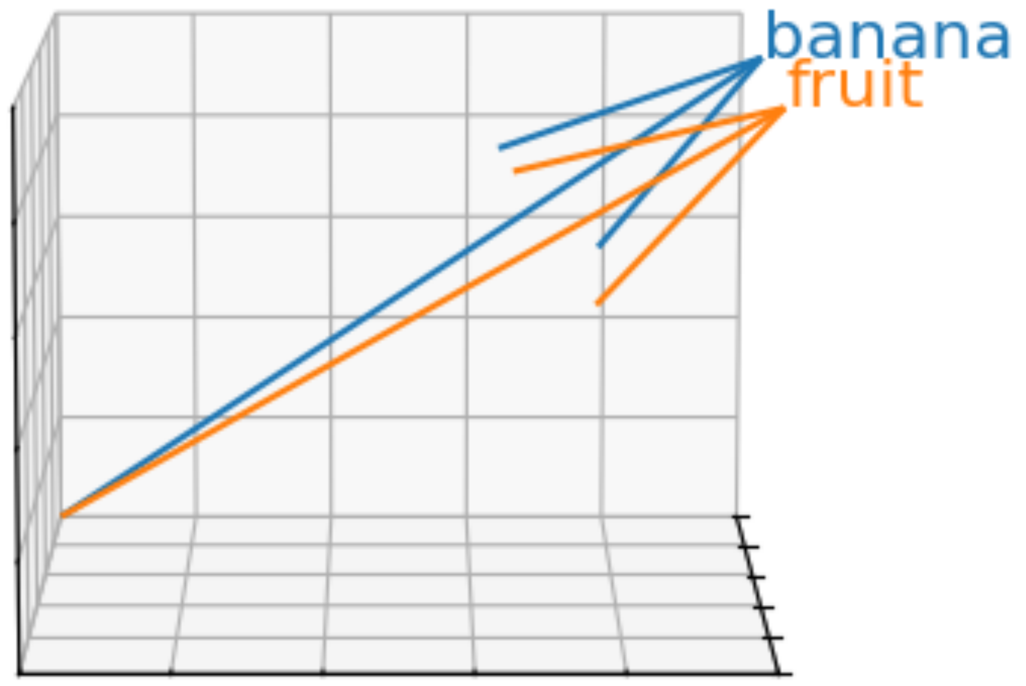

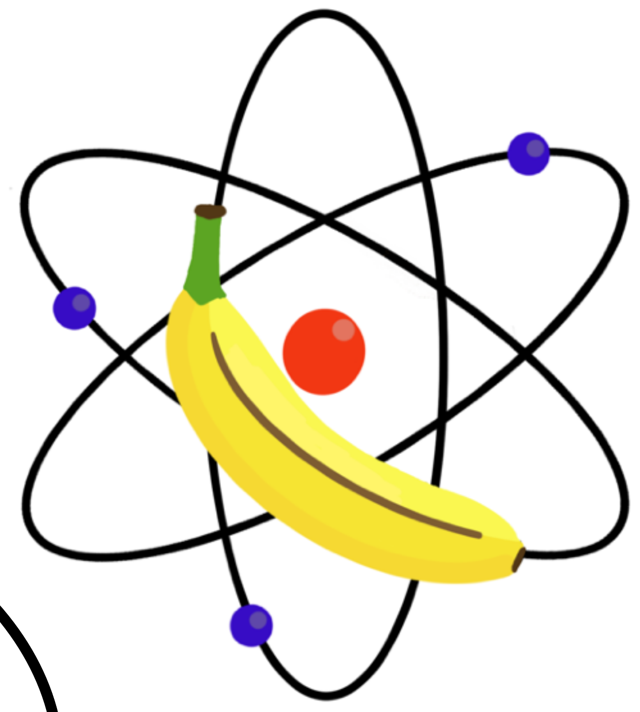

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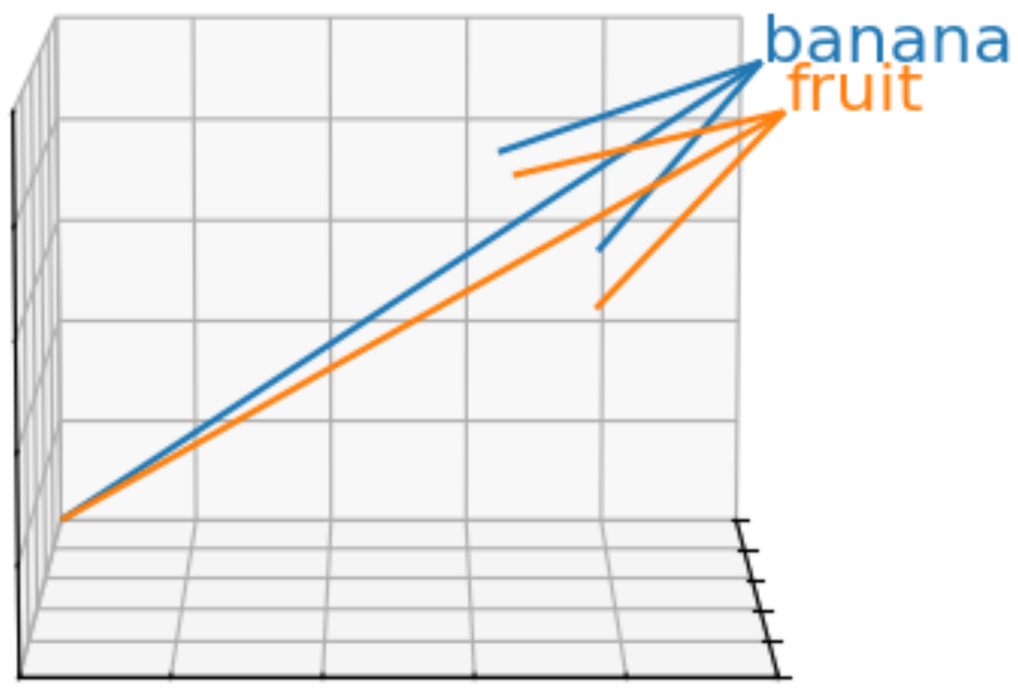
ChatGPT:



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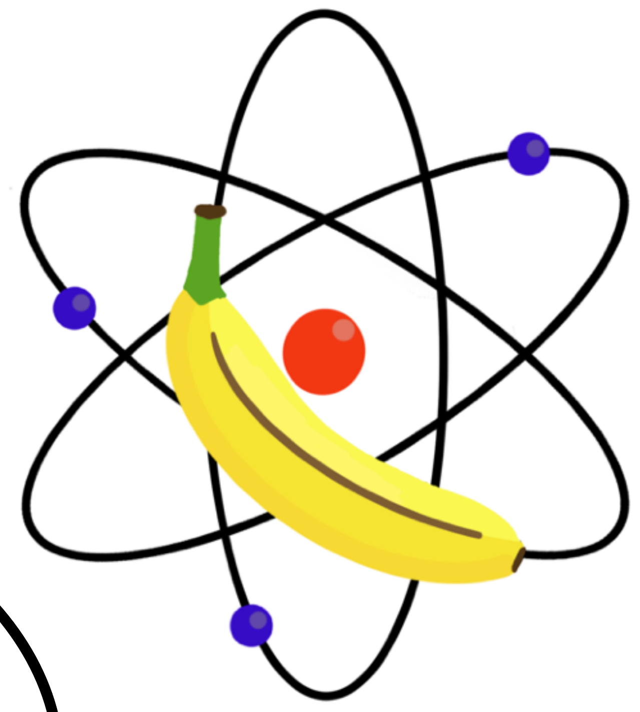

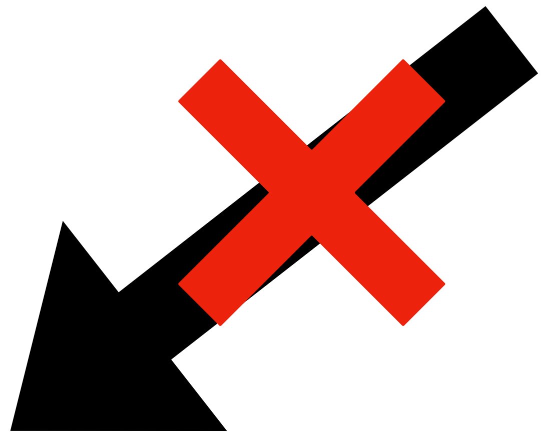

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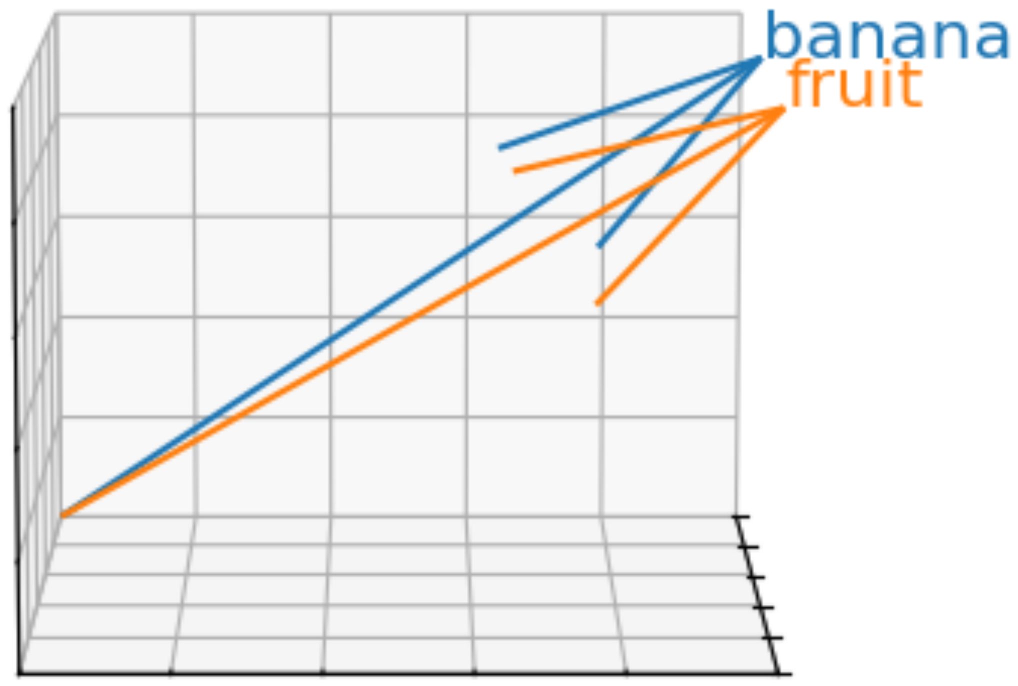

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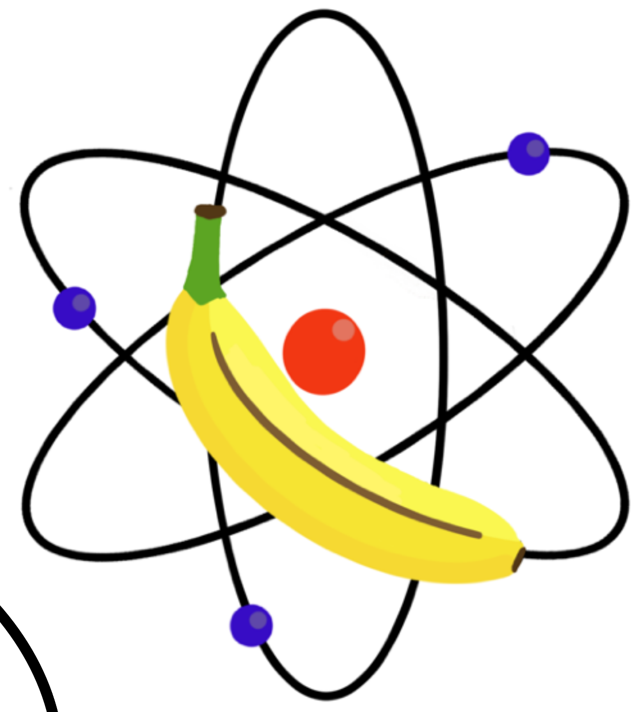

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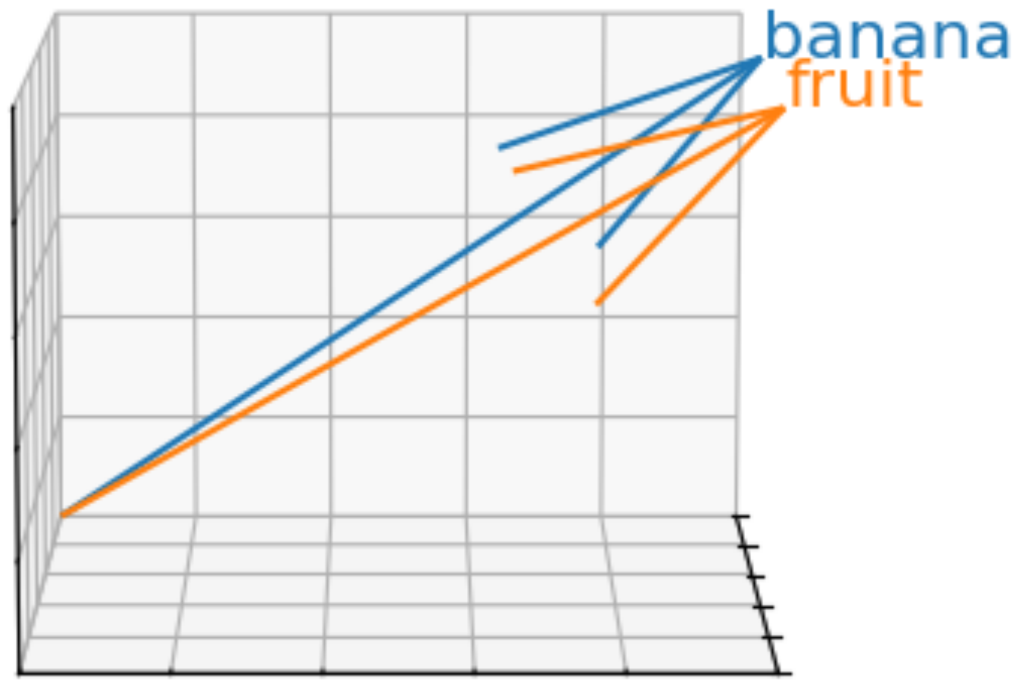

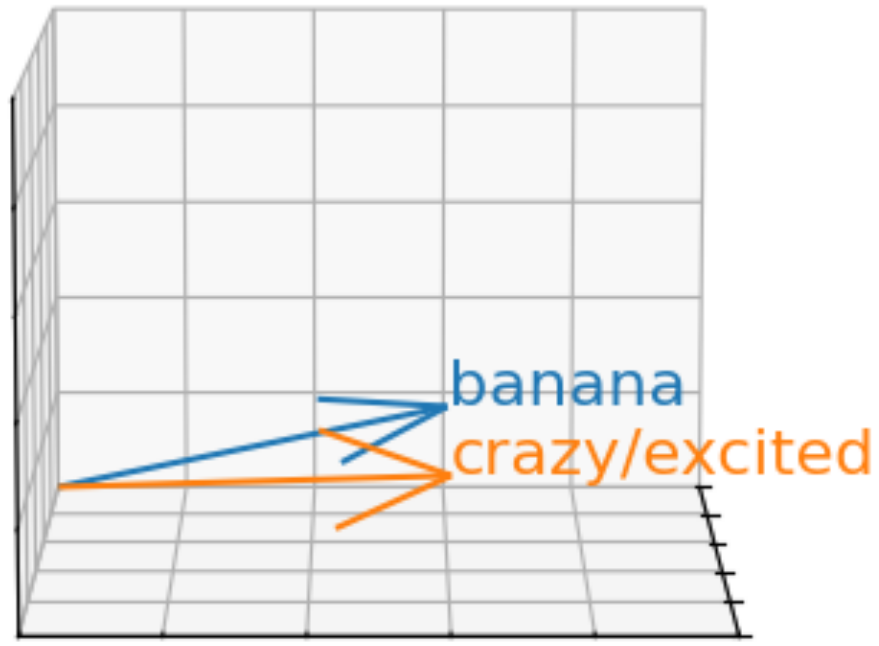

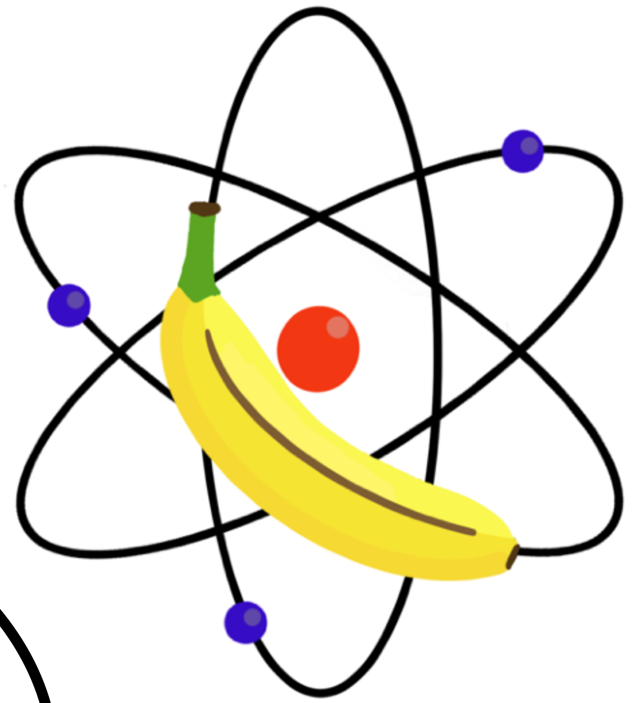

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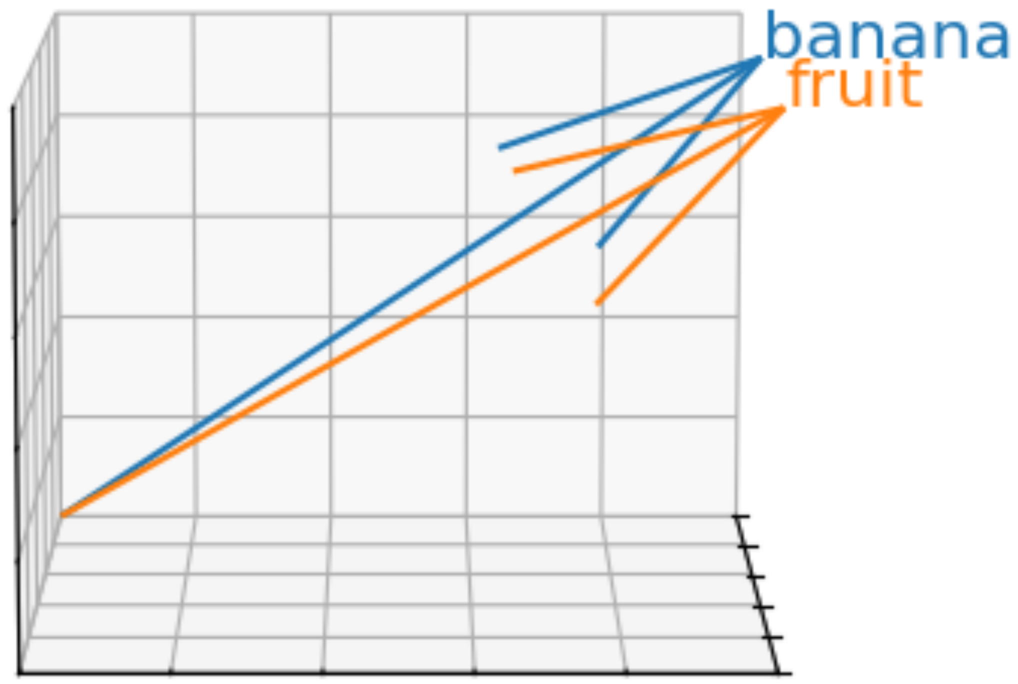

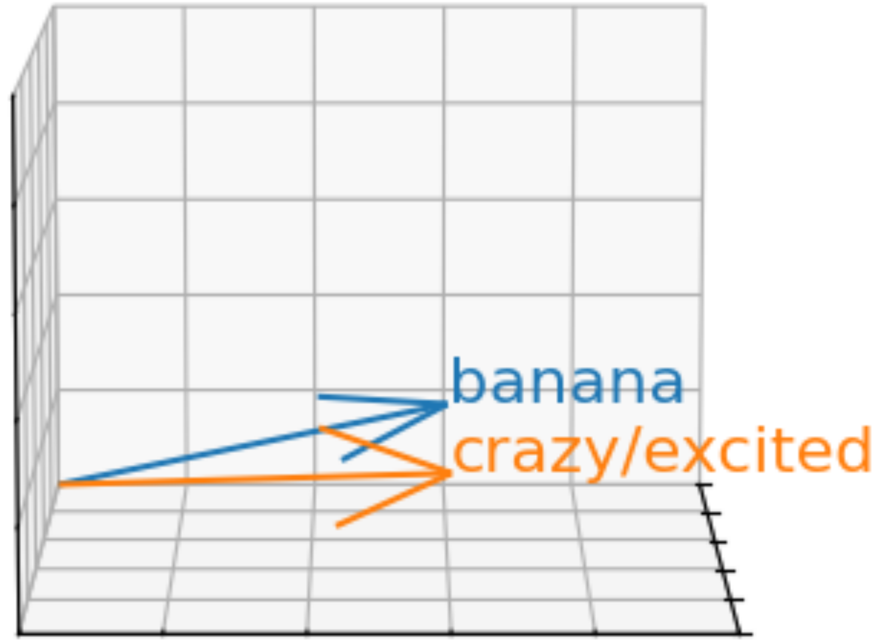

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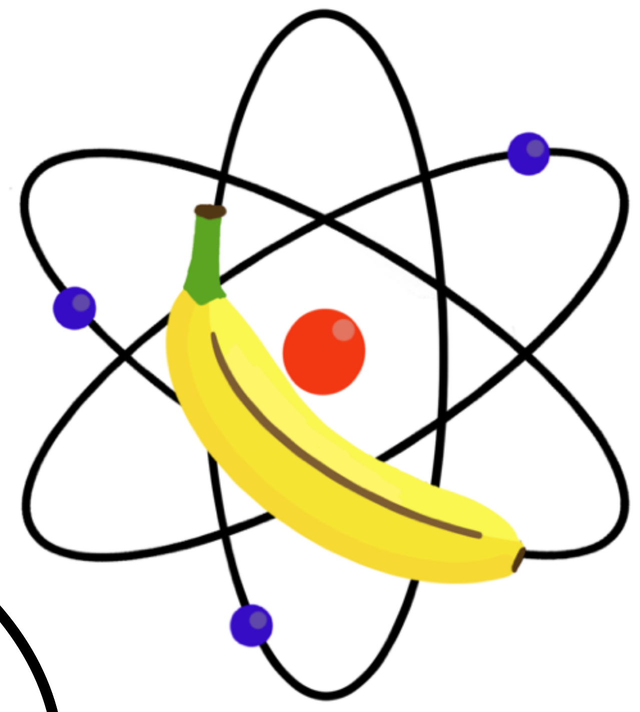

ChatGPT:

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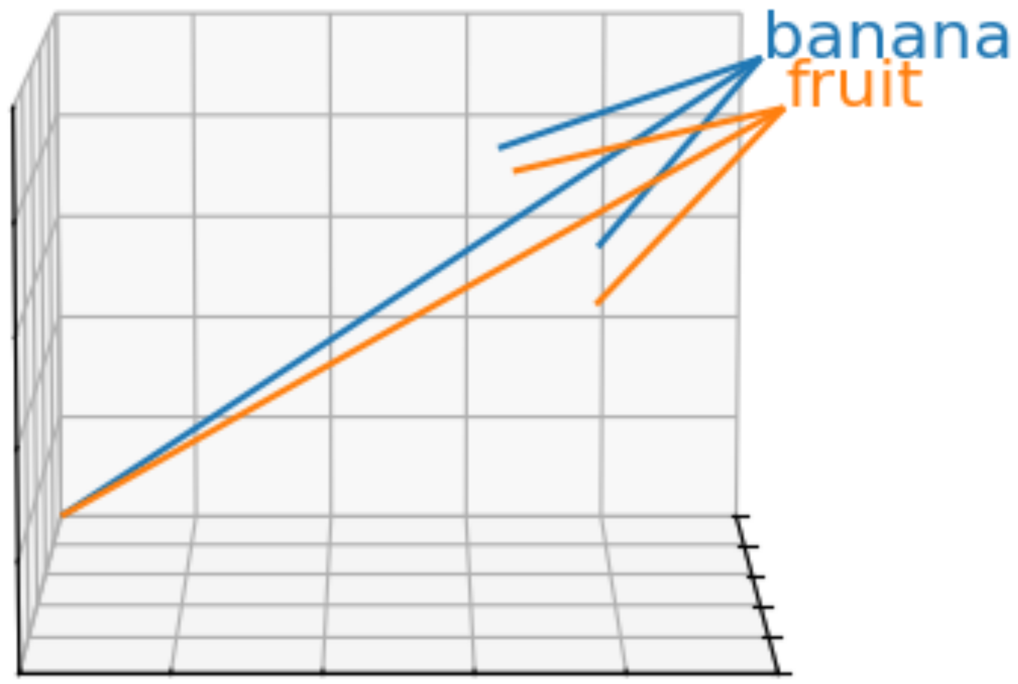
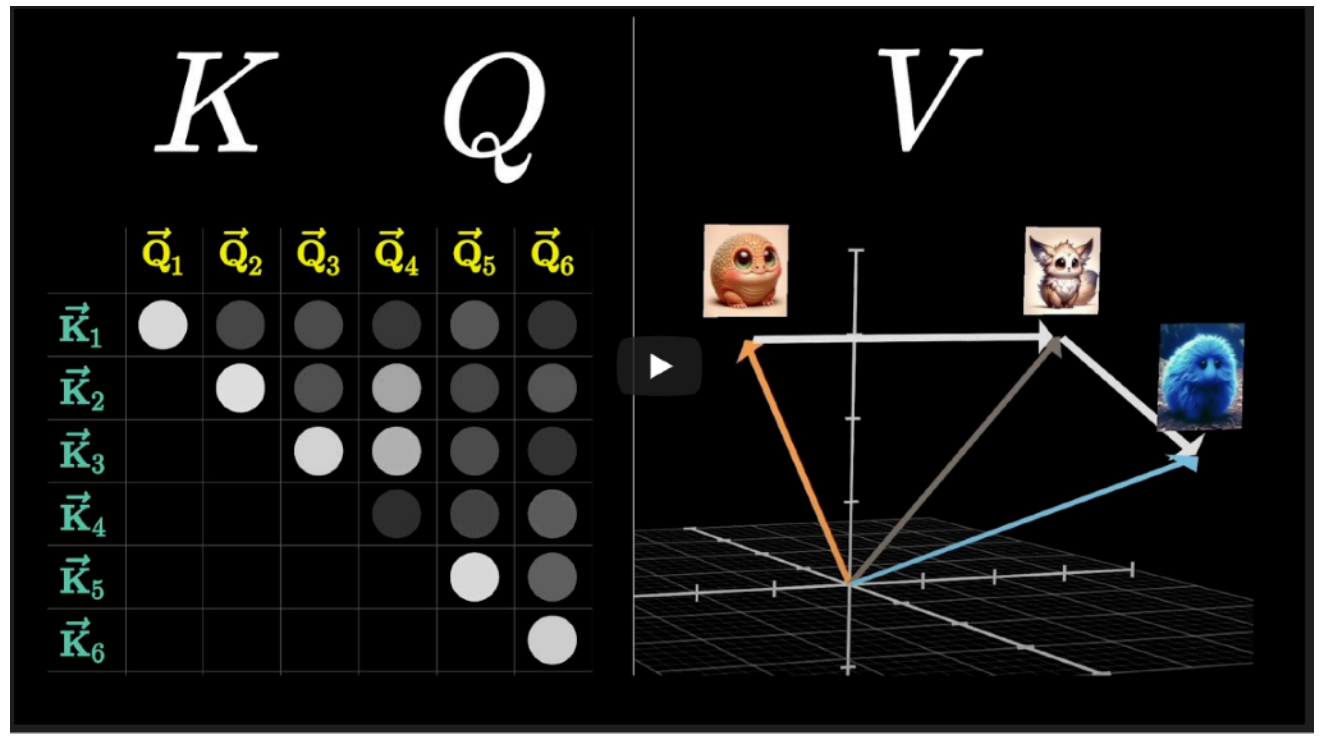
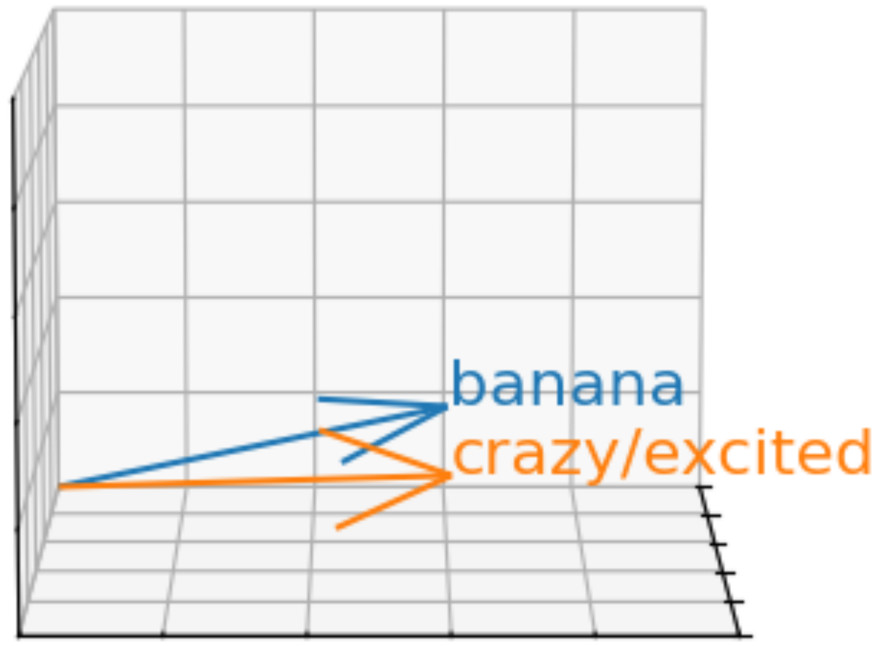
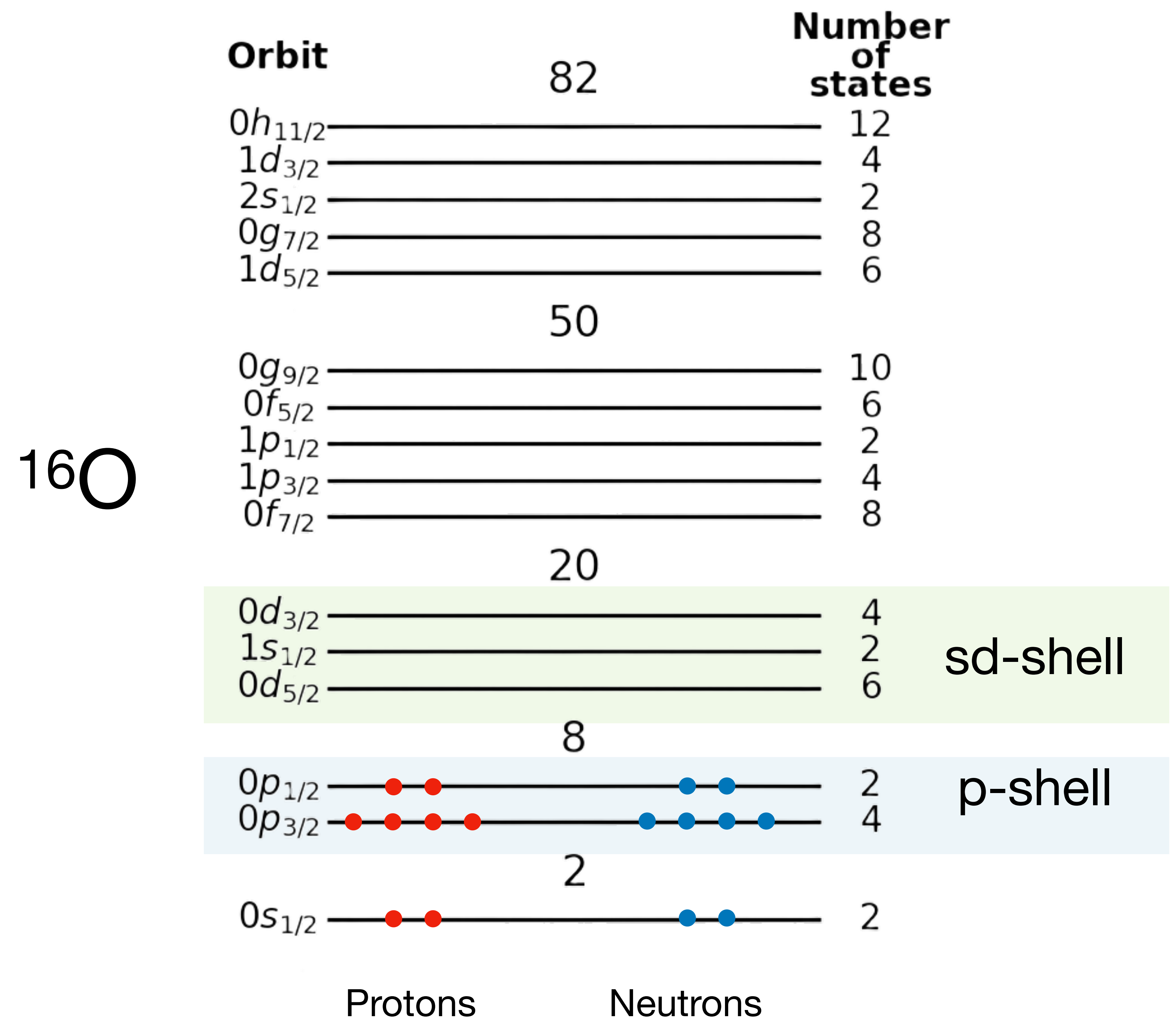


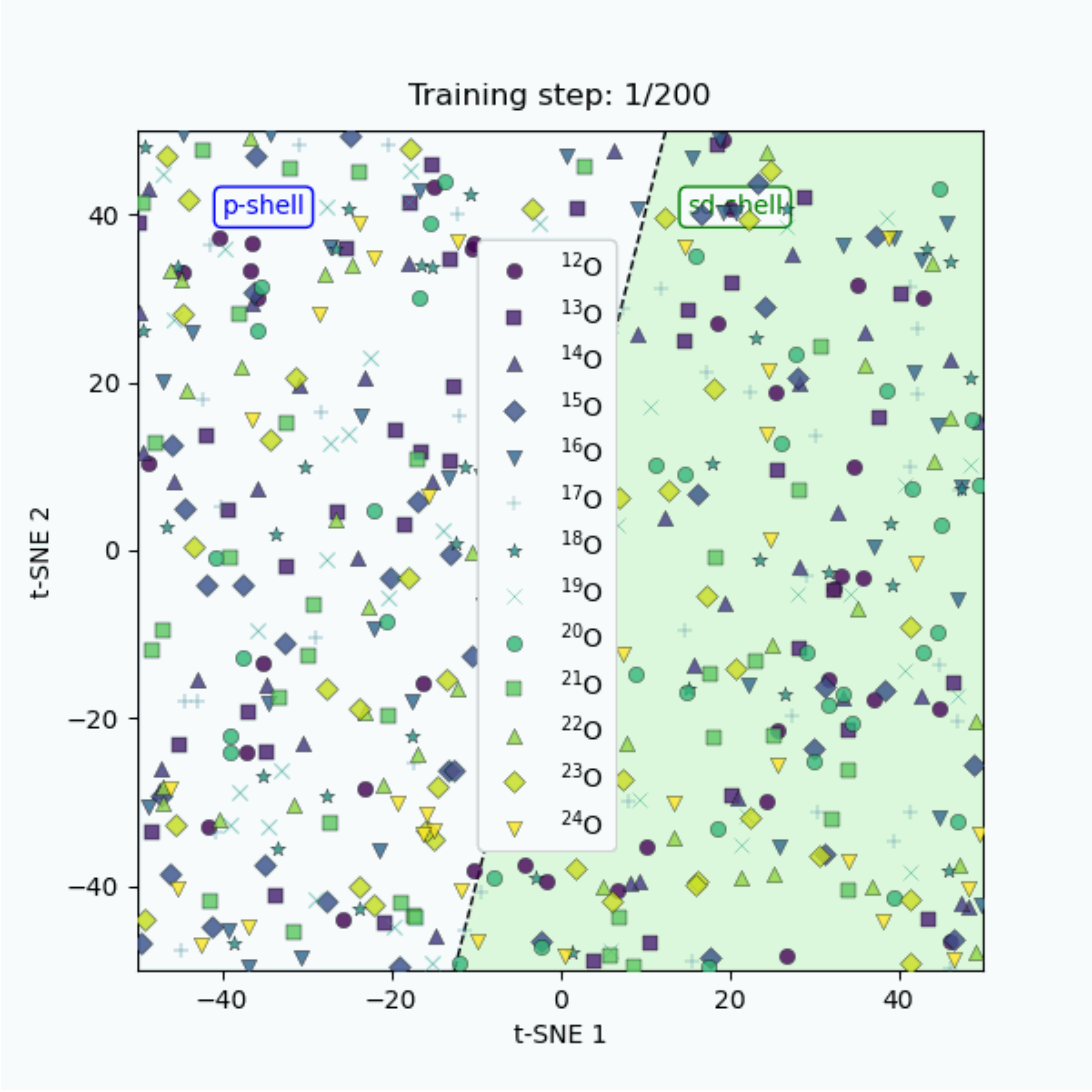




Visualizing the Embeddings

- Projection of embeddings from the attention mechanism.
- Model is learning nuclear shells!





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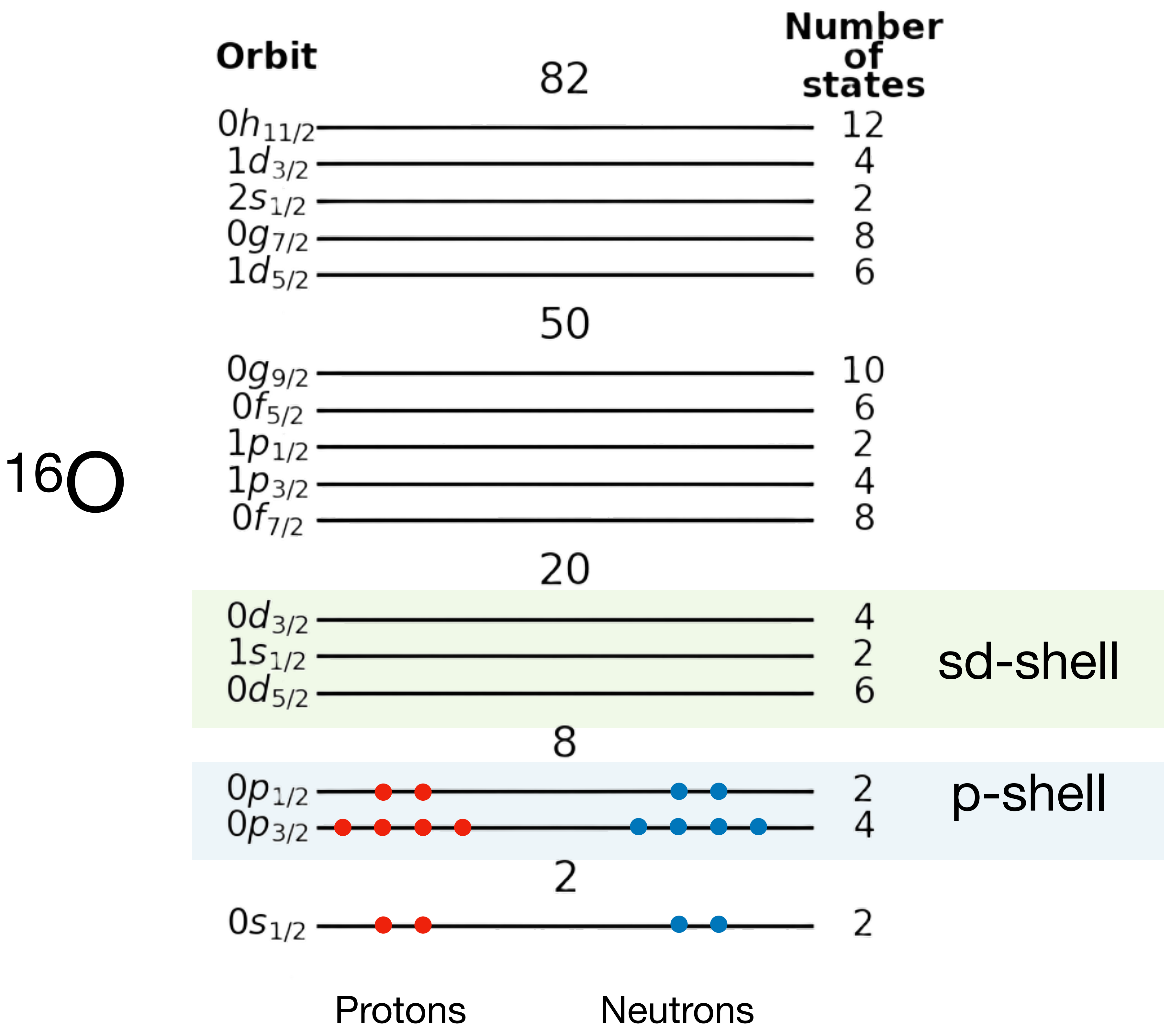
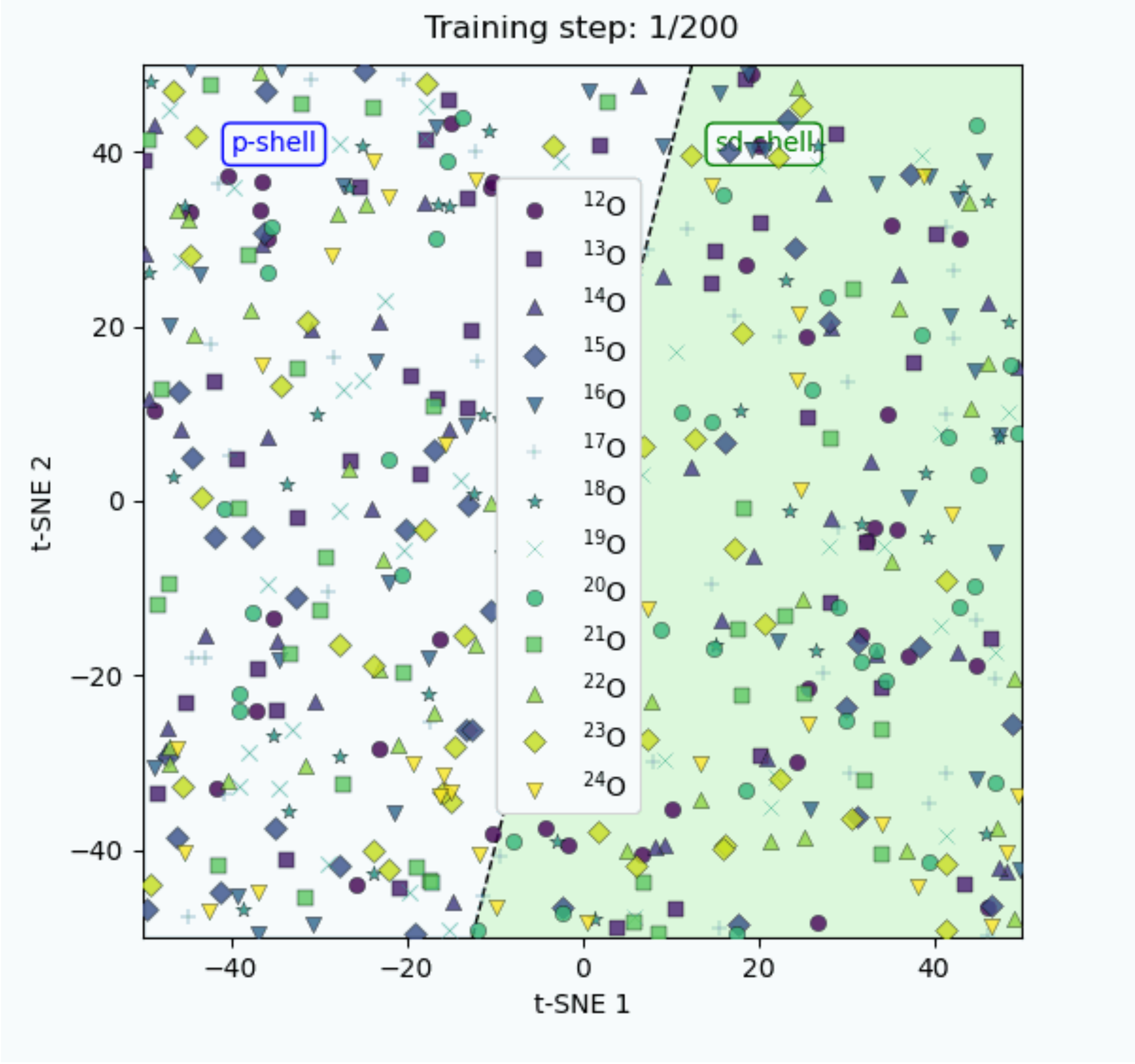
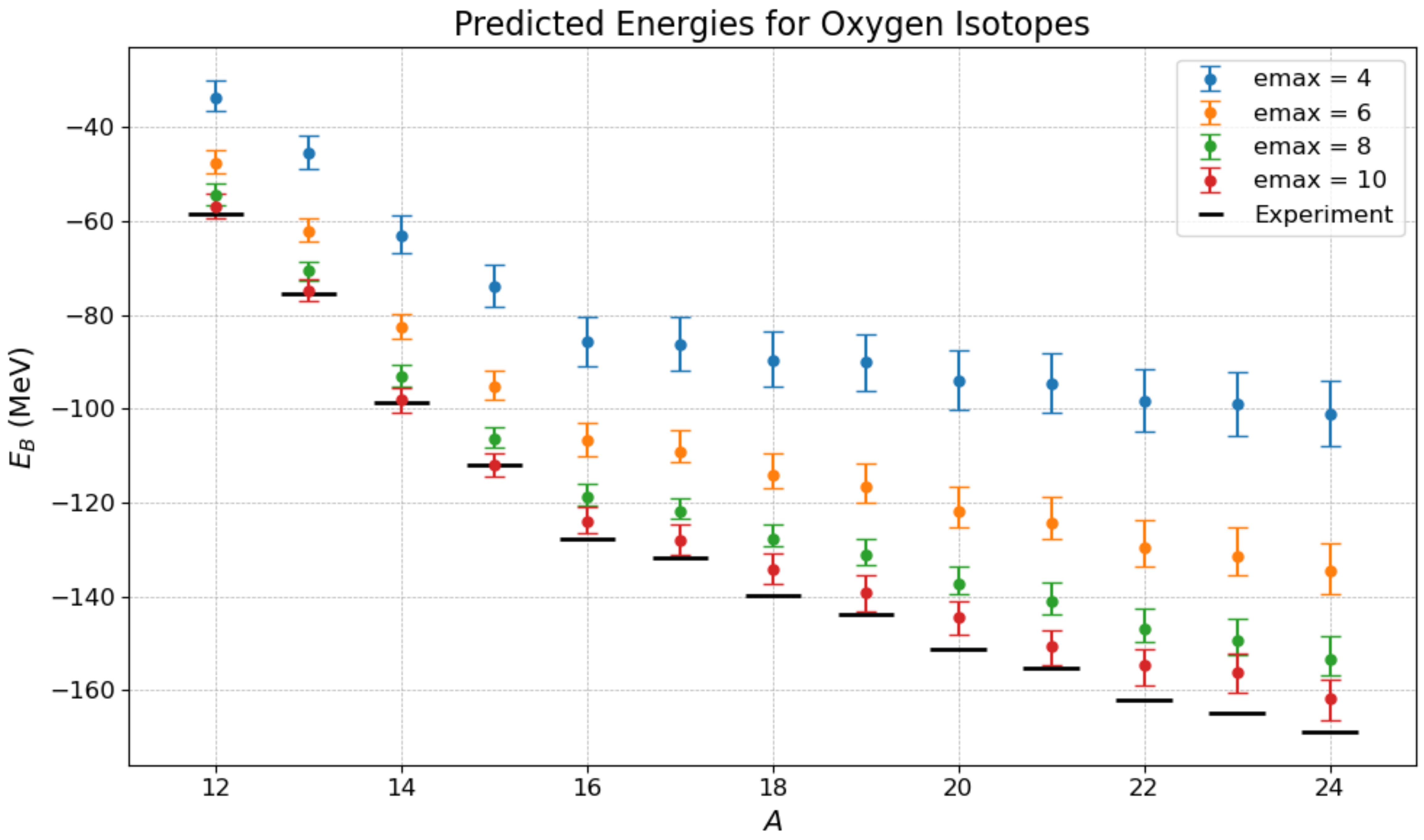
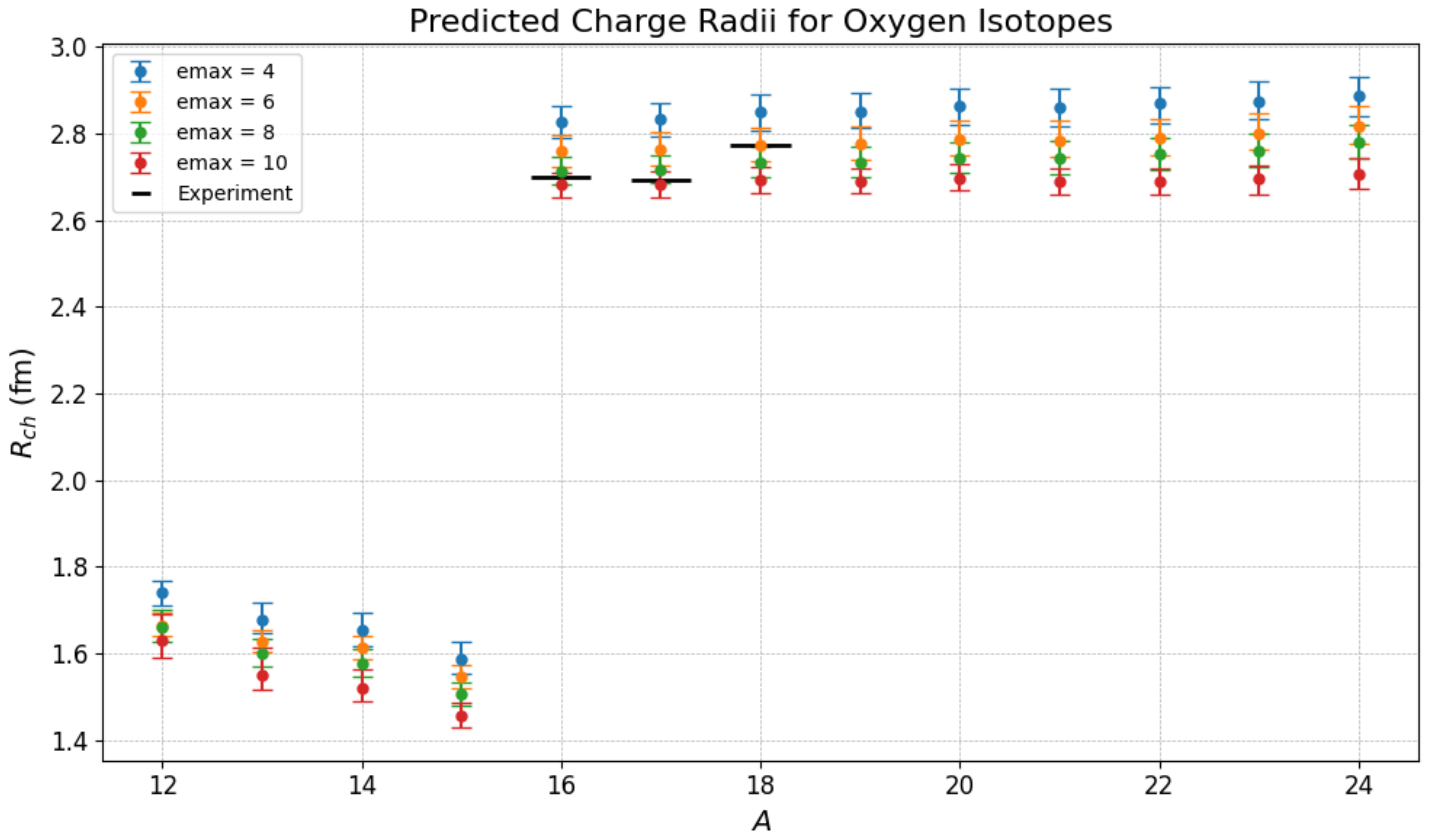




Emulating Multiple Isotopes

Belley, **Munoz,** Garcia., arxiv:2502.20363

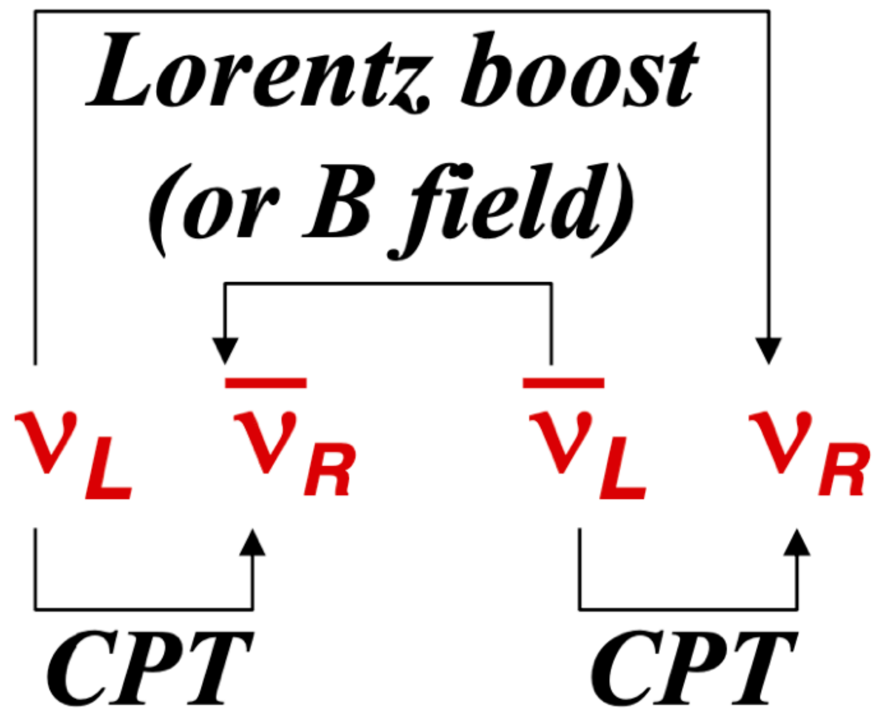




Combining this with previous UQ technique, we can predict observables with associated uncertainties over the full isotopic chains in a few minutes.

The Nature of the Neutrino Puzzle

The classic picture: The Dirac neutrino



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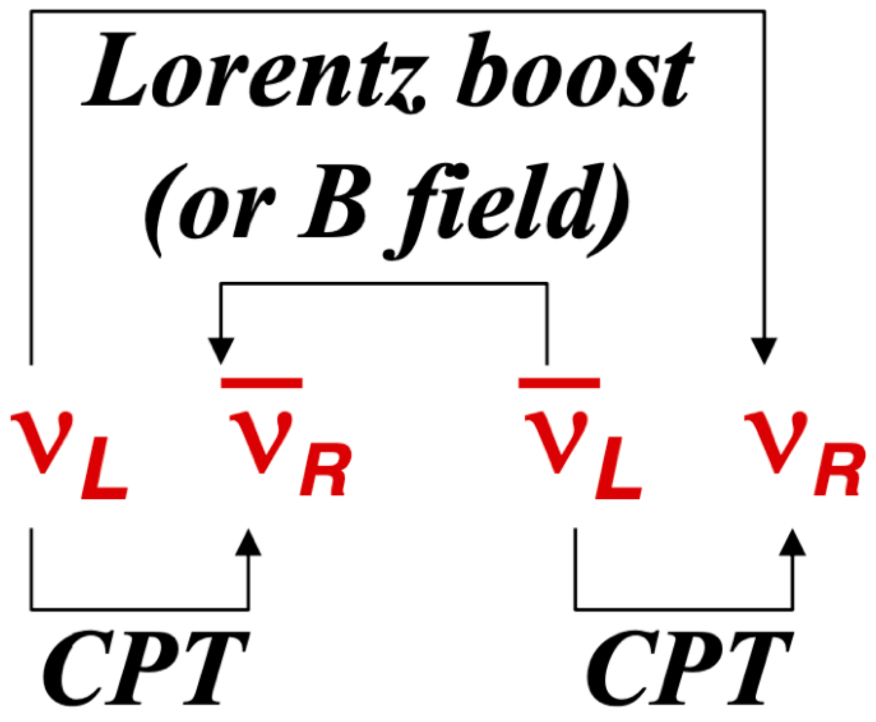

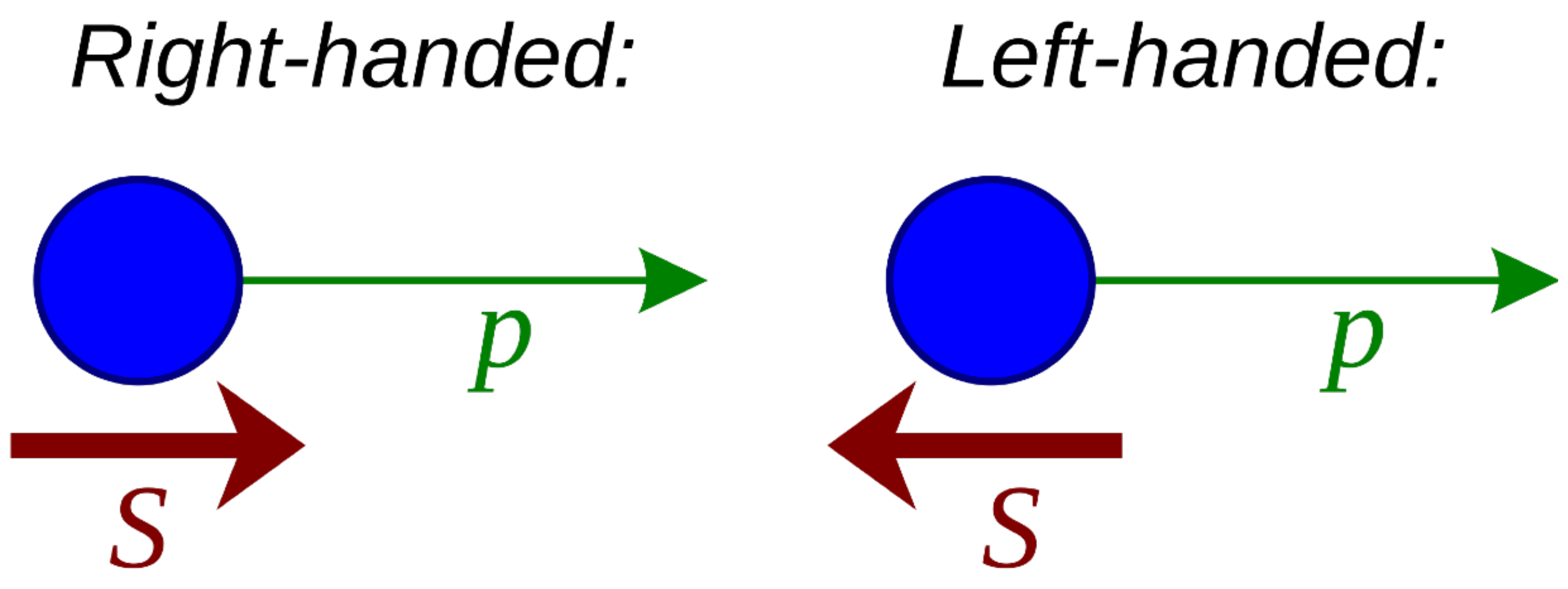

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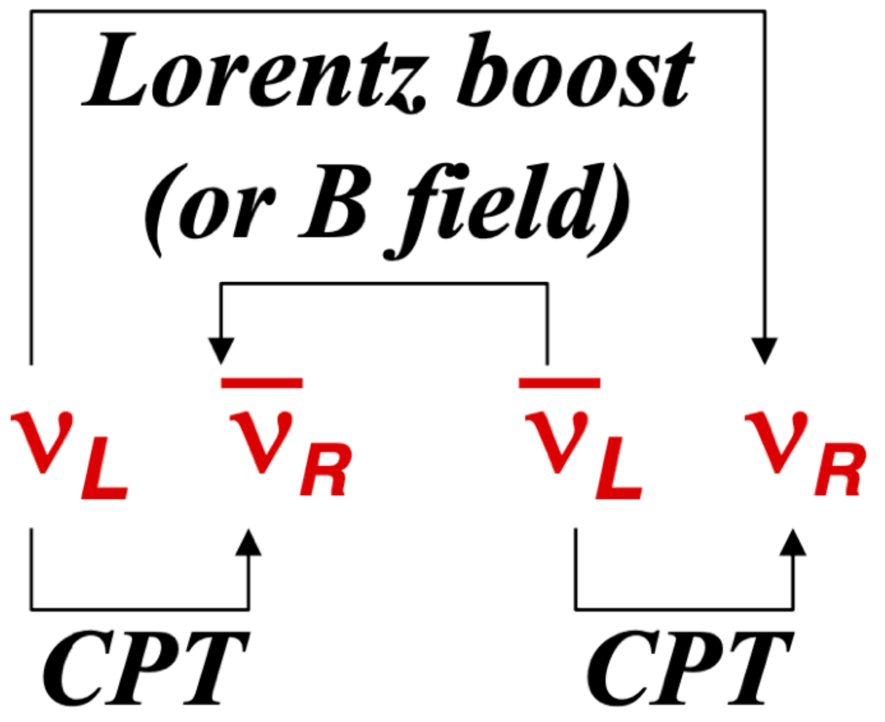
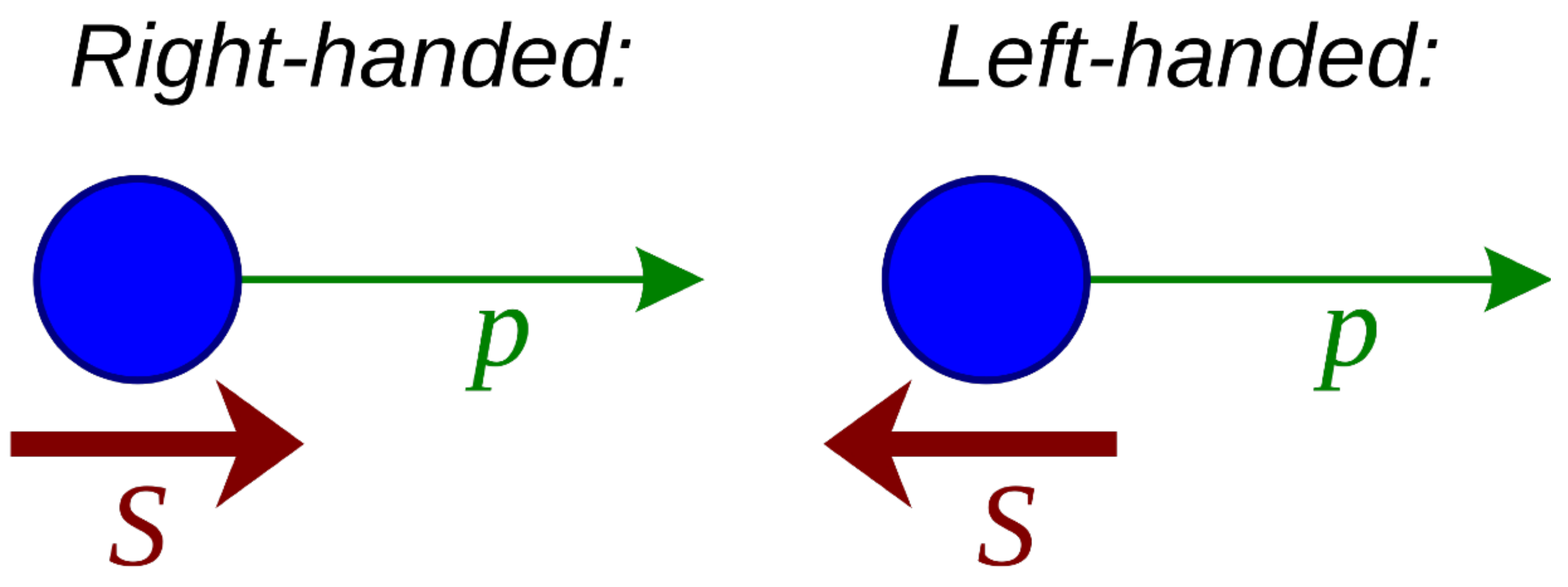
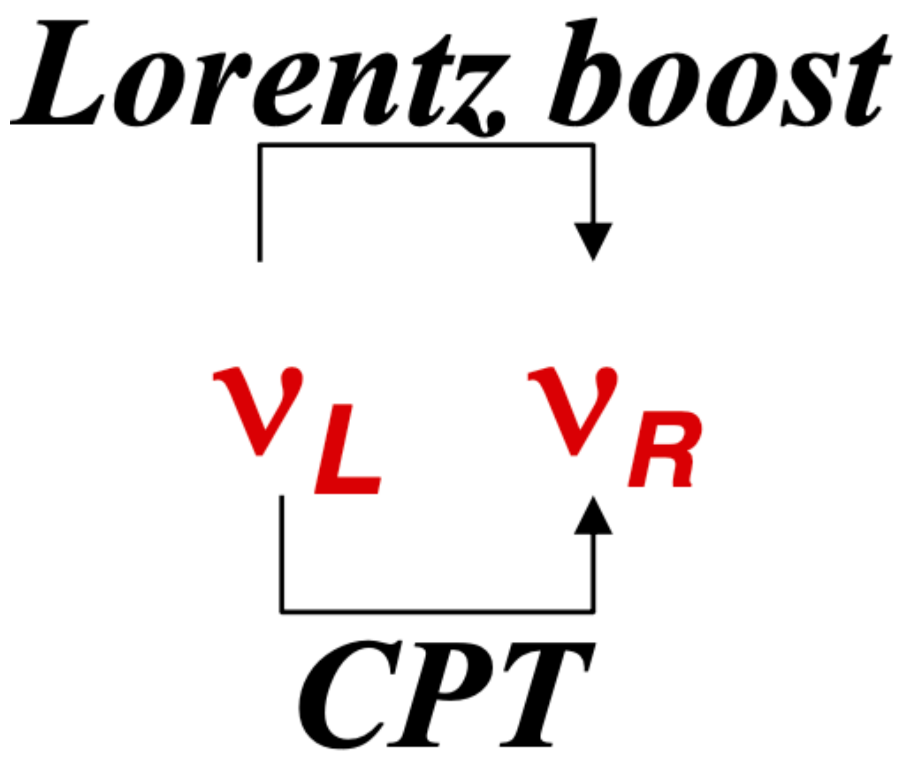




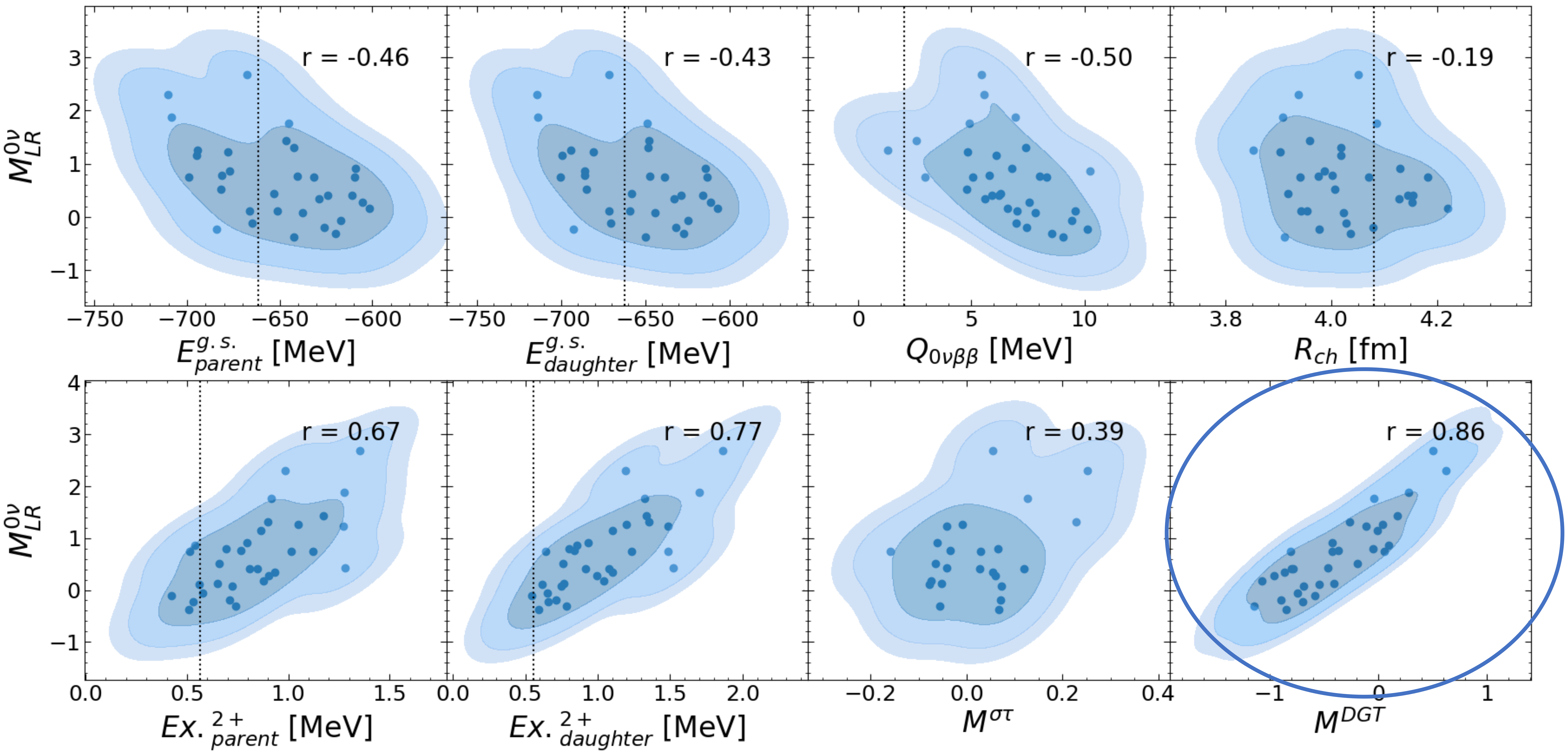
The Majorana neutrino



Correlations Between Observables

In ^{76}Ge:

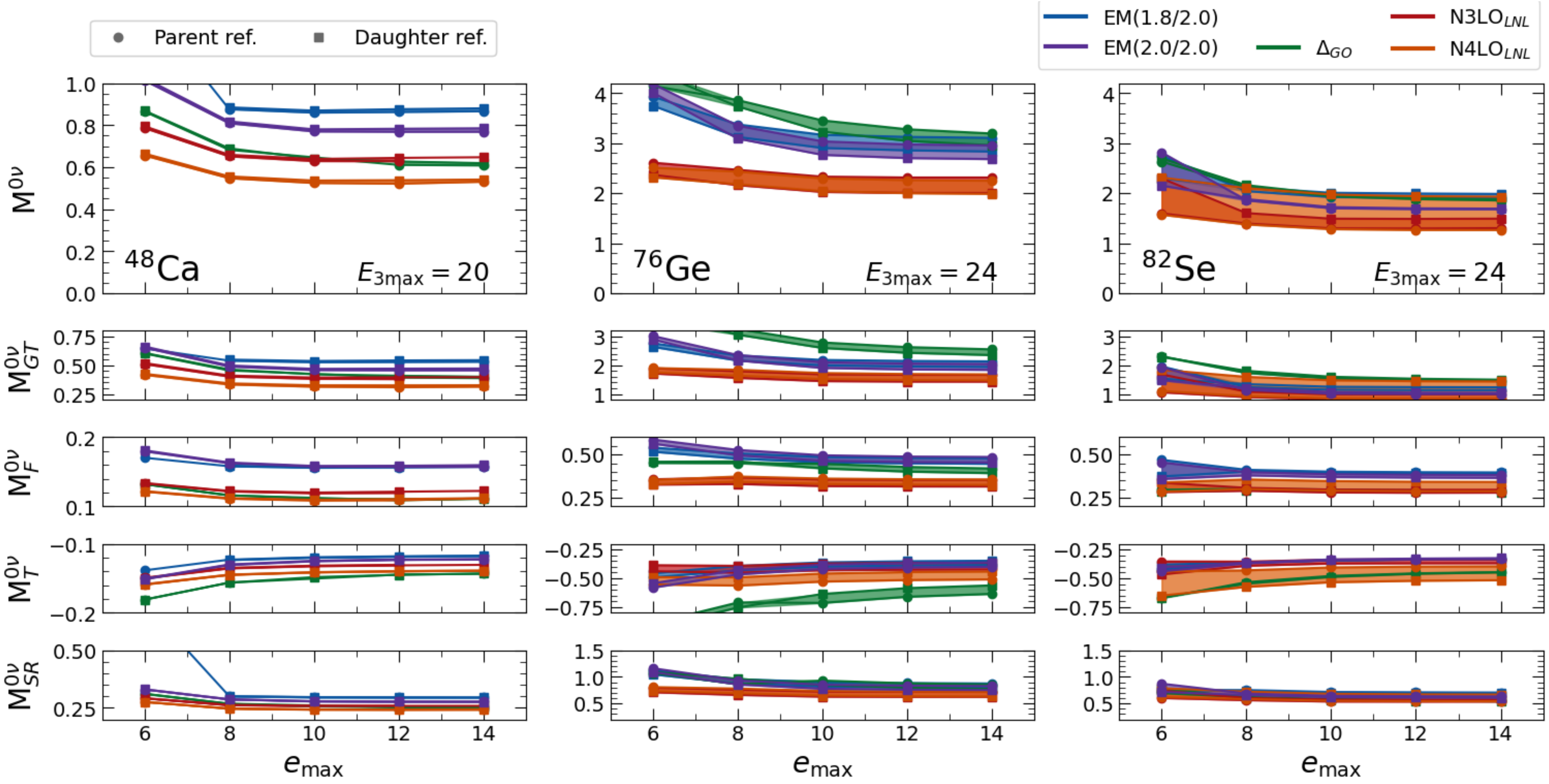
Belley et al., arXiv:2210.05809



Only correlation seen in multiple nuclei is with the unobserved double Gamow-Teller transition NME.

Ab Initio 0νββ Decay: ^{48}Ca, ^{76}Ge and ^{82}Se

Results with 5 different input Hamiltonians to study uncertainty from interaction choice.



Belley, et al., PRL126.042502