

Muon-electron conversion from Lorentz- and CPT-violating operators

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Work in collaboration with V. A. Kostelecký, E. Passemar, and N. Sherrill Based on arXiv:2501.05986 (in review)

Why Violate Symmetries?

- Standard Model works well, but we know it's incomplete
- SM is based on symmetries, but symmetries have a habit of being broken (e.g., electroweak, chiral, CP)
 - Even small violations can have important consequences
 - Good place to look for new physics



Modified from Boyle (2014)



Lepton Flavor Violation

- Lepton flavor conservation is an accidental symmetry of SM
 - $U(1)_{L_e} \times U(1)_{L_{\mu}} \times U(1)_{L_{\tau}}$
- Lepton flavor conservation is violated by neutrino oscillations
 - Massive neutrinos give rise to PMNS mixing (analogous to CKM for quarks)



de Salas et al. (2018)



Charged Lepton Flavor Violation

- If LFV exists for neutrinos, then charged lepton flavor violation must also exist
- Process is highly suppressed in SM + neutrino oscillations: BR $\lesssim 10^{-54}$
 - Loop process
 - Smallness of neutrino masses
 - GIM mechanism





Experimental Progress in CLFV







Nuclear $\mu - e$ Conversion Searches

Basic idea: Collide muon beam with atomic target; some muonic atoms will be formed

• Primary background is decay $\mu \rightarrow e v_{\mu} \overline{v_e}$

- Three-body decay spectrum
- *Hope* to observe clean, monoenergetic signal from rare $\mu \rightarrow e$ decay
 - $E_e^{conv} = m_\mu E_{bind} E_{recoil}$
 - ≈ 105 MeV for Al, ≈ 95.6 MeV for Au







Nuclear $\mu - e$ Conversion Searches

Observable of interest is

$$R_{\mu e} = \frac{\omega_{conv}}{\omega_{capt}} = \frac{\Gamma\left[\mu + \frac{A}{Z}N \to e + \frac{A}{Z}N\right]}{\Gamma\left[\mu + \frac{A}{Z}N \to \nu_{\mu} + \frac{A}{Z-1}N\right]}$$

- N.b.: We focus on coherent conversion; incoherent also possible (Haxton *et al.* 2024)
- Current best limit is from SINDRUM II (2006): $R_{\mu e} < 7 \ge 10^{-13}$ on ${}^{197}_{79}$ Au (90% C.L.)
- Near-future improvement expected by COMET Phase I $(R_{\mu e} < 7 \ge 10^{-15})$ and Mu2e Run I $(R_{\mu e} < 6.2 \ge 10^{-16})$, both on $^{27}_{13}$ Al



van der Schaaf (2003)



Lorentz Violation

- Lorentz symmetry is a *fundamental* symmetry of the SM
- Lorentz invariance is a statement of isotropy in spacetime (symmetry under rotations and boosts)
- Small violations of Lorentz invariance may exist and would be of enormous interest for moving beyond the SM
- Presence of such violations introduces meaningful distinction between observer and particle transformations

In a Lorentz-invariant theory...





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In a Lorentz-violating theory...





Standard Model Extension

- *Very* general effective field theory starting from SM+GR and built from known particles
- Allows for generic Lorentz- and CPT-violating interactions
 - CPTV \Rightarrow LV (Greenberg 2002), so CPTV automatically included
- LV can occur spontaneously or explicitly
- No assumptions about flavor structure, so possible CLFV interactions are also included
- Still requires *coordinate-independent* physics, so Lagrangian is observer Lorentz and general-coordinate scalar



Nuclear $\mu - e$ Conversion in the SME

- Most relevant for nuclear μ e conversion are EM (mass dim. 5) and 4-point quark-lepton (mass dim. 6) operators
- Lagrangian ($Q \in \{u, d, s\}, q \in \{u, d\}$; red = CPT-odd) is given by

$$\mathcal{L} \supset -\frac{1}{2} F_{\alpha\beta} \overline{\psi_e} \left(\left(m_F^{(5)} \right)_{e\mu}^{\alpha\beta} + i \left(m_{5F}^{(5)} \right)_{e\mu}^{\alpha\beta} \gamma_5 + \left(a_F^{(5)} \right)_{e\mu}^{\mu\alpha\beta} \gamma_{\mu} + \left(b_F^{(5)} \right)_{e\mu}^{\mu\alpha\beta} \gamma_5 \gamma_{\mu} + \frac{1}{2} \left(H_F^{(5)} \right)_{e\mu}^{\mu\nu\alpha\beta} \sigma_{\mu\nu} \right) \psi_{\mu}$$

$$+ \overline{\psi_Q} \psi_Q \overline{\psi_e} \left(\left(k_{SV}^{(6)} \right)_{QQe\mu}^{\lambda} \gamma_{\lambda} + \left(k_{SA}^{(6)} \right)_{QQe\mu}^{\lambda} \gamma_5 \gamma_{\lambda} + \frac{1}{2} \left(k_{ST}^{(6)} \right)_{QQe\mu}^{\kappa\lambda} \sigma_{\kappa\lambda} \right) \psi_{\mu}$$

$$+ \overline{\psi_q} \gamma_0 \psi_q \overline{\psi_e} \left(\left(k_{VS}^{(6)} \right)_{qqe\mu}^{0} + i \left(k_{VP}^{(6)} \right)_{qqe\mu}^{0} \gamma_5 + \frac{1}{2} \left(k_{VV}^{(6)} \right)_{qqe\mu}^{0\lambda} \gamma_{\lambda} + \left(k_{VA}^{(6)} \right)_{qqe\mu}^{0\lambda} \gamma_5 \gamma_{\lambda} + \frac{1}{2} \left(k_{VT}^{(6)} \right)_{qqe\mu}^{0\kappa\lambda} \sigma_{\kappa\lambda} \right) \psi_{\mu}$$



Leptonic Wavefunctions

• The bound muon (1s, $\kappa = -1$) and the plane-wave electron wavefunctions are found by numerically solving the Dirac equation

$$\frac{d}{dr} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} -\kappa/r & W - V(r) + m \\ -(W - V(r) - m) & \kappa/r \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

- *W* is the energy; $\kappa = \mp (J + 1/2)$; *V*(*r*) is the electric potential; $u_1(r)$ and $u_2(r)$ are related to the radial components of the wavefunction
- Spherically-symmetric models fitting experimental data exist for the nuclear charge distributions of various isotopes (e.g., de Vries *et al.* 1987)



Conversion Rates

Conversion rates involve the square of the transition amplitude

$$\omega_{\rm conv} = \frac{1}{2} \sum_{s=\pm\frac{1}{2}} \sum_{\kappa=\pm 1} \sum_{s'=\pm\frac{1}{2}} \left| \int d^3 x F_{\alpha\beta} \overline{\psi}^{(e)}_{\kappa,s'} \mathcal{O}^{\alpha\beta} \psi^{(\mu)}_s \right|^2$$

$$\omega_{\rm conv} = \frac{1}{2} \sum_{s,s',\kappa} \left| \int d^3x \left(\alpha \overline{\psi}_{\kappa,s'}^{(e)} \mathcal{K} \psi_s^{(\mu)} + \beta \overline{\psi}_{\kappa,s'}^{(e)} \mathcal{K}_0 \psi_s^{(\mu)} \right) \right|^2$$

The nuclear matrix elements $\alpha = \langle N | \overline{q} q | N \rangle$ and $\beta = \langle N | \overline{q} \gamma_0 q | N \rangle$ contributing to coherent conversion have already been computed (e.g., Kosmas *et al.* 2001)



Consequences of Lorentz Violation

- Lorentz violation means location, time, and orientation of the experiment matter
 - Leading effect is rotation of Earth with respect to fixed SME background coefficients
- Conversion rates easiest to calculate in lab frame in spherical coordinates
- Bounds have to be translated to a standard reference frame to be meaningful





Sun-Centered Frame

- Standard reference frame for LV studies
 - *Z*-axis towards celestial north
 - *X*-axis from Earth to Sun at 2000 vernal equinox
- Rotation from SCF to lab-frame coefficients is time-dependent and relies on laboratory location and apparatus orientation





Obtaining Bounds

- Bounds are taken one-at-a-time on the various components of each relevant SME coefficient in the sun-centered frame
- The conversion frequency can be decomposed into a time-independent component and time-dependent harmonics in the sidereal frequency of the Earth
 - Our results are time-averaged, since we use only the time-integrated experimental limit on the rate
- Sidereal analysis of time-stamped data would provide greater sensitivity to some components of the coefficients
 - Sidereal oscillations in data could provide "smoking gun" signal of LV



Constraints and Projections

Coefficient Type	SINDRUM II Constraints (90% C.L.)	COMET Phase I (projected)	Mu2e Run I (projected)
Electromagnetic	$< 6 - 8 \ge 10^{-12} \text{ GeV}^{-1}$	$< 0.9 - 1 \ge 10^{-12} \text{ GeV}^{-1}$	$< 0.2 \text{ x} 10^{-12} \text{ GeV}^{-1}$
Quark-Lepton, $\overline{\psi_{u,d}}\psi_{u,d}$	$< 6 - 7 \ge 10^{-13} \text{GeV}^{-2}$	$< 1 \mathrm{x} 10^{-13} \mathrm{GeV^{-2}}$	$< 0.2 \text{ x} 10^{-13} \text{ GeV}^{-2}$
Quark-Lepton, $\overline{\psi_s}\psi_s$	$< 10 - 15 \ge 10^{-13} \text{ GeV}^{-2}$	$< 2 \ge 10^{-13} \text{GeV}^{-2}$	< 0.4 x 10 ⁻¹³ GeV ⁻²
Quark-Lepton, $\overline{\psi_{u,d}}\gamma_0\psi_{u,d}$	$< 20 - 30 \ge 10^{-13} \text{ GeV}^{-2}$	$< 4 \ge 10^{-13} \text{GeV}^{-2}$	< 0.7 x 10 ⁻¹³ GeV ⁻²



Outlook

- Better constraints on EM operators by about an order of magnitude already obtained from MEG (Kostelecký, Passemar, Sherrill 2022)
- MEG II will keep $\mu \rightarrow e\gamma$ competitive for the EM operators in the short term; final results from COMET and Mu2e could surpass those bounds
- Quark-lepton operators constrained uniquely in this channel (first bounds); nuclear conversion complementary to other channels
- With their full sensitivities, COMET and Mu2e should improve bounds about two orders beyond SINDRUM II







Mu2e Collaboration (2023)







Current/Future $\mu \rightarrow e$ Conversion Limits

Channel	Current Limit: BR<	Future Sensitivity: BR<
$\mu ightarrow e \gamma$	4.2 x 10 ⁻¹³ (MEG)	6 x 10 ⁻¹⁴ (MEG II)
$\mu \rightarrow 3e$	1.0 x 10 ⁻¹² (SINDRUM)	10 ⁻¹⁶ (Mu3e)
$\mu N \rightarrow e N$	7 x 10 ⁻¹³ (SINDRUM II)	10^{-14} (DeeMe) 7 x 10^{-15} / 7 x 10^{-17} (COMET) 6.2 x 10^{-16} / 8 x 10^{-17} (Mu2e)



Coordinate Information

The transformation to the stationary SCF is accomplished, at lowest order, by applying the rotation

$$\mathcal{R}_{\text{tot.}} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\chi\cos\omega_{\oplus}T_{\oplus} & \cos\chi\sin\omega_{\oplus}T_{\oplus} & -\sin\chi \\ -\sin\omega_{\oplus}T_{\oplus} & \cos\omega_{\oplus}T_{\oplus} & 0 \\ \sin\chi\cos\omega_{\oplus}T_{\oplus} & \sin\chi\sin\omega_{\oplus}T_{\oplus} & \cos\chi \end{pmatrix}$$

to each spatial index. Here, χ is the colatitude, ω_{\oplus} is the rotational frequency of Earth, and T_{\oplus} is local sidereal time, $T_{\oplus} \approx T - \frac{66.25^{\circ} - \lambda}{360^{\circ}}$ (23.934 hrs) for longitude λ . A final improper rotation sets the z-axis along the direction of the beamline; ψ is the angle of the laboratory z-axis, measured north of east.

Experiment	χ	λ	T_0	ψ	c
SINDRUM II	42.5°	8.2°	$3.86~{ m h}$	242°	0.44
COMET	53.6°	140.6°	-4.94 h	188°	0.34
Mu2e	48.2°	-88.2°	$10.27~\mathrm{h}$	122°	0.50



Harmonic Dependences

$$\begin{split} \omega_{\rm conv} \supset (\zeta_{TJ}^{(0)} + \zeta_{TJ}^{(2c)} c_{2\omega_{\oplus}T_{\oplus}} + \zeta_{TJ}^{(2s)} s_{2\omega_{\oplus}T_{\oplus}}) \\ & \times (|(m_F^{(5)})_{\mu e}^{TJ}|^2 + |(m_{5F}^{(5)})_{\mu e}^{TJ}|^2) \\ & + (\zeta_{\alpha TJ}^{(0)} + \zeta_{\alpha TJ}^{(2c)} c_{2\omega_{\oplus}T_{\oplus}} + \zeta_{\alpha TJ}^{(2s)} s_{2\omega_{\oplus}T_{\oplus}} \\ & + \zeta_{\alpha TJ}^{(4c)} c_{4\omega_{\oplus}T_{\oplus}} + \zeta_{\alpha TJ}^{(4s)} s_{4\omega_{\oplus}T_{\oplus}}) \\ & \times (|(a_F^{(5)})_{\mu e}^{\alpha TJ}|^2 + |(b_F^{(5)})_{\mu e}^{\alpha TJ}|^2) \\ & + (\zeta_{\alpha \beta TJ}^{(0)} + \zeta_{\alpha \beta TJ}^{(2c)} c_{2\omega_{\oplus}T_{\oplus}} + \zeta_{\alpha \beta TJ}^{(2s)} s_{2\omega_{\oplus}T_{\oplus}} \\ & + \zeta_{\alpha \beta TJ}^{(4c)} c_{4\omega_{\oplus}T_{\oplus}} + \zeta_{\alpha \beta TJ}^{(4s)} s_{4\omega_{\oplus}T_{\oplus}}) |(H_F^{(5)})_{\mu e}^{\alpha \beta TJ}|^2 \end{split}$$

 $\omega_{\rm conv} \supset \left((\zeta_q)_{\alpha}^{(0)} + (\zeta_q)_{\alpha}^{(2c)} c_{2\omega_{\oplus}T_{\oplus}} + (\zeta_q)_{\alpha}^{(2s)} s_{2\omega_{\oplus}T_{\oplus}} \right)$ $\times (|(k_{SV}^{(6)})_{ageu}^{\alpha}|^2 + |(k_{SA}^{(6)})_{ageu}^{\alpha}|^2)$ $+ (\zeta_a^0)_T^{(0)} (|(k_{VS}^{(6)})_{aae\mu}^T|^2 + |(k_{VP}^{(6)})_{qqe\mu}^T|^2)$ + $\left[((\zeta_a)_{T,I}^{(0)} + (\zeta_a)_{T,I}^{(2c)} c_{2\omega_{\oplus}T_{\oplus}} + (\zeta_a)_{T,I}^{(2s)} s_{2\omega_{\oplus}T_{\oplus}} \right]$ $\times |(k_{ST}^{(6)})_{ageu}^{TJ}|^2 + (TJ \rightarrow JZ)]$ + $[(\zeta_q)_{TZ}^{(0)}|(k_{ST}^{(6)})_{age\mu}^{TZ}|^2 + (TZ \to JK)]$ + $((\zeta_{q}^{0})_{T\alpha}^{(0)} + (\zeta_{q}^{0})_{T\alpha}^{(2c)}c_{2\omega\oplus T\oplus} + (\zeta_{q}^{0})_{T\alpha}^{(2s)}s_{2\omega\oplus T\oplus})$ $\times (|(k_{VV}^{(6)})_{ageu}^{T\alpha}|^2 + |(k_{VA}^{(6)})_{ageu}^{T\alpha}|^2)$ + $\left[((\zeta_a^0)_{TTJ}^{(0)} + (\zeta_a^0)_{TTJ}^{(2c)} c_{2\omega_{\oplus}T_{\oplus}} + (\zeta_q^0)_{TTJ}^{(2s)} s_{2\omega_{\oplus}T_{\oplus}} \right]$ $\times |(k_{VT}^{(6)})_{ageu}^{TTJ}|^2 + (TTJ \rightarrow TJZ)]$ + $\left[(\zeta_{a}^{0})_{TTZ}^{(0)} | (k_{VT}^{(6)})_{age\mu}^{TTZ} |^{2} + (TTZ \rightarrow TJK) \right]$

Constraints and Projections

Coefficients	SINDRUM I	I COMET	Mu2e
$ (m_F^{(5)})_{\mu e}^{TJ} , (m_{5F}^{(5)})_{\mu e}^{TJ} $	< 8	< 1	< 0.2
$ (m_F^{(5)})_{\mu e}^{TZ} , (m_{5F}^{(5)})_{\mu e}^{TZ} $	< 8	< 0.9	< 0.2
$ (a_F^{(5)})_{\mu e}^{TTJ} , (b_F^{(5)})_{\mu e}^{TTJ} $	< 6	< 1	< 0.2
$ (a_F^{(5)})_{\mu e}^{TTZ} , (b_F^{(5)})_{\mu e}^{TTZ} $	< 6	< 0.9	< 0.2
$ (a_F^{(5)})_{\mu e}^{JTJ} , (b_F^{(5)})_{\mu e}^{JTJ} $	< 6	< 1	< 0.2
$ (a_F^{(5)})_{\mu e}^{JTK} , (b_F^{(5)})_{\mu e}^{JTK} $	< 8	< 1	< 0.2
$ (a_F^{(5)})_{\mu e}^{JTZ} , (b_F^{(5)})_{\mu e}^{JTZ} $	< 8	< 0.9	< 0.2
$ (a_F^{(5)})_{\mu e}^{ZTJ} , (b_F^{(5)})_{\mu e}^{ZTJ} $	< 8	< 1	< 0.2
$ (a_F^{(5)})_{\mu e}^{ZTZ} , (b_F^{(5)})_{\mu e}^{ZTZ} $	< 7	< 0.9	< 0.2
$ (H_F^{(5)})_{\mu e}^{TJTJ} , (H_F^{(5)})_{\mu e}^{JZTK} $	< 7	< 1	< 0.2
$ (H_F^{(5)})_{\mu e}^{TJTK} , (H_F^{(5)})_{\mu e}^{JZTJ} $	< 6	< 1	< 0.2
$ (H_F^{(5)})_{\mu e}^{TJTZ} , (H_F^{(5)})_{\mu e}^{JZTZ} $	< 6	< 0.9	< 0.2
$ (H_F^{(5)})_{\mu e}^{TZTJ} , (H_F^{(5)})_{\mu e}^{XYTJ} $	< 6	< 1	< 0.2
$ (H_F^{(5)})_{\mu e}^{TZTZ} , (H_F^{(5)})_{\mu e}^{XYTZ} $	< 7	< 0.9	< 0.2

d=5 in units of $10^{-12} { m GeV}^{-1}$
d=6 in units of 10^{-13} GeV ⁻²
$J \in \{X, Y\}$

Coefficients	SINDRUM II	COMET	Mu2e
$ (k_{SV}^{(6)})_{uue\mu}^T , (k_{SA}^{(6)})_{uue\mu}^T $	< 6	< 1	< 0.2
$ (k_{SV}^{(6)})_{uue\mu}^{J} , (k_{SA}^{(6)})_{uue\mu}^{J} $	< 7	< 1	< 0.2
$ (k_{SV}^{(6)})_{uue\mu}^Z , (k_{SA}^{(6)})_{uue\mu}^Z $	< 7	< 1	< 0.2
$ (k_{SV}^{(6)})_{dde\mu}^T , (k_{SA}^{(6)})_{dde\mu}^T $	< 6	< 1	< 0.2
$ (k_{SV}^{(6)})_{dde\mu}^J , (k_{SA}^{(6)})_{dde\mu}^J $	< 7	< 1	< 0.2
$ (k_{SV}^{(6)})_{dde\mu}^Z , (k_{SA}^{(6)})_{dde\mu}^Z $	< 7	< 1	< 0.2
$ (k_{SV}^{(6)})_{sse\mu}^T , (k_{SA}^{(6)})_{sse\mu}^T $	< 10	< 2	< 0.4
$ (k_{SV}^{(6)})_{sse\mu}^J , (k_{SA}^{(6)})_{sse\mu}^J $	< 15	< 2	< 0.4
$ (k_{SV}^{(6)})_{sse\mu}^Z , (k_{SA}^{(6)})_{sse\mu}^Z $	< 15	< 2	< 0.4
$ (k_{VS}^{(6)})_{uue\mu}^T , (k_{VP}^{(6)})_{uue\mu}^T $	< 30	< 4	< 0.8
$ (k_{VS}^{(6)})_{dde\mu}^{T} , (k_{VP}^{(6)})_{dde\mu}^{T} $	< 30	< 4	< 0.7
$ (k_{ST}^{(6)})_{uue\mu}^{TJ} , (k_{ST}^{(6)})_{uue\mu}^{JZ} $	< 7	< 1	< 0.2
$ (k_{ST}^{(6)})_{uue\mu}^{TZ} , (k_{ST}^{(6)})_{uue\mu}^{XY} $	< 7	< 1	< 0.2
$ (k_{ST}^{(6)})_{dde\mu}^{TJ} , (k_{ST}^{(6)})_{dde\mu}^{JZ} $	< 7	< 1	< 0.2
$ (k_{ST}^{(6)})_{dde\mu}^{TZ} , (k_{ST}^{(6)})_{dde\mu}^{XY} $	< 7	< 1	< 0.2
$ (k_{ST}^{(6)})_{sse\mu}^{TJ} , (k_{ST}^{(6)})_{sse\mu}^{JZ} $	< 15	< 2	< 0.4
$ (k_{ST}^{(6)})_{sse\mu}^{TZ} , (k_{ST}^{(6)})_{sse\mu}^{XY} $	< 15	< 2	< 0.4
$ (k_{VV}^{(6)})_{uue\mu}^{TT} , (k_{VA}^{(6)})_{uue\mu}^{TT} $	< 20	< 4	< 0.7
$ (k_{VV}^{(6)})_{uue\mu}^{TJ} , (k_{VA}^{(6)})_{uue\mu}^{TJ} $	< 25	< 4	< 0.7
$ (k_{VV}^{(6)})_{uue\mu}^{TZ} , (k_{VA}^{(6)})_{uue\mu}^{TZ} $	< 25	< 4	< 0.7
$ (k_{VV}^{(6)})_{dde\mu}^{TT} , (k_{VA}^{(6)})_{dde\mu}^{TT} $	< 20	< 4	< 0.7
$ (k_{VV}^{(6)})_{dde\mu}^{TJ} , (k_{VA}^{(6)})_{dde\mu}^{TJ} $	< 20	< 4	< 0.7
$ (k_{VV}^{(6)})_{dde\mu}^{TZ} , (k_{VA}^{(6)})_{dde\mu}^{TZ} $	< 20	< 4	< 0.7
$ (k_{VT}^{(6)})_{uue\mu}^{TTJ} , (k_{VT}^{(6)})_{uue\mu}^{TJZ} $	< 25	< 4	< 0.7
$ (k_{VT}^{(6)})_{uue\mu}^{TTZ} , (k_{VT}^{(6)})_{uue\mu}^{TXY} $	< 25	< 4	< 0.7
$ (k_{VT}^{(6)})_{dde\mu}^{TTJ} , (k_{VT}^{(6)})_{dde\mu}^{TJZ} $	< 20	< 4	< 0.7
$ (k_{VT}^{(6)})_{dde\mu}^{TTZ} , (k_{VT}^{(6)})_{dde\mu}^{TXY} $	< 20	< 4	< 0.7

