Asymptotic Theory for Short-Range Correlations

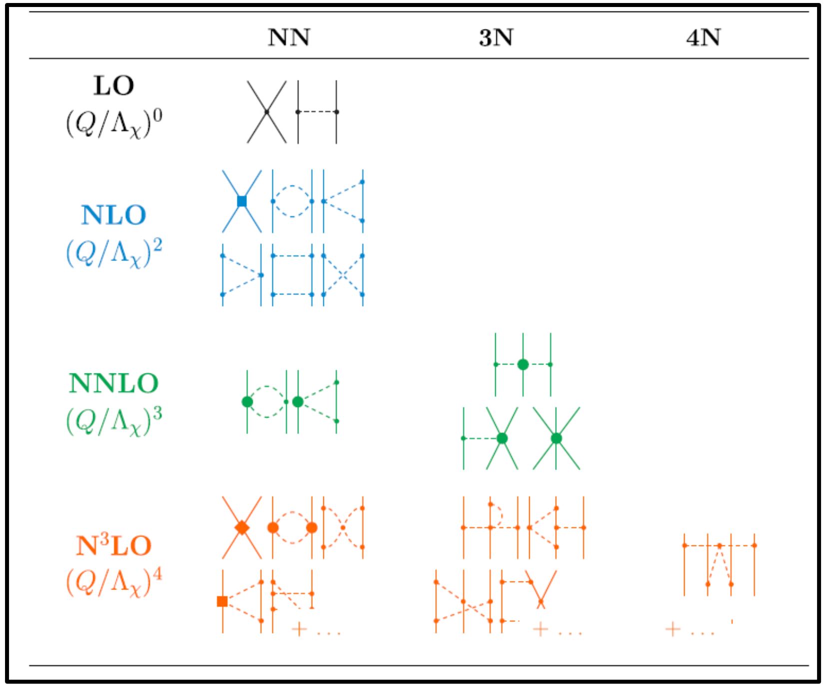
Ronen Weiss

Washington University in St. Louis

CIPANP 2025, June 10, 2025

Nuclear Systems: Theory

- **Significant progress** in the last years (numerical methods, computing power, interaction models)



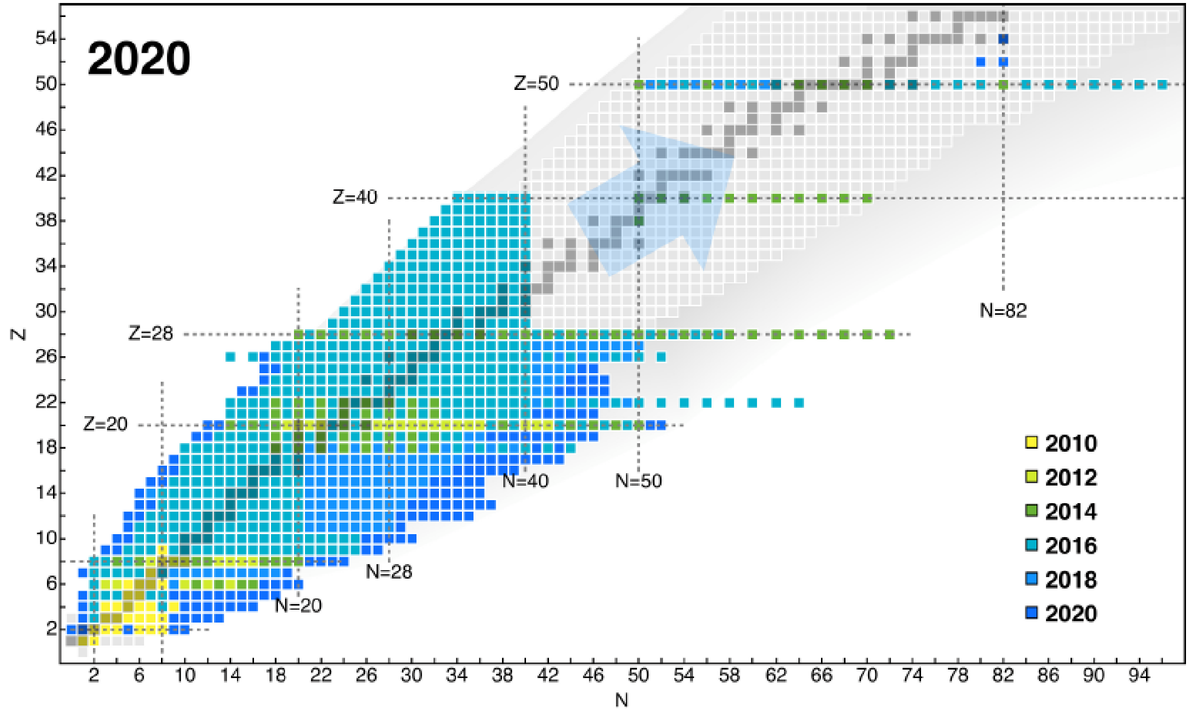


FIGURE 1 | Progress in *ab initio* nuclear structure calculations over the past decade. The blue arrow indicates nuclei that will become accessible with new advances for open-shell nuclei in the very near term (see section 2.3).

H. Hergert, Frontiers in Physics 8, 379 (2020)

Nuclear Systems: Theory

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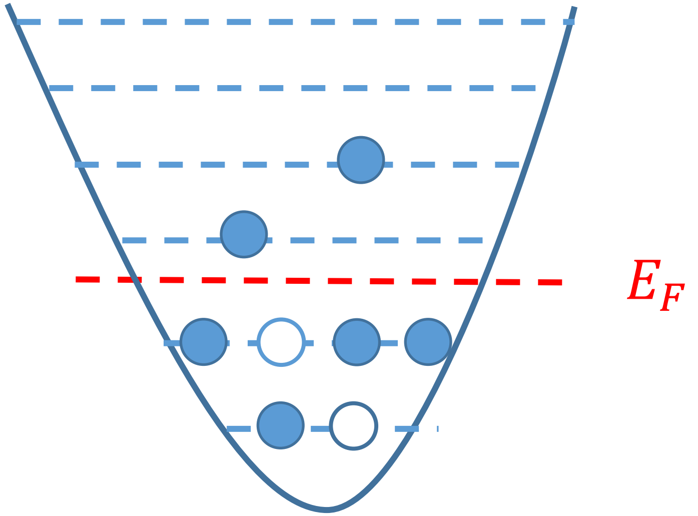
- **But**,
  - **Structure calculations** beyond medium-mass nuclei: **limited to "soft" interaction models**



Nuclear Systems: Theory

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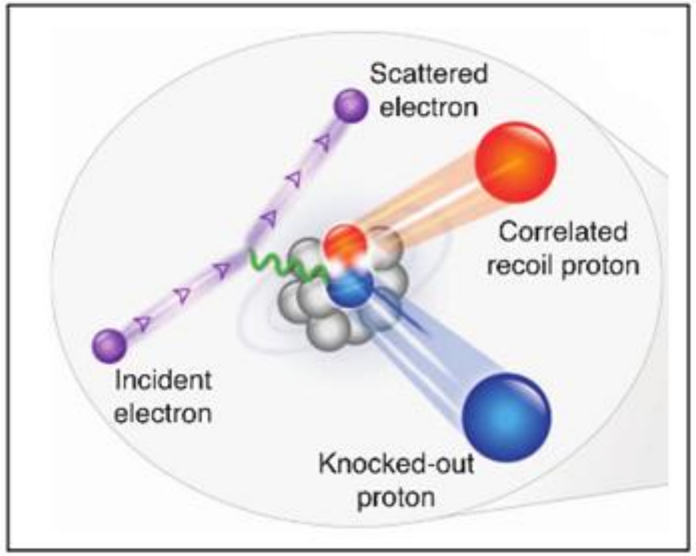
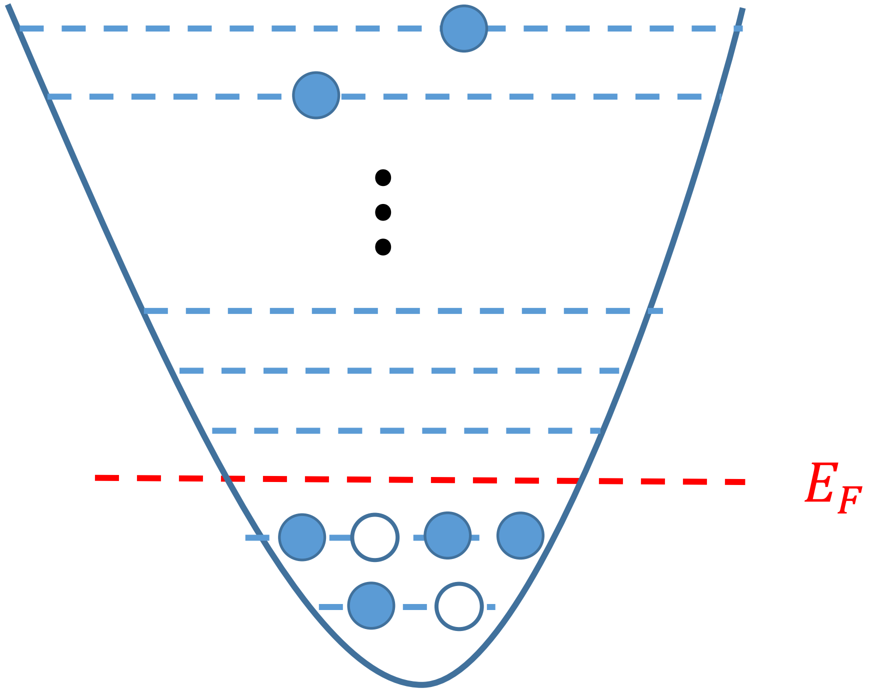
- **But**,
  - **Structure calculations** beyond medium-mass nuclei: **limited to "soft" interaction models**
  - Many **reactions** are **limited to light nuclei** or involve uncontrolled approximations



O. Hen et al., Science 346, 614 (2014)

Research Focus

- **My current focus:**
  - **Short-range physics** (high energy excitations, universality)



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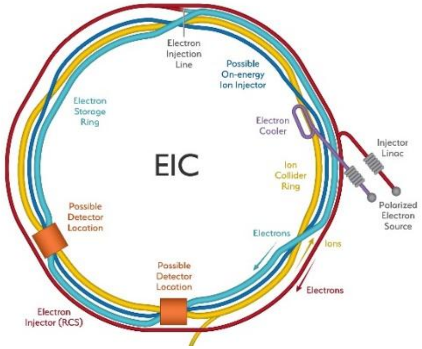
- Advances are needed to **support major experimental efforts**



Neutrino oscillation experiments

Neutrinoless double beta decay

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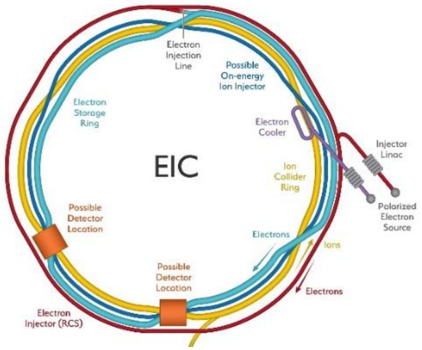
  - **Quantum computing** – Algorithms for reactions
  - **Classical computing methods** (for reactions) ⇦ See talk by Lorenzo Andreoli: Wed 4:30
- Advances are needed to **support major experimental efforts**

FRIB



Jefferson Lab

GSI

DUNE

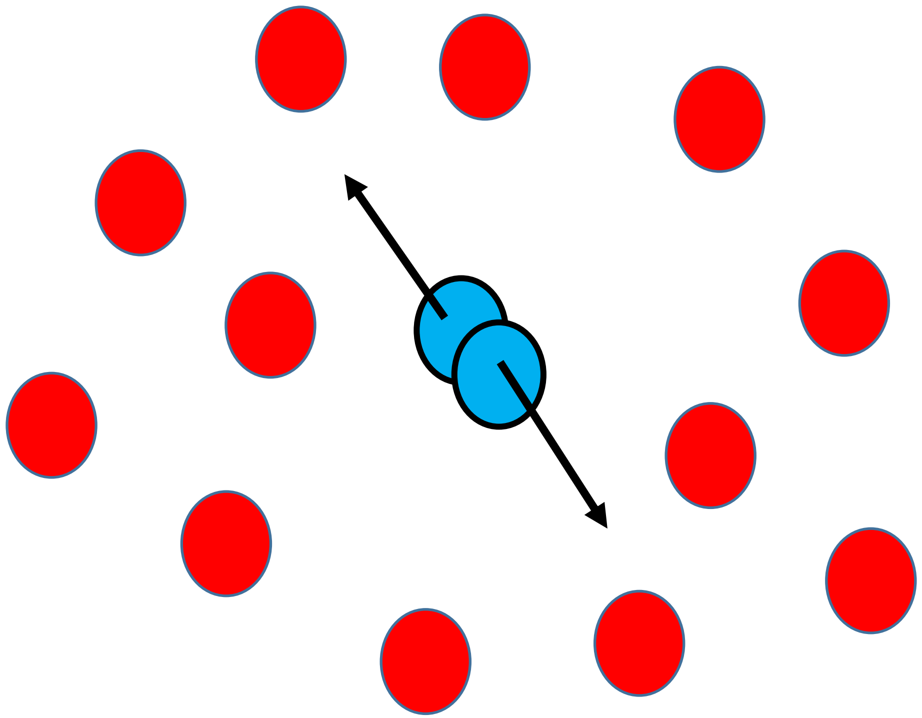
Neutrino oscillation experiments

Neutrinoless double beta decay

Short-range physics

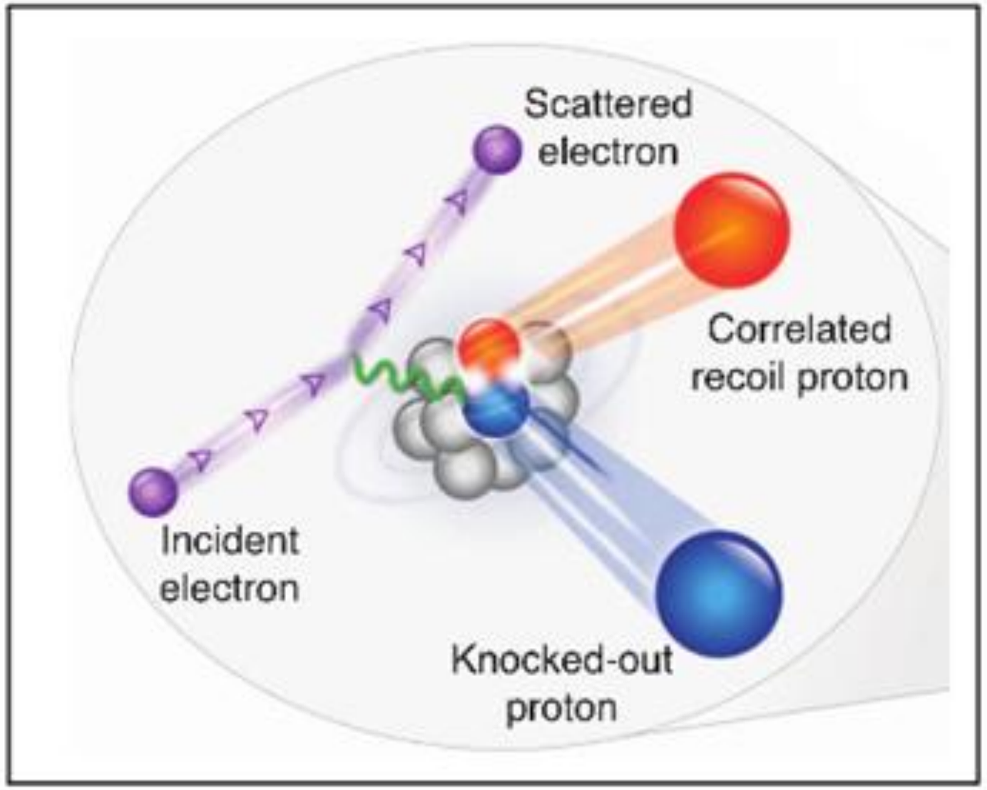
Short-range correlations (SRCs)

What happens when few particles get close to each other?



SRCs in Nuclear Systems

Studied **experimentally** using large momentum transfer quasi-elastic reactions



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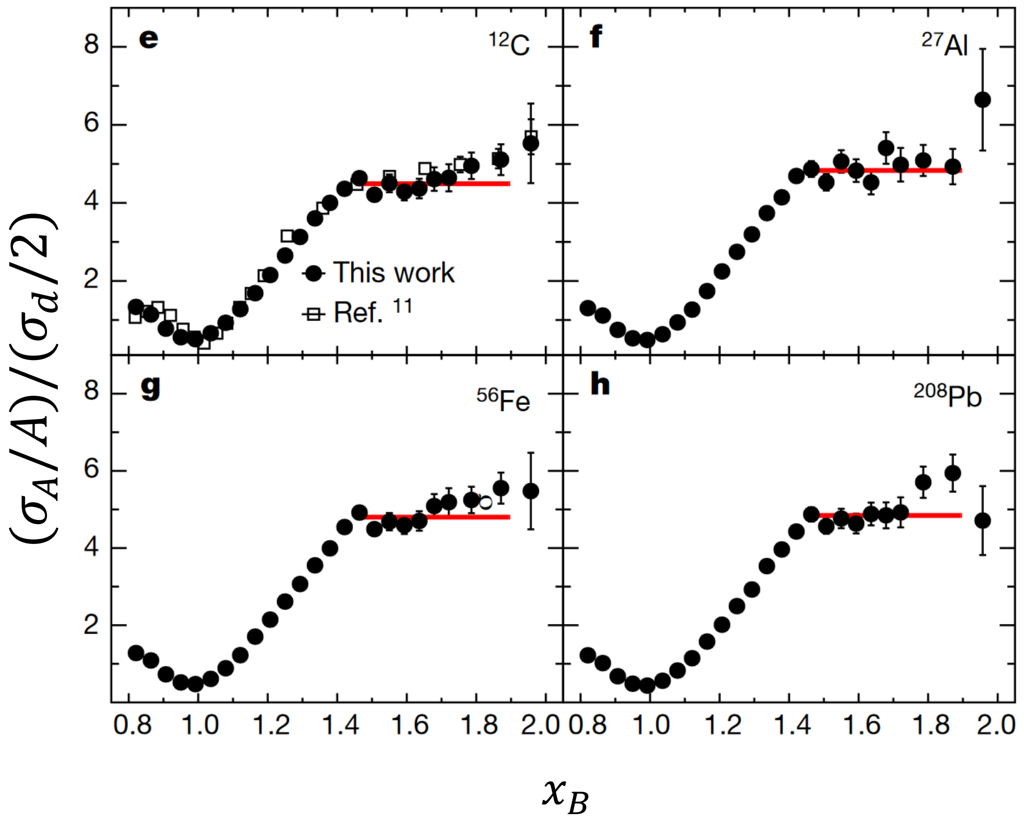

B. Schmookler et. al. (CLAS Collaboration), Nature 566, 354 (2019)

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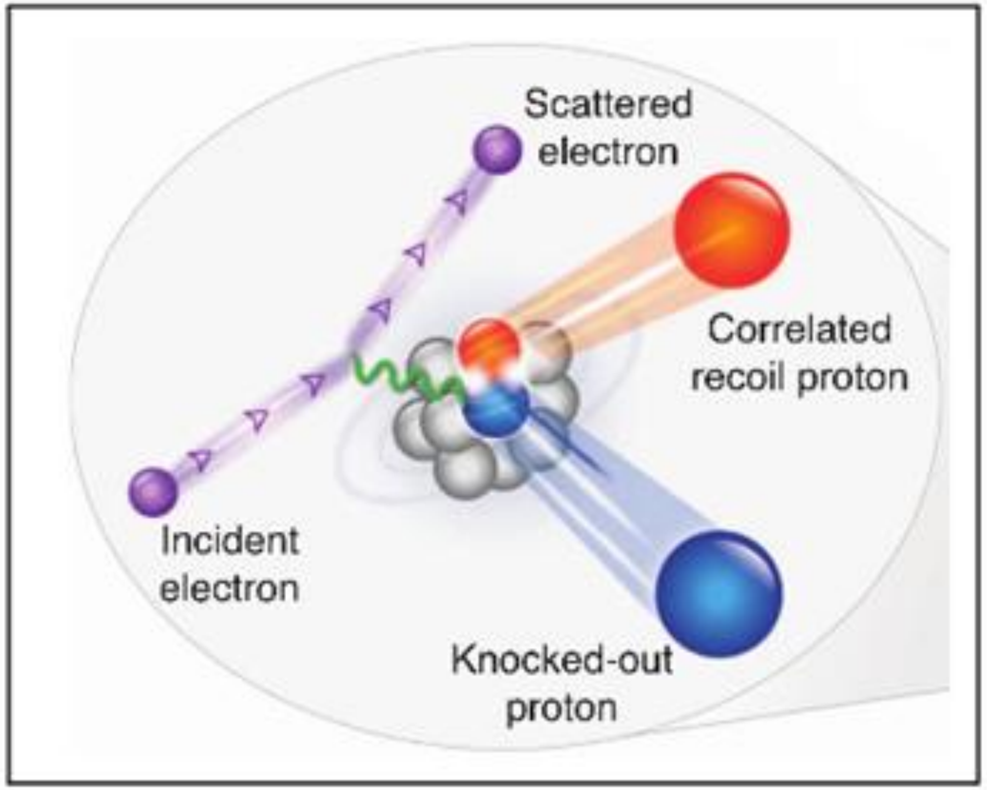
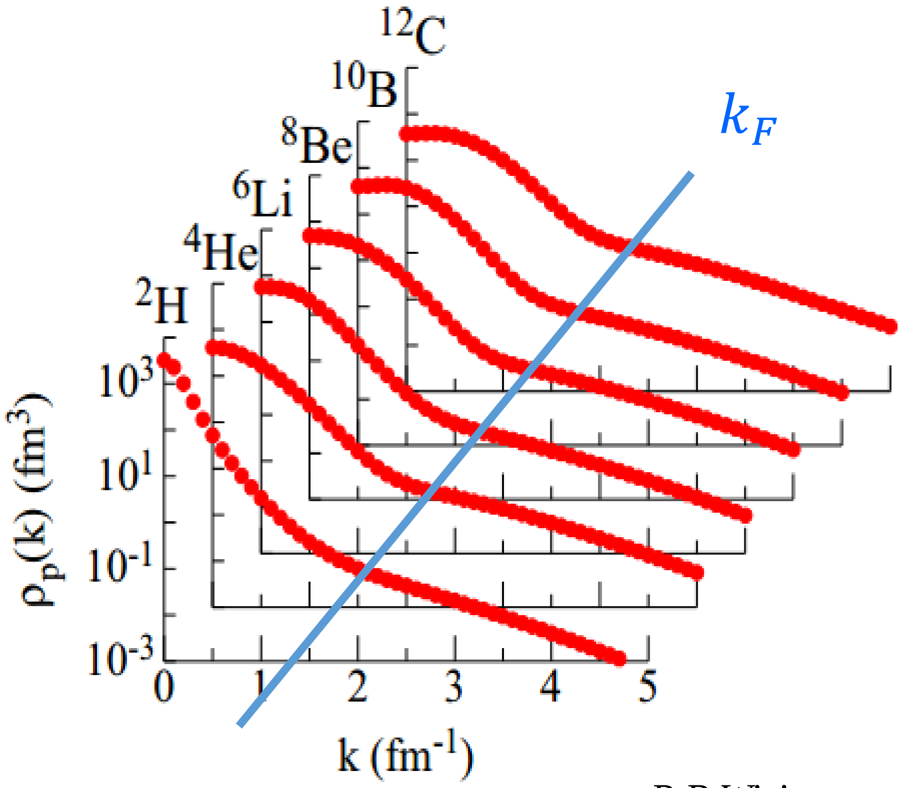


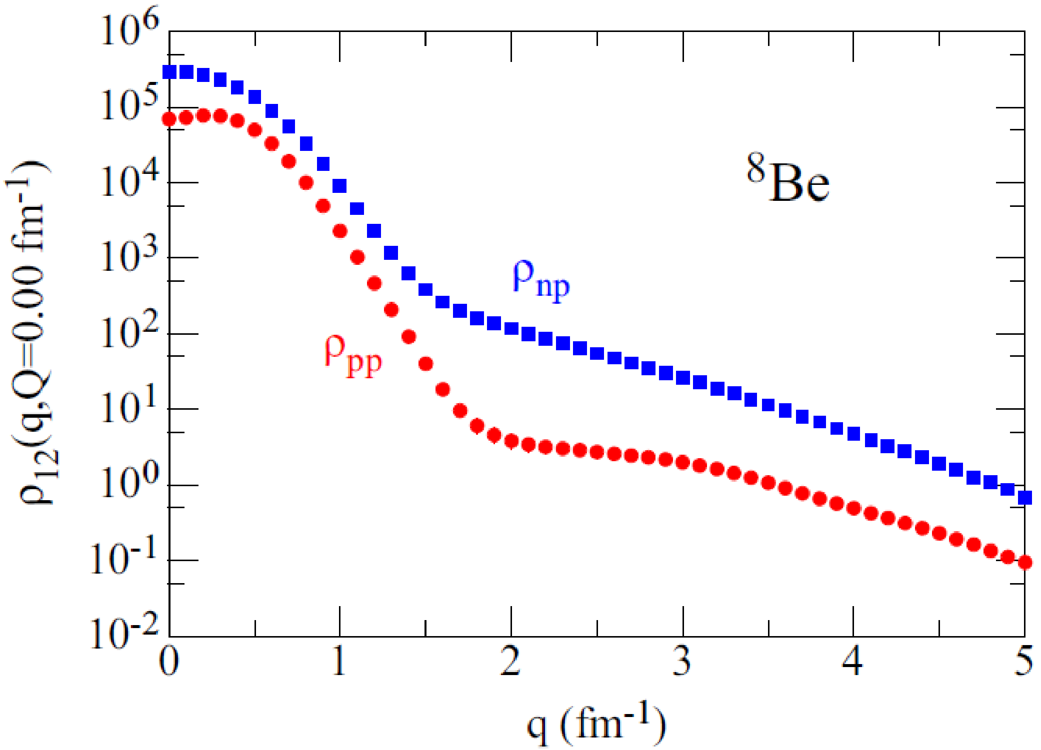
O. Hen et al., Science 346, 614 (2014)

- **High momentum** particles with **back-to-back** configuration
- Universal behavior – **"isolated pair"**
- Neutron-proton dominance
- Significant **deviations from mean-field models**

SRCs in Nuclear Systems

Similar features are seen in **ab-initio calculations**:





R.B Wiringa et. al., Phys. Rev. C 89, 024305 (2014)

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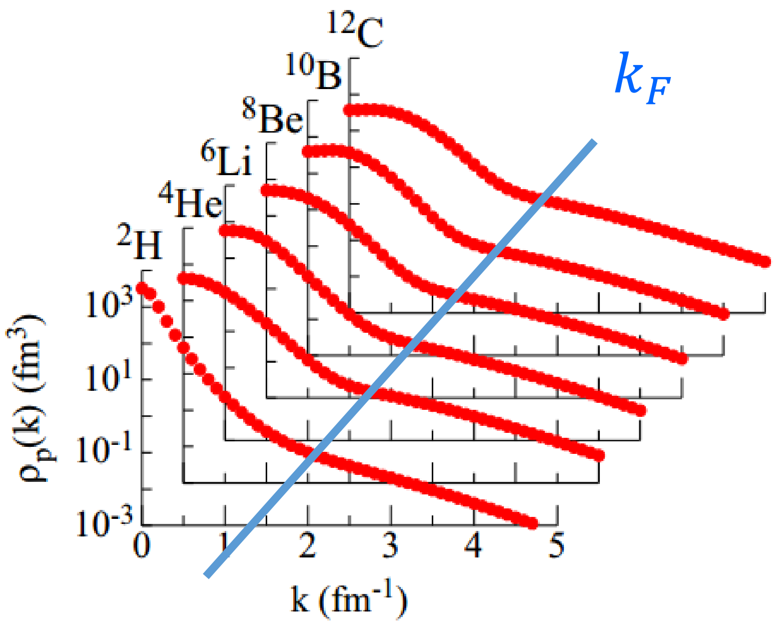

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- **Ground-state properties**
- **Limited to light nuclei**
- **Do not describe relevant reactions**

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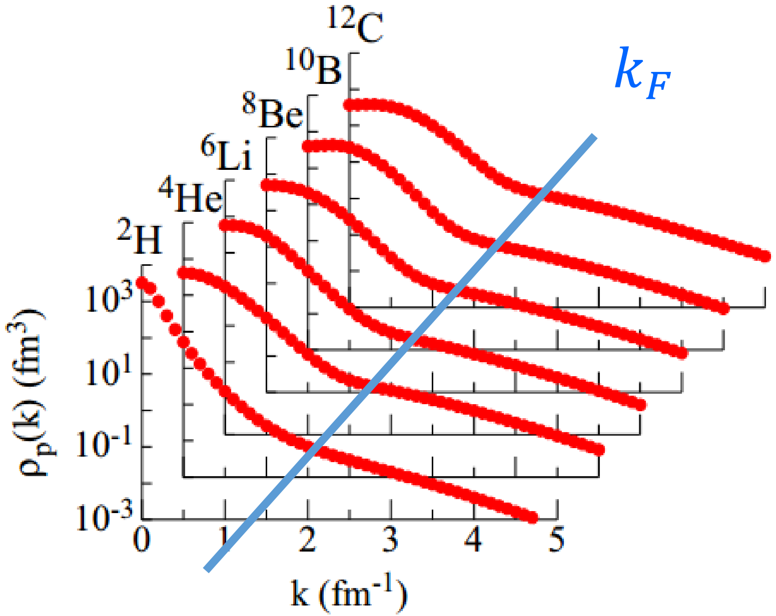

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- **Ground-state properties**
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- **How to compare theory and experiments?**
- **How to utilize information regarding SRCs?**

Towards a systematic short-range description

RW, D. Lonardoni, S. Gandolfi, PLB 857, 138974 (2024)

Short-range expansion – two-body system

- **The two-body system:**

$$\left[-\frac{\hbar^2}{m}\nabla^2 + V(r)\right]\varphi^E(r) = E\varphi^E(r)$$

- **For $r \to 0$:** The energy becomes negligible $E \ll \frac{\hbar^2}{mr^2}$

$$\varphi^E(r) = \varphi^{E=0}(r)$$

Two particles close together behave in the same way, regardless of the energy

(in a two-body system)

Short-range expansion – two-body system

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- Taylor expansion around $E = 0$:

$$\varphi^E(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$

Short-range expansion – two-body system

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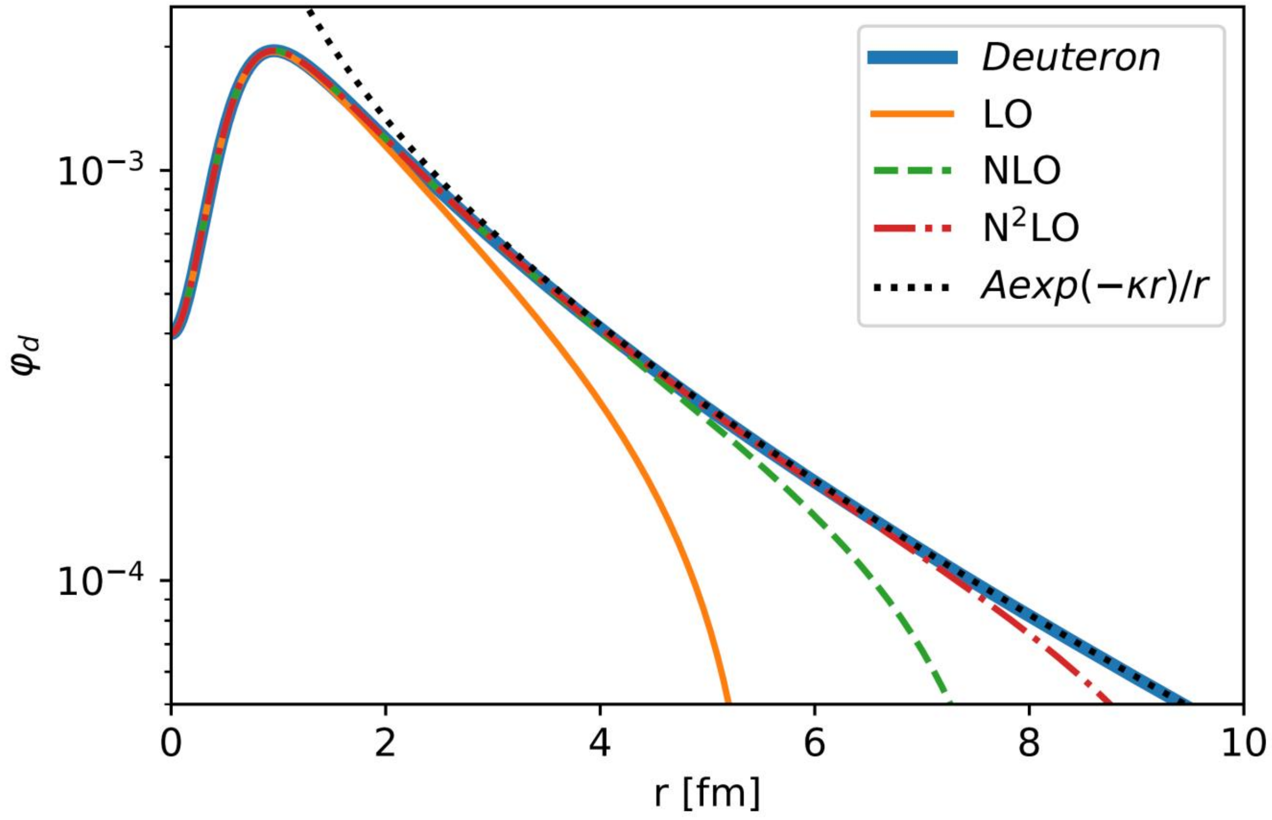

AV4'
Deuteron channel
Bound state

Short-range expansion – many-body system

- **The many-body case:** Exact expansion

$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots, \boldsymbol{r}_A) = \sum_{E,\alpha} \varphi_\alpha^E(\boldsymbol{r}_{12}) A_\alpha^E(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \ldots, \boldsymbol{r}_A)$$

(α – quantum numbers)

Complete set of
two-body functions

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$$\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \dots, \boldsymbol{r}_A) = \sum_{\alpha} \varphi_\alpha^{E=0}(\boldsymbol{r}_{12}) A_\alpha^{(0)}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \boldsymbol{r}_4, \dots) + \sum_{\alpha}\left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right) A_\alpha^{(1)}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \boldsymbol{r}_4, \dots) + \dots$$

Leading order factorization

Subleading terms

Short-range expansion – many-body system

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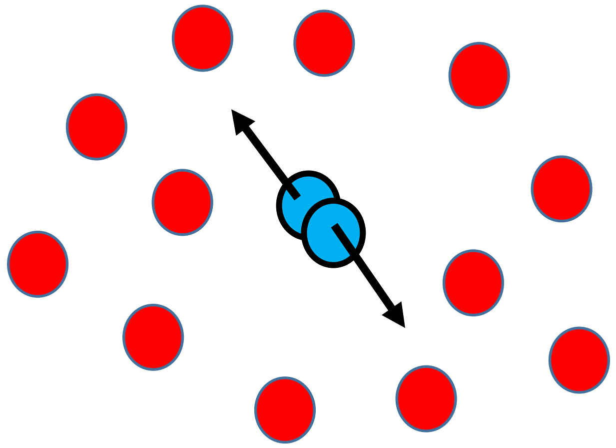

Short-range expansion – many-body system

- **The many-body case:** (α – quantum numbers)

$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots,\boldsymbol{r}_A)=\sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12})A_\alpha^{(0)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\boldsymbol{r}_4,\dots)+\sum_\alpha\left(\frac{d}{dE}\varphi_\alpha^{E=0}(\boldsymbol{r})\right)A_\alpha^{(1)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\boldsymbol{r}_4,\dots)+\dots$$

Leading order factorization — Subleading terms

- **Two-body observables:**

$$\langle\Psi|\hat{O}_2|\Psi\rangle=\sum_{\alpha,\beta}\left(\langle\varphi_\alpha^{E=0}(r)|\hat{O}_2|\varphi_\beta^{E=0}(r)\rangle C_{\alpha\beta}^{00}+\langle\varphi_\alpha^{E=0}(r)|\hat{O}_2|\frac{d}{dE}\varphi_\beta^{E=0}(r)\rangle C_{\alpha\beta}^{01}+\cdots\right)$$

- **Nuclear "contacts":**

$$C_{\alpha\beta}^{mn}\propto\langle A_\alpha^{(m)}|A_\beta^{(n)}\rangle$$

See original definition of the contact:
S. Tan, Ann. Phys. (N.Y.) 323, 2952 (2008)

Short-range expansion – many-body system

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- Two-body dynamics
- Universal for all nuclei

- Depends on the nucleus
- Independent of the operator

Short-range expansion – many-body system

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- **Power counting** is needed
- Two relevant parameters:
  - Number of energy derivatives
  - Orbital angular momentum $(s, p, d, \ldots)$
- Can be analyzed analytically for the two-body system

Short-range expansion – many-body system

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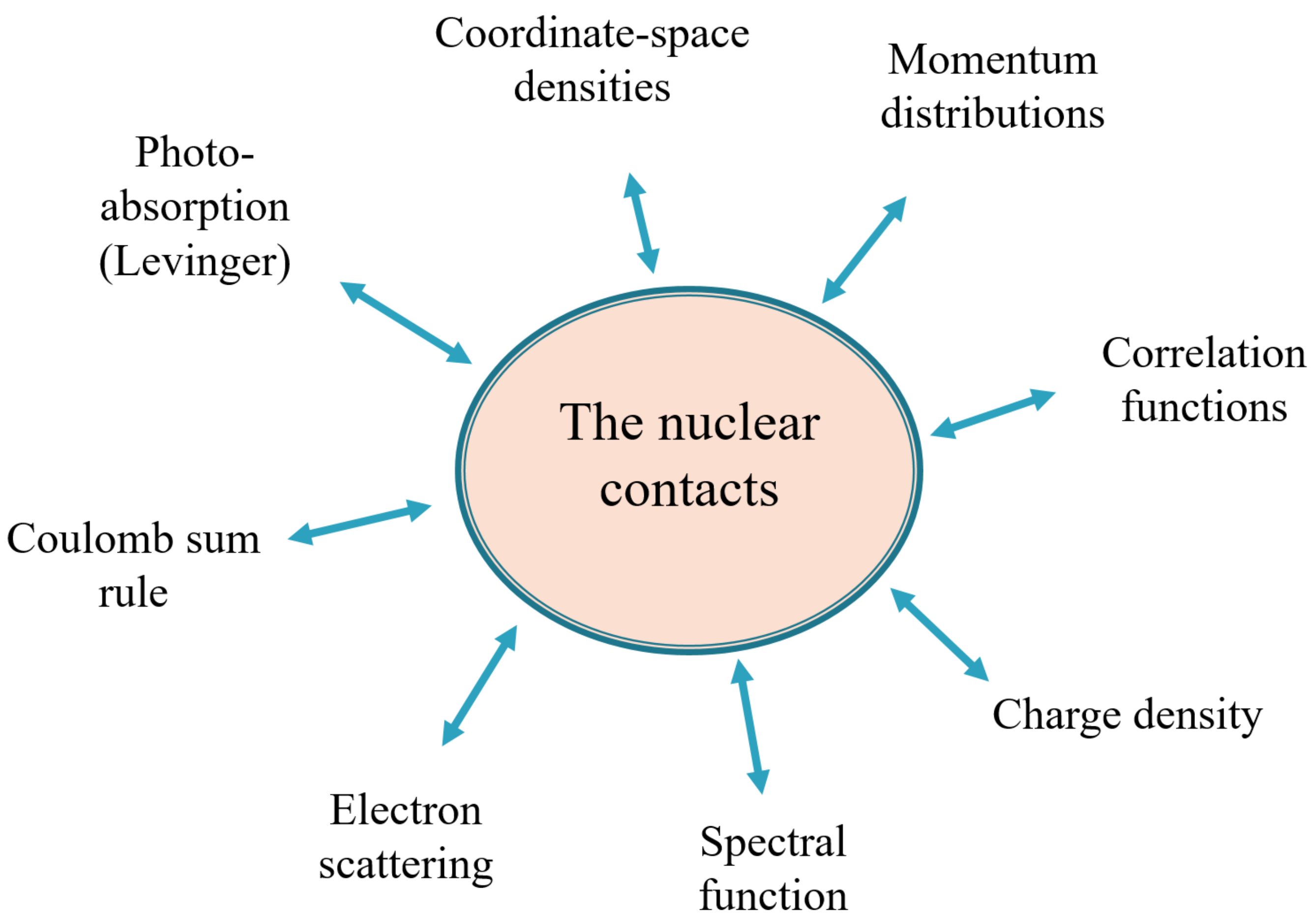
Leading order channels

spin-one: $\ell_2 = 0,2\,;\, s_2 = 1\,;\, j_2 = 1\,;\, t_2 = 0$ (only **np pairs**)

spin-zero: $\ell_2 = 0\,;\, s_2 = 0\,;\, j_2 = 0\,;\, t_2 = 1$ (**All pairs**)

The nuclear contact relations



RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

RW, B. Bazak, N. Barnea, PRL 114, 012501 (2015)

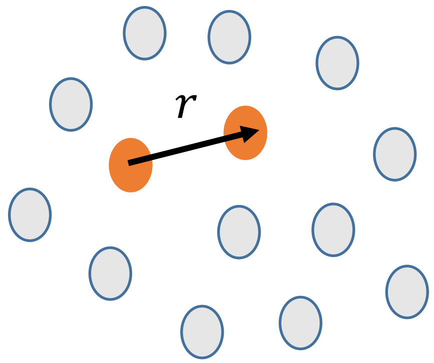
RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

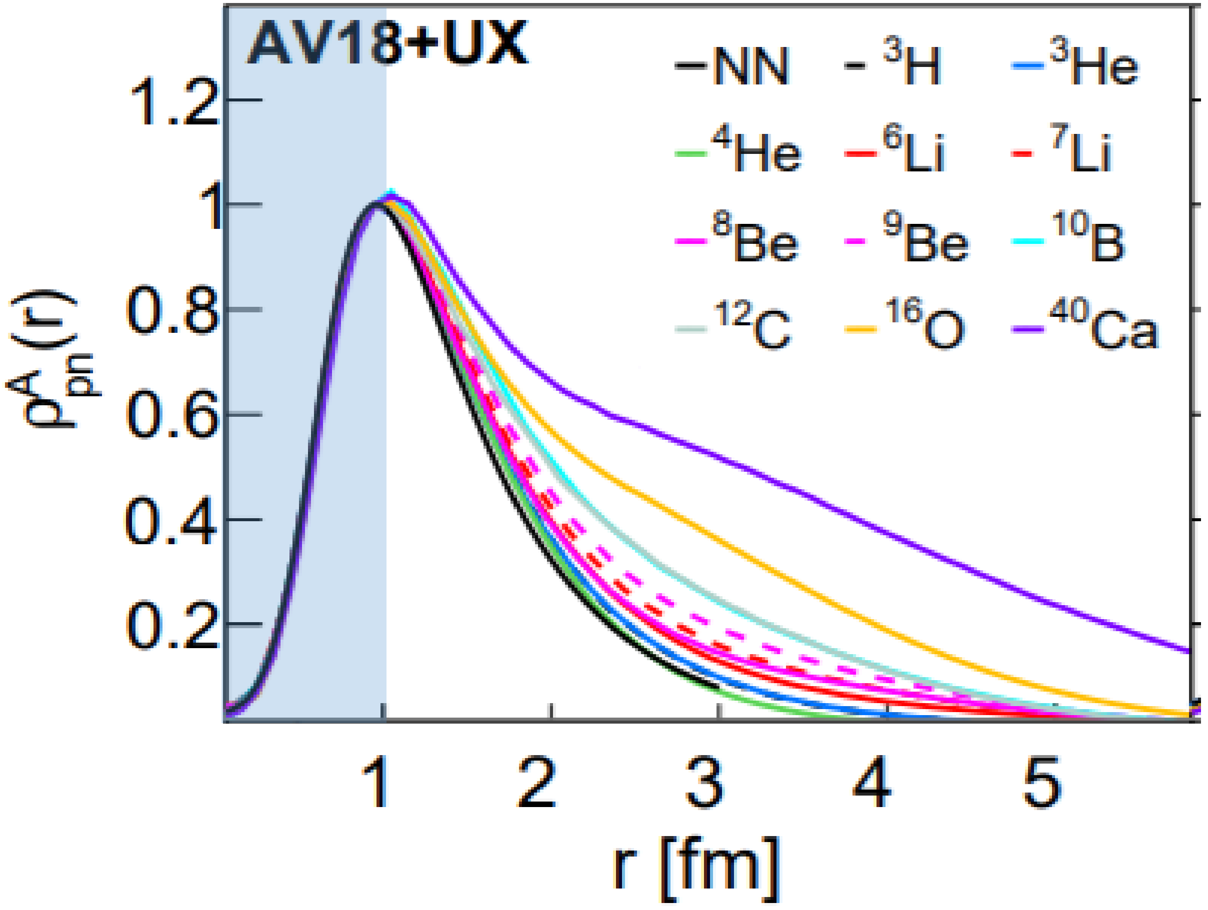
A. Schmidt, J.R. Pybus, RW, et al., Nature 578, 540 (2020)

Two-body density



$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C$$

$$\rho_{NN}(\boldsymbol{r}) \xrightarrow{r \to 0} C |\varphi(\boldsymbol{r})|^2$$



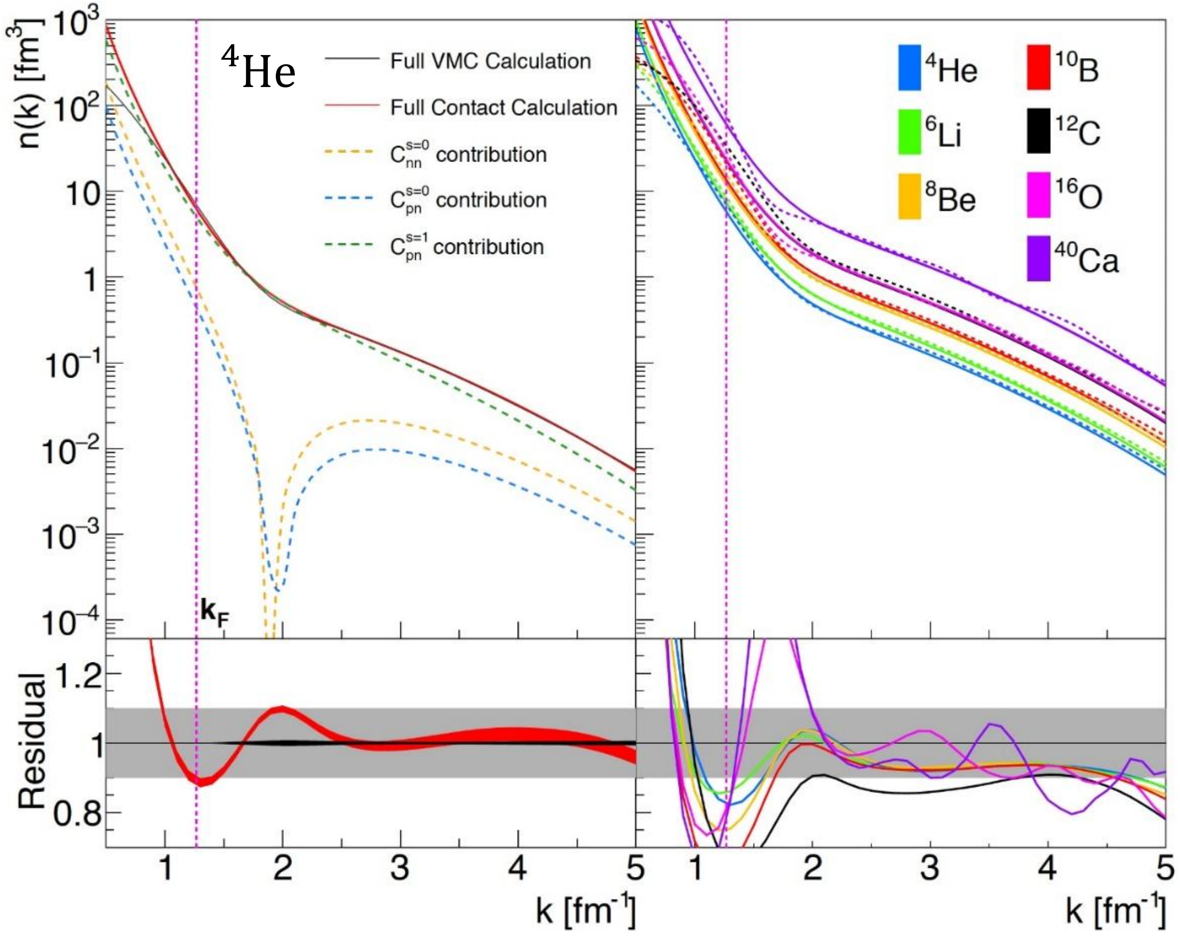
Shows the validity of the factorization (LO term)

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

One-body momentum distribution

$$n_p(k) \xrightarrow{k\to\infty} C_{pn}^{1}\left|\tilde{\varphi}_{pn}^{1}(k)\right|^2 + C_{pn}^{0}\left|\tilde{\varphi}_{pn}^{0}(k)\right|^2 + 2C_{pp}^{0}\left|\tilde{\varphi}_{pp}^{0}(k)\right|^2$$

$n_p(k)$

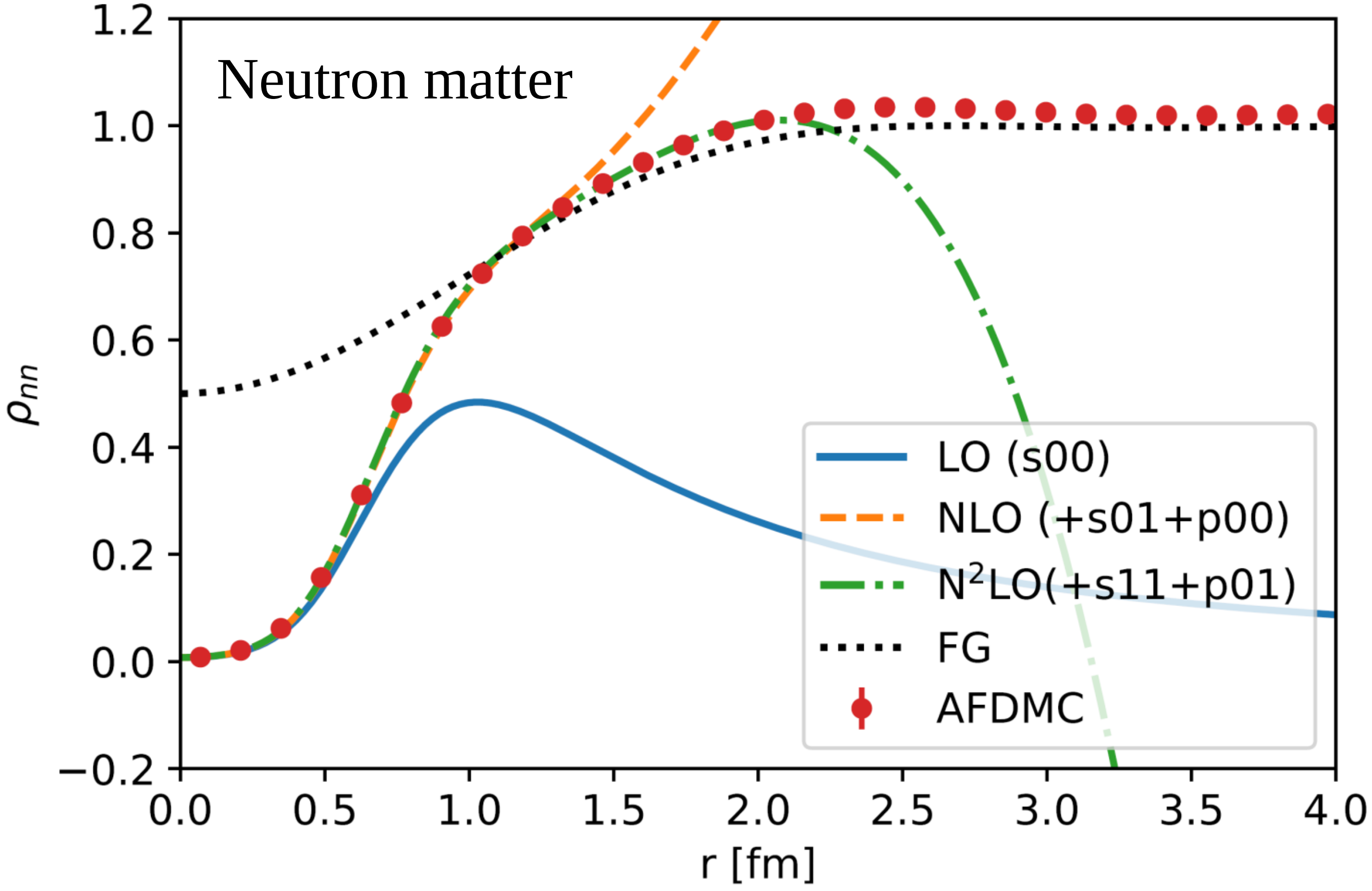


No free parameters!

RW, R. Cruz-Torres, N. Barnea, E. Piasetzky and O. Hen, PLB 780, 211 (2018)

Beyond leading order

$$\langle\Psi|\hat{O}_2|\Psi\rangle = \sum_\alpha \left(\langle\varphi_\alpha^{E=0}(r)|\hat{O}_2|\varphi_\alpha^{E=0}(r)\rangle C_\alpha^{00} + \langle\varphi_\alpha^{E=0}(r)|\hat{O}_2|\frac{d}{dE}\varphi_\alpha^{E=0}(r)\rangle C_\alpha^{01} + \cdots \right)$$



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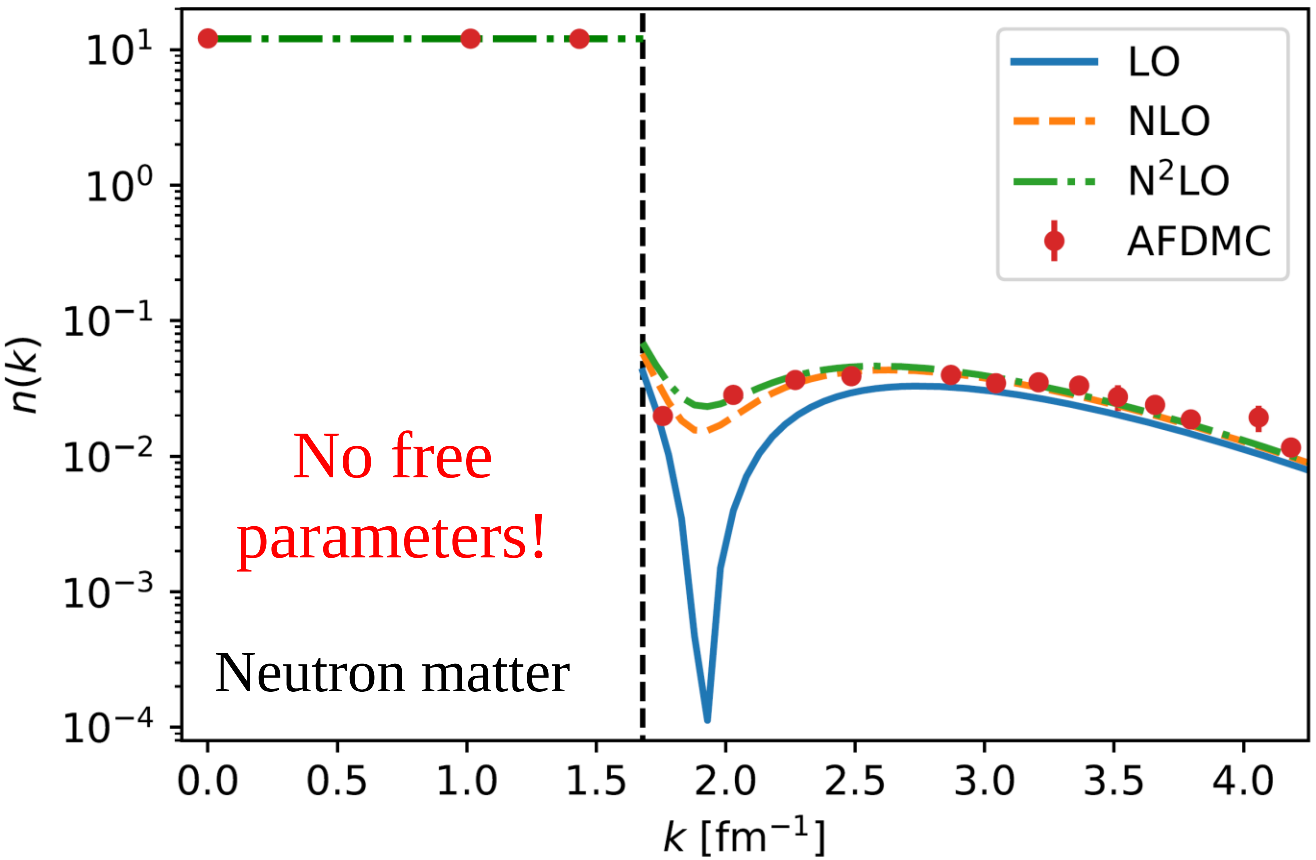
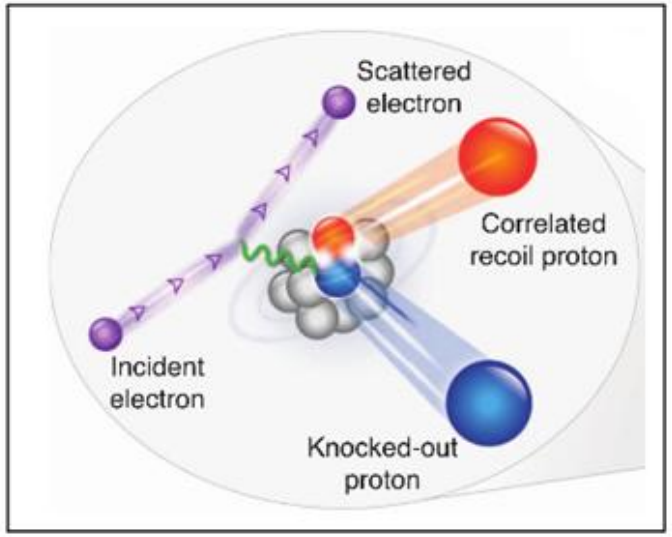


RW, D. Lonardoni, S. Gandolfi,
PLB 857, 138974 (2024)

Electron-scattering experiments

- $A(e,e'N)$ and $A(e,e'NN)$ cross sections

- $S(\boldsymbol{p_1},\epsilon_1)$ = **spectral function**
  The probability to find nucleon with momentum $\boldsymbol{p}_1$ and energy ϵ_1 in the nucleus



$$S^N(\boldsymbol{p_1},\epsilon_1) = \sum_{s_1}\sum_f \delta\left(\epsilon_1 + E_f^{A-1} - E_i^A\right)\left|\langle\Psi_f^{A-1}|a_{\boldsymbol{p}_1,s_1}^N|\Psi_i^A\rangle\right|^2$$

RW, I. Korover, E. Piasetzky, O. Hen and N. Barnea, PLB 791, 242 (2019)

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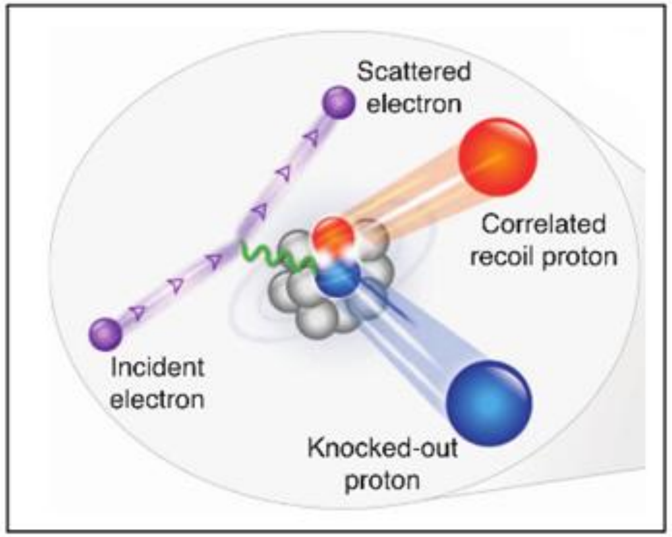

- Using the leading-order factorization:

$$S^p(\boldsymbol{p_1},\epsilon_1) \xrightarrow{p_1\to\infty} C^1_{pn}S^1_{pn}(\boldsymbol{p_1},\epsilon_1) + C^0_{pn}S^0_{pn}(\boldsymbol{p_1},\epsilon_1) + 2C^0_{pp}S^0_{pp}(\boldsymbol{p_1},\epsilon_1)$$

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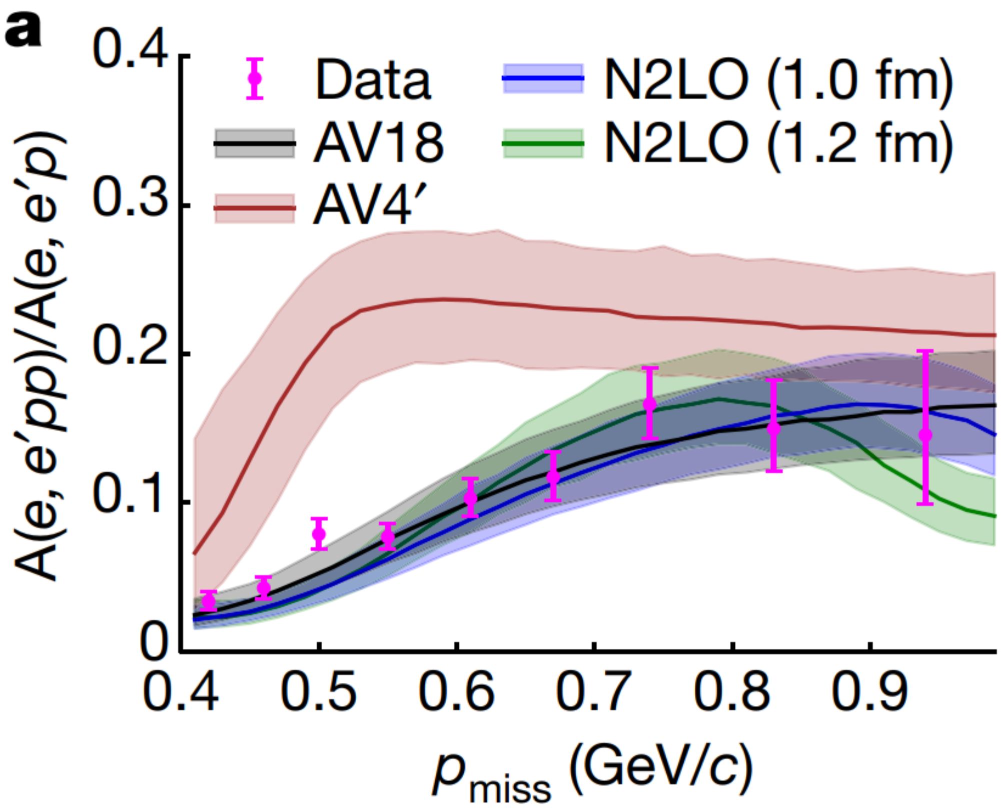
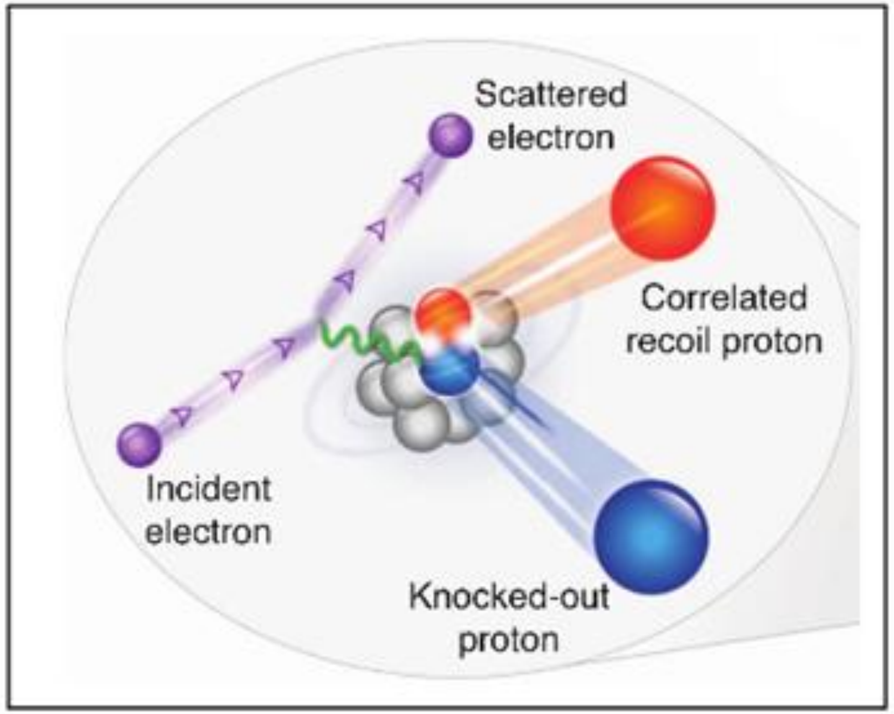
Electron-scattering experiments

- Good description of experimental data (^{12}C):





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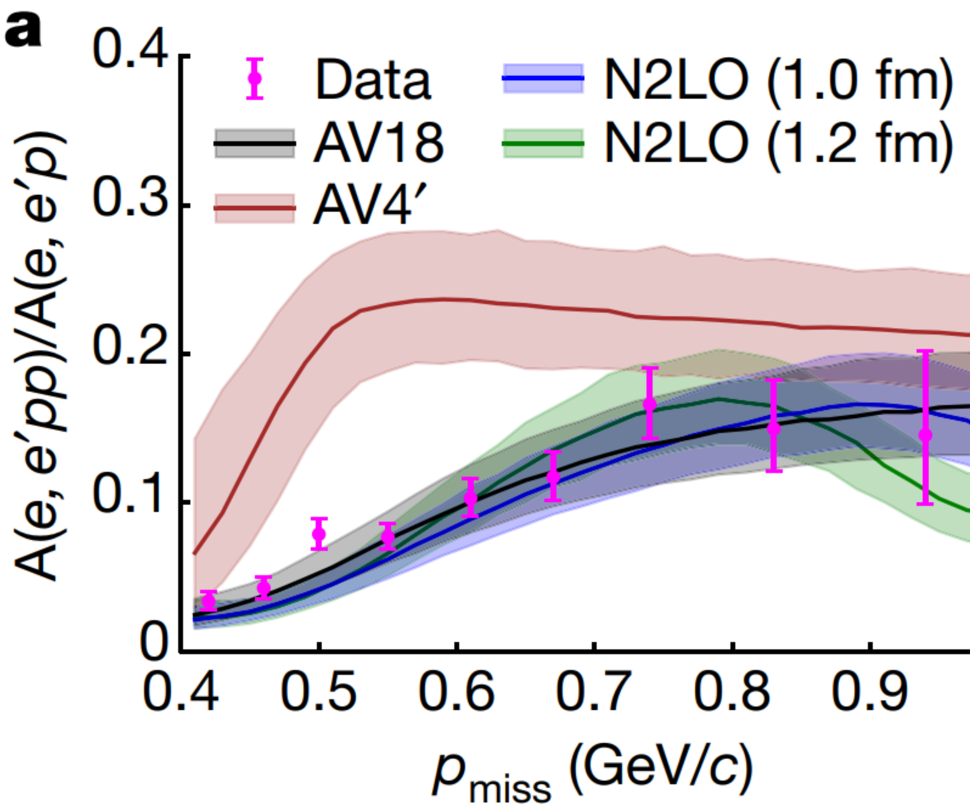

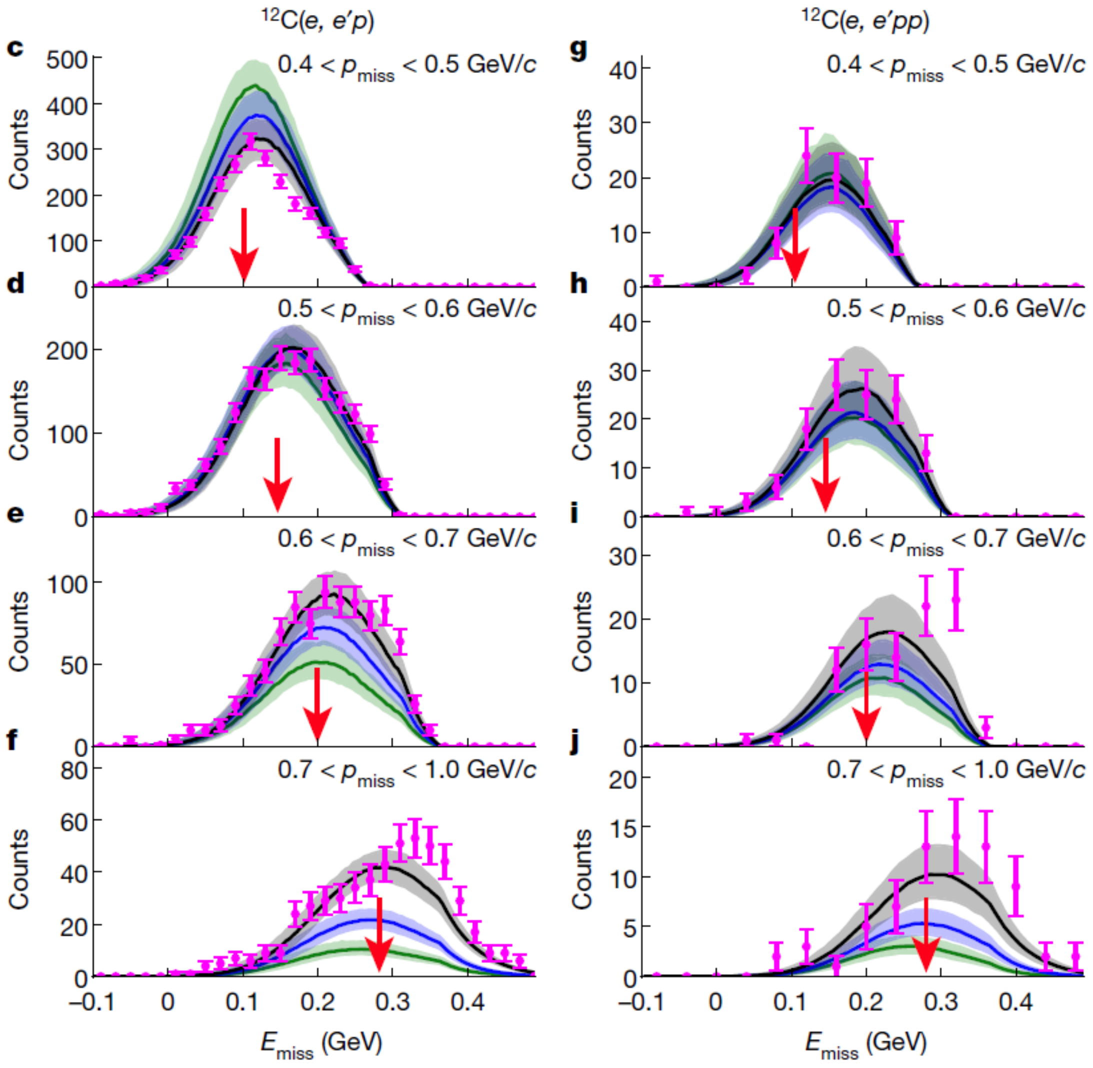

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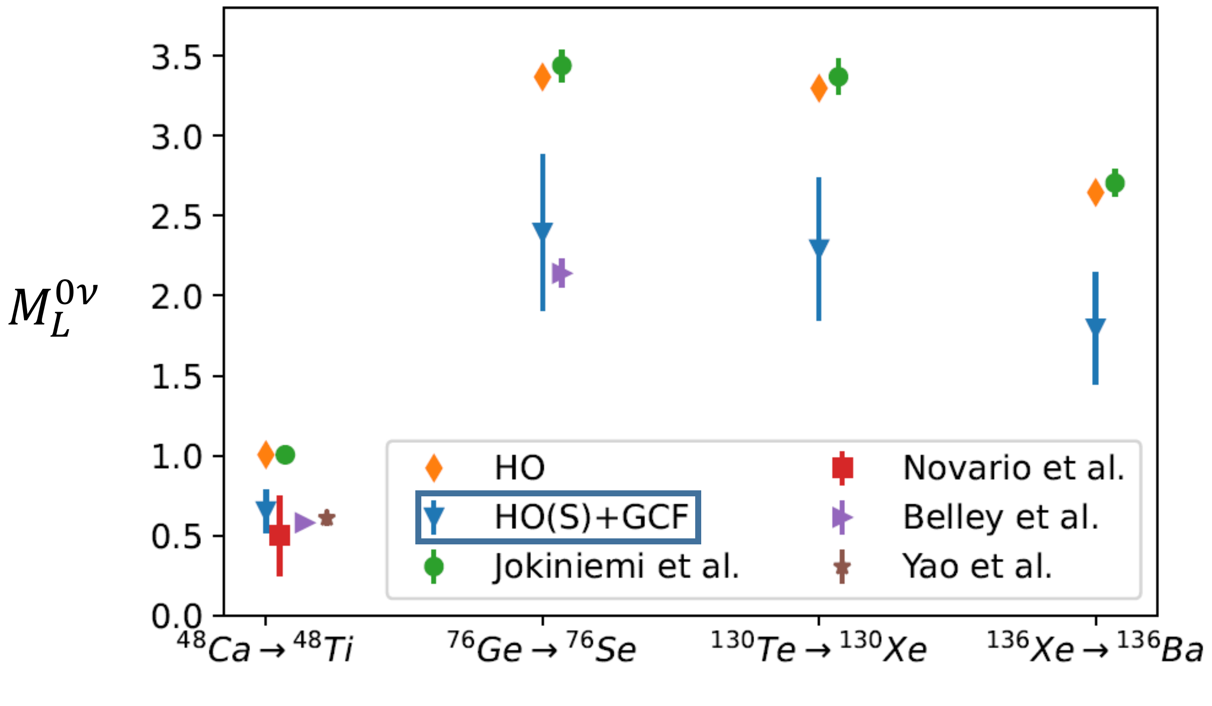
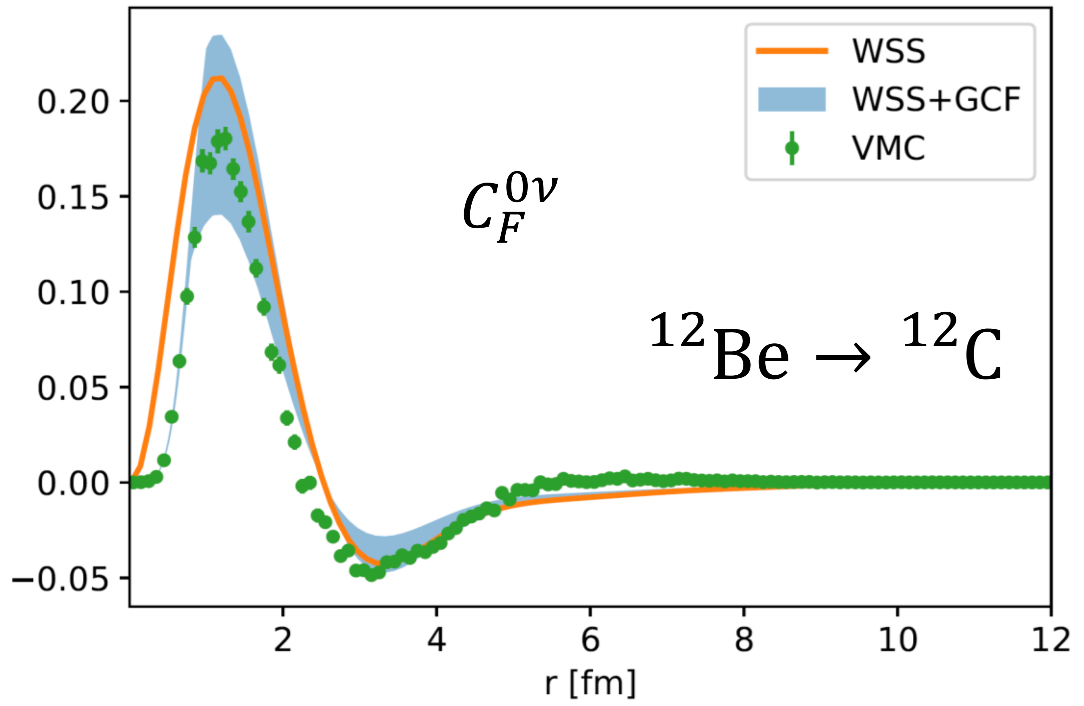




$$M_F + M_{GT} + M_T$$

Significant reduction due to SRCs

RW, P. Soriano, A. Lovato, J. Menendez, R. B. Wiringa, PRC 106, 065501 (2022)

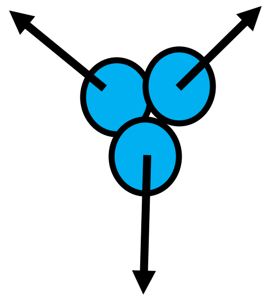
Three-body correlations

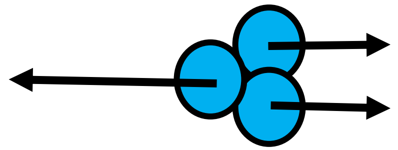
RW and S. Gandolfi, Phys. Rev. C 108, L021301 (2023)

Three-body correlations

Various open questions:

- What are the **dominant configurations**?
- Are 3N SRCs sensitive to the **three-body force**?
- Are they **universal**? What is their **abundance**?
- What is their **contribution to different observables**?

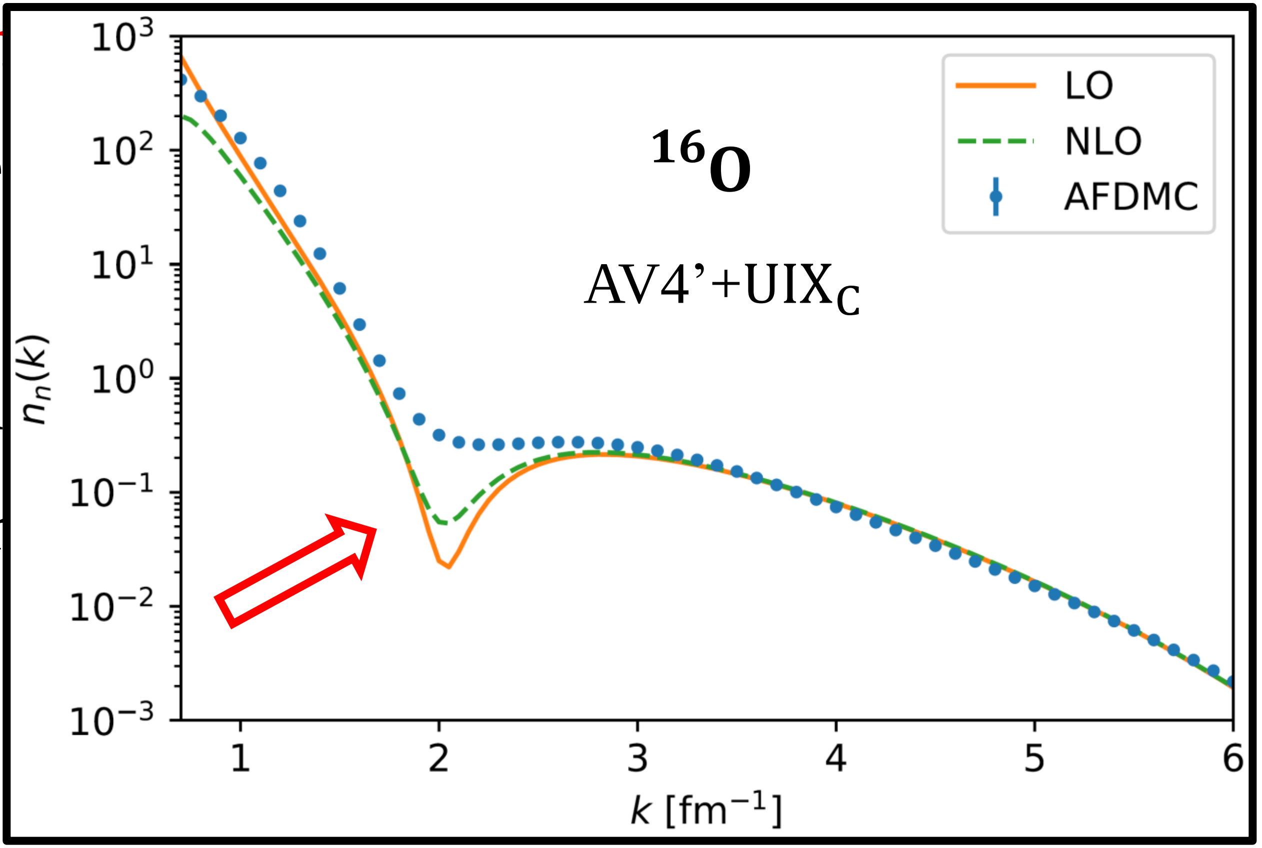


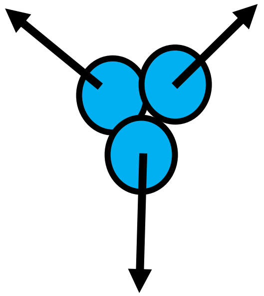


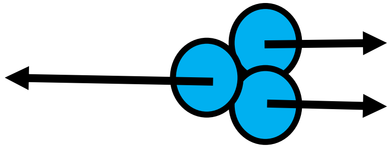
Three-body correlations

Various oper

- What are the
- Are 3N SRC
- Are they **uni**
- What is thei







Three-body correlations

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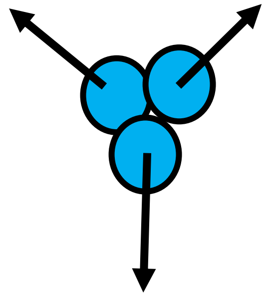
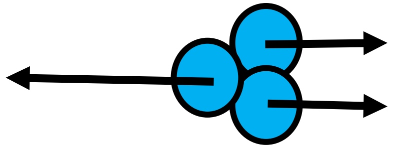
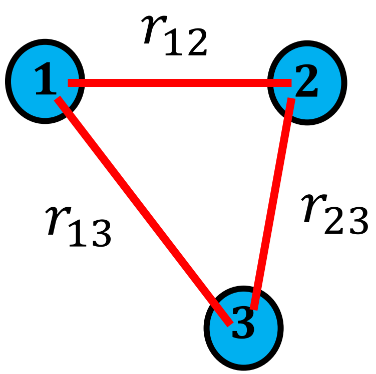




We performed first ab-initio calculations of 3N SRC

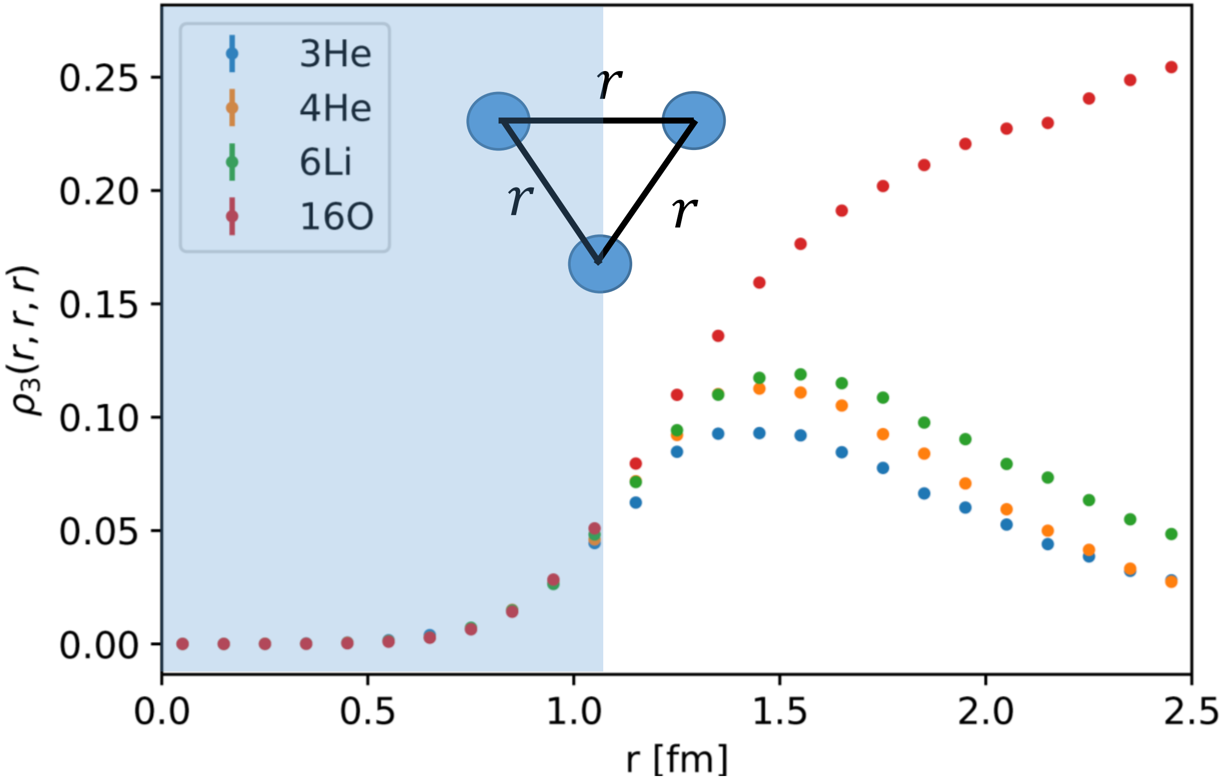
$$\rho_3(r_{12}, r_{13}, r_{23}) \equiv \binom{A}{3} \langle \Psi | \delta(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r_{12}) \delta(|\boldsymbol{r}_1 - \boldsymbol{r}_3| - r_{13}) \delta(|\boldsymbol{r}_2 - \boldsymbol{r}_3| - r_{23}) | \Psi \rangle$$

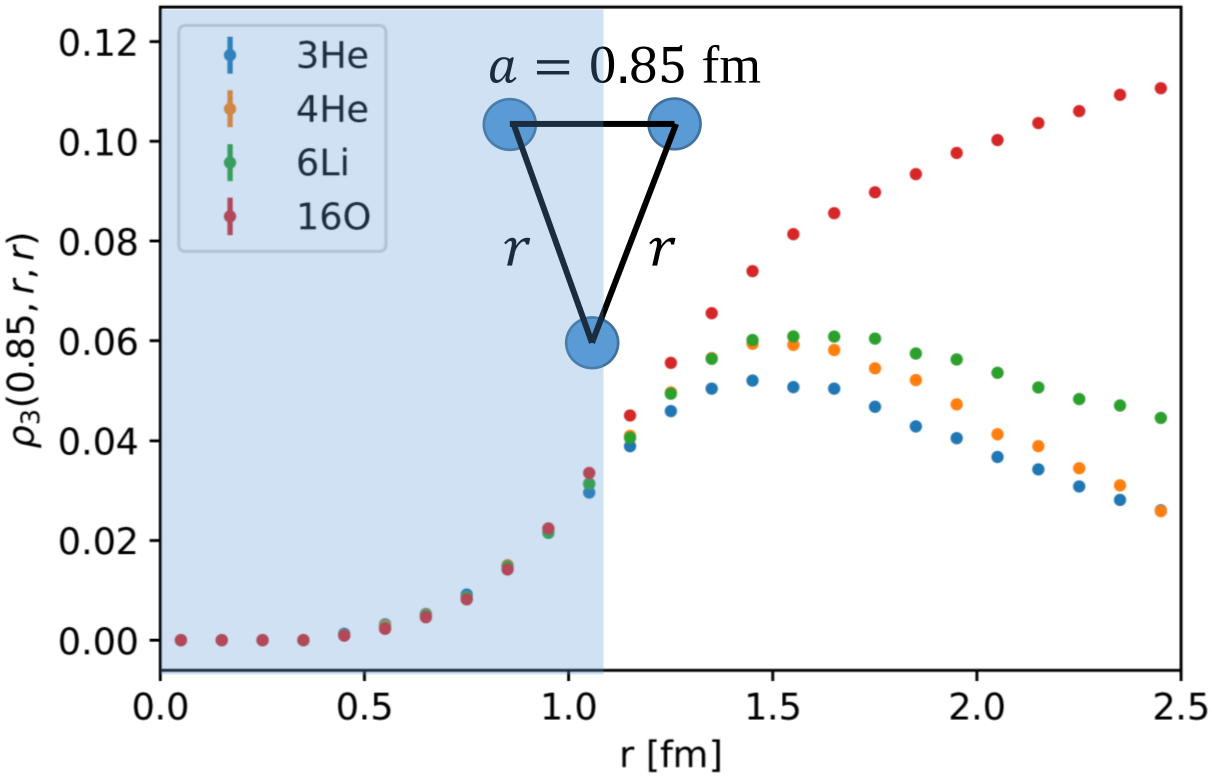


Three-body density

Universality

Same scaling factor for all geometries!





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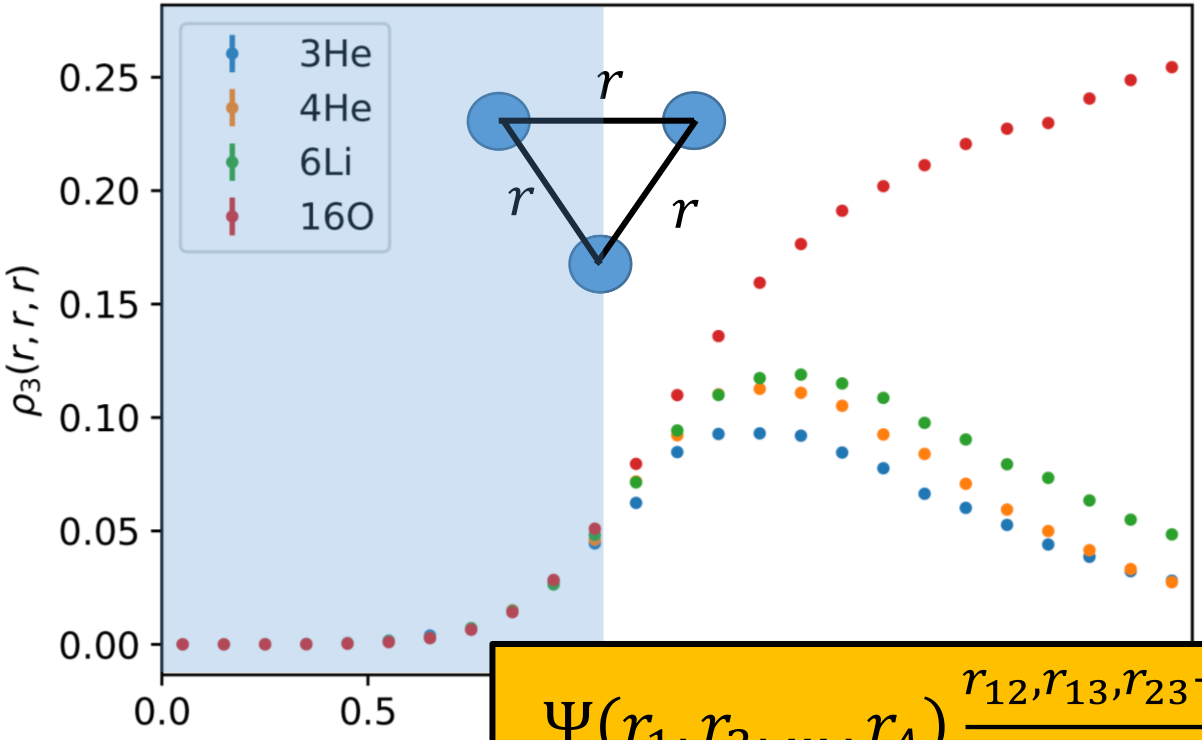
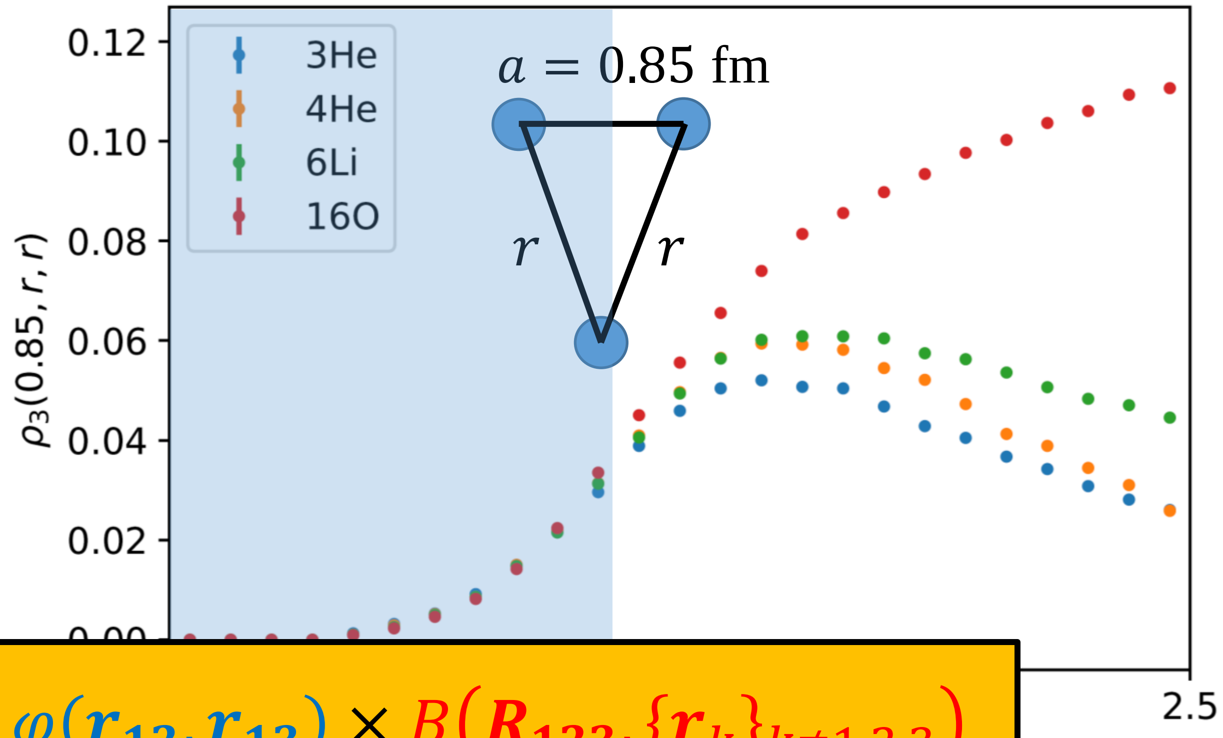




$$\Psi(r_1, r_2, \dots, r_A) \xrightarrow{r_{12}, r_{13}, r_{23} \to 0} \varphi(\boldsymbol{r}_{12}, \boldsymbol{r}_{13}) \times B\left(\boldsymbol{R}_{123}, \{\boldsymbol{r}_k\}_{k \neq 1,2,3}\right)$$

universal function

Three-body contact values

$$\frac{C(^4\mathrm{He})}{C(^3\mathrm{He})} \times \frac{3}{4} = 3.8 \pm 0.3 \qquad \frac{C(^6\mathrm{Li})}{C(^3\mathrm{He})} \times \frac{3}{6} = 3.1 \pm 0.3 \qquad \frac{C(^{16}\mathrm{O})}{C(^3\mathrm{He})} \times \frac{3}{16} = 4.2 \pm 0.5$$

Can be compared **to inclusive cross section ratios** (in the appropriate kinematics)

$$a_3(A) = \frac{3}{A} \frac{\sigma_{eA}}{(\sigma_{e\,^3\mathrm{He}} + \sigma_{e\,^3\mathrm{H}})/2}$$

For a **symmetric nucleus *A***

$$a_3(A) = \frac{3}{A} \frac{C(A)}{C(^3\mathrm{He})}$$

Future work: short-range physics

- **Three-body** correlations
- Improved **reaction dynamics**
- Imbedding SRC features in **ab-initio approaches**
- Applications:
  - **Beta decay**
  - **Neutrino-nucleus scattering**
  - …

Neutrino oscillation experiments

Summary

- **Nuclear short-range correlations**
- **Short-range expansion:** $\Psi(\boldsymbol{r}_1, \boldsymbol{r}_2, \dots, \boldsymbol{r}_A) = \sum_\alpha \varphi_\alpha^{E=0}(\boldsymbol{r}_{12}) A_\alpha^{(0)}(\boldsymbol{R}_{12}, \boldsymbol{r}_3, \boldsymbol{r}_4, \dots) + \cdots$
  - **Systematic and comprehensive** description of short-range physics
  - Allows to compare **nuclear interactions**, ab-initio **structure calculations**, and **experiments**
  - Different **applications** (neutrinoless double beta decay)
- First result about **three-body correlations**

BACKUP

Generalized Contact Formalism

$$\Psi(r_1, r_2, \dots, r_N) \xrightarrow{r_{12}\to 0} \left(\frac{1}{r_{12}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{12}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right) \qquad r_0 \ll a, d$$

$$\Psi(r_1, r_2, \dots, r_N) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}_{12}) \times A\left(\boldsymbol{R}_{12}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right) \qquad r_0 \lesssim d$$

$$\Psi \xrightarrow{r_{ij}\to 0} \sum_\alpha \varphi_{ij}^\alpha(\boldsymbol{r}_{ij}) A_{ij}^\alpha(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C}_{ij}^{\alpha\beta} \propto \langle A_{ij}^\alpha | A_{ij}^\beta \rangle$$

Channels $\alpha = (\ell_2 S_2) j_2 m_2$

Two-body functions

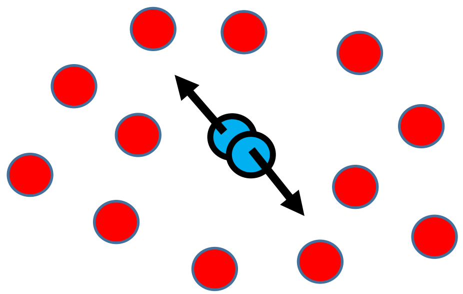
The pair kind $ij \in \{pp, nn, pn\}$

3 matrices of Nuclear Contacts

Nuclear systems

Generalized Contact Formalism

$$\Psi(r_1, r_2, \ldots, r_A) \xrightarrow{r_{12}\to 0} \varphi(\boldsymbol{r}) \times A\left(\boldsymbol{R}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$



universal function

For any **short-range** two-body operator $\hat{O}$

$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \, C \qquad C \propto \langle A | A \rangle$$

- Two-body dynamics
- Universal for all nuclei
- Simply calculated

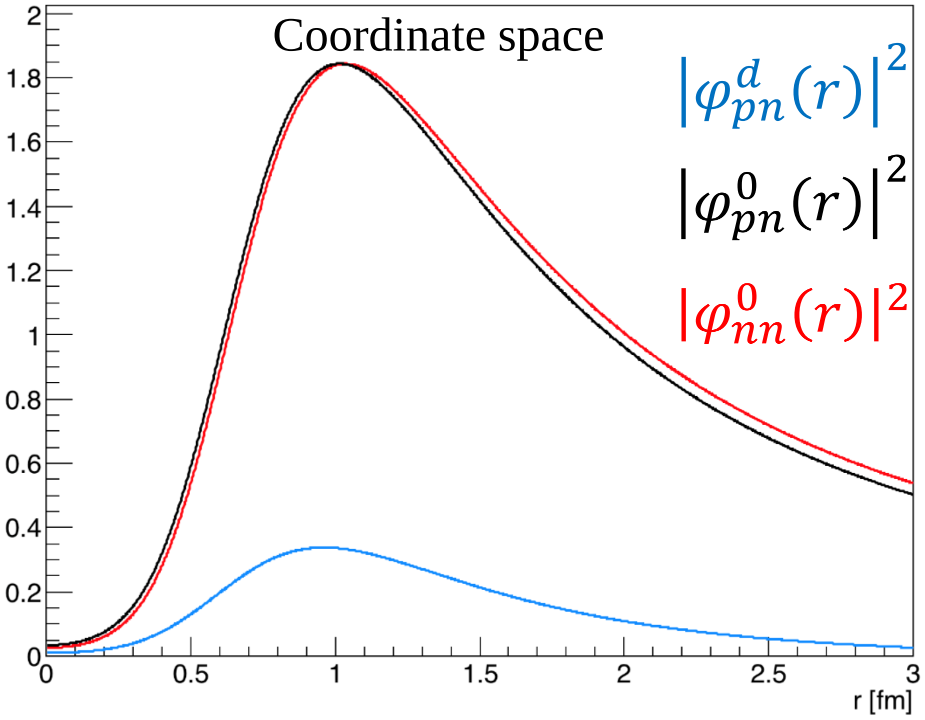
- The "contact"
- Number of correlated pairs
- Depends on the nucleus
- Independent of the operator

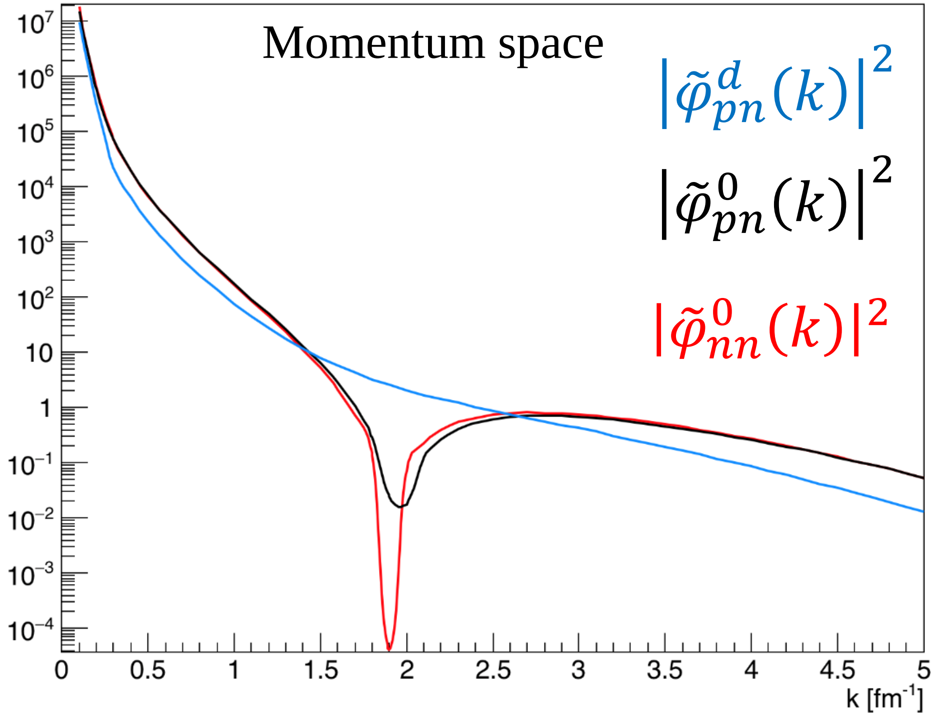
RW, B. Bazak, N. Barnea, PRC 92, 054311 (2015)

Generalized Contact Formalism

$$\Psi \xrightarrow{r_{ij\to 0}} \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k\neq i,j}) \quad ; \quad \boldsymbol{C_{ij}^{\alpha\beta}} \propto \langle A_{ij}^{\alpha} | A_{ij}^{\beta} \rangle$$

The universal functions using the AV18 potential





The spectral function

$$S(\boldsymbol{p_1}, \epsilon_1) = \sum_{s} \sum_{f_{A-1}} \delta(\epsilon_1 + E_f^{A-1} - E_0) \left|\langle f_{A-1} | a_{\boldsymbol{p_1},s} | \psi_0 \rangle\right|^2$$

The initial wave function

$$\psi_0 \rightarrow \sum_{\alpha} \varphi_{ij}^{\alpha}(\boldsymbol{r}_{ij}) A_{ij}^{\alpha}(\boldsymbol{R}_{ij}, \{\boldsymbol{r}_k\}_{k \neq i,j})$$

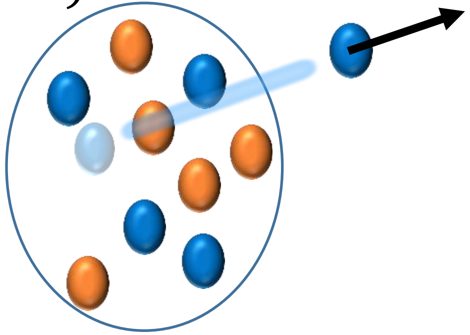
The final wave function

$$|\psi_f^{12}\rangle = a_{\boldsymbol{p_1},s}^{\dagger} | f_{A-1}\rangle \propto |\Psi_{\nu}^{A-2}\rangle e^{i\boldsymbol{p_1}\cdot r_1 + i\boldsymbol{p}_2 \cdot r_2} \chi_{s_1} \chi_{s_2}$$

Energy conservation:

$$E_f^{A-1} = \epsilon_2 + (A-2)m - B_f^{A-2} + \frac{P_{12}^2}{2m(A-2)}$$

$$B_f^{A-2} \approx \langle B_f^{A-2} \rangle$$



The spectral function

$p_1 > k_F$

$$S^p(\boldsymbol{p_1},\epsilon_1) = C^1_{pn}S^1_{pn}(\boldsymbol{p_1},\epsilon_1) + C^0_{pn}S^0_{pn}(\boldsymbol{p_1},\epsilon_1) + 2C^0_{pp}S^0_{pp}(\boldsymbol{p_1},\epsilon_1)$$

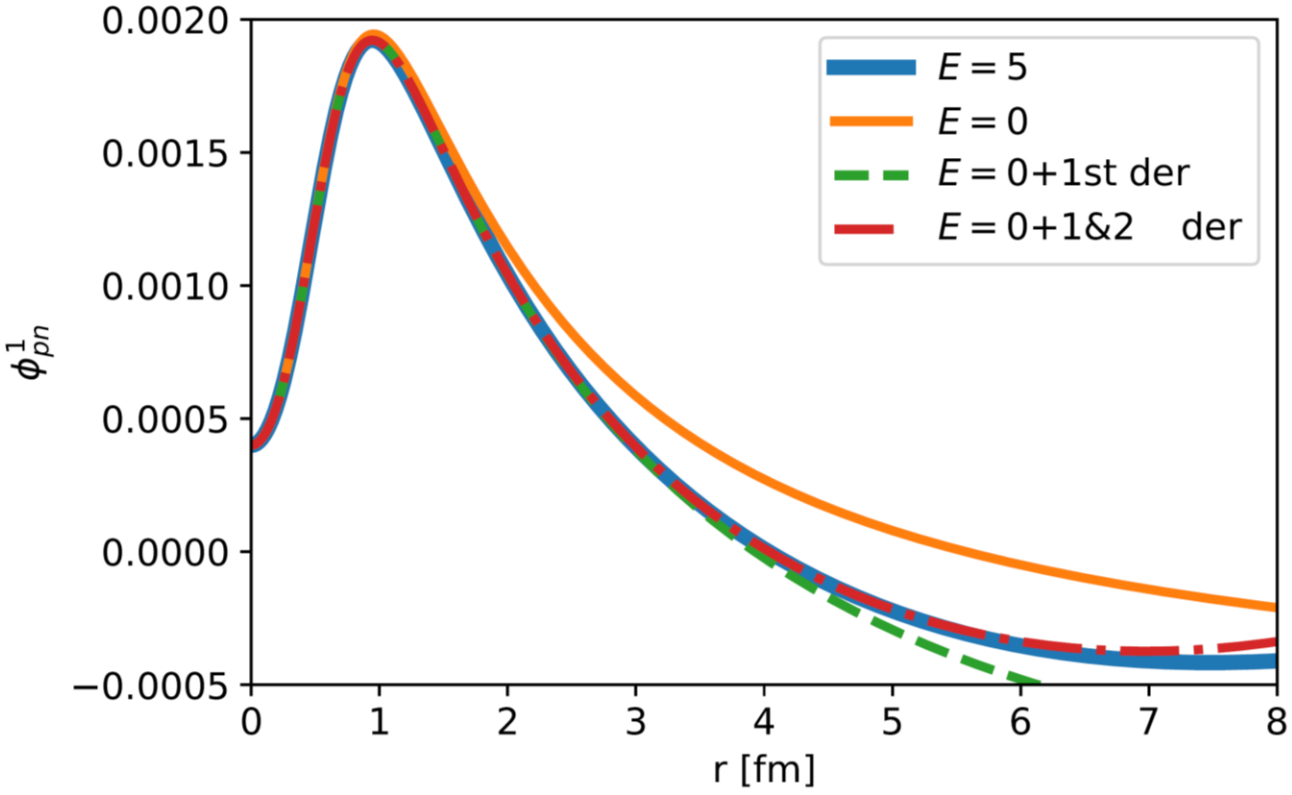
$$S^\alpha_{ab}(\boldsymbol{p_1},\epsilon_1) = \frac{1}{4\pi}\int \frac{d^3p_2}{(2\pi)^3} \underbrace{\delta(f(\boldsymbol{p}_2))}_{\text{Energy conservation}} \underbrace{n_{CM}(\boldsymbol{p_1}+\boldsymbol{p_2})}_{\text{CM momentum distribution (Gaussian)}} \underbrace{|\tilde{\varphi}^\alpha_{ab}(|\boldsymbol{p_1}-\boldsymbol{p_2}|/2)|^2}_{\text{Two-body function}}$$

Similar to the convolution model

C. Ciofi degli Atti, S. Simula, L. L. Frankfurt, and M. I. Strikman, Phys. Rev. C 44, R7(R) (1991),
C. Ciofi degli Atti and S. Simula PRC ***53,*** *1689 (1996)*

Short-range expansion

$$\varphi^{E}(\boldsymbol{r}) = \varphi^{E=0}(\boldsymbol{r}) + \left(\frac{d}{dE}\varphi^{E=0}(\boldsymbol{r})\right)E + \frac{1}{2!}\left(\frac{d^2}{dE^2}\varphi^{E=0}(\boldsymbol{r})\right)E^2 + \cdots$$



AV4'
Deuteron channel
Scattering state

Short-range expansion: Next order terms

The many-body case:

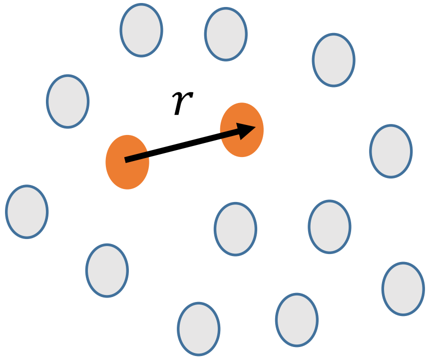
$$\Psi(\boldsymbol{r}_1,\boldsymbol{r}_2,\dots,\boldsymbol{r}_A)=\sum_{\alpha}\varphi_{\alpha}^{E=0}(\boldsymbol{r}_{12})A_{\alpha}^{(0)}+\sum_{\alpha}\left(\frac{d}{dE}\varphi_{\alpha}^{E=0}(\boldsymbol{r})\right)A_{\alpha}^{(1)}+\sum_{\alpha}\left(\frac{d^2}{dE^2}\varphi_{\alpha}^{E=0}(\boldsymbol{r})\right)A_{\alpha}^{(2)}+\cdots$$

$$A_{\alpha}^{(0)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)=\sum_{E}A_{\alpha}^{E}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

$$A_{\alpha}^{(1)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)=\sum_{E}E\,A_{\alpha}^{E}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

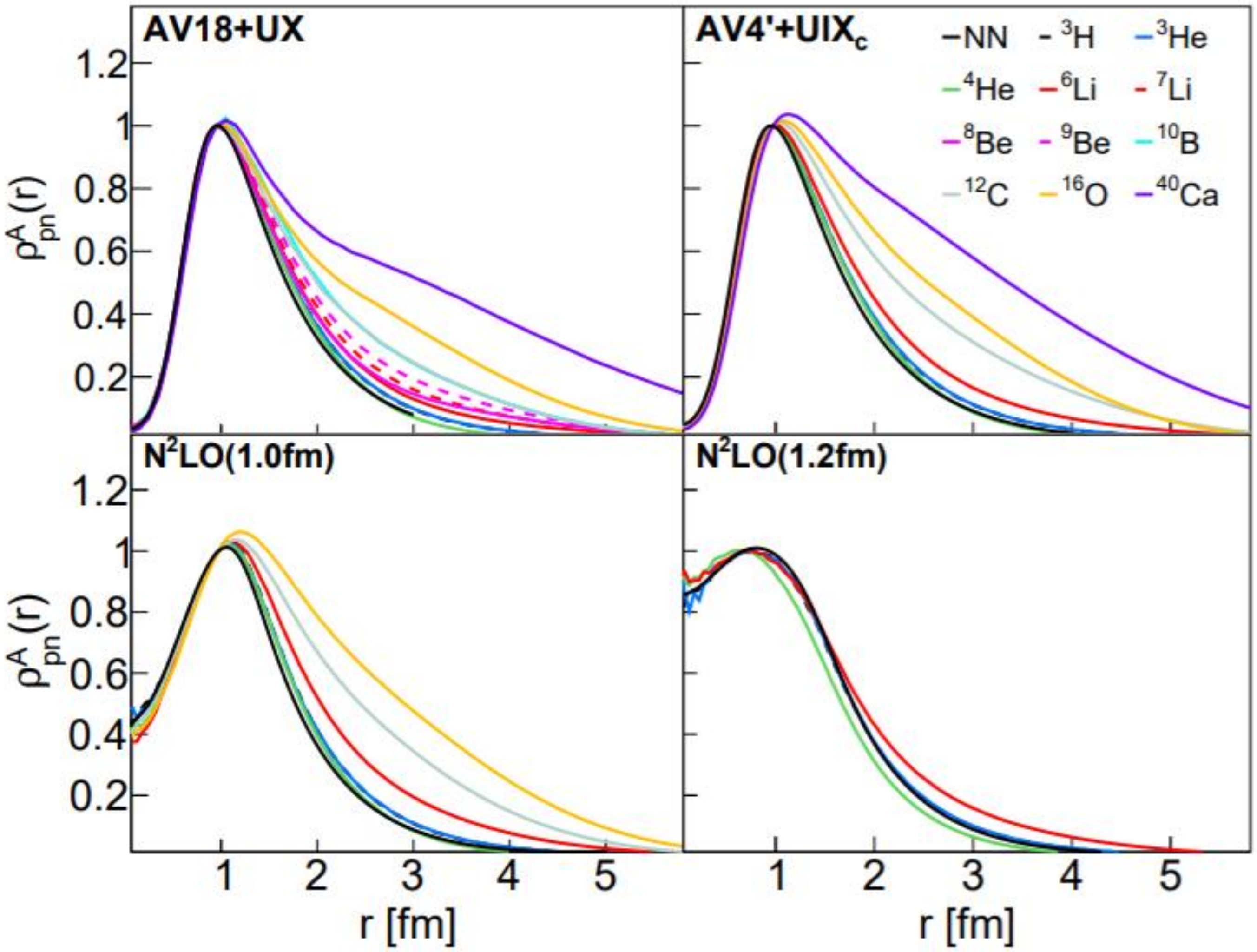
$$A_{\alpha}^{(2)}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)=\frac{1}{2!}\sum_{E}E^2A_{\alpha}^{E}(\boldsymbol{R}_{12},\boldsymbol{r}_3,\dots,\boldsymbol{r}_A)$$

Two-body density



$$\langle \hat{O} \rangle = \langle \varphi | \hat{O}(r) | \varphi \rangle \; C$$

$$\rho_{NN}(\boldsymbol{r}) \xrightarrow{r \to 0} C |\varphi(\boldsymbol{r})|^2$$



Shows the validity of the factorization

R. Cruz-Torres, D. Lonardoni, RW, et al., Nature Physics (2020)

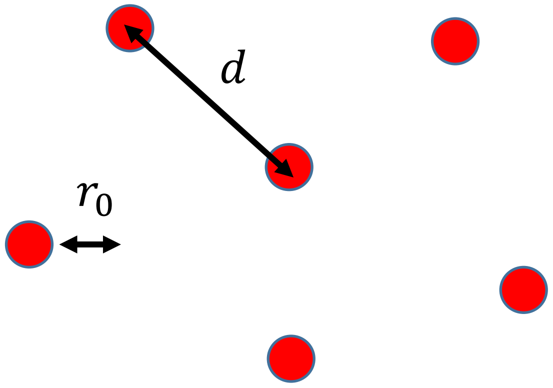
The Contact Theory

- Dilute systems - with **negligible interaction range**

- Zero-range condition:

$$r_0 \ll a, d$$

Interaction range — Scattering length — Average distance between particles



- Zero-range model: Non-interacting particles with boundary condition

Independent of the details of the interaction

$$\Psi(r_1, r_2, \ldots, r_N) \xrightarrow{r_{12}\to 0} \left(\frac{1}{r_{12}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{12}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

The Contact Theory

$$\Psi(r_1, r_2, \ldots, r_N) \xrightarrow{r_{12}\to 0} \left(\frac{1}{r_{12}} - \frac{1}{a}\right) \times A\left(\boldsymbol{R}_{\mathbf{12}}, \{\boldsymbol{r}_k\}_{k\neq 1,2}\right)$$

- A parameter – **the contact** – can be defined:

$$\boldsymbol{C} \propto \langle A|A\rangle$$

- C ≈ **number of SRC pairs** in the system
- Connected to many quantities in the system

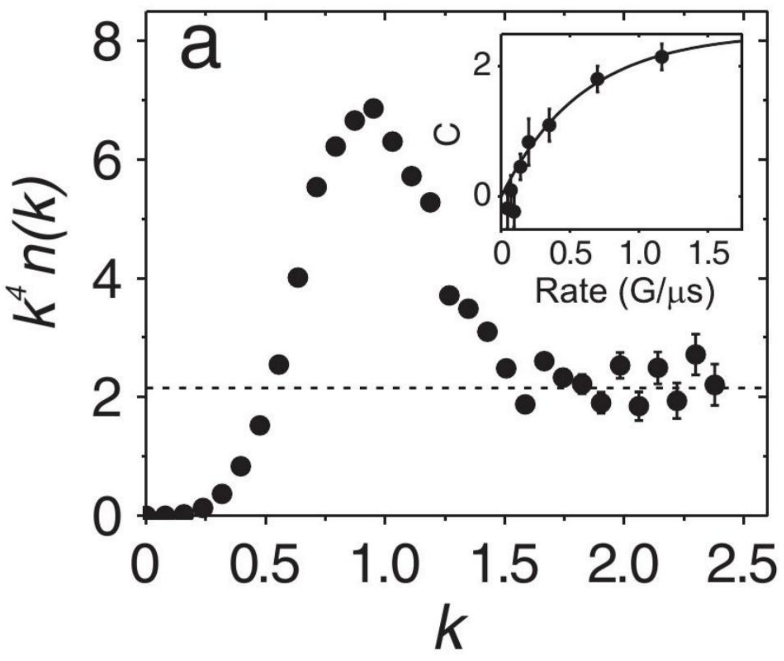
$$n(k) \xrightarrow{k\to\infty} \boldsymbol{C}/k^4$$

$$T + U = \frac{\hbar^2}{4\pi m a}\boldsymbol{C} + \sum_\sigma \frac{d^3k}{(2\pi)^3}\frac{\hbar^2 k^2}{2m}\left(n_\sigma(\boldsymbol{k}) - \frac{\boldsymbol{C}}{k^4}\right) \qquad \ldots$$

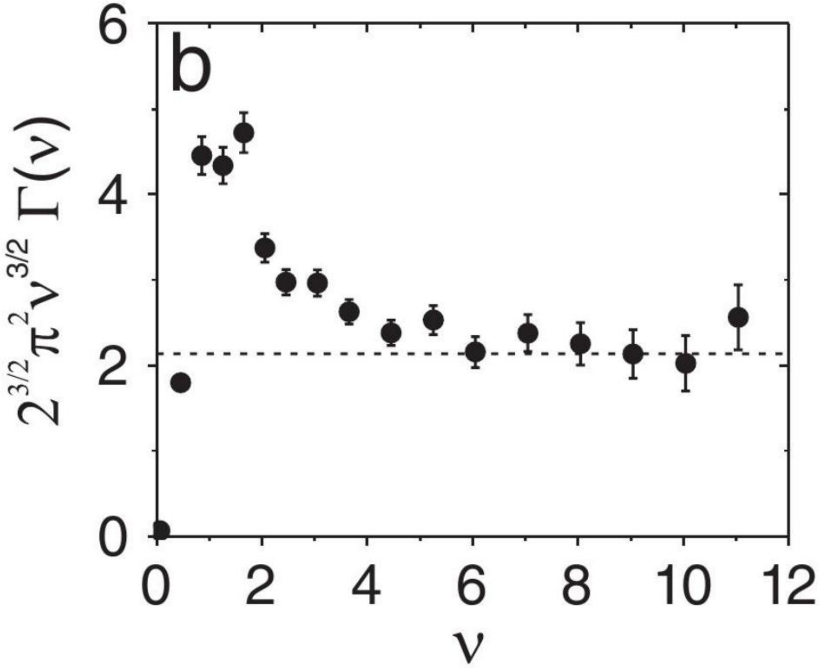
The Contact Theory

- Verified experimentally: (ultra-cold atomic systems)

Momentum distribution



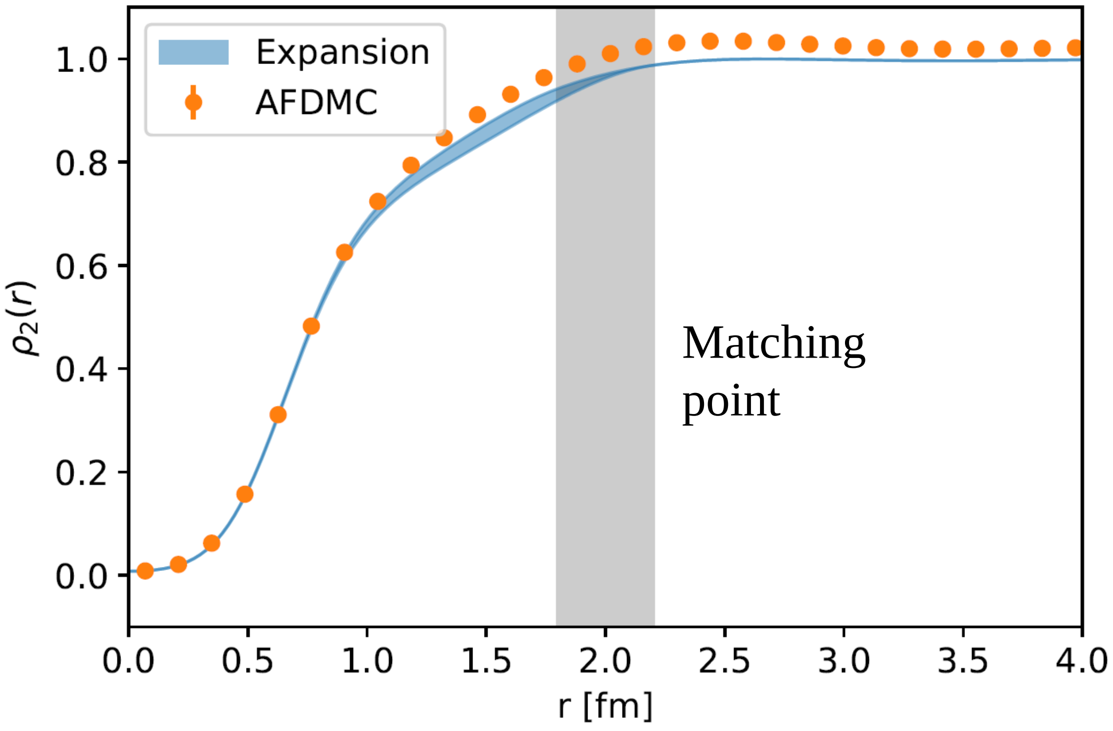
RF line shape



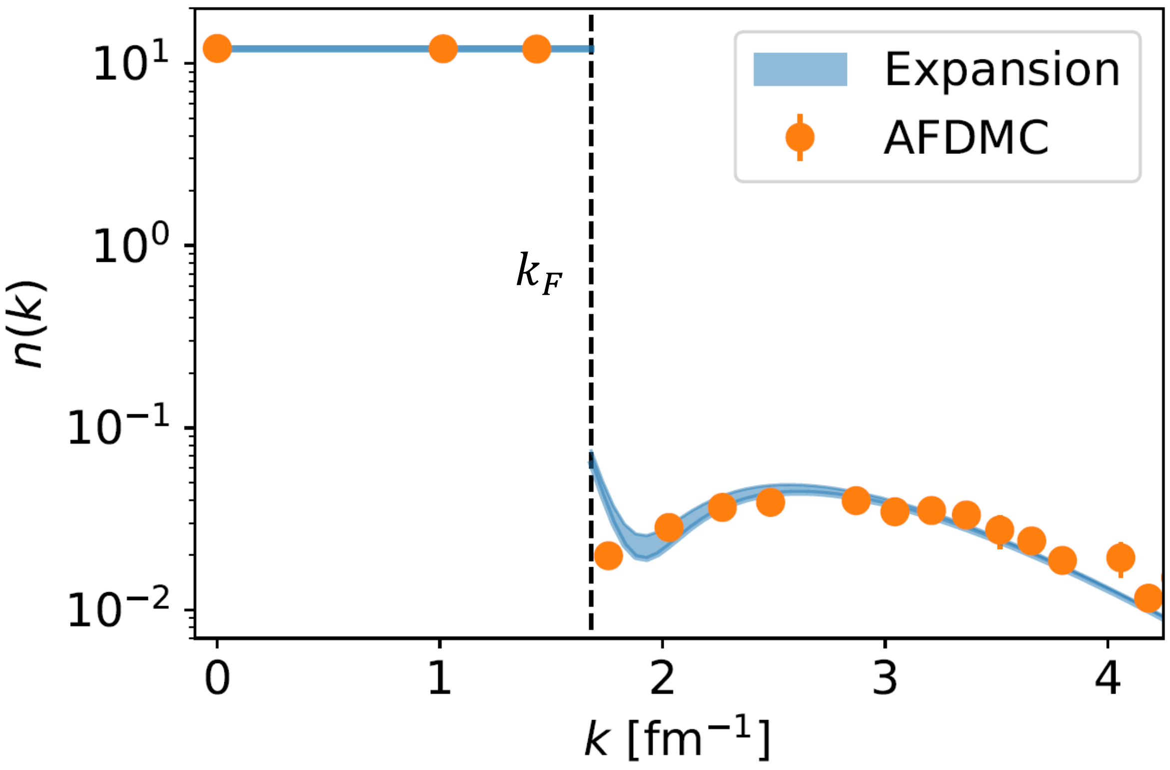
J. T. Stewart, J. P. Gaebler, T. E. Drake, and D. S. Jin, Phys. Rev. Lett. 104, 235301 (2010)

Matching to long-range model

Fitting only the LO contact and matching to FG:



Obtained NN Potential Energy $\langle V_2 \rangle = -29.2 \pm 1.2$ MeV
Exact NN Potential Energy $\langle V_2 \rangle = -30.1$ MeV



Obtained Kinetic Energy $\langle T \rangle = 43.3 \pm 0.2$ MeV
Exact Kinetic Energy $\langle T \rangle = 43.3$ MeV