QUANTUM TOMOGRAPHY OF CHIRAL MEDIA: FROM QUARK-GLUON PLASMA TO WEYL SEMIMETALS TO AXIONS

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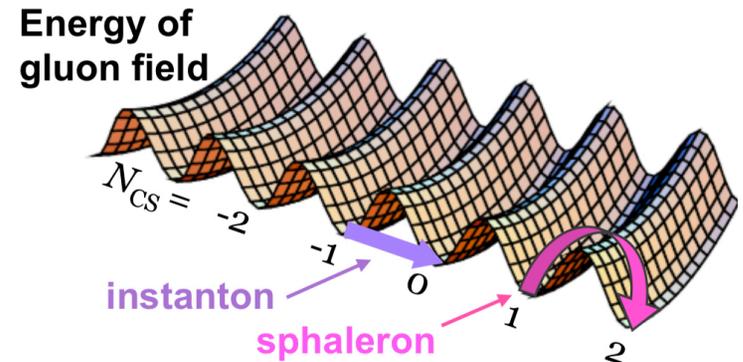OF SCIENCE AND TECHNOLOGY

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CHIRAL MAGNETIC EFFECT

Sphaleron transitions in hot nuclear matter $\Rightarrow$

chiral imbalance $\Rightarrow$ Chiral magnetic effect:

$$j = b_0 B \qquad b_0 = \frac{e^2 \mu_5}{2\pi^2}$$

Kharzeev, McLerran Warringa (2008), Fukushima, Kharzeev, Warringa (2008)

$b_0 \propto \mu_5$ = chiral chemical potential

How does the chiral magnetic current impact jets in QGP?

Closely related problems:

- Fast charges in Weyl semimetals

- Cosmic rays in the axion field
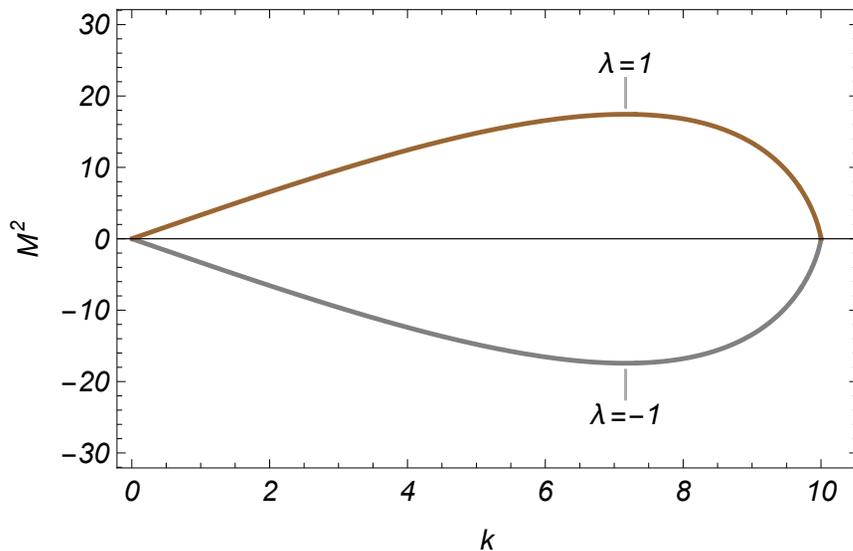
GLUON/PHOTON DISPERSION RELATION

- It modifies Amper's law: $\boldsymbol{\nabla} \times \boldsymbol{B} = \dot{\boldsymbol{E}} + \boldsymbol{j} + b_0 \boldsymbol{B}$

  such that the wave equation becomes: $\nabla^2 \boldsymbol{A} = \partial_t^2 \boldsymbol{A} - b_0 \boldsymbol{\nabla} \times \boldsymbol{A}$

- The plane-wave solutions: $\boldsymbol{A} = \boldsymbol{\epsilon}_\lambda e^{-i\omega t + i\boldsymbol{k} \cdot \boldsymbol{x}}$ λ=±1 is Right or Left polarization.

  where $\omega^2 - \boldsymbol{k}^2 = -\lambda b_0 |\boldsymbol{k}|$

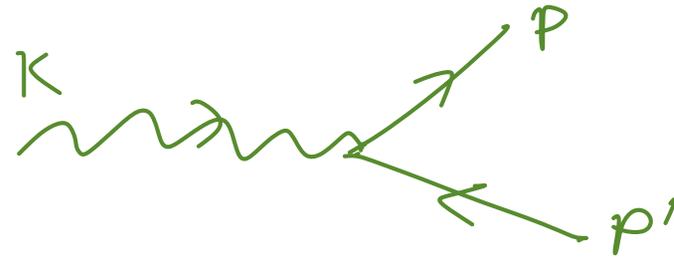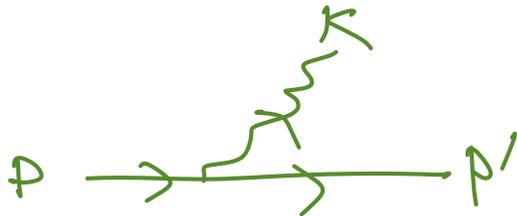- Thus, the dynamical gluon or photon mass can be real or imaginary.

The effective photon mass *M* squared at finite chiral chemical potential in EM plasma.

Akamatsu, Yamamoto (2013)

NEW PROCESSES INDUCED BY CME

Energy-momentum conservation: $k^2 = (p \pm p')^2 = 2m(m \pm E)$

Photon dispersion: $\omega^2 - \boldsymbol{k}^2 = -\lambda b_0 |\boldsymbol{k}|$ $\qquad b_0 \propto \mu_5 =$ chiral chemical potential

$\Rightarrow$ 1→2 processes are forbidden if *b₀=0*

- Pair production is allowed if *λb₀<0*

- Gauge boson radiation is allowed if *λb₀>0* *(Chiral Cherenkov radiation)*

NEW CHANNELS IN QED

1. Photon production

$$\frac{dW^{e\to\gamma^+ e}}{dx} = \frac{\alpha b_0}{4}\left(\frac{(1-x)^2+1}{x} - \frac{2m^2}{b_0 E}\right) \qquad x \le \frac{1}{1+\frac{m^2}{b_0 E}}$$

Only right-handed photons are produced with transverse momentum

$$k_\perp^2 = -x^2 m^2 + (1-x)x\lambda b_0 E$$

2. Pair production

$$\frac{dW^{\gamma^-\to e\bar{e}}}{dx} = \frac{\alpha b_0}{4}\left(x^2 + (1-x)^2 + \frac{2m^2}{\omega b_0}\right) \qquad x_1 \le x \le x_2$$

Only left-handed photons decay.

$$x_{1,2} = \frac{1}{2}\left(1\mp\sqrt{1-\frac{4m^2}{b_0\omega}}\right)$$

- These processes induce the overall polarization of a jet and contribute to the energy loss.

$$-\frac{dE}{dz} = \int_0^1 \frac{dW^{e\to\gamma^+ e}}{dx} xE dx$$

NEW CHANNELS IN QCD

A free gluon is has the same dispersion is a free photon.
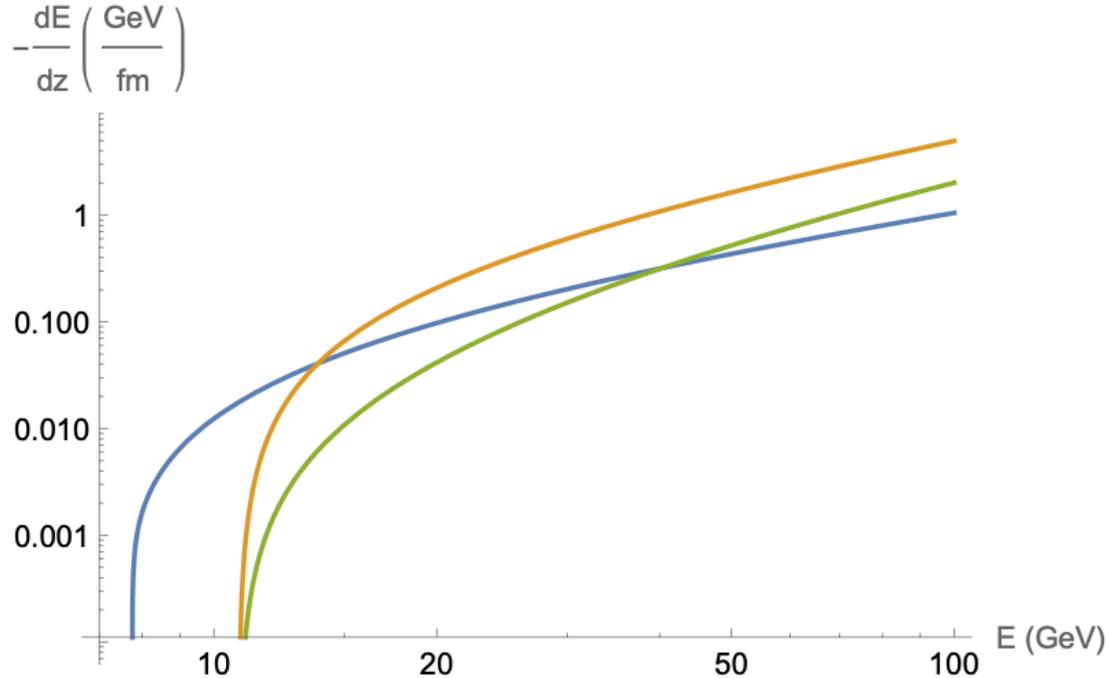
New non-Abelian term in the Lagrangian: $b_0 \epsilon^{ijk} \dfrac{g}{6} f^{abc} A_i^a A_j^b A_k^c$

$$i\mathcal{M}^A_{g_R \to g_R g_R} = \frac{k_\perp}{x(1-x)}\,,$$

$$iM^A_{g_L \to g_R g_L} = \frac{(1-x)k_\perp}{x}\,, \qquad +\mathcal{O}\left(\frac{b_0, m}{E}\right)$$

$$iM^A_{g_L \to g_L g_R} = \frac{x k_\perp}{(1-x)}\,,$$

$$iM^A_{g_L \to g_R g_R} = \mathcal{O}(k_\perp/E, b_0/E)\,,$$

$$iM^B_{g \to gg} = \frac{-b_0 k_\perp (\lambda(1-x) + \lambda' x + \lambda_0)}{x(1-x)E} \quad \text{(small)}$$

- Both right and left handed gluons can be produced and decay, but with different amplitudes.

Hansen, KT (2024)

$$-\frac{dE_{q\to qg}}{dz} = \frac{4\alpha_s g^2 b_0 E}{9},$$

$$-\frac{dE_{g_R\to gg}}{dz} = \frac{3\alpha_s g^2 b_0 E}{4}\left(\ln\frac{b_0 E}{\omega_p^2} - 1\right),$$

$$-\frac{dE_{g_L\to gg}}{dz} = \frac{3\alpha_s g^2 b_0 E}{4}\left(\ln\frac{b_0 E}{\omega_p^2} - \frac{17}{6}\right).$$

Fig. 3. The rate of energy loss due to the Color Chiral Cherenkov radiation for $q \to qg$ (blue), $g_R \to gg$ (orange), and $g_L \to gg$ (green). Parameters: $g = 2$, $T = 300$ MeV, $b_0 = 50$ MeV.

FACTORIZATION OF SUCCESSIVE EMISSIONS

Double photon emission:

$$W^{e \to e\gamma\gamma} = \frac{1}{8E} \sum_{\text{spins}} \sum_{\lambda_1, \lambda_2} \int \frac{d^3 p'}{2E'(2\pi)^3} \int \frac{d^3 k_1}{2\omega_1 (2\pi)^3} \int \frac{d^3 k_2}{2\omega_2 (2\pi)^3} (2\pi)^4 \delta(p - k_1 - k_2 - p')$$

$$\times \left| \frac{e^2 \bar{u}' \slashed{\epsilon}_2^* (\slashed{p} - \slashed{k}_1 + m) \slashed{\epsilon}_1^* u}{(p - k_1)^2 - m^2 + 2iE_q/\tau} + \frac{e^2 \bar{u}' \slashed{\epsilon}_1^* (\slashed{p} - \slashed{k}_2 + m) \slashed{\epsilon}_2^* u}{(p - k_2)^2 - m^2 + 2iE_q/\tau} \right|^2,$$

The intermediate propagator has a resonance corresponding to χCherenkov:

$$\frac{1}{-2p \cdot k_1 + k_1^2 + 2iE_q/\tau} = \frac{1}{\omega_1 E \left(\frac{m^2}{E^2 \omega_1} \frac{\omega_1 - x_0 E}{1 - x_0} + \vartheta^2 \right) + 2iE_q/\tau}$$

If the resonance is dominant then: $\left| \dfrac{1}{(p - k_1)^2 - m^2 + 2iE_q/\tau} \right|^2 \approx \dfrac{\pi\tau}{2E_q} \delta\left((p - k_1)^2 - m^2 \right)$

Factorization: $\quad W^{e \to e\gamma\gamma} = \tau \displaystyle\int_0^E d\omega_1 \frac{dW^{e(p) \to e(q)\gamma(k_1)}}{d\omega_1} \int_0^{E_q} d\omega_2 \frac{dW^{e(q) \to e(p')\gamma(k_2)}}{d\omega_2}$

EVOLUTION EQUATIONS OF CHIRAL CASCADE IN QED
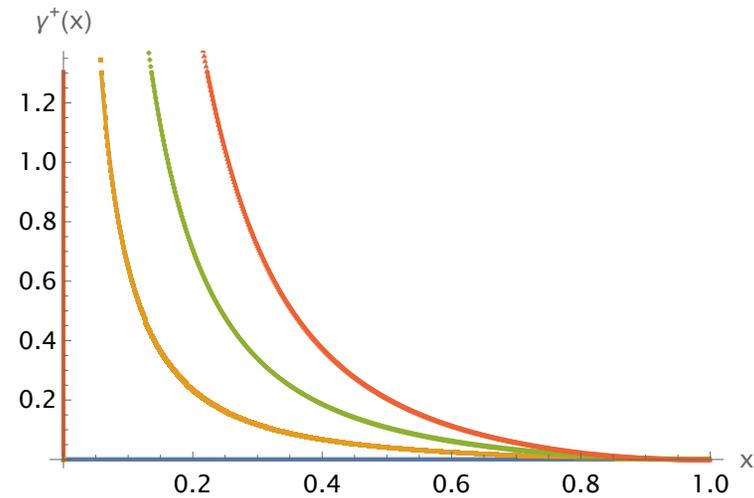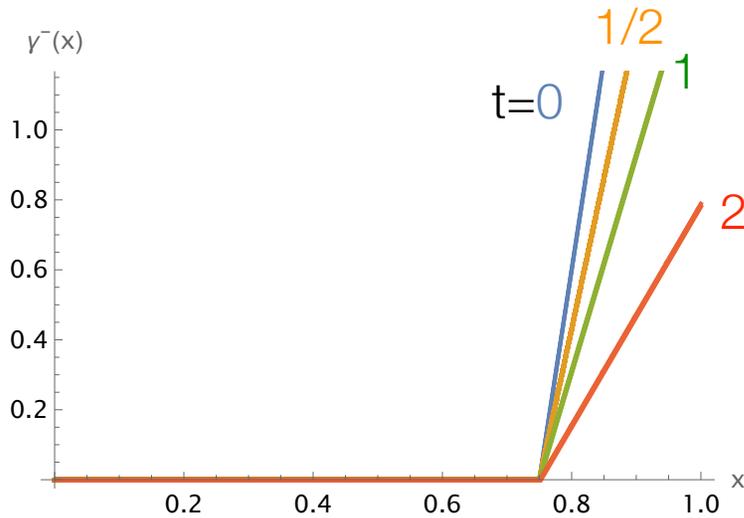
$$\frac{de(y,t)}{dt} = \frac{1}{\tau} \int_y^1 \frac{dx}{x} \left\{ P_{ee}(x)e\left(\frac{y}{x},t\right) + P_{e\gamma^-}(x)\gamma^-\left(\frac{y}{x},t\right) \right\},$$

$$\frac{d\bar{e}(y,t)}{dt} = \frac{1}{\tau} \int_y^1 \frac{dx}{x} \left\{ P_{ee}(x)\bar{e}\left(\frac{y}{x},t\right) + P_{e\gamma^-}(x)\gamma^-\left(\frac{y}{x},t\right) \right\}, \qquad \frac{1}{\tau} = \frac{\alpha b_0}{4}$$

$$\frac{d\gamma^+(y,t)}{dt} = \frac{1}{\tau} \int_y^1 \frac{dx}{x} \left\{ P_{\gamma^+ e}(x)e\left(\frac{y}{x},t\right) + P_{\gamma^+ e}(x)\bar{e}\left(\frac{y}{x},t\right) \right\},$$

$$\frac{d\gamma^-(y,t)}{dt} = \frac{1}{\tau} \int_y^1 \frac{dx}{x} P_{\gamma^-\gamma^-}(x)\gamma^-\left(\frac{y}{x},t\right)$$

- These are NOT standard evolution equations - they describe evolution due to anomaly and contain no conventional terms.

- The standard splitting functions are due to 1→2 processes.

- There is clear asymmetry between right and left photons.
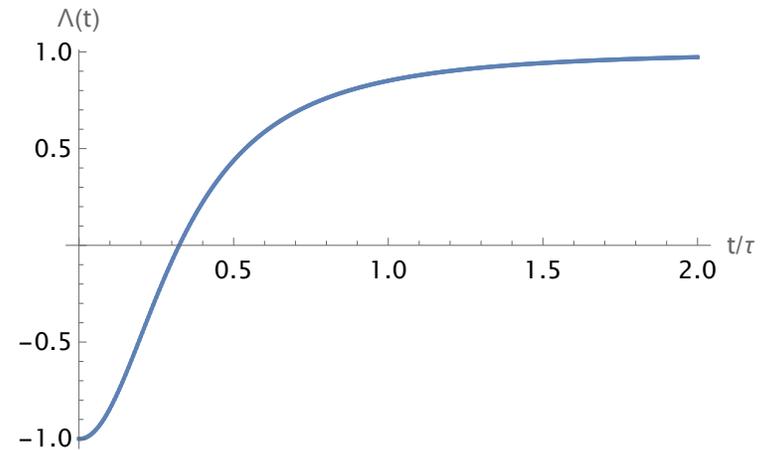
The initial condition: only left-handed photons. At late times: only right-handed ones.

The degree of polarization:

$$\Lambda(t) = \frac{\int_0^1 \left(\gamma^+(x,t) - \gamma^-(x,t)\right) dx}{\int_0^1 \left(\gamma^+(x,t) + \gamma^-(x,t)\right) dx}$$

$$\frac{dq(y,t)}{dt} = \frac{1}{\tau} \int_y^1 \frac{dx}{x} \left\{ P_{qq}(x) q\left(\frac{y}{x}, t\right) + P_{qg^-}(x) g^-\left(\frac{y}{x}, t\right) \right\} ,$$

$$\frac{dg^+(y,t)}{dt} = \frac{1}{\tau} \int_y^1 \frac{dx}{x} \left\{ N_f P_{g^+q}(x) q\left(\frac{y}{x}, t\right) + N_f P_{g^+q}(x) \bar{q}\left(\frac{y}{x}, t\right) \right.$$
$$\left. + P_{g^+g^+}(x) g^+\left(\frac{y}{x}, t\right) + P_{g^+g^-}(x) g^-\left(\frac{y}{x}, t\right) \right\} ,$$

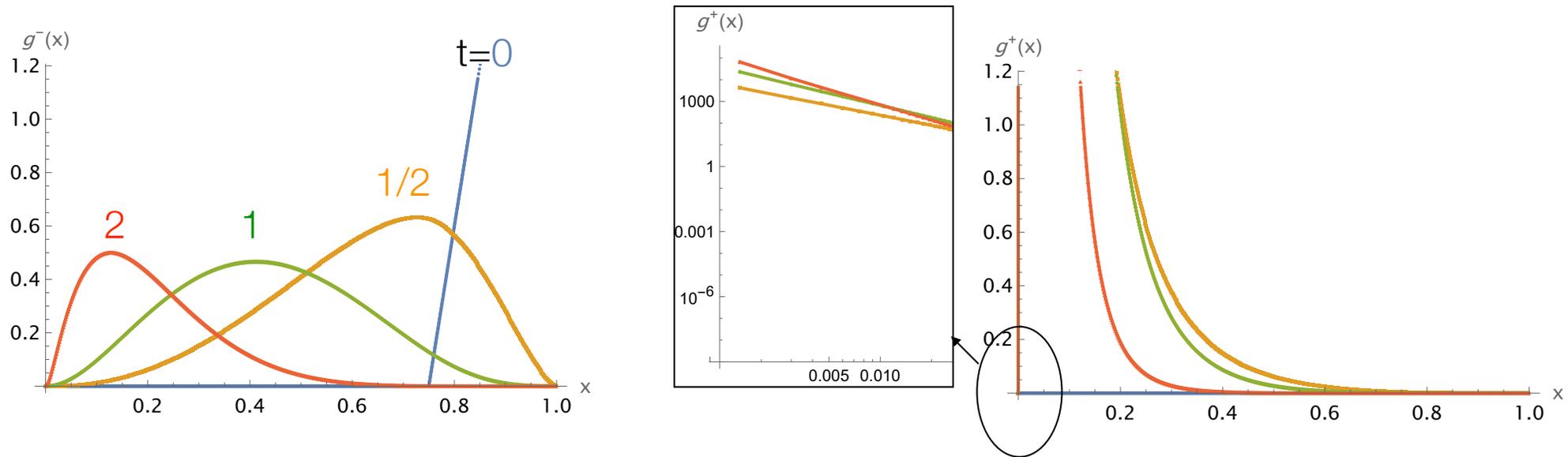$$\frac{dg^-(y,t)}{dt} = \frac{1}{\tau} \int_y^1 \frac{dx}{x} P_{g^-g^-}(x) g^-\left(\frac{y}{x}, t\right)$$

$$P_{g^+g^+} = \frac{N_c}{x} \left[\frac{1}{(1-x)_+} + C^+ \delta(1-x) \right] ,$$

$$P_{g^-g^-} = N_c \left[\frac{x^3}{(1-x)_+} + C^- \delta(1-x) \right]$$

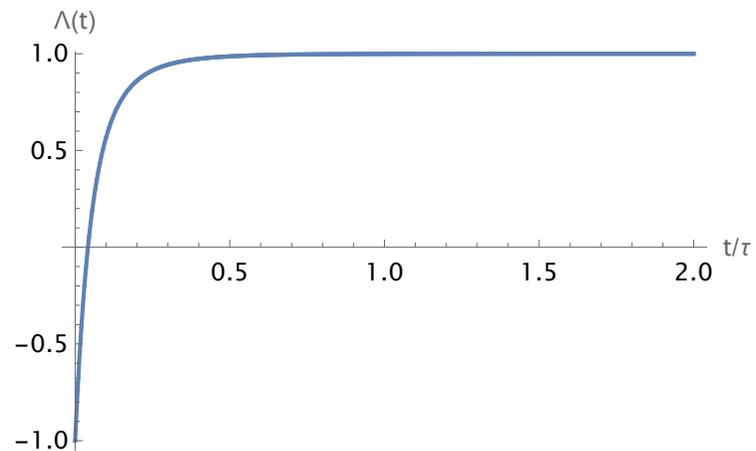Charge and energy conservation: $C^+ = 0 , \quad N_c C^- = \frac{11 N_c}{6} - \frac{N_f}{3}$

Hansen, KT (2025)

The initial condition: only left-handed gluons.

The degree of polarization:

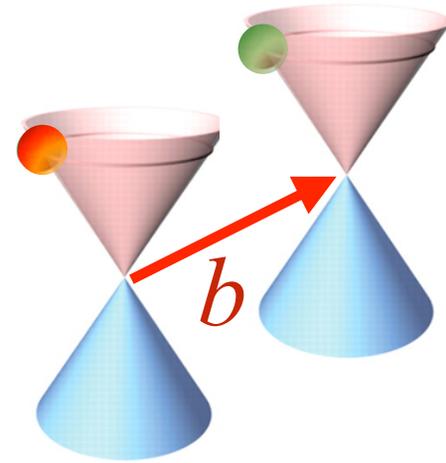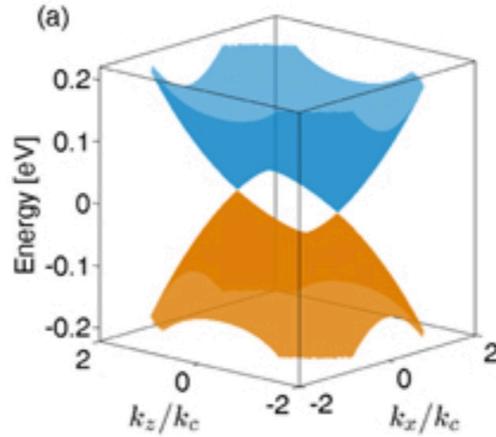Emergent chiral domains in Weyl semimetals:

(a)

TaAs

NbAs

NbP

TaP

$$\begin{cases} \nabla \cdot \boldsymbol{B} = 0 \,, \\ \nabla \cdot \boldsymbol{E} = -\boldsymbol{b} \cdot \boldsymbol{B} \,, \\ \nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B} \,, \\ \nabla \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{b} \times \boldsymbol{E} \,. \end{cases}$$

$\Rightarrow$

$$\begin{cases} \nabla \cdot \boldsymbol{B} = 0 \,, \qquad \nabla \cdot \boldsymbol{D} = 0 \\ \nabla \times \boldsymbol{E} = i\omega \boldsymbol{B} \,, \qquad \nabla \times \boldsymbol{B} = -i\omega \boldsymbol{D} \end{cases}$$

where $\boldsymbol{D} = \boldsymbol{E} + \dfrac{i}{\omega}\boldsymbol{b} \times \boldsymbol{E} = 0$.

Dielectric tensor

Dirac semimetal
(doubly degenerated bands)

$$\begin{pmatrix} 1 & -ib/\omega_{\boldsymbol{k}\lambda} & 0 \\ ib/\omega_{\boldsymbol{k}\lambda} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Qui, Cao, Huang (2017)

$ZrTe_5$

Na_3Bi,

Cd_3As_2

- Dispersion: $\left| k_i k_j - k^2 \delta_{ij} + \omega_{\boldsymbol{k}\lambda}^2 \varepsilon_{ij}(\omega_{\boldsymbol{k}\lambda}) \right| = 0$

$$\omega^2 - \boldsymbol{k}^2 = \frac{1}{2}b^2 - \lambda \operatorname{sgn}(\boldsymbol{k} \cdot \boldsymbol{b})\sqrt{(\boldsymbol{k} \cdot \boldsymbol{b})^2 + \frac{1}{4}b^4}$$

- Now the photon "mass" depends on the photon direction w.r.t. $\boldsymbol{b}$.

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- Photon spectrum:

$$\frac{dW^{e \to \gamma + e}}{dx} = \frac{\alpha \lambda b \cos \beta}{4} \left(\frac{(1-x)^2 + 1}{x} \right)$$

Dirac semimetal
(doubly degenerated bands)

Kinematic constraints:

$$x > \frac{m^2}{\cos \beta \, 2E}$$

$$\beta > 0$$

ZrTe$_5$

Na$_3$Bi,

Cd$_3$As$_2$

$$\partial_\mu J_5^\mu$$

⇒ • The right/left-polarized photons emitted in the forward/backward direction.

Magnetic Weyl semimetals as a source of circularly polarized THz radiation

Jeremy Hansen, Kazuki Ikeda, Dmitri E. Kharzeev, Qiang Li, Kirill Tuchin (May 17, 2024)

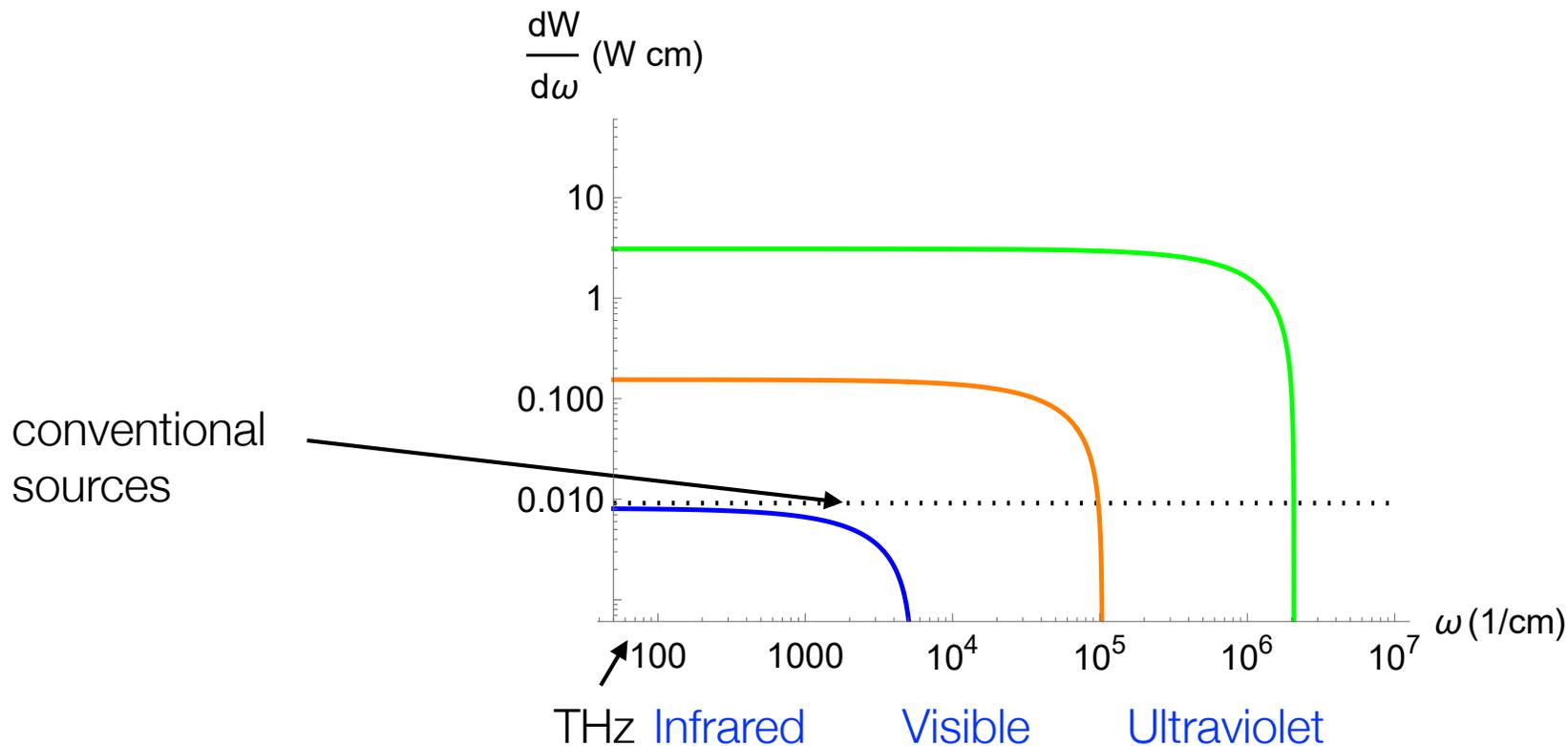e-Print: 2405.11076 [cond-mat.mtrl-sci]

Fig. 3. Intensity spectrum of Chiral Cherenkov radiation by a pulse of $N = 10^{11}$ electrons of energy $E = 3$ MeV, $\beta = 0$ in a Weyl semimetal with $b = 3.8$ eV (green), $b = 0.19$ eV (orange), and $b = 0.01$ eV (blue). The spectra are given by Eq. (14) with $\varepsilon_b = 10$, dotted line: bremsstrahlung spectrum in $Co_3Sn_2S_2$ computed according to [9].

AXION ELECTRODYNAMICS

Sikivie (84), Wilczek (87), Carroll et al (90)

$$\mathcal{L}_{\mathrm{MCS}} = \mathcal{L}_{\mathrm{QED}} + c_A \theta(x) \vec{E} \cdot \vec{B}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \,,$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho - c\,\boldsymbol{\nabla}\theta \cdot \boldsymbol{B} \,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\partial_t \boldsymbol{B} \,,$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \partial_t \boldsymbol{E} + \boldsymbol{j} + c(\partial_t \theta\, \boldsymbol{B} + \boldsymbol{\nabla}\theta \times \boldsymbol{E}) \,,$$

Anomalous Hall Effect

Chiral magnetic effect: $\quad \boldsymbol{j} = b_0 \boldsymbol{B}$

- If $b_0 \approx$ const, then a fast proton moving in the classical axion field will emit highly polarized chiral Cherenkov radiation.

- A new way to search for axions.

BREMSSTRAHLUNG IN CHIRAL MEDIA

- Photon/gluon propagator (static limit):

$$D_{00}(\boldsymbol{q}) = \frac{i}{\boldsymbol{q}^2}\,,$$

$$D_{0i}(\boldsymbol{q}) = D_{0i}(\boldsymbol{q}) = 0\,,$$

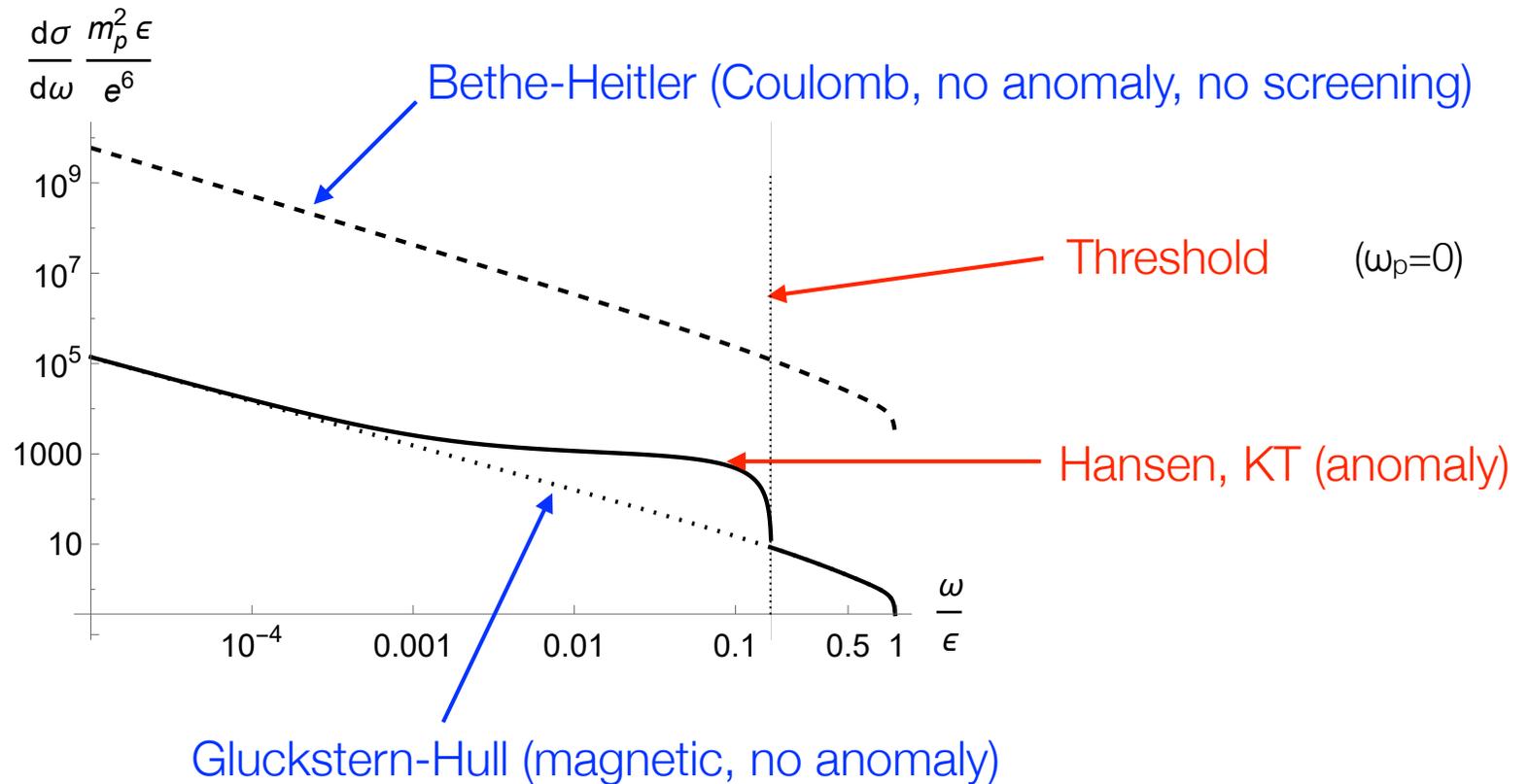$$D_{ij}(\boldsymbol{q}) = -\frac{i\delta_{ij}}{\boldsymbol{q}^2 - b_0^2} - \frac{\epsilon_{ijk}q^k}{b_0(\boldsymbol{q}^2 - b_0^2)} + \frac{\epsilon_{ijk}q^k}{b_0\boldsymbol{q}^2}$$

resonance!

- D_{ij} couples only to the magnetic moment of the target $\quad \boldsymbol{J}(\boldsymbol{x}) = \boldsymbol{\nabla} \times (\boldsymbol{\mu}\delta(\boldsymbol{x}))$

Hansen, KT (2022)

$\dfrac{d\sigma}{d\omega}\dfrac{m_p^2\,\epsilon}{e^6}$

Bethe-Heitler (Coulomb, no anomaly, no screening)

Threshold ($\omega_p=0$)

Hansen, KT (anomaly)

Gluckstern-Hull (magnetic, no anomaly)

$$\blacktriangleleft \cdots\cdots\cdots\cdots \omega^2 = \mathbf{k}^2 + k^2 = \mathbf{k}^2 - \lambda b_0 |\mathbf{k}|$$

$$\blacktriangleleft \cdots\cdots\cdots$$
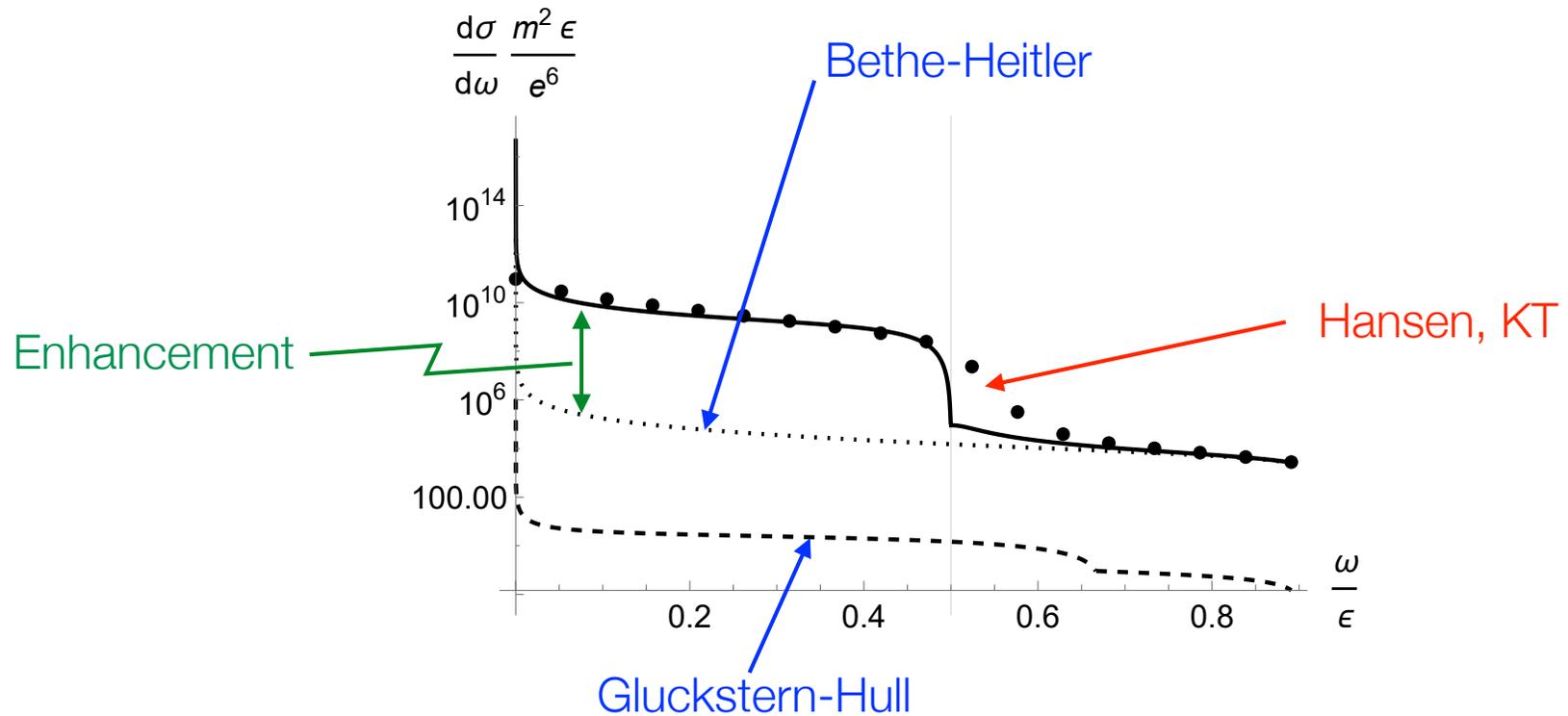
- Fermion propagator: $\quad \dfrac{1}{2p \cdot k - k^2 + iE/\tau} = \dfrac{1}{\omega E \left(\frac{m^2}{E\omega} \frac{\omega - \omega^*}{E - \omega^*} + \theta^2 + \frac{i}{\omega\tau} \right)} \quad$ with $\quad \omega^* = \dfrac{\lambda b_0 E^2}{\lambda b_0 E + m^2}$
  $b_0 \lambda > 0$

- The resonance emerges when $\omega < \omega^*$ due to the anomaly in the photon/gluon dispersion relation.

- The photon/gluon propagator shows similar behavior: $\quad \mathbf{q}^2_{\min} = \dfrac{1}{4} \dfrac{\omega^2 E^2}{E'^2} \left[\dfrac{m^2(\omega - \omega^*)}{\omega E(E - \omega^*)} + \theta^2 \right]^2$

Weak screening limit $\mu \ll m$ μ-Debye mass

• Only one photon polarization ($b_0\lambda > 0$) is enhanced!

SUMMARY

- In the presence of the chiral magnetic effect, the jets exhibit polarization and a distinctive transverse momentum dependence.

- Relativistic heavy-ion collisions can serve as a testing ground not only for the Early Universe but also for the Dark Matter.

Additional material

QUANTIZATION OF EM FIELD

$$\boldsymbol{A}(\boldsymbol{x},t) = \sum_{\boldsymbol{k}\lambda}(a_{\boldsymbol{k}\lambda}\boldsymbol{A}_{\boldsymbol{k}\lambda} + a_{\boldsymbol{k}\lambda}^{\dagger}\boldsymbol{A}_{\boldsymbol{k}\lambda}^{*}) + \sum_{\boldsymbol{k}\nu}(a_{\boldsymbol{k}\nu}\boldsymbol{A}_{\boldsymbol{k}\lambda} + a_{\boldsymbol{k}\nu}^{\dagger}\boldsymbol{A}_{\boldsymbol{k}\lambda}^{*})$$

Transverse EM waves   Longitudinal EM waves (exist only if there is spatial dispersion)

$$\boldsymbol{A}_{\boldsymbol{k}\lambda} = \boldsymbol{e}_{\boldsymbol{k}\lambda}\left(\frac{k\,v_{\boldsymbol{k}\lambda}}{2\omega_{\boldsymbol{k}\lambda}^{2}\varepsilon_{ij}e_{\boldsymbol{k}\lambda i}^{*}e_{\boldsymbol{k}\lambda j}V}\right)^{1/2}e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_{\boldsymbol{k}\lambda}t} \qquad \boldsymbol{A}_{\boldsymbol{k}\nu} = \hat{\boldsymbol{k}}\left(\frac{k^{2}}{\omega_{\boldsymbol{k}\nu}^{2}k_{i}k_{j}\partial\varepsilon_{ij}/\partial\omega_{\boldsymbol{k}\nu}V}\right)^{1/2}e^{i\boldsymbol{k}\cdot\boldsymbol{x}-i\omega_{\boldsymbol{k}\nu}t}$$

Chiral Cherenkov radiation: $\dfrac{dW}{d\Omega d\omega} = \dfrac{\alpha Q^{2}}{16\pi}\sum_{\lambda}\delta(\omega+\varepsilon'-\varepsilon)\dfrac{k^{3}}{\varepsilon\varepsilon'\omega^{2}\varepsilon_{ij}e_{\boldsymbol{k}\lambda i}^{*}e_{\boldsymbol{k}\lambda j}}\sum_{ss'}|\mathcal{M}_{0}|^{2}$

At high energies equations simplify: $\qquad e_{\boldsymbol{k}\pm i}^{*}e_{\boldsymbol{k}\pm j} \to \dfrac{1}{2}\left(\delta^{ij}-\dfrac{k_{i}k_{j}}{k^{2}}\right)$

STRONG SCREENING $\mu \gg m$

• The effect of anomaly is reduced by screening.