Quantum Monte Carlo calculations of lepton-nucleus scattering

CIPANP 2025

15th Conference on the Intersections of Particle and Nuclear Physics

June 11, 2025

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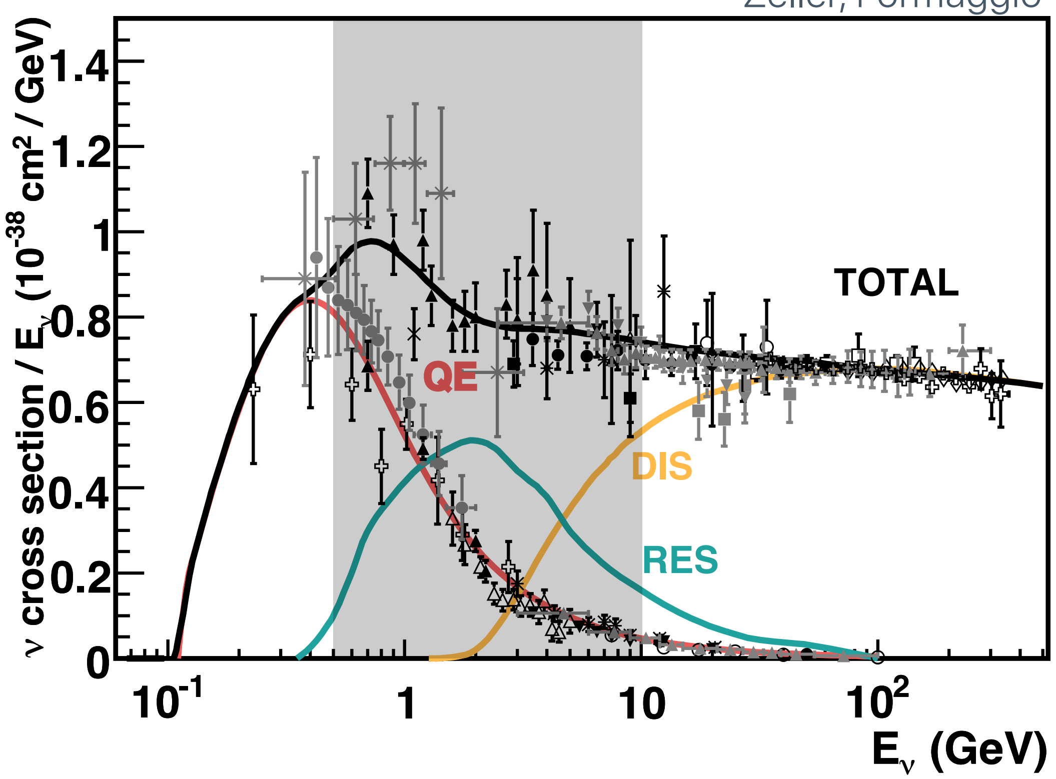
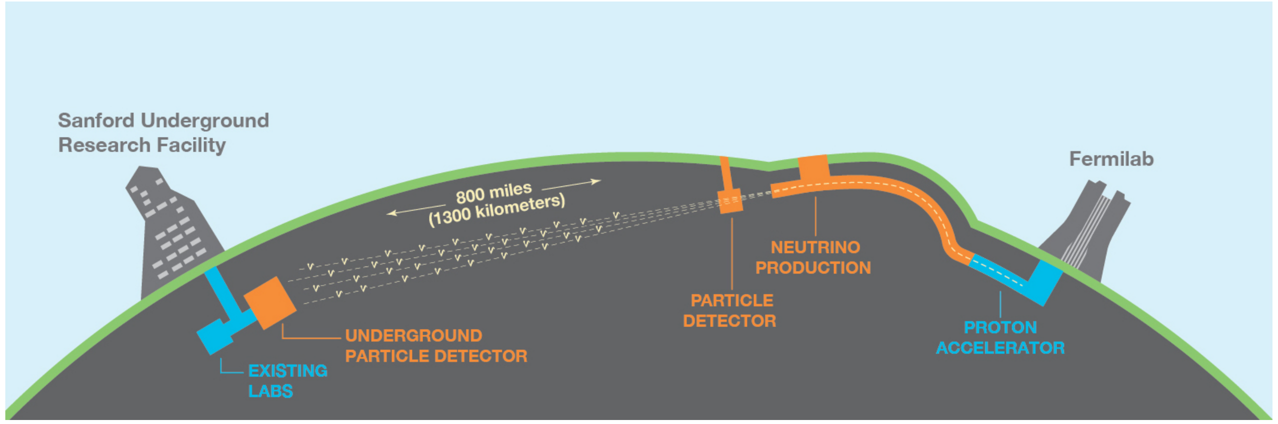
Lepton-nucleus scattering

Theoretical understanding of **nuclear effects** is extremely important for **electron** and **neutrino** experimental programs: oscillation experiments require accurate calculations of cross sections

Zeller, Formaggio



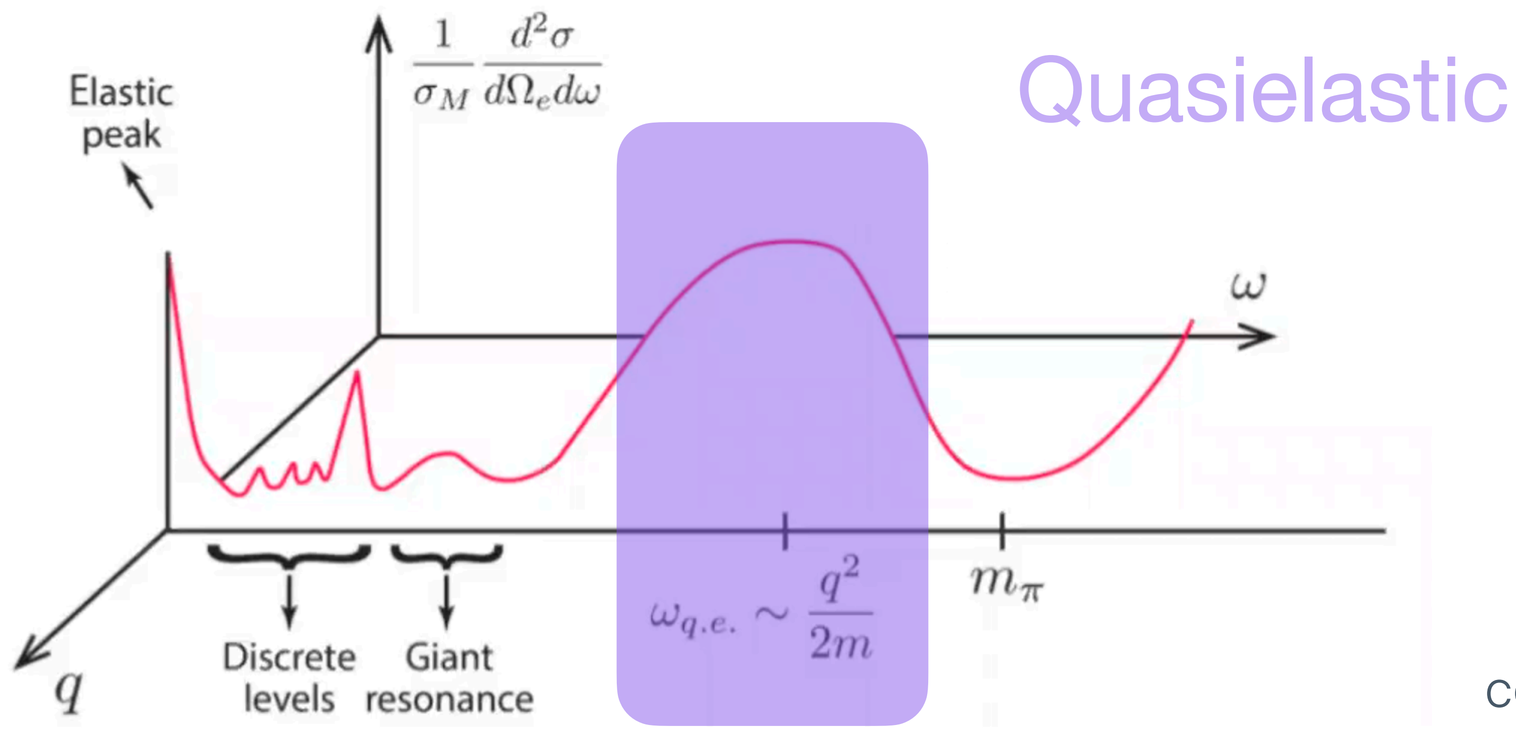


- **QE**: dominated by single-nucleon knockout
- **RES**: excitation to nucleonic resonant states which decay into mesons
- **DIS**: neutrino resolves the nucleonic quark content

PB: reliably bridging the transition regions which use different degrees of freedom. It requires knowledge of the nuclear ground state, electroweak coupling and propagation of the struck nucleons, hadrons, or partons

Lepton-nucleus scattering

Theoretical understanding of **nuclear effects** is extremely important for **electron** and **neutrino** experimental programs: oscillation experiments require accurate calculations of cross sections



courtesy of S. Pastore

Electron scattering can be used to test our nuclear model (**e4nu**):

- **same nuclear effects, ground state, FSI, similar interactions**
- **no need to reconstruct energies, monochromatic beam, high statistics**
- **abundant experimental data**

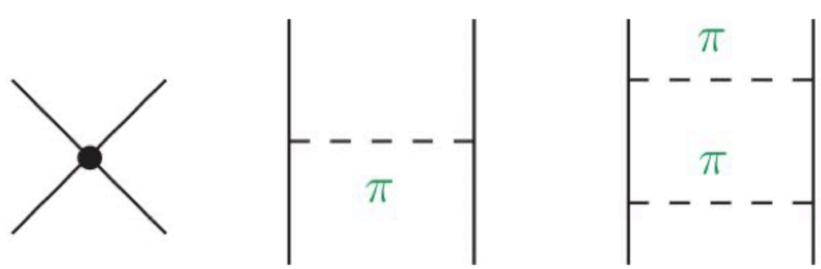
- **Ab initio description of nuclei:**
  - Nuclear interaction and ground state wave functions
  - Electromagnetic interaction of leptons with nucleons and clusters of correlated nucleons
- **Lepton-nucleus scattering:**
  - Inclusive processes
  - Short-Time Approximation
  - Relativistic effects
- **Conclusions**

Many-body nuclear interactions

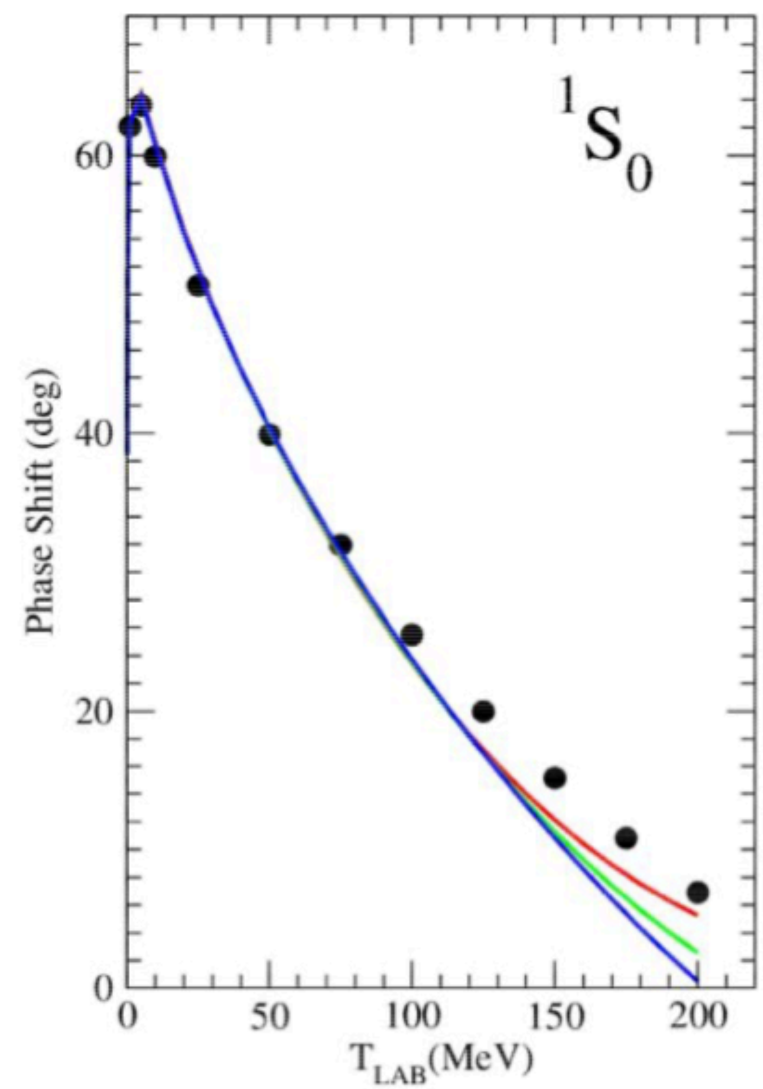
Many-body Nuclear Hamiltonian in coordinate space: Argonne v_{18} + Urbana X

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

v_{ij} and V_{ijk} are two- and three-nucleon operators based on experimental data fitting. Fitted parameters subsume underlying QCD dynamics



Contact term: short-range
Two-pion range: intermediate-range $r \propto (2\,m_\pi)^{-1}$
One-pion range: long-range $r \propto m_\pi^{-1}$



SP et al. PRC80(2009)034004

Many-body nuclear problem

Many-body Nuclear Hamiltonian in coordinate space: Argonne v_{18} + Urbana X

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_A, s_1, s_2, \ldots, s_A, t_1, t_2, \ldots, t_A)$$

ψ are complex spin-isospin vectors in 3A dimensions with components $2^A \times \frac{A!}{Z!(A-Z)!}$

http://exascaleage.org/np/

^{4}He: 96

^{6}Li: 1280

^{8}Li: 14336

^{12}C: 540572

Develop Computational Methods to solve (numerically) exactly or within approximations that are under control the many-body nuclear problem

Many-body nuclear interactions

Many-body Nuclear Hamiltonian in coordinate space: Argonne v_{18} + Urbana X

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

Quantum Monte Carlo method:
Use nuclear wave functions that minimize the expectation value of E

$$E_V = \frac{\langle\psi|H|\psi\rangle}{\langle\psi|\psi\rangle} \geq E_0$$

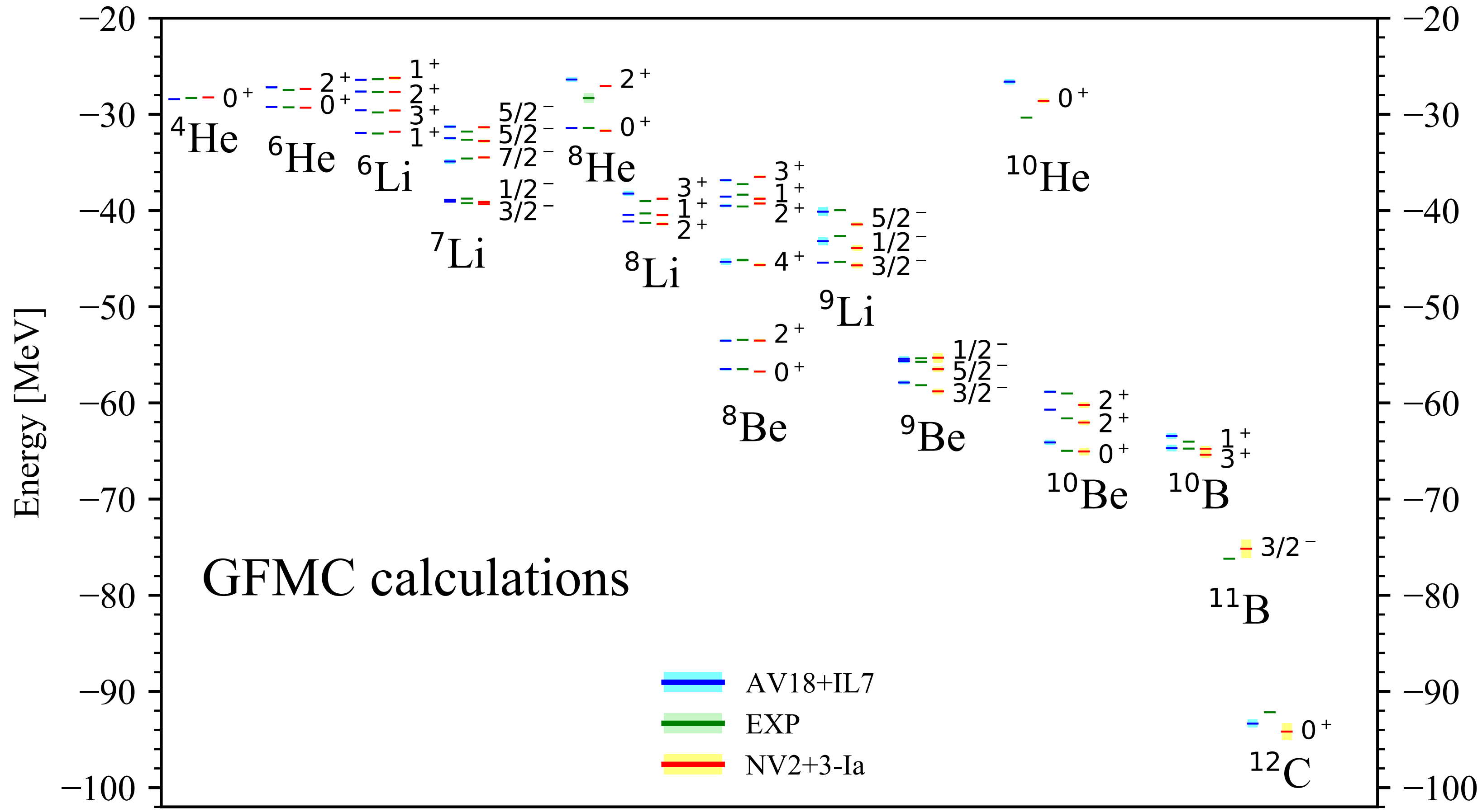
The evaluation is performed using Metropolis sampling

$$|\psi\rangle = \mathcal{S}\prod_{i<j}^{A}\left[1 + U_{ij} + \sum_{k\neq i,j}^{A} U_{ijk}\right]\left[\prod_{i<j} f_c(r_{ij})\right]|\Phi(JMTT_3)\rangle$$

Many-body nuclear interactions

Many-body Nuclear Hamiltonian in coordinate space: Argonne v_{18} + Urbana X

$$H = \sum_i T_i + \sum_{i<j} v_{ij} + \sum_{i<j<k} v_{ijk}$$

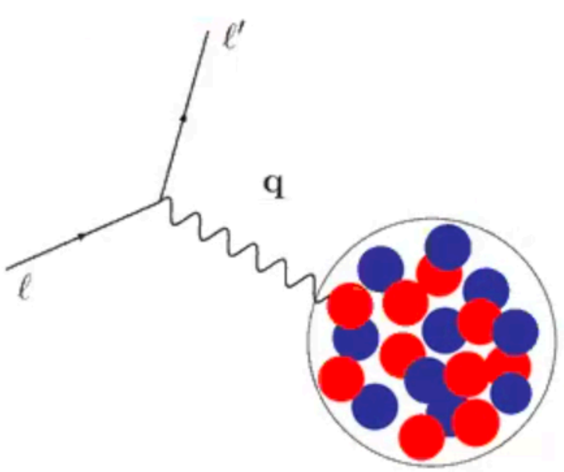


Spectra of light nuclei
Piarulli et al. PRL120(2018)052503

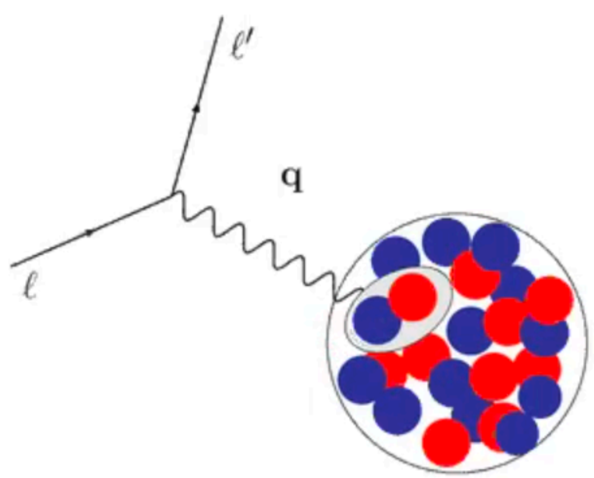
Electromagnetic interactions

Phenomenological Hamiltonian for NN and NNN

The interaction with external probes is described in terms on one- and two-body charge and current operators



one-body



two-body

Charge operators

$$\rho = \sum_{i=1}^{A} \rho_i + \sum_{i<j} \rho_{ij} + \dots$$

Current operators

$$\mathbf{j} = \sum_{i=1}^{A} \mathbf{j}_i + \sum_{i<j} \mathbf{j}_{ij} + \dots$$

Two-body currents are a manifestation of two-nucleon correlations

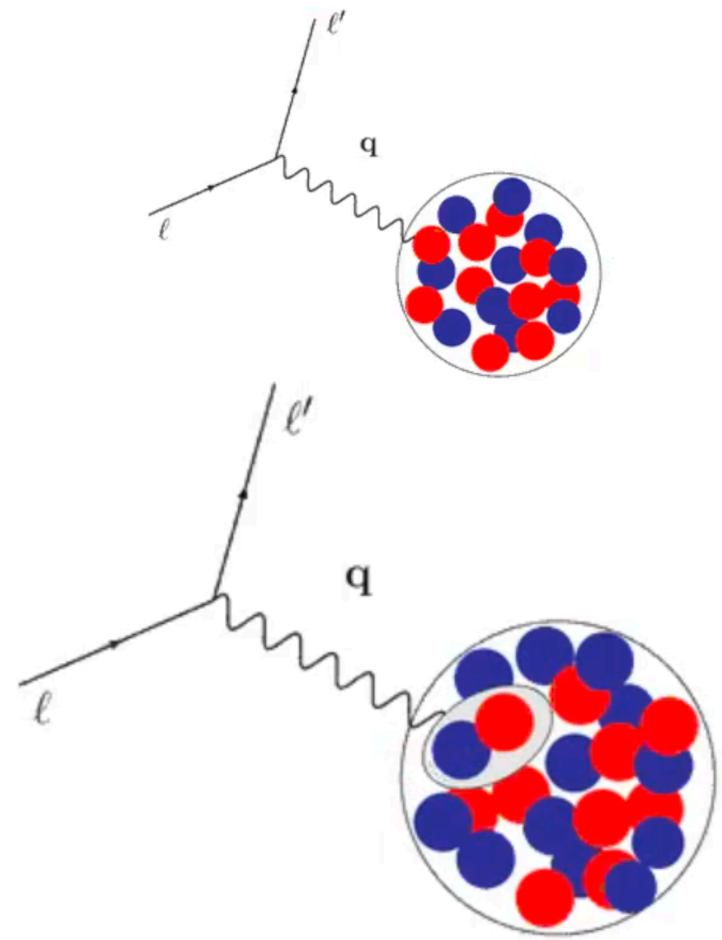
Electromagnetic interactions

- One body-currents: non-relativistic reduction of covariant nucleons' isoscalar and isovector currents ($q \ll m$)

- Two-body currents: modeled on MEC currents constrained by commutation relation with the nuclear Hamiltonian

$$\mathbf{q} \cdot \mathbf{j} = [H, \rho] = [t_i + v_{ij} + V_{ijk}, \rho]$$

- Argonne v_{18} two-nucleon and Urbana potentials, together with these currents, provide a quantitatively successful description of many nuclear electroweak observables, including charge radii, electromagnetic moments and transition rates, charge and magnetic form factors of nuclei with up to A = 12 nucleons



Carlson, Schiavilla 1992, Marcucci et al. 2005

Lepton-Nucleus scattering: Inclusive Processes

Electromagnetic Nuclear Response Functions

$$R_\alpha(q,\omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$

Longitudinal response induced by the charge operator $O_L = \rho$

Transverse response induced by the current operator $O_T = \mathbf{j}$

$$\frac{d^2\sigma}{d\omega d\Omega} = \sigma_M [v_L R_L(\mathbf{q},\omega) + v_T R_T(\mathbf{q},\omega)]$$

5 responses in neutrino-nucleus scattering

One can exploit integral properties of the response functions to **avoid explicit calculation** of the final states: CC + Lorentz Integral Transform, GFMC + Euclidean

Short-time approximation

S. Pastore et al. PRC101(2020)044612

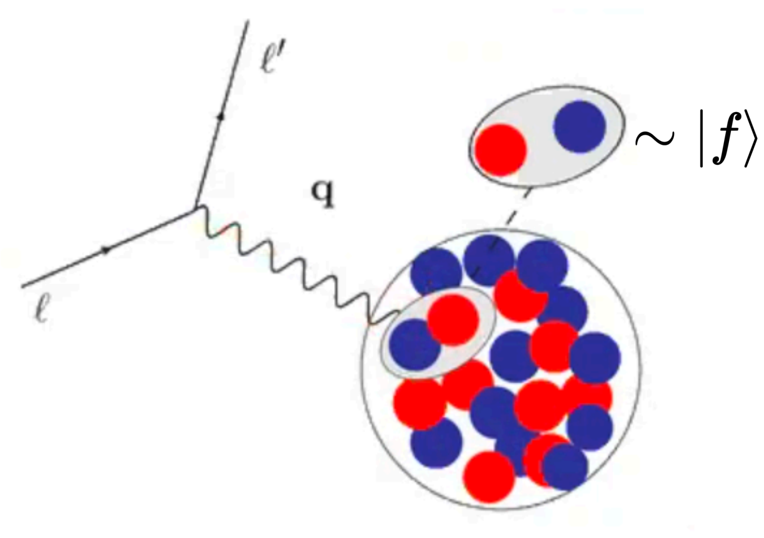
Factorization scheme:

- describe electroweak scattering from $A \geq 12$ without losing **two-body physics**
- account for **exclusive processes**
- incorporate **relativistic effects**

Short-time approximation

S. Pastore et al. PRC101(2020)044612

Factorization scheme: describe electroweak scattering from $A \geq 12$ without losing **two-body physics**, account for **exclusive processes**, incorporate **relativistic effects**



The sum over all final states is replaced by a two nucleon propagator

Response functions

$$R_\alpha(q,\omega) = \sum_f \delta(\omega + E_0 - E_f) \left| \langle f | O_\alpha(\mathbf{q}) | 0 \rangle \right|^2$$

$$R_\alpha(q,\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{i(\omega+E_i)t} \left\langle \Psi_i \middle| O_\alpha^\dagger(\mathbf{q}) e^{-iHt} O_\alpha(\mathbf{q}) \middle| \Psi_i \right\rangle$$

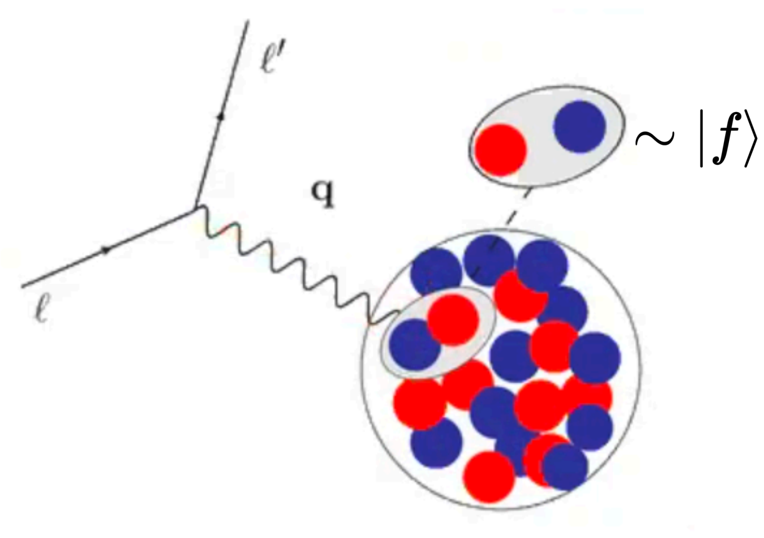
$$\begin{aligned} O^\dagger e^{-iHt} O &= \left(\sum_i O_i^\dagger + \sum_{i<j} O_{ij}^\dagger\right) e^{-iHt} \left(\sum_{i'} O_{i'} + \sum_{i'<j'} O_{i'j'}\right) \\ &= \sum_i O_i^\dagger e^{-iHt} O_i + \sum_{i \neq j} O_i^\dagger e^{-iHt} O_j \\ &\quad + \boxed{\sum_{i \neq j} \bigl(O_i^\dagger e^{-iHt} O_{ij} + O_{ij}^\dagger e^{-iHt} O_i} \\ &\quad + O_{ij}^\dagger e^{-iHt} O_{ij} \bigr) + \dots \end{aligned}$$

correctly accounts for interference

Short-time approximation

S. Pastore et al. PRC101(2020)044612

Factorization scheme: describe electroweak scattering from $A \geq 12$ without losing **two-body physics**, account for **exclusive processes**, incorporate **relativistic effects**



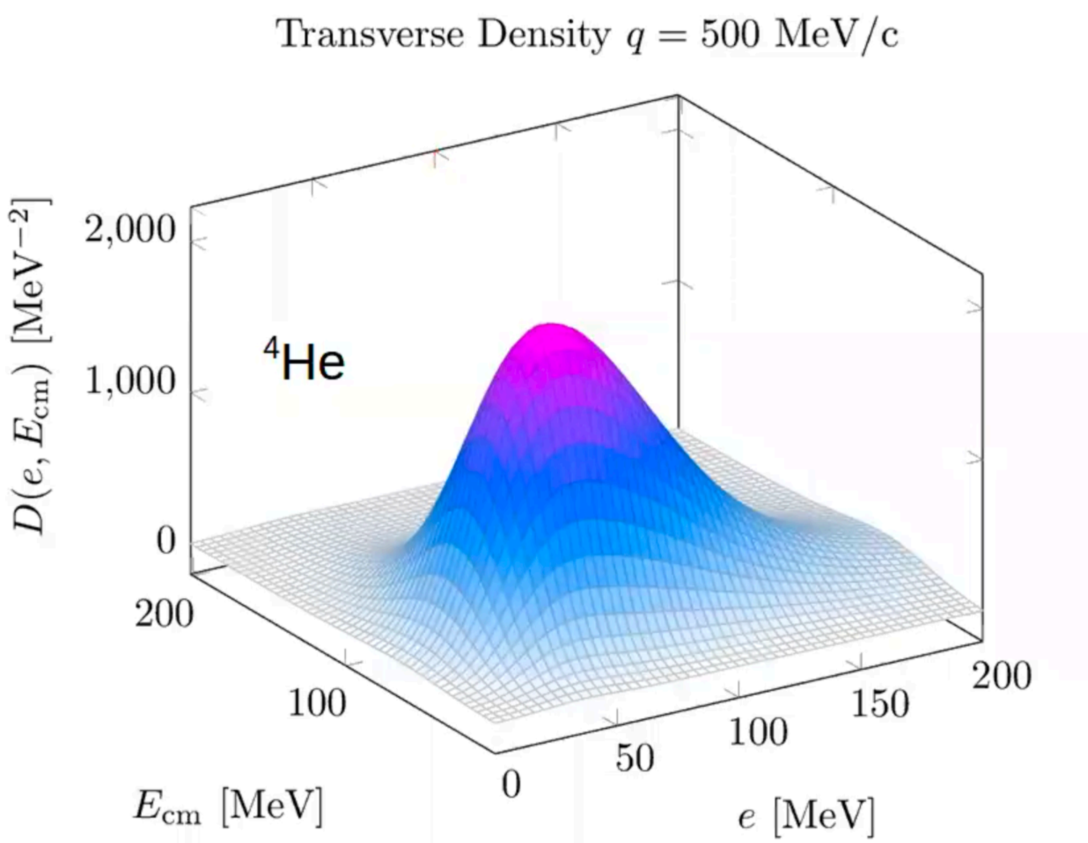
Response functions

$$R_\alpha(q,\omega) = \sum_f \delta(\omega + E_0 - E_f) |\langle f | O_\alpha(\mathbf{q}) | 0 \rangle|^2$$

Response densities

$$R^{\rm STA}(q,\omega) \sim \int \delta(\omega + E_0 - E_f) de\, dE_{cm} \mathcal{D}(e, E_{cm}; q)$$

Transverse response density

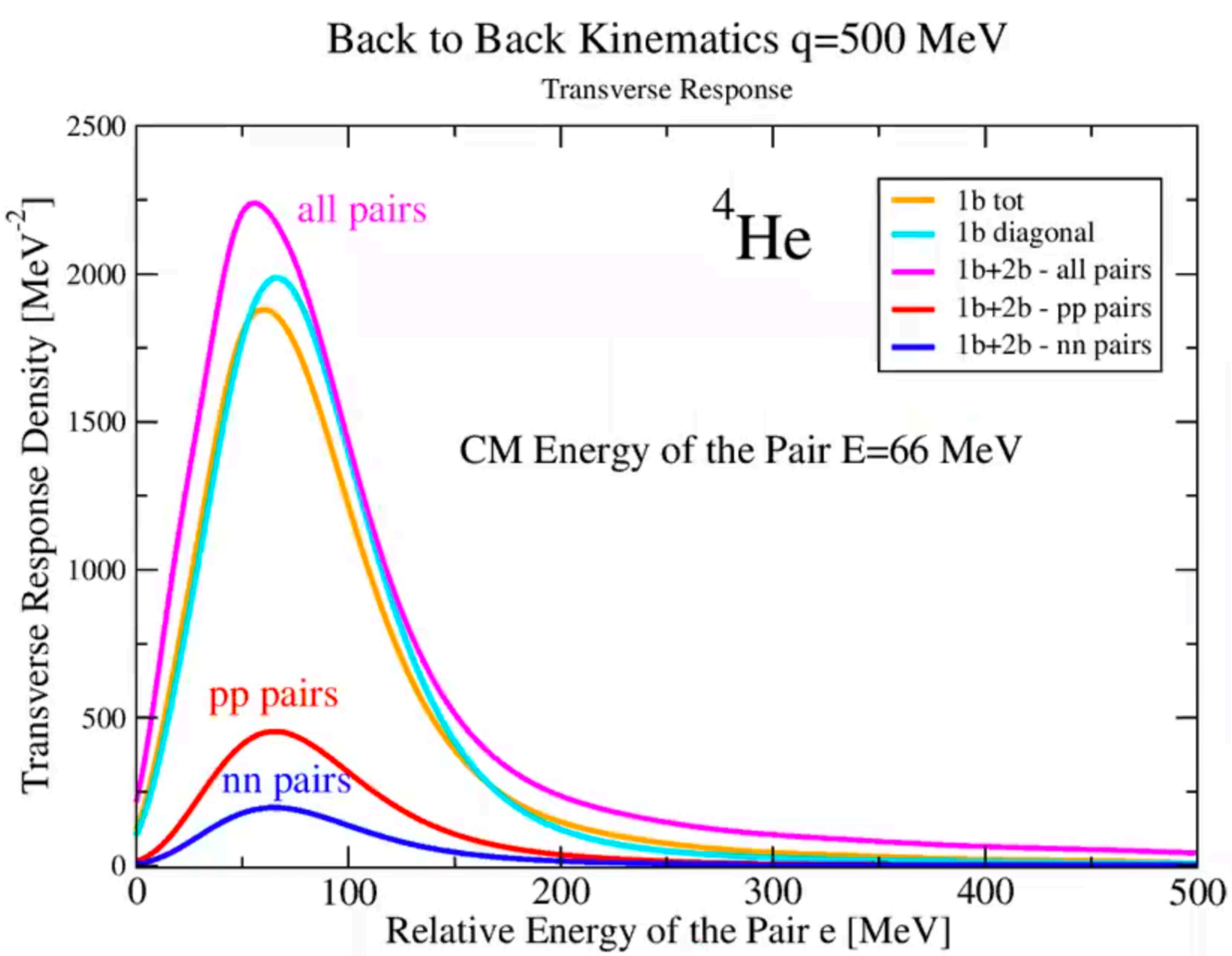


Electron scattering from ^4He in the STA:

- Provides “more” **exclusive** information in terms of nucleon-pair kinematics via the Response Densities as functions of (E,e)
- Give access to particular kinematics for the struck nucleon pair

PRC101(2020)044612

Back-to-back kinematic



Integrating response densities, we obtain response functions

We can select a particular kinematic, and assess the contributions from different particle identities

np dominance at high relative energy,

Also seen in SRC pairs:
Subedi et al. Science 320(2008)1475

PRC101(2020)044612

Benchmark

L.A et al. PRC105(2022)014002

- We benchmarked three different methods based on the **same description of nuclear dynamics of the initial target state: GFMC, STA, SF**
- Compared to the experimental data for the longitudinal and transverse electromagnetic response functions of ^{3}He, and the inclusive cross sections of both ^{3}He and ^{3}H
- Comparing the results allows for a precise quantification of the uncertainties inherent to factorization schemes

Benchmark

L.A. et al. PRC105(2022)014002

Green's function Monte Carlo

$$|\Psi_0\rangle \propto \lim_{\tau\to\infty} \exp[-(H-E_0)\tau]|\Psi_T\rangle$$

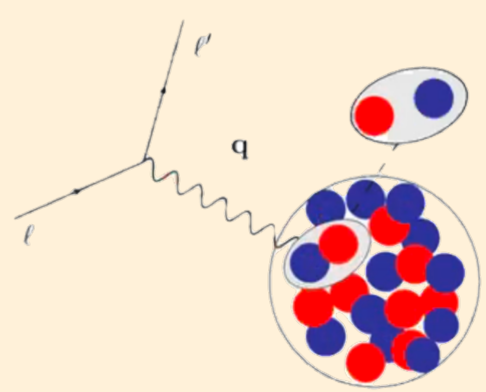
$$E_\alpha(\mathbf{q},\tau) = \int_{\omega_{\rm th}}^{\infty} d\omega e^{-\omega\tau} R_\alpha(\mathbf{q},\omega), \quad \alpha = L, T$$

$$E_\alpha(\mathbf{q},\tau) = \left\langle \Psi_0 \middle| J_\alpha^\dagger(\mathbf{q}) e^{-(H-E_0)\tau} J_\alpha(\mathbf{q}) \middle| \Psi_0 \right\rangle - |F_\alpha(\mathbf{q})|^2 e^{-\omega_{el}\tau}$$

Stort-time approximation

$$R_\alpha(\mathbf{q},\omega) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} \mathrm{e}^{i(\omega+E_0)t} \times \left\langle \Psi_0 \middle| J_\alpha^\dagger(\mathbf{q}) \mathrm{e}^{-iHt} J_\alpha(\mathbf{q}) \middle| \Psi_0 \right\rangle$$

$$J^\dagger \mathrm{e}^{-iHt} J = \sum_i J_i^\dagger \mathrm{e}^{-iHt} J_i + \sum_{i\neq j} J_i^\dagger \mathrm{e}^{-iHt} J_j + \sum_{i\neq j} \left(J_i^\dagger \mathrm{e}^{-iHt} J_{ij} + J_{ij}^\dagger \mathrm{e}^{-iHt} J_i + J_{ij}^\dagger \mathrm{e}^{-iHt} J_{ij} \right) + \cdots$$



Spectral function

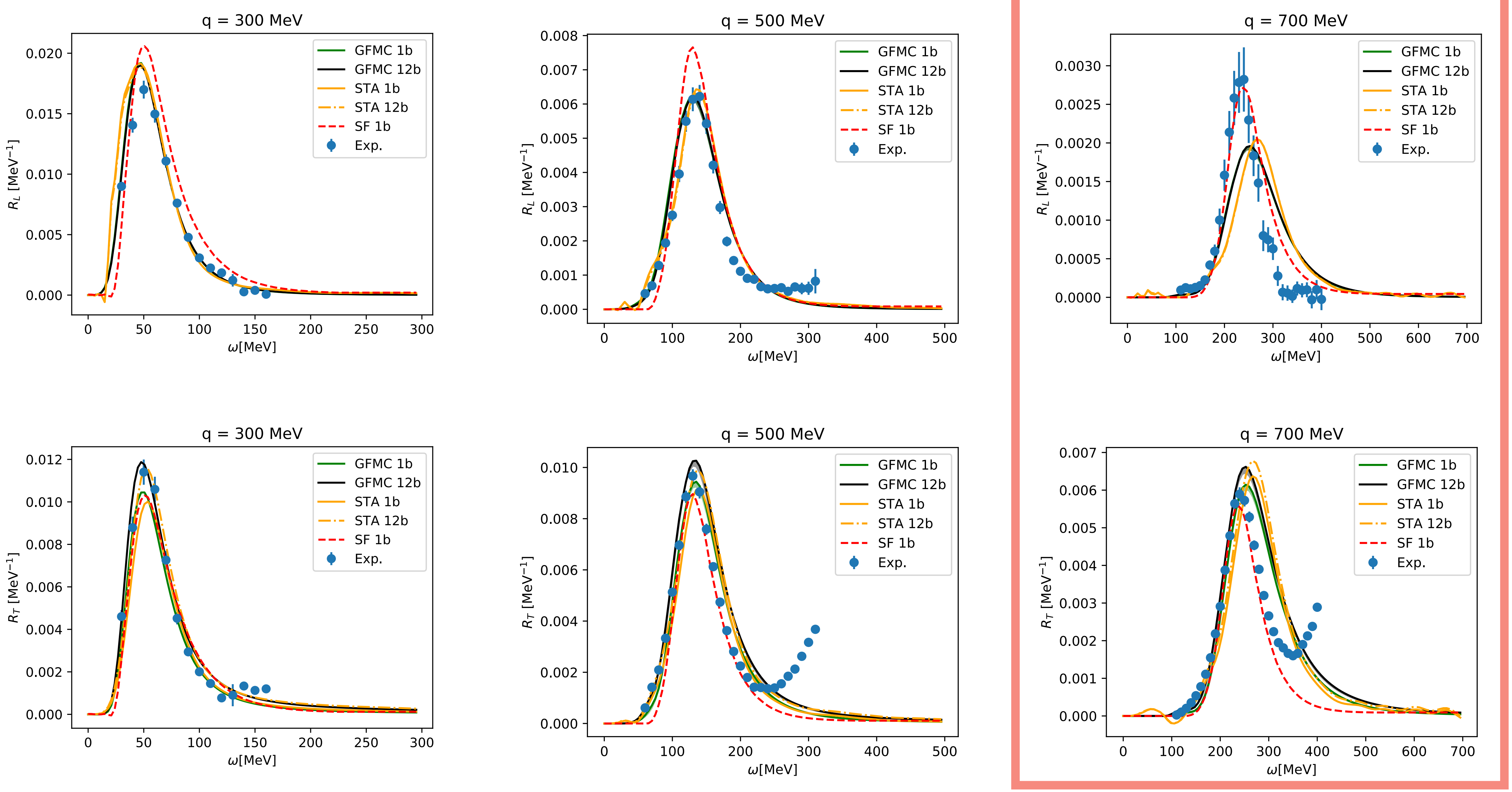
$$|\Psi_f\rangle = |\mathbf{p}\rangle \otimes \left|\Psi_n^{A-1}\right\rangle$$

$$R_\alpha(\mathbf{q},\omega) = \sum_{\tau_k=p,n} \int \frac{d^3k}{(2\pi)^3} dE [P_{\tau_k}(\mathbf{k},E) \times \frac{m_N^2}{e(\mathbf{k})e(\mathbf{k}+\mathbf{q})} \sum_i \left\langle k \middle| j_{i,\alpha}^\dagger \middle| k+q \right\rangle \langle p | j_{i,\alpha} | k \rangle \times \delta(\tilde{\omega} + e(\mathbf{k}) - e(\mathbf{k}+\mathbf{q}))]$$

P_{τ_k} is the probability distribution of removing a nucleon with momentum k and isospin $\tau_k = p, n$ from the target nucleus

Benchmark

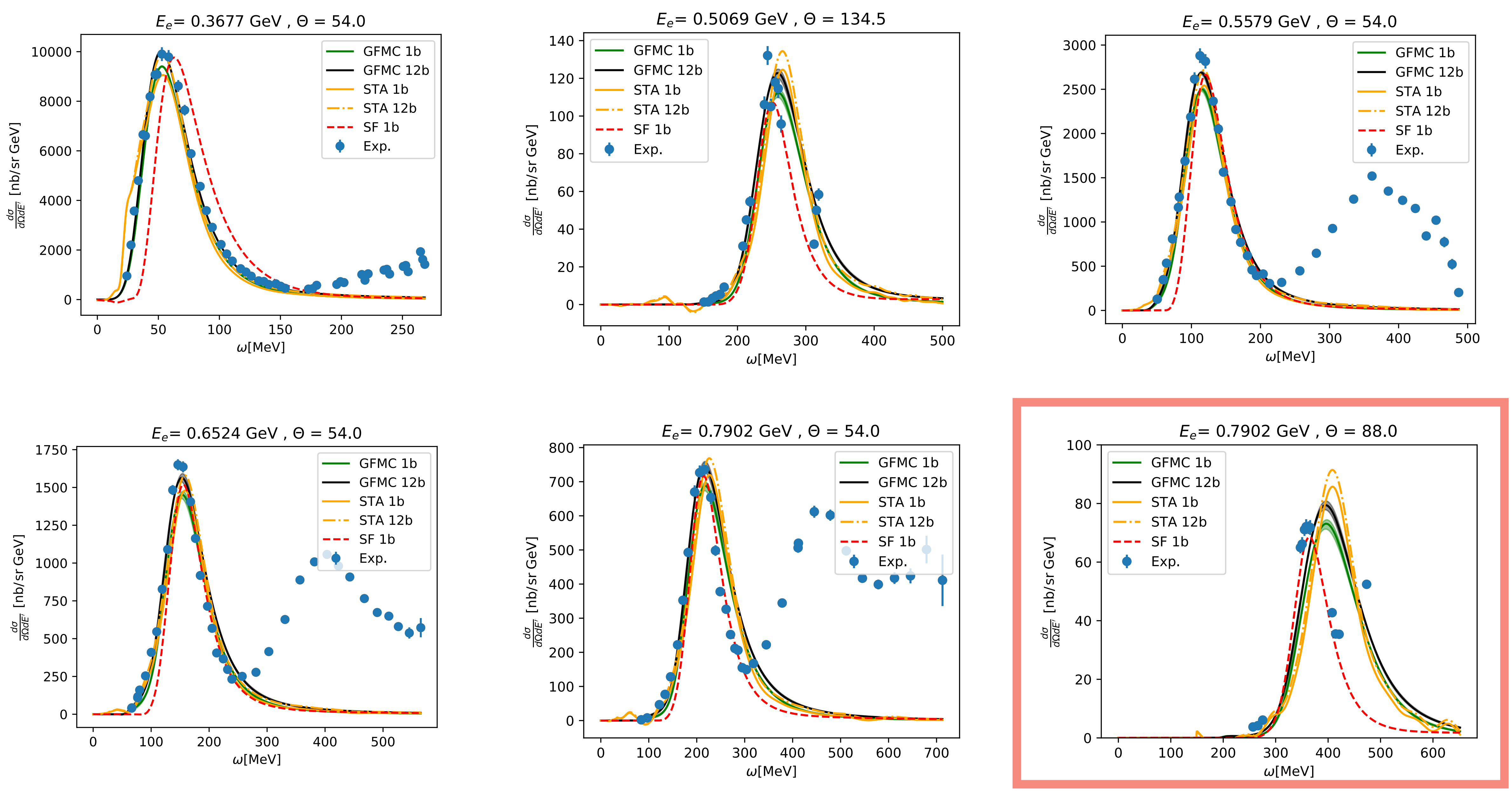
Longitudinal and transverse response function in ^{3}He



PRC105(2022)014002

Benchmark

^3H



PRC105(2022)014002

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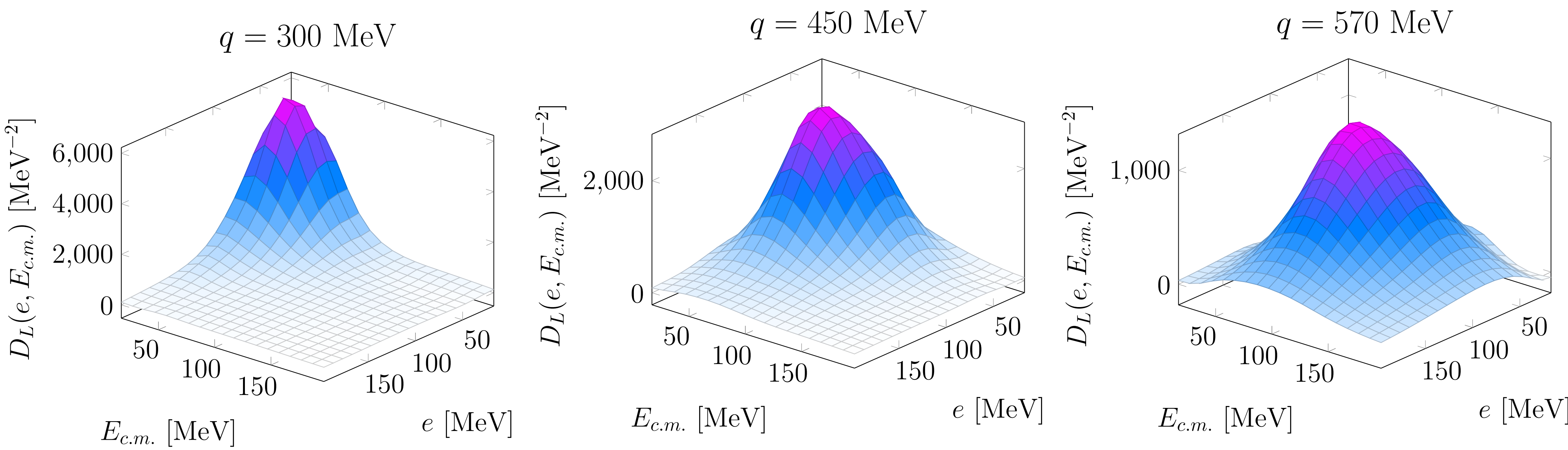
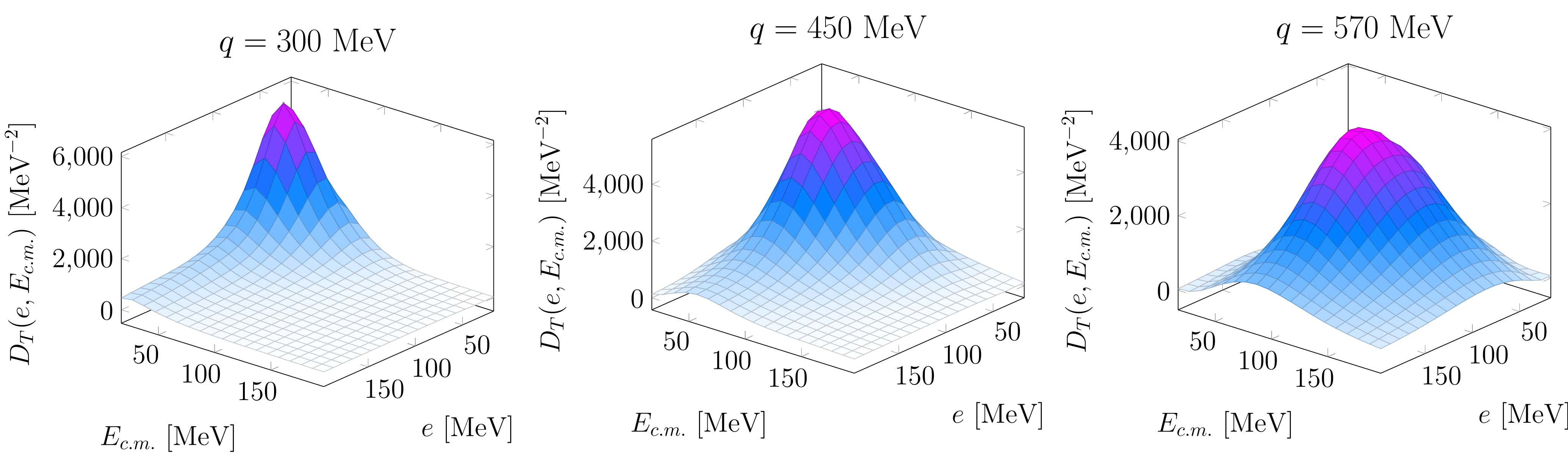
Responses for ^{12}C

Response densities are calculated for different values of momenta in the range **300 < q < 800 MeV**:

Longitudinal



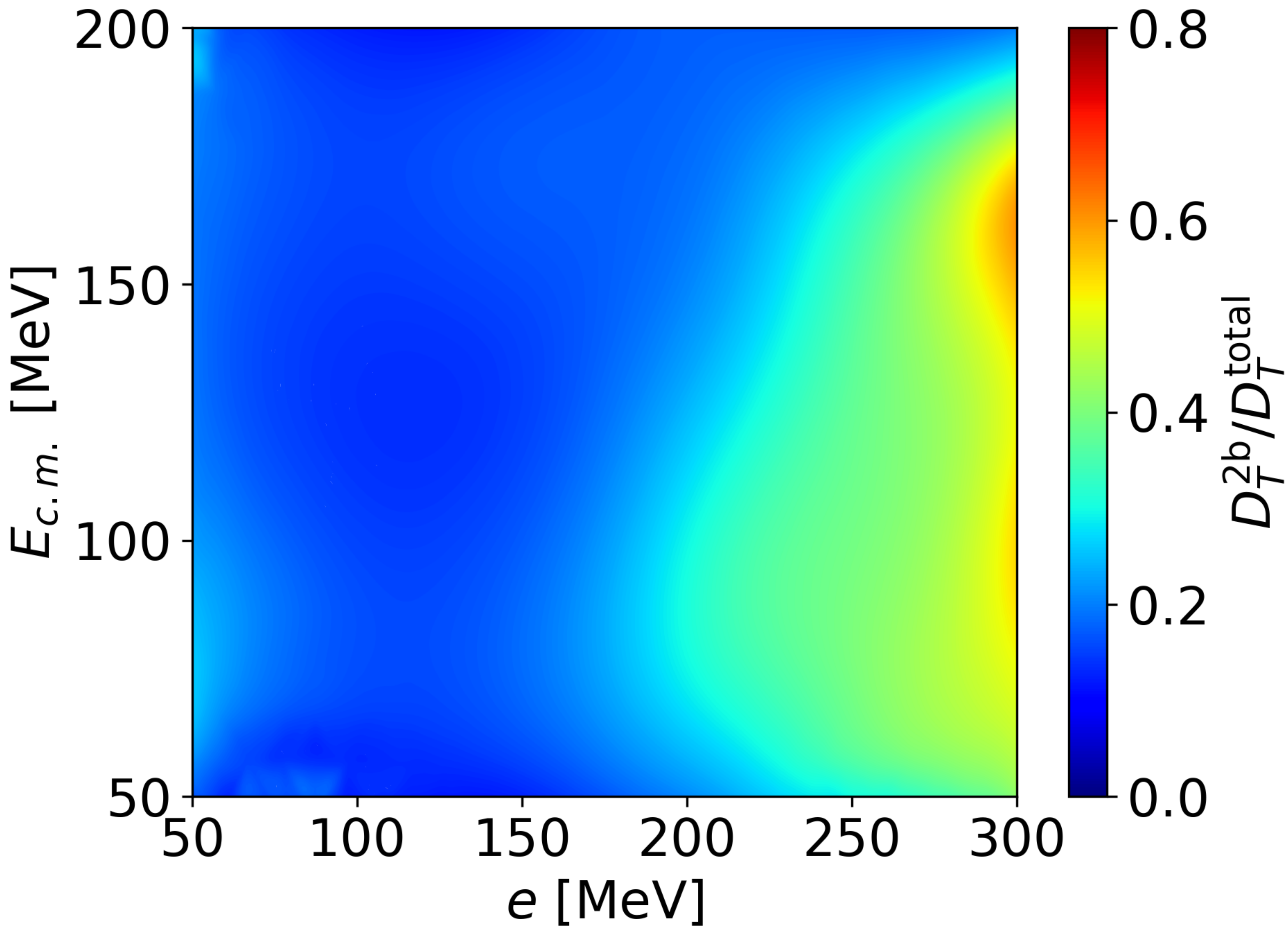
Transverse



Phys. Rev. C 110, 064004

Two-body contributions

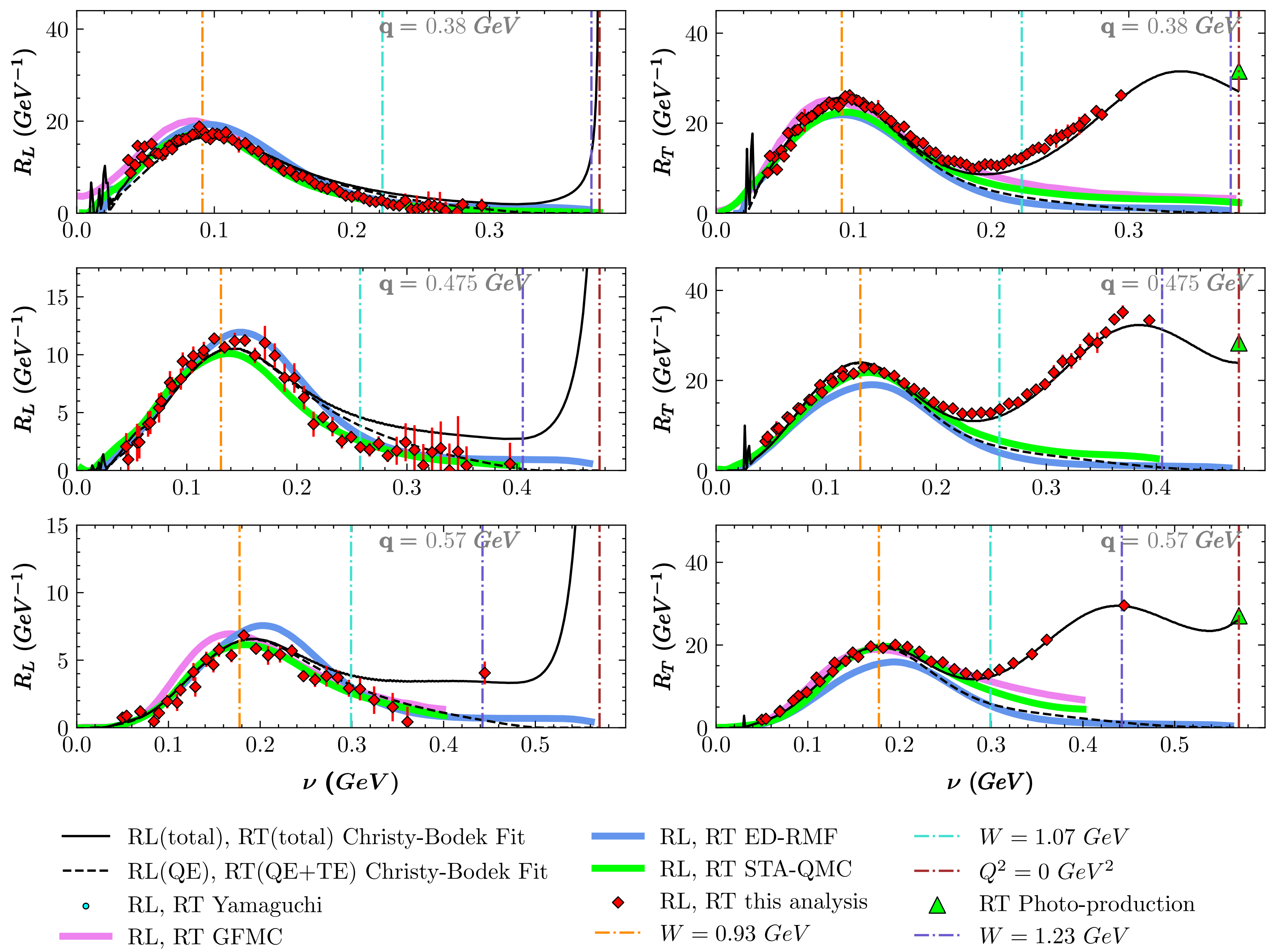
Transverse response density at q=570 MeV:



Phys. Rev. C 110, 064004

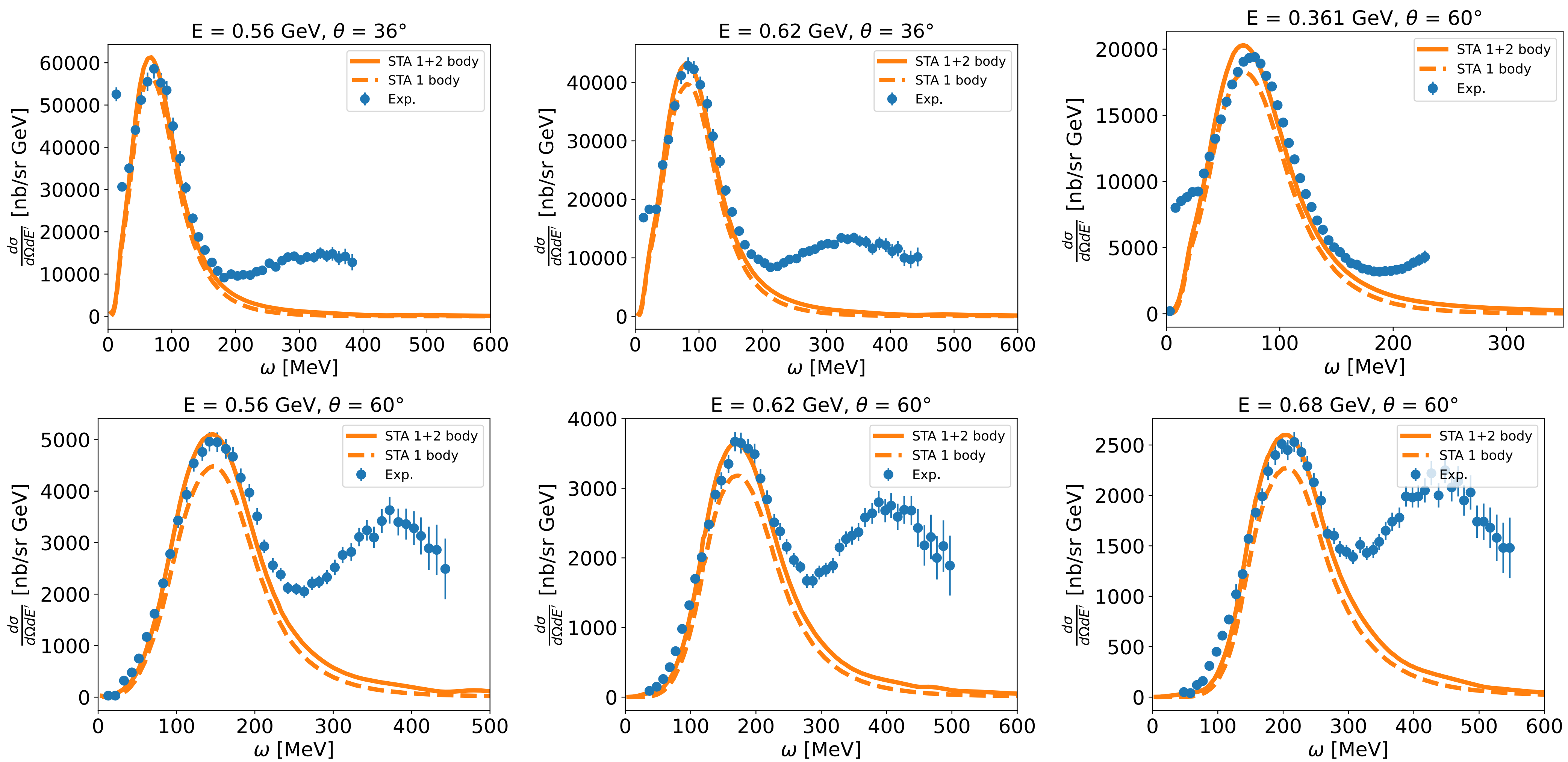
Global Extraction of the ^{12}C Nuclear Electromagnetic Response Functions ($\mathcal{R}_L$ and $\mathcal{R}_T$) and Comparisons to Nuclear Theory and Neutrino/Electron Monte Carlo Generators

Arie Bodek,[1] M. E. Christy,[2] Zihao Lin,[1] Giulia-Maria Bulugean,[1] Amii Matamoros Delgado,[1] Artur M. Ankowski,[3] G. D. Megias,[4] and Julia Tena Vidal[5]



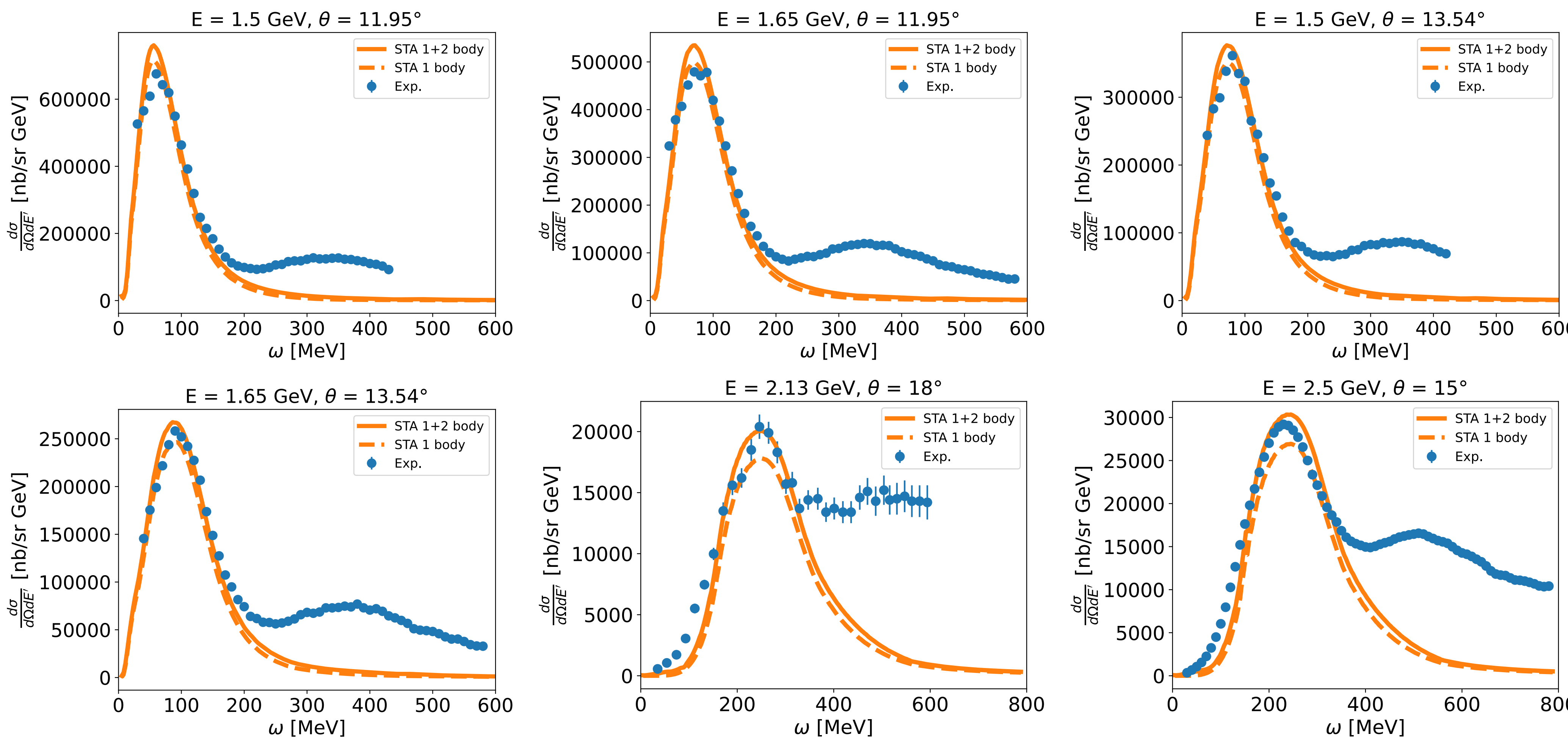
A. Bodek, M. E. Christy et al. https://arxiv.org/abs/2409.10637

Cross sections results for ^{12}C



Phys. Rev. C 110, 064004

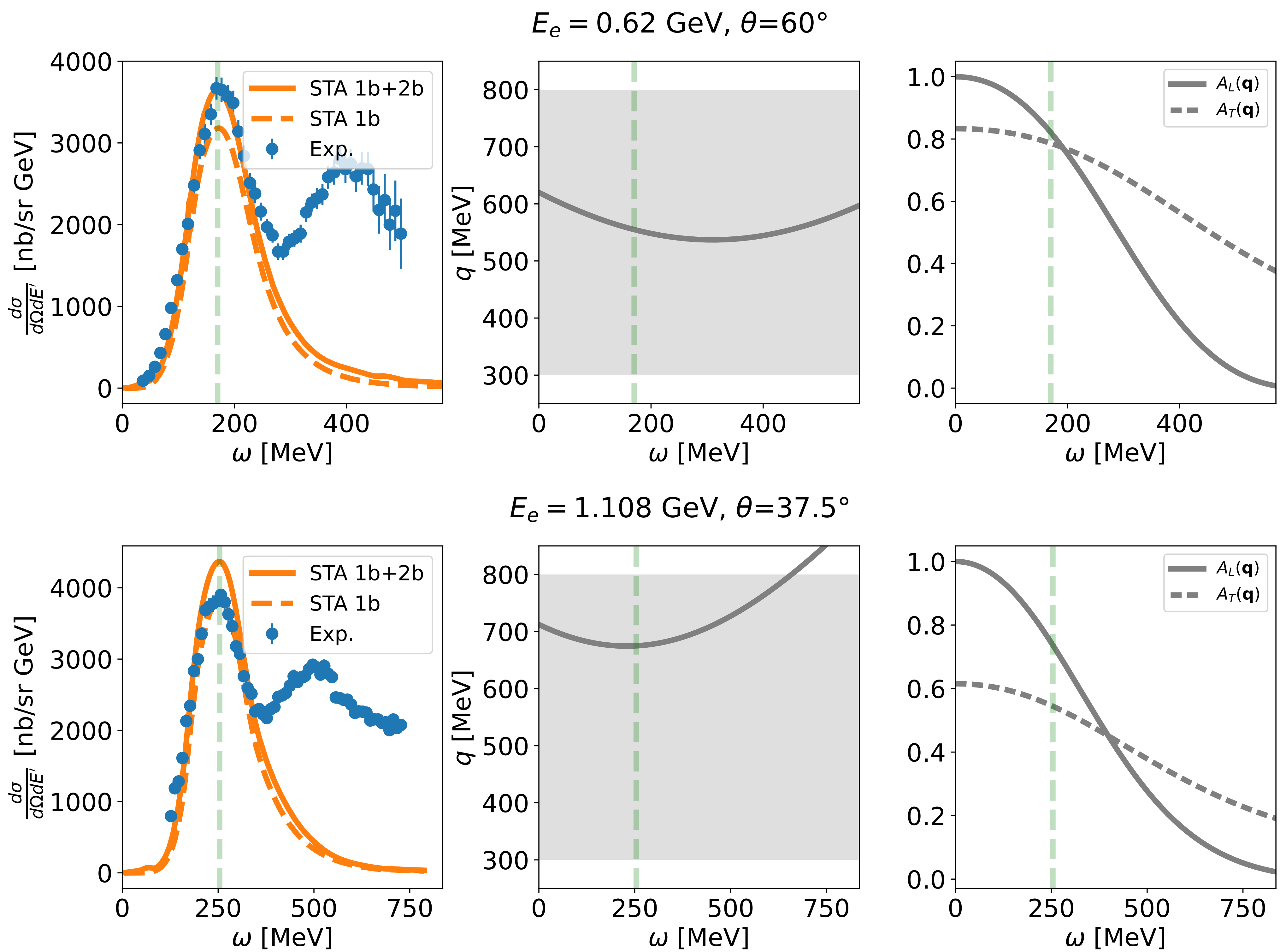
Cross sections results for ^{12}C



Phys. Rev. C 110, 064004

Data from Quasielastic Electron Nucleus Scattering Archive hosted at University of Virginia
http://discovery.phys.virginia.edu/research/groups/qes-archive/index.html

Cross sections results for ^{12}C



Phys. Rev. C 110, 064004

Relativistic corrections

Necessary to include relativistic correction at higher momentum q.

We are currently working on including **relativistic corrections within the STA formalism**:

A. Gnech (ODU), R. Weiss, G. Chambers-Wall, S. Pastore (WashU), S. Gandolfi, J. Carlson (LANL)

- **Relativistic currents**: expansion for a large value of the momentum transfer **q**
- **Relativistic kinematical effects**: allowed by STA factorization scheme

Relativistic currents

- Original expansion: one body-currents are the non-relativistic reduction of covariant nucleons' isoscalar and isovector currents ($q \ll m$)

$$j^\mu = e\bar{u}(\boldsymbol{p}'\boldsymbol{s}')\left(e_N\gamma^\mu + \frac{i\kappa_N}{2m_N}\sigma^{\mu\nu}q_\nu\right)u(\boldsymbol{p}\boldsymbol{s})$$

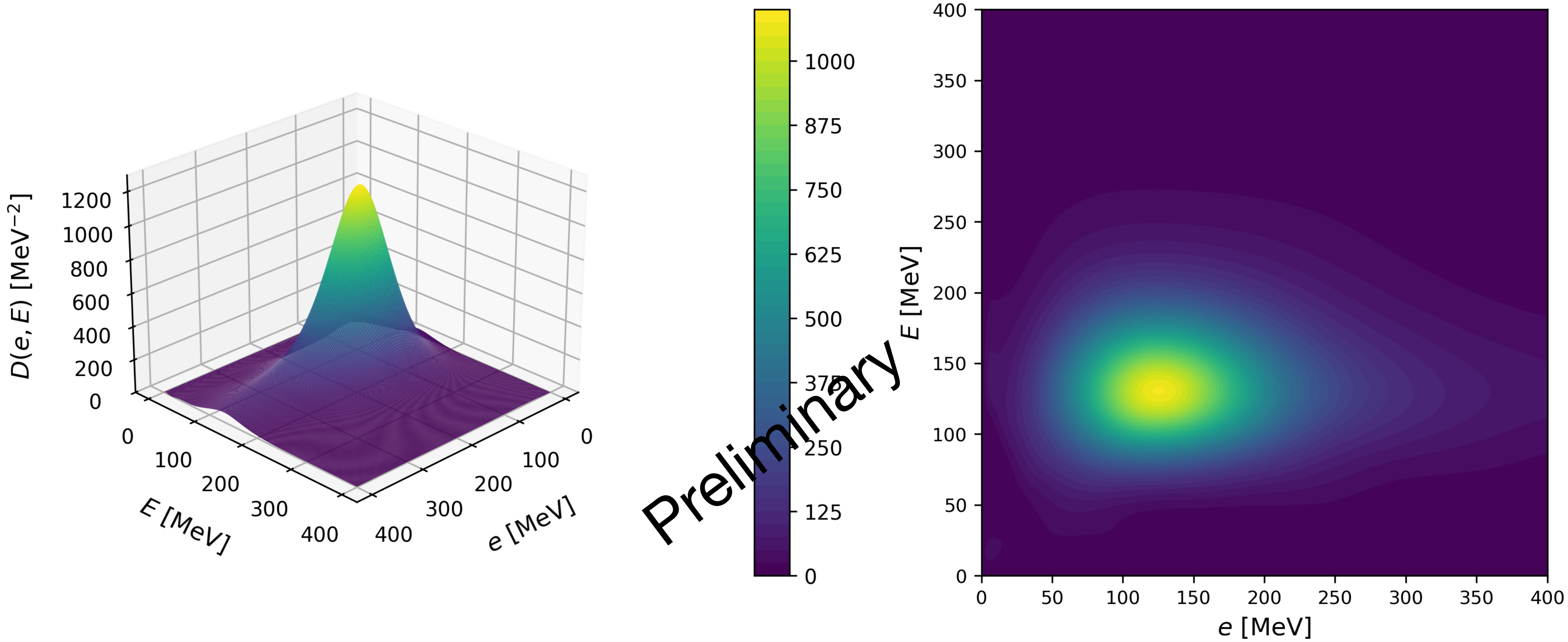
$$\boldsymbol{p}' = \boldsymbol{p} + \boldsymbol{q}$$

- Large momenta $q, p' \sim m$

$$j^0_{p^0} = \alpha(q)G_E\left(Q^2_{qe}\right)e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$

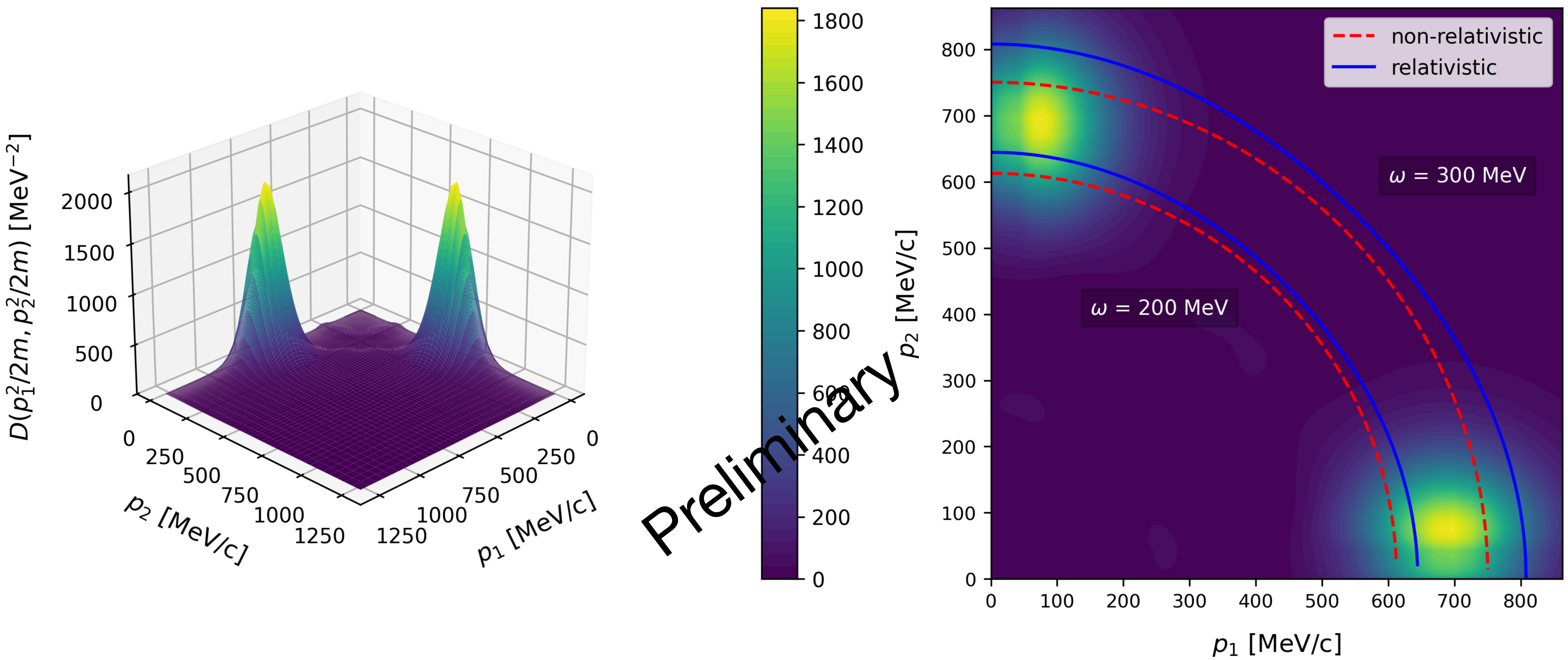
$$\boldsymbol{j}^\perp_{p^0} = \frac{2m\tau_{qe}}{q^2}G_M\left(Q^2_{qe}\right)\alpha(q)i(\boldsymbol{\sigma}\times\boldsymbol{q})e^{i\boldsymbol{q}\cdot\boldsymbol{r}_i}$$

Kinematical relativistic effects



$$R_\alpha(\boldsymbol{q},\omega) = \int de dE_{cm} D(e, E_{cm})\delta(\omega - E_{cm} - e)$$

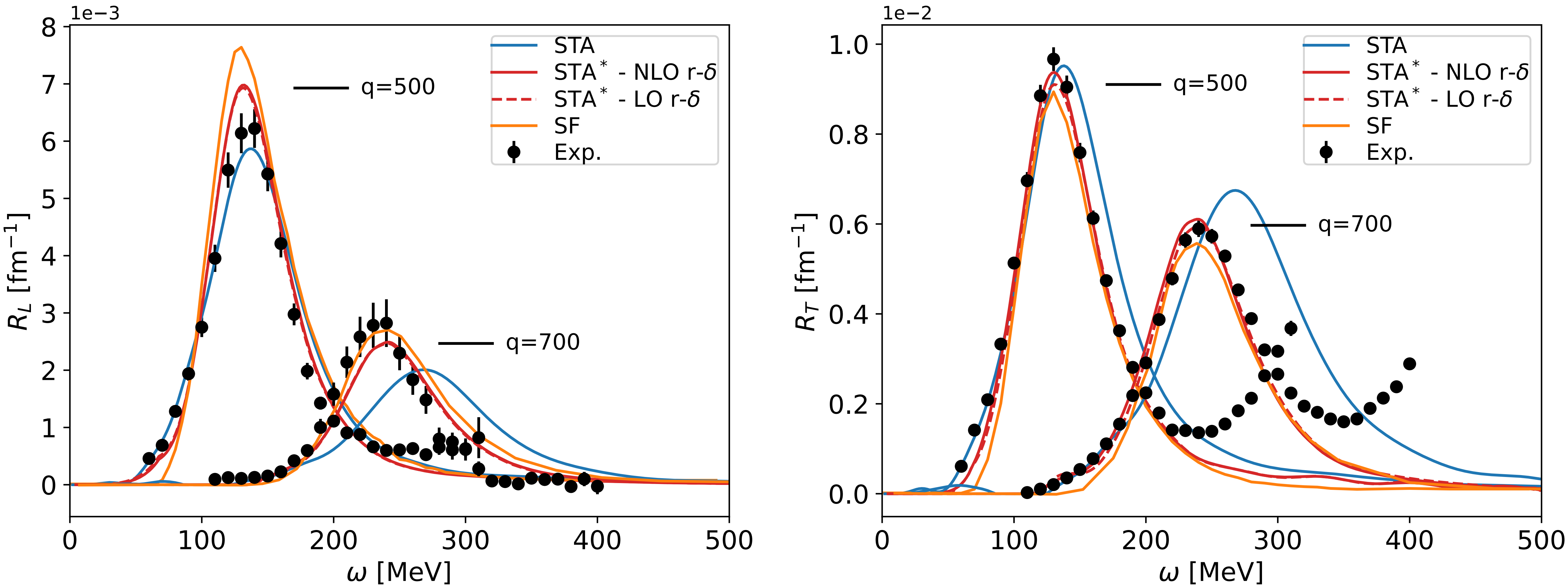
Kinematical relativistic effects



$$R(\mathbf{q},\omega) = \int_0^\infty dp_1 \int_0^\infty dp_2 D(p_1,p_2)\delta\left(\omega - \sqrt{p_1^2+m^2} - \sqrt{p_2^2+m^2} + 2m\right)$$

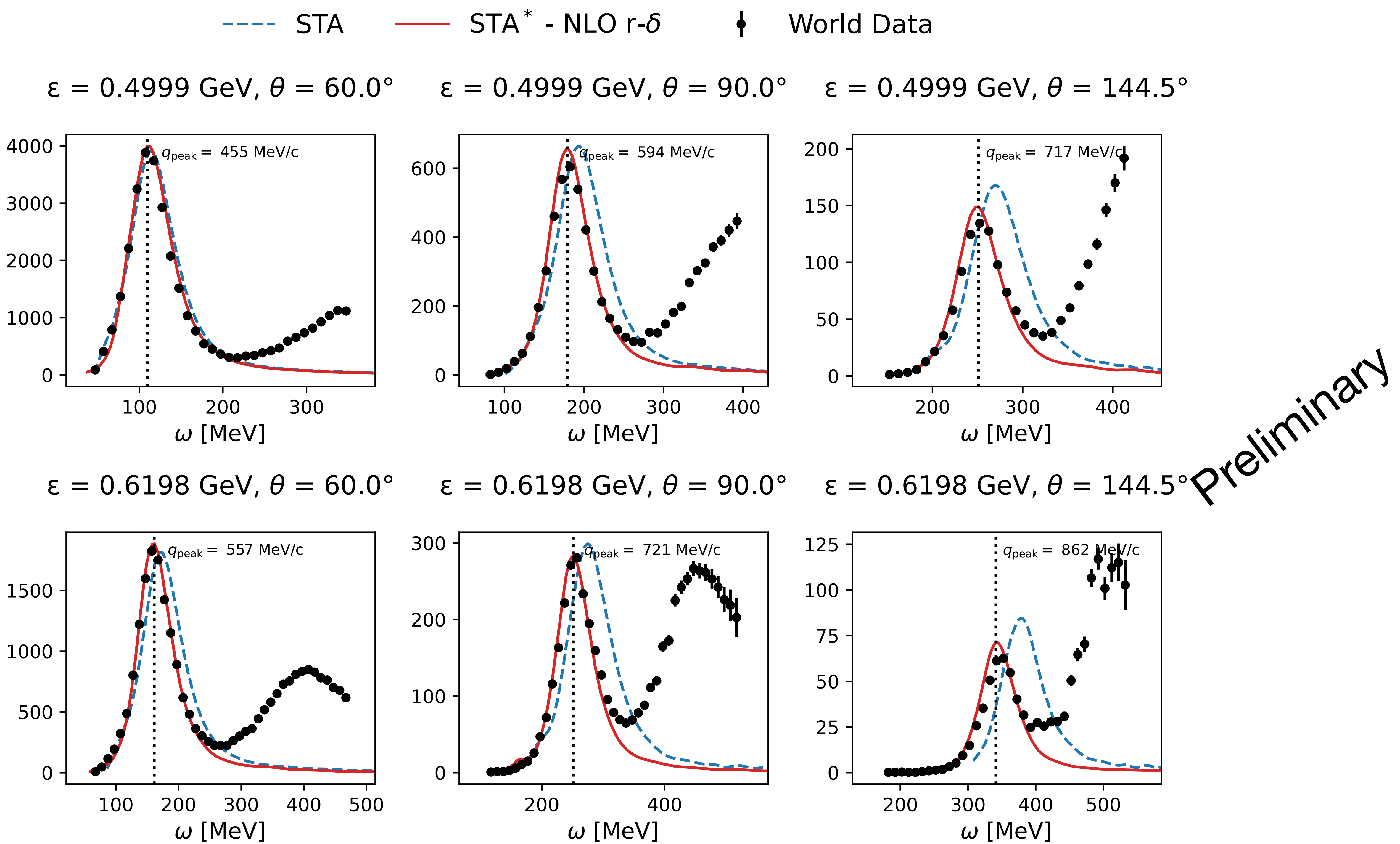
- Also, currently working on retaining **angle** information $D(p_1, p_2, \theta)$

^3He response functions



Preliminary

^3He cross sections



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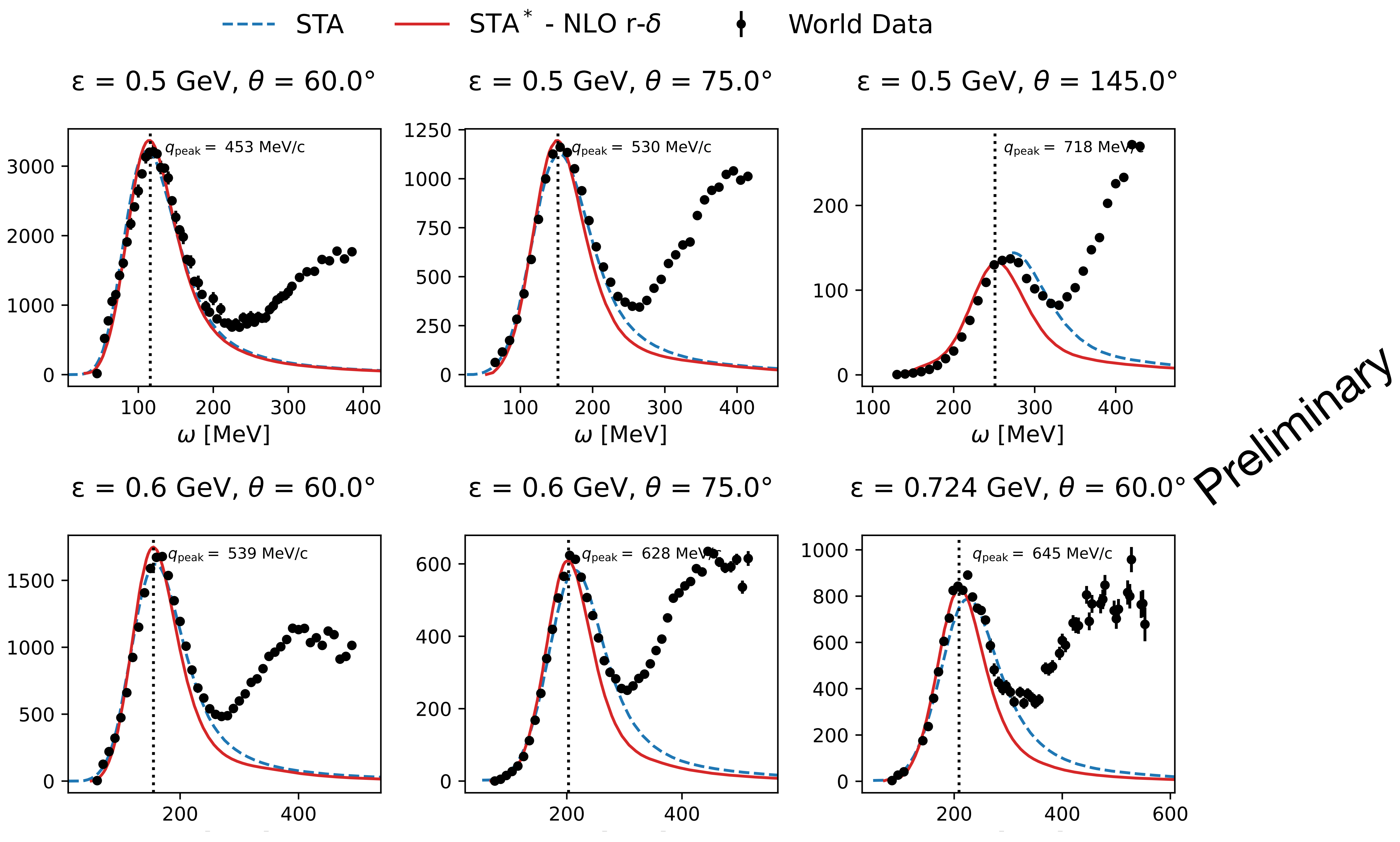
^4He cross sections



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GENIE validation using e-scattering

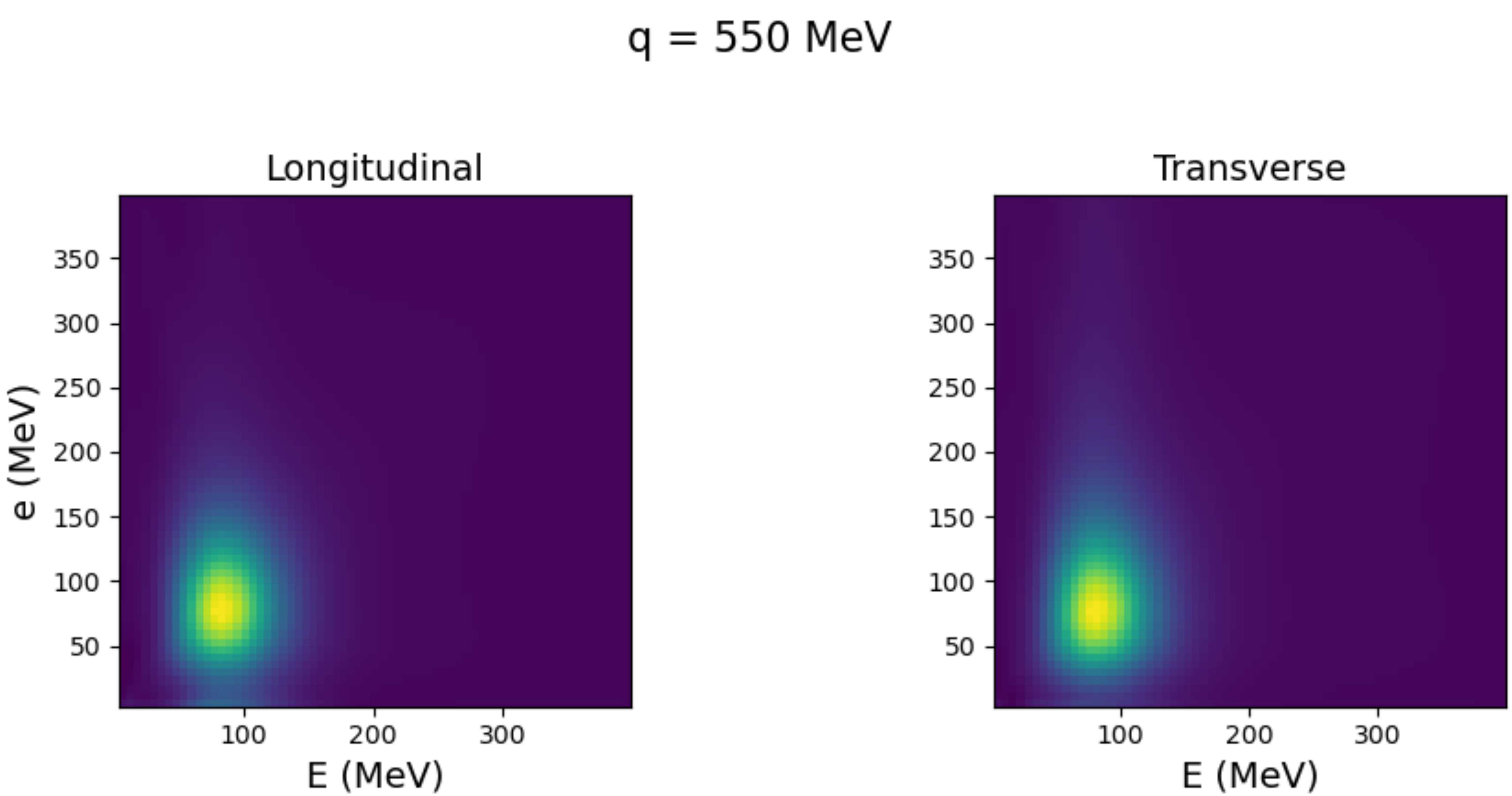
- STA responses used to build the cross sections
- Cross sections are used to generate events in GENIE
- Electromagnetic processes (for which data are available) are used to validate the generator

Barrow, Gardiner, Pastore, Betancourt et al. PRD 103 (2021) 5, 052001

GENIE HadronTensorModelI Class: https://internal.dunescience.org/doxygen/classgenie_1_1HadronTensorModelI.html

GENIE validation using e-scattering

- Directly use r**esponse densities** in GENIE event generator
- Moment morphing technique to generate response densities for arbitrary values of e, E, q



Conclusion:

- The STA responses (and cross sections) for $^{3}He,^{4}He,^{12}C$ are in good agreement with the data, and are accurate up to moderate values of q (and consequently to moderate values of incoming electron beam for cross sections calculations)
- Properly including relativistic **currents** and **kinematical effects** extends the range of our calculations to higher values of q
- It is a promising method to describe electromagnetic scattering from $A \geq 12$ accounting for two-body physics, both **currents** and **correlations**, in the Quasielastic regime and beyond

Currently working/future projects:

- Neutrino-nucleus scattering
- Incorporate **pion production** within the STA formalism
- Use of information from **response densities** in event generators: collaboration with GENIE Monte Carlo event generator
- Address **heavier nuclei:** the STA is exportable to other QMC methods to address larger nuclei, e.g. AFDMC

Thank you!

Collaborators:

ODU: A. Gnech

WashU: G. Chambers-Wall, R. Weiss, S. Pastore and M. Piarulli

LANL: G. King, S. Gandolfi, J. Carlson

ANL: R. B. Wiringa

FNAL: J. Barrow, M. Betancourt, S. Gardiner