Merging EMD holographic model with vdW HRG

Tulio Eduardo Restrepo Medina

In collaboration with:
Musa Khan, Prachi Garella, Joaquin Grefa, Mauricio Hippert, Jorge Noronha, Claudia Ratti, Romulo Rougemont and Yumu Yang

CIPAMP 2025

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NuSTEAM
Nuclear Science in Texas to Enhance and Advance Minorities

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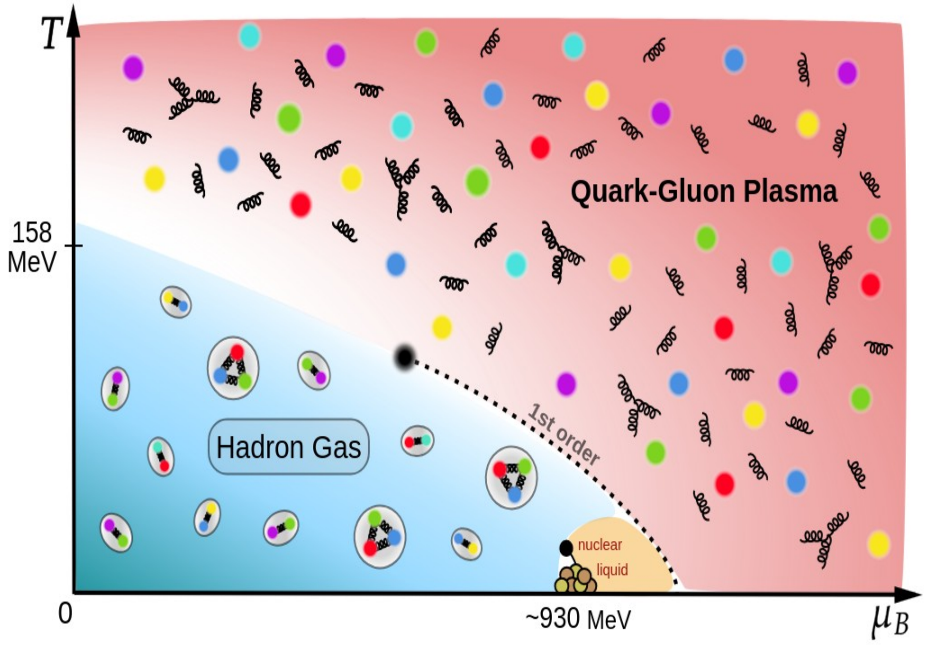
muses

QCD phase diagram



QCD phase diagram

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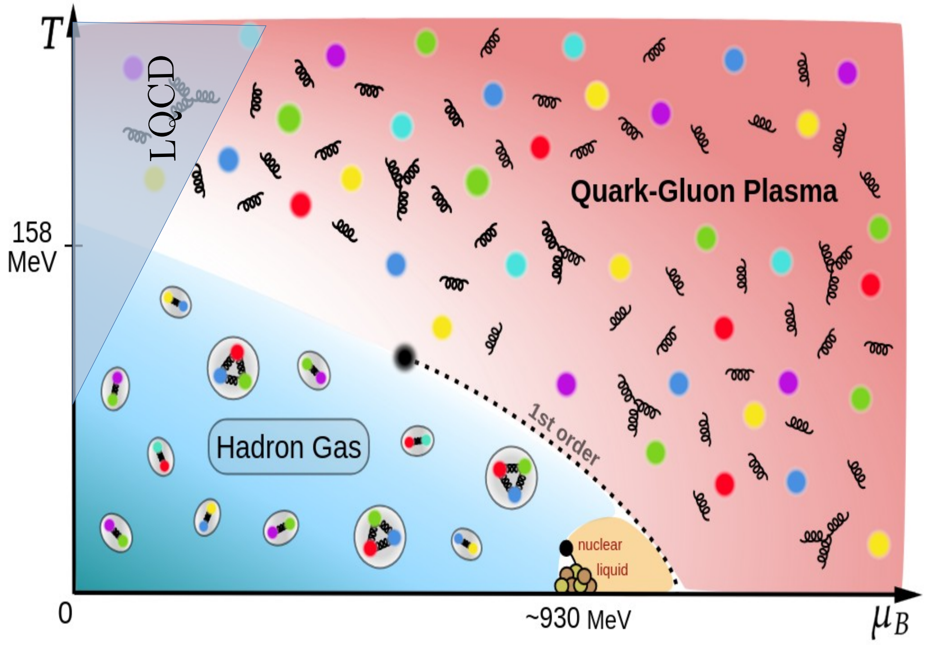

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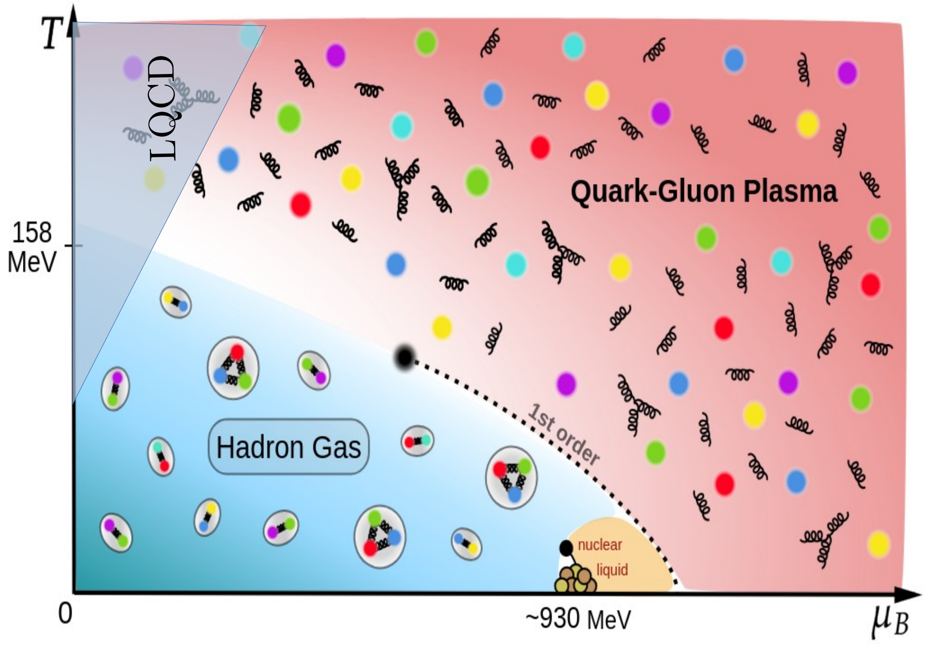

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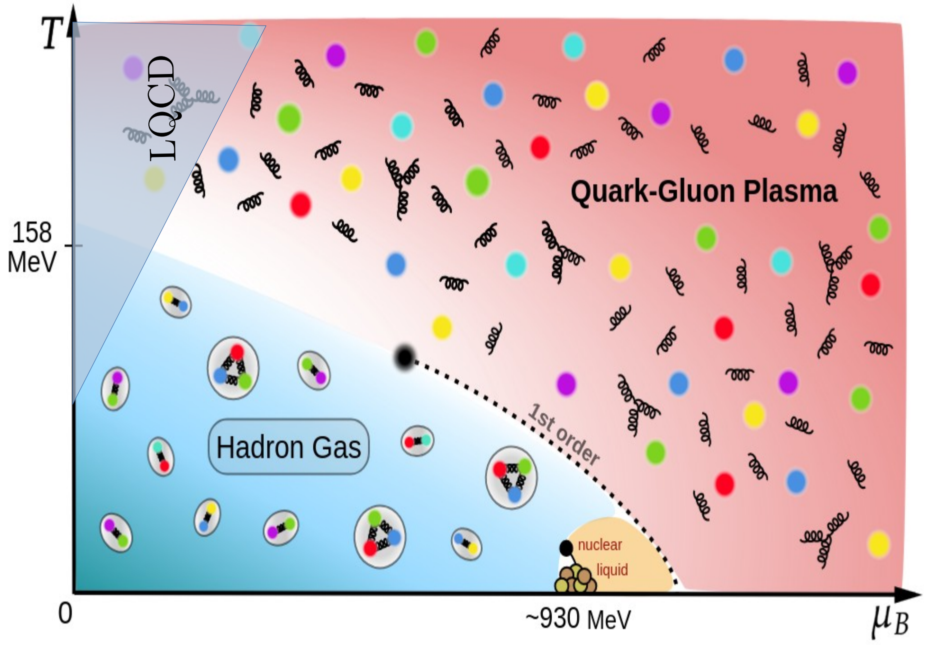

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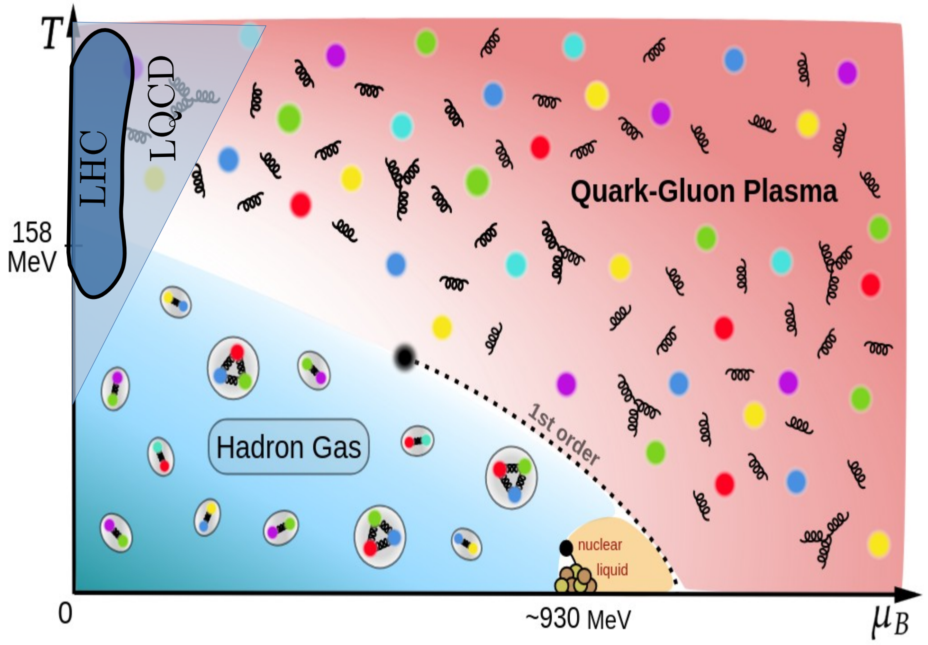

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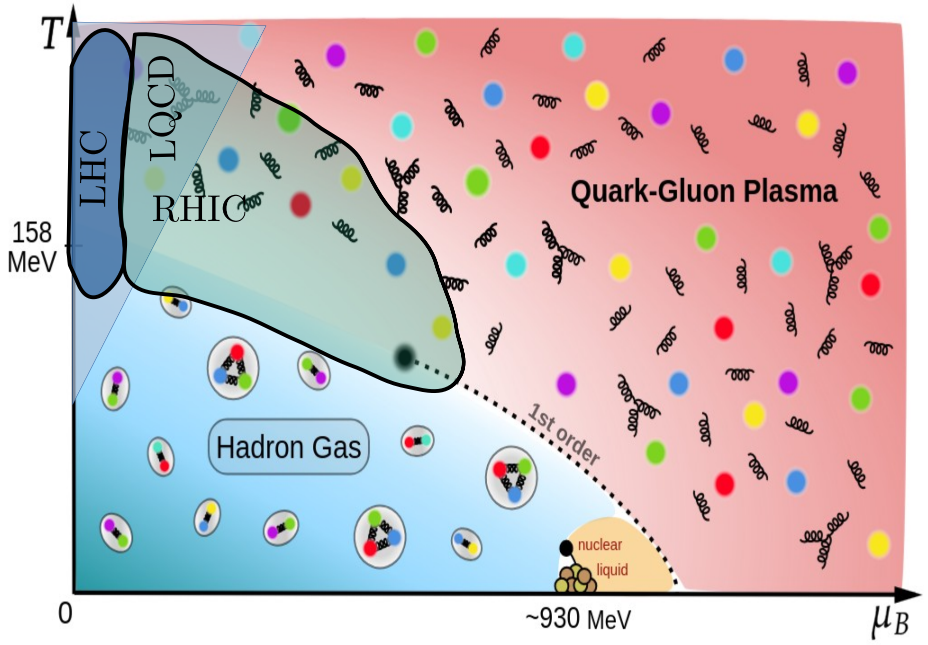

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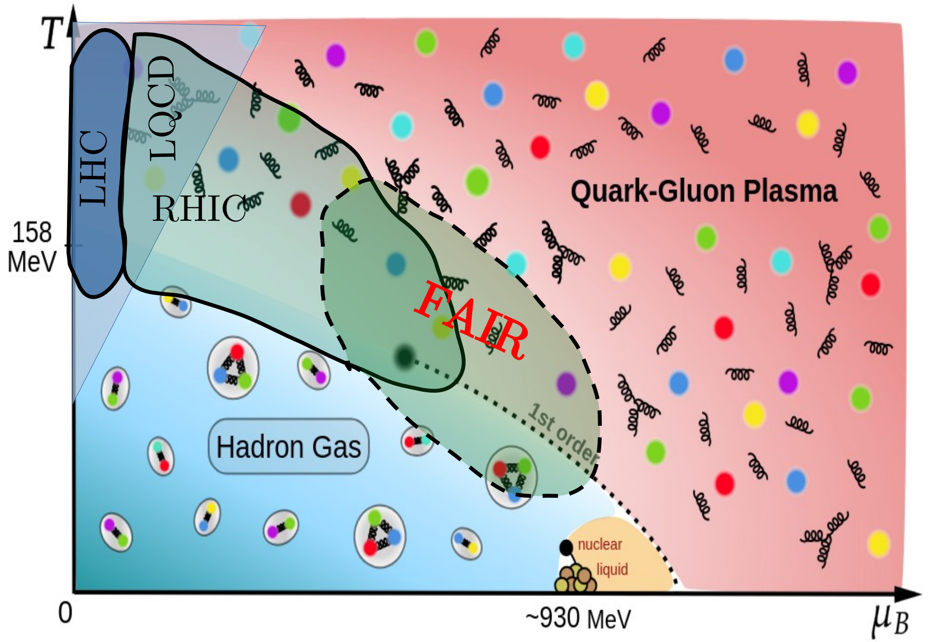

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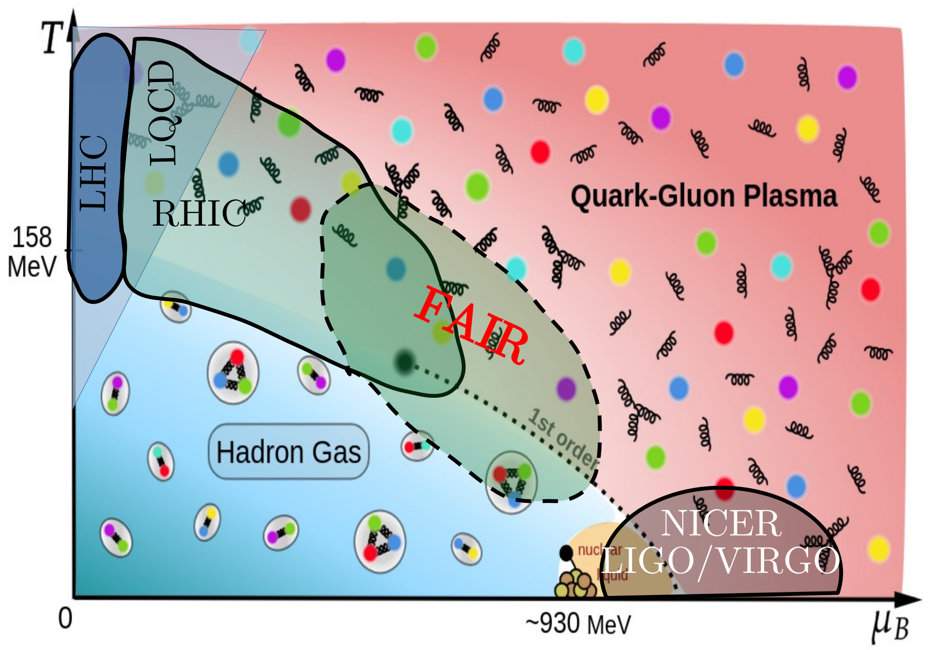
- At very high densities and low temperatures, we are in the realm of neutron stars.



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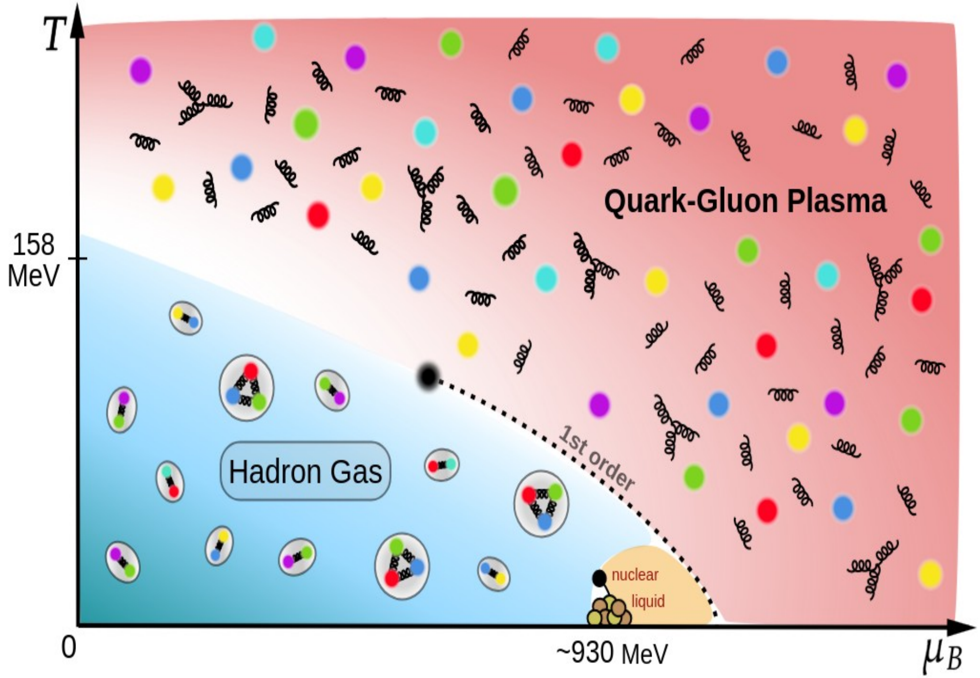

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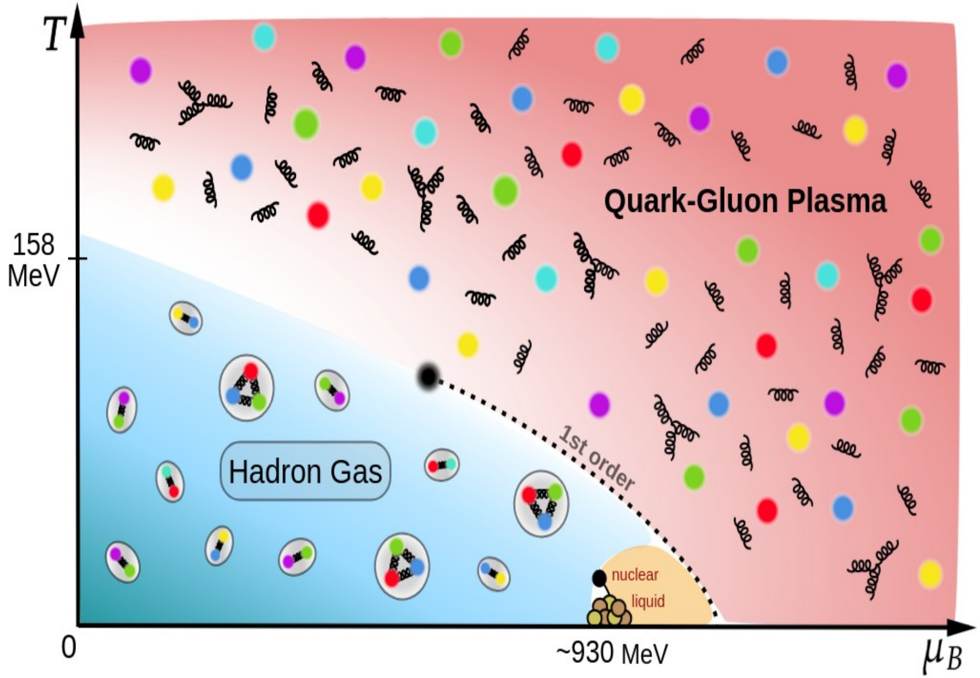

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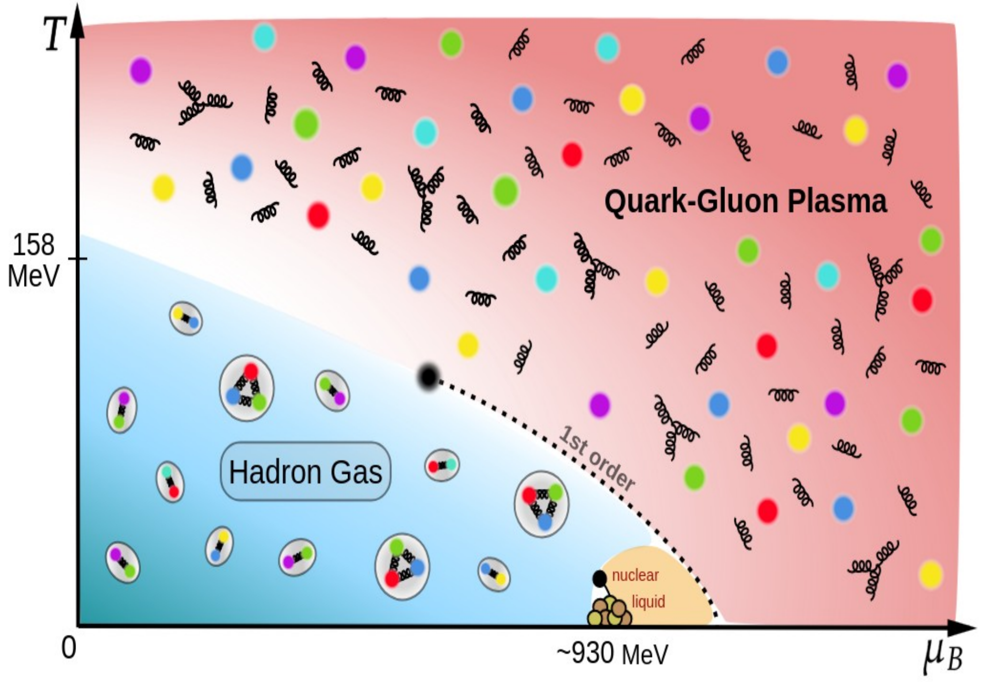

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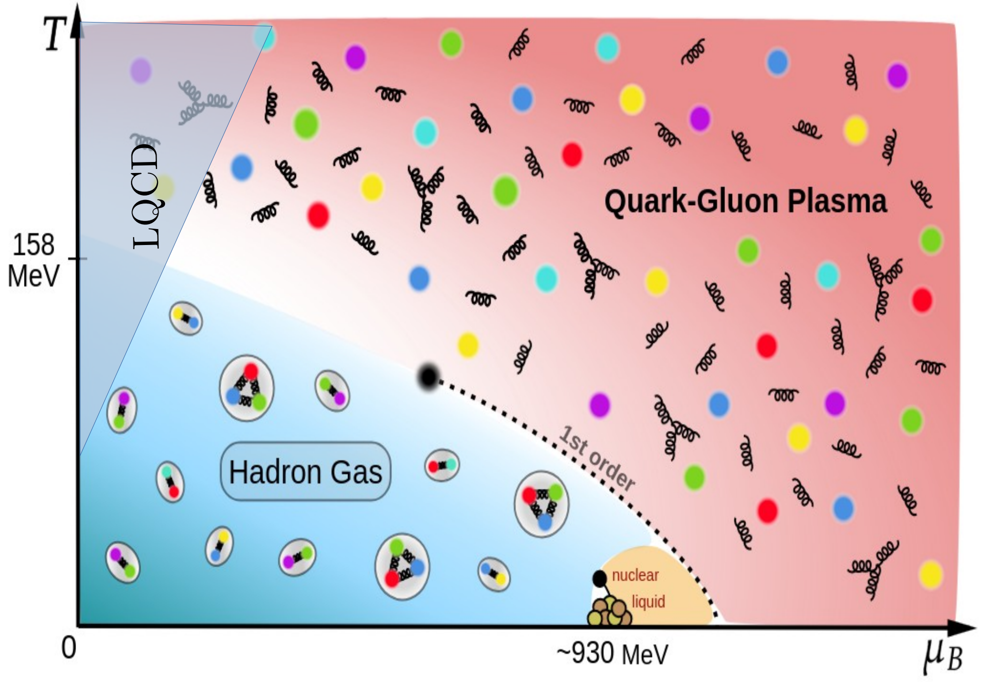

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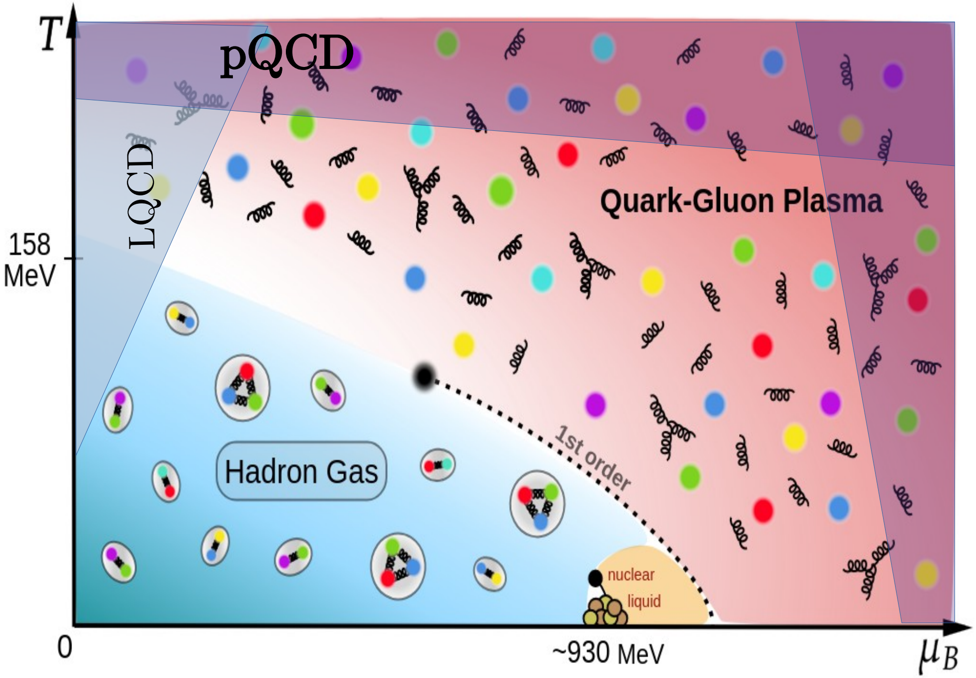

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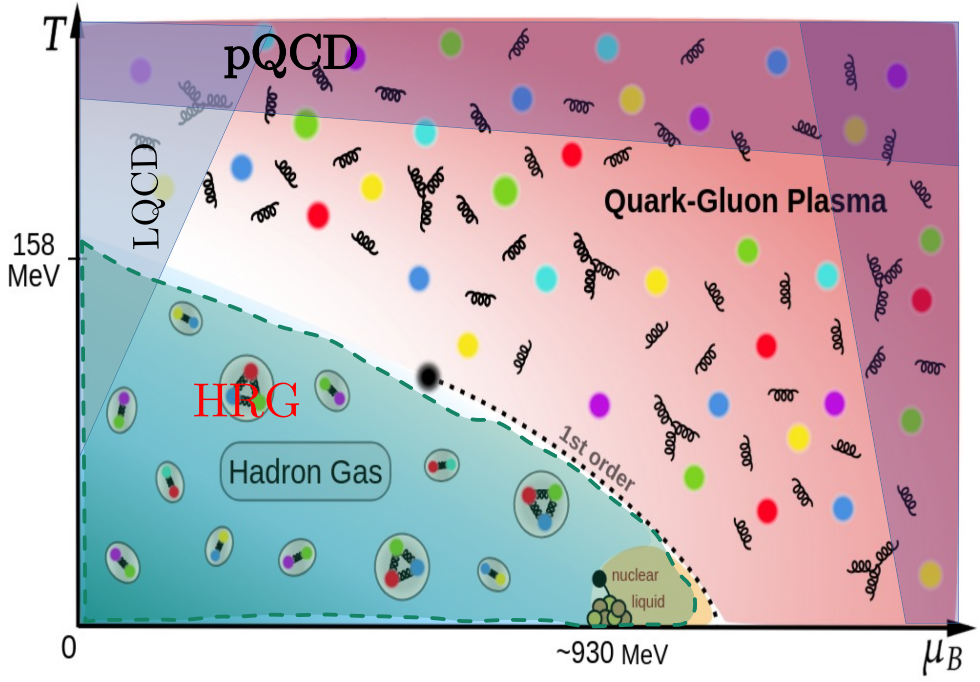
- To cover a large region of the phase diagram, we merge HRG and BH.



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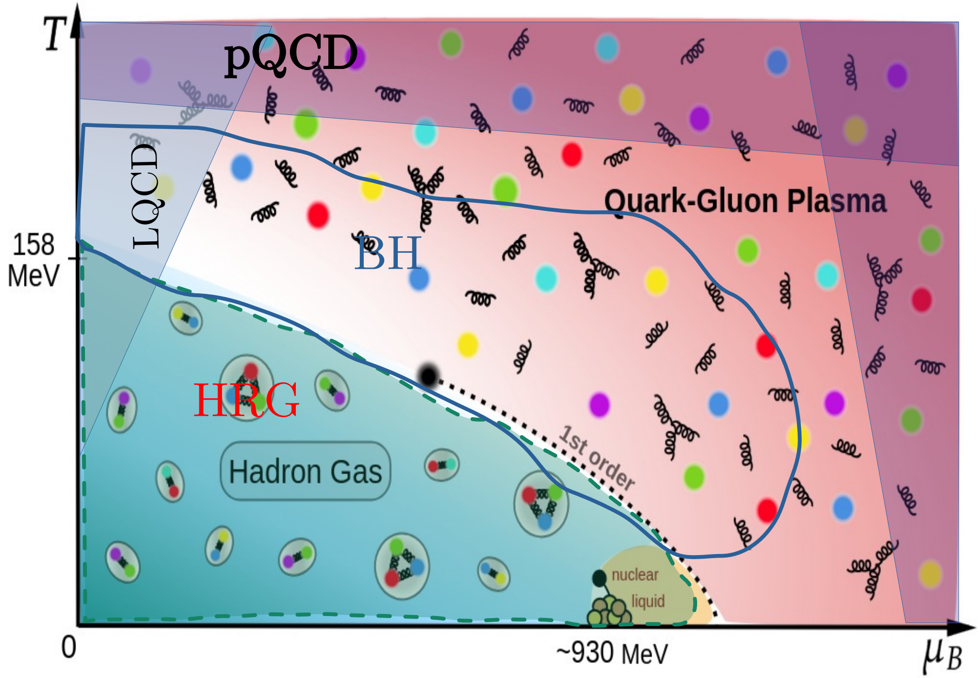

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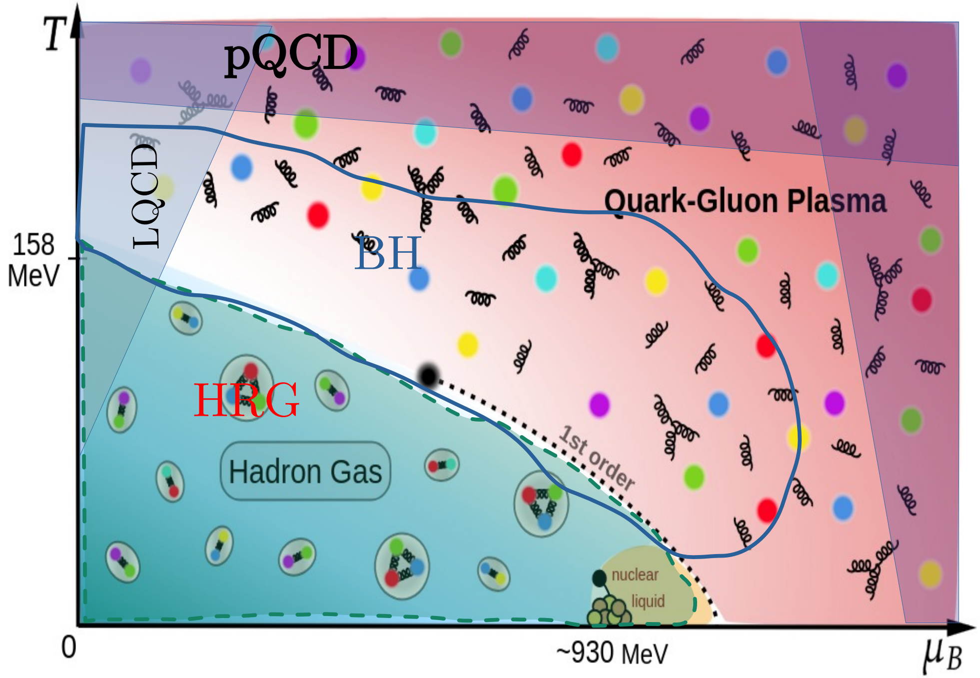

For more about the phase diagram, see Joaquin Grefa's talk, after coffee break.

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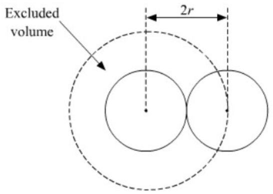

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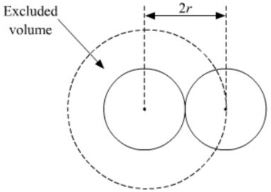

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Vovchenko, Anchishkin, Gorenstein, Phys. Rev. C 91 (2015), 064314

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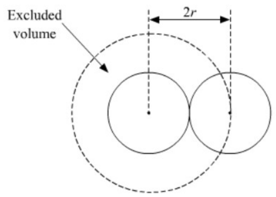

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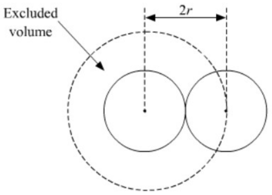

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Vovchenko, Gorenstein, Stoecker, PRL 118 (2017), 182301

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Non-int mesons

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$$P_B(T,\boldsymbol{\mu}) = \sum_{j\in B} p_j^{\mathrm{id}}(T,\mu_j^{B*}) - a\,n_B^2$$

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$$n_{B(\bar{B})} \;=\; (1 - b\, n_{B(\bar{B})}) \sum_{j \in B(\bar{B})} n_j^{\rm id}(T, \mu_j^{B(\bar{B})*})$$

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$$\mu_j^{B(\overline{B})*} = \mu_j - bP_{B(\overline{B})} - abn^2_{B(\overline{B})} + 2an_{B(\overline{B})}$$

Effective chemical potential

Vovchenko, Gorenstein, Stoecker, PRL 118 (2017), 182301

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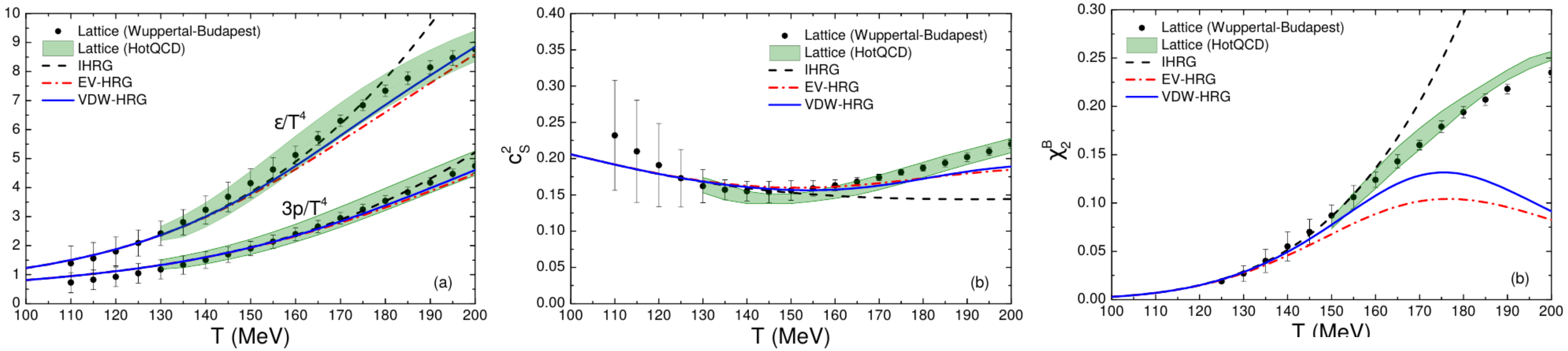

Vovchenko, Gorenstein, Stoecker, PRL 118 (2017), 182301

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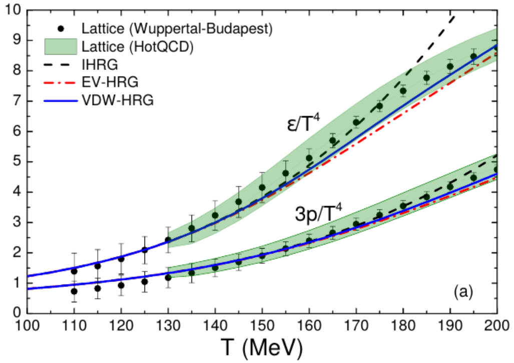

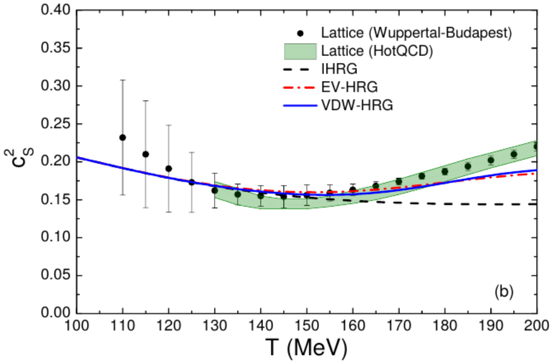

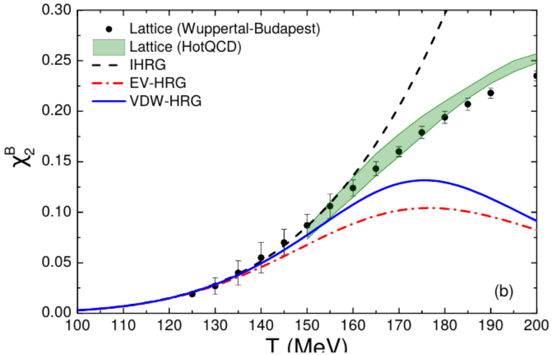

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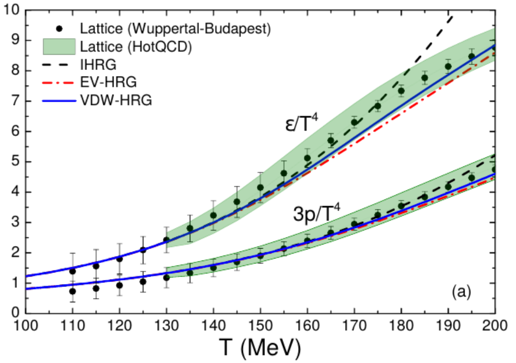
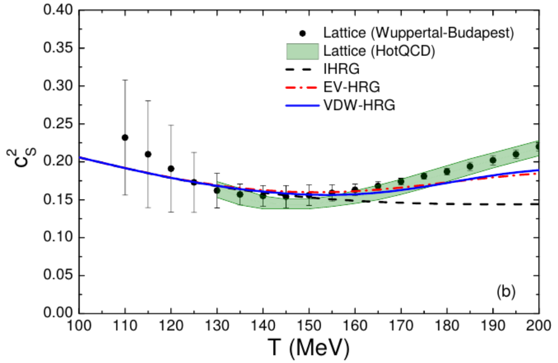
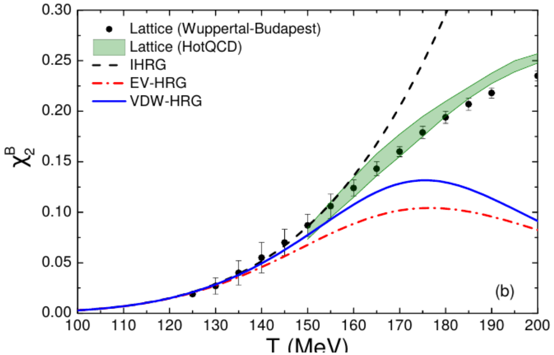
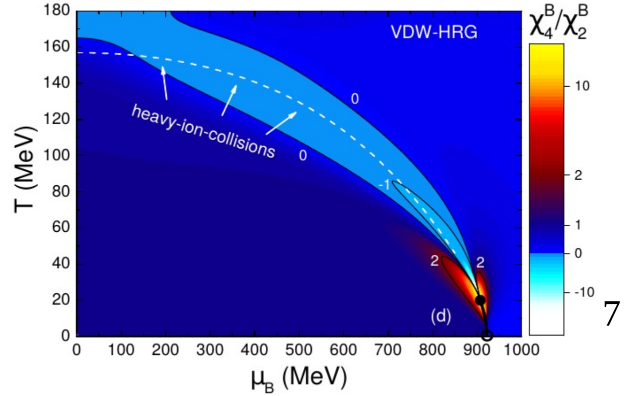
- The vdW HRG model is in very good agreement with LQCD at low T (μ_B=0).
- It does not exhibit deconfinement or chiral transition.
- Nuclear liquid-gas transition at low temperature and high chemical potential, with a critical point at $T_c \approx 19.7$ MeV, $\mu_c \approx 908$ MeV.









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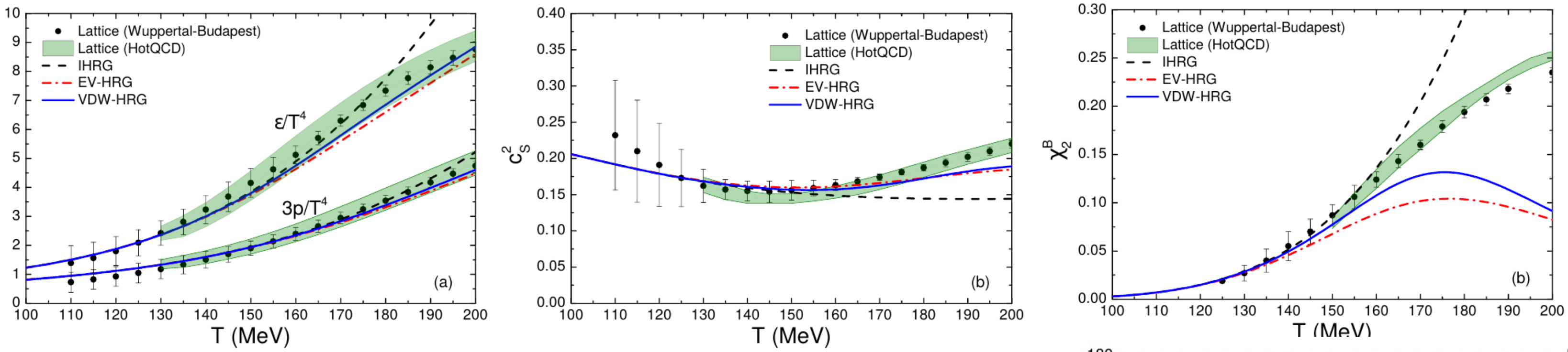
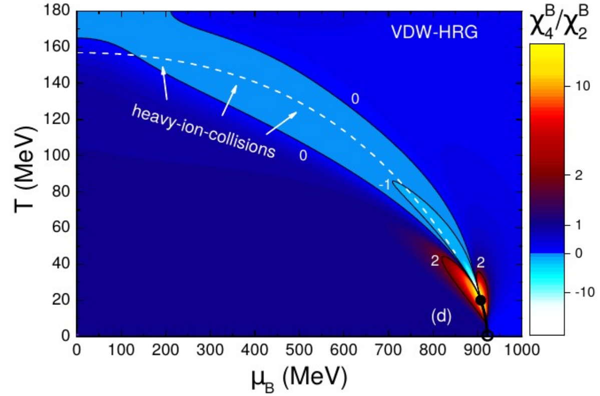


- In this work, we use the model to describe the hadronic phase.



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Einstein-Maxwell-Dilaton Model

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- Where ϕ is a dilaton field.

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Maxwell action

Newton constant

Dewolfe, Gubser and Rosen, Phys. Rev. D 83, 086005 (2011)

- Action of a 5-dimensional gravitational theory.
- In order to have an AdS/QCD correspondence, one needs to break conformal symmetry.
- For that, an arbitrary potential $V(\phi)$ is introduced to break conformal symmetry
- Where ϕ is a dilaton field.
- This action introduces an interaction term between the Maxwell action and dilaton field, $f(\phi)$, which produces a finite chemical potential in the dual QCD-like model.

EMD Holographic model

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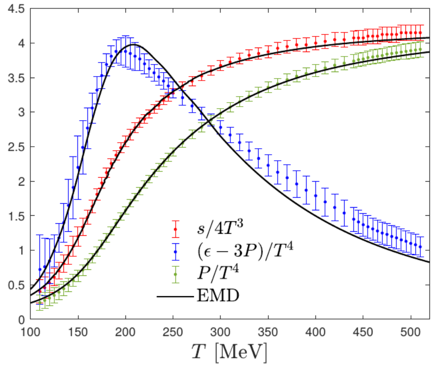

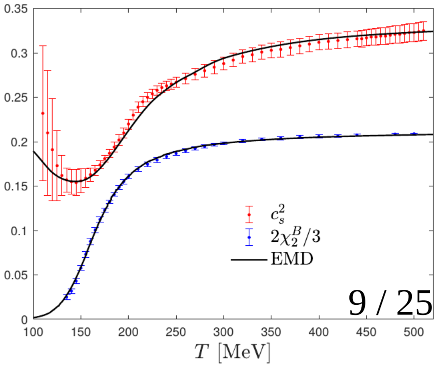

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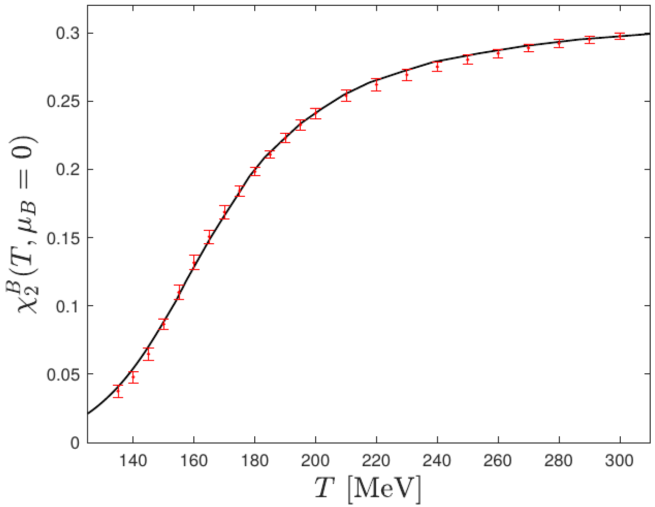

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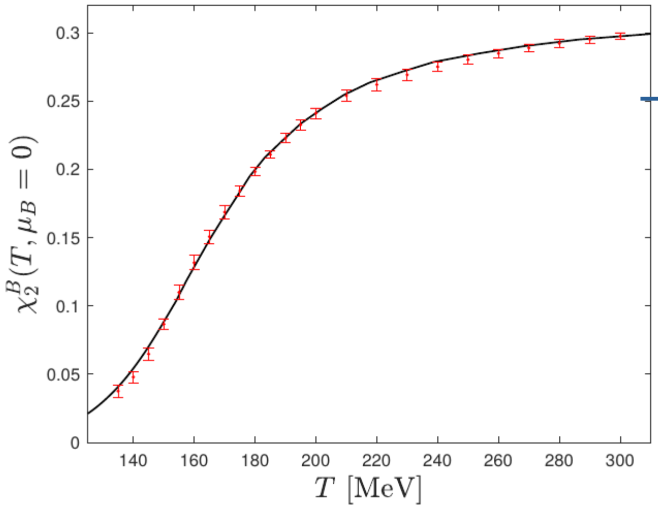

$\rightarrow$ Fix $f(\phi)$

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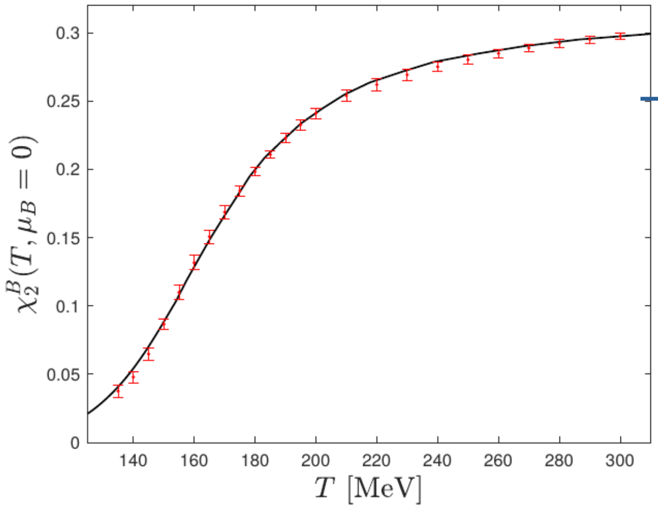

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Grefa et al. Phys.Rev.D 104 (2021) 3, 034002

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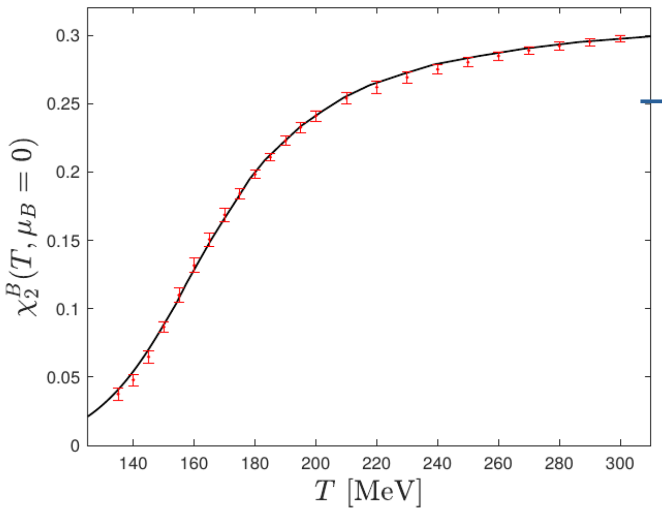

Fix $f(\phi)$

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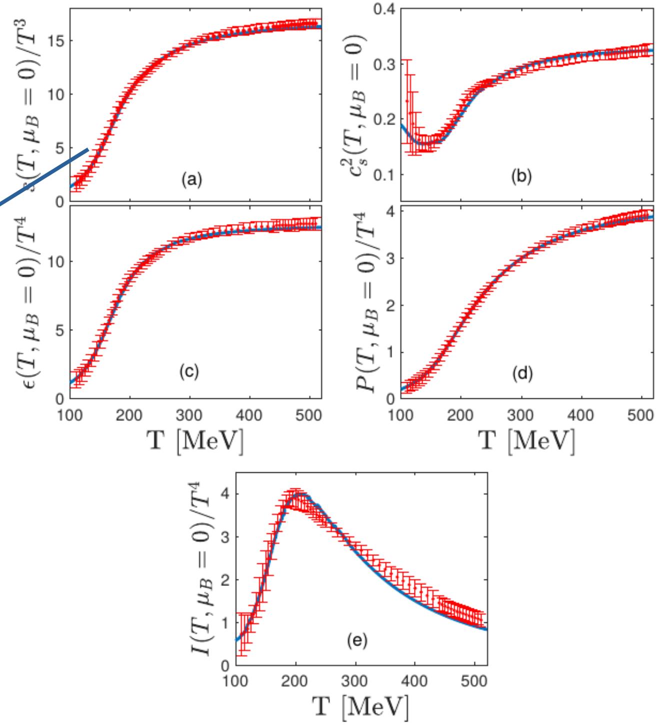

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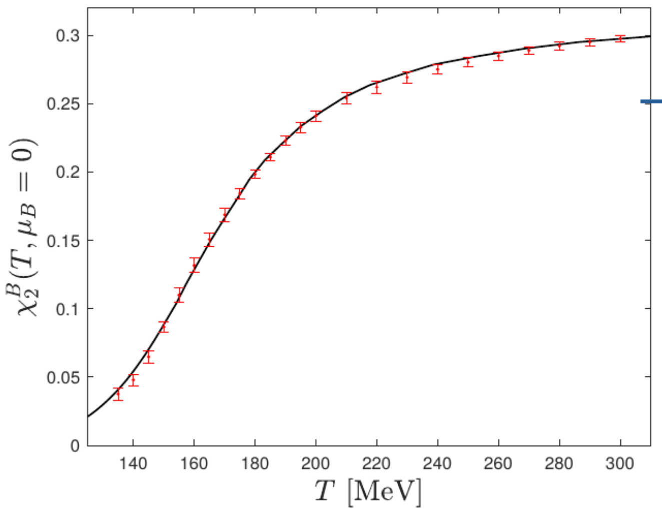

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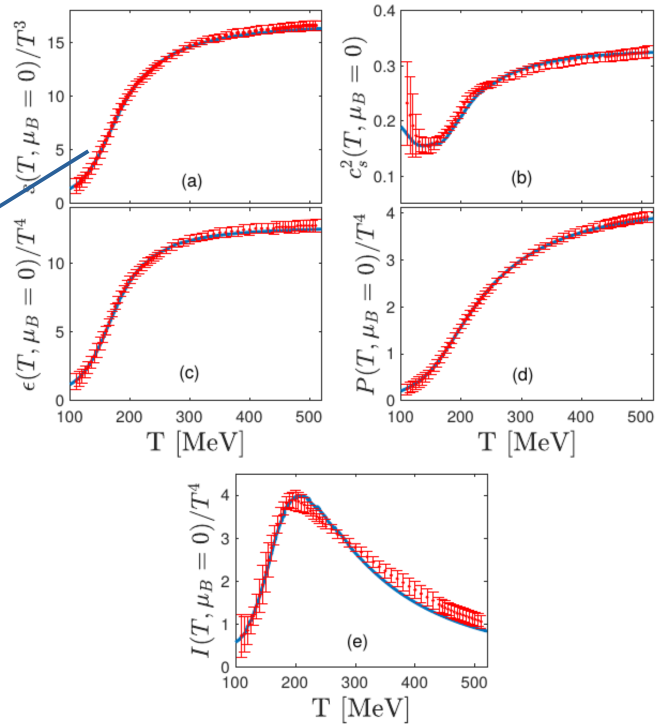

Grefa et al. Phys.Rev.D 104 (2021) 3, 034002

- Limitation: BH exhibits poor behavior at low T values!

Bayesian fine tuning

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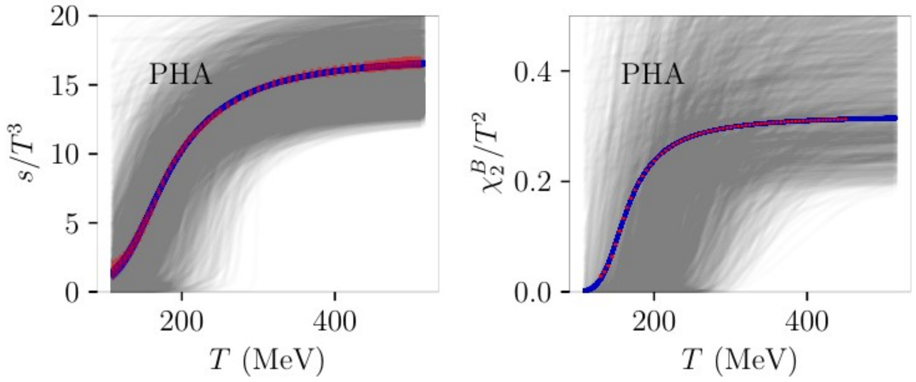

Hippert et al. PRD 110 (2024) 9, 094006

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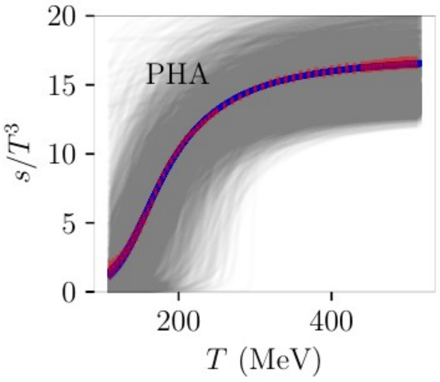
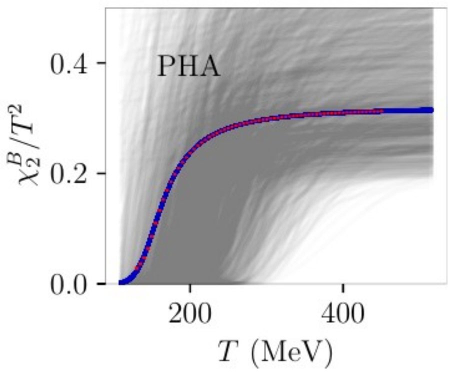
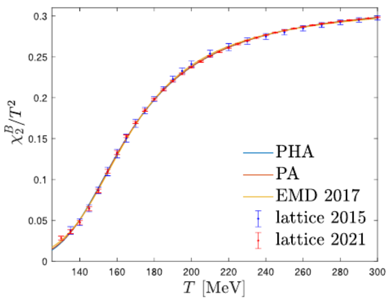
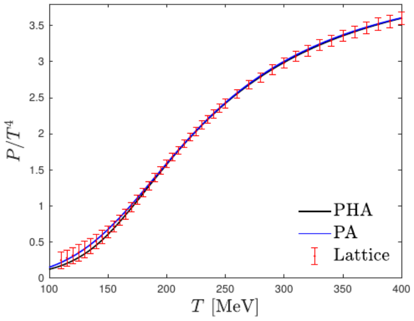




Best fit parameters





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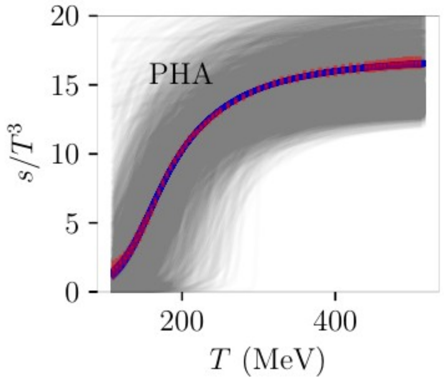
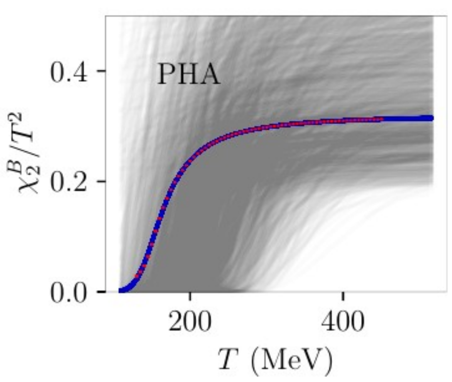
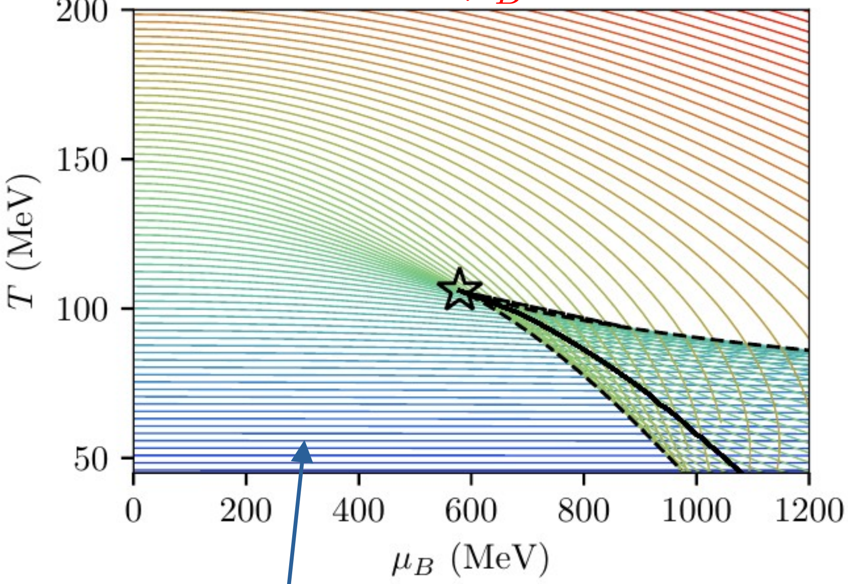




$T_{CEP} = 107$ MeV, $\mu_B^{CEP} = 571$ MeV



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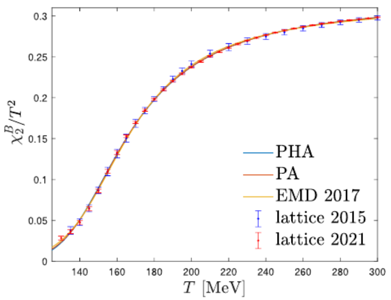

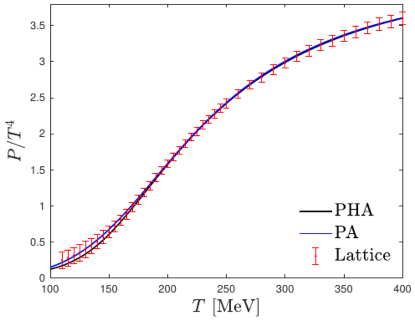

Lines of constant ϕ_0.

Hippert et al. PRD 110 (2024) 9, 094006

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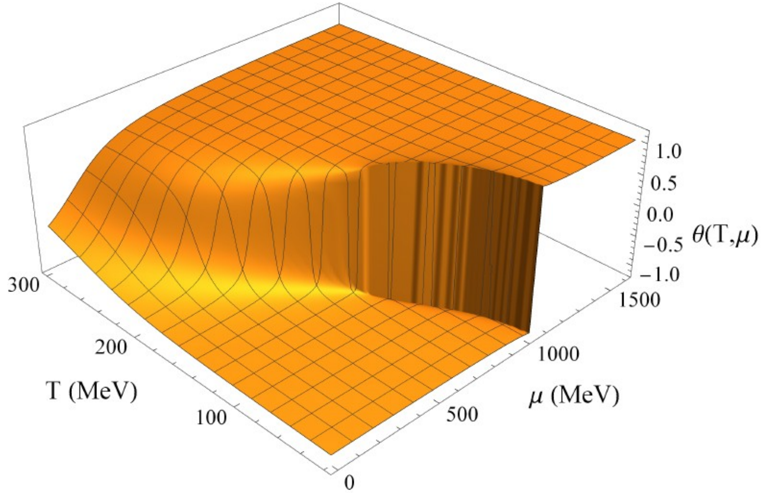

Plot from Kapusta, Welle. Phys.Rev.C 106 (2022) 4, 044901

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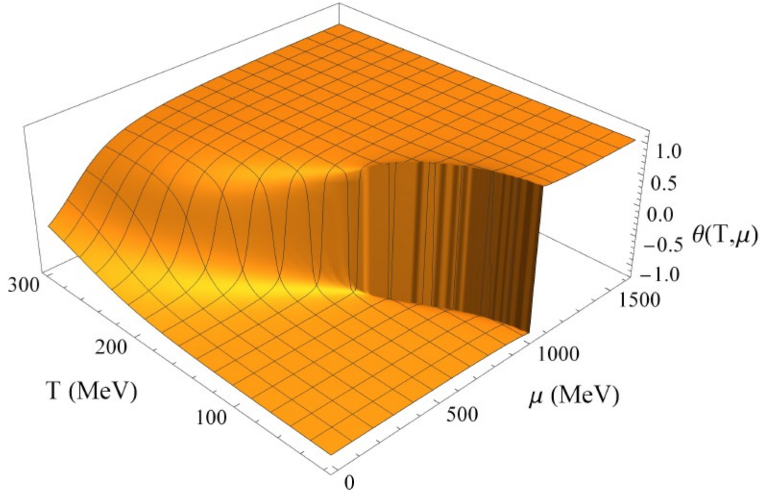
Previous versions in Albright, Kapusta, Young. Phys.Rev.C 90 (2014) 2, 024915 and Plumberg, Welle, Kapusta PoS CORFU2018 (2018) 157.



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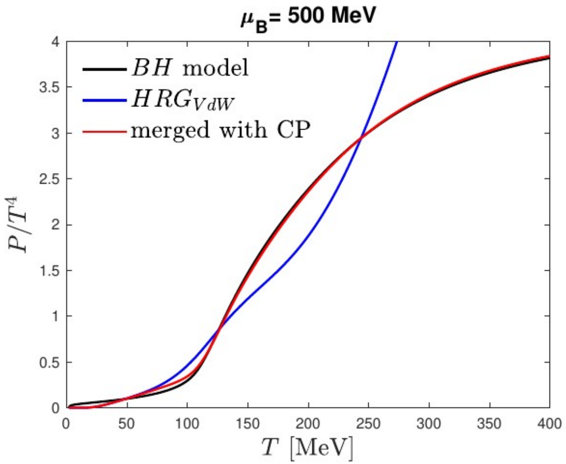

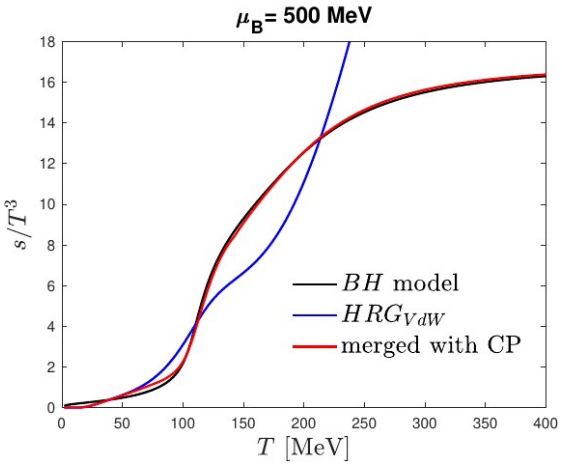

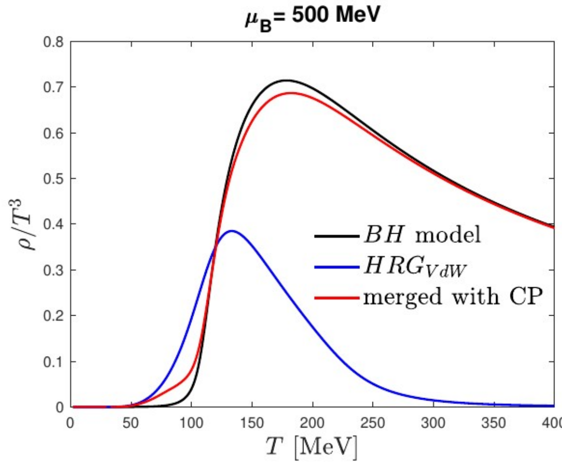

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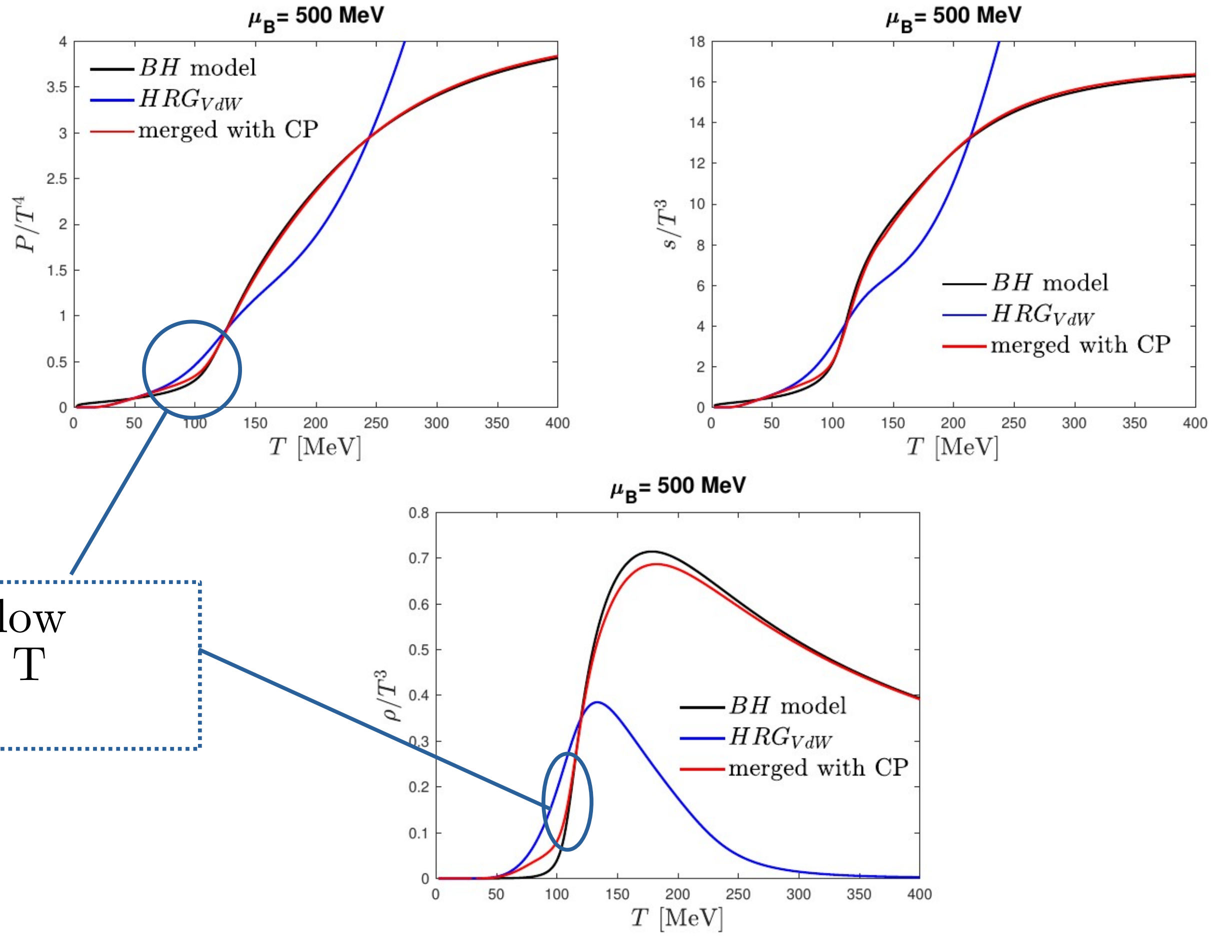
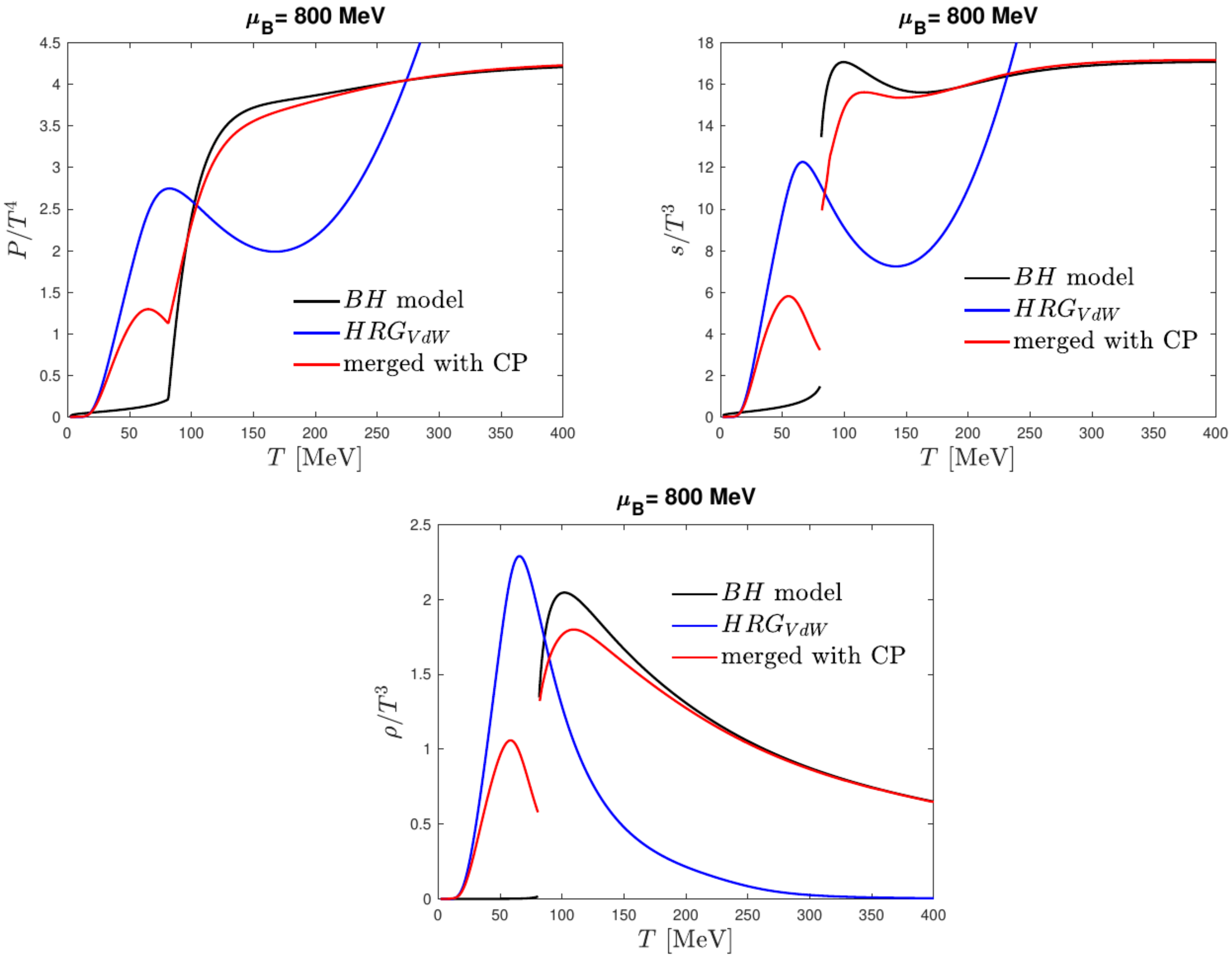


Does not follow
HRG at low T



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P/T^4
T [MeV]
BH model
HRG_{VdW}
merged with CP
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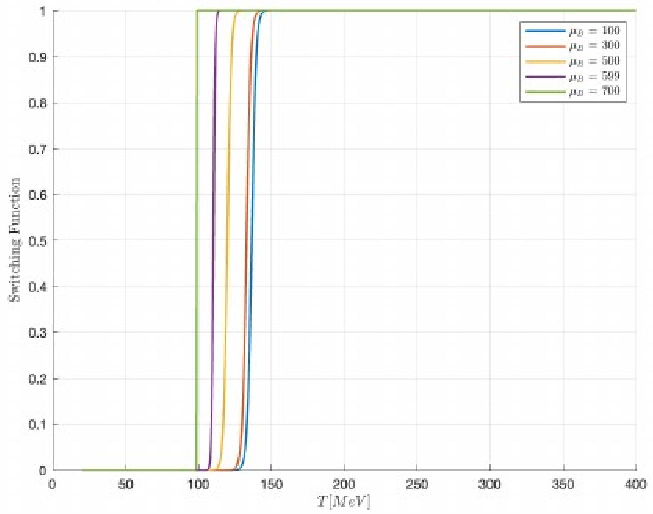

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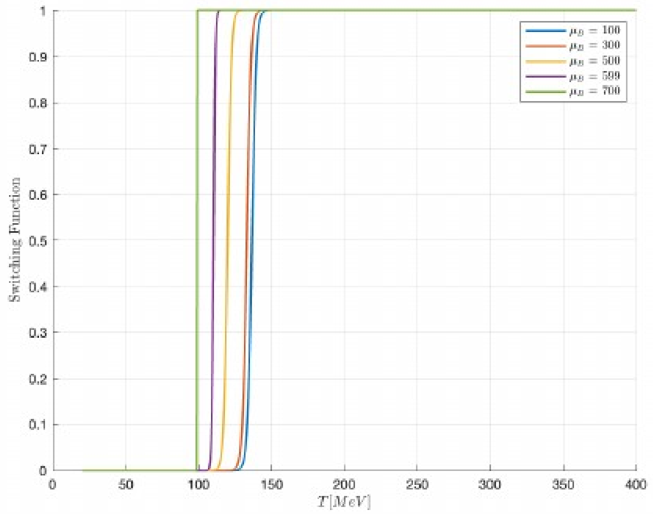

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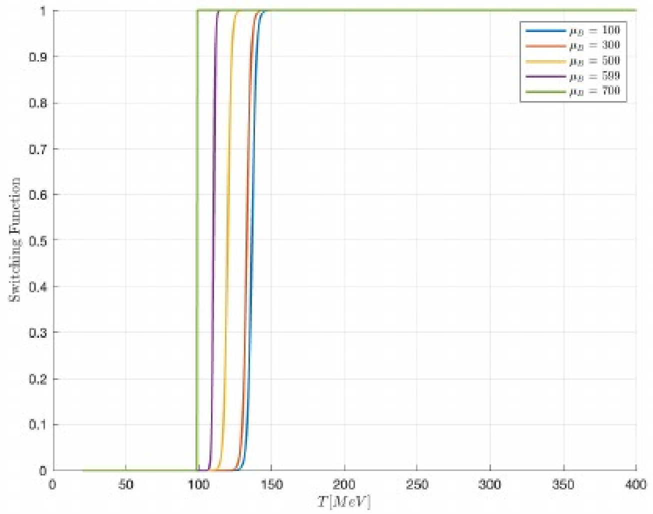

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- Applied on entropy, when applied on pressure, negative contribution from the switching funtion.

Why merging entropy instead of pressure?

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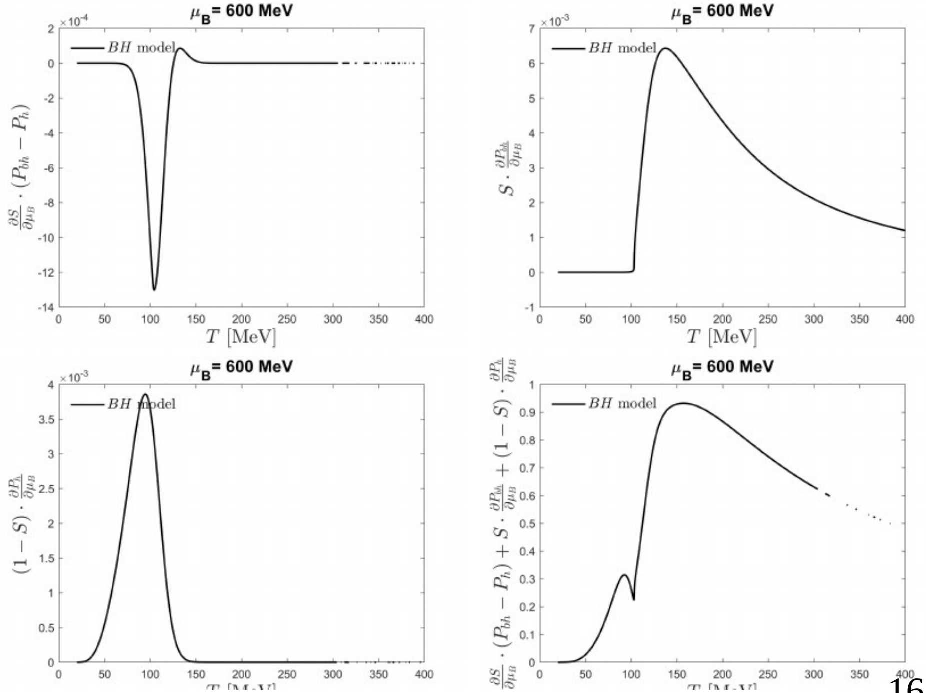

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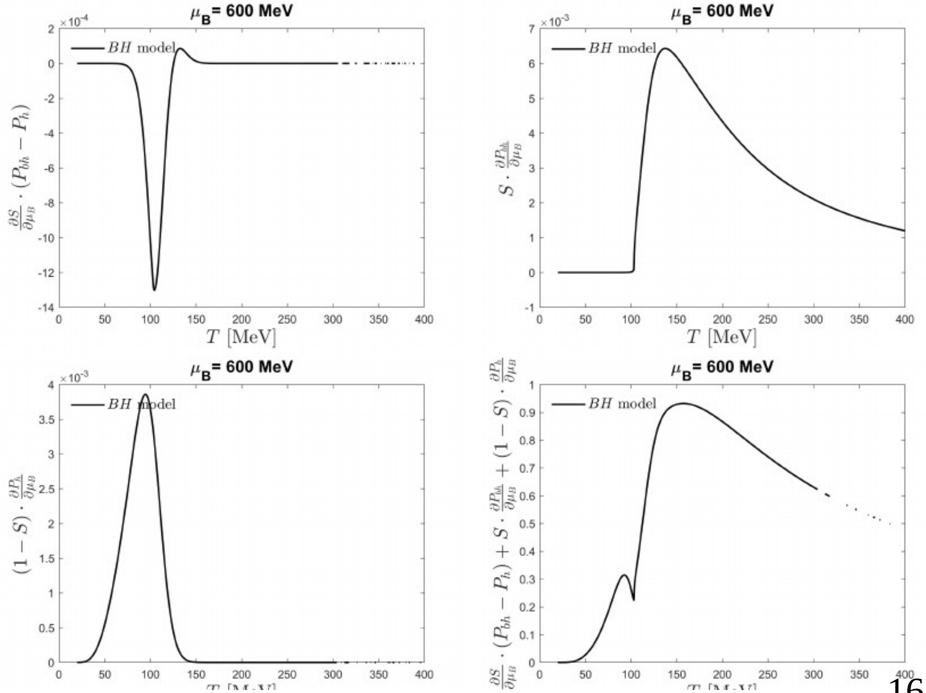
- The negative contribution of the term in red produces an artificial minimum in density, affecting other thermodynamic quantities.
- Since the switching function depends explicitly on T and μ_B, it affects the thermodynamics!!



Results for pressure

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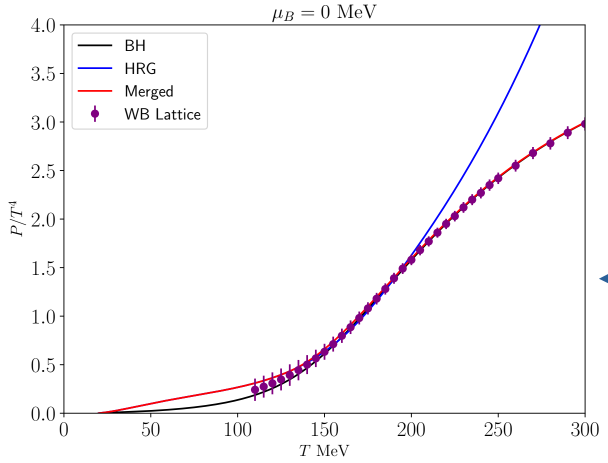
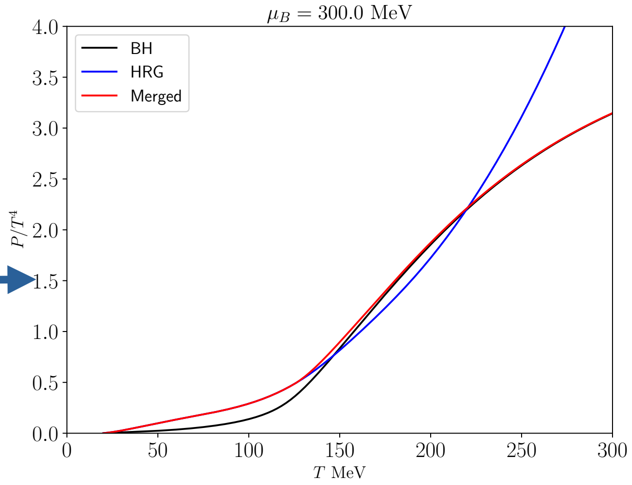
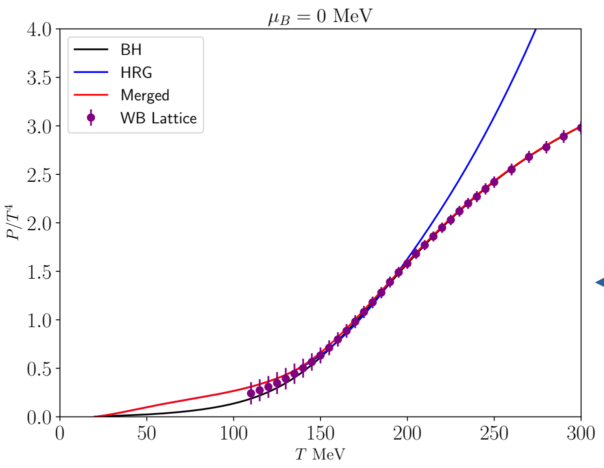


Crossover



Results for pressure



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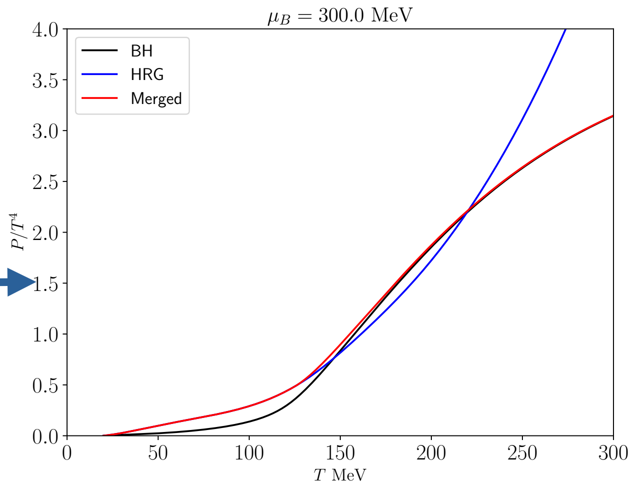
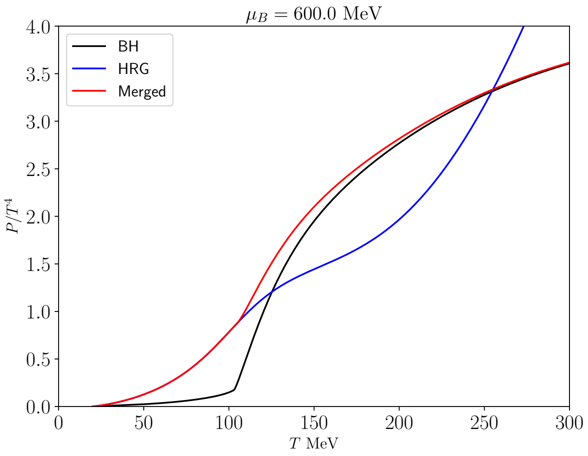
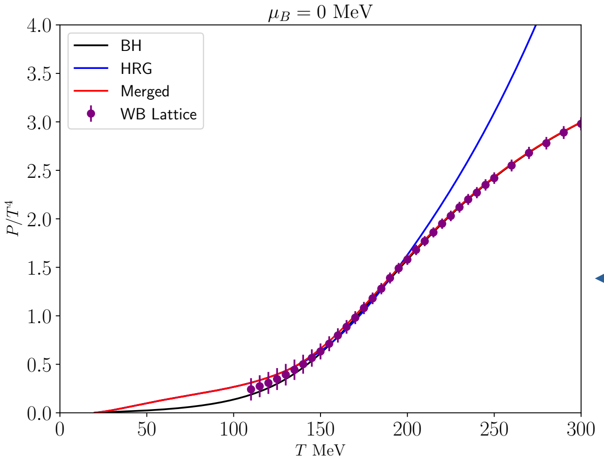
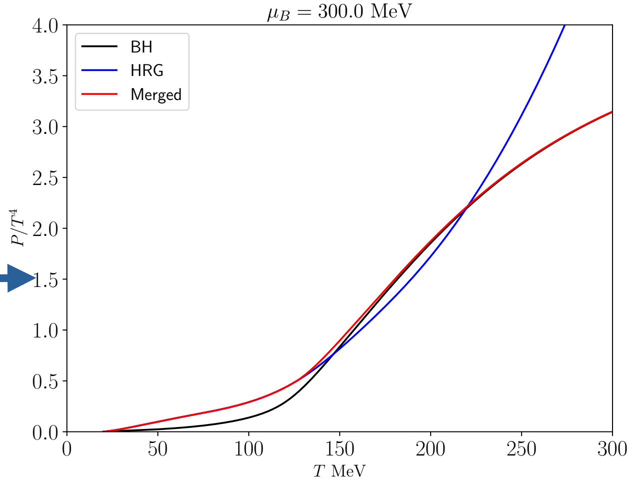


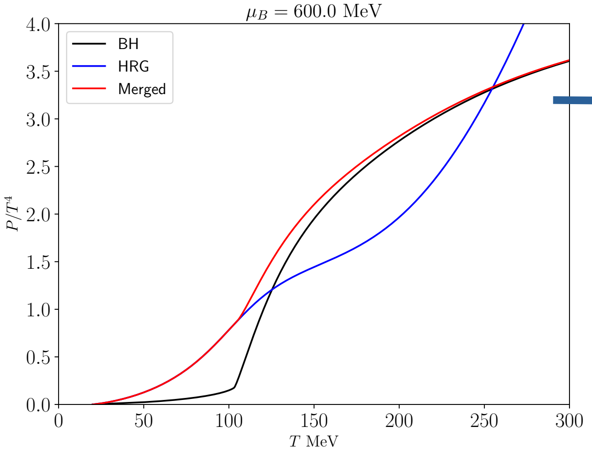


Results for pressure



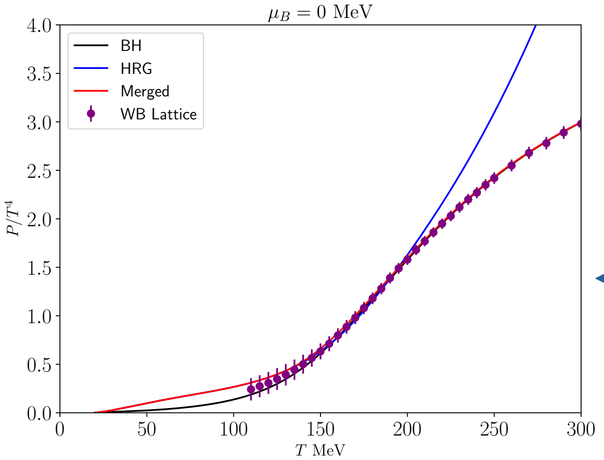


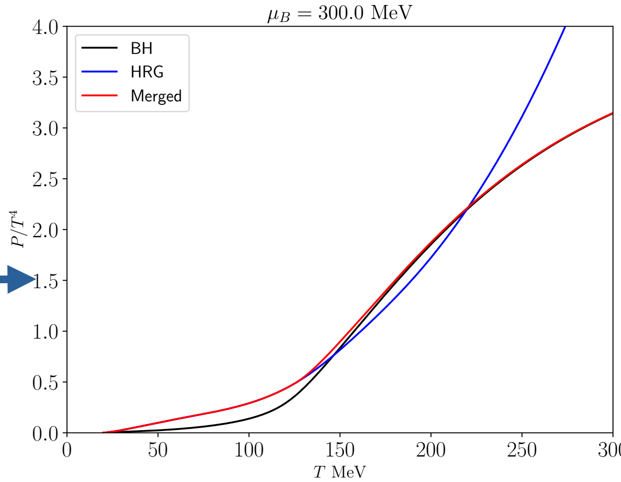
Crossover



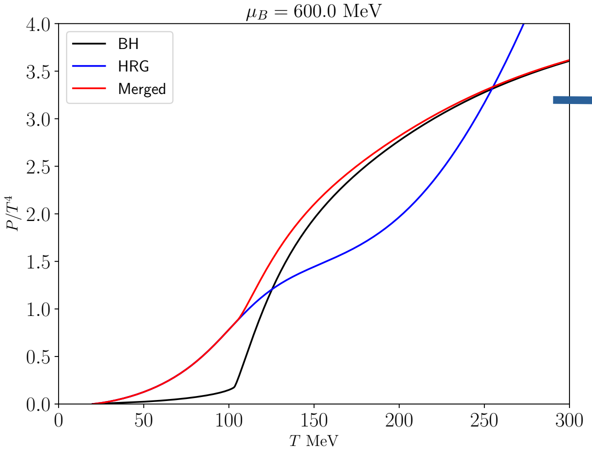
Near the critical point

Results for pressure

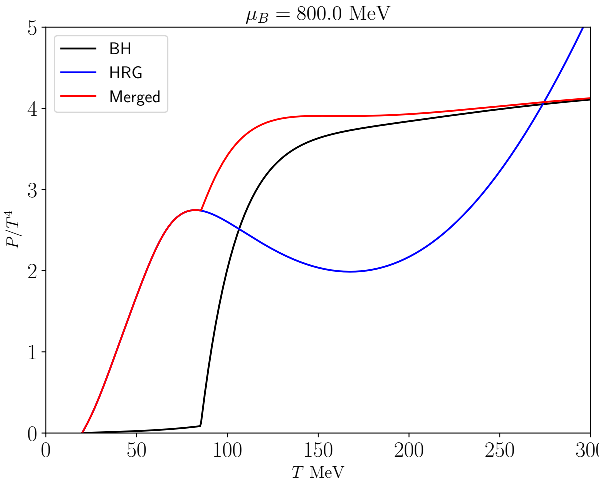




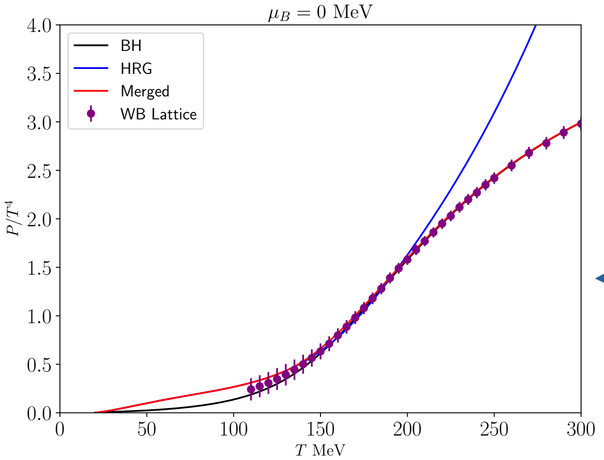
Crossover



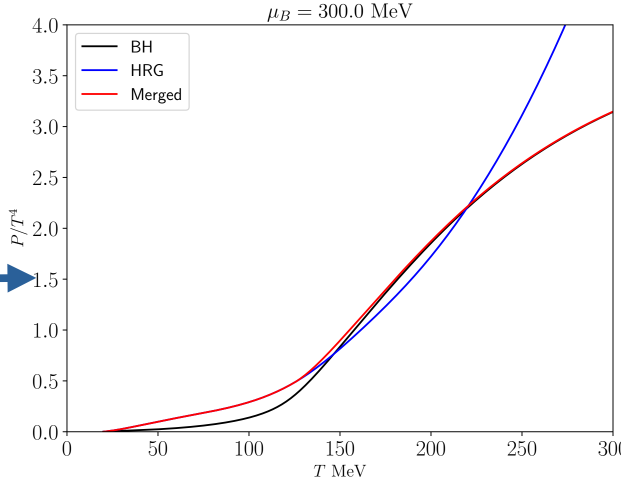
Near the critical point

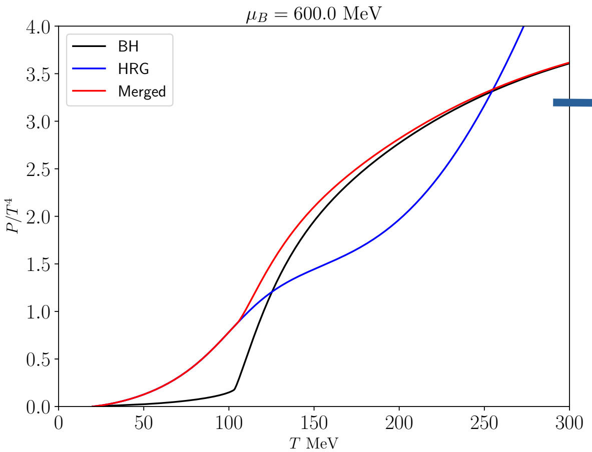


Results for pressure



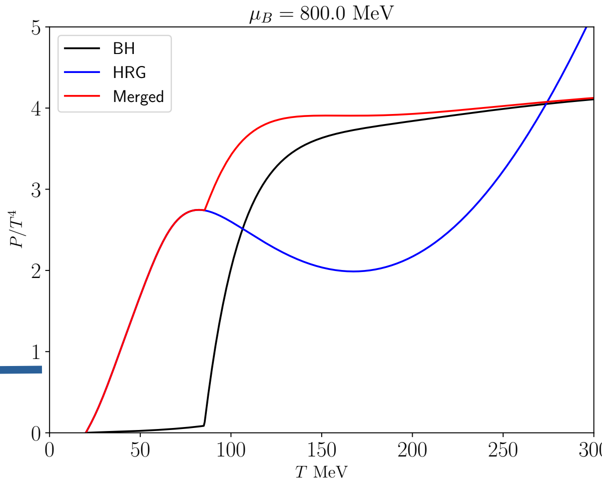
Crossover





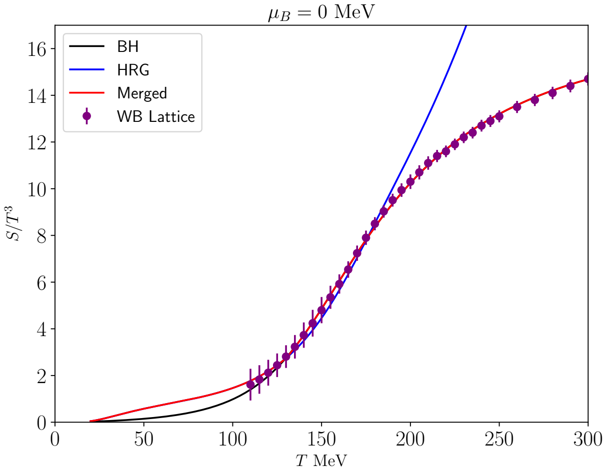
Near the critical point

First-order

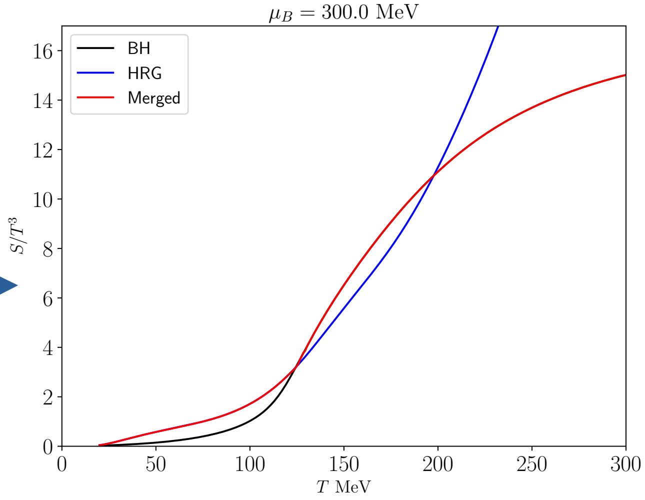


Results for entropy

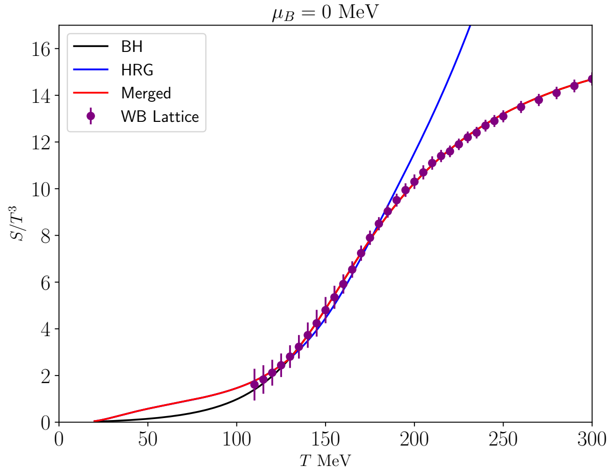
Results for entropy



Crossover



Results for entropy



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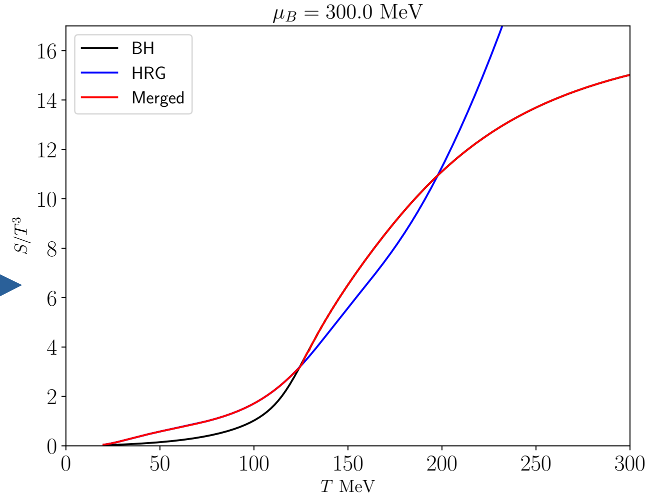
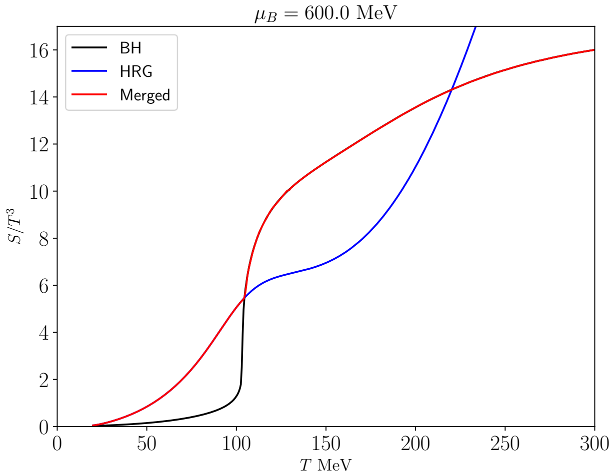
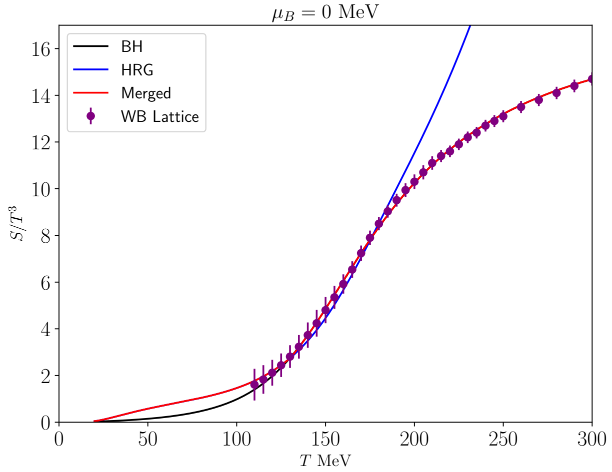
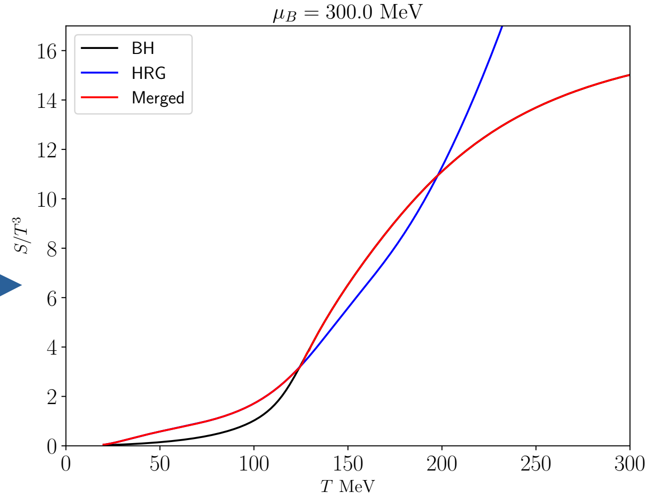




Results for entropy





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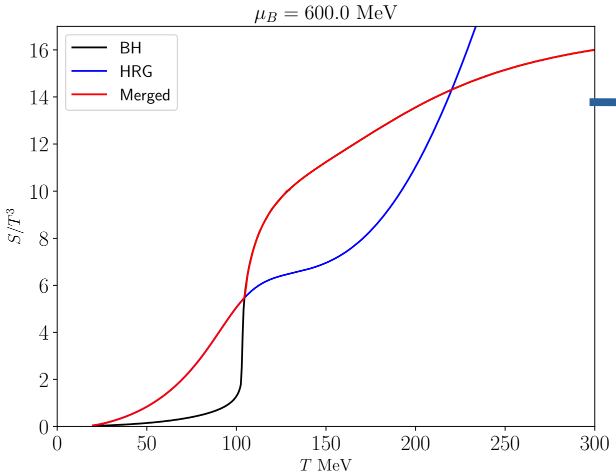
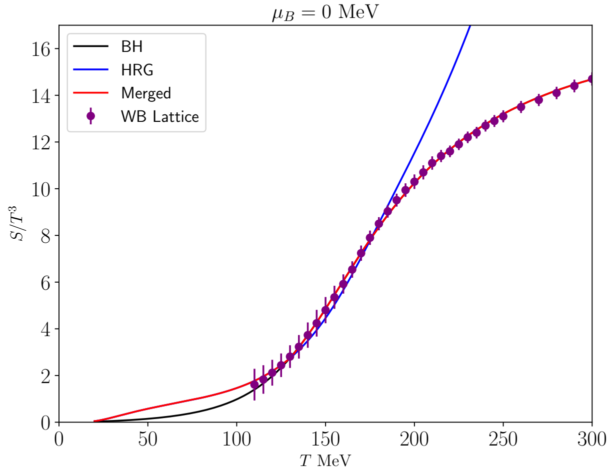
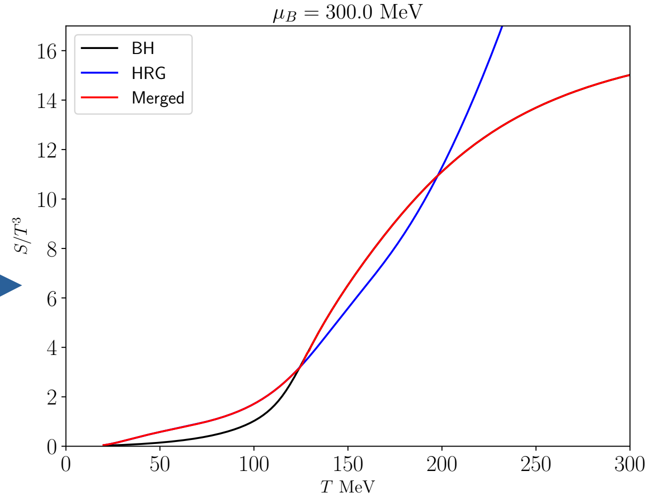


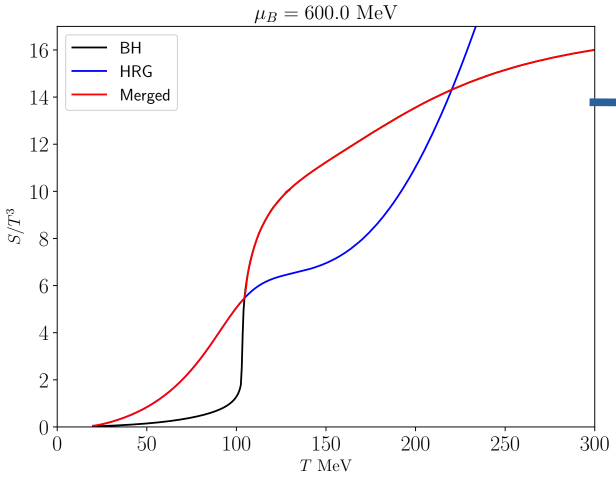
Near the critical point

Results for entropy

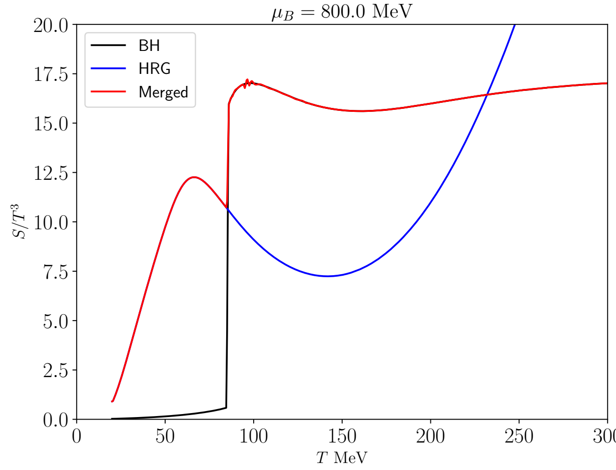




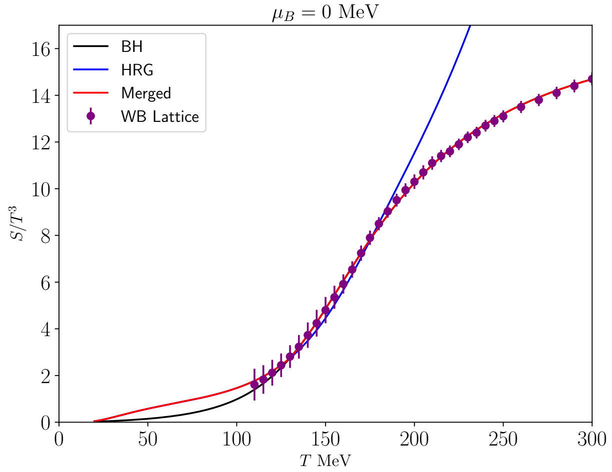
Crossover



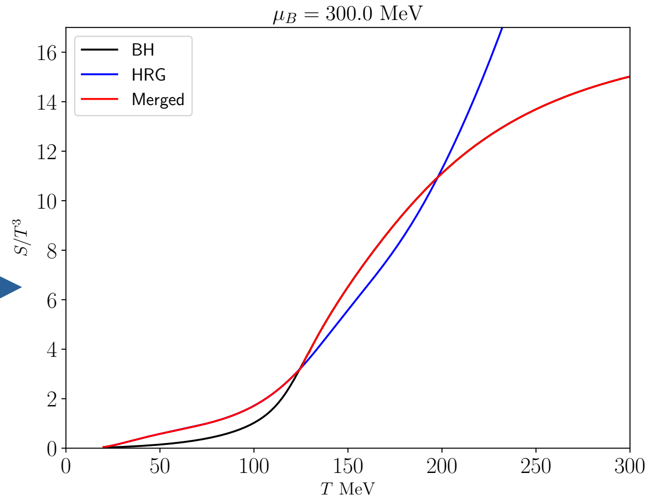
Near the critical point

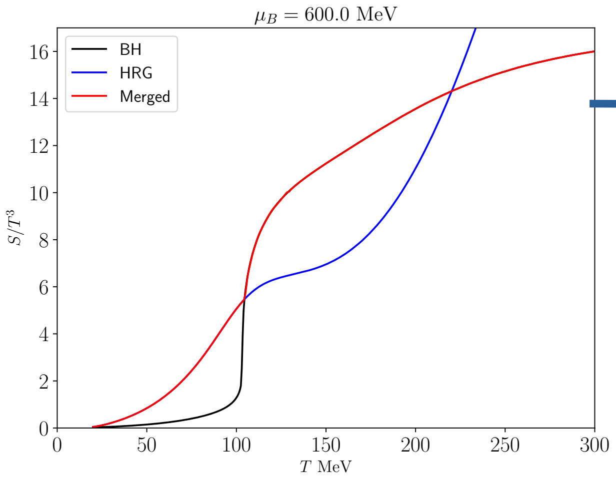


Results for entropy



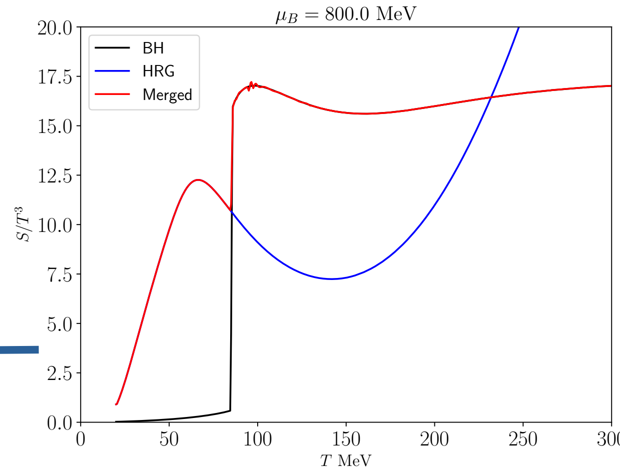
Crossover





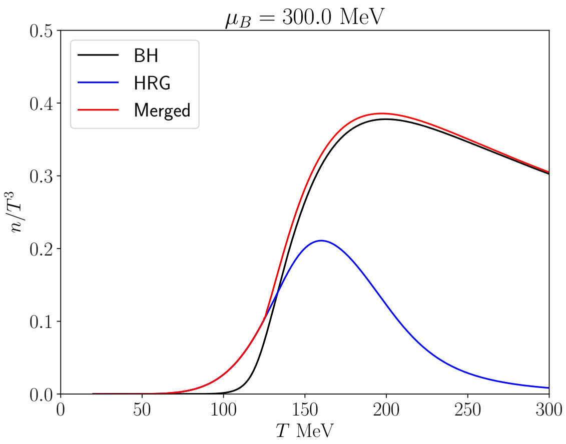
Near the critical point

First-order

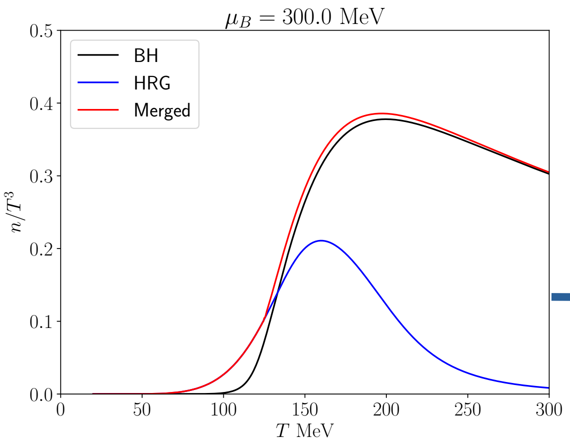


Results for density

Results for density

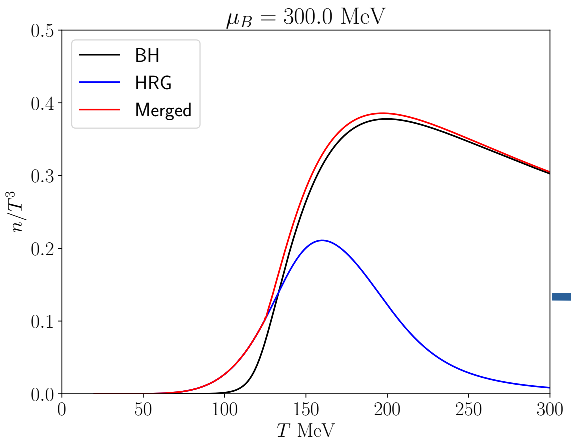


Results for density

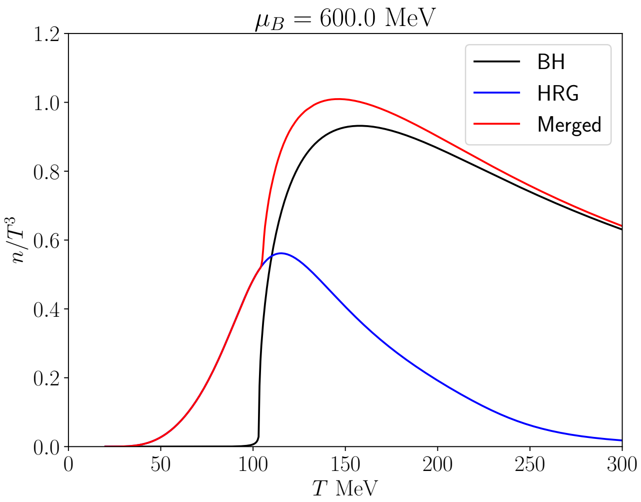


Crossover

Results for density



Crossover



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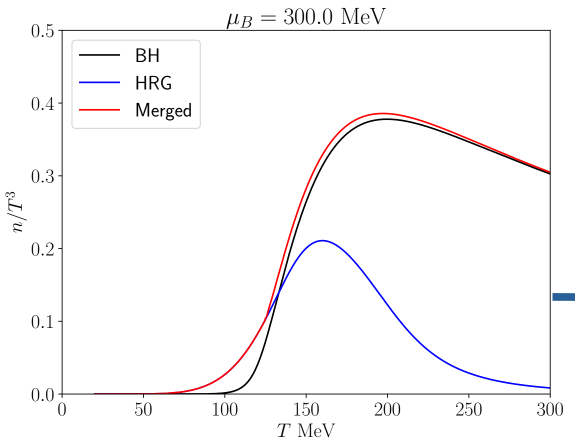
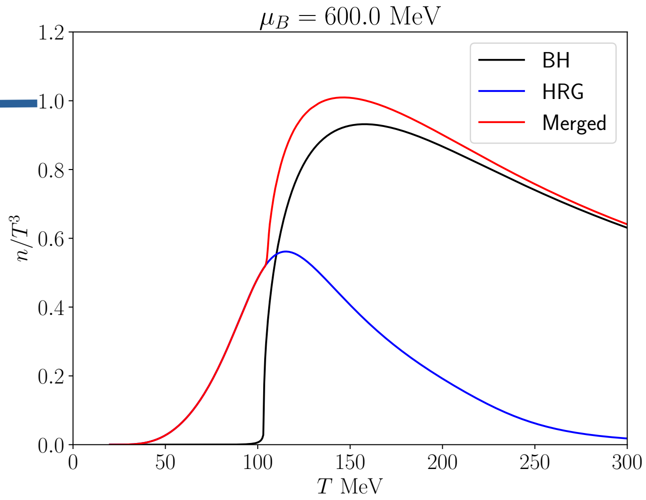
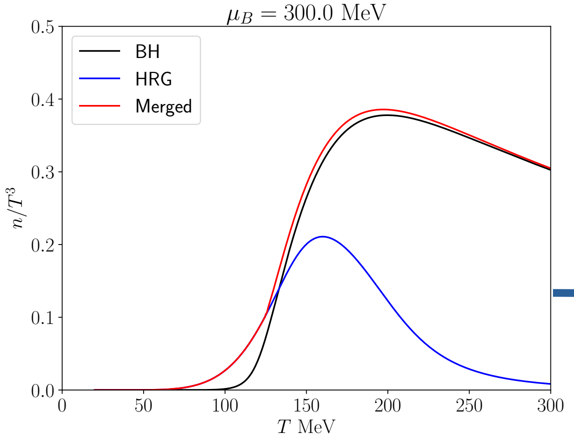


Crossover

Near the
critical point

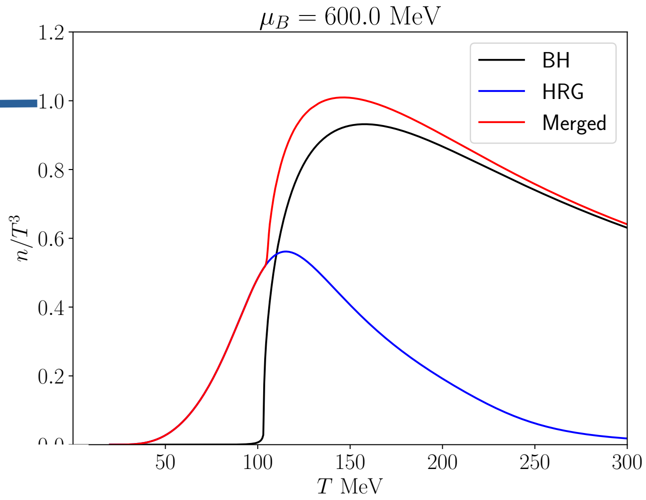


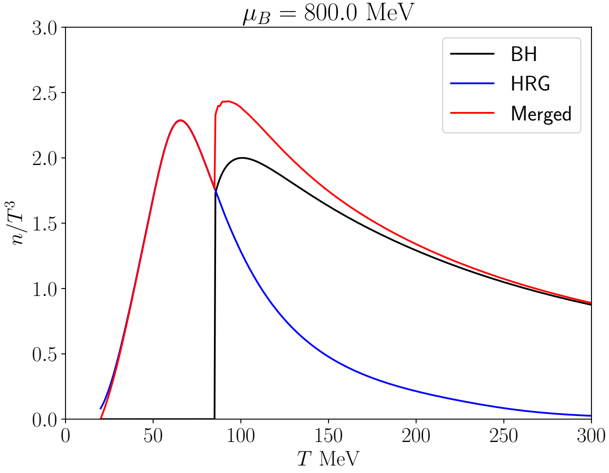
Results for density



Crossover

Near the
critical point





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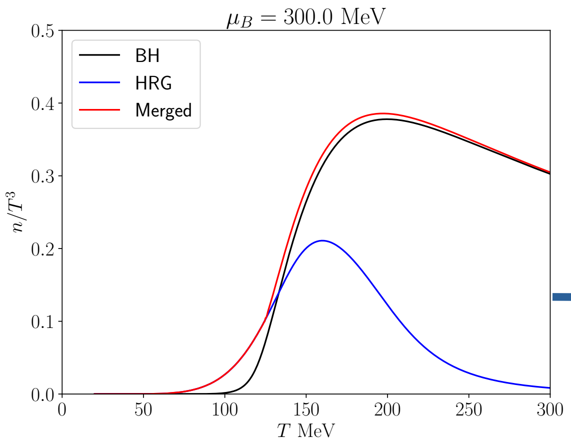
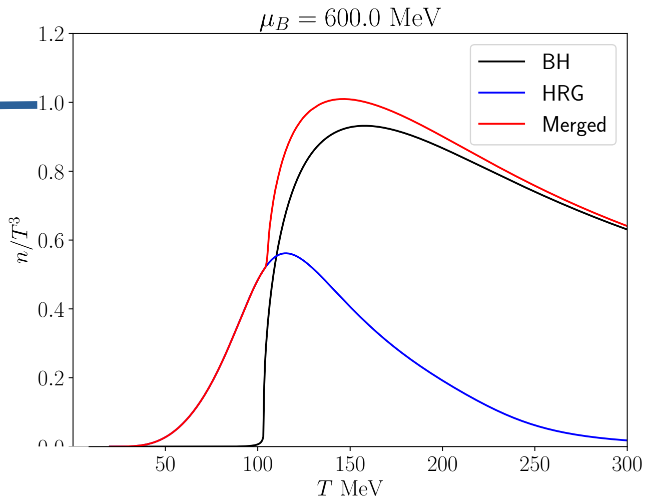
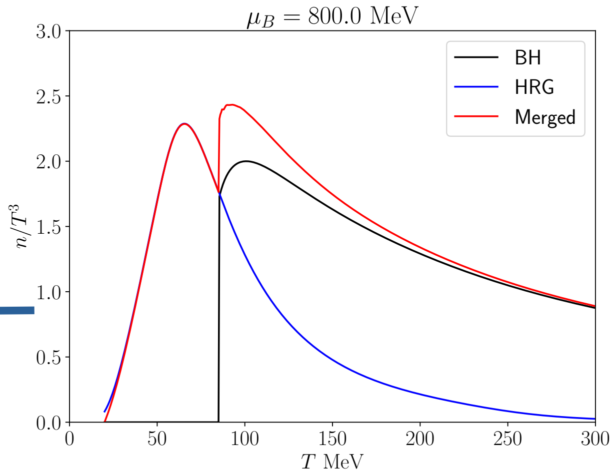


Crossover

Near the critical point

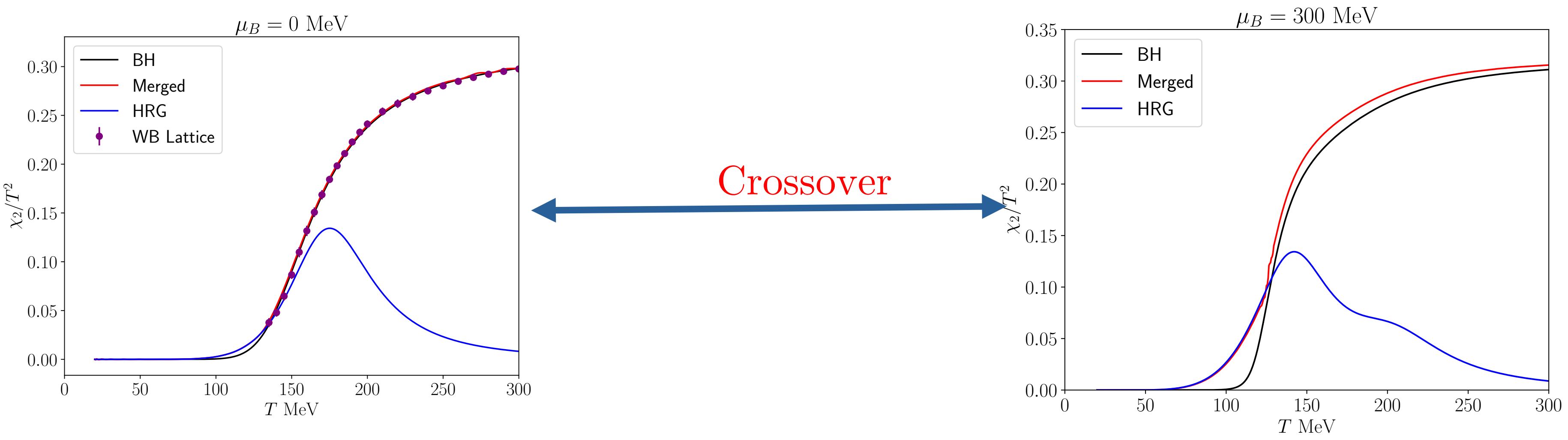


First-order

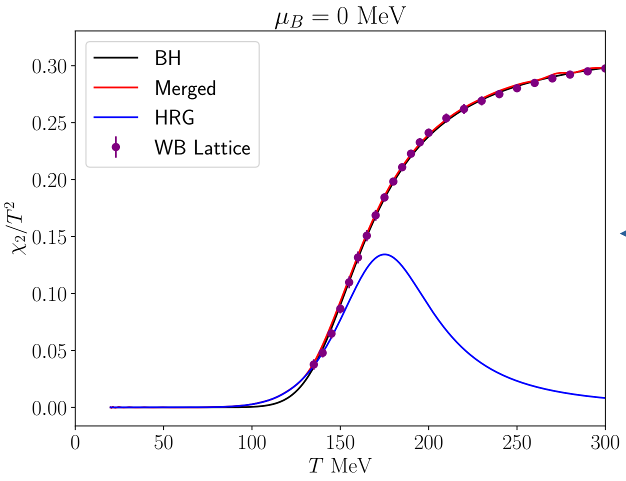


Results for susceptibility

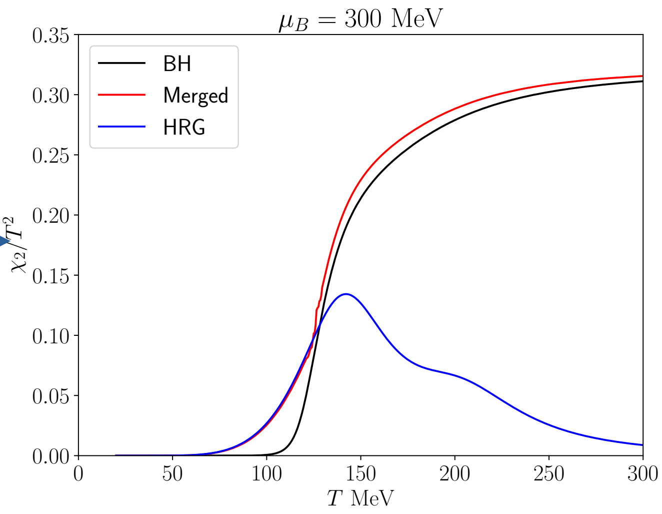
Results for susceptibility

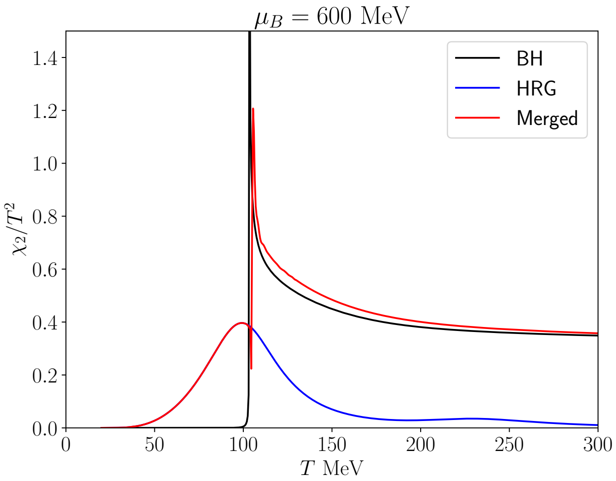


Results for susceptibility

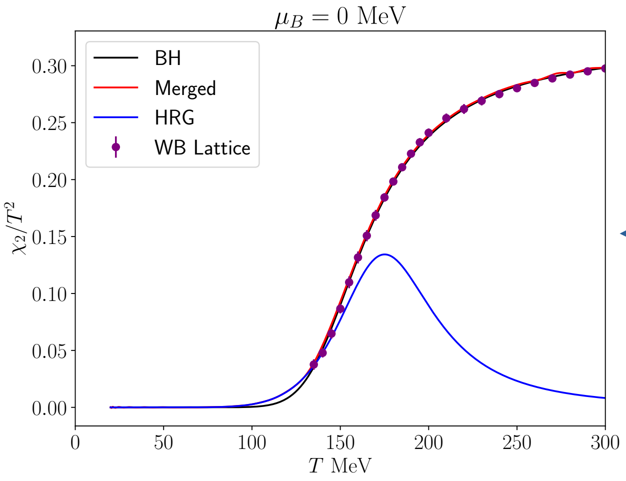


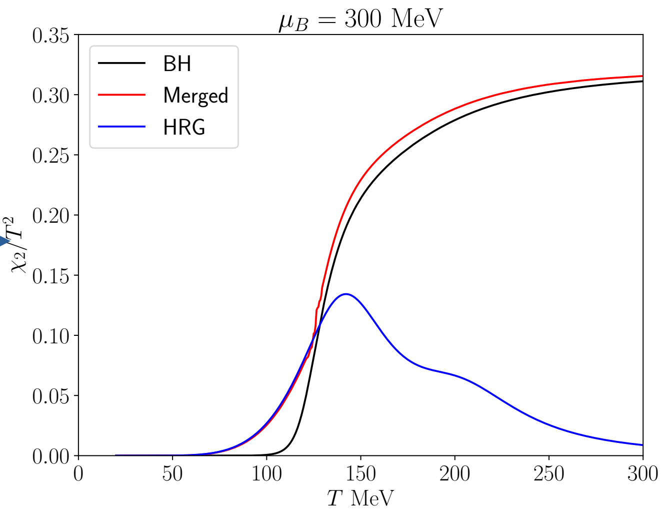
Crossover



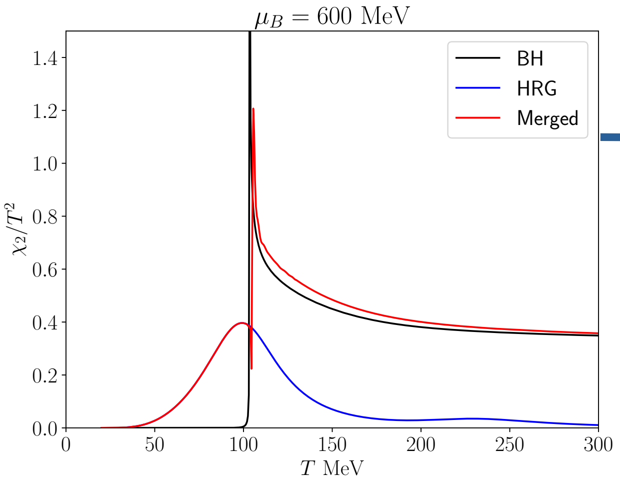


Results for susceptibility



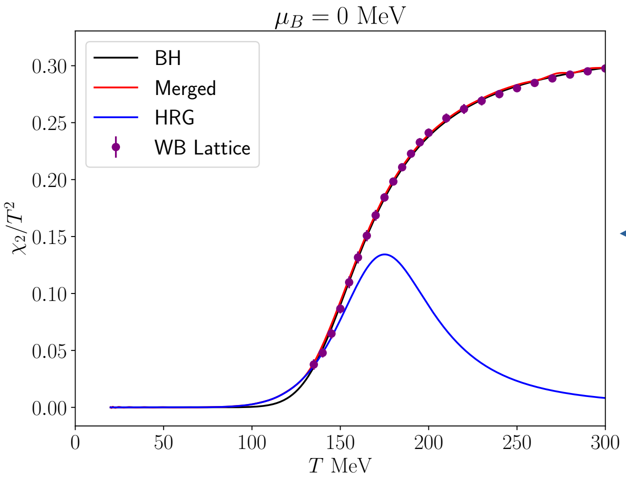


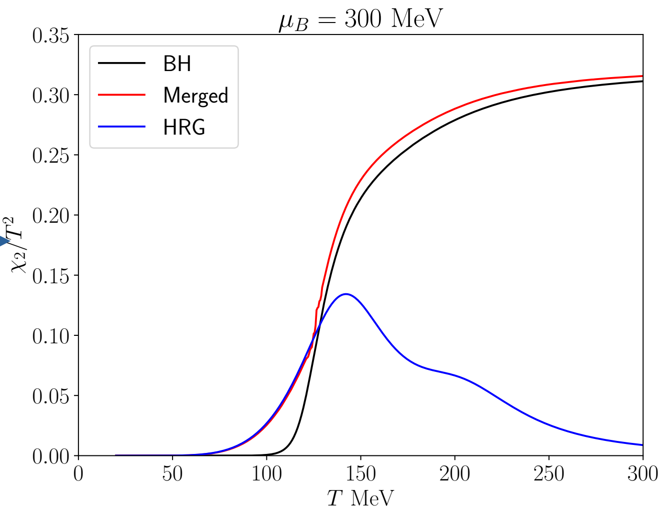
Crossover



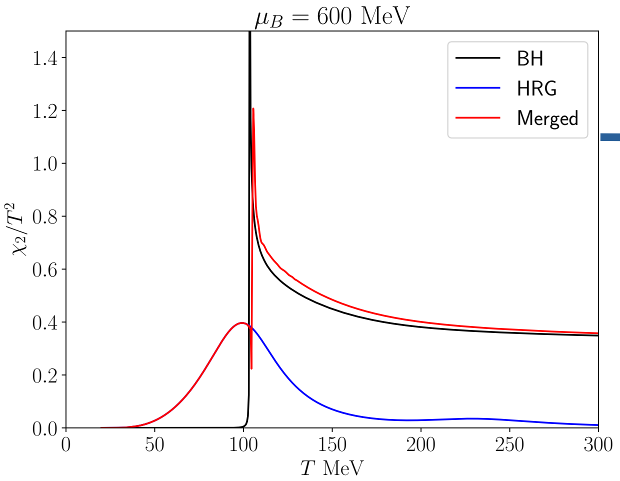
Near the
critical point

Results for susceptibility

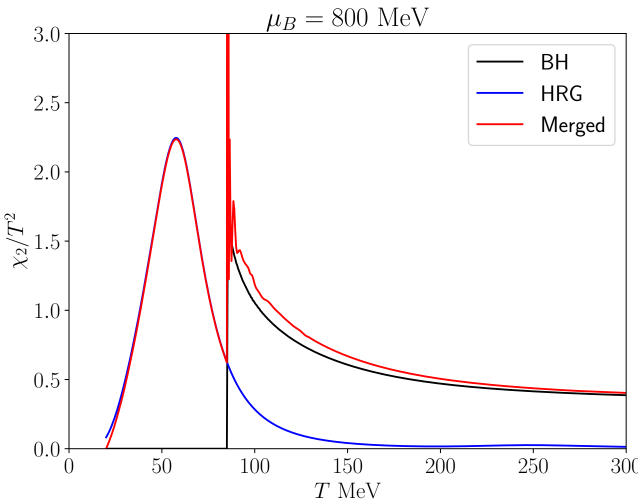




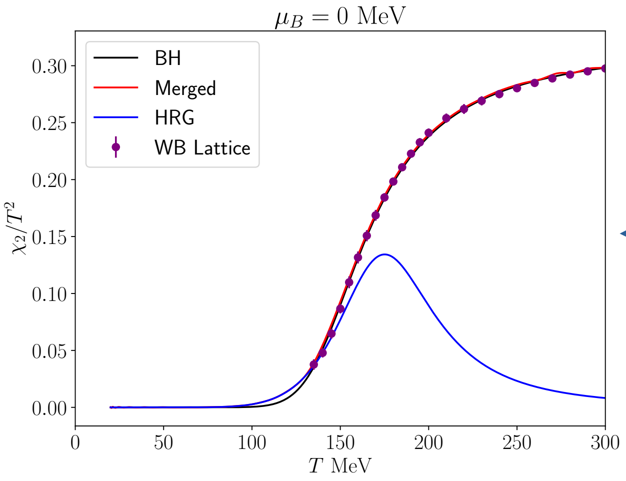
Crossover



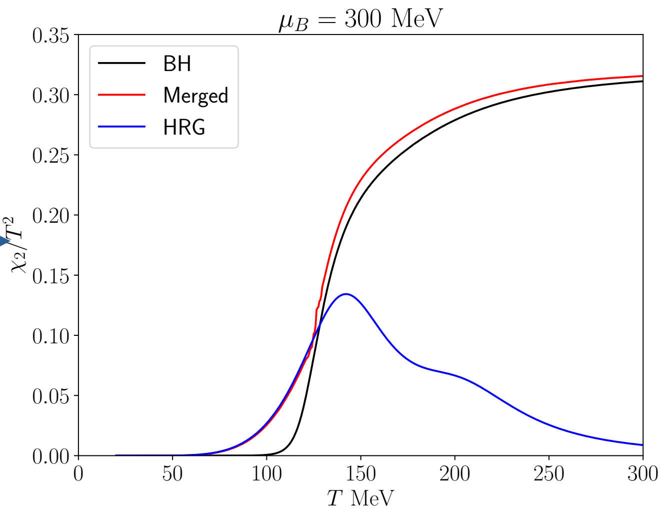
Near the critical point

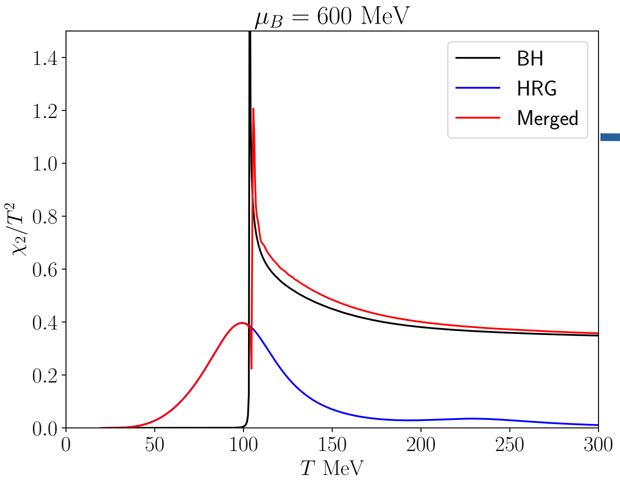


Results for susceptibility



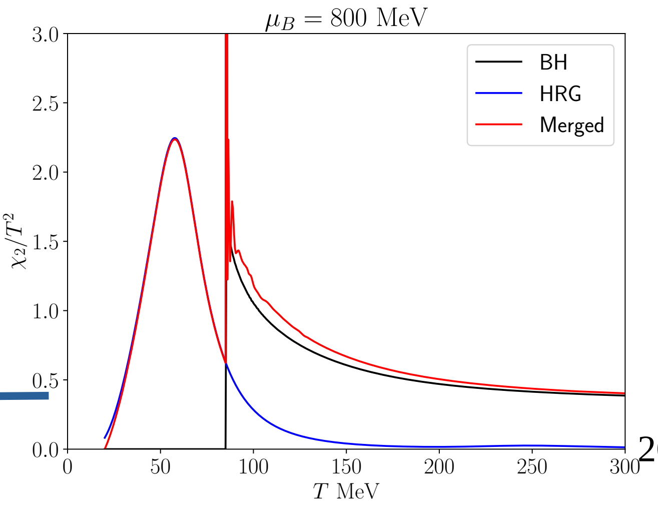
Crossover





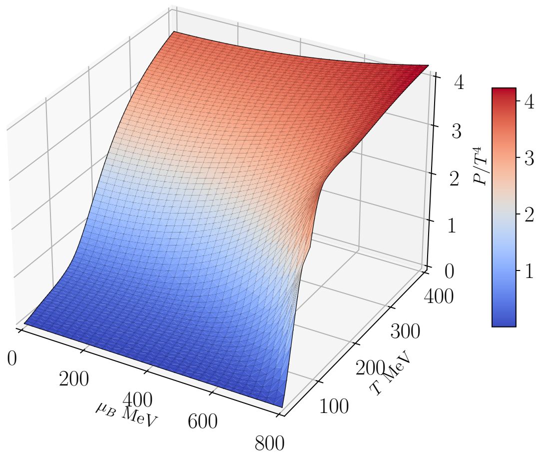
Near the critical point

First-order

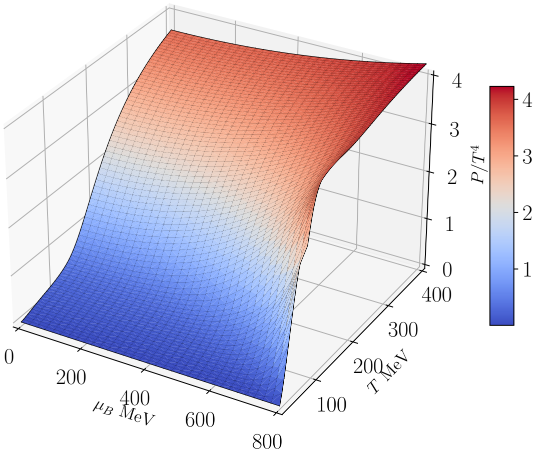


3d plots

3d plots



3d plots



0
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P/T^4
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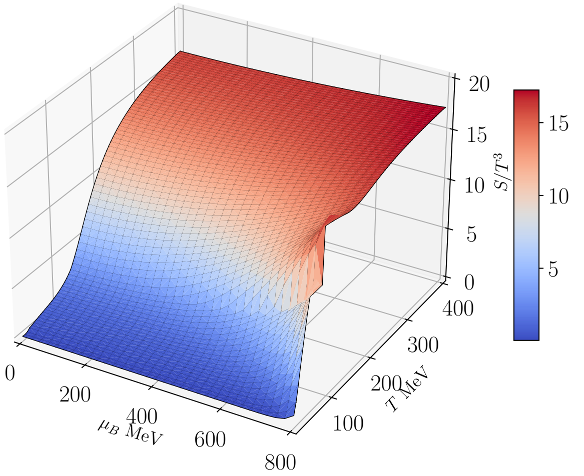

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μ_B MeV
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20
S/T^3
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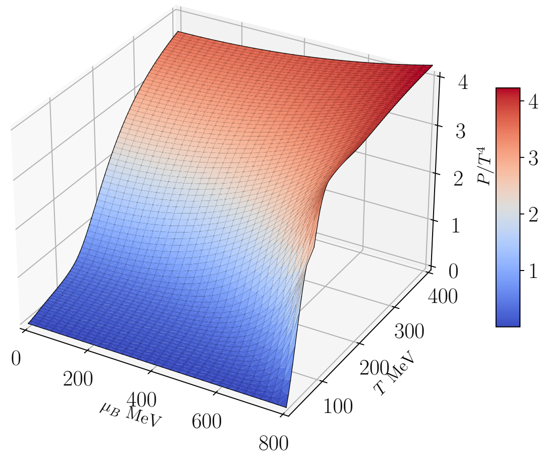
15

3d plots



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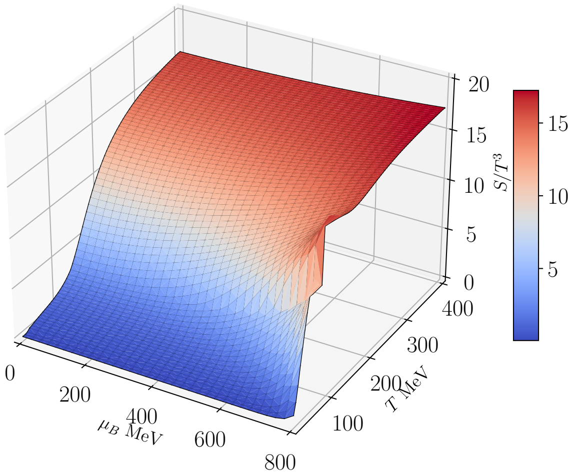

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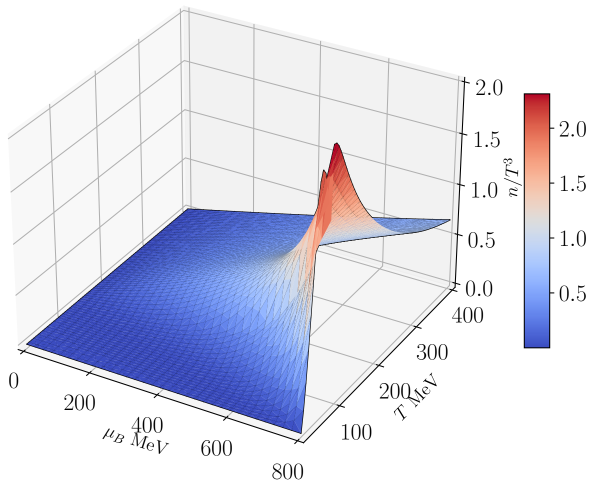
S/T^3



0
200
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μ_B MeV
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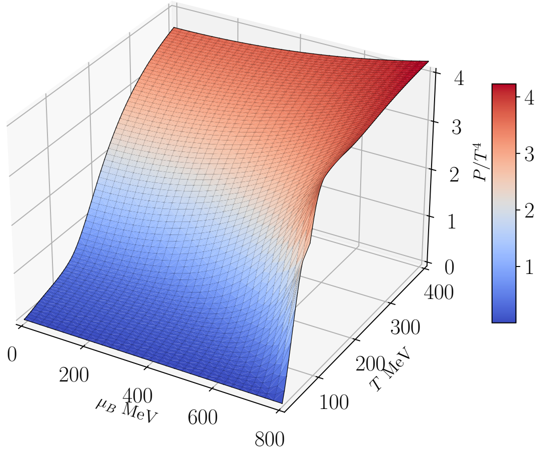
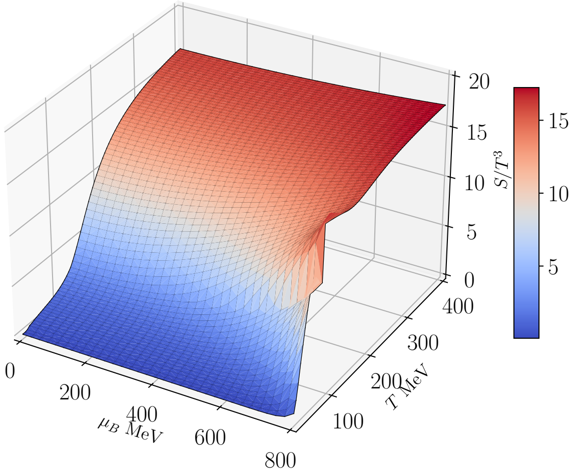
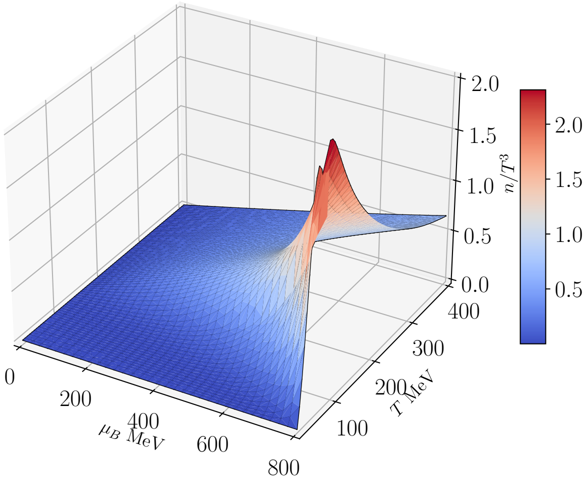
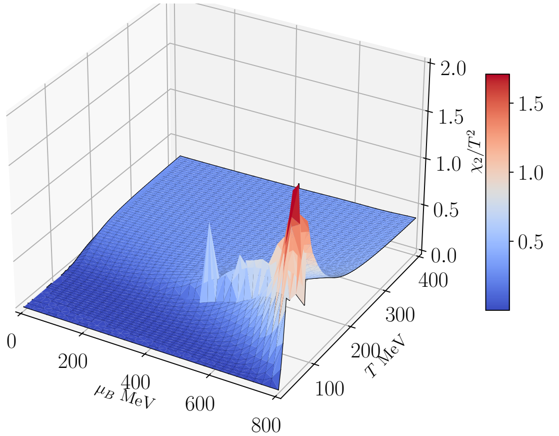
400
T MeV
0.0
0.5
1.0
1.5
2.0
n/T^3

3d plots


P/T^4
μ_B MeV
T MeV


S/T^3
μ_B MeV
T MeV


n/T^3
μ_B MeV
T MeV


χ_2/T^2
μ_B MeV
T MeV

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- We are working on obtaining smoother results for higher-order derivatives.

Thank you!!

Back up

Switching function 1(Kapusta)

$$P(T,\mu) = P_{BG}(T,\mu)R(T,\mu)$$

$$P_{BG} = S(T,\mu)P_{qg} + [1 - S(T,\mu)]P_h(T,\mu)$$

$$\boxed{S(T,\mu) = \exp\left[-\left(T^2/T_s^2 + \mu^2/\mu_s^2\right)^{-2}\right]}$$

$$Q_\pm = \left\{\left[(\Delta^2(T))^2 + r^2(T,\mu)\right]^{1/2} \pm r(T,\mu)\right\}^k$$

where

$$r(T,\mu) = \frac{\mu^m - \mu_x^m(T)}{\mu^m + \mu_x^m(T)}$$

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When $T > T_c$

$$R(T,\mu) = 1 - a(T)\left(\sqrt{\Delta^4 + 1} + 1\right)^k$$

$$- a(T)\left(\sqrt{\Delta^4 + 1} - 1\right)^k + a(T)(Q_+ + Q_-)$$

$T > T_c$ and $\mu \geq \mu_x$

$$R_Q = 1 + a(T)Q_+(T,\mu) - a(T)\left(\sqrt{\Delta^4 + 1} + 1\right)^k$$

When $T < T_c$ and $\mu \leq \mu_x$

$$R_H = 1 + a(T)Q_-(T,\mu) - a(T)\left(\sqrt{\Delta^4 + 1} + 1\right)^k$$