Amplitude analysis in hadron spectroscopy experiments

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On behalf of JPAC

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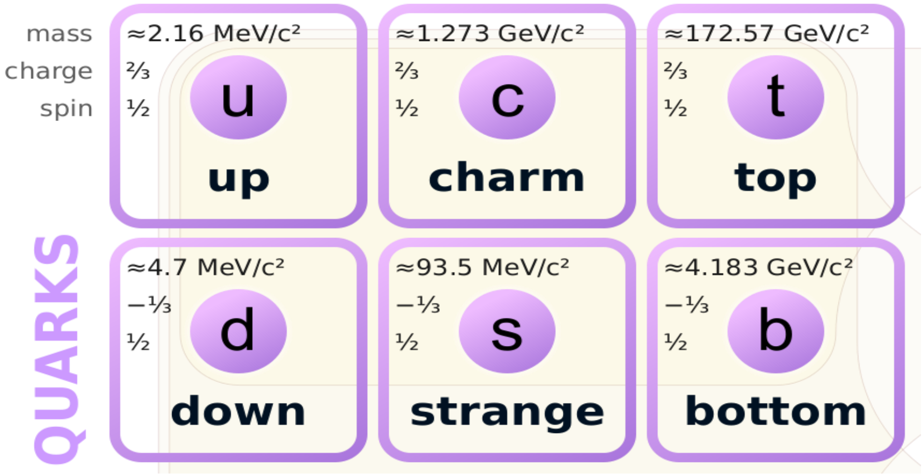
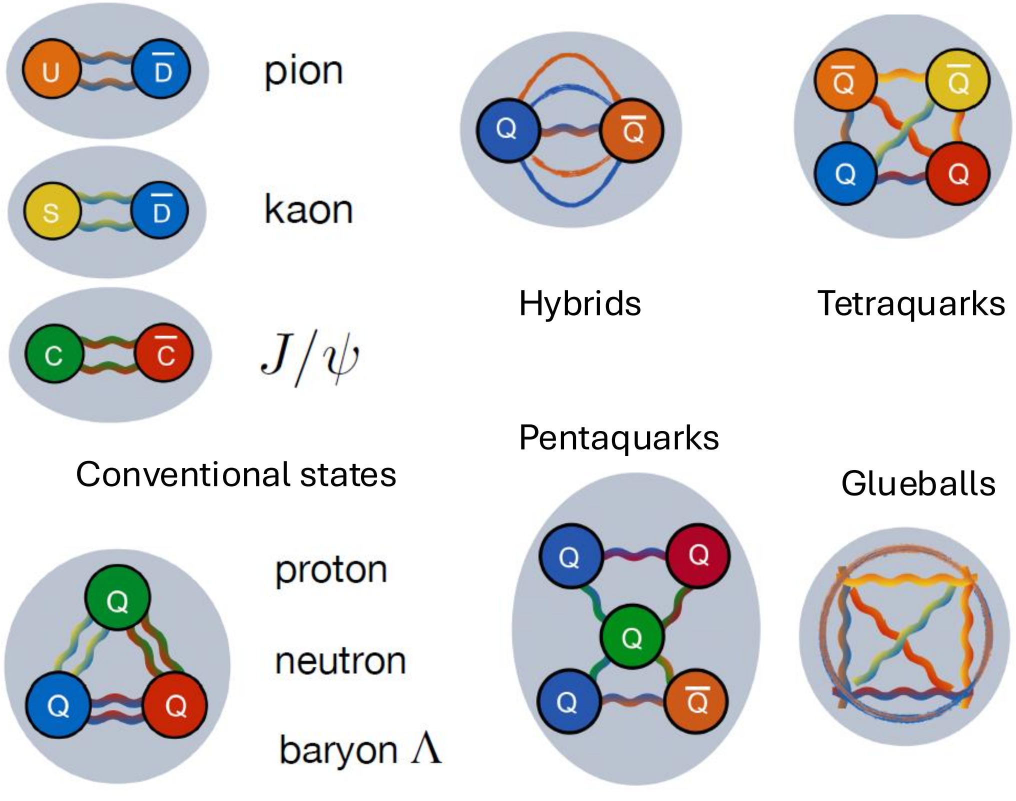
Hadron spectroscopy



Cush, Public domain, via Wikimedia Commons

- A plethora of states formed by quarks and gluons
- Mesons ($\bar{q}q$) and baryons (qqq) are conventional hadrons
- Other unconventional states possible and observed
  - Hybrids ($\bar{q}qg$) e.g: $\pi_1(1600), \eta_1(1855)$
  - Tetraquarks ($\bar{q}q\bar{q}q$), pentaquarks ($\bar{q}q\bar{q}qq$), hexaquark states ($qqqqqq$), etc e.g: T_{cc}^{+}, and other T_{xx} states, P_{xx} states, etc.
  - Glueballs gg, ggg, etc
  - Molecular states
- Many have been observed, many more yet to be found
- COMPASS, BESIII, LHCb, GlueX, and more!

- Standard model provides six quarks and a force carrier (gluon)
- Fundamental states of QCD can have three possible charges (colors)
  - Only color neutral states are observable (hadrons)
  - Confinement and asymptotic freedom



Courtesy: Vincent Mathieu, U. Barcelona

Hadron spectroscopy

Properties hadrons

Decays

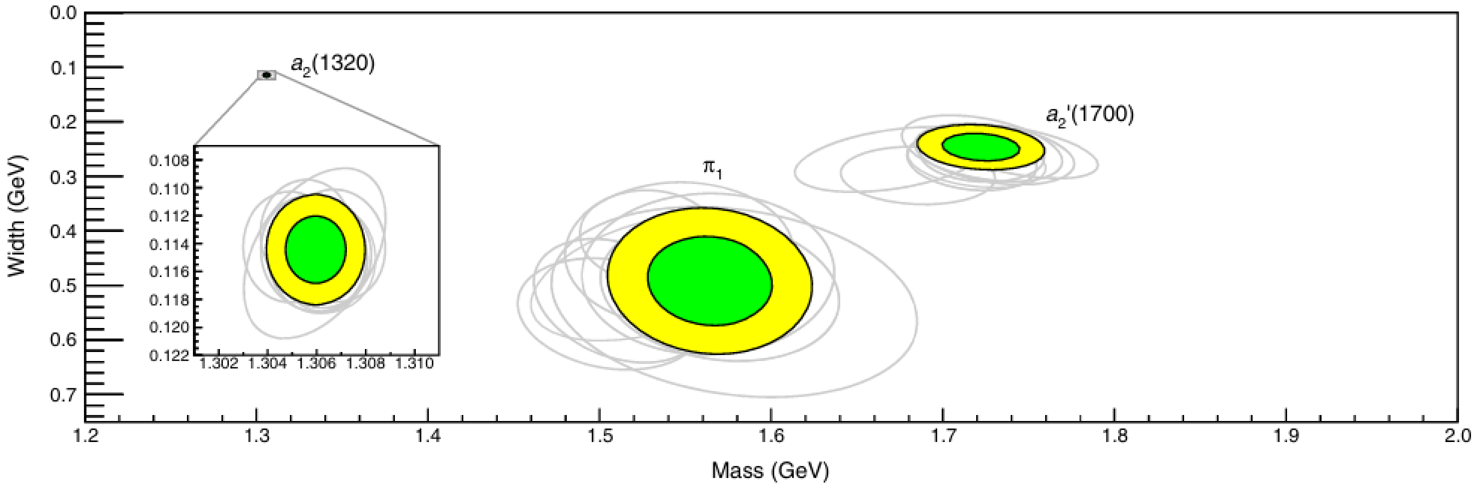
- Mass and decay widths
    - Pole positions
- Allowed transitions
- Line shapes
    - Nature of states

Production

- Production mechanisms
- Cross section
- Polarization observables
    - Density matrix elements
    - Asymmetries

Interactions

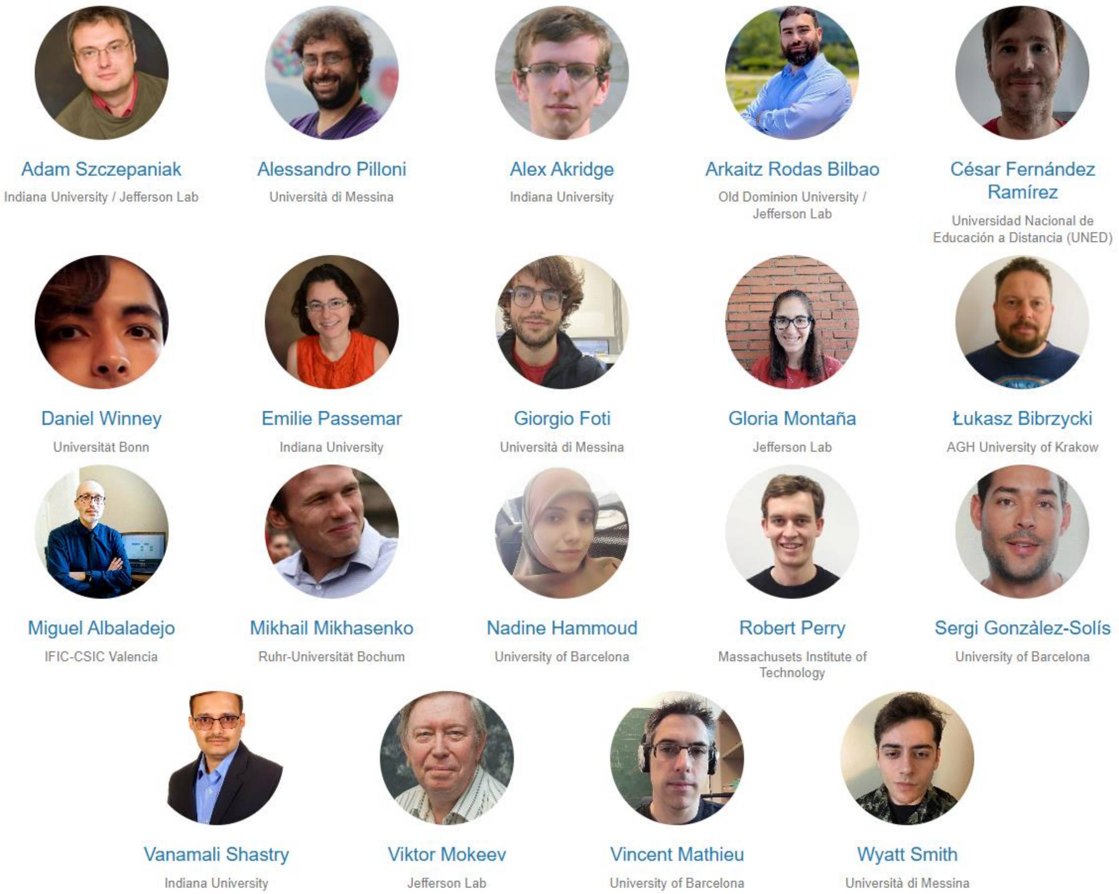
- Phase shifts
- Unitarizations
    - New states
    - Nature of known states
    - Pole positions



Pole positions of three $\eta\pi$ resonances extracted by JPAC (Phys.Rev.Lett. 122 (2019) 042002) using experimental data from COMPASS (Phys. Lett. B 740, 303 (2015).)

All three poles now in PDG; The light hybrid conundrum now solved

Joint Physics Analysis Center (JPAC)



More than a decade of physics

Phenomenology Lattice AI/ML

- Emphasize the need for amplitude analysis
- Pole positions extracted from data included in PDG
  - $a_2(1320), a_2(1700), \pi_1(1600)$ (Rodas, *et al* PRL 122 (2019) 4, 042002)
- Production mechanisms
  - $\pi\pi$ production at large energies
  - $\eta\pi$ production at large energies and double Regge exchange
  - More complicated photoproduction processes

Full members

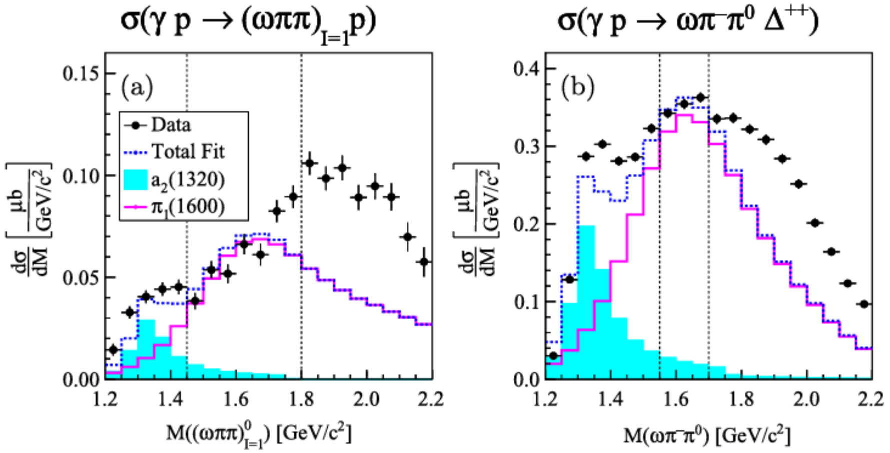
Schools & workshops

Train the future generation

Affiliate members

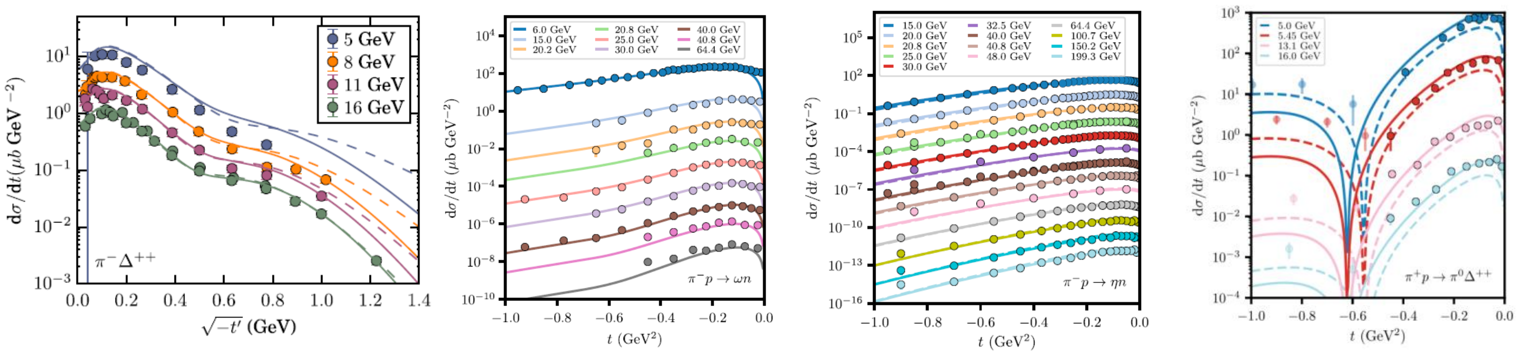
Production mechanisms

- Large-s production of resonances
  - COMPASS: $\pi^- + p \ \rightarrow (3\pi)^- + p$ at
    $E_\pi \approx 190\ GeV^2$
  - GlueX: $\vec{\gamma} + p \ \rightarrow X^{0(-)} + p(\Delta^{++})$ where X is any resonance at $s \approx 17\ GeV^2$
  - Dominated by Regge (CEX) and Pomeron (neutral) exchanges
  - Broad features known from analysis of cross sections; finer details to be understood
  - Many unanswered questions

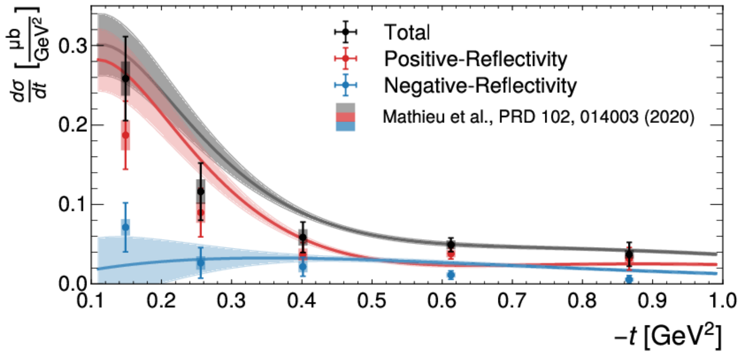


GlueX, *Phys.Rev.Lett.* 133 (2024) 26, 26

Why is the $\pi_1(1600)$ more likely to be produced in a charge exchange reaction?



J. Nys *et al* (JPAC): *Phys.Lett.B* 779 (2018) 77-81 and *Phys.Rev.D* 98 (2018) 3, 034020

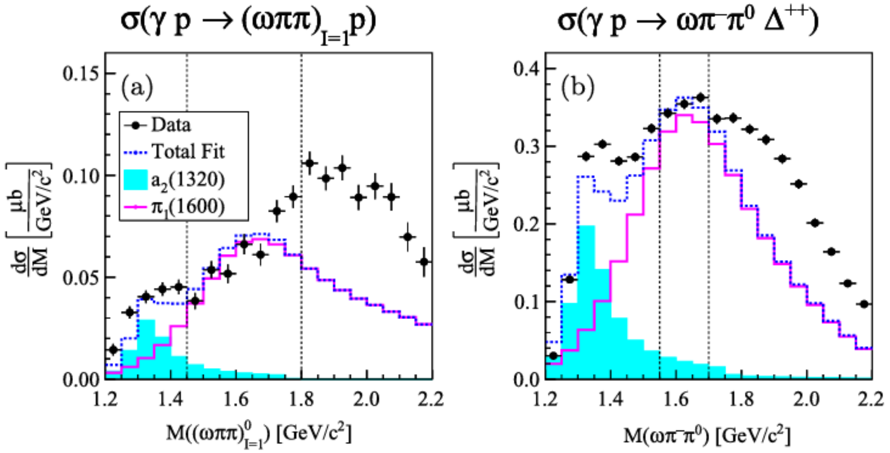


$a_2^0(1320)$ photoproduction cross section
GlueX 2501.03091

Production mechanisms

Goal: understand the mechanism behind the photoproduction of $\pi_1(1600)$

One pion $\Longrightarrow$ Two pions $\Rightarrow$ ρp, $\pi\Delta$, $\pi\pi p$ $\Downarrow$ $\omega\pi p (b_1 p)$ $\Longleftarrow$ $\omega\pi\pi p$ $(b_1\Delta, \pi_1 p)$ $\Longleftarrow$ $\pi_1\Delta$



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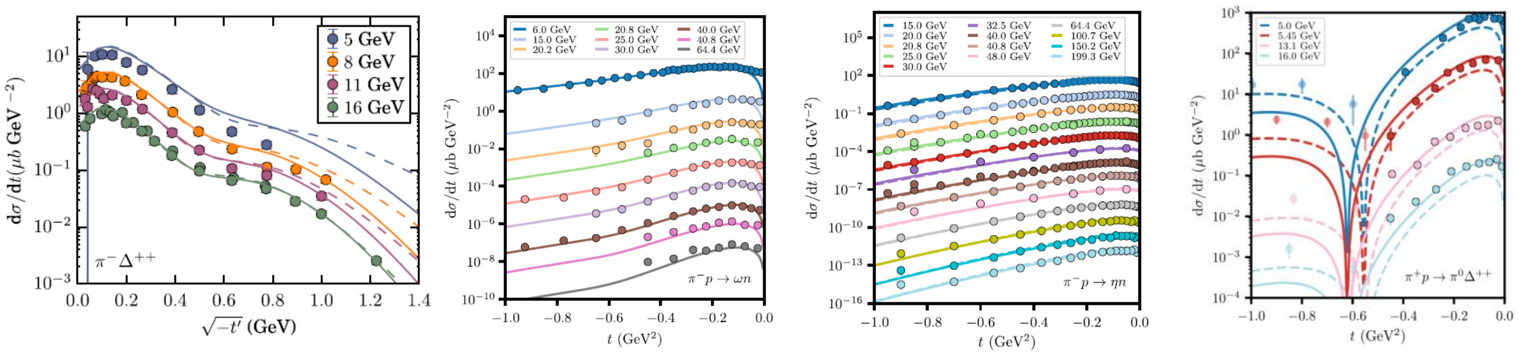

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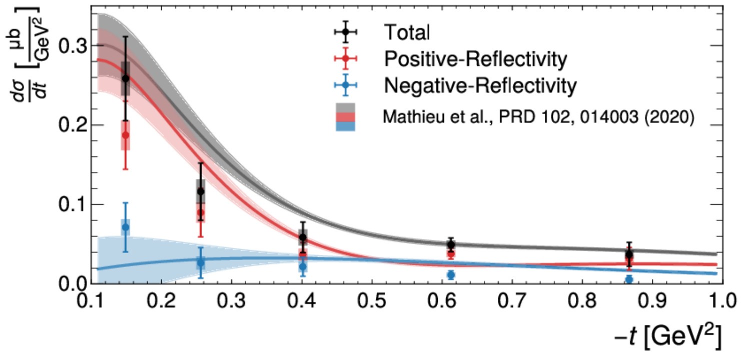
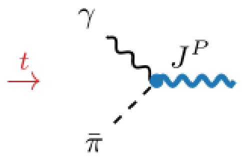


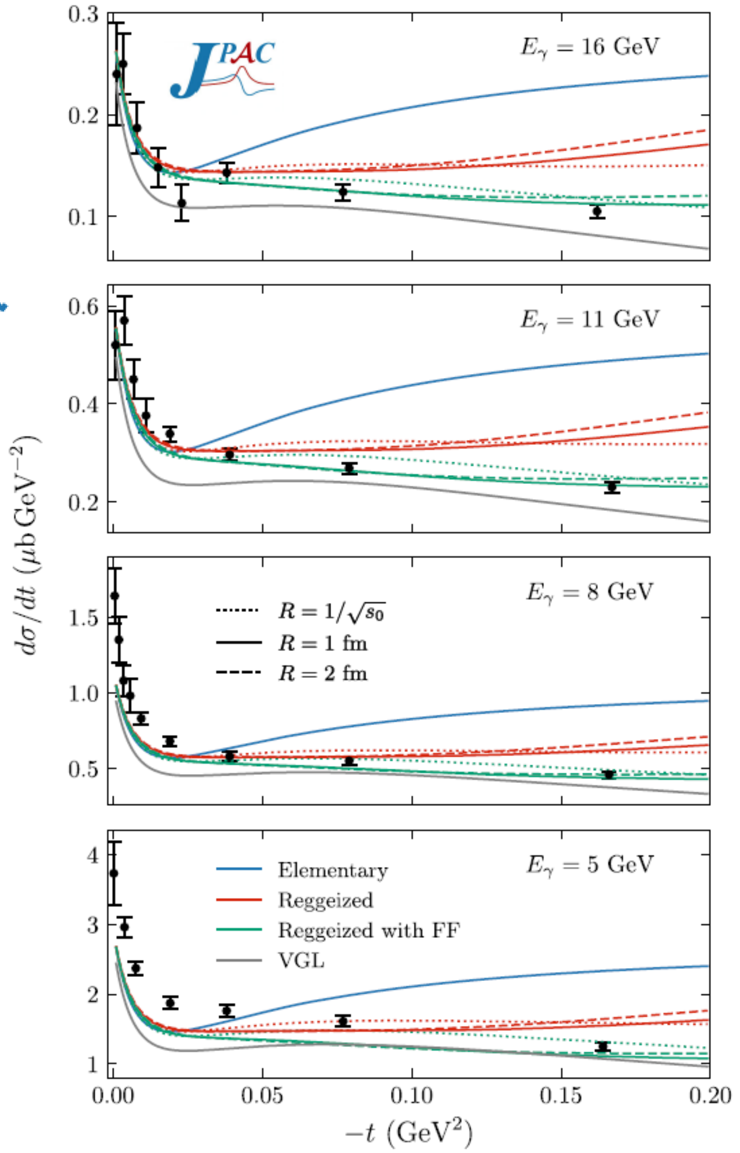
$a_2^0(1320)$ photoproduction cross section
GlueX 2501.03091

Single pion photoproduction (G. Montaña *et al* (JPAC) *Phys.Rev.D* 110 (2024) 11, 114012)

Reggeization and gauge invariance

- The pion exchange amplitude in the t-channel rest frame vanishes.
  - How to explain gauge invariance?
- Pion exchange contributions are "hidden" among other J^P = (even)$^-$ exchanges.
  - Recovered from spin-J exchange via analytical continuation
  - Interpreted as cross-channel contributions
- Nucleon and pion poles are connected by gauge invariance
  - s- and u-channel exchanges are essential to conserve the current at the level of the Born diagrams
  - This carries over to the Regge limit as well.





$$A^{e}_{\mu_\gamma\mu_i\mu_f} = 2\sqrt{2}\, g_{\pi NN}\Big[e_\pi\Big(\frac{\epsilon\cdot(p_\pi - k/2)}{t-m_\pi^2} + \frac{\epsilon\cdot P}{s-u}\Big) + \frac{1}{2} e_{N_i}\Big(\frac{\epsilon\cdot p_\pi}{s-m_N^2} + \frac{\epsilon\cdot P}{s-u}\,\frac{t-m_\pi^2-k^2}{s-m_N^2}\Big) - \frac{1}{2} e_{N_f}\Big(\frac{\epsilon\cdot p_\pi}{u-m_N^2}+\frac{\epsilon\cdot P}{s-u}\,\frac{t-m_\pi^2-k^2}{u-m_N^2}\Big)\Big]\bar{u}(p_f,\mu_f)\gamma_5 u(p_i,\mu_i).$$

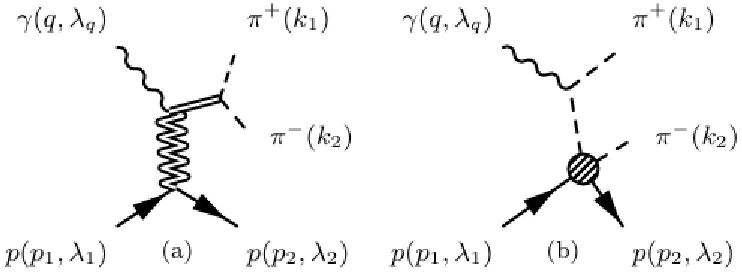
Born amplitude

$$\mathcal{A}^{J\to 0}(s,t) = i\,\frac{\alpha'}{\alpha(t)}\,\frac{z_t}{\sqrt{1-z_t^2}}$$

t-channel pion exchange amplitude obtained via analytical continuation (excluding the Regge propagator)

Two pion photoproduction (N. Hammoud *et al* (JPAC) *Phys.Rev.D* 111 (2025) 1, 014002)

- Resonant $\pi\pi$ production involves two possible contributions
  - Resonance in the upper vertex (e.g., ρp production)
  - Resonance in the lower vertex (e.g., $\pi\Delta$ production)
- Additional nonresonant contributions.
- Nonresonant and πN resonances included as Deck amplitude
- Resonant and Nonresonant contributions add coherently.
- Pomeron exchange dominates at large s when $\pi\pi$ is neutral
  - Additional corrections from resonance exchanges appear for smaller s.
  - Angular moments are useful in analyzing these processes.

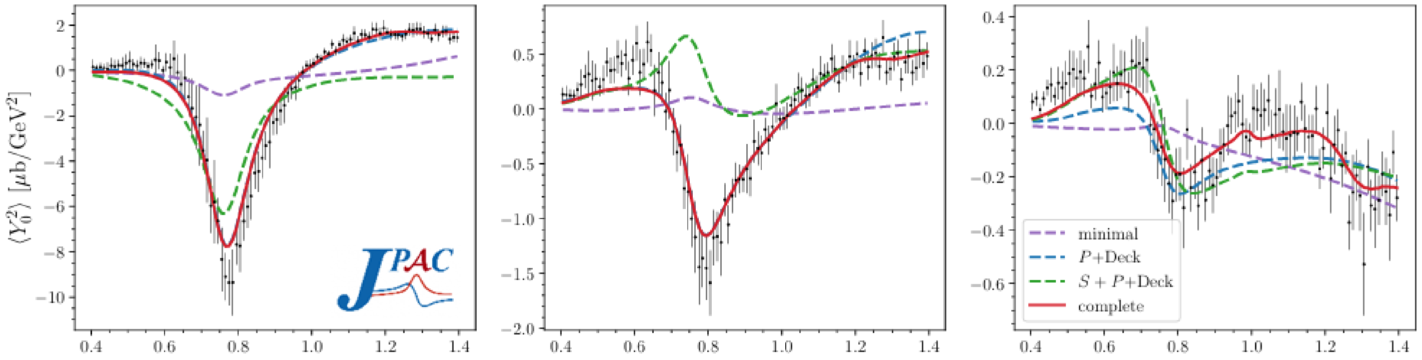


Resonant and Deck contributions

- $\pi\pi$ resonances include $\sigma, f_0(980), \rho, f_0(1370), f_2(1270)$
- Pomeron and natural parity Regge exchanges
  - 4 models using combinations of exchanges
- Models fitted to CLAS data with $E_\gamma = 3.7\ GeV$

$$< Y_L^M >= \sqrt{4\pi} \int d\Omega^H \frac{d\sigma}{dtdm_{12}d\Omega^H} \mathrm{Re}\, Y_{LM}(\Omega^H)$$

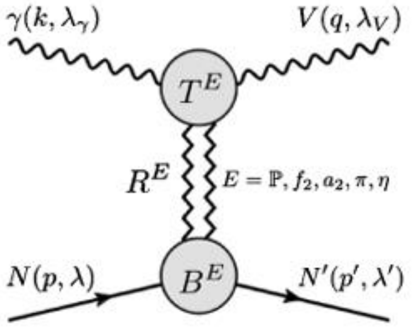
Full model nicely fits the data



Angular moment $< Y_0^2 >$ as function of $\pi\pi$ invariant mass for 3 different t-values

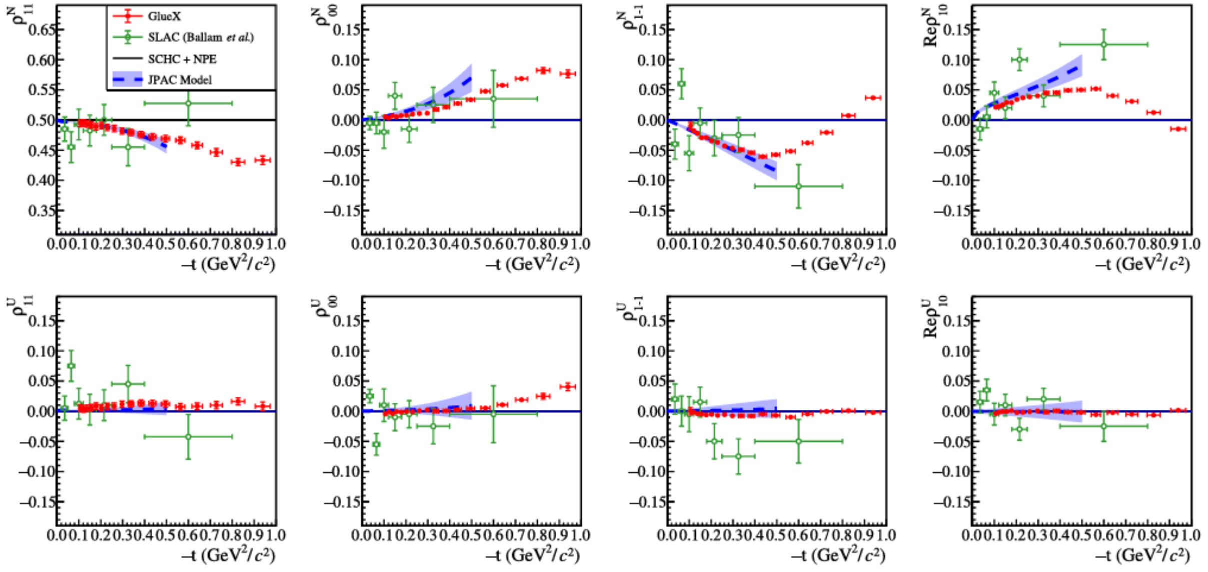
- Potential extension to electroproduction
- Mass distribution of the SDMEs

Two pion photoproduction – Vector meson (V. Mathieu *et al* *Phys.Rev.D* 97 (2018) 9, 094003)



Spin density matrix elements when the $\pi\pi$ is a vector meson

- Dominated by Pomeron exchange.
- Pomeron coupling constants estimated using SLAC data on SDMEs and PDG data on proton and deuteron Compton scattering



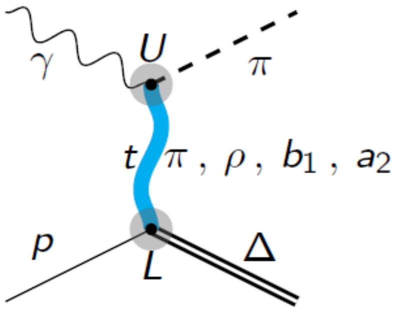
Natural and Unnatural parity exchange SDMEs. JPAC model compared to GlueX and SLAC data. GlueX, *Phys.Rev.C* 108 (2023) 5, 055204

- The model explains the SLAC data quite well
- Pomeron dominance implies dominant natural exchange contributions
- The model was used a prediction for GlueX measurements
- The GlueX data for the SDMEs in the small $|t|$ region is very well explained by the model.

Dependence of the SDMEs on $\pi\pi$ invariant mass is being analyzed

Two pion photoproduction – $\pi\Delta$ (V. Shastry *et al* (JPAC) *In progress*)

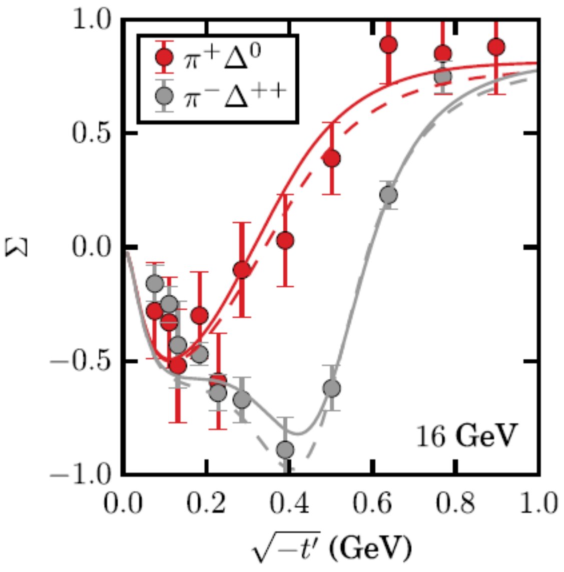
Spin density matrix elements when one of the pions comes from the lower vertex



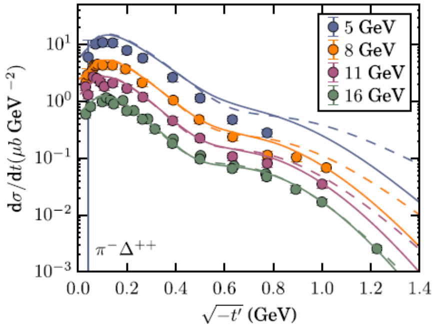
- Amplitude factorizes at large s.
- Base model has vertices modelled using effective Lagrangians.
  - Production cross sections, and beam asymmetries are well explained
  - Unnatural parity exchange dominates the small $|t|$ region and natural parity exchange dominates the large $|t|$ region

$$T_{\lambda_\gamma,\lambda_1,\lambda_\Delta}(s,t) = \sum_{\times}\left[\xi_{\lambda_\gamma\lambda_1\lambda_\Delta}\, T^{\times}_{\lambda_\gamma,\lambda_1,\lambda_\Delta}(s,t)\right] \quad ; \qquad \times \in \{\pi,\rho,b_1,a_2\}$$

$$T^{\times}_{\lambda_\gamma,\lambda_1,\lambda_\Delta}(s,t) = \sqrt{-t}^{\;|\lambda_\gamma|}\sqrt{-t}^{\;|\lambda_1-\lambda_\Delta|}\;\hat{\beta}^{\times,U}_{\lambda_\gamma}(t)\;\hat{\beta}^{\times,L}_{\lambda_1,\lambda_\Delta}(t)\;\mathcal{P}^{\times}_{R}(s,t)\;\mathcal{S}_{\times}(t)$$



- Relative phases (including the signs of $\beta_{\lambda_1\lambda_\Delta}$) unconstrained by cross section alone
  - SDMEs solve most of this problem
- Needs a reparameterization of the model



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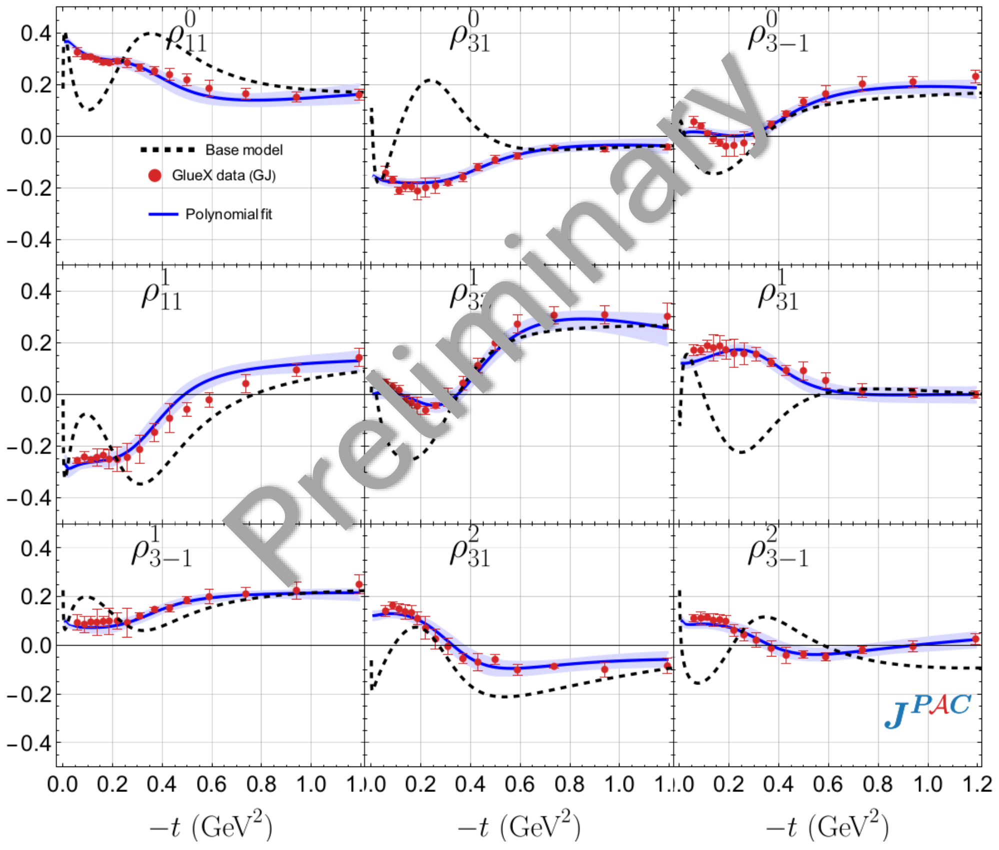
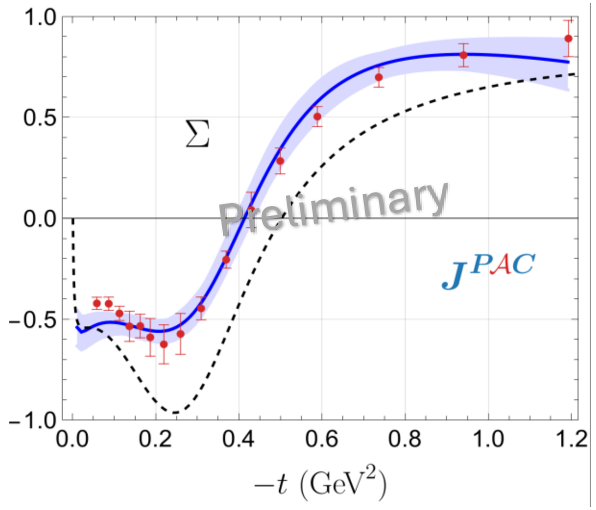


- Instead of Lagrangians, chose polynomials (linear in *t*).
- Fit the coefficients to the data
  - Takes care of any higher order term left out of the model.
- Can be used in any future work involving Δ
- EXD broken for natural exchange
- Very forward region ($|t| \leq 0.1\ GeV^2$) shows absorption
- Not effectively described by PMA
- More analysis needed and in progress.

Structure of the amplitudes and their singularities

t-channel Residues and scalar amplitudes



GlueX data from F. Afzal *et al* (GlueX), *Phys.Lett.B* 863 (2025) 139368

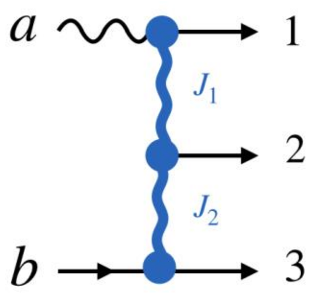
Other pseudoscalar meson pairs - $\eta^{(\prime)}\pi$

- Resonance in the upper vertex: $a_2 p$ photoproduction
- Non-resonant photoproduction ($2 \rightarrow 3$)

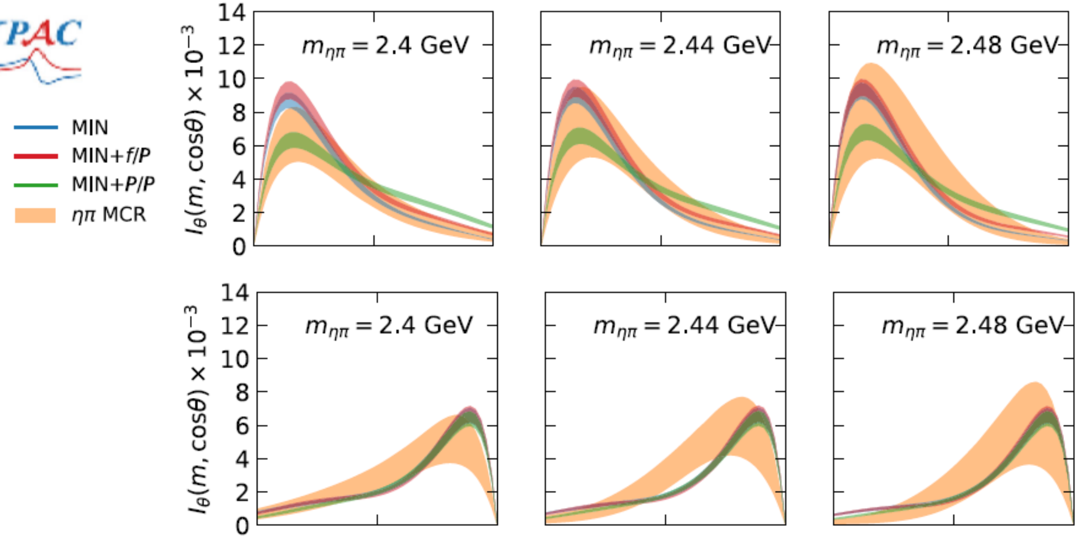
Double Regge exchange in $\pi^- + p \rightarrow \eta^{(\prime)}\pi^- p$
(EPJC, (2021) 81:647)

- Three models examined:
  - Minimal model with $a_2 P + a_2 f_2 + f_2 f_2$
  - $MIN + f_2 P$
  - $MIN + PP$
- Models fitted to COMPASS partial wave data.
- All three models explain the forward region equally well
- Minimal model insufficient to explain the broad backward peak in the $\eta' p$ production
  - $MIN + PP$ gives the broadest backward peak

Interesting interplay between Pomeron and Regge exchanges.
Better background and uncertainty estimates needed.
How does this work in a photoproduction process?



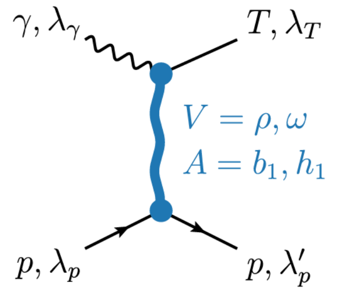
Non-resonant production of two pseudoscalar mesons



Intensity vs $\cos\theta$: Model results (EPJC, (2021) 81:647) compared with COMPASS data (Phys. Lett. B **740**, 303 (2015)).

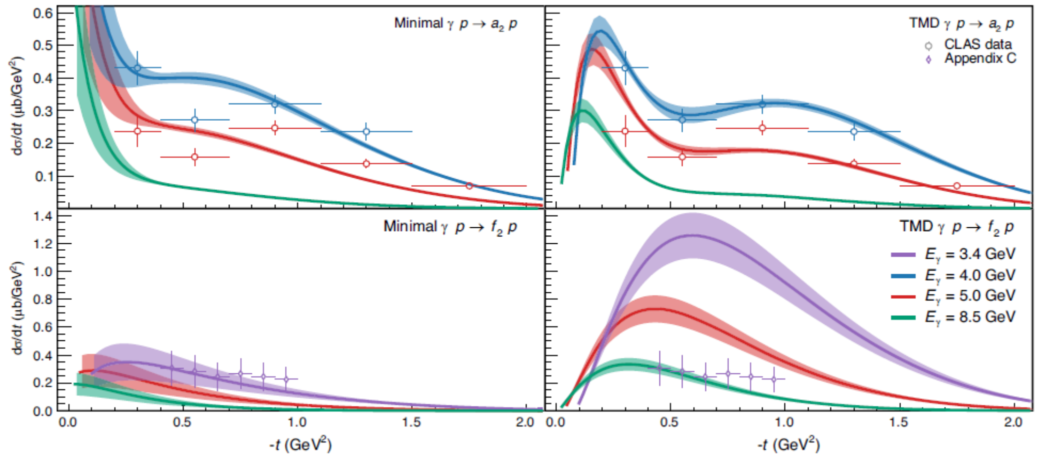
Ongoing (event-level) analyses in collaboration with COMPASS and GlueX;
Preliminary results from COMPASS presented in Hadron2025 (H. Pekeler's talk)

$a_2 p$ photoproduction (V. Mathieu *et al* (JPAC) *Phys.Rev.D* 102 (2020) 014003)

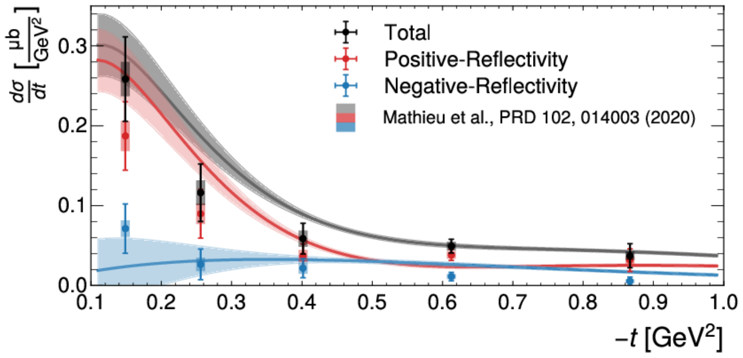


- Vector and axial vector couplings to $T\gamma$ system
- Minimal model has only the momentum independent couplings
  - Gauge invariance to be imposed explicitly
- Tensor meson dominance (TMD) model has vector couplings from the full gauge invariant Lagrangian
- Both models are fit to the CLAS a_2 photoproduction cross section data, f_2 photoproduction cross section is the prediction

- TMD required to explain the dip in the a_2 production cross section
- Unnatural parity exchanges dominate the production mechanism according to the minimal model
  - Counterintuitive to phenomenological predictions, corrected by TMD
- TMD over-estimates f_2 production cross section



Two models for a_2 and f_2 photoproduction (Phys.Rev.D 102 (2020) 014003), data from CLAS (Phys.Rev.C 102, 032201 (2020))



Recent GlueX analysis (2501.03091)

Recent results from GlueX support TMD predictions for cross section

Parity asymmetry needs refinement

Summary and future directions

- Understand the underlying mechanism in the high energy photoproduction of mesons
- Provide predictions that can be tested in ongoing and future experiments
  - Data driven modeling
  - Many predictions tested and validated
- Close collaboration with experimentalists on each of these projects
- Gauge invariance in single pion photoproduction has been studied
- Some aspects of $\pi\pi$ photoproduction has been understood
  - Analysis of the $\pi\Delta$ production amplitude and extraction of residues in progress
  - Analysis of mass dependence the SDMEs in progress
- More complicated systems currently being studied include $b_1 p$, $b_1\Delta$, ρp, and $\rho\Delta$ photoproduction
- Interesting physics from $\eta\pi$ system in the form of double Regge exchanges
  - Double Regge exchanges in the photoproduction of $\eta\pi$ in progress
- All analyses with the goal of modeling $\pi_1\Delta$ and $\pi_1 p$ photoproduction