

Nuclear-Structure Corrections in Superaligned Beta Decay

Transition Densities with Coupled-Cluster Theory

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CIPANP, June 12, 2025



V_{ud} from Superallowed β -Decays

$$\frac{1}{ft} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} (1 + \Delta_R^V) (1 + \delta'_R) (1 - \delta_C + \delta_{NS}) \quad \frac{1}{\mathcal{F}t} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2}$$

V_{ud} from Superaligned β -Decays

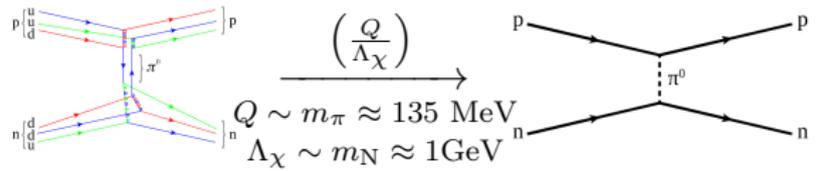
$$\frac{1}{ft} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} \left(\overset{\text{Single-Nucleon}}{1 + \Delta_R^V} \right) \left(\underset{\text{Outer-Correction}}{1 + \delta'_R} \right) \left(\underset{\text{Nucleus-Dependent}}{1 - \delta_C + \delta_{NS}} \right) \frac{1}{\mathcal{F}t} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2}$$

- **Single-Nucleon:** Short-distance electroweak loop effects, universal to all decays
- **Outer-Correction:** QED correction depending on electron energy and nuclear charge
- **Isospin:** Accounts for imperfect isospin symmetry in real nuclei
- **Nucleus-Dependent:** Radiative correction sensitive to nuclear internal structure

Chiral Effective Field Theory

1) Nuclear Interaction

Chiral Effective Field Theory:



NN

NNN

LO



NLO



NNLO



$$\underbrace{V_{1\pi} + V_{ct}^{(0)}}_{(Q/\Lambda_\chi)^0}$$

$$+ \underbrace{V_{2\pi}^{(2)} + V_{ct}^{(2)}}_{(Q/\Lambda_\chi)^2}$$

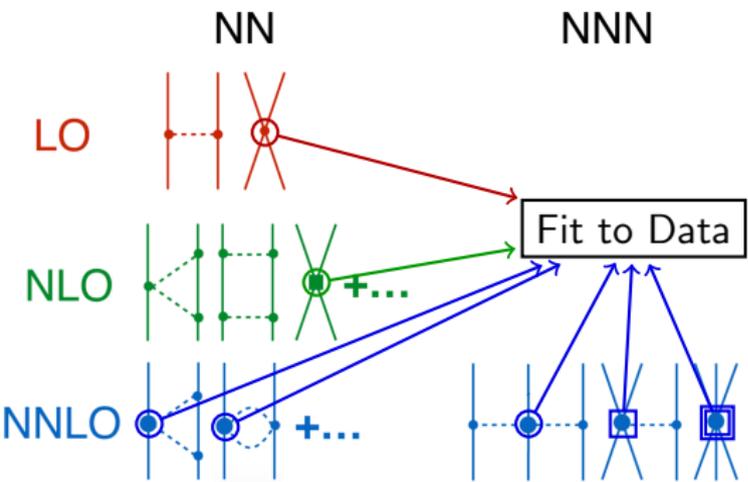
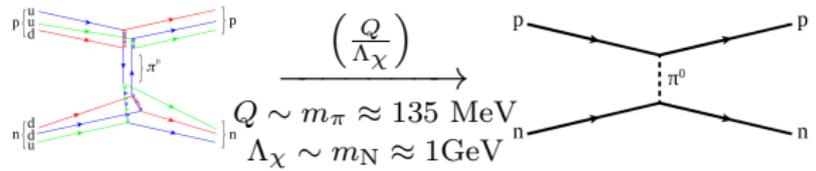
$$+ \underbrace{V_{2\pi}^{(3)} + V_{2\pi}^{3N} + V_{1\pi}^{3N} + V_{ct}^{3N}}_{(Q/\Lambda_\chi)^3} + \dots$$

W.G. Jiang, et al. 2020. Phys. Rev. C 102, 054301.

Chiral Effective Field Theory

1) Nuclear Interaction

Chiral Effective Field Theory:



$$\begin{aligned}
 & \underbrace{V_{1\pi} + V_{ct}^{(0)}}_{(Q/\Lambda_\chi)^0} \\
 & + \underbrace{V_{2\pi}^{(2)} + V_{ct}^{(2)}}_{(Q/\Lambda_\chi)^2} \\
 & + \underbrace{V_{2\pi}^{(3)} + V_{2\pi}^{3N} + V_{1\pi}^{3N} + V_{ct}^{3N}}_{(Q/\Lambda_\chi)^3} + \dots
 \end{aligned}$$

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Independent Particle Model

2) Many-body Quantum Mechanics

Many-Body Schrödinger Equation: $\hat{H}|\Psi\rangle = E|\Psi\rangle$

Many-Body Hamiltonian: $\hat{H} \equiv \hat{H}_0 + \hat{V}_{\text{NN}} + \hat{V}_{\text{NNN}} + \dots$

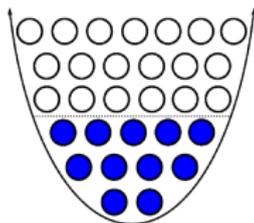
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Independent Particle Basis



$$\hat{H}_0 \phi_1(\mathbf{r}) = \varepsilon_0 \phi_1(\mathbf{r})$$

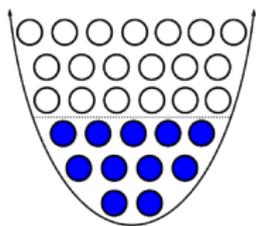
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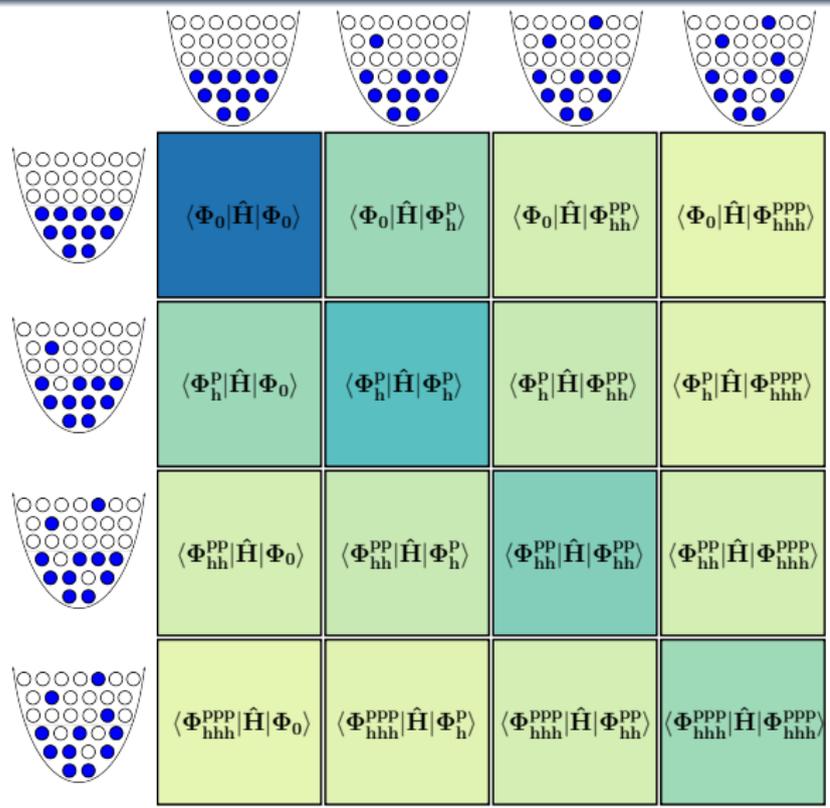
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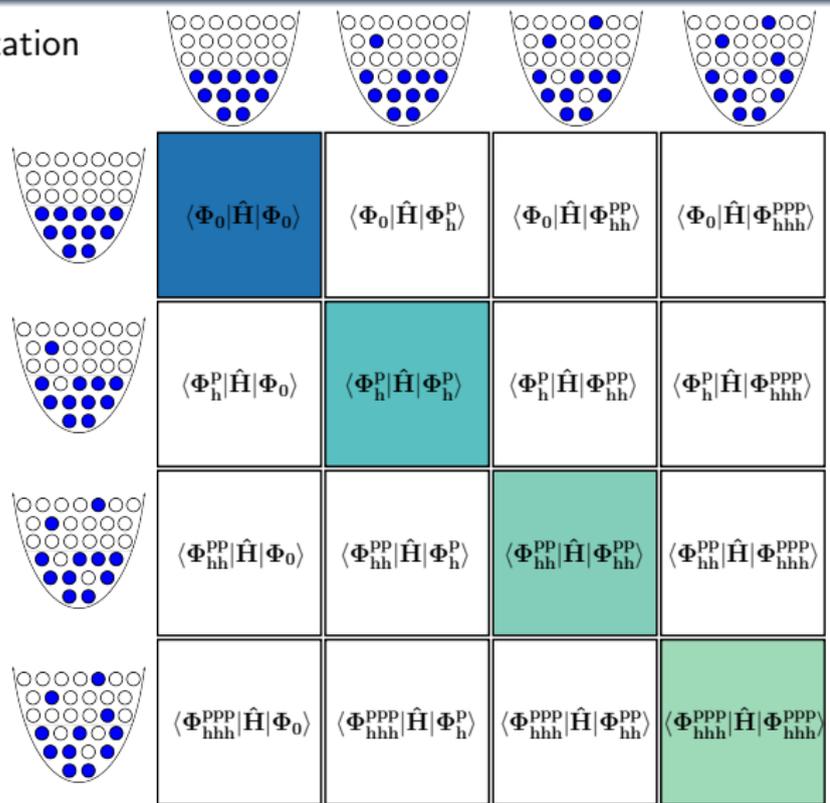
$$|\Psi\rangle = c_0 \begin{array}{c} \text{Well with 6 blue particles} \end{array} + \sum_{1p1h} c_1 \begin{array}{c} \text{Well with 7 blue particles} \end{array} + \sum_{2p2h} c_2 \begin{array}{c} \text{Well with 8 blue particles} \end{array} + \sum_{3p3h} c_3 \begin{array}{c} \text{Well with 9 blue particles} \end{array} + \dots$$

Many-Body Hamiltonian



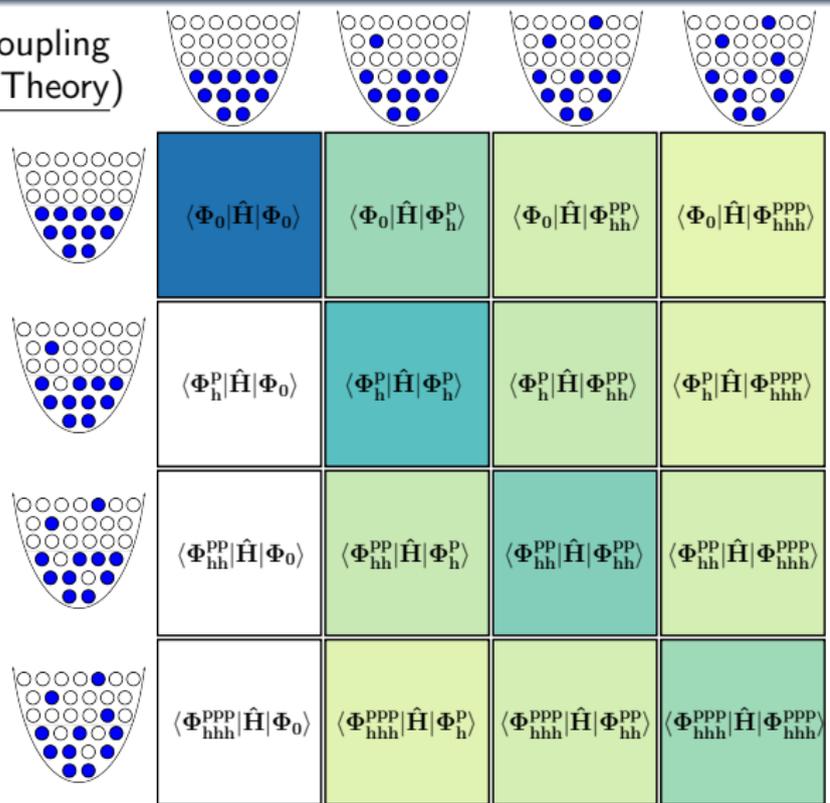
Many-Body Hamiltonian

Full Diagonalization
(NCSM)



Many-Body Hamiltonian

Asymmetric Decoupling
(Coupled Cluster Theory)



Coupled-Cluster Theory

Exponential Ansatz: $|\Psi\rangle = e^{\hat{T}} |\Phi_0\rangle$

$$|\Phi_0\rangle \equiv \begin{array}{c} \text{○○○○○○} \\ \text{○○○○○○} \\ \text{○○○○○○} \\ \text{●●●●●●} \\ \text{●●●●●●} \\ \text{●●●●●●} \end{array} \quad \hat{T} \equiv \sum_{1p1h} t_1 \begin{array}{c} \text{○○○○○○} \\ \text{○○○○○○} \\ \text{○○○○○○} \\ \text{●●●●●●} \\ \text{●●●●●●} \\ \text{●●●●●●} \end{array} + \sum_{2p2h} t_2 \begin{array}{c} \text{○○○○○○} \\ \text{○○○○○○} \\ \text{○○○○○○} \\ \text{●●●●●●} \\ \text{●●●●●●} \\ \text{●●●●●●} \end{array} + \sum_{3p3h} t_3 \begin{array}{c} \text{○○○○○○} \\ \text{○○○○○○} \\ \text{○○○○○○} \\ \text{●●●●●●} \\ \text{●●●●●●} \\ \text{●●●●●●} \end{array} + \dots$$

Coupled-Cluster Theory

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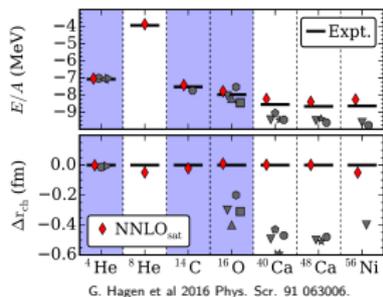
CC Hamiltonian: $\bar{H} \equiv e^{-\hat{T}} \hat{H} e^{\hat{T}}$

- Decouple excitations: $\langle \Phi' | \bar{H} | \Phi_0 \rangle \rightarrow 0$
- Correlated ground state: $\langle \Phi_0 | \bar{H} | \Phi_0 \rangle \rightarrow E_{gs}$
- Scales polynomially
- Anti-Hermitian Hamiltonian

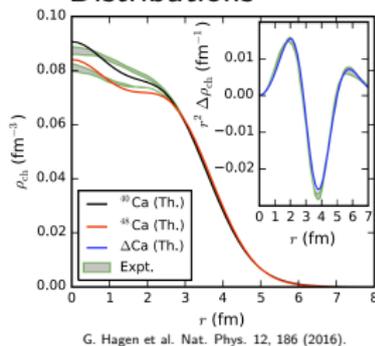
$\langle \Phi_0 \hat{H} \Phi_0 \rangle$	$\langle \Phi_0 \hat{H} \Phi_h^p \rangle$	$\langle \Phi_0 \hat{H} \Phi_{hh}^{pp} \rangle$	$\langle \Phi_0 \hat{H} \Phi_{hhh}^{ppp} \rangle$
$\langle \Phi_h^p \hat{H} \Phi_0 \rangle$	$\langle \Phi_h^p \hat{H} \Phi_h^p \rangle$	$\langle \Phi_h^p \hat{H} \Phi_{hh}^{pp} \rangle$	$\langle \Phi_h^p \hat{H} \Phi_{hhh}^{ppp} \rangle$
$\langle \Phi_{hh}^{pp} \hat{H} \Phi_0 \rangle$	$\langle \Phi_{hh}^{pp} \hat{H} \Phi_h^p \rangle$	$\langle \Phi_{hh}^{pp} \hat{H} \Phi_{hh}^{pp} \rangle$	$\langle \Phi_{hh}^{pp} \hat{H} \Phi_{hhh}^{ppp} \rangle$
$\langle \Phi_{hhh}^{ppp} \hat{H} \Phi_0 \rangle$	$\langle \Phi_{hhh}^{ppp} \hat{H} \Phi_h^p \rangle$	$\langle \Phi_{hhh}^{ppp} \hat{H} \Phi_{hh}^{pp} \rangle$	$\langle \Phi_{hhh}^{ppp} \hat{H} \Phi_{hhh}^{ppp} \rangle$

CC Theory: Success and Progress

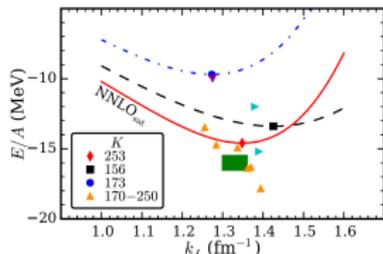
Energies and Radii



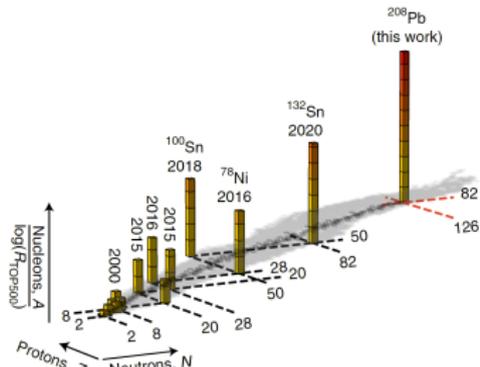
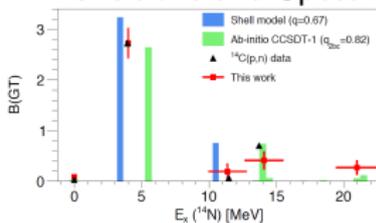
Distributions



Infinite Matter

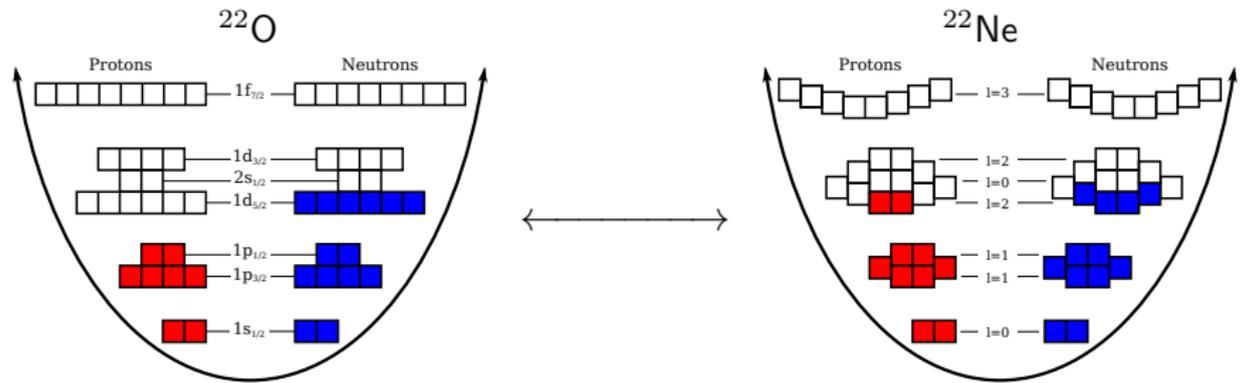


Transitions and Spectra



● Open-shell nuclei reached with **Deformed Coupled Cluster**.

Deformed Coupled Cluster Theory



- Restricted to closed-shell, Spherical
- \hat{J}^2 is good quantum number
- Computational discount

- Unrestricted, Quadrupole Deformation
- \hat{J}_z is good quantum number
- Computational penalty

Daughter State: Equation-of-motion method

EOM method builds biorthogonal eigenvectors of \bar{H} on the closed-shell ground state,

$$\bar{H} \hat{R}_\mu |\Phi_0\rangle = E_\mu \hat{R}_\mu |\Phi_0\rangle \quad \hat{R} \equiv r_0 + \sum_{1p1h} r_1 \begin{array}{c} \text{shell diagram} \\ \text{(1p1h)} \end{array} + \sum_{2p2h} r_2 \begin{array}{c} \text{shell diagram} \\ \text{(2p2h)} \end{array} + \dots \sim \hat{L}^\dagger$$

$\langle \Phi_0 \hat{H} \Phi_0 \rangle$	$\langle \Phi_0 \hat{H} \Phi_h^p \rangle$	$\langle \Phi_0 \hat{H} \Phi_{hh}^{pp} \rangle$	$\langle \Phi_0 \hat{H} \Phi_{hhh}^{ppp} \rangle$
$\langle \Phi_h^p \hat{H} \Phi_0 \rangle$	$\langle \Phi_h^p \hat{H} \Phi_h^p \rangle$	$\langle \Phi_h^p \hat{H} \Phi_{hh}^{pp} \rangle$	$\langle \Phi_h^p \hat{H} \Phi_{hhh}^{ppp} \rangle$
$\langle \Phi_{hh}^{pp} \hat{H} \Phi_0 \rangle$	$\langle \Phi_{hh}^{pp} \hat{H} \Phi_h^p \rangle$	$\langle \Phi_{hh}^{pp} \hat{H} \Phi_{hh}^{pp} \rangle$	$\langle \Phi_{hh}^{pp} \hat{H} \Phi_{hhh}^{ppp} \rangle$
$\langle \Phi_{hhh}^{ppp} \hat{H} \Phi_0 \rangle$	$\langle \Phi_{hhh}^{ppp} \hat{H} \Phi_h^p \rangle$	$\langle \Phi_{hhh}^{ppp} \hat{H} \Phi_{hh}^{pp} \rangle$	$\langle \Phi_{hhh}^{ppp} \hat{H} \Phi_{hhh}^{ppp} \rangle$

$$\langle \Phi_0 | \hat{L}_\mu \bar{H} = \langle \Phi_0 | \hat{L}_\mu E_\mu$$

$$\langle \Phi_0 | \hat{L}_\mu \hat{R}_\mu | \Phi_0 \rangle = 1$$

\hat{R}, \hat{L} target appropriate quantum numbers, τ_\pm for charge-exchange.

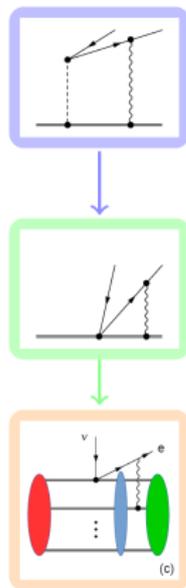
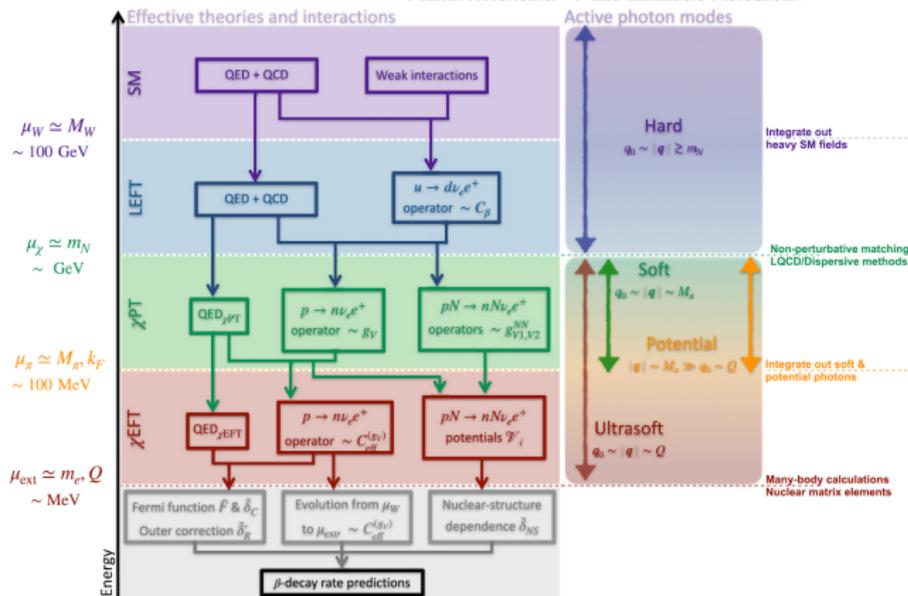
$$\bar{O} \equiv e^{-\hat{T}} \hat{O} e^{\hat{T}} \quad |M_\beta|^2 = \langle \Phi_0 | \hat{L}_0 \bar{O}^\dagger \hat{R}_k | \Phi_0 \rangle \langle \Phi_0 | \hat{L}_k \bar{O} | \Phi_0 \rangle$$

Effective Field Theory for Radiative Corrections

PHYSICAL REVIEW C **110**, 055502 (2024)

Ab initio electroweak corrections to superallowed β decays and their impact on V_{ud}

Vincenzo Cirigliano¹, Wouter Dekens¹, Jordy de Vries^{2,3}, Stefano Gandolfi⁴,
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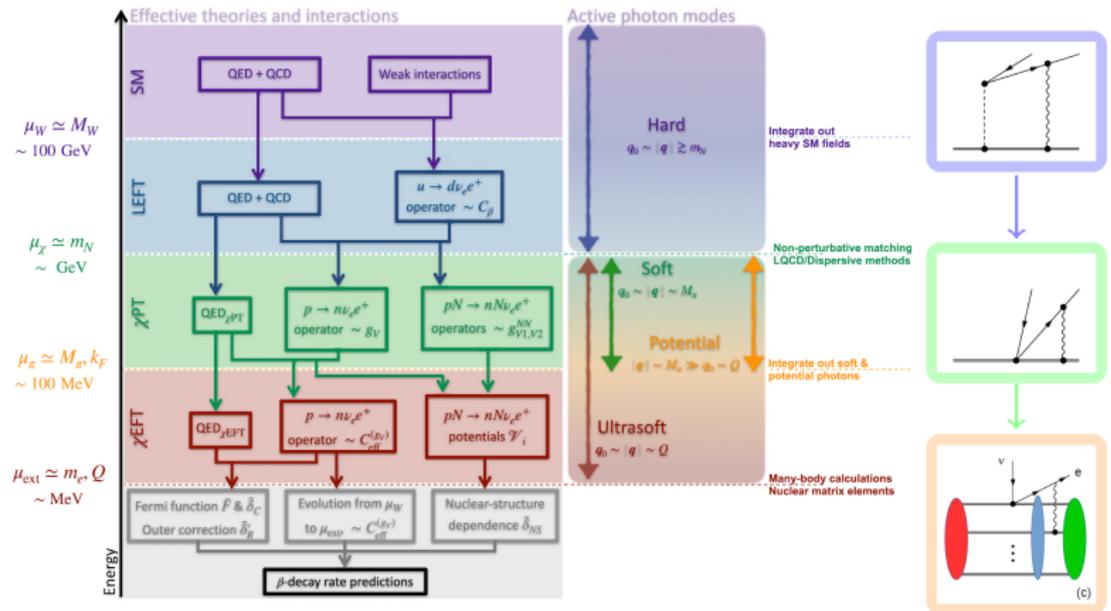


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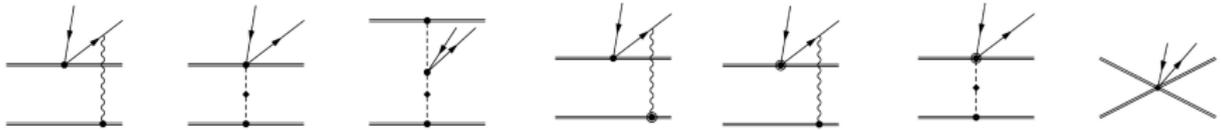
Exploit different separation of scales to write correction as expansion in α and Q/Λ_χ

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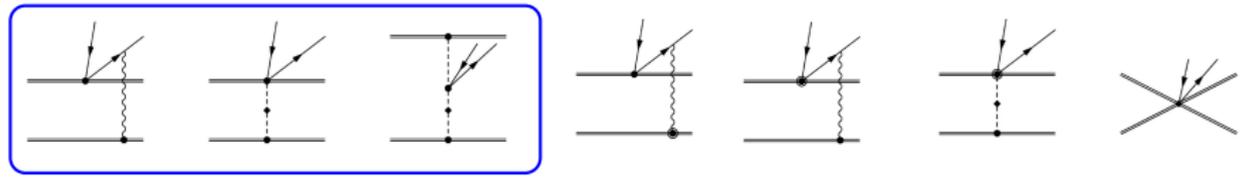


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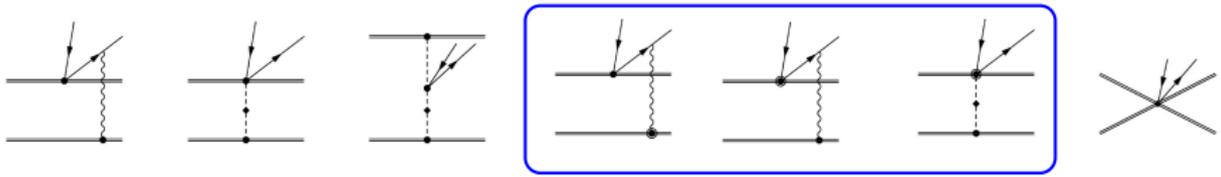
- Leading-order EM and Weak interactions, $\mathcal{O}(\alpha q_{ex}/M_\pi)$
- Energy-dependent, proportional to external momentum
- Replaces charge-radius corrections, grows with A

Effective Field Theory for Radiative Corrections

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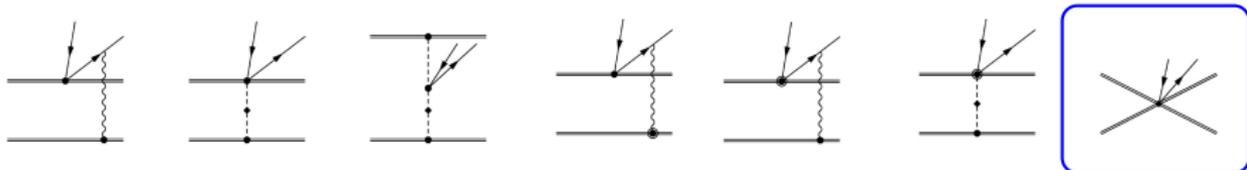
- NLO EW interactions, $\mathcal{O}(\alpha M_\pi/\Lambda_\chi)$
- Energy-independent, Long-range
- Corrections from magnetic moments and recoil
- Induces UV sensitivity and cutoff dependence

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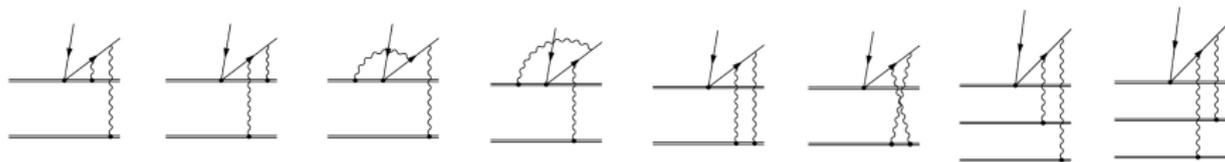
- Cutoff dependence absorbed by contact terms, $\mathcal{O}(F_\pi^{-2} \Lambda_\chi^{-1})$
- LECs could be extracted from LQCD or other models
- Can also be used as free parameters for a global V_{ud} fit

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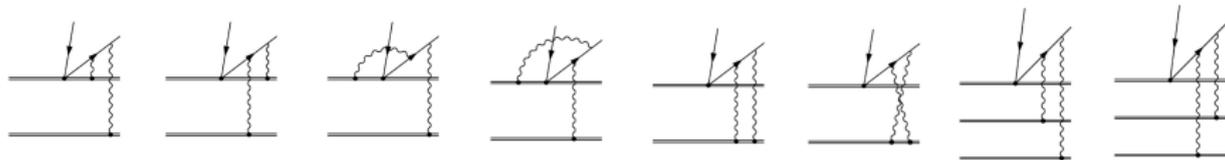
- Mixed-scale photons important for required precision, $\mathcal{O}(\alpha^2)$
- Include corrections to Fermi function
- Soft photons also induce three-body terms (excluded for now)

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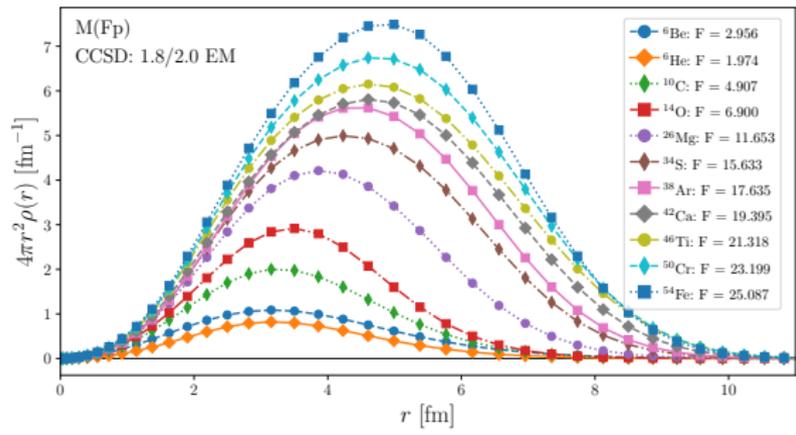
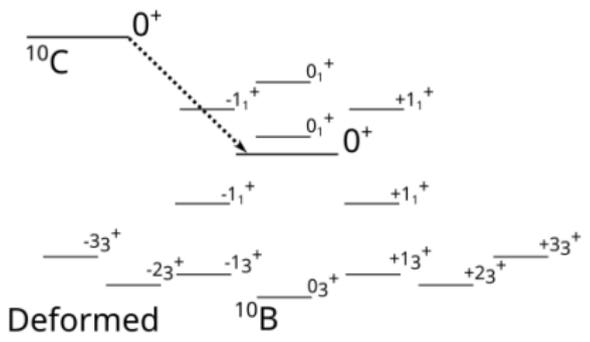
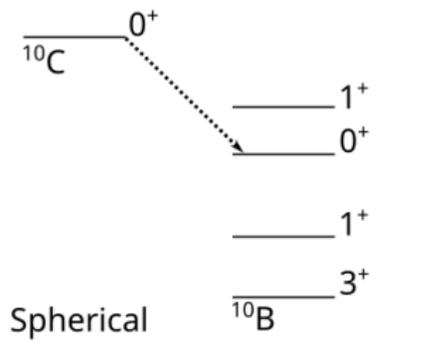
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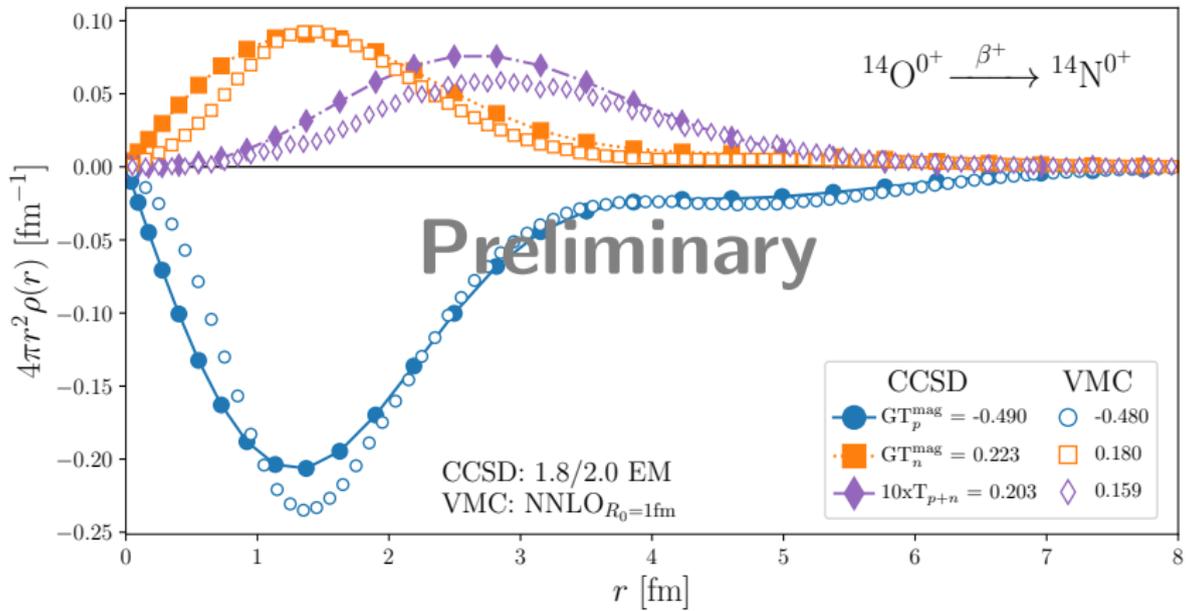
$$M_{\beta, k=p, n}^{\mathcal{O}} = \int d\vec{r}_{ij} \langle \mathcal{F} | \sum_{i < j} \hat{h}_{\beta, k=p, n}^{\mathcal{O}}(r_{ij}) \left[\tau_i^+ \hat{P}_j^k + (i \leftrightarrow j) \right] | \mathcal{I} \rangle, \quad \beta = \text{F, GT, T, SO}$$

Deformed Transition Target State



$$\hat{O}_{F,p} = \tau_i^+ \hat{P}_j^p + (i \leftrightarrow j)$$

Superaligned: $^{14}\text{O} \rightarrow ^{14}\text{N}$

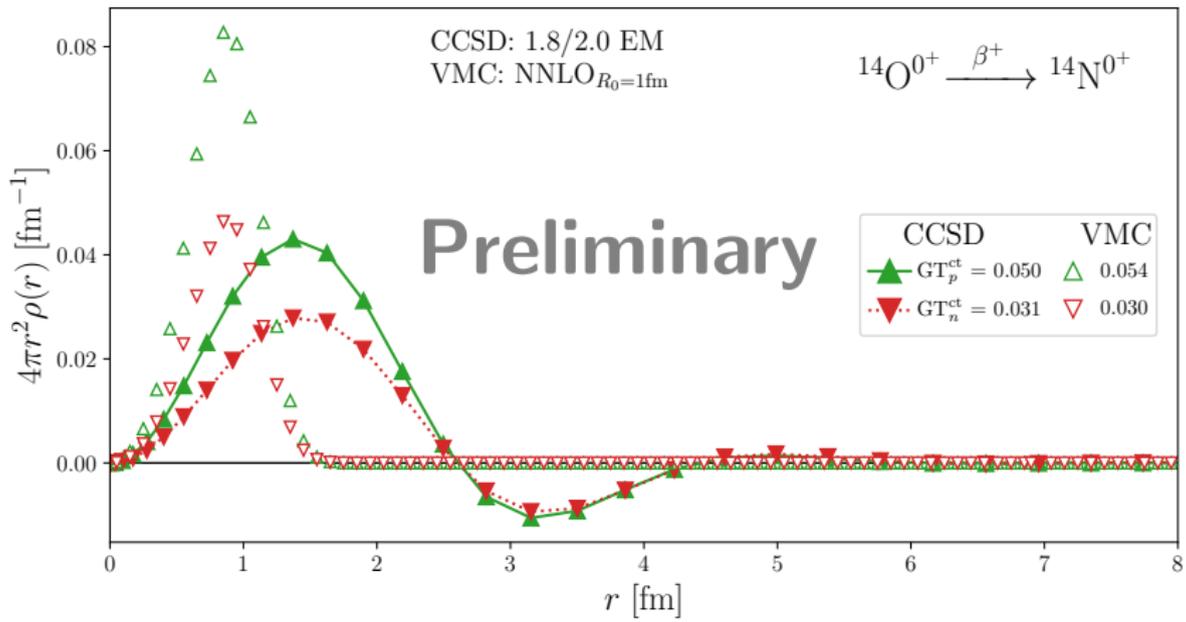


$T = 1$ transition. Closed-shell reference state: J-scheme.

Gamow-Teller components dominate over tensor.

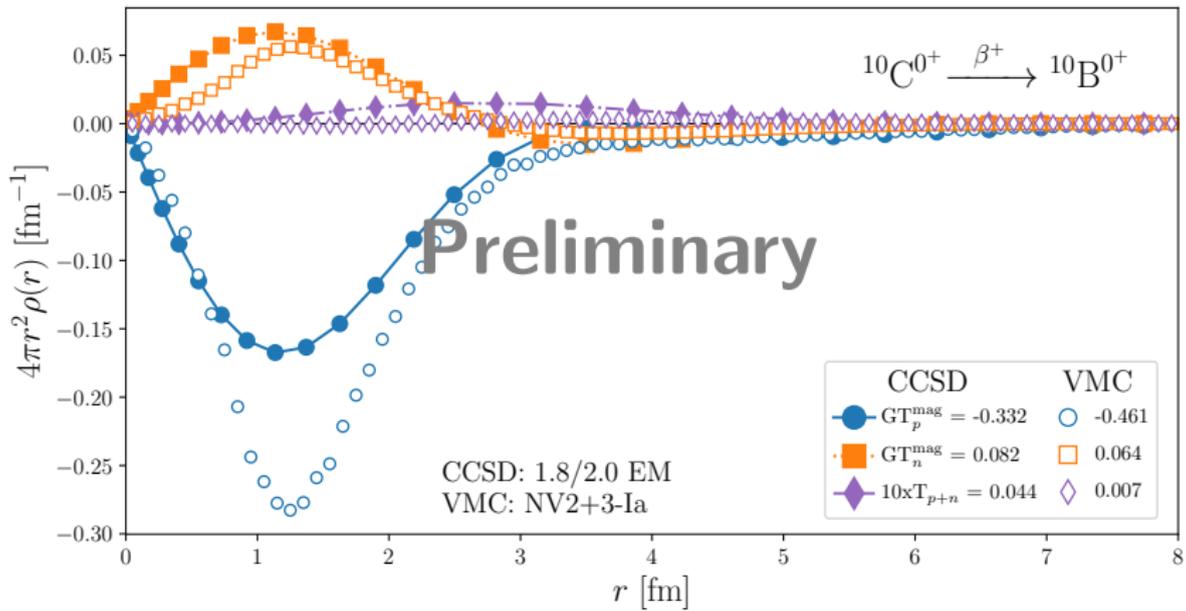
Short-range behavior doesn't agree likely due to interaction regulation.

Superaligned: $^{14}\text{O} \rightarrow ^{14}\text{N}$



Regulation: $\delta(r) \propto e^{-r^4/R_0^4}$ vs. $f_\Lambda(r) \propto \int dk k^2 j_l(kr) f_\Lambda(k)$

Superaligned: $^{10}\text{C} \rightarrow ^{10}\text{B}$

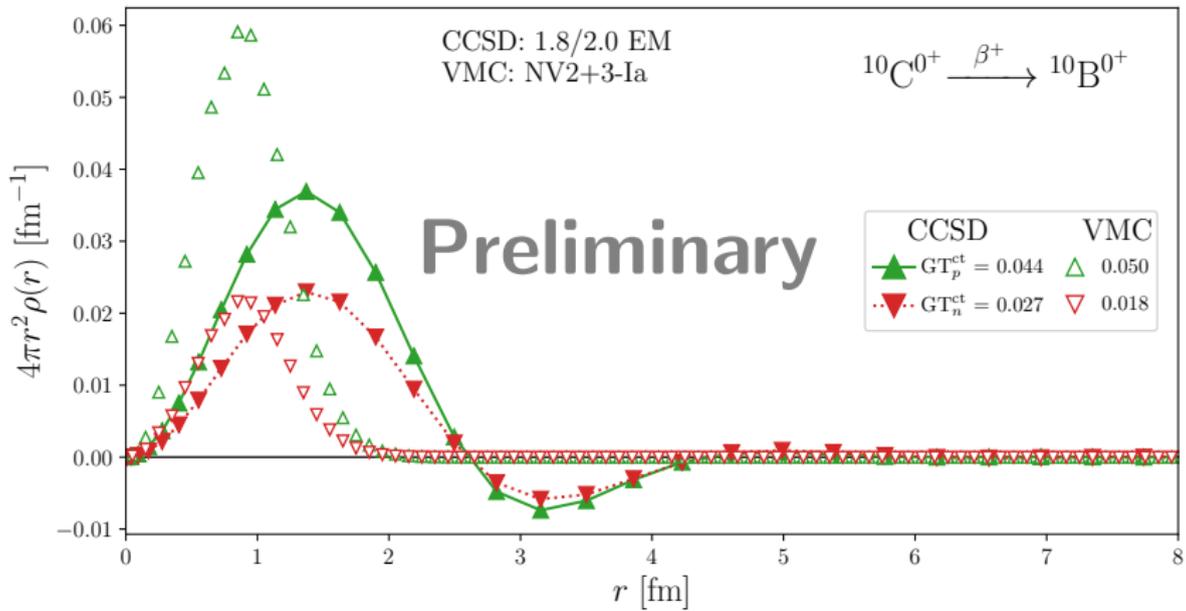


$T = 1$ transition. Deformed reference state: M-scheme.

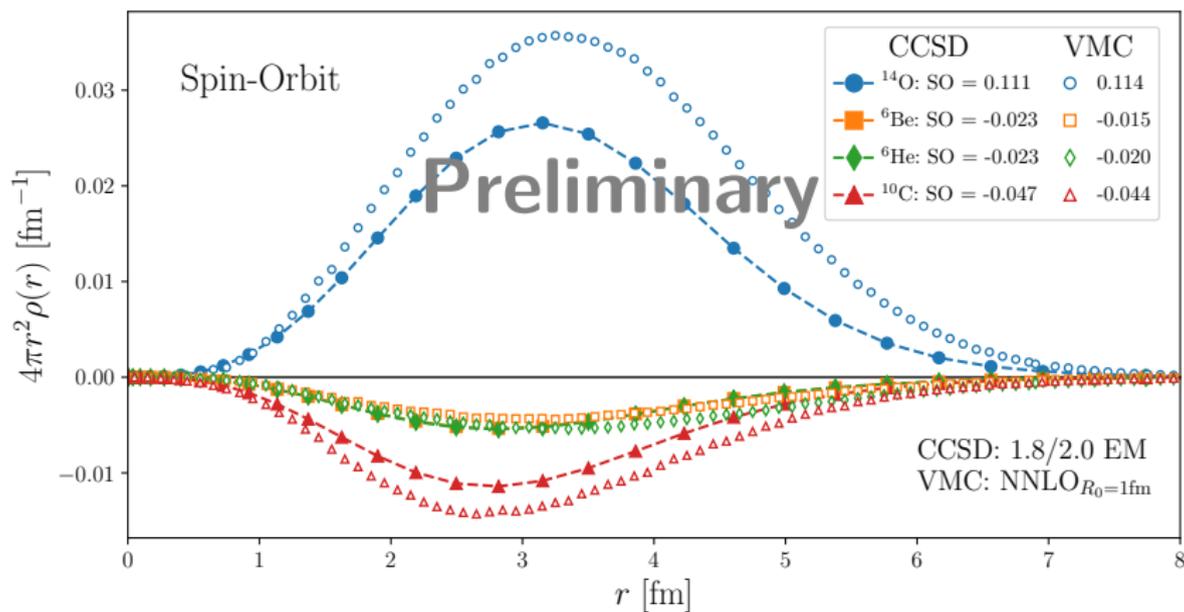
Deformation probably causes disagreements. Add triples correlations?

Qualitative agreement promising given different methods and interactions.

Superaligned: $^{10}\text{C} \rightarrow ^{10}\text{B}$

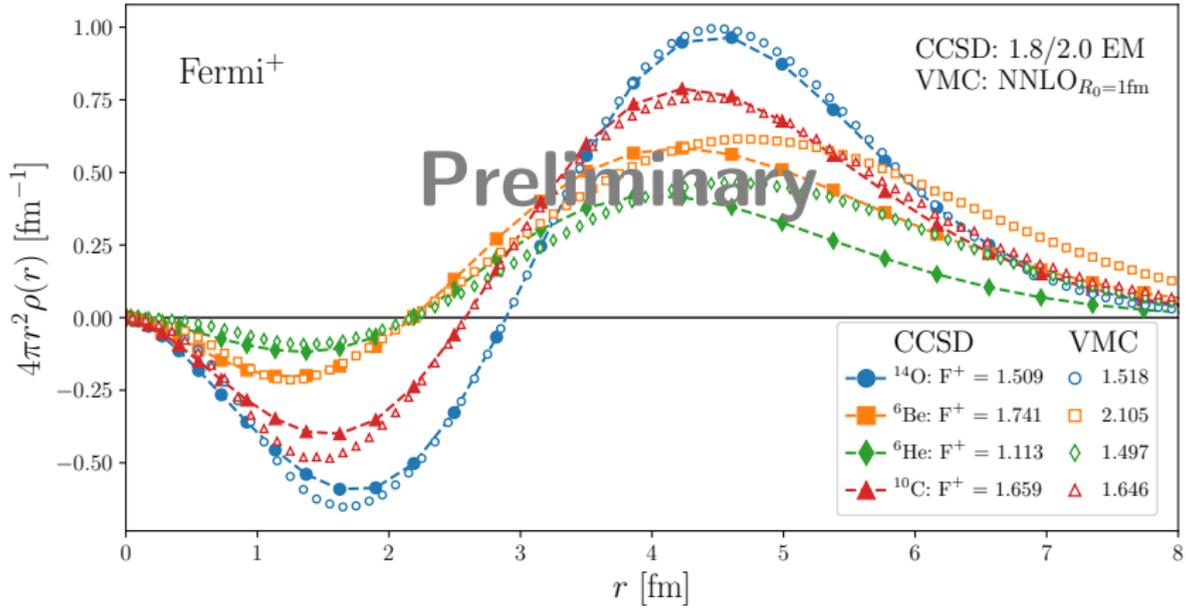


Regulation: $\delta(r) \propto e^{-r^4/R_0^4}$ vs. $f_\Lambda(r) \propto \int dk k^2 j_l(kr) f_\Lambda(k)$

Spin-Orbit Correction to δ_{NS} 

$$\hat{O}_{\text{SO},p} = \tau_i^+ \hat{P}_j^p \left(-i \hat{r}_{ij} \times \left(\vec{\nabla}_i - \vec{\nabla}_j \right) \right) \cdot \vec{\sigma}_j + (i \leftrightarrow j)$$

Fermi Correction to δ_{NS}



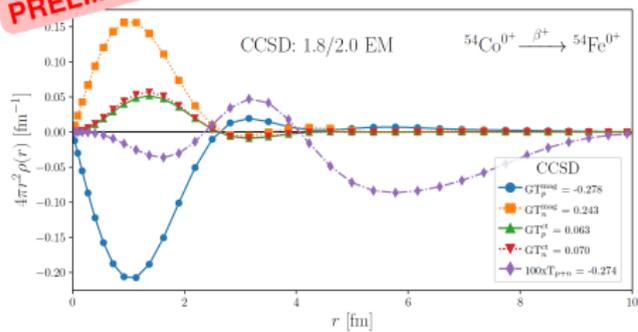
$$\hat{O}_{F^+,p} = -\log(r/R_A) \left[\tau_i^+ \hat{P}_j^p + (i \leftrightarrow j) \right]$$

Extracting V_{ud}

$$^{14}\text{O} \rightarrow ^{14}\text{N} : V_{ud} = 0.97364(10)_{\text{exp}}(12)_{\Delta V_R}(22)_{\text{out}}(12)_{\delta_C}(43)_{\delta_{\text{NS}}^{\text{ct}}}(20)_{\delta_{\text{NS}}}$$

$$V_{ud} = 0.97364(56)_{\text{tot}}, V_{ud} = 0.97405(37)_{\text{tot}} \text{ (Towner, Hardy)}$$

PRELIMINARY



Transition	$M_{\text{GT}+\text{T},\text{p}}^{\text{mag}}$	$M_{\text{GT}+\text{T},\text{n}}^{\text{mag}}$	$M_{\text{GT},\text{p}}^{\text{ct}}$	$M_{\text{GT},\text{n}}^{\text{ct}}$
$^{10}\text{C} \rightarrow ^{10}\text{B}$	-0.327	0.082	0.044	0.027
$^{14}\text{O} \rightarrow ^{14}\text{N}$	-0.486	0.239	0.050	0.031
$^{26}\text{Al} \rightarrow ^{26}\text{Mg}$	-0.221	0.229	0.048	0.060
$^{34}\text{Cl} \rightarrow ^{34}\text{S}$	-0.368	0.305	0.047	0.056
$^{38}\text{K} \rightarrow ^{38}\text{Ar}$	-0.407	0.330	0.048	0.059
$^{42}\text{Sc} \rightarrow ^{42}\text{Ca}$	-0.229	0.218	0.053	0.062
$^{46}\text{V} \rightarrow ^{46}\text{Ti}$	-0.268	0.225	0.058	0.066
$^{50}\text{Mn} \rightarrow ^{50}\text{Cr}$	-0.270	0.231	0.060	0.067
$^{54}\text{Co} \rightarrow ^{54}\text{Fe}$	-0.273	0.235	0.062	0.070

Need consistent δ_C , Need uncertainty quantification, Fit contact terms to check accuracy and precision of V_{ud}

