Probing QGP via local charge fluctuations in heavy-ion collisions

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INTERSECTIONS – CIPANP 2025

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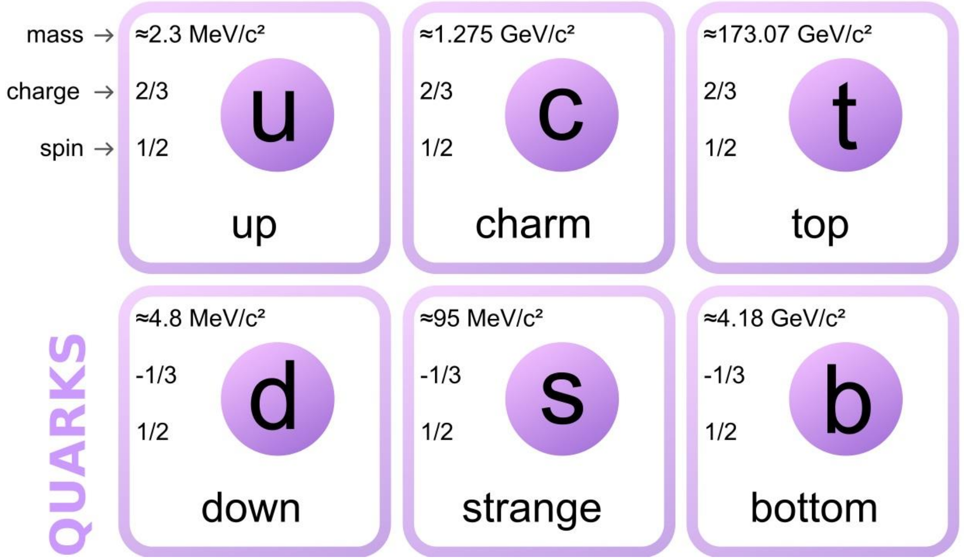
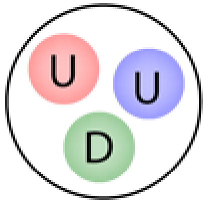
Phase Diagram of QCD Matter

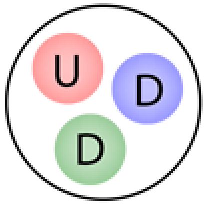




Proton

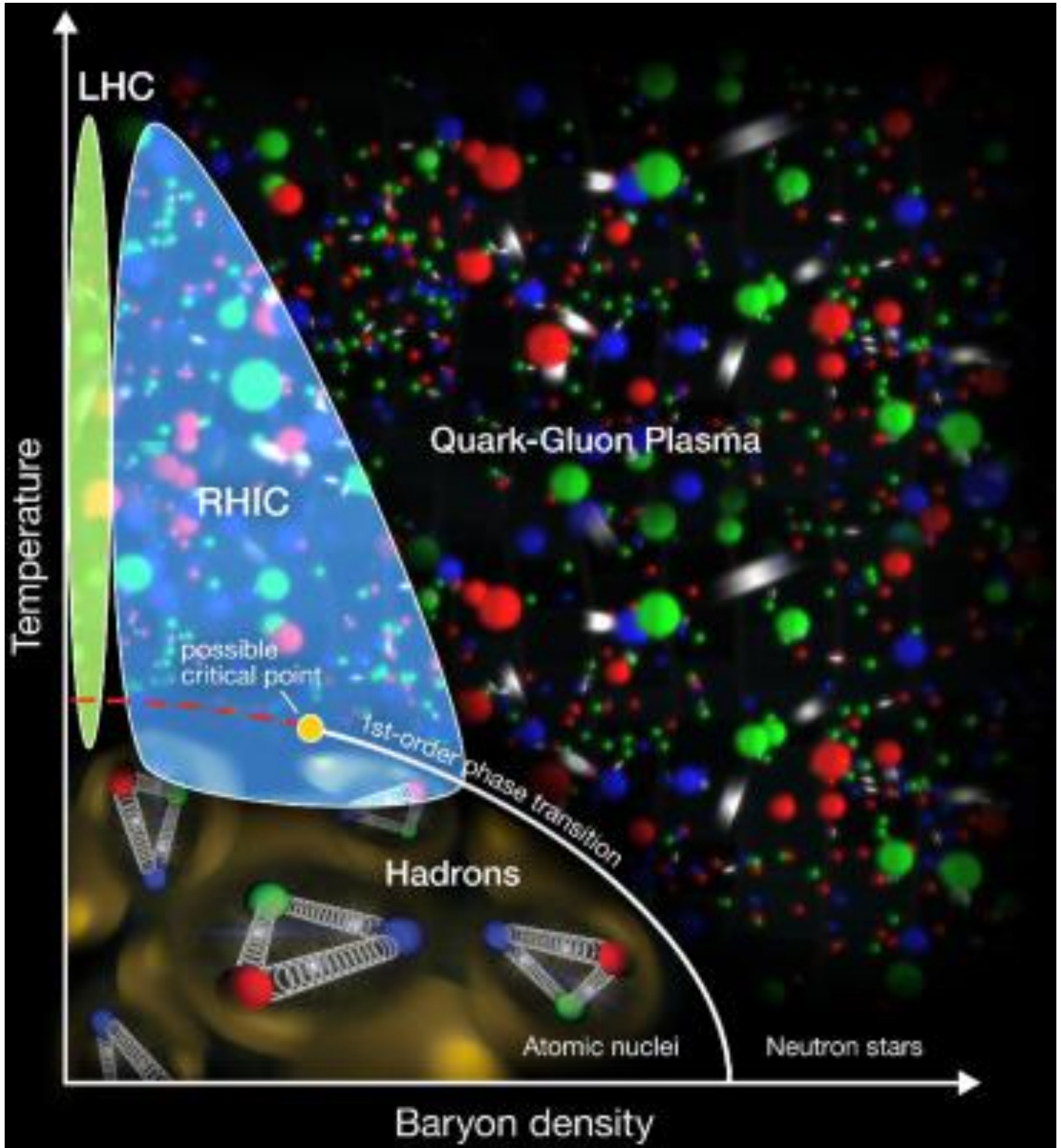
U = "up" quark $+\frac{2}{3}\,e$

D = "down" quark $-\frac{1}{3}\,e$



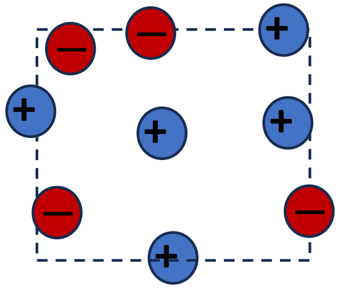
Neutron

Quarks make up hadrons!



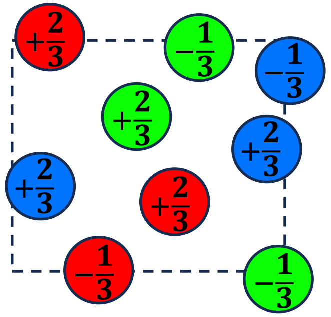
The Main Idea

While hadrons carry ***integer*** electric charges, quarks carry ***fractional*** electric charges.



Hadrons

vs



Quarks

D_{QGP} < D_{HG} → Distinct signal for QGP in heavy-ion collisions

Quantified by:

$$D = 4\frac{\kappa_2[Q]}{\langle N_{\mathrm{ch}}\rangle}$$

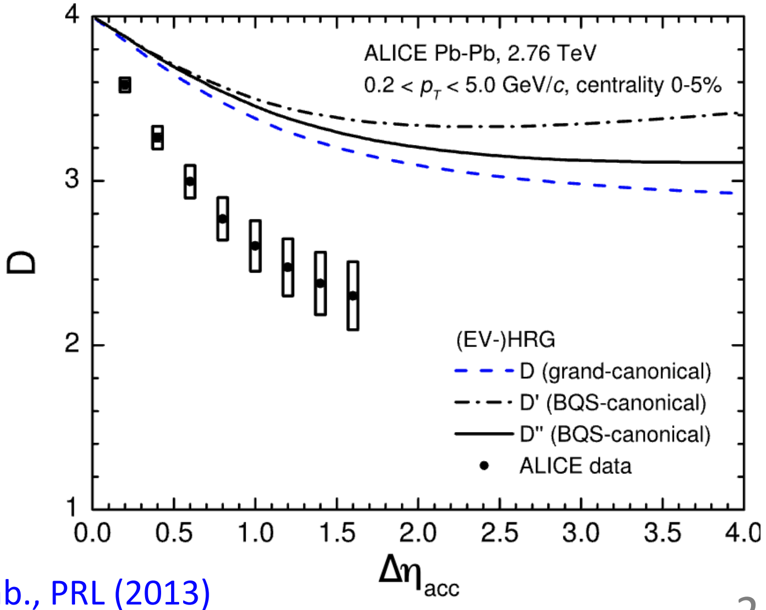
Here $\kappa_2[Q] = \langle Q^2\rangle - \langle Q\rangle^2$

$$N_{\mathrm{ch}} = N_+ + N_-$$

Previous estimates using grand canonical ensemble (GCE) limit:

- $D_{HG} \approx 4$
- $D_{QGP} \approx 1$

No quantitative calculations have been done for QGP w/o GCE limit



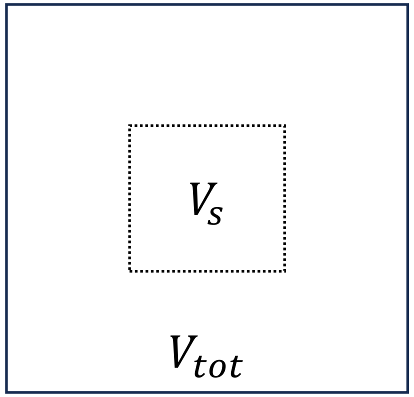
Koch, Jeon, PRL (2000);

ALICE Collab., PRL (2013)
Vovchenko, Koch, PRC (2021)

GCE vs Heavy-Ion Collisions

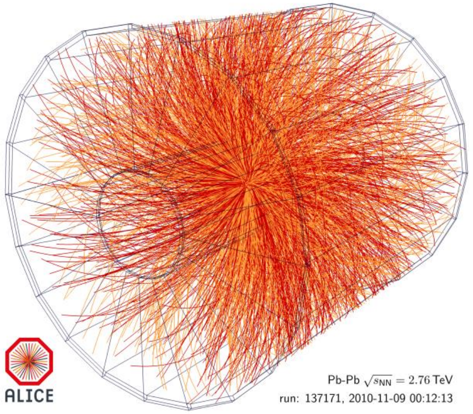
In Grand Canonical Ensemble (GCE):

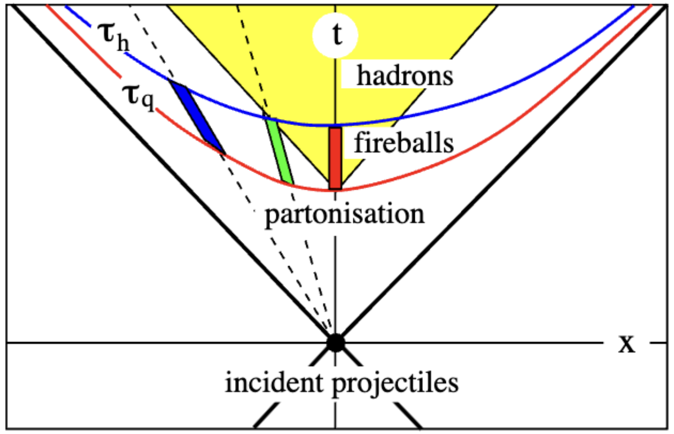
- Coordinate space measurements
- Subvolume much smaller than total volume, $V_s \ll V_{tot}$
  - Charge conservation effects are neglected
  - Charges are free to fluctuate



In heavy-ion collisions:

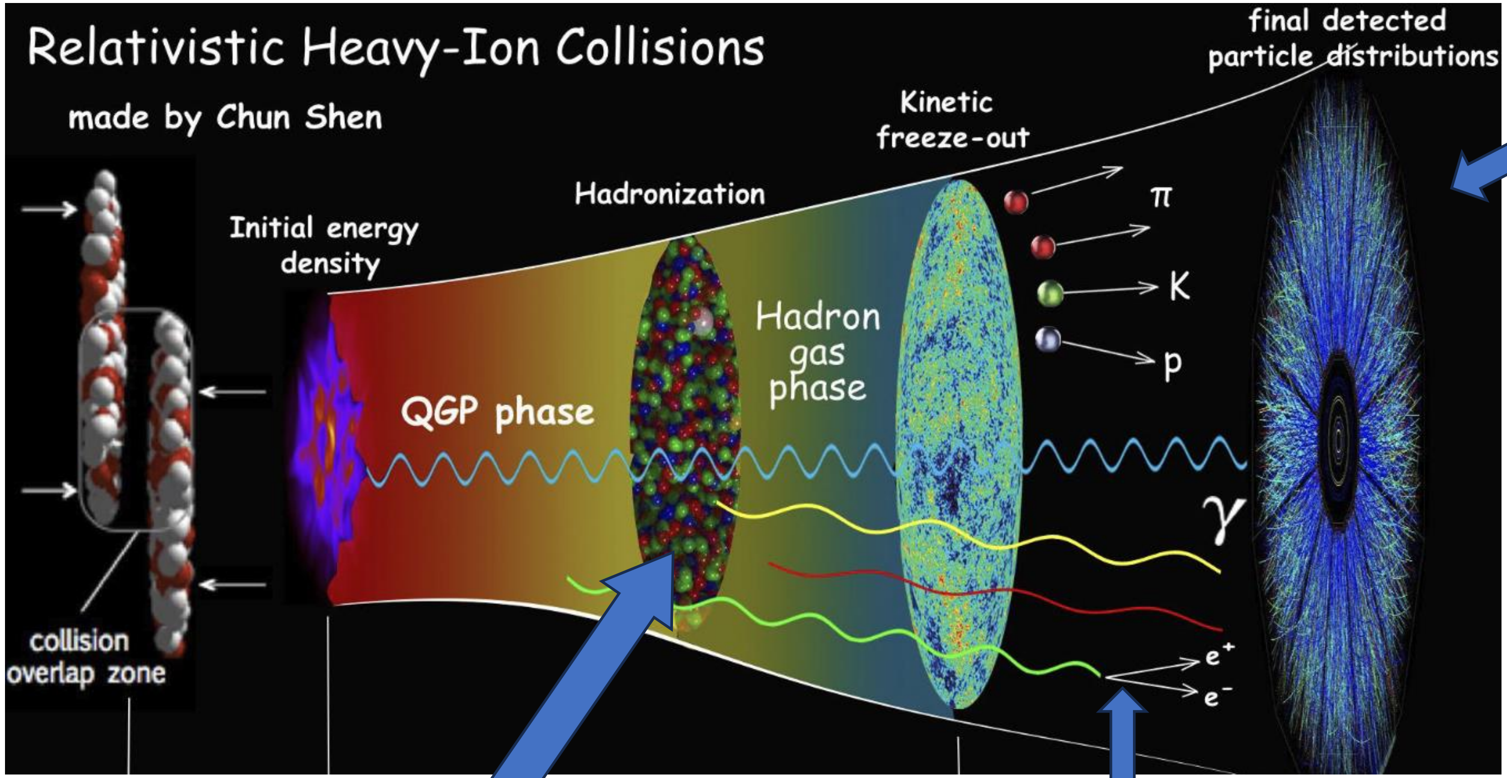
- Momentum acceptance cuts
- V_s is comparable to V_{tot}
  - Global charge conservation effects present
- Presence of causally disconnected regions of fireball
  - Local conservation effects come into play





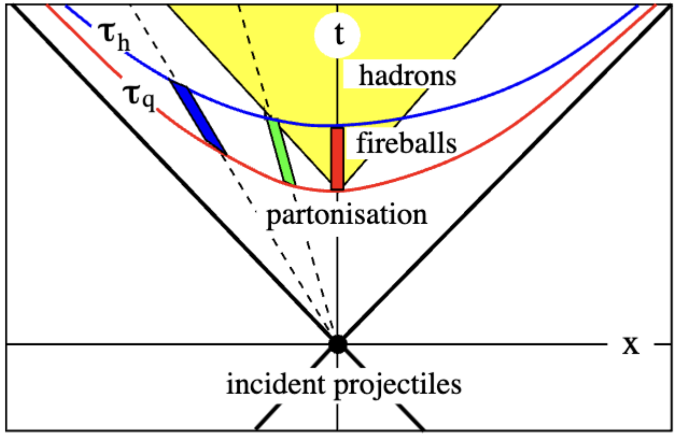
Castorina, Satz, IJMPE Vol. 23 No. 4 (2014)

Overview



4. Momentum acceptance coordinates

3. Locality of charge conservation

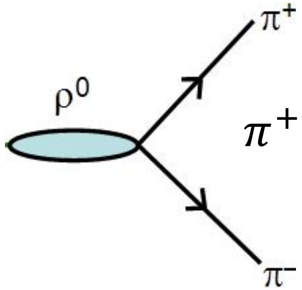


Castorina, Satz, IJMPE Vol. 23 No. 4 (2014)

1. Fluctuations at hadronization (primordial charges)

ω – Distinguishes HG from QGP

2. Resonance decays



$\pi^+\pi^-$ are correlated!

Charge Susceptibility, χ_2^Q with Resonance Decays, γ_Q

In GCE, $\kappa_2[Q] = VT^3\chi_2^Q$

$$\chi_2^Q = \langle n_{\mathrm{ch}}^{\mathrm{prim}} \rangle + \varphi_2^{Q,\mathrm{prim}}$$

Skellam baseline Correction

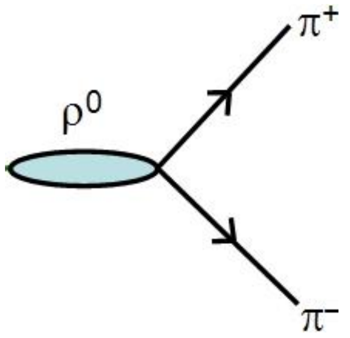
Before decays, strength of interactions is parametrized by ω:

$$\chi_2^Q = \omega \langle n_{\mathrm{ch}}^{\mathrm{prim}} \rangle \qquad \omega = \frac{\kappa_2[Q]}{\langle N_{\mathrm{ch}}^{\mathrm{prim}} \rangle} \qquad \varphi_2^{Q,\mathrm{prim}} = (\omega - 1)\langle n_{\mathrm{ch}}^{\mathrm{prim}} \rangle$$

After resonance decays, net charge remains conserved but multiplicities of + and – charges increase:

$$\langle n_{\mathrm{ch}} \rangle = \gamma_Q \langle n_{\mathrm{ch}}^{\mathrm{prim}} \rangle$$

$\gamma_Q \approx 1.67$ Thermal FIST



Correlated!

$$\boxed{\chi_2^Q = \langle n_{\mathrm{ch}} \rangle + \left(\frac{\omega}{\gamma_Q} - 1 \right) \langle n_{\mathrm{ch}} \rangle}$$

Grand-Canonical Fluctuations in Hadron Gas

Electric charge fluctuations can freeze-out in the hadron gas phase or as early as in the QGP phase. Particles detected in experiment may contain the memory of where this occurs.

This distinguishes whether a signal is obtained for HG or QGP.

variance

$$\omega = \frac{\kappa_2[Q]}{\langle N_{\mathrm{ch}}^{\mathrm{prim}} \rangle}$$

charged hadron multiplicity

For hadron gas scenario, ω can be estimated by Poisson statistics, with

$$\kappa_2[Q] \approx \langle N_{\mathrm{ch}}^{\mathrm{prim}} \rangle \rightarrow \omega_{HG} = 1$$

But ω is enhanced by Bose-Einstein statistics for pions and the presence of multi-charged hadrons. An HRG model calculation (Thermal-FIST) gives

$$\boxed{\omega_{HG} = 1.1}$$

Thermal FIST

Grand-Canonical Fluctuations in QGP

But for QGP (quarks), we must take a closer look:

In GCE, $\kappa_2[Q] = V\chi_2^Q$

$$\omega = \frac{\kappa_2[Q]}{\langle N_{\rm ch}^{\rm prim}\rangle} = \frac{VT^3\chi_2^Q}{S}\frac{S}{\langle N_{\rm ch}^{\rm prim}\rangle}$$

$$= \frac{\chi_2^Q}{s/T^3}\frac{S}{\langle N_{\rm ch}\rangle}\frac{\langle N_{\rm ch}\rangle}{\langle N_{\rm ch}^{\rm prim}\rangle}$$

Given by free QGP limit:

$(\chi_2^Q)_{\rm QGP} = 2/3$ $\quad (s/T^3)_{\rm QGP} = \frac{19\pi^2}{9}$

(Stefan-Boltzmann limit)

$\gamma_Q \approx 1.67$ calculated using a thermal model estimate at T = 155 MeV (Thermal-FIST)

$S/N_{\rm ch} = 6.7 \pm 0.8$ at LHC energies

P. Hanus, A. Mazeliauskas, K. Reygers, PRC (2019)

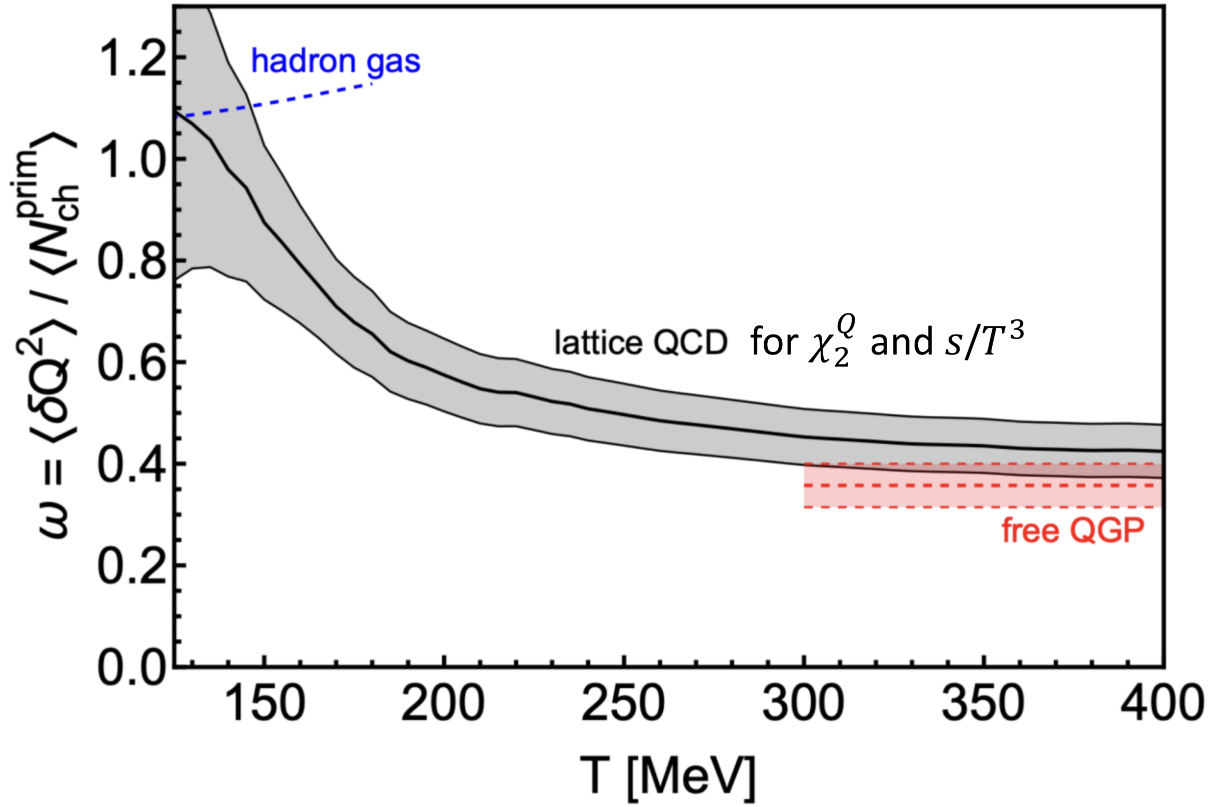
$$\boxed{\omega_{QGP} = 0.36 \pm 0.04}$$

Compared to $\omega_{HG} = 1.1$

Grand-Canonical Fluctuations (General Case)

Using Lattice QCD data (first principles calculations), we can justify our estimates for ω

Temperature dependence of ω



$\omega_{HG} \approx 1.1$ around T < 160 MeV (hadron gas regime)

ω_{QGP} approaches the Stefan-Boltzmann limit ($\omega_{QGP} \approx 0.36$), justifying our value for ω_{QGP}.

Correlations and Charge Conservation Implementation, C_2^Q

To account for exact charge conservation, we use the 2-point density correlator,

$$C_2^Q(\mathbf{r}_1, \mathbf{r}_2) \equiv \langle \delta\rho_Q(\mathbf{r}_1)\delta\rho_Q(\mathbf{r}_2)\rangle \quad \text{where} \quad \delta\rho_Q(\mathbf{r}_i) = \rho_Q(\mathbf{r}_i) - \langle \rho_Q(\mathbf{r}_i)\rangle$$

V. Vovchenko, PRC (2024)

$$C_2^Q(\mathbf{r}_1, \mathbf{r}_2) = \chi_2^Q \left[\delta(\mathbf{r}_1 - \mathbf{r}_2) - \frac{\varkappa(\mathbf{r}_1, \mathbf{r}_2)}{V_{\text{tot}}}\right] \qquad \mathbf{r} = \eta$$

local correlation balancing contribution

Gaussian local charge conservation:

$$\varkappa(\eta_1, \eta_2) \propto \exp\left[-\frac{(\eta_1 - \eta_2)^2}{2\sigma_\eta^2}\right]$$

Global:

$$\sigma_\eta \to \infty : \varkappa(\eta_1, \eta_2) \to 1$$

$$C_2^Q(\eta_1, \eta_2) = \chi_2^Q \left[\delta(\eta_1 - \eta_2) - \frac{1}{V}\right]$$

Integrating C_2^Q yields the net-charge variance

$$\kappa_2[Q]|_{|\eta|<\eta_{\text{cut}}} = \int_{-\eta_{\text{cut}}/2}^{\eta_{\text{cut}}/2} d\eta_1 \int_{-\eta_{\text{cut}}/2}^{\eta_{\text{cut}}/2} d\eta_2 C_2^Q(\eta_1, \eta_2)$$

Putting everything together we arrive at…

Momentum Acceptance

Combining the previous equations,

$$\kappa_2[Q]|_{|\eta|<\eta_{\text{cut}}} = \int_{-\eta_{\text{cut}}/2}^{\eta_{\text{cut}}/2} d\eta_1 \int_{-\eta_{\text{cut}}/2}^{\eta_{\text{cut}}/2} d\eta_2 C_2^Q(\eta_1, \eta_2)$$

$$C_2^Q(\mathbf{r}_1, \mathbf{r}_2) = \chi_2^Q \left[\delta(\mathbf{r}_1 - \mathbf{r}_2) - \frac{\varkappa(\mathbf{r}_1, \mathbf{r}_2)}{V_{\text{tot}}} \right]$$

$$\chi_2^Q = \langle n_{\text{ch}} \rangle + \left(\frac{\omega}{\gamma_Q} - 1 \right) \langle n_{\text{ch}} \rangle$$

However, fluctuations in heavy-ion collisions are measured in momentum acceptance coordinates rather than spatial coordinates. Incorporating this,

$$D = 4\frac{\kappa_2[Q]}{\langle Q^+ + Q^- \rangle} = 4 \left\{ \underbrace{1}_{\text{Skellam baseline}} - \underbrace{\left(1 - \frac{\omega}{\gamma_Q}\right) \frac{\langle p^2(\eta) \rangle}{\langle p(\eta) \rangle}}_{\text{local 2-particle correlations}} - \underbrace{\frac{\omega}{\gamma_Q} \frac{\langle p(\eta_1) p(\eta_2) \rangle_\varkappa}{\langle p(\eta) \rangle}}_{\text{long-range 2-particle correlations}} \right\}$$

where,

$$\langle p^n(\eta) \rangle = \frac{1}{2\eta_{\max}} \int d\eta p^n(\eta)$$

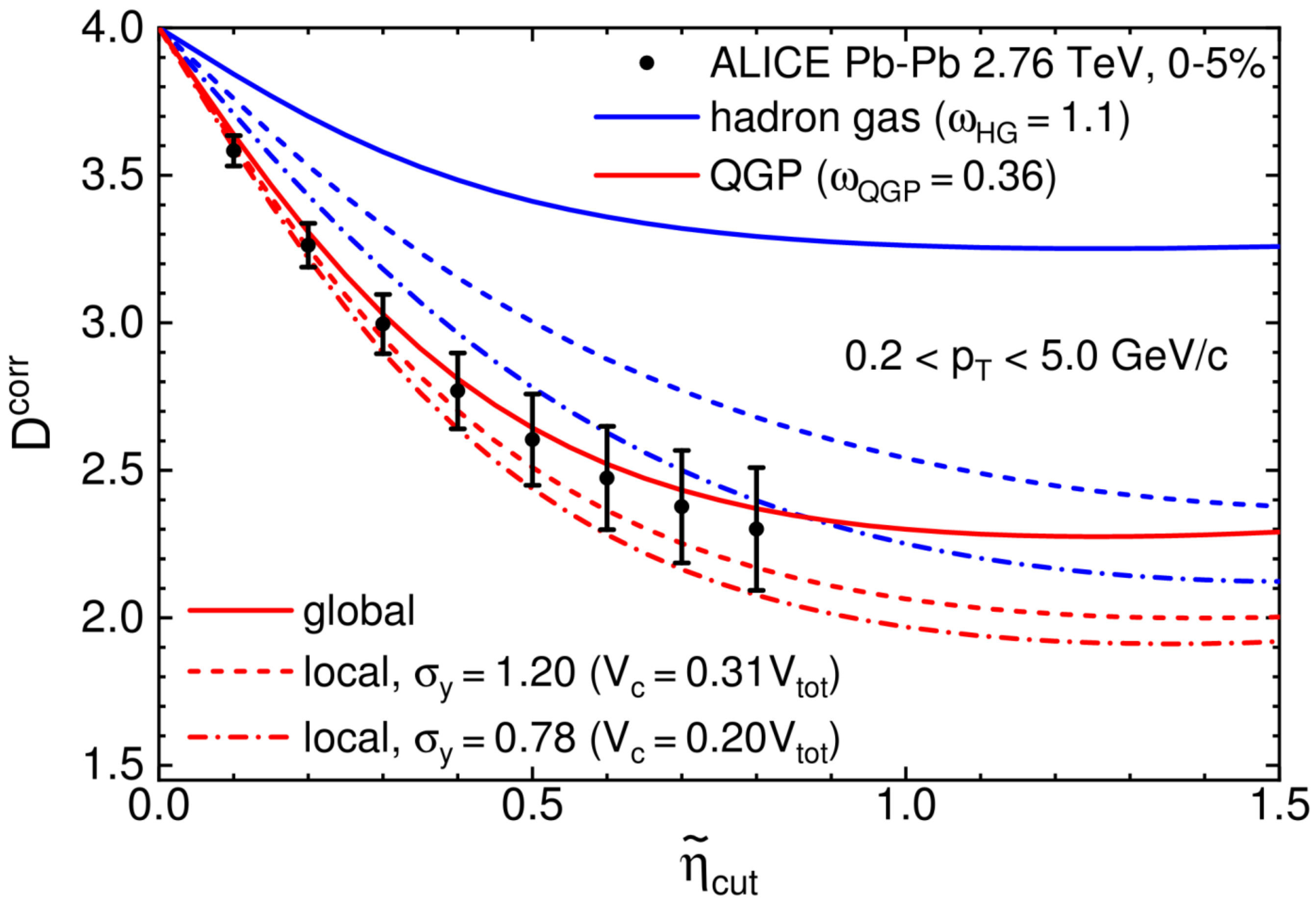
and p(η) is found using BW model calculations

$$\langle p(\eta_1) p(\eta_2) \rangle_\varkappa = \frac{1}{4\eta_{\max}^2} \iint d\eta_1 d\eta_2 p(\eta_1) p(\eta_2) \varkappa(\eta_1, \eta_2)$$

Parra, RP, Koch, Ratti, Vovchenko, 2504.02085 (2025)

Comparison to ALICE Run 1 Data

$$D = 4\left\{1 - \left(1 - \frac{\omega}{\gamma_Q}\right)\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle} - \frac{\omega}{\gamma_Q}\frac{\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}}{\langle p(\eta)\rangle}\right\}$$



Parameters used:

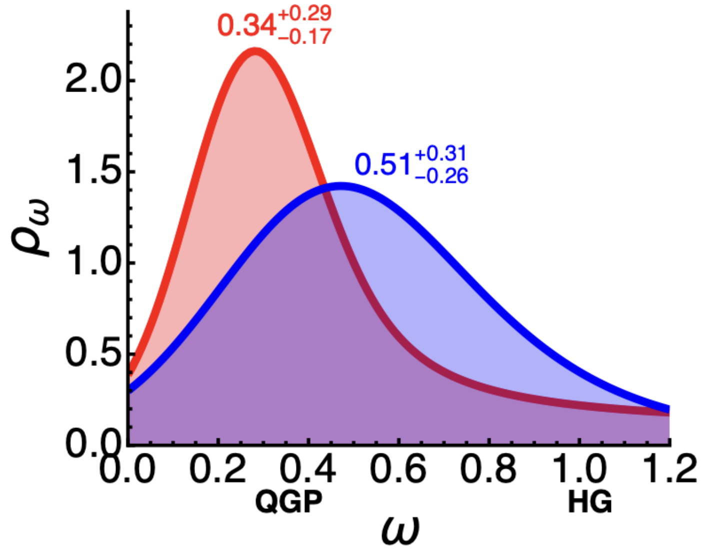
$\omega_{HG} = 1$ $\omega_{QGP} = 0.36$

$\gamma_Q = 1.67$

- Vary σ_y for global and local charge conservation

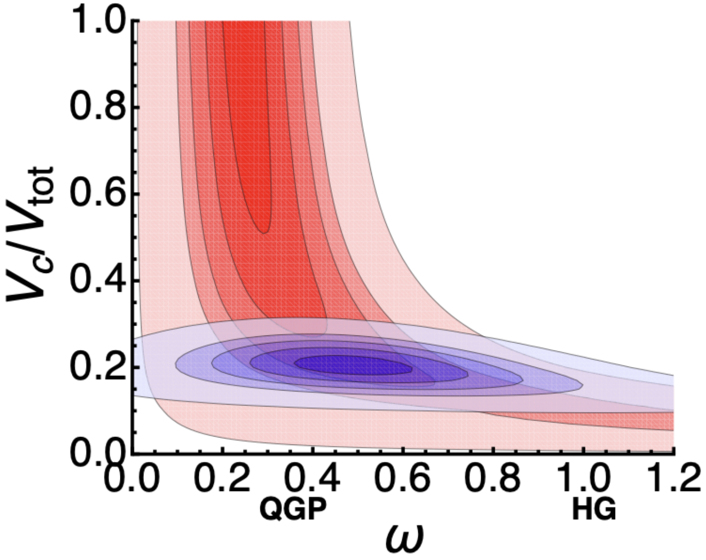
Parra, RP, Koch, Ratti, Vovchenko, 2504.02085 (2025)

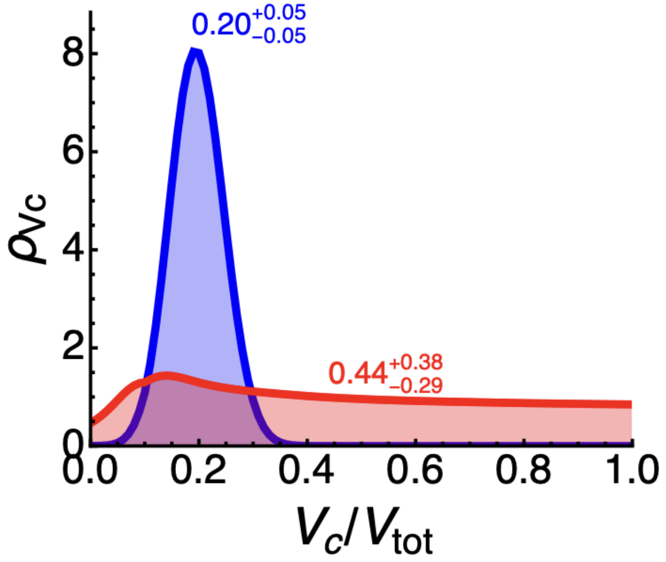
Bayesian analysis of ω and σ parameters in D-measure



Uniform Prior
$\omega \sim U(0,1.2)$, $V_C/V_{tot} \sim U(0,1)$
$B_{QGP/HG} = 9.7$

Local Conservation Prior
$\omega \sim U(0,1.2)$, $V_C/V_{tot} \sim \mathcal{N}(0.20,0.05^2)$
$B_{QGP/HG} = 4.7$





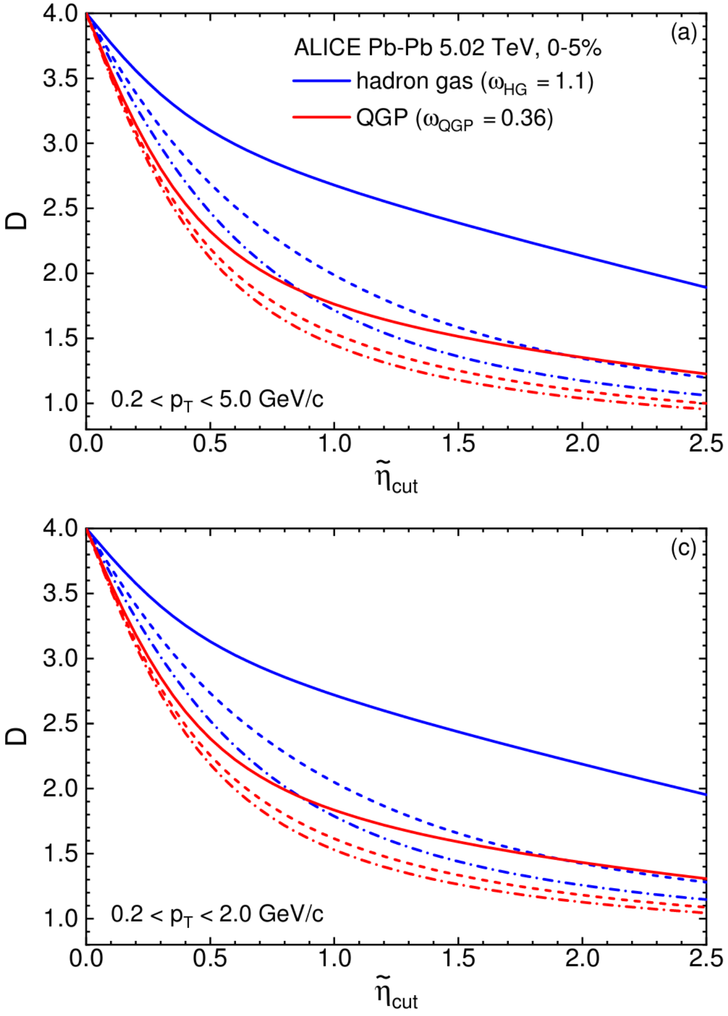
→ **Moderate** evidence for freeze-out of charge fluctuations in QGP phase (ω).

Bayes factor BF_{12} for H_1 over H_2	Evidence category
> 100	Extreme evidence for H_1 over H_2
30 - 100	Very strong evidence for H_1 over H_2
10 - 30	Strong evidence for H_1 over H_2
3 - 10	Moderate evidence for H_1 over H_2
1 - 3	Anecdotal evidence for H_1 over H_2
1	No evidence over H_2

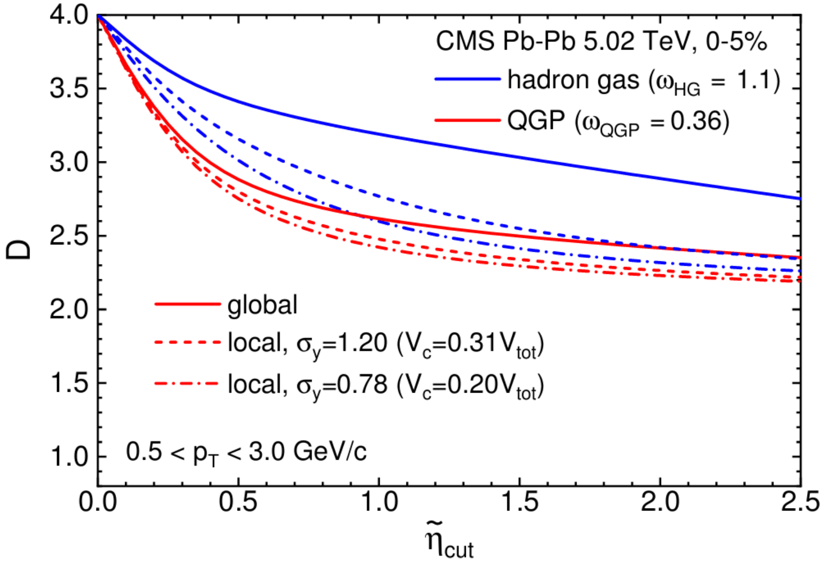
Parra, RP, Koch, Ratti, Vovchenko, 2504.02085 (2025)

LHC Run 2 predictions

ALICE



CMS



Conclusion and Outlook

- We performed quantitative calculations for charge fluctuations for QGP and hadronic scenario.
  - To do this, we incorporated global/local charge conservation effects, resonance decays, and momentum acceptance.
- We obtained moderate evidence for freeze-out of charge fluctuations in QGP phase.
- We presented predictions for ALICE Run 2 using our formalism.
- Looking forward to ALICE Run 2 data to obtain stronger evidence.
- This formalism can be extended to RHIC energies.

Conclusion and Outlook

- We performed quantitative calculations for charge fluctuations for QGP and hadronic scenario.
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Thank you for your attention!

Backup Slides

Blast-Wave Model

To calculate acceptance probabilities, we use the blast-wave model:

$$\sqrt{s_{\mathrm{NN}}} = 2.76\ \mathrm{TeV} \qquad p(\eta) \approx 0.84 p_\pi(\eta) + 0.12 p_K(\eta) + 0.04 p_p(\eta)$$

Single-particle momentum distribution:

$$\boxed{\frac{d^3 N_i}{dp_T d\tilde{\eta}}(\eta) = \frac{E(\tilde{\eta},\eta) p_T^2 \cosh\tilde{\eta}}{\sqrt{(p_T \cosh\tilde{\eta})^2 + m^2}} \int_0^1 \zeta d\zeta \, \exp\left[-\frac{E(\tilde{\eta},\eta)\ \cosh\rho}{T}\right] I_0\left(\frac{p_T \sinh\rho}{T}\right)}$$

$$E(\tilde{\eta},\eta) \equiv m_T\ \cosh(y-\eta) = m_T\ \cosh\left(\operatorname{arcsinh}\left[\frac{p_T\ \sinh\tilde{\eta}}{m_T}\right] - \eta\right)$$

$$\rho = \tanh^{-1}(\beta_s \zeta^{n_{\mathrm{BW}}})$$

$$m_T = \sqrt{p_T^2 + m_i^2}$$

Acceptance probability:

$$p_i(\eta) = \frac{\int_{p_T^{\min}}^{p_T^{\max}} dp_T \int_{-\tilde{\eta}_{\mathrm{cut}}}^{\tilde{\eta}_{\mathrm{cut}}} d\tilde{\eta} \frac{d^3 N}{dp_T d\tilde{\eta}}(\eta)}{\int_0^\infty dp_T \int_{-\infty}^{\infty} d\tilde{\eta} \frac{d^3 N}{dp_T d\tilde{\eta}}(\eta)}$$

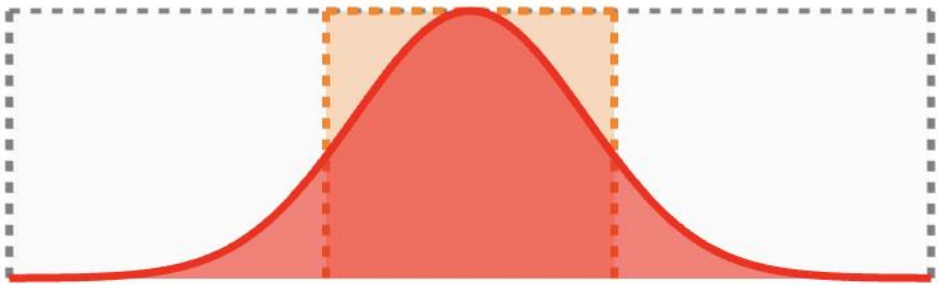
Truncated fireball vs Gaussian correlation

Truncated Fireball

V_c

vs

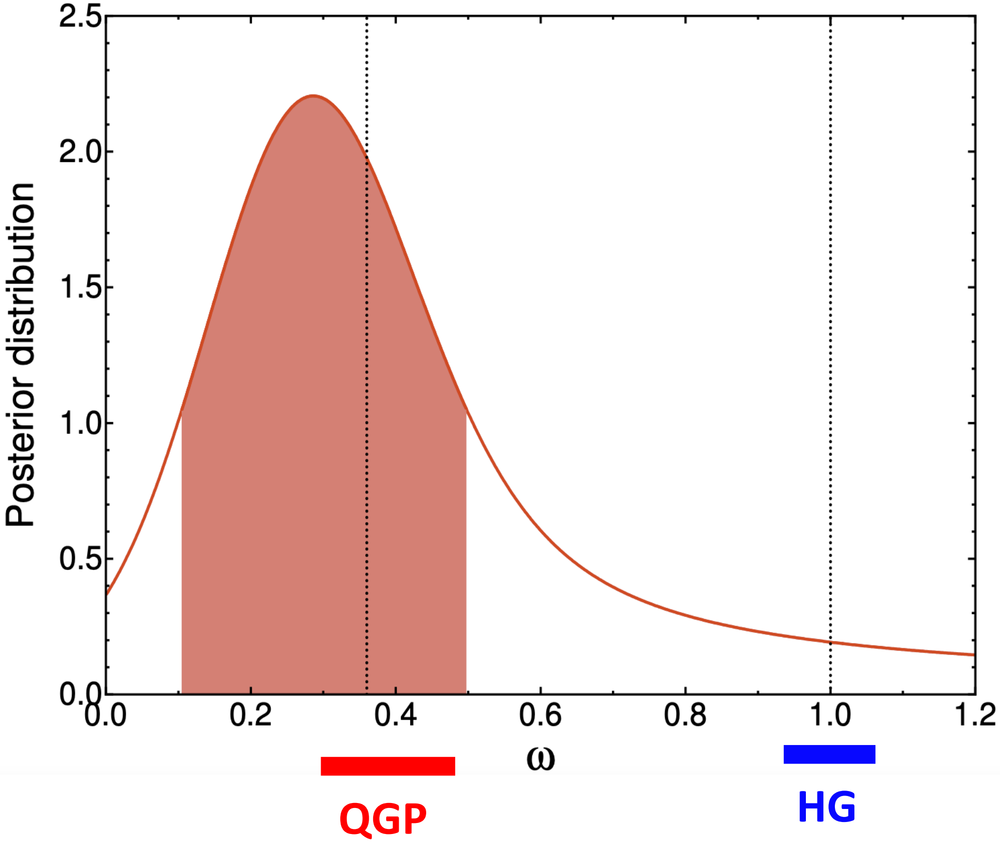
Gaussian correlation

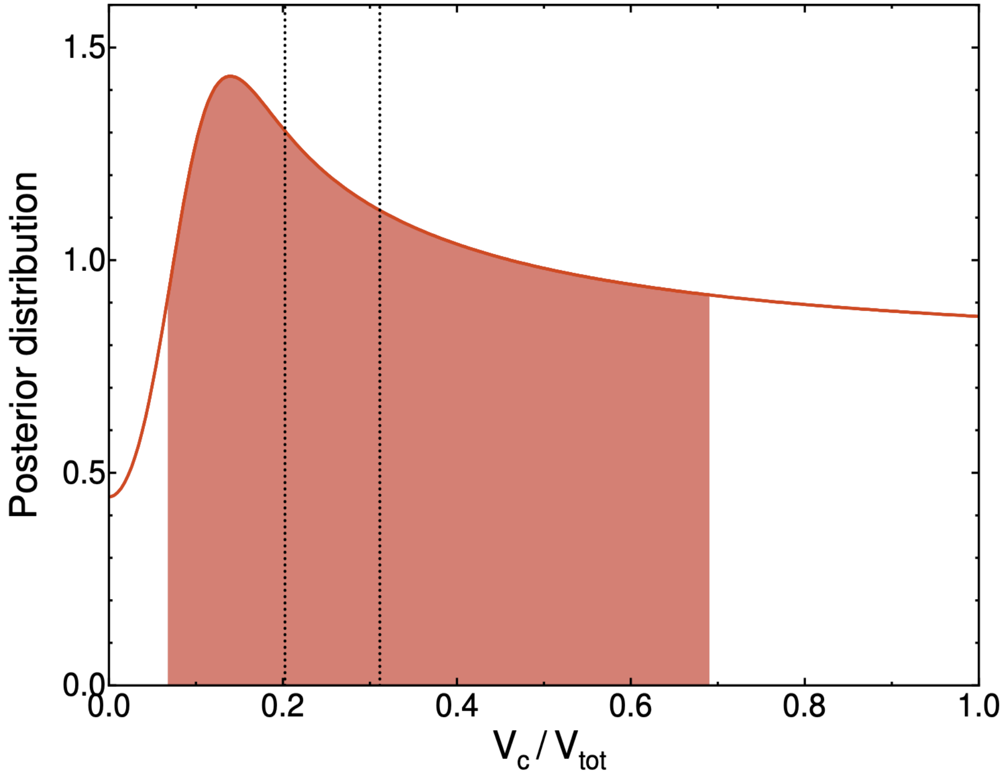


From V. Vovchenko PHENOmenal talk

With Gaussian correlation, hadrons at forward/backward rapidities also contribute to the system

Bayesian analysis of ω and σ parameters in D-measure





$$k(\sigma_y) = \sqrt{2\pi}\sigma_y \mathrm{erf}\left(\frac{\eta_{\max}}{\sqrt{2}\sigma_y}\right) \approx \sqrt{2\pi}\sigma_y$$

$$V_c = k dV/dy$$

Moderate evidence for freeze-out of charge fluctuations in QGP (ω).

Bayes factor with uniform priors: 9.8

J. Parra, R. Poberezhniuk, et al., in preparation

Grand canonical net-charge susceptibility

Poisson baseline

correction (two-particle local correlations)

$$\chi_2^Q = \langle n_+ + n_- \rangle + \tilde{\chi}_2^Q$$

Parametrization: $$\chi_2^Q = \omega \langle n_+ + n_- \rangle$$

$$\tilde{\chi}_2^Q = (\omega - 1)\langle n_+ + n_- \rangle$$

Resonance decays: $$\chi_2^{Q,\text{after}} = \gamma_+ \langle n_+ \rangle + \gamma_- \langle n_- \rangle + \tilde{\chi}_2^{Q,\text{after}}$$

$$Q = \gamma_+ n_+ - \gamma_- n_- = n_+ - n_-$$

$$\chi_2^{Q,\text{after}} = \chi_2^{Q,\text{before}}$$

$$\chi_2^Q = \gamma_+ \langle n_+ \rangle + \gamma_- \langle n_- \rangle + (\omega - \gamma_+)\langle n_+ \rangle + (\omega - \gamma_-)\langle n_- \rangle$$

$$\gamma_Q = \frac{\gamma_+ \langle n_+ \rangle + \gamma_- \langle n_- \rangle}{\langle n_+ \rangle + \langle n_- \rangle}$$

$$\chi_2^Q = \gamma_Q \langle n_+ + n_- \rangle + (\omega - \gamma_Q)\langle n_+ + n_- \rangle$$

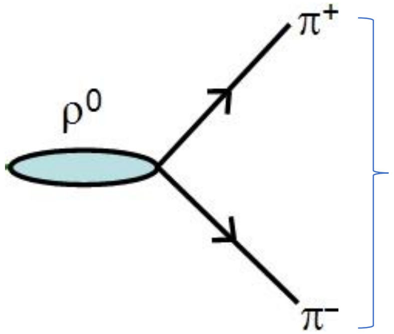
Charge Susceptibility, χ_2^Q and Resonance Decays, γ_Q

Next, we take resonance decays into account.

Decays like $\rho^0 \rightarrow \pi^+\pi^-$ introduce correlations between charged particles

Charge susceptibility after decays:

$$\chi_2^{Q,\mathrm{final}} = \gamma_+\langle n_+\rangle + \gamma_-\langle n_-\rangle + \tilde{\chi}_2^{Q,\mathrm{final}}$$
$$= \langle n_+ + n_-\rangle_{\mathrm{final}} + \tilde{\chi}_2^{Q,\mathrm{final}},$$



Correlated!

But net electric charge density is conserved overall

$$n_Q = \gamma_+ n_+ - \gamma_- n_- = n_+ - n_-$$
$$\chi_2^{Q,\mathrm{final}} = \chi_2^{Q,\mathrm{prim}}$$

Implementing interaction parameter ω:

$$\chi_2^Q = \gamma_+\langle n_+\rangle + \gamma_-\langle n_-\rangle + (\omega - \gamma_+)\langle n_+\rangle + (\omega - \gamma_-)\langle n_-\rangle$$

$\approx 5/3$

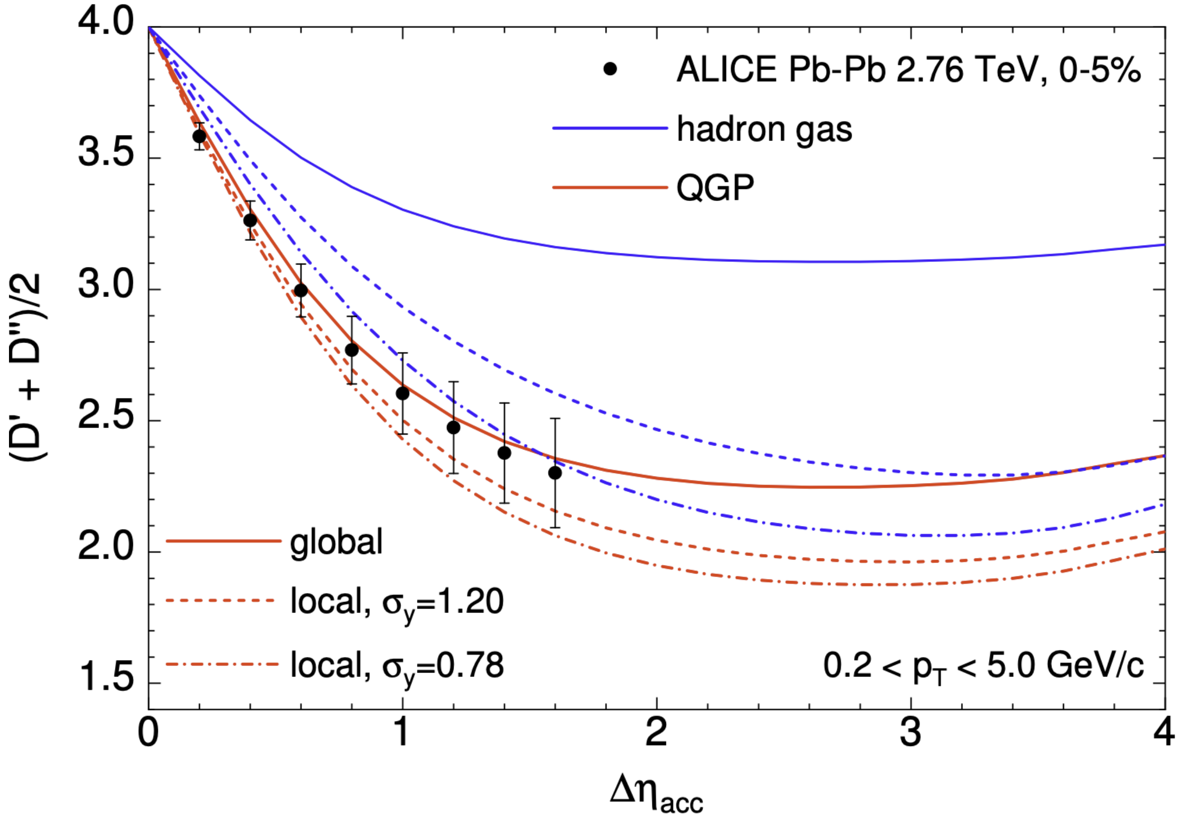
$$\boxed{\begin{aligned}\chi_2^Q &= \gamma_Q\langle n_+ + n_-\rangle + (\omega - \gamma_Q)\langle n_+ + n_-\rangle \\ &= \langle n_+ + n_-\rangle_{\mathrm{final}} + \left(\frac{\omega}{\gamma_Q} - 1\right)\langle n_+ + n_-\rangle_{\mathrm{final}}\end{aligned}}$$

Comparison to ALICE Run 1 Data

D-measure

$$D = 4\left\{1 - \left(1 - \frac{\omega}{\gamma_Q}\right)\frac{\langle p^2(\eta)\rangle}{\langle p(\eta)\rangle} - \frac{\omega}{\gamma_Q}\frac{\langle p(\eta_1)p(\eta_2)\rangle_{\varkappa}}{\langle p(\eta)\rangle}\right\}$$

D as a function of $\Delta\eta_{\mathrm{acc}} = 2\tilde{\eta}_{\mathrm{cut}}$



D-measure Corrected for Experiment

$$\frac{D'_{\mathrm{HG}} + D''_{\mathrm{HG}}}{2} \quad \text{and} \quad \frac{D'_{\mathrm{QGP}} + D''_{\mathrm{QGP}}}{2}$$

Where,

$$D'_{\mathrm{HG}} = D_{\mathrm{HG}} + 4\langle p(\eta)\rangle, \quad D''_{\mathrm{HG}} = \frac{D_{\mathrm{HG}}}{1 - \langle p(\eta)\rangle},$$

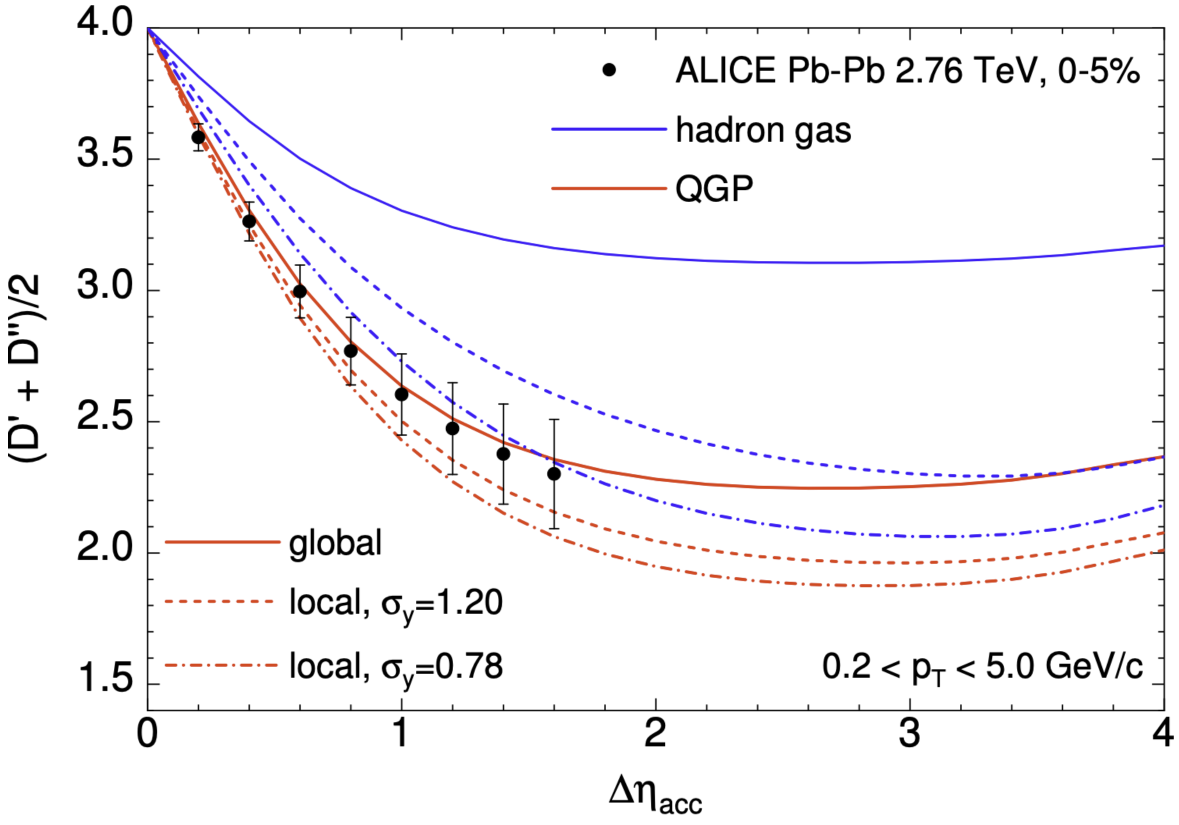
$$D'_{\mathrm{QGP}} = D_{\mathrm{QGP}} + 4\langle p(\eta)\rangle, \quad D''_{\mathrm{QGP}} = \frac{D_{\mathrm{QGP}}}{1 - \langle p(\eta)\rangle}$$

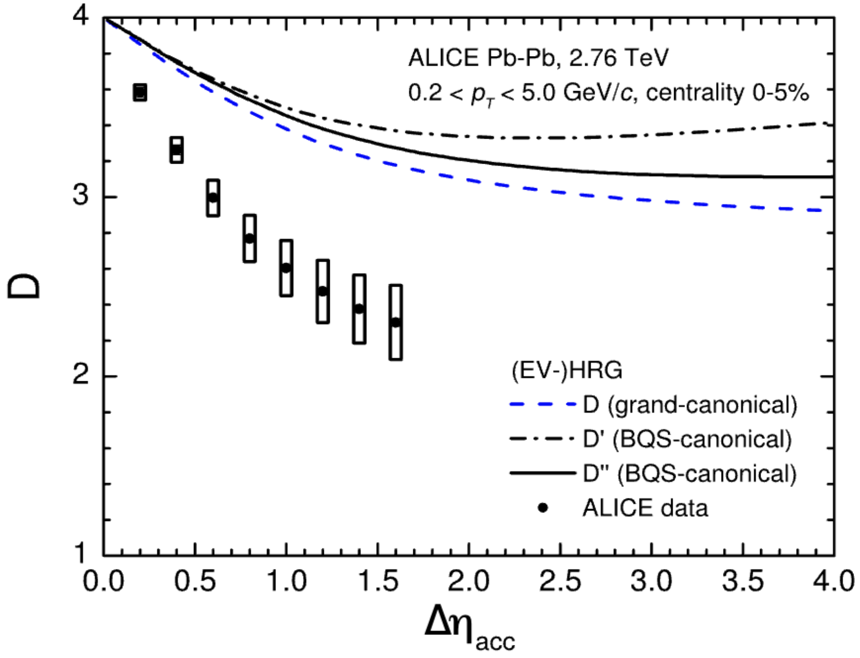
Parameters used:

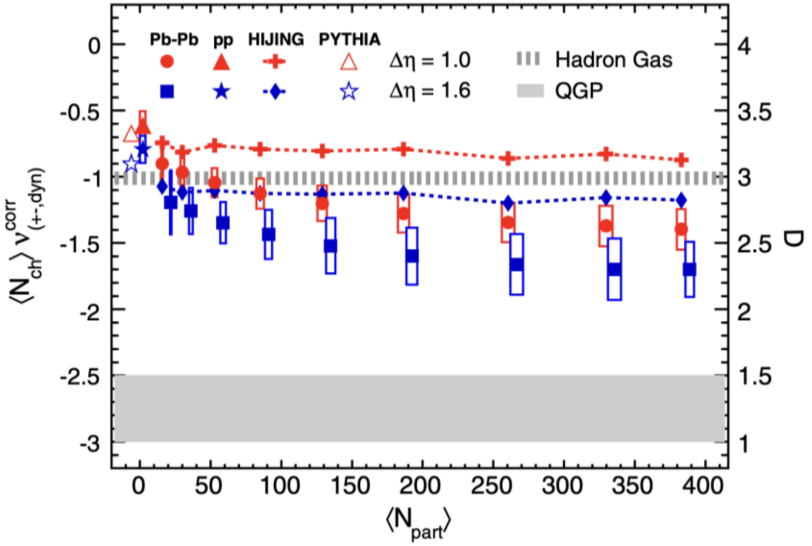
$\omega_{HG} = 1$ $\omega_{QGP} = 0.36$ γ_Q = 5/3

J. Parra, R. Poberezhniuk, et al., in preparation

Comparison to previous estimates



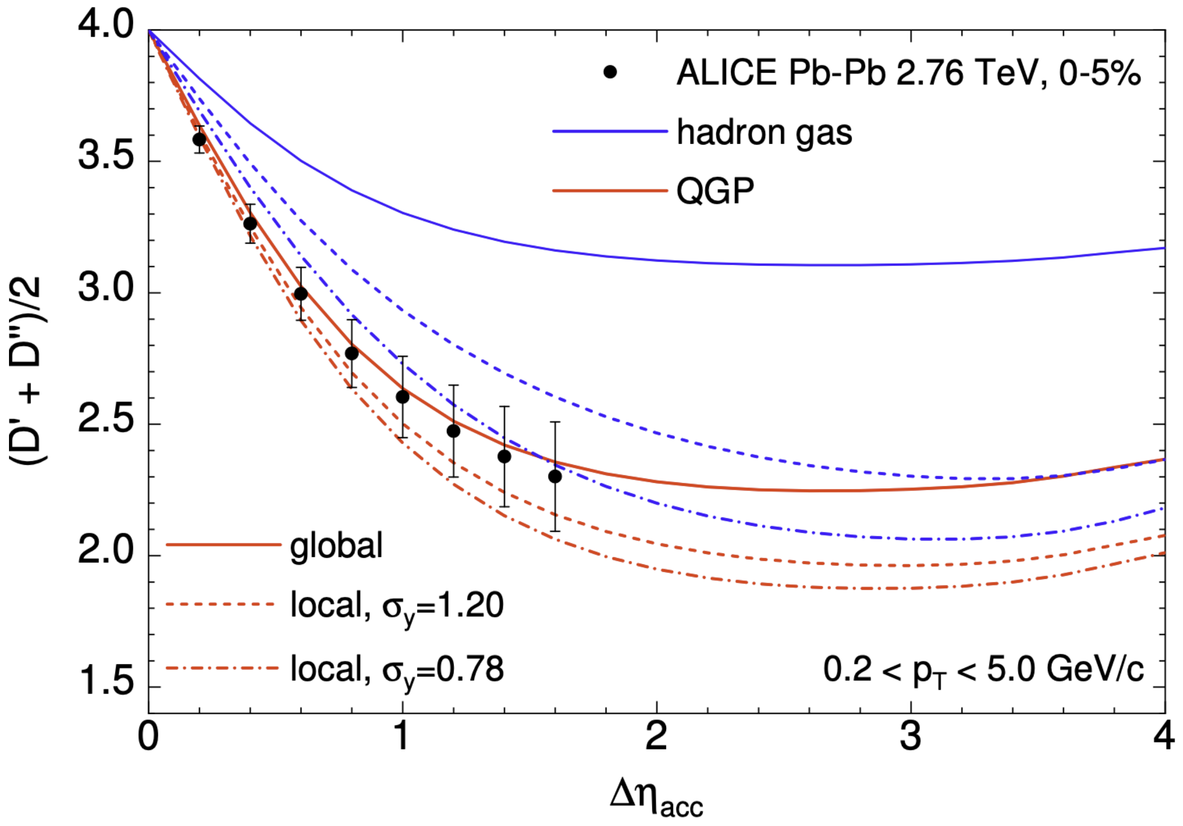




Bayesian analysis of ω and σ parameters in D-measure

Posterior probability = likelihood function → $p(\omega, \sigma_\eta | \vec{D}) = p(\vec{D} | \omega, \sigma_\eta)$

$$\rightarrow p(\omega, \sigma_\eta | \vec{D}) = p(\vec{D} | \omega, \sigma_\eta) = \frac{e^{-\chi^2/2}}{\sum_{\omega,\sigma_\eta} e^{-\chi^2/2}}$$



$$\chi^2 = \frac{(D_{0.1} - D_{0.1}(\omega, \sigma_\eta))^2}{\sigma_{0.1}^2} + \frac{(D_{0.8} - D_{0.8}(\omega, \sigma_\eta))^2}{\sigma_{0.8}^2}$$

Where, from the ALICE data points,
$D_{0.1}$ = 3.5833, $D_{0.8}$ =2.301, and
$\sigma_{0.1}$ = 0.051456, $\sigma_{0.8}$ = 0.208166

$$D_{0.1}(\omega, \sigma_\eta) = \frac{D'_{0.1}(\omega, \sigma_\eta) + D''_{0.1}(\omega, \sigma_\eta)}{2}$$

$$D_{0.8}(\omega, \sigma_\eta) = \frac{D'_{0.8}(\omega, \sigma_\eta) + D''_{0.8}(\omega, \sigma_\eta)}{2}$$

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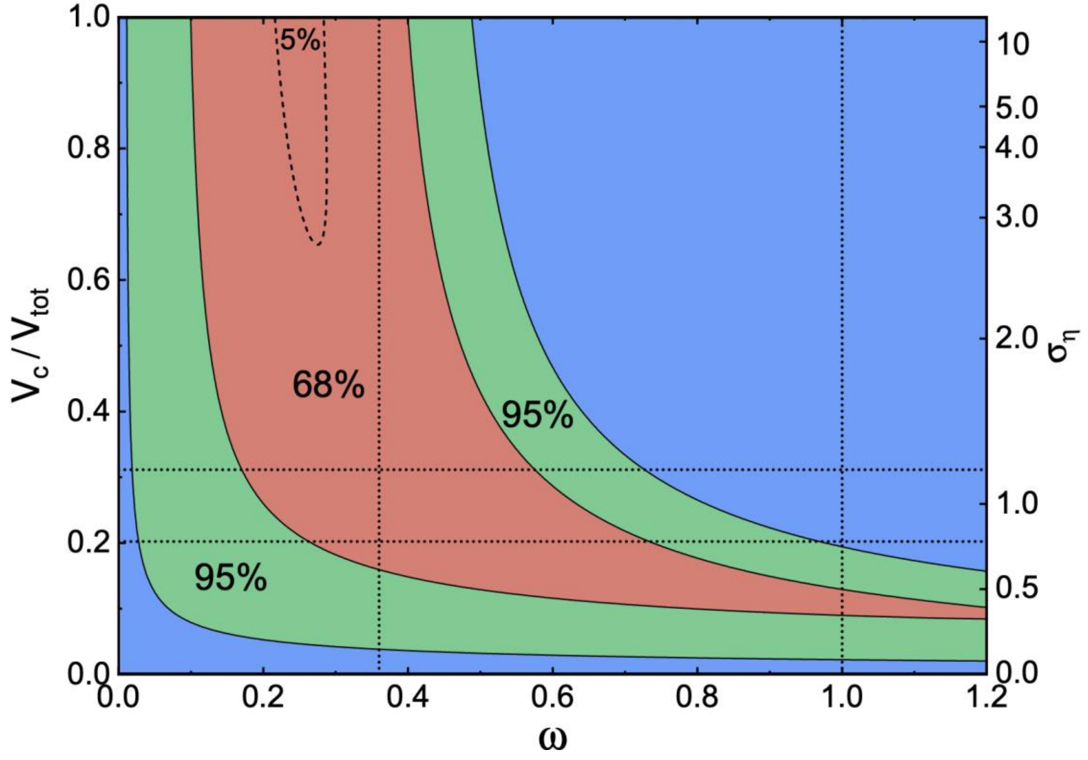
Bayesian analysis of ω and σ parameters in D-measure

2-D Contour Plot of Posterior Probabilities



Regions where the sum,

$$\sum p(\omega, \sigma_\eta | \vec{D}) \Delta\omega \Delta\sigma_\eta$$

reach 0.68 and 0.95.

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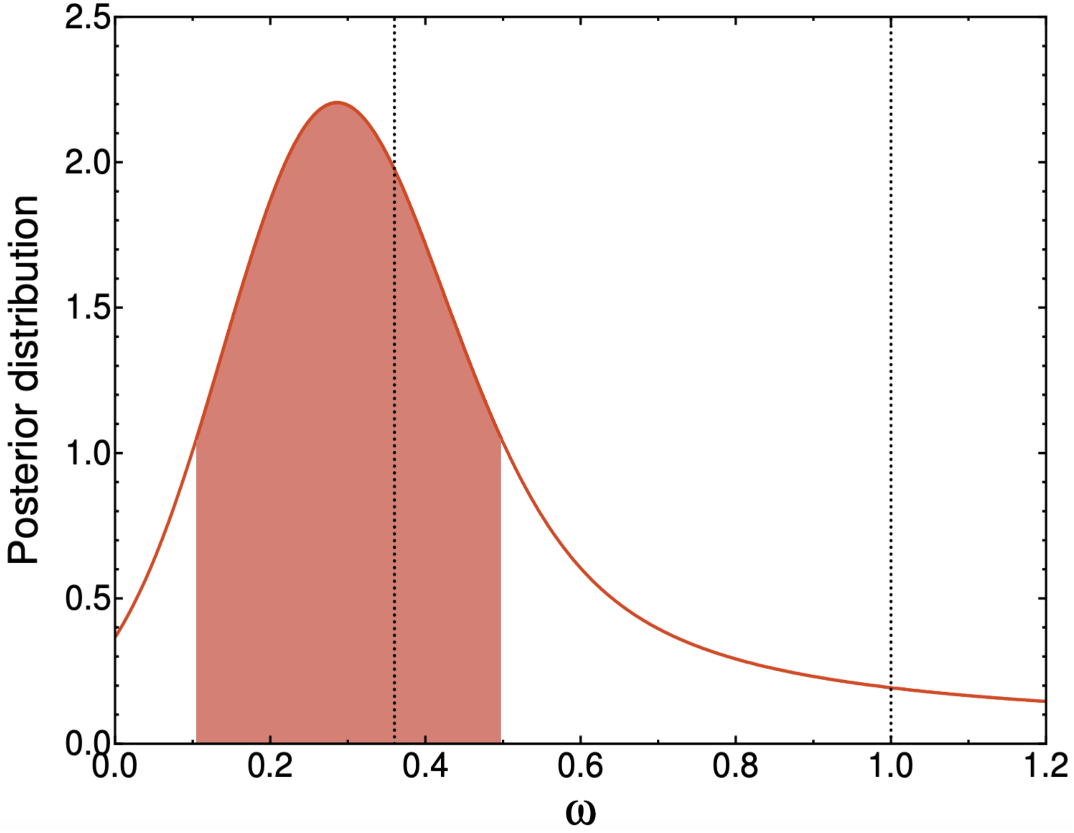
Bayesian analysis of ω and σ parameters in D-measure

1-D plot of marginalized probability p($\omega|D$)



$$p(\omega|\vec{D}) = \int_{\sigma_\eta^{\min}}^{\sigma_\eta^{\max}} d\sigma_\eta \; p(\omega, \sigma_\eta|\vec{D})$$

Found where the sum,

$$\sum p(\omega|\vec{D})\Delta\omega$$

reaches 0.68 and 0.95.

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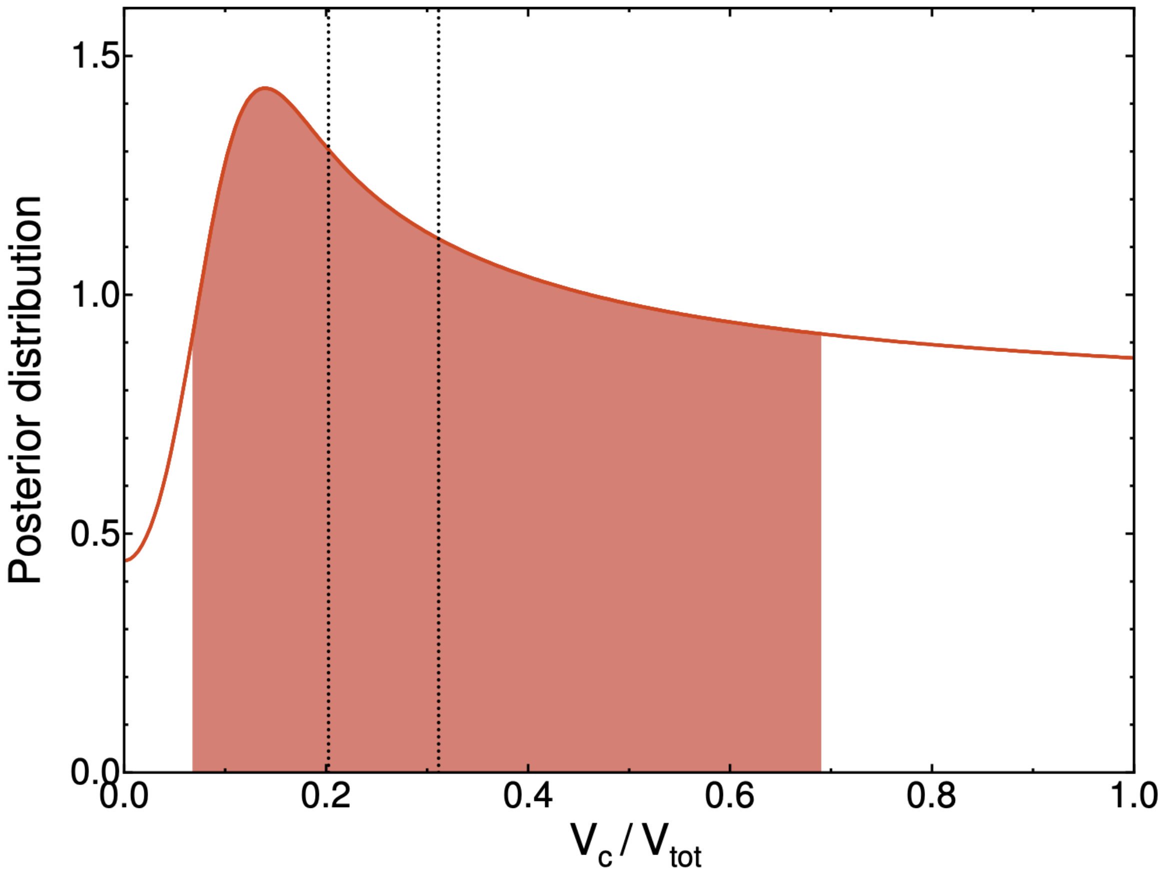
Bayesian analysis of ω and σ parameters in D-measure

1-D plot of marginalized probability p($\sigma|D$)



$$p(\sigma_\eta|\vec{D}) = \int_{\omega_{\min}}^{\omega_{\max}} d\omega\ p(\omega, \sigma_\eta|\vec{D})$$

Found where the sum,

$$\sum p(\sigma_\eta|\vec{D})\Delta\sigma_\eta$$

reaches 0.68 and 0.95.

$$V_c = kdV/dy \quad k(\sigma_y) = \sqrt{2\pi}\sigma_y \mathrm{erf}\left(\frac{\eta_{\max}}{\sqrt{2}\sigma_y}\right) \approx \sqrt{2\pi}\sigma_y$$

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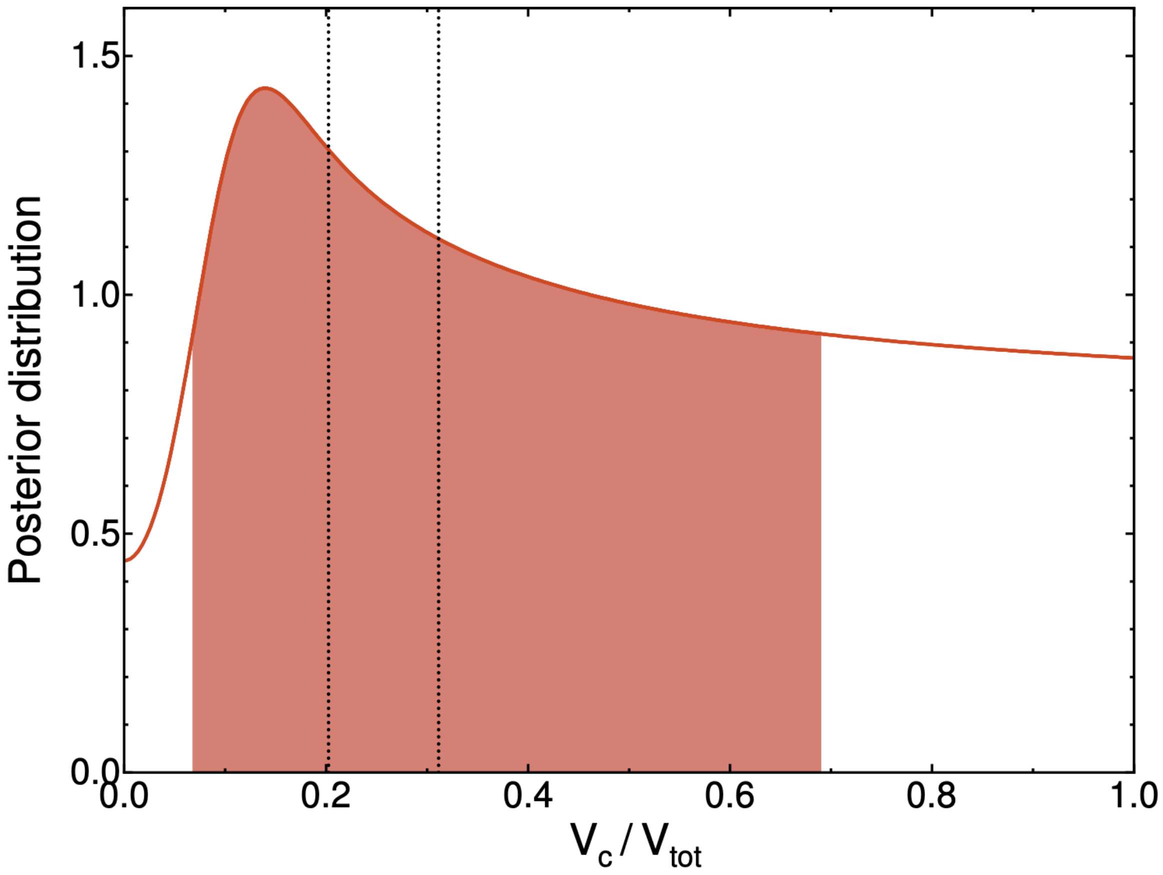
Bayesian analysis of ω and σ parameters in D-measure

1-D plot of marginalized probability p($\sigma|D$)



$$p(\sigma_\eta|\vec{D}) = \int_{\omega_{\min}}^{\omega_{\max}} d\omega\ p(\omega, \sigma_\eta|\vec{D})$$

Found where the sum,

$$\sum p(\sigma_\eta|\vec{D})\Delta\sigma_\eta$$

reaches 0.68 and 0.95.

$$V_c = kdV/dy \quad k(\sigma_y) = \sqrt{2\pi}\sigma_y \mathrm{erf}\left(\frac{\eta_{\max}}{\sqrt{2}\sigma_y}\right) \approx \sqrt{2\pi}\sigma_y$$

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Non-uniform volume distribution

$$\mathcal{C}_2^Q(\eta_1,\eta_2) = \frac{dV}{d\eta}(\eta_1)\chi_2^Q\left[\delta(\eta_1-\eta_2) - \frac{dV}{d\eta}(\eta_2)\frac{\varkappa(\eta_1,\eta_2)}{V_{\rm tot}}\right]$$
$$= \frac{dV}{d\eta}(\eta_1)\langle n_+ + n_-\rangle_{\rm final}\delta(\eta_1-\eta_2)$$
$$+ \frac{dV}{d\eta}(\eta_1)\left(\frac{\omega}{\gamma_Q}-1\right)\langle n_+ + n_-\rangle_{\rm final}\delta(\eta_1-\eta_2) - \frac{dV}{d\eta}(\eta_1)\frac{dV}{d\eta}(\eta_2)\frac{\omega}{\gamma_Q}\langle n_+ + n_-\rangle_{\rm final}\frac{\varkappa(\eta_1,\eta_2)}{V_{\rm tot}}$$

$$\langle p^n(\eta)\rangle = \frac{1}{V_{\rm tot}}\int d\eta \frac{dV}{d\eta}(\eta)p^n(\eta)$$

$$\langle p(\eta_1)p(\eta_2)\rangle_\varkappa = \frac{1}{V_{\rm tot}^2}\int d\eta_1\frac{dV}{d\eta}(\eta_1)\int d\eta_2\frac{dV}{d\eta}(\eta_2)p(\eta_1)p(\eta_2)\varkappa(\eta_1,\eta_2)$$

$$V_{\rm tot} = \frac{dV}{dy}\sqrt{2\pi}\sigma_V \qquad \frac{dV}{d\eta}(\eta) = \frac{dV}{dy}\exp\left(-\frac{\eta^2}{2\sigma_V^2}\right)$$

$$\sigma_V = 3.86$$