Recent results on hadron spectroscopy from the lattice

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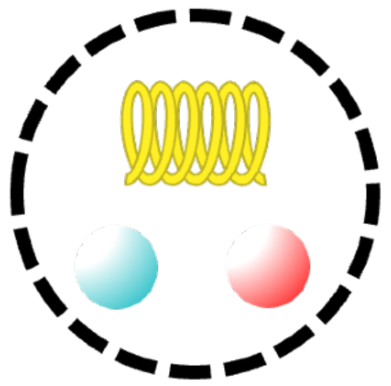
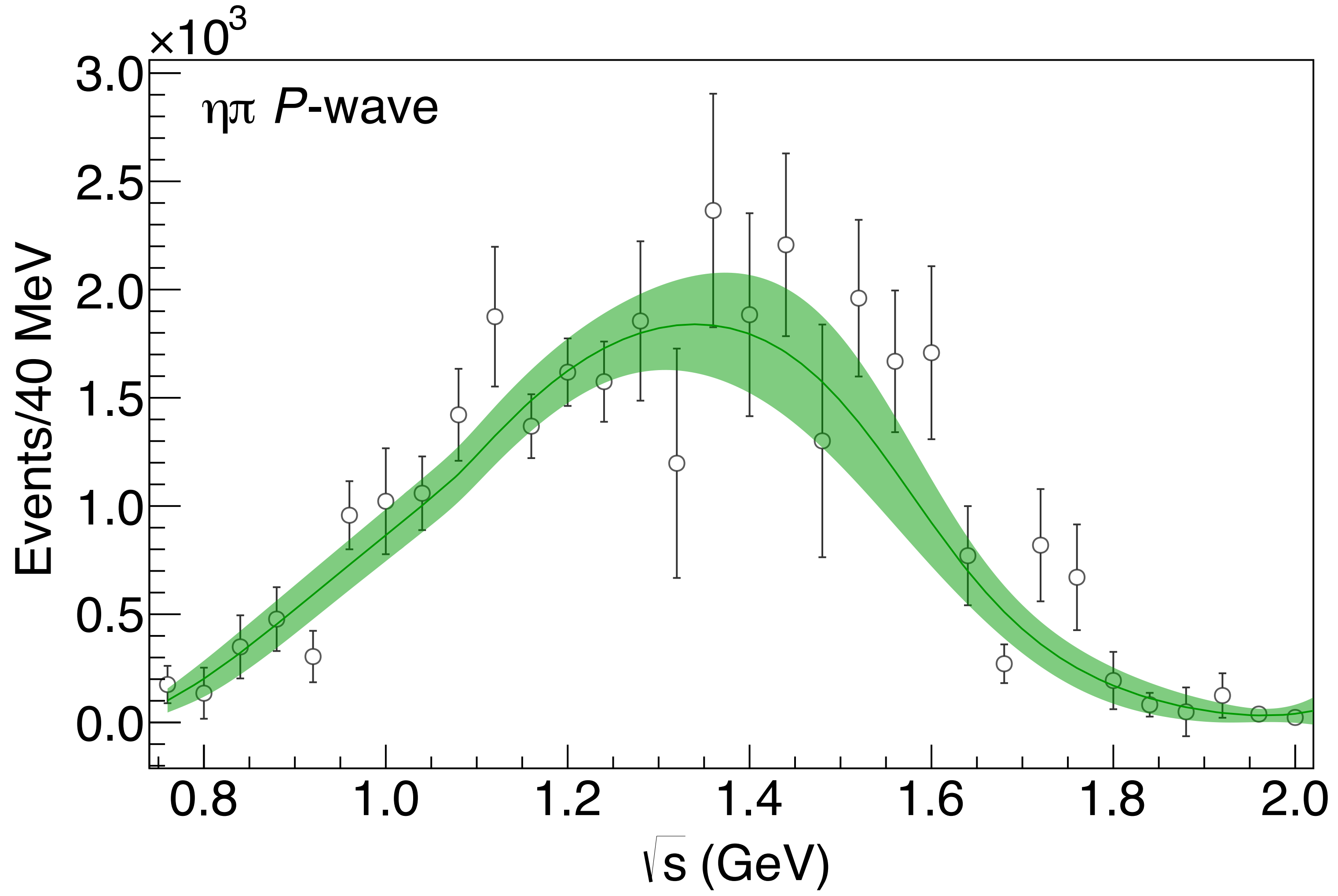
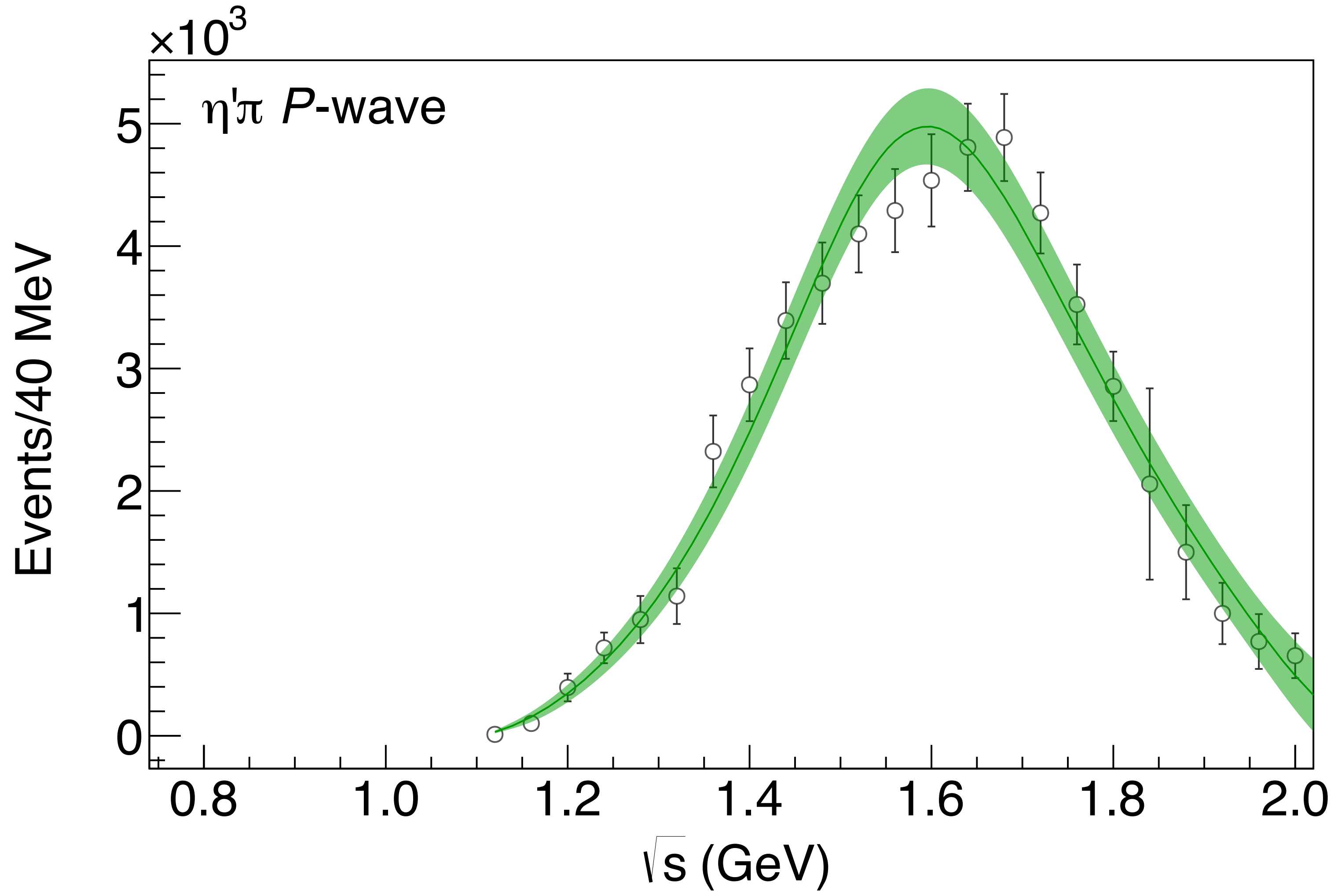
Hadron spectroscopy

π_1

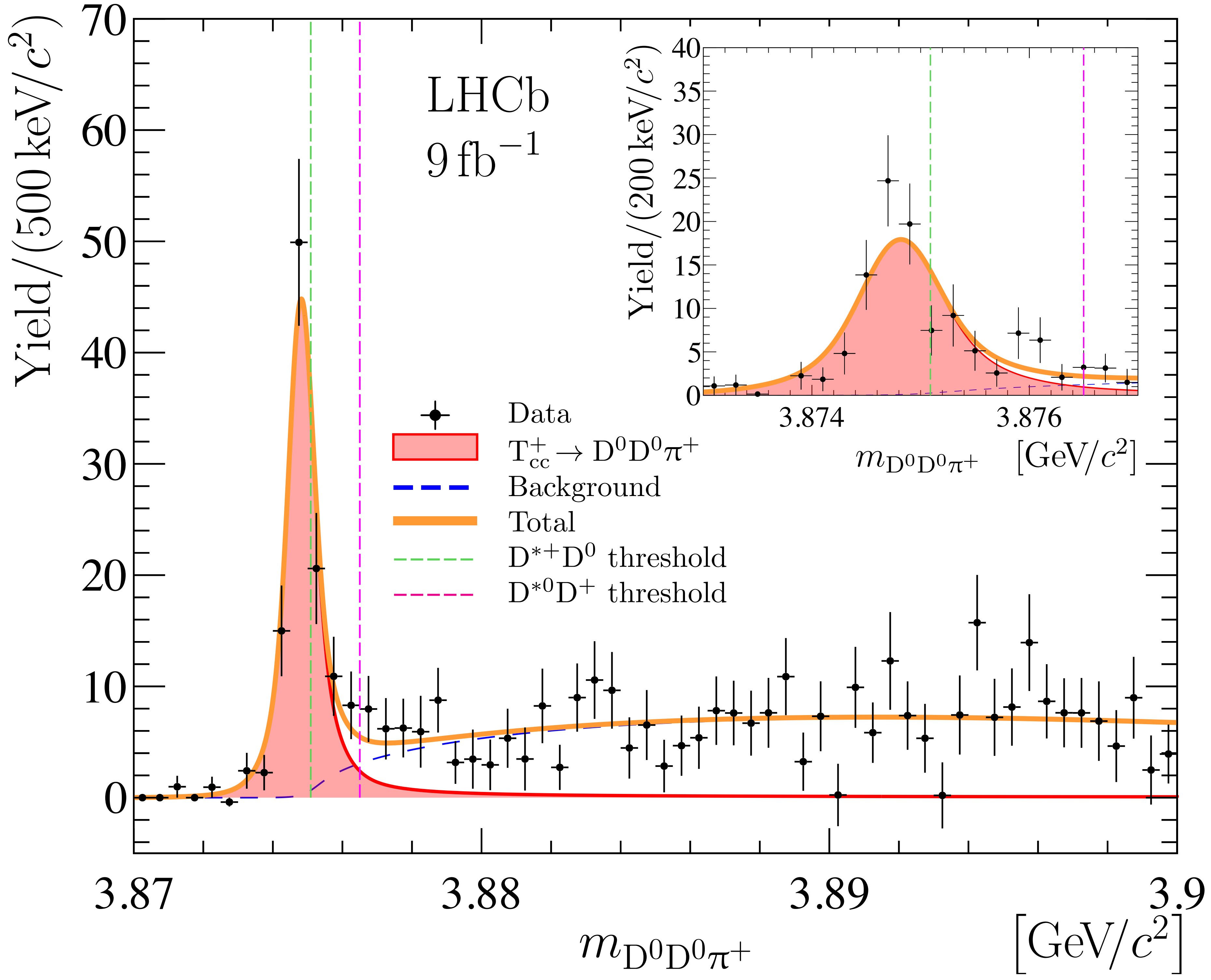






[(JPAC) 1810.04171]

T_{cc}^{+}



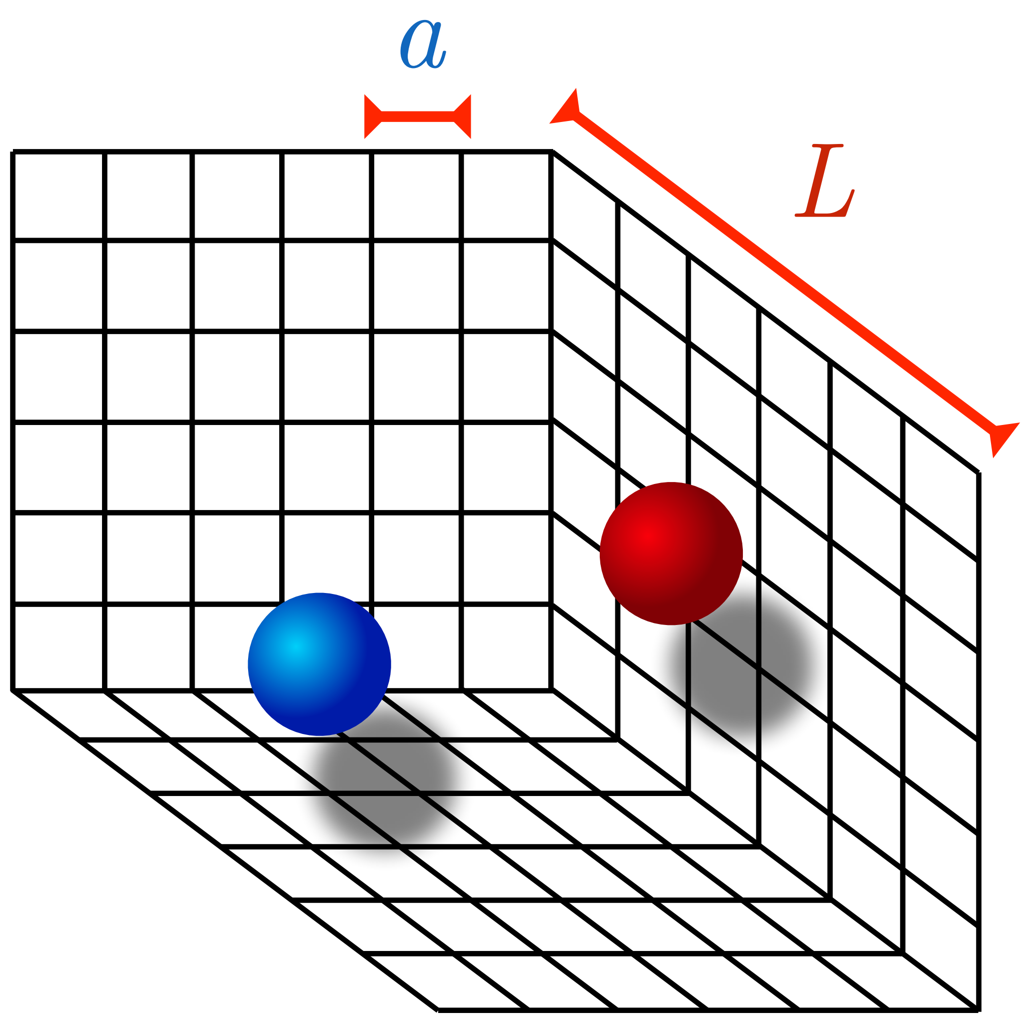
[(LHCb) 2109.01038]

Non-perturbative dynamics: Lattice QCD

- QCD Path Integral
- Finite Lattice spacing
- Finite Volume
- Wick rotated: Euclidean spacetime
- (Almost) any quark masses

  eg. $SU(3)$ point: $m_u = m_d = m_s$

$$\langle O_1 O_2 \ldots \rangle = \frac{\int \mathcal{D}[\psi\bar{\psi}G] e^{-S_{\rm QCD}} O_1 O_2 \ldots}{\int \mathcal{D}[\psi\bar{\psi}G] e^{-S_{\rm QCD}}}$$



- Discrete FV Spectrum:

$$\left\langle O_i(t) O_j^\dagger(0) \right\rangle = \sum_{n=0} Z_i^{n*} Z_j^n e^{-E_n t}$$

- Local matrix elements:

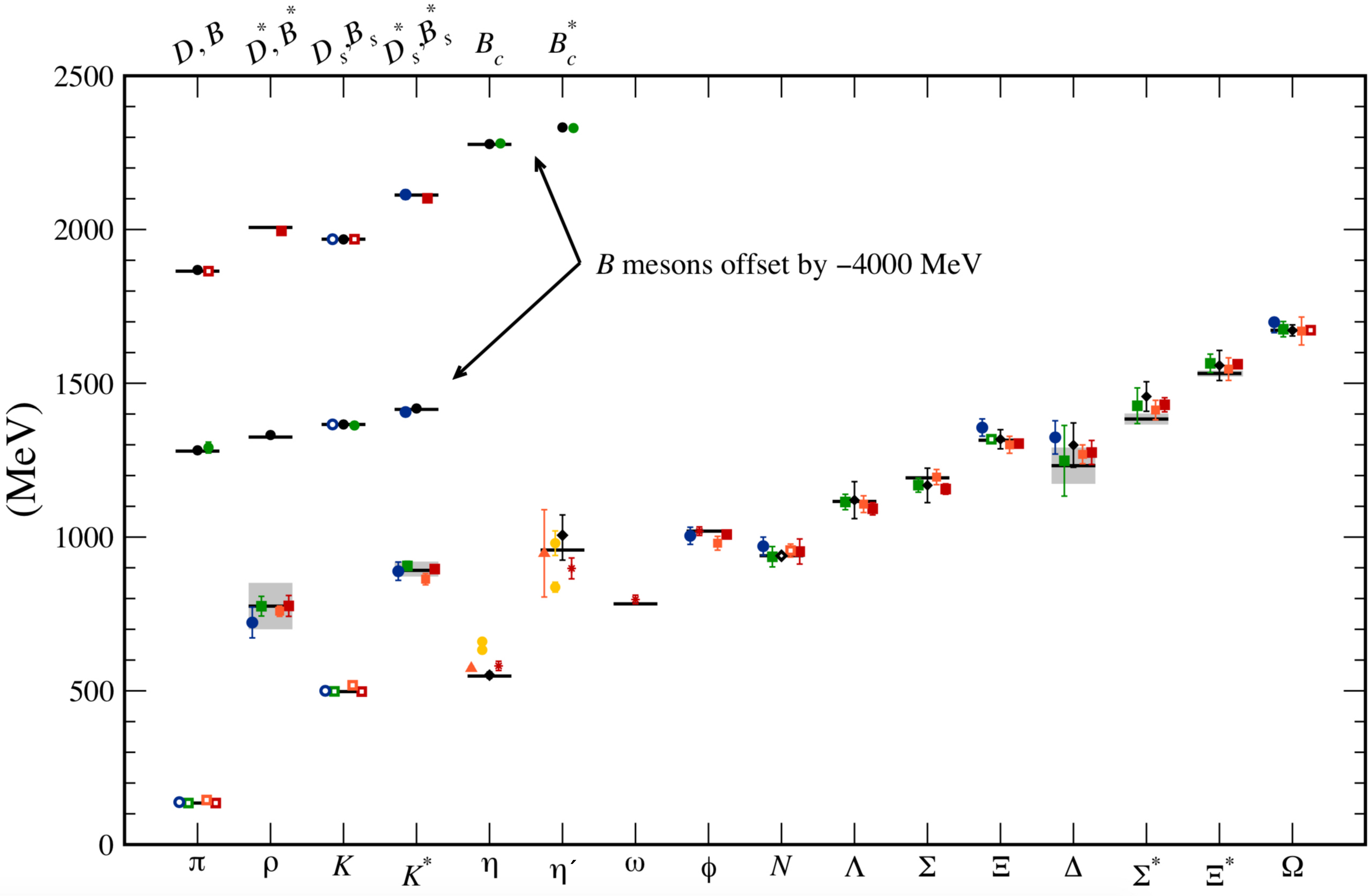
$$\left\langle O_i(\Delta t) \mathcal{J}^\mu(t) O_j^\dagger(0) \right\rangle = \sum_{n,m=0} \langle n | \mathcal{J}^\mu(0) | m \rangle \, Z_j^m Z_i^{n*} e^{-E_n(\Delta t - t)} e^{-E_m t}$$

QCD spectrum at physical masses

Comparison:

LQCD (symbols)

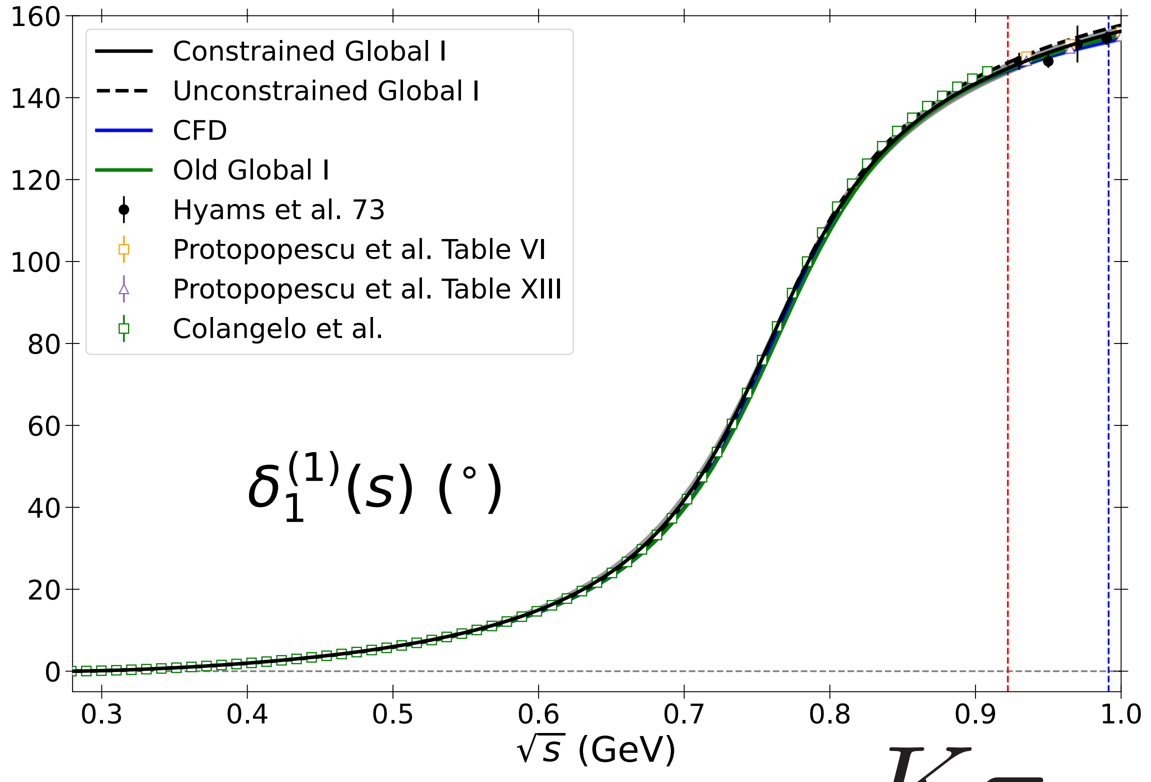
Experiment (lines)



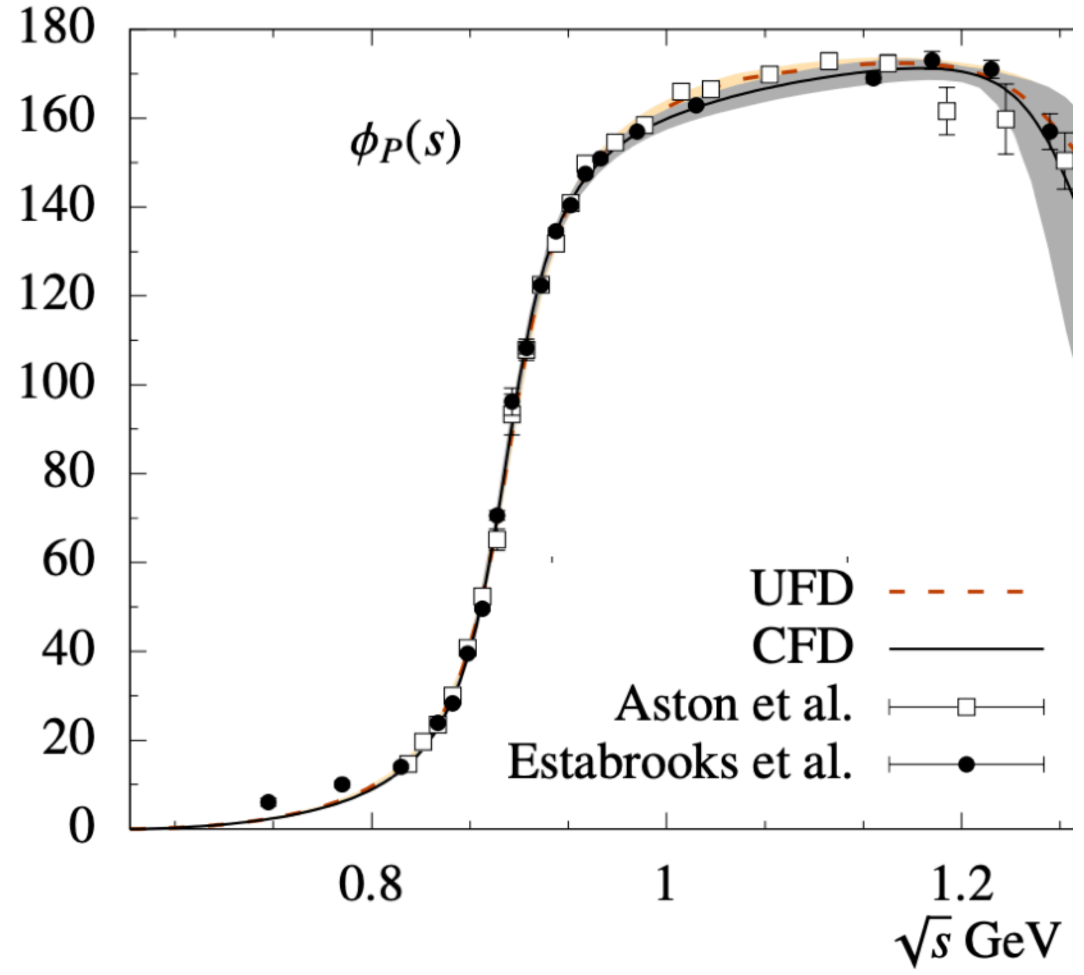
QCD spectrum at physical masses

Experimental data:

$\pi\pi \to \rho \to \pi\pi$



$K\pi \to K^\star \to K\pi$

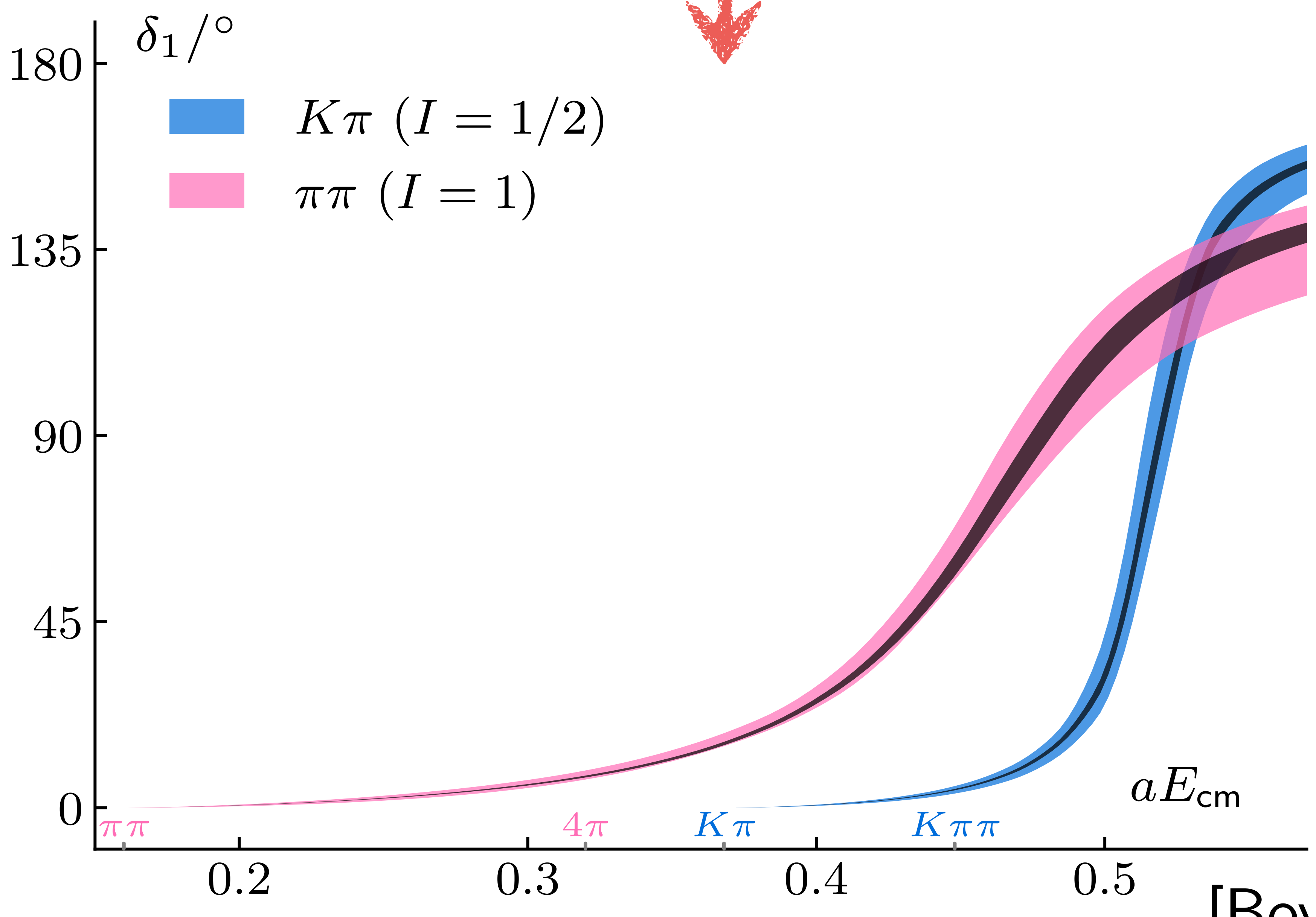


[Peláez et al. 2412.15327]

[Peláez, Rodas 2010.11222]

$$\mathcal{L} = -\frac{1}{4}F^2 + \bar{\psi}(i\not{D} - m)\psi$$

Lattice QCD



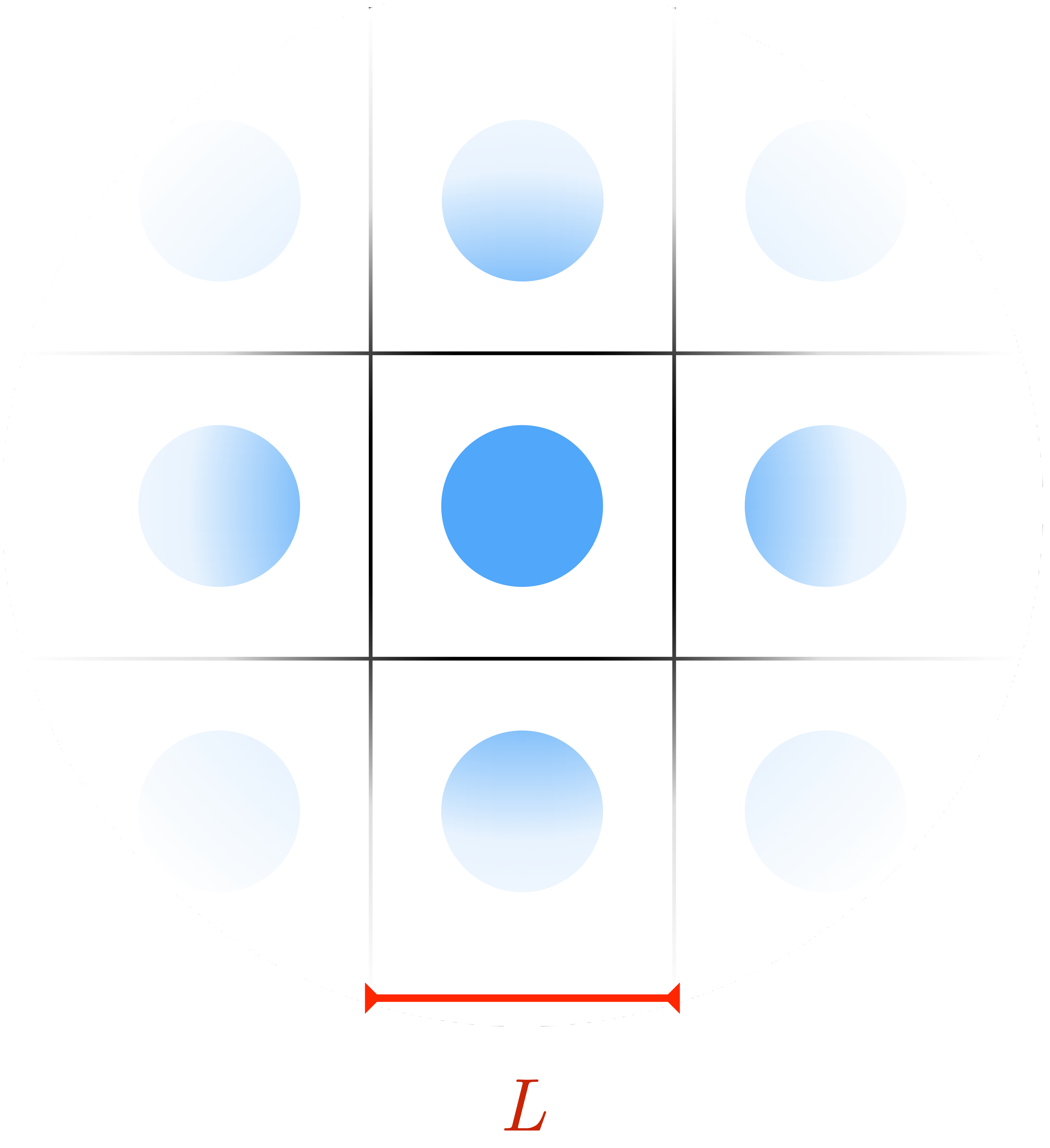
[Boyle et al. 2406.19193]

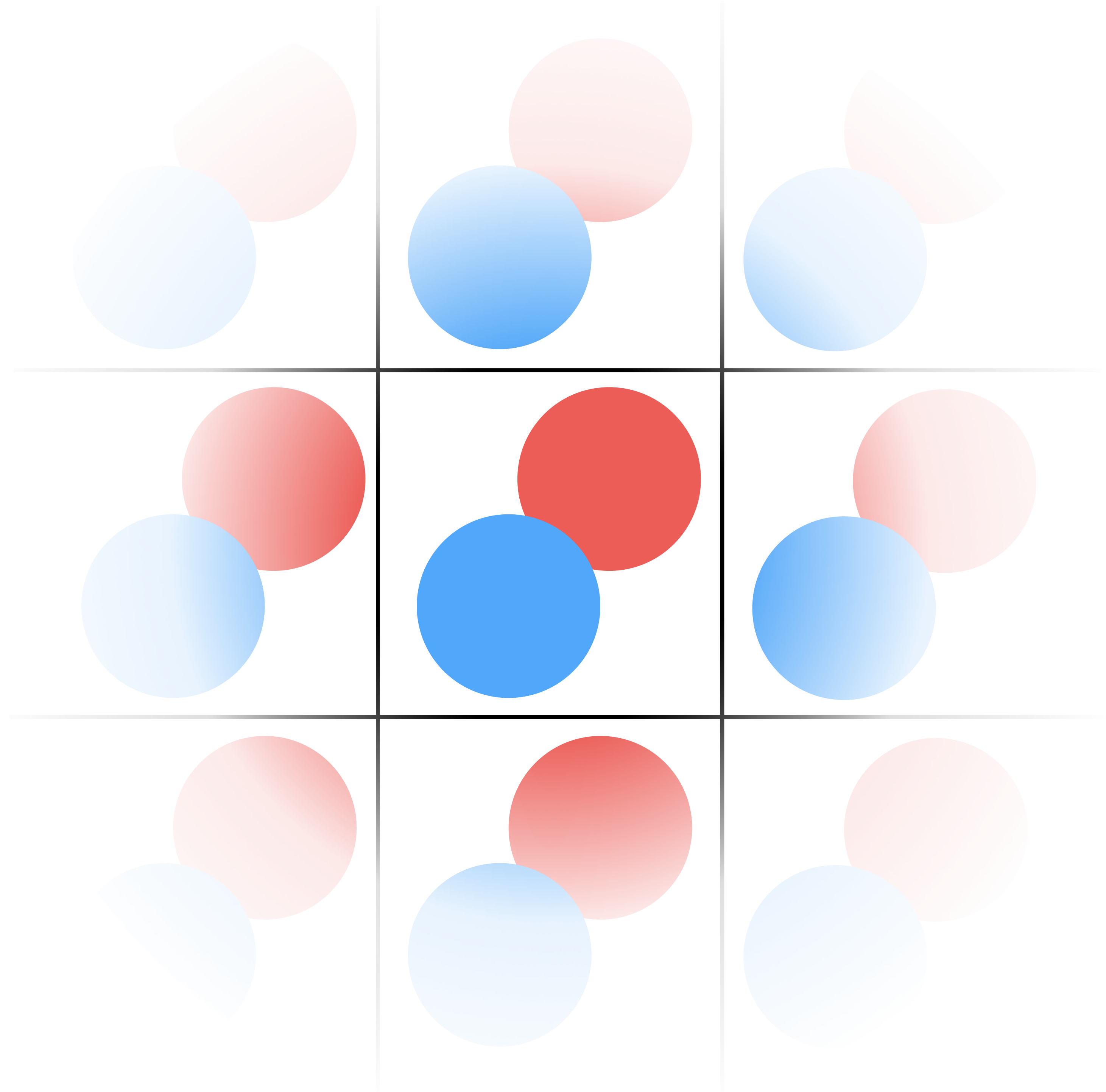
Good agreement with experimental values!

Spectrum in a finite volume torus

$$\langle \pi, L|\hat{H}|\pi, L\rangle = m_\pi + \mathcal{O}(e^{-m_\pi L})$$

$$\langle n, L|\hat{H}|n, L\rangle = E_n(L) \gtrsim 2m_\pi$$





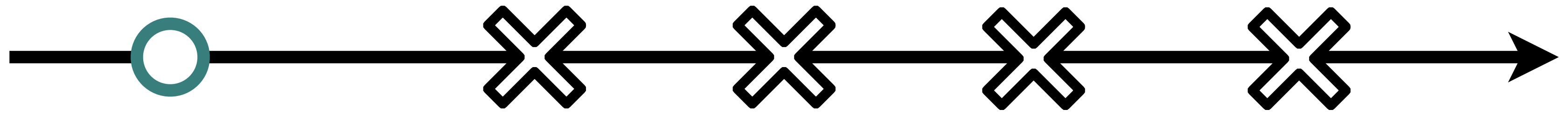
Strong interaction for $r \gg 1 \sim e^{-m_\pi r}$

Is the FV spectrum related to scattering amplitudes?

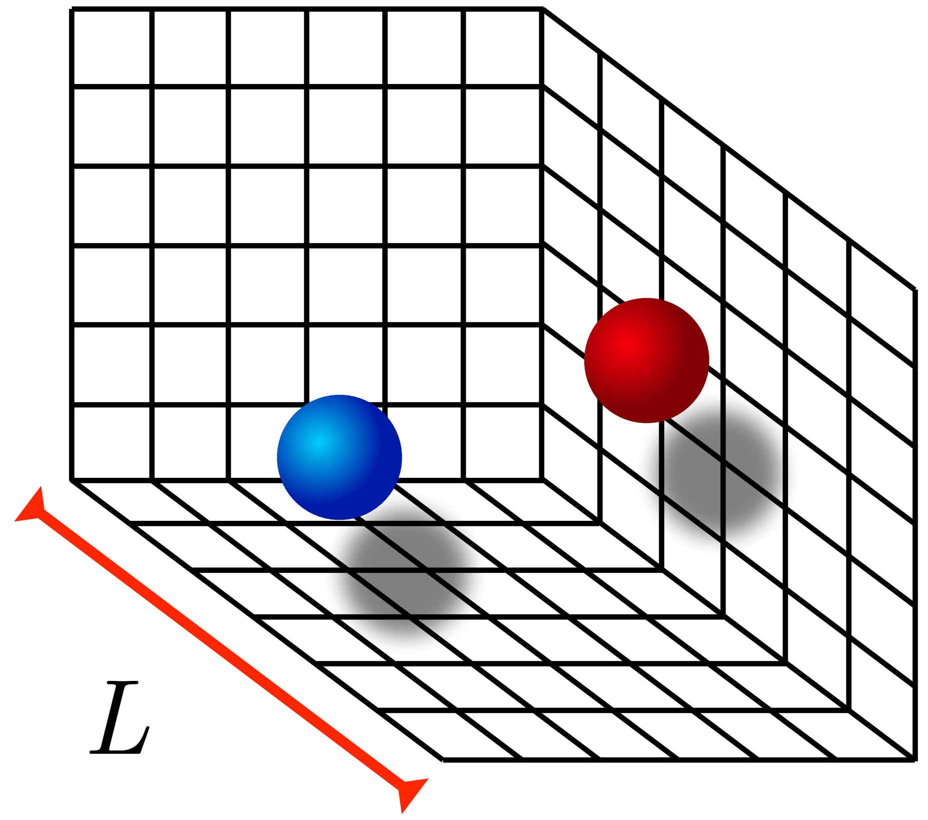
Lüscher formalism

$$\left\langle O_i(t) O_j^\dagger(0) \right\rangle = \sum_{n=0} Z_i^{n*} Z_j^n e^{-E_n t}$$

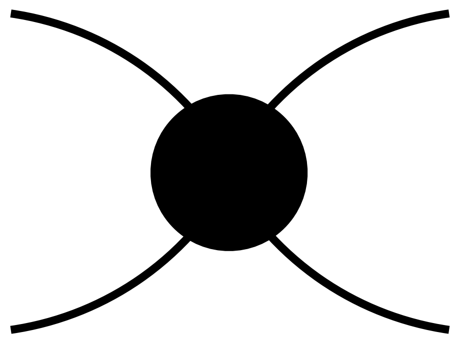
$C_L(P^2)$

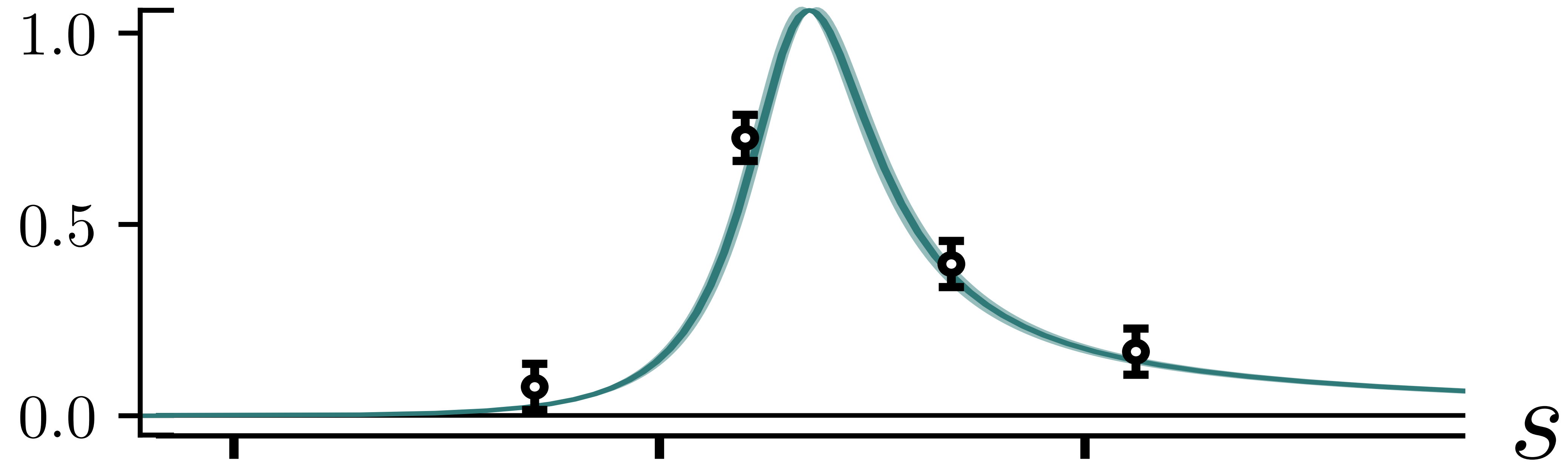


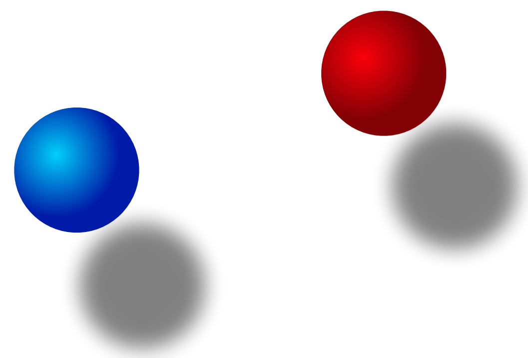
$E_n^{\star 2}(L)$



$$\det(F(s,L) + \mathcal{M}^{-1}(s)) = 0 \quad + \mathcal{O}(e^{-m_\pi L})$$

$\mathcal{M}(s) =$ 





Alternative approach with spatial wf
[Ishii et al. (HALQCD) 1203.3642]

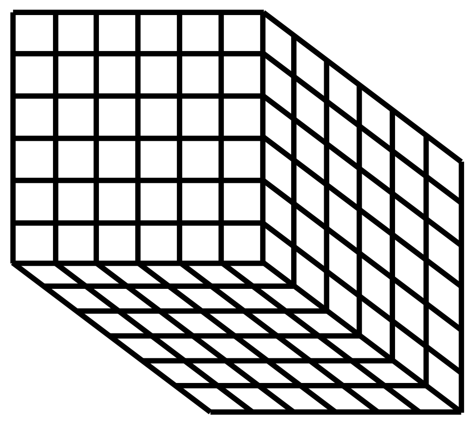
Finite volume spectrum extraction

$$C_{ij}(t) \equiv \left\langle O_i(t) O_j^\dagger(0) \right\rangle = \sum_{n=0} Z_i^{n*} Z_j^n e^{-E_n t}$$

Optimize computation, e.g. *distillation*.
[Peardon et al. (HadSpec) 0905.2160]

Comprehensive set of operators:

- $q\bar{q}$ *like* (single-meson) operators ρ ρ_2 b_1 ...
- Two-meson *like* operators $\pi\pi$ $K\overline{K}$



Mixes J^P into *cubic irreps*

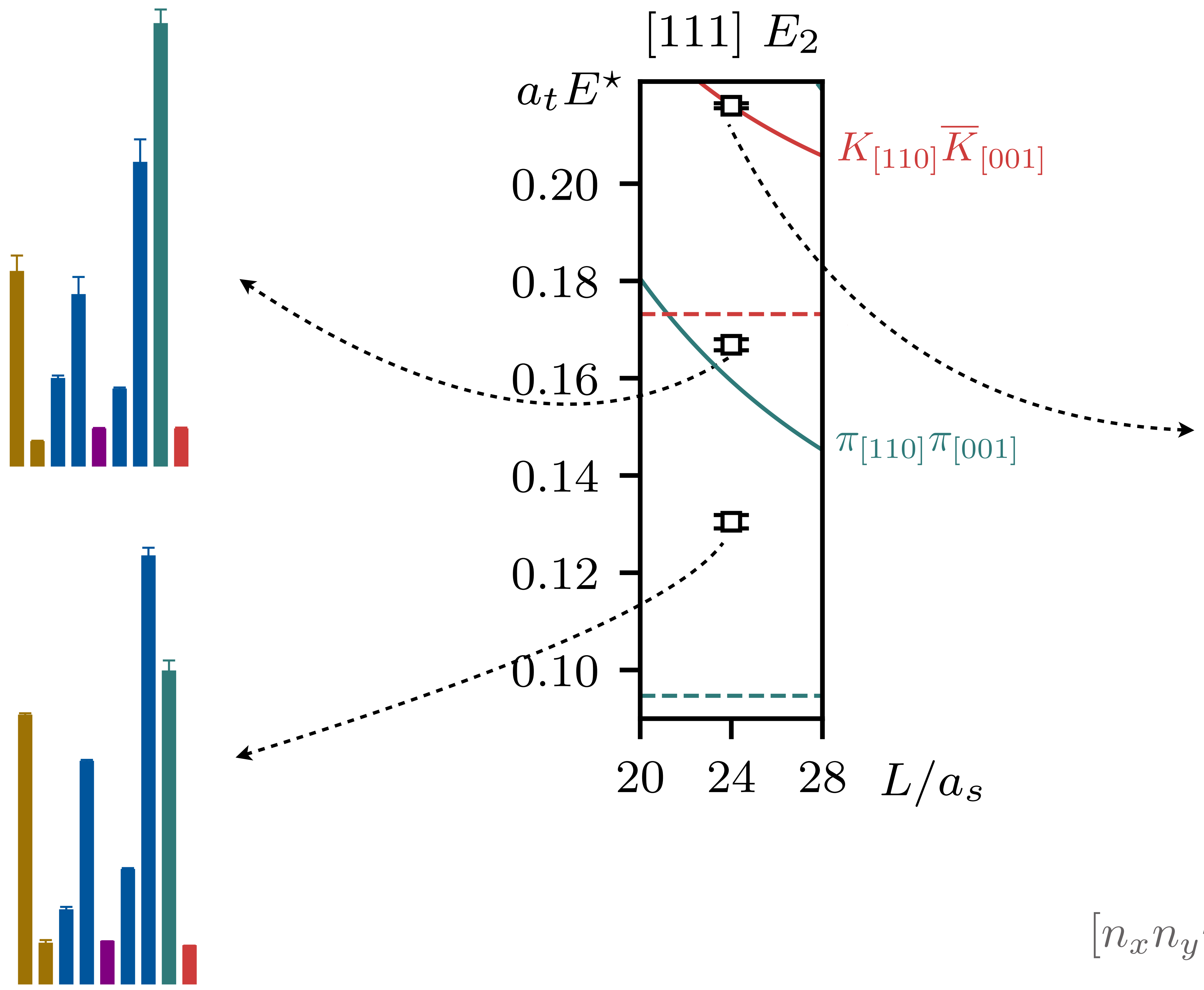
1. Generalized Eigenvalue Problem (GEVP)

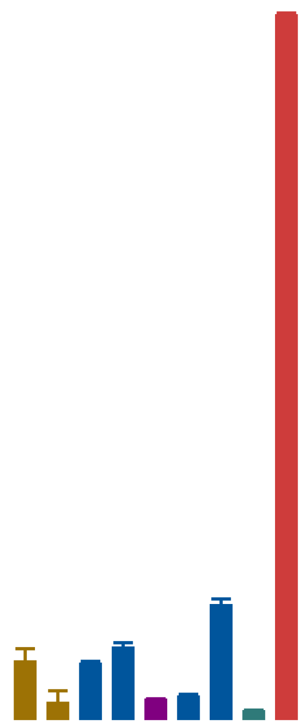
$$C_{ij}(t) v_j^n = C_{ij}(t_0) v_j^n \lambda_n(t - t_0)$$

2. Fit eigenvalues $\lambda_n(t - t_0) = e^{-E_n(t-t_0)}$

3. (Optional) Overlaps Z from eigenvectors

$$Z_j^n = e^{E_n t_0/2} \sum_i v_i^n C_{ij}(t_0)$$



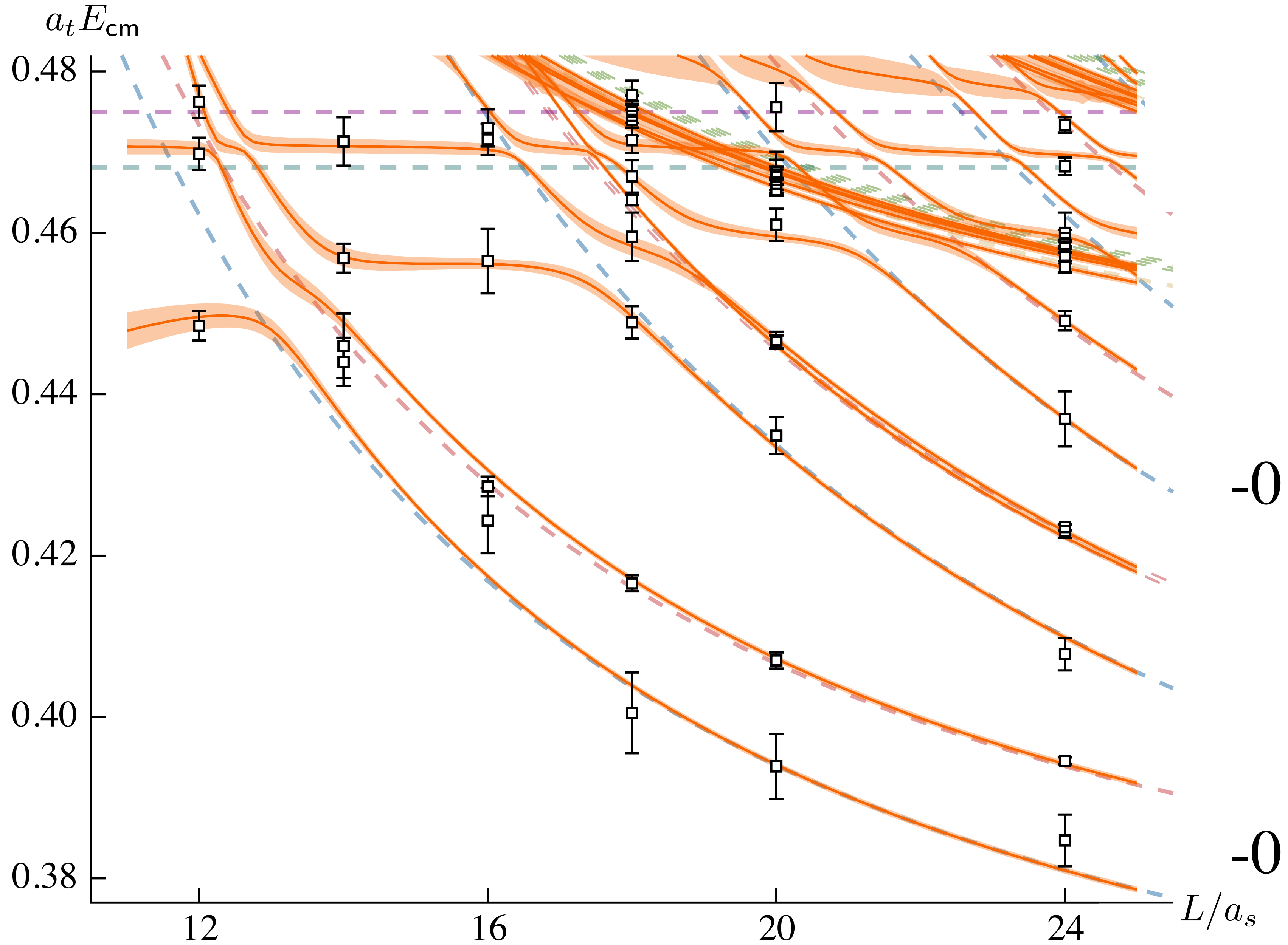


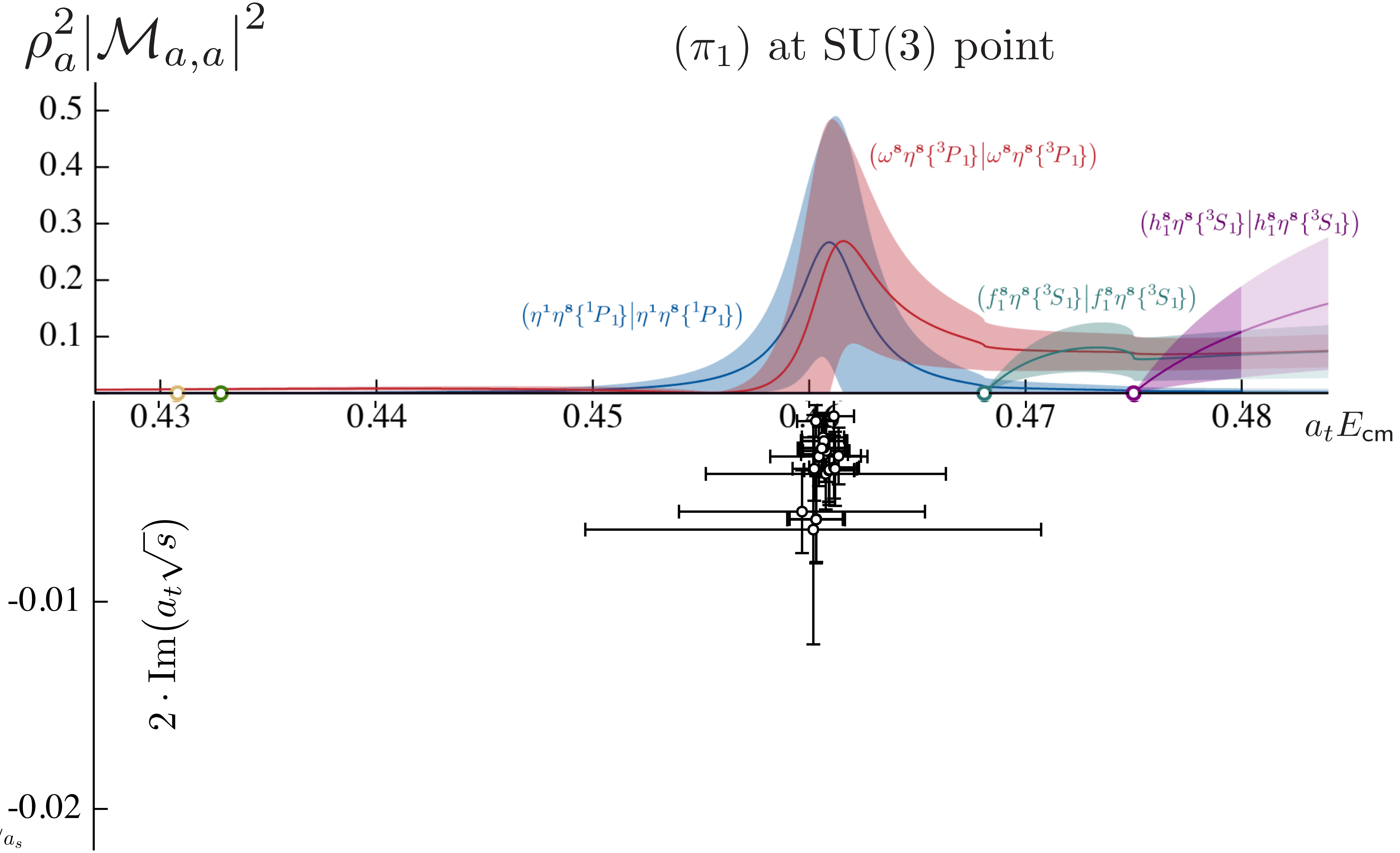
$$[n_x n_y n_z] = \frac{L}{2\pi}\vec{P}$$

Lowest lying hybrid meson

	m/MeV
π	~ 700
K	~ 700

- Meson-meson and qqbar
- 6 volumes [$12a_s$,…,$24a_s$]
- Spectrum for P=[000]
- 53 energy levels

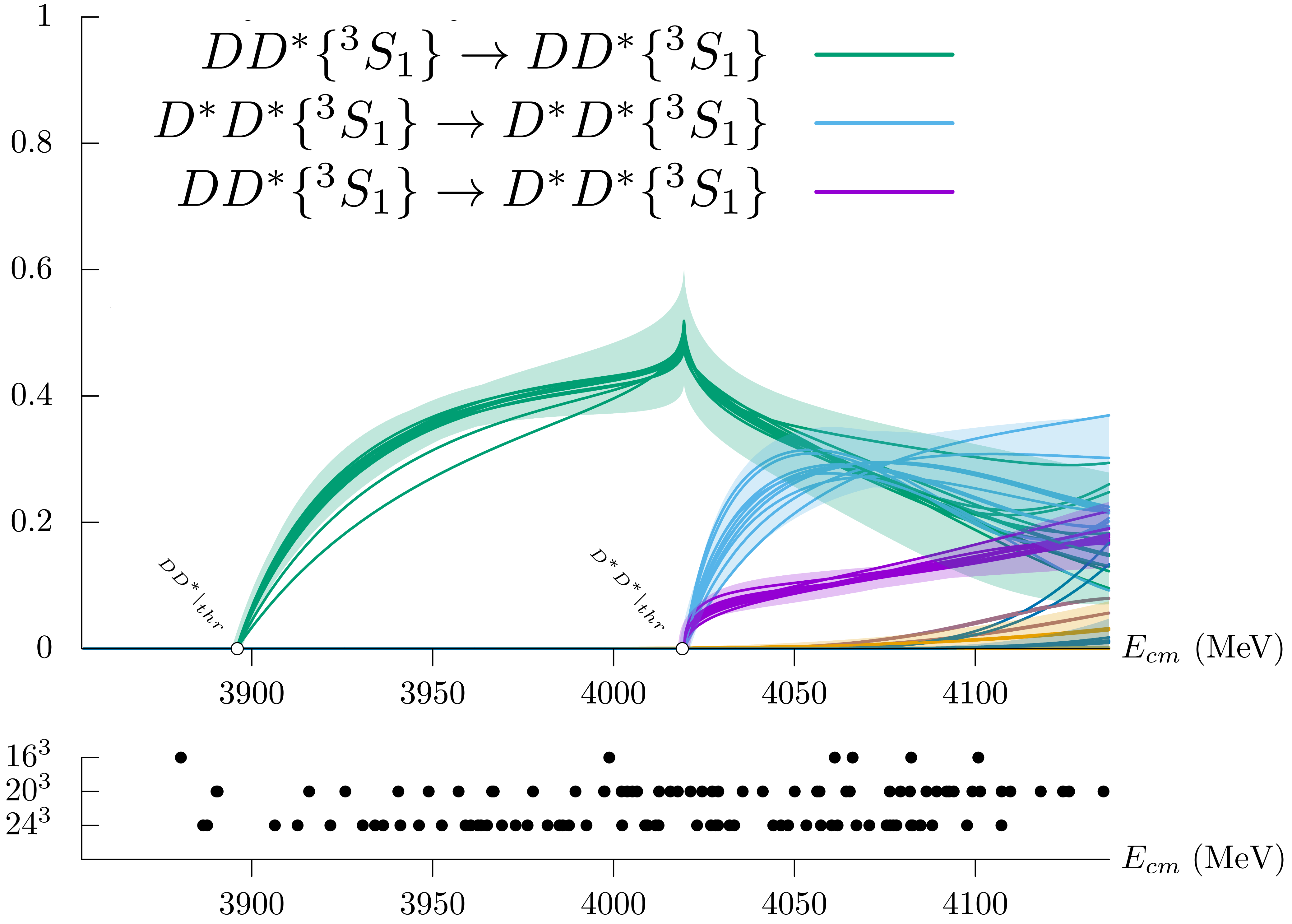




Double charmed tetraquark $T_{cc}\,(I=0)$

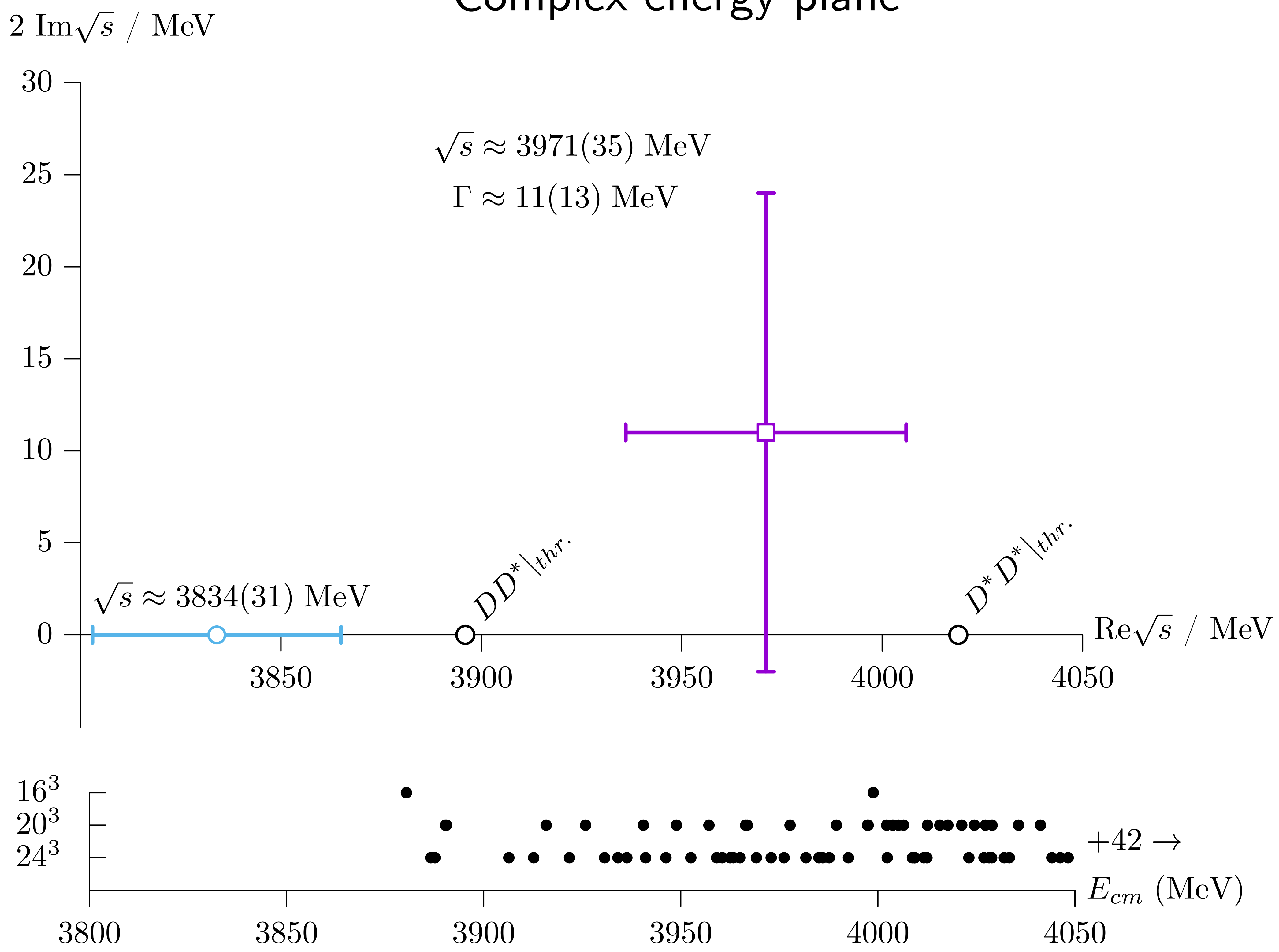
	m/MeV
π	390
D	1,886
D^*	2,009

$$\rho_a \rho_b \left| \mathcal{M}_{a,{}^{2S+1}\ell_J;b,{}^{2S'+1}\ell'_J} \right|^2$$



Pole singularities

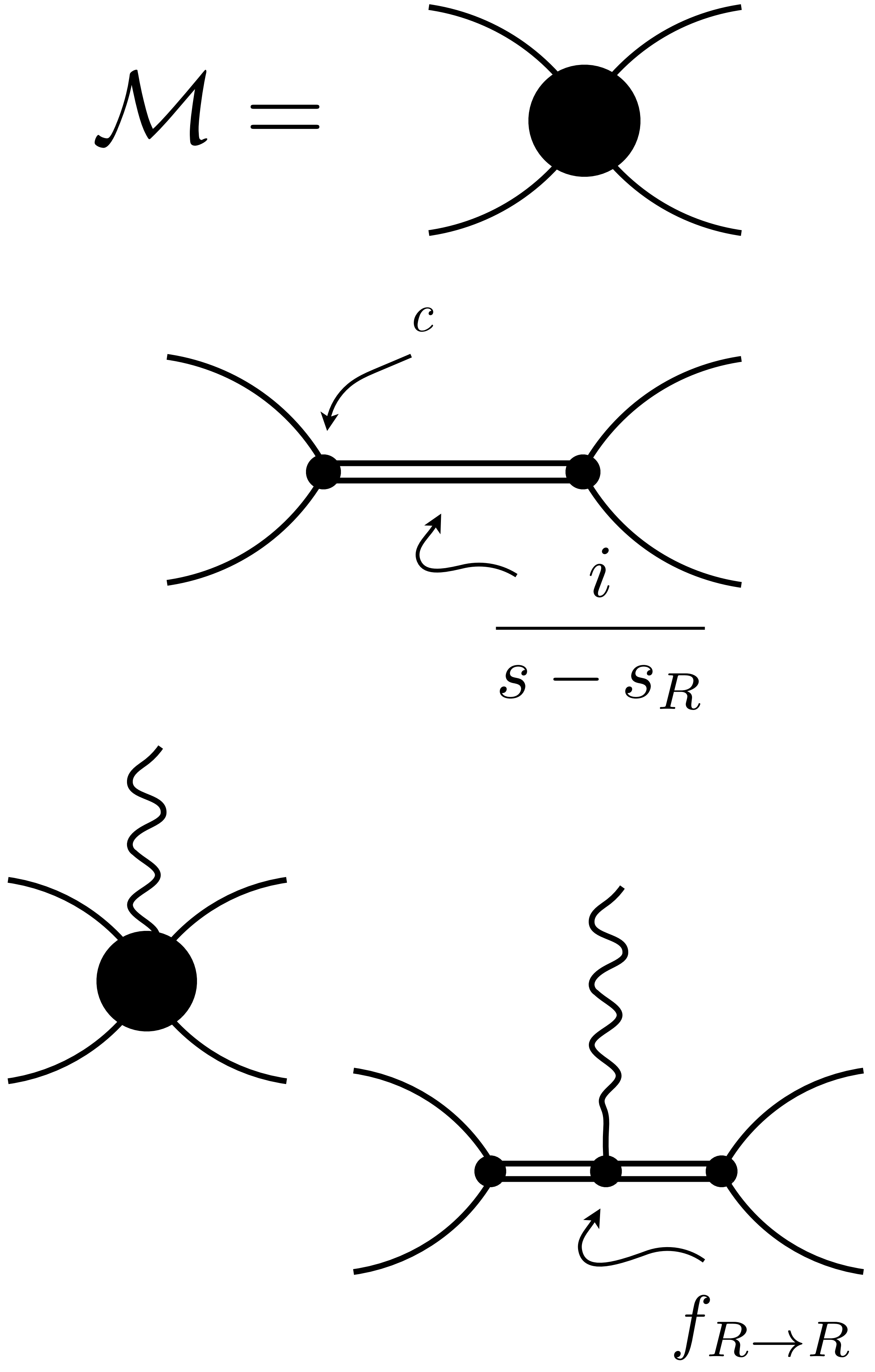
Complex energy plane



109 energy levels across 3 vols, 9 irreps

Summary and outlook

- Scattering observables can be accessed from LQCD:
  - Multiple two-meson channels.
  - Physical quark mass calculations.
  - Three-hadron scattering.
  - Inclusion of left-hand cut effects.
- Coupling of electroweak currents to resonances
  - Predictions of production rates, neutrino scattering, ...
  - Study of internal structure of resonances.



[Ortega-Gama et al., 1812.10504]

Extra slides

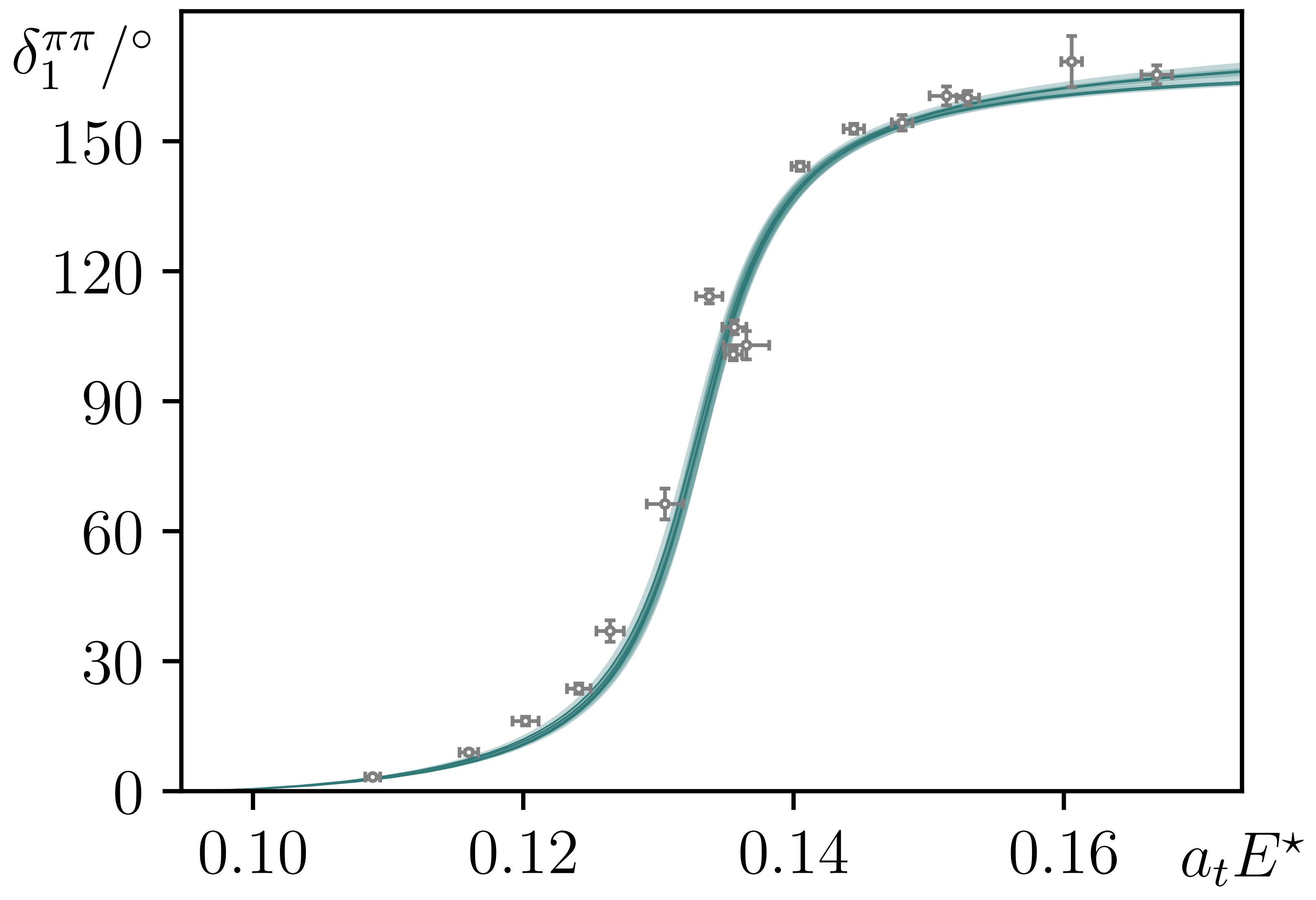
Elastic $\pi\pi$ P-wave

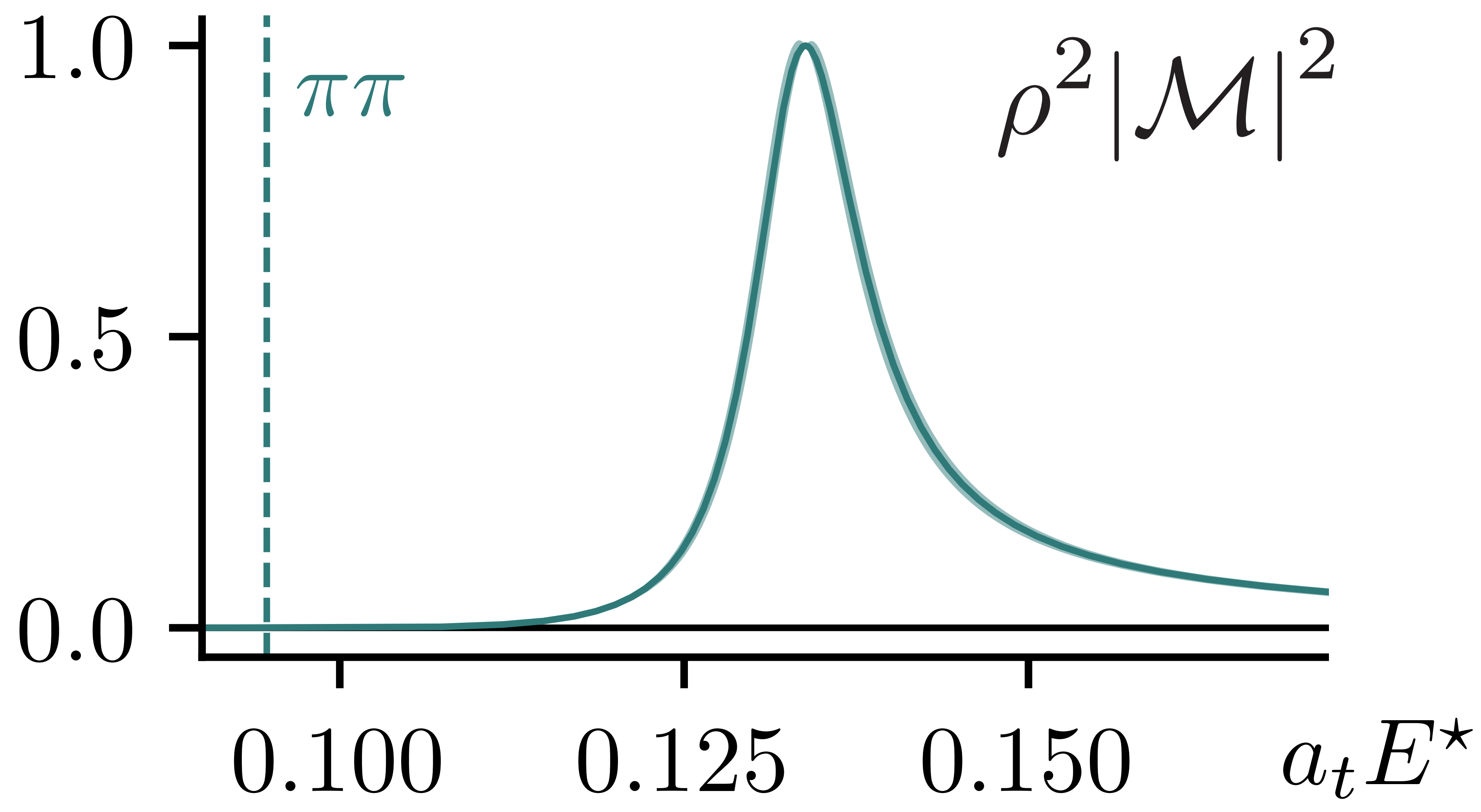
	m/MeV
π	284
K	519

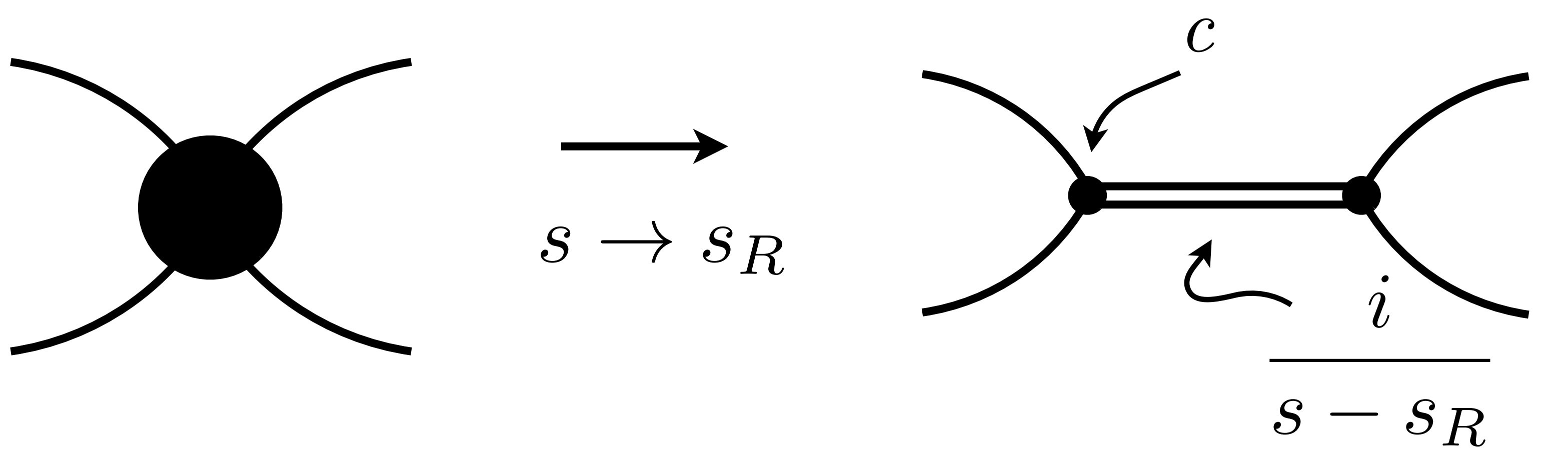
$|\mathcal{M}_{\ell>1}| \ll 1$

$$\mathcal{M}(s) = F^{-1}(s, L)$$

$$\mathcal{M} = \frac{1}{\rho}\sin(\delta_1^{\pi\pi})\, e^{i\delta_1^{\pi\pi}}$$



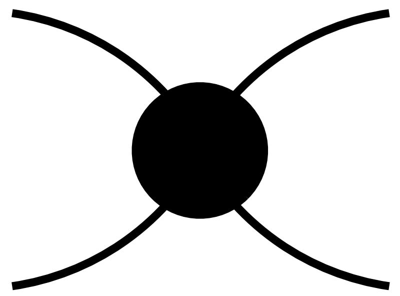




$$\rho(1^{--}): \quad \mathrm{Re}(\sqrt{s_R}) = 797 \pm 2.6\ \mathrm{MeV}$$

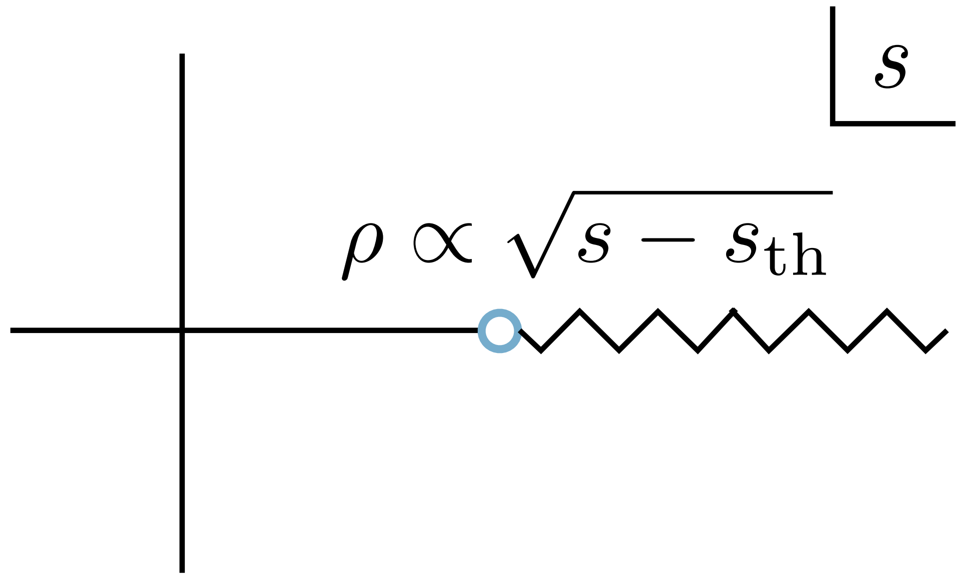
$$\mathrm{Im}(\sqrt{s_R})/2 = 28.5 \pm 1\ \mathrm{MeV}$$

Resonances in scattering amplitudes

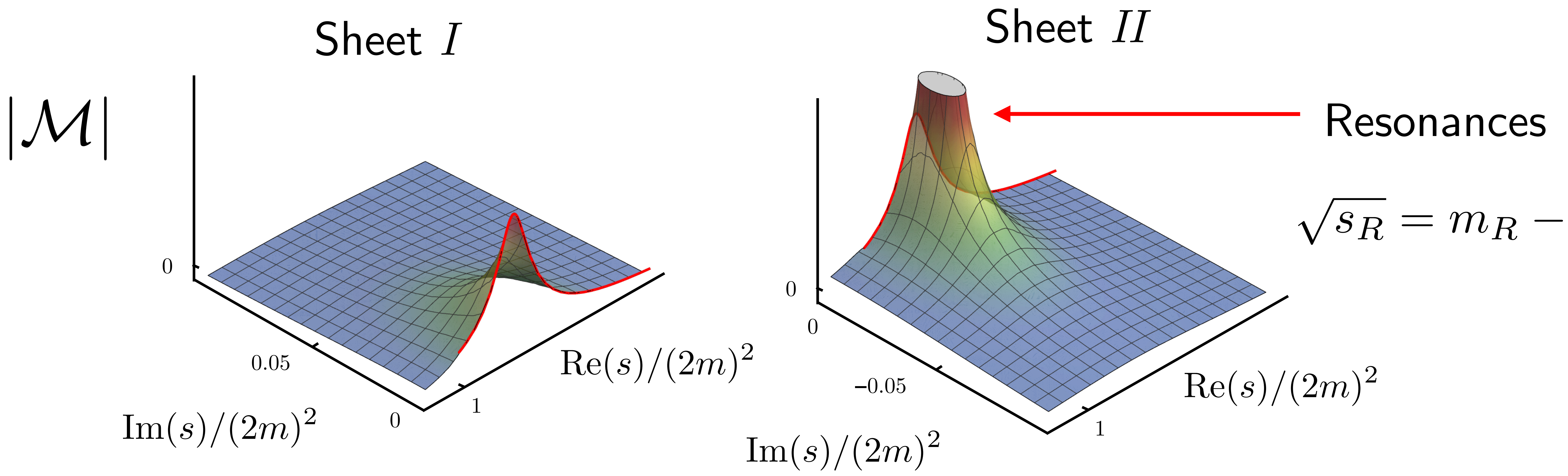
$\mathcal{M}(s) =$ 

s : Energy squared of the hadron pair

- Unitarity: $\text{Im}(\mathcal{M}^{-1}) = -\rho$ (two-particle phase space)



- Causality: analyticity in Sheet I



$$\sqrt{s_R} = m_R - i\frac{\Gamma_R}{2}$$