How Weak Interactions Suppress g-modes in Hot Neutron Stars (NS)

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N3AS Network for Neutrinos, Nuclear Astrophysics, and Symmetries
PHYSICS FRONTIER CENTER

NSF

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INSTITUTE for NUCLEAR THEORY

A brief history

- In 1941, Cowling's classification of stellar oscillations.
- Early 1960s, Chandrasekhar extended it to compact objects.
- In 1967, Thorne linked oscillations to gravitational waves.
- In 1983, Lindlom, Detweiler, and McDermott conducted numerical studies with various equations of state (EOSs).
- In the 1990s, various g-modes and their detectability: discontinuity, entropy, composition, …

Oscillations of NS

Fluid perturbations

- Radial oscillation (l=0): $\varepsilon^r = R_n^r(r)e^{\mathbf{i}\omega t}$
  don't couple to gravitational waves

- Non-radial oscillation (l>=2):

$$\xi_{\text{even}}^{r,\,\theta,\,\phi} = \nabla\left(R_n(r)Y_m^l(\theta,\phi)e^{\mathbf{i}\omega t}\right)$$

$$\xi_{odd}^{\theta,\,\phi} = \hat{r}\times\nabla\left(R_n(r)Y_m^l(\theta,\phi)e^{\mathbf{i}\omega t}\right)$$

  f-mode (fundamental n=0) (even),
  p-modes (pressure n=1,2,…) (even)
  g-modes (gravity n=1,2,…) (even)
  r-modes (rotation m=+-1,+-2,…) (odd)

$$\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$$

	ν (kHz)	τ (s)
f-mode	1.3-2.8	0.1-1
g-mode	<0.8	>100
p-mode	>2.7	1-1000
r-mode	~ spin	<0
w-mode	~10	~1E-5

- **Spacetime perturbations**:
  Family I w-modes (even)
  Family II w-modes (odd)
  important for BBH ring-down

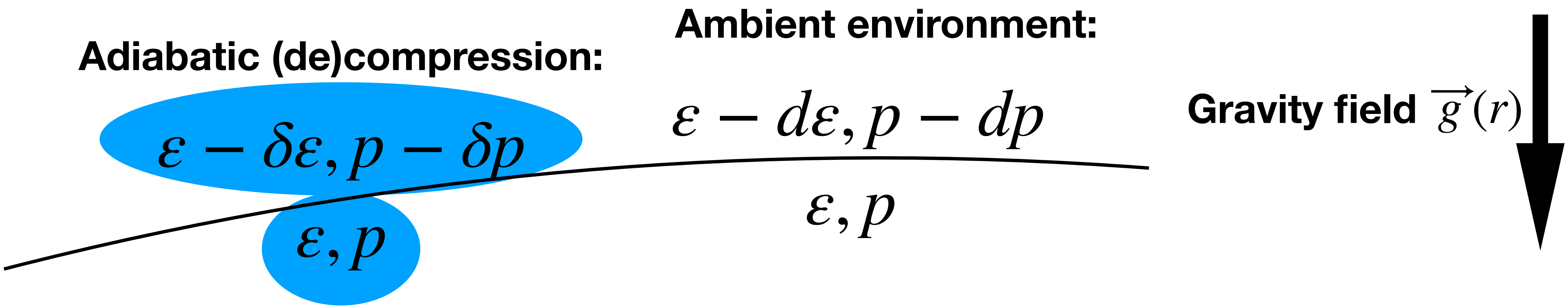
even-parity (polar mode)

$$h_{\mu\nu}^{even} = \begin{pmatrix} H_0 & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & r^2K & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta\ K \end{pmatrix} Y_{lm}(\theta,\phi)$$

odd-parity (axial modes)

$$h_{\mu\nu}^{odd} = \begin{pmatrix} 0 & 0 & \frac{h_0\partial_\phi}{\sin\theta} & -h_0\sin\theta\ \partial_\theta \\ 0 & 0 & \frac{h_1\partial_\phi}{\sin\theta} & -h_1\sin\theta\ \partial_\theta \\ \cdots & \cdots & 0 & 0 \\ \cdots & \cdots & 0 & 0 \end{pmatrix} Y_{lm}(\theta,\phi)$$

gravity mode (g-mode)



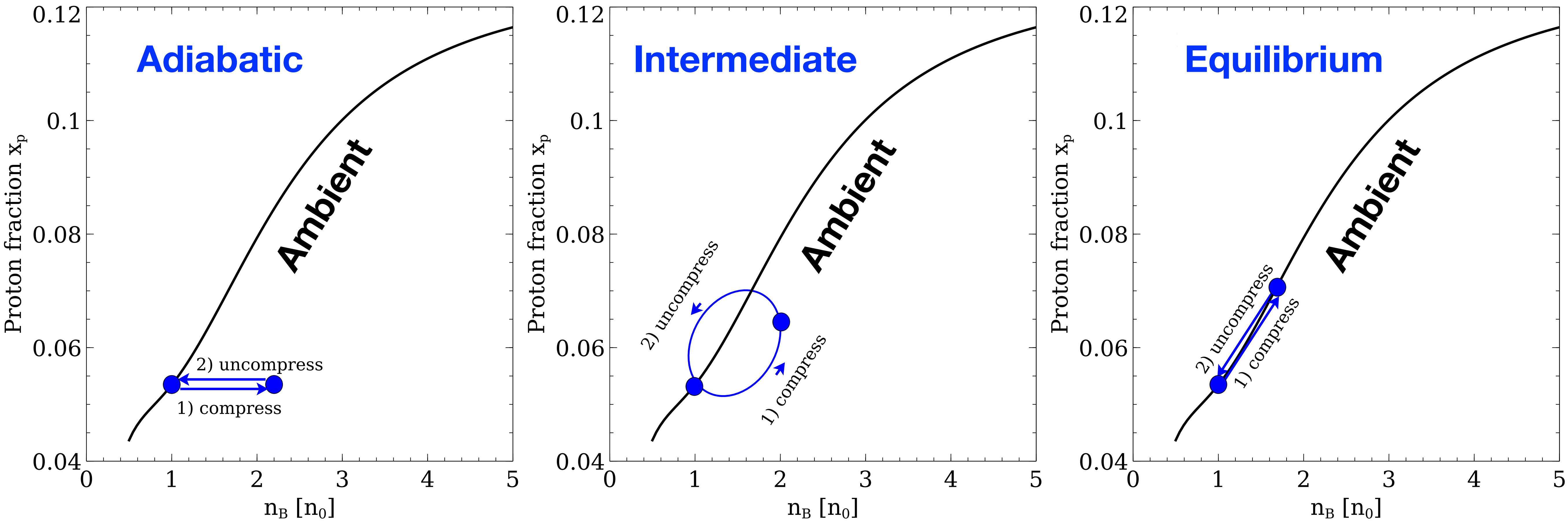
- Mechanical equilibrium: $p - \delta p = p - dp$
- Gravity as recovering: $\varepsilon - \delta\varepsilon > \varepsilon - d\varepsilon$
- Non-convecting(stable): $c_{ad}^2 = \dfrac{\delta p}{\delta \varepsilon} > \dfrac{dp}{d\varepsilon} = c_{eq}^2$

$$c_{ad}^2 = \left[\frac{\partial p}{\partial n_B}\right]_{\{x\}} \left[\frac{\partial \varepsilon}{\partial n_B}\right]_{\{x\}}^{-1} \qquad c_{eq}^2 = \frac{dp}{dr}\left[\frac{d\varepsilon}{dr}\right]^{-1}$$

fix composition & entropy, e.g. $\{x\} = \{\frac{n_p}{n_B}, \frac{n_n}{n_B}, S, \ldots\}$

Dissipation from β-reactions

- Path of a fluid element as it is compressed and uncompressed.



- Path in $\{x_p, n_B\}$ plane maps to path in {P, V} plane.

  Traversing the path leads to dissipative work, $\oint P \cdot dV$

Dissipation from β-reactions

- Nucleon weak β-equilibrium at low neutrino opacity,
  $(N+)\ n \to p + e^- + \bar{\nu}$
  $(N+)\ p + e^- \to n + \nu$
  leads to $\delta_\mu = \mu_n - \mu_p - \mu_e \to 0$, and $x = \dfrac{n_p}{n_B} \to x_{eq}$.

- off β-equilibrium, when $\delta_x = x - x_{eq}$,
  $$\partial_t \delta_x = \frac{1}{n_B}(\Gamma_{n\to p} - \Gamma_{p\to n})$$
  $$\Gamma^{\rm dUrca}_{n\to p} \sim \int \left(\prod_{n,p,e,\bar{\nu}} \frac{d^3 p_i}{2E_i} \right) \delta^4 \left(\sum_{n,p,e,\bar{\nu}} p_i \right) \sum_{\rm spins} |\mathscr{M}|^2 f_n \left(1 - f_p\right) \left(1 - f_e\right) \propto T^5$$
  $$\Gamma^{\rm dUrca}_{p\to n} \sim \int \left(\prod_{n,p,e,\nu} \frac{d^3 p_i}{2E_i} \right) \delta^4 \left(\sum_{n,p,e,\nu} p_i \right) \sum_{\rm spins} |\mathscr{M}|^2 \left(1 - f_n\right)\ f_p\ f_e\ \propto T^5$$
  $\Gamma^{\rm mUrca}_{n\to p}$ and $\Gamma^{\rm mUrca}_{p\to n}$ have additional factors of $d^3 p_N^{in} d^3 p_N^{out} f_N (1 - f_N) \propto T^2$

- Treat dUrca+mUrca jointly: Nucleon Width Appro. (Alford, Haber, & Zhang 2406.13717)

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- off β-equilibrium, when $\delta_x = x - x_{eq}$,

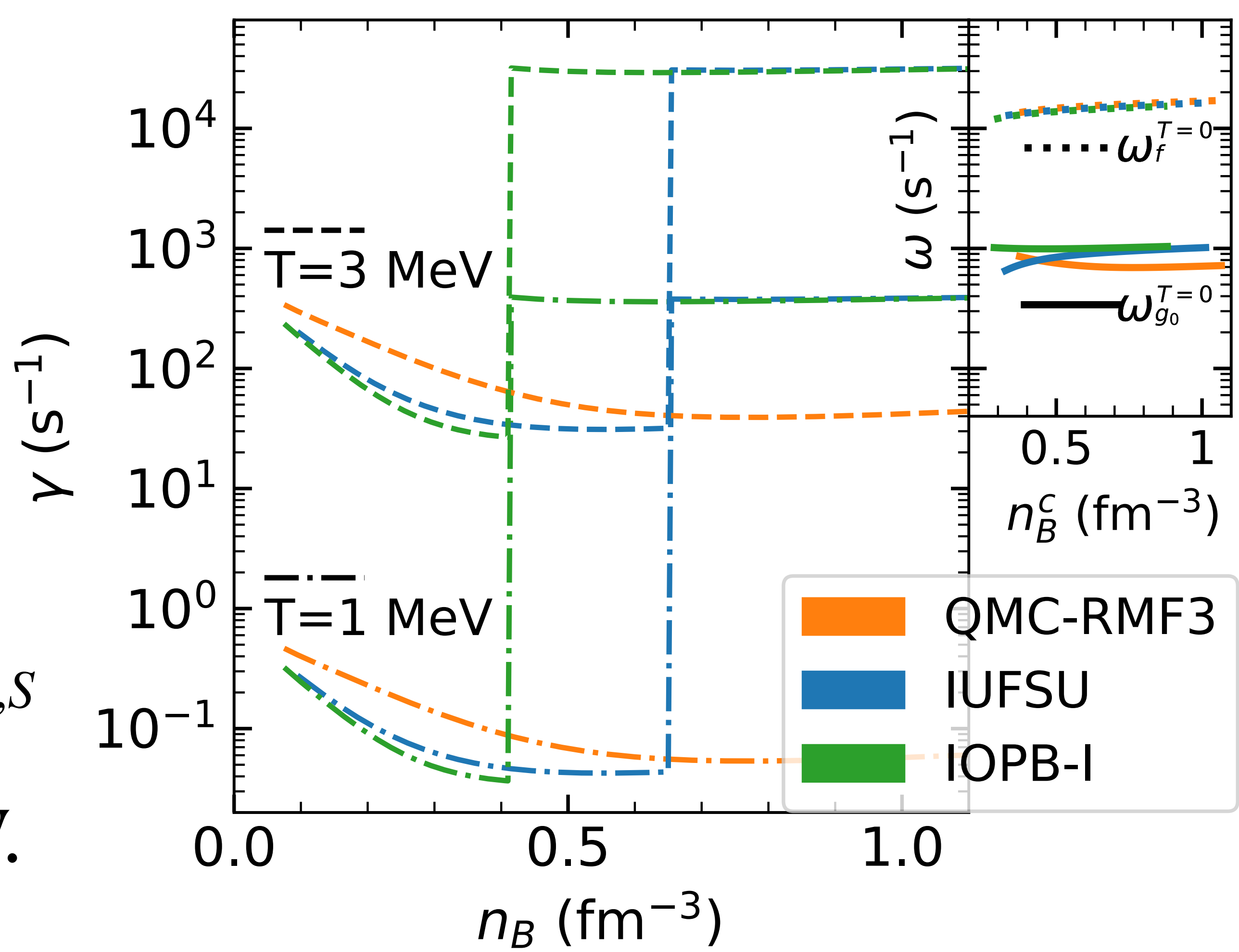
  $$\partial_t \delta_x = \frac{1}{n_B}(\Gamma_{n\to p} - \Gamma_{p\to n}) \approx -\gamma\delta_x\,,$$

  (linear order)

  where γ is the weak relaxation rate,

  $$\gamma = -\frac{1}{n_B}\frac{\partial(\Gamma_{n\to p} - \Gamma_{p\to n})}{\partial\delta_\mu}\bigg|_{n_B,S}\frac{\partial\delta_\mu}{\partial\delta_x}\bigg|_{n_B,S}$$

- γ reaches kHz frequency at T~3MeV.



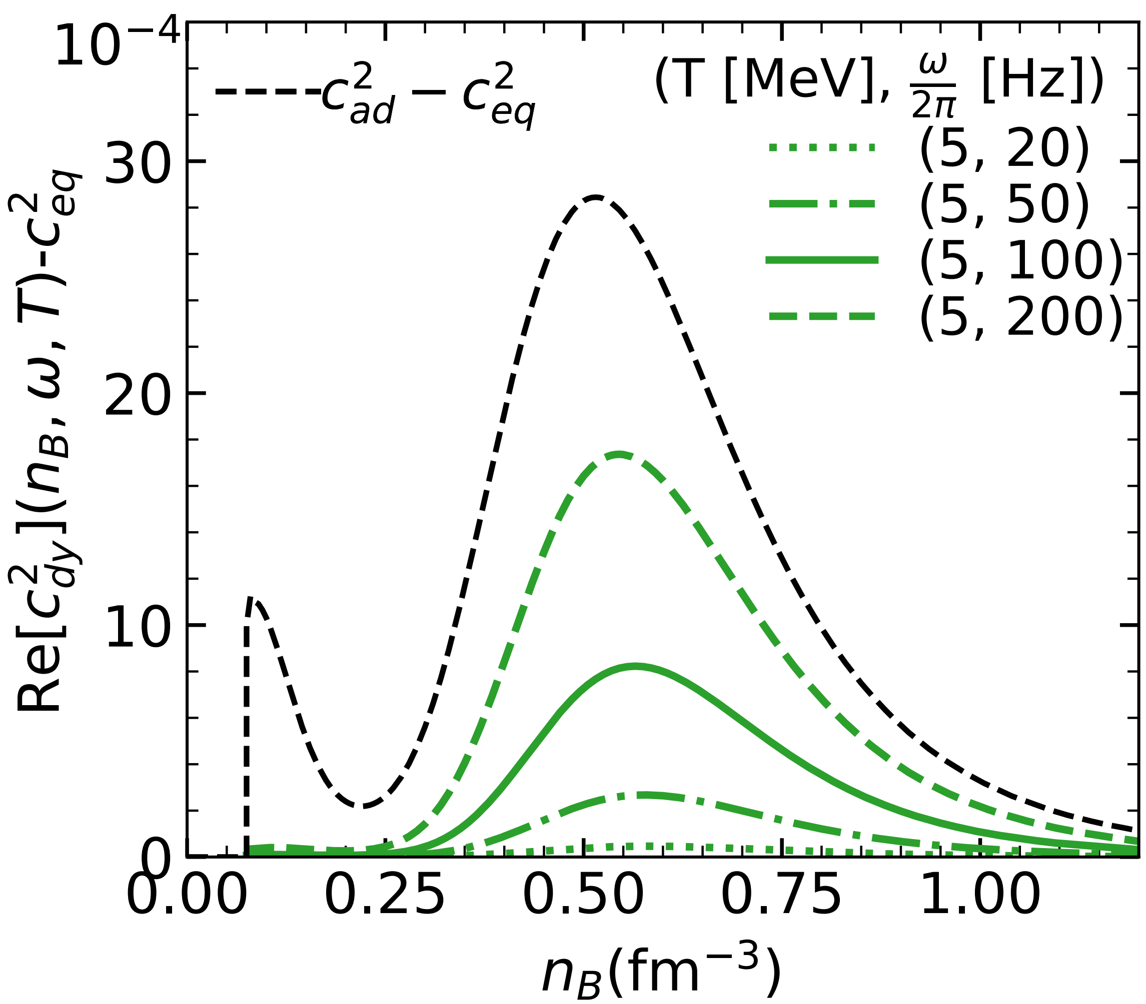
Dynamic sound speed

$$c_{dy}^2 = c_{eq}^2 + \frac{c_{ad}^2 - c_{eq}^2}{1 - i\frac{\gamma}{\omega}}$$

- Complex, frequency-dependent quantity derived from the period ansatz that captures both **restoring force** and **damping**. $n_B(t) = n_B^0 + \delta n_B \cos(\omega t)$

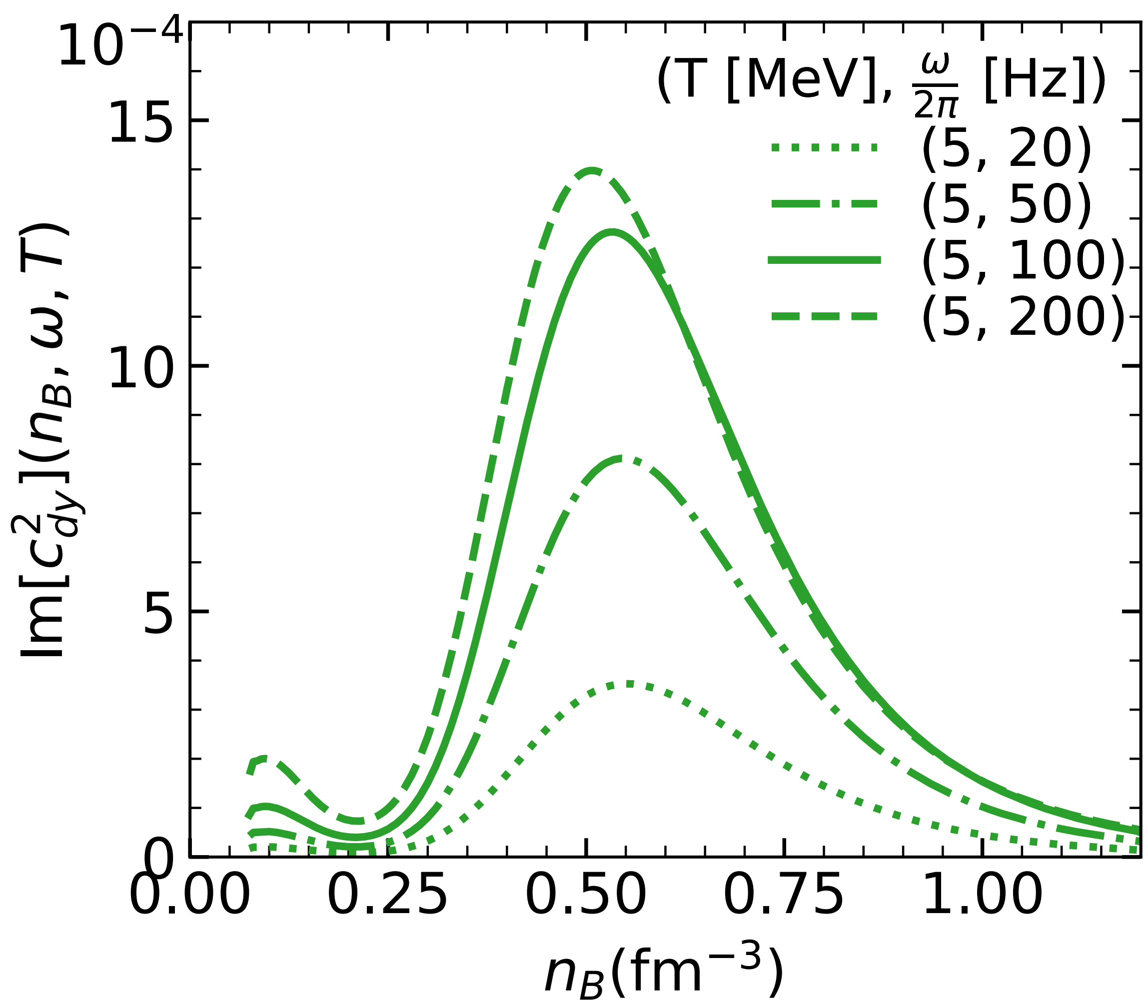
1. Restoring force: $c_{eq}^2 < \mathrm{Re}\left[c_{dy}^2\right] < c_{ad}^2$

$$\mathrm{Re}\left[c_{dy}^2\right] = c_{eq}^2 + \left(c_{ad}^2 - c_{eq}^2\right)\frac{\omega^2}{\omega^2 + \gamma^2}$$



2. Dissapative damping: resonance at $\gamma \approx \omega$

$$\mathrm{Im}\left[c_{dy}^2\right] = \left(c_{ad}^2 - c_{eq}^2\right)\frac{\omega\gamma}{\omega^2 + \gamma^2}$$



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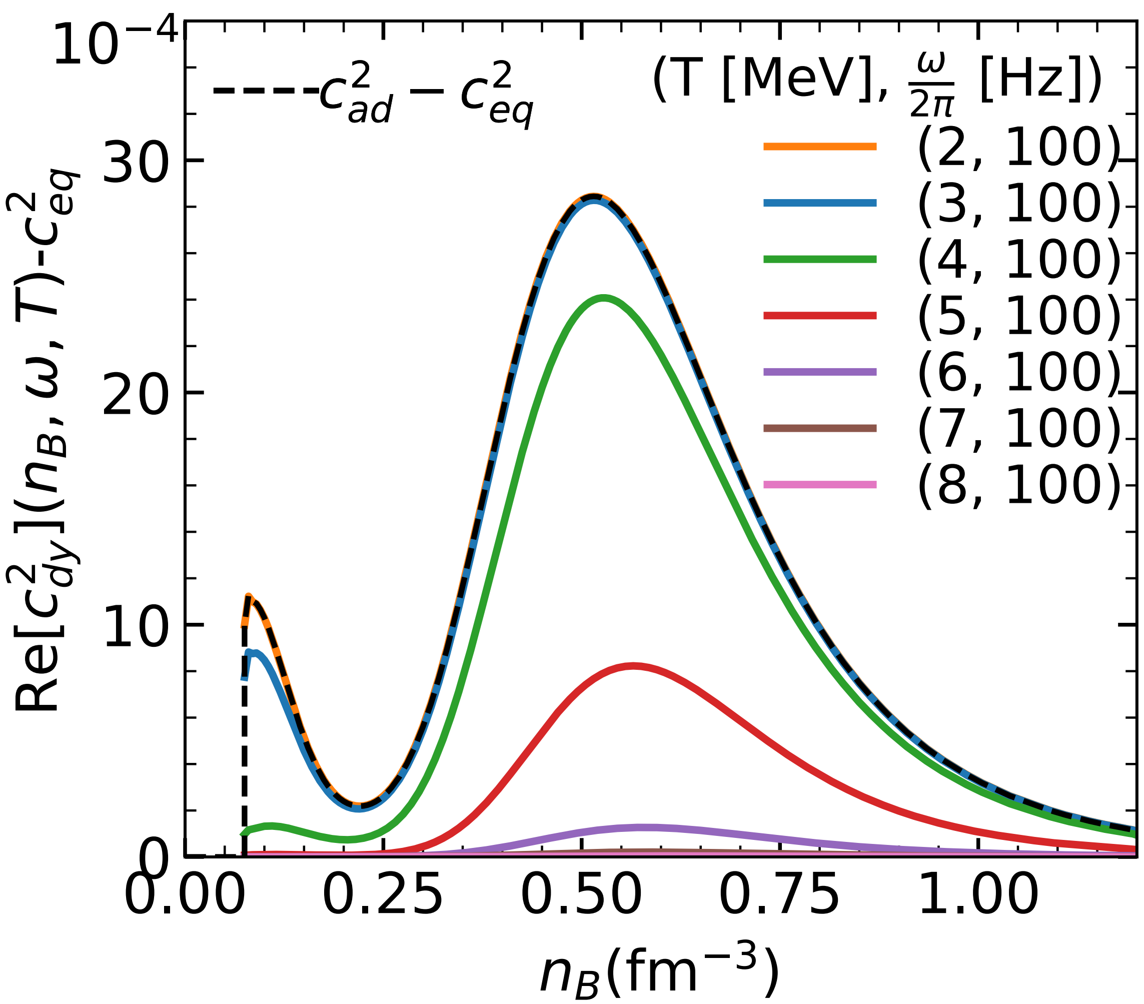
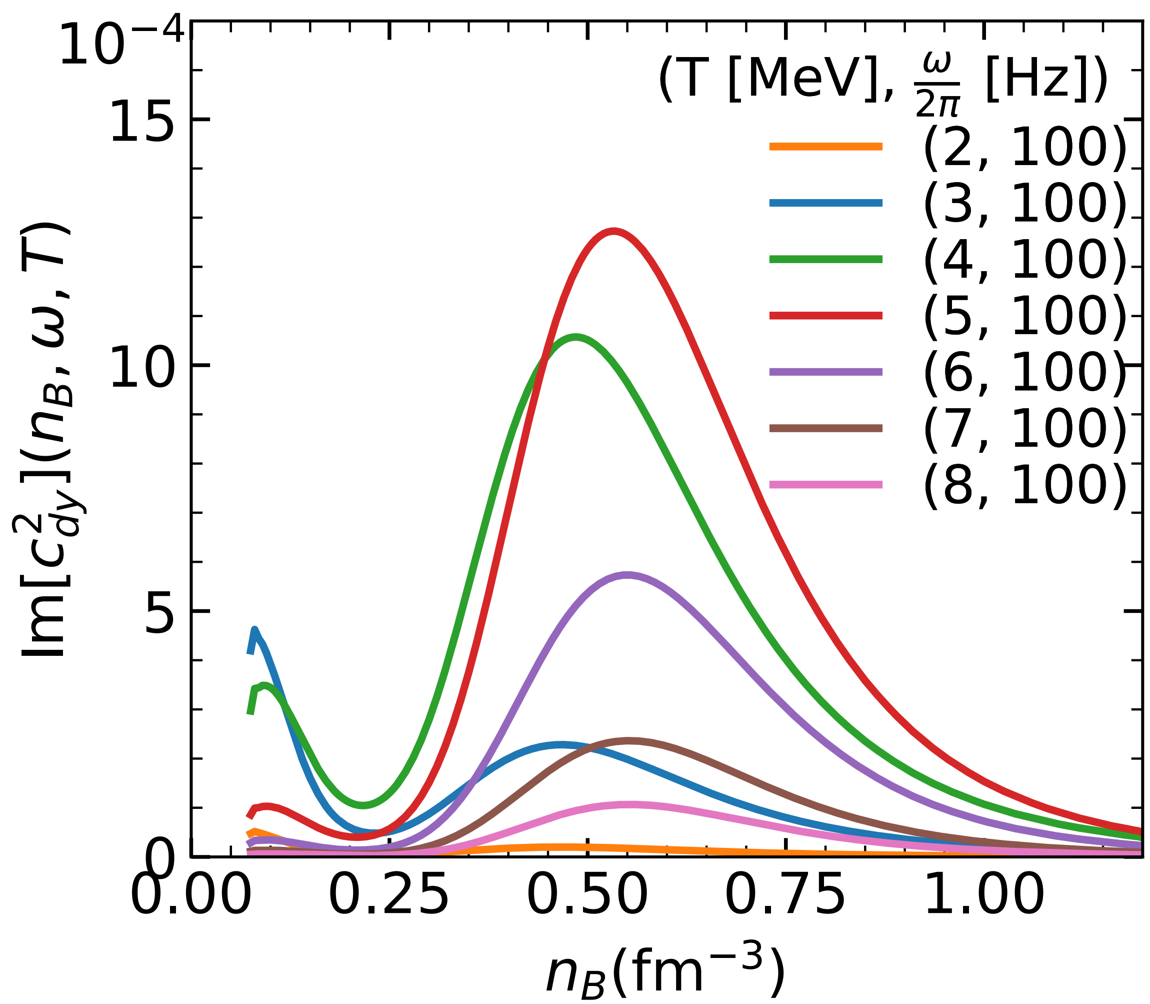
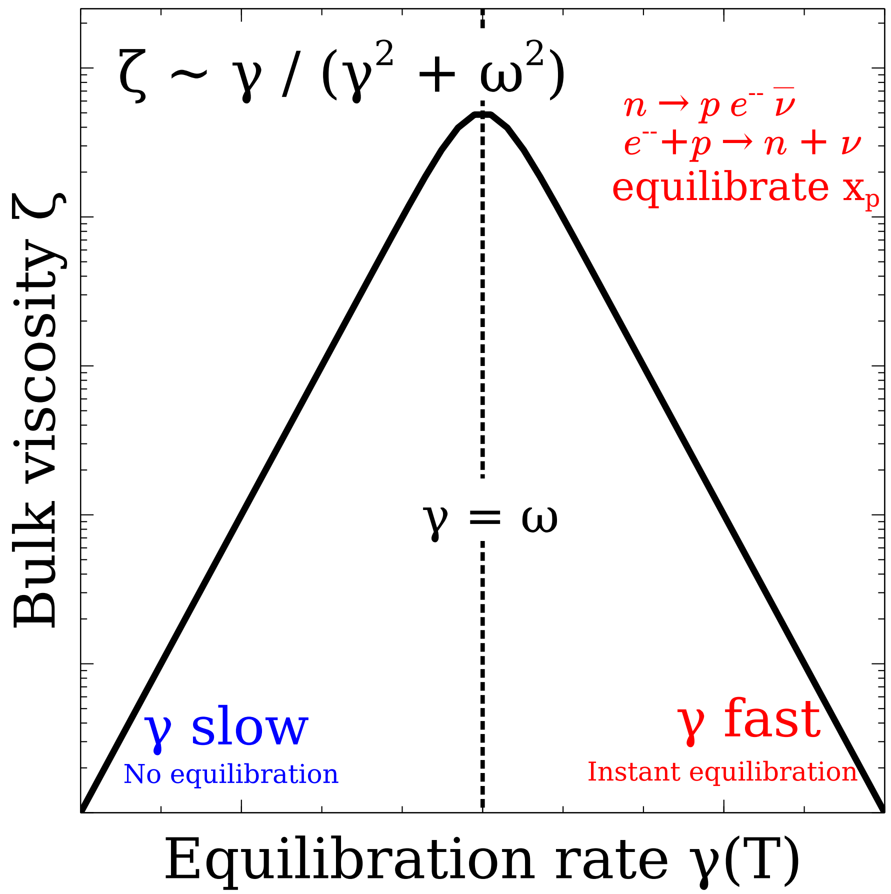


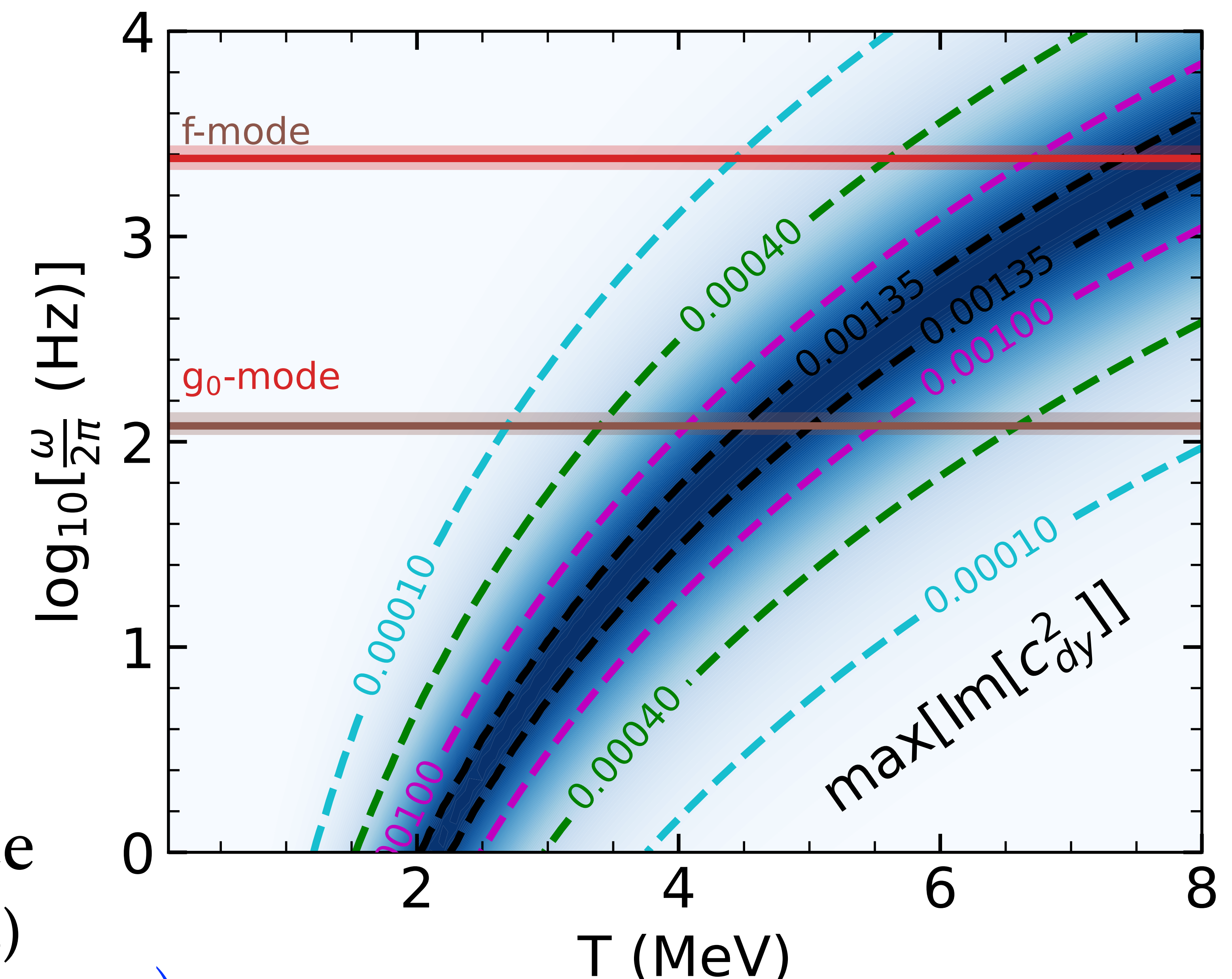


Bulk Viscosity

- Bulk viscosity $\zeta = \frac{\varepsilon + p}{\omega}\mathrm{Im}\left[c_{dy}^2\right] = \frac{\gamma}{\omega^2 + \gamma^2}(c_{ad}^2 - c_{eq}^2)(\varepsilon + p)$ quantifies energy dissipation from compressional flow,

$$\frac{d\varepsilon}{dt} = -\zeta(\nabla \cdot \mathbf{v})^2.$$

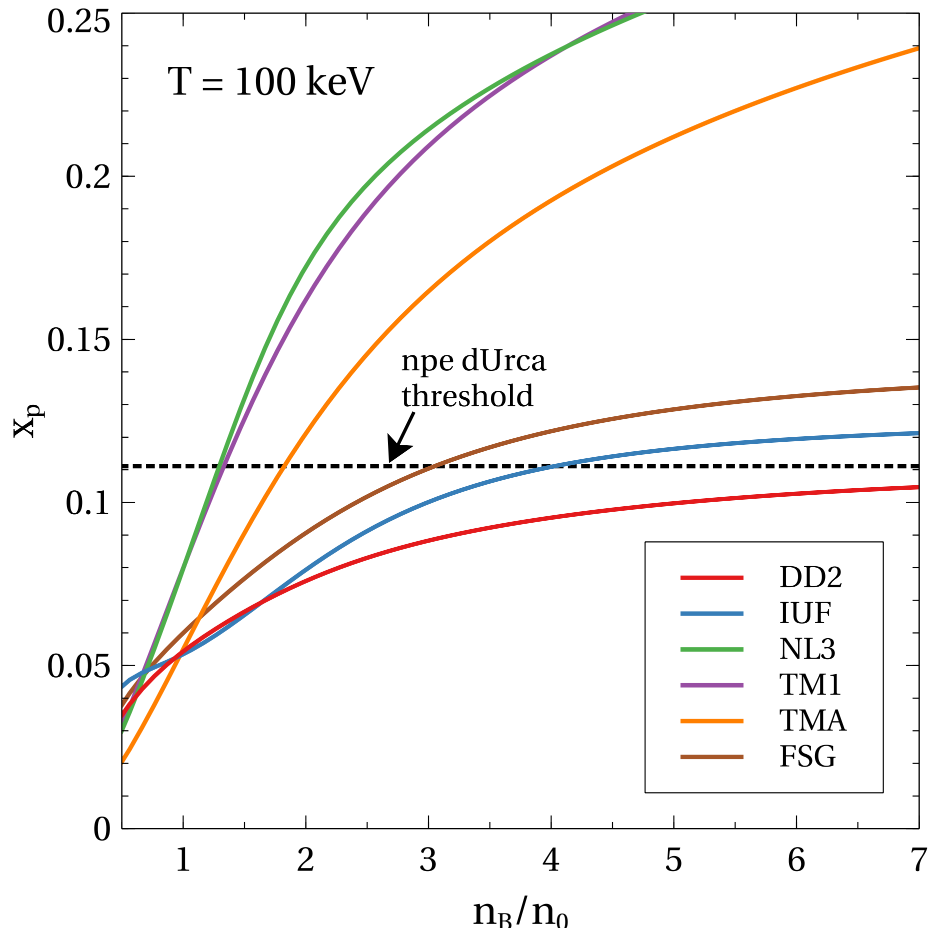




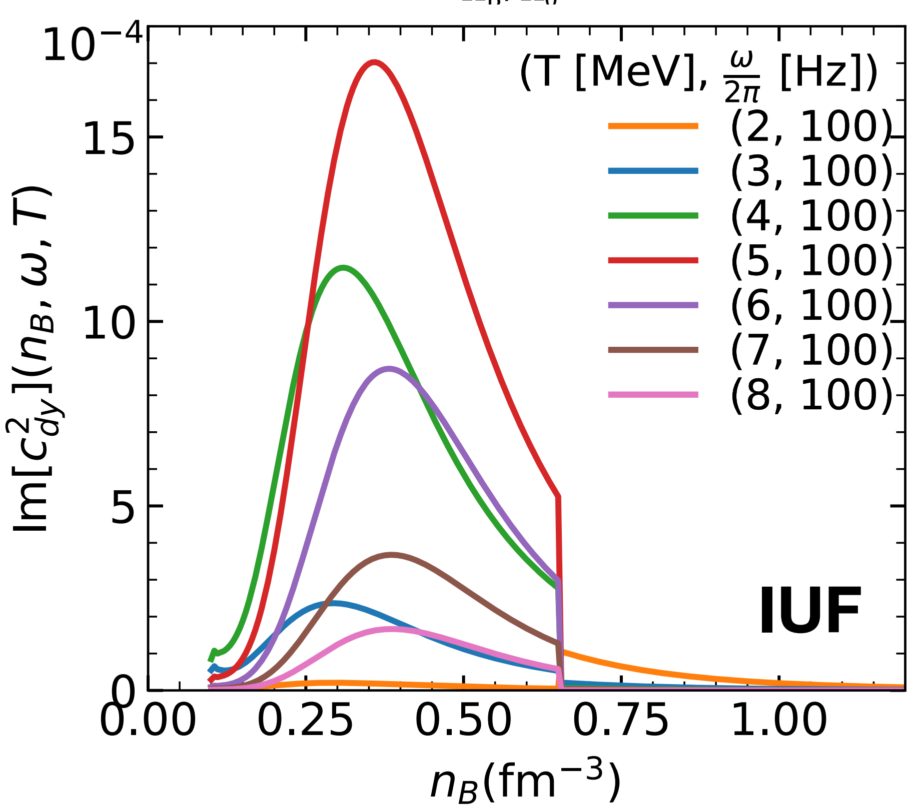
- Even modes have divergence (f-mode, g-modes, p-modes)

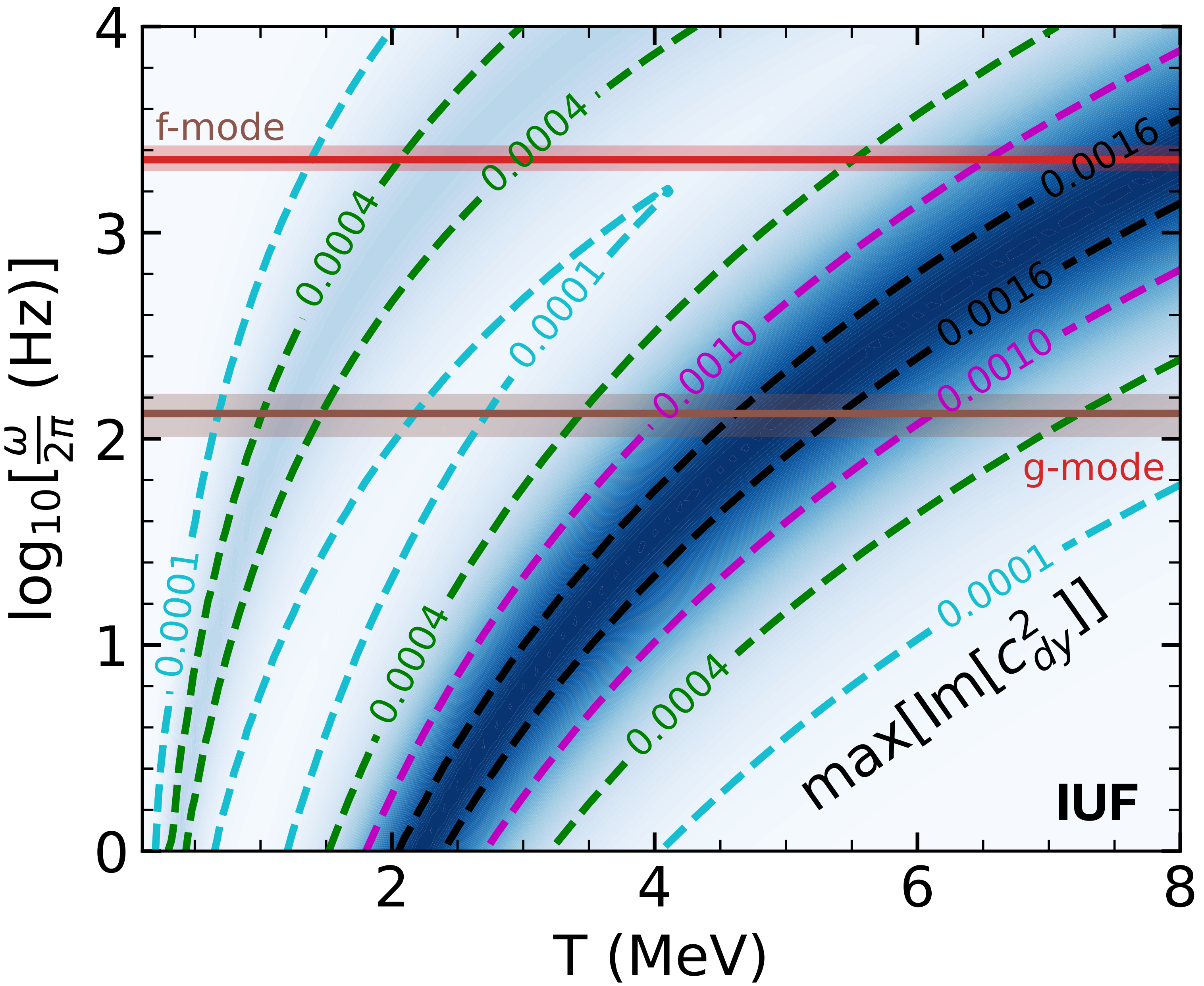
$$\nabla \cdot \xi_{\mathrm{even}}^{r,\,\theta,\,\phi} = \nabla \cdot \left(\nabla \left(R_n(r) Y_m^l(\theta, \phi) e^{\mathbf{i}\omega t} \right) \right)$$

Direct Urca Process



- Threshold: $k_n < k_e + k_p = 2k_p$, equivalent to $n_n/n_p > 8 \longrightarrow x_p > 1/9$





ODEs of Non-radial Oscillation

Eigen value problem of even quasi-normal modes

- Solve Einstein's equations:

$$8\pi\delta\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) = \delta R_{\mu\nu}$$

with fluid perturbations, and metric perturbations:

$$\xi_{\text{even}}^{r,\,\theta,\,\phi} = \nabla\left(R_n(r)Y_m^l(\theta,\phi)e^{\mathbf{i}\omega t}\right)$$

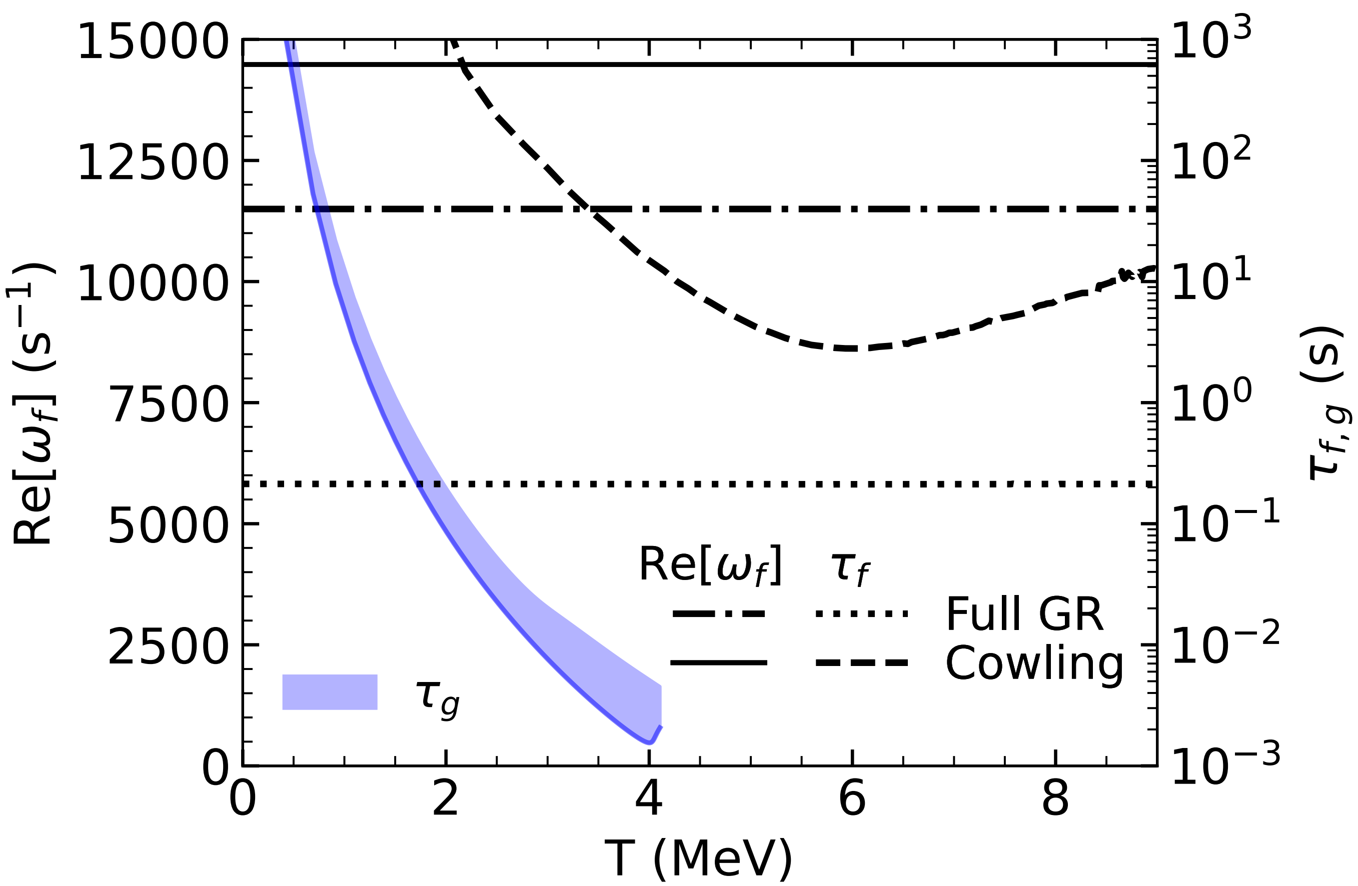
$\delta T_{\mu\nu}$ **from** $\xi^{r,\,\theta,\,\phi}$ **and** c_{dy}^2

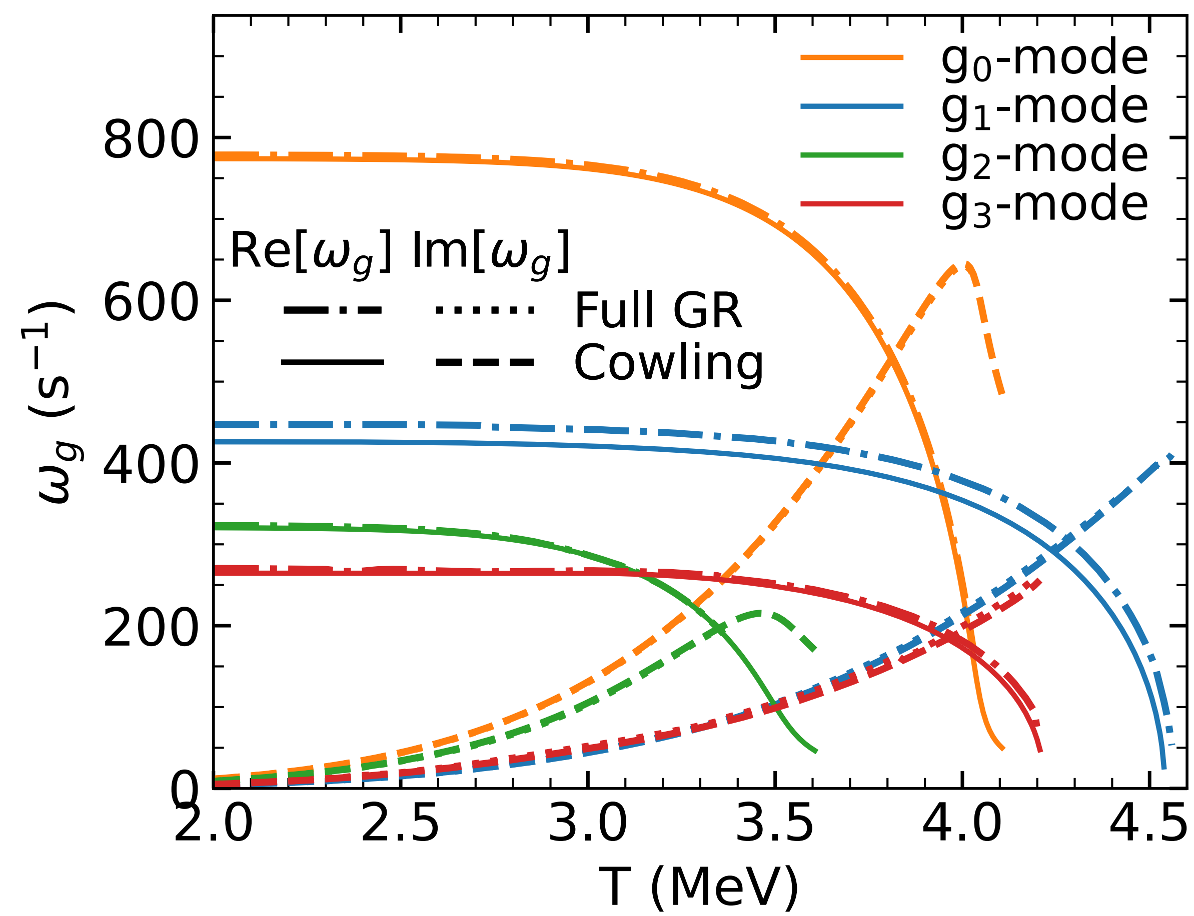
$$\delta g_{\mu\nu}^{even} = \begin{pmatrix} H_0 e^{\nu} & H_1 & 0 & 0 \\ H_1 & H_2 e^{\lambda} & 0 & 0 \\ 0 & 0 & r^2 K & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta\ K \end{pmatrix} Y_{lm}(\theta,\phi)e^{\mathbf{i}\omega t}$$

- To get eigenvalue $\omega = 2\pi\nu + \frac{\mathbf{i}}{\tau}$,

and eigenfunction $\xi_{\text{even}}^{r,\,\theta,\,\phi}$ and H_0, H_1, H_2, K

Impact of bulk viscosity on $\omega = \mathrm{Re}[\omega] + \frac{i}{\tau}$

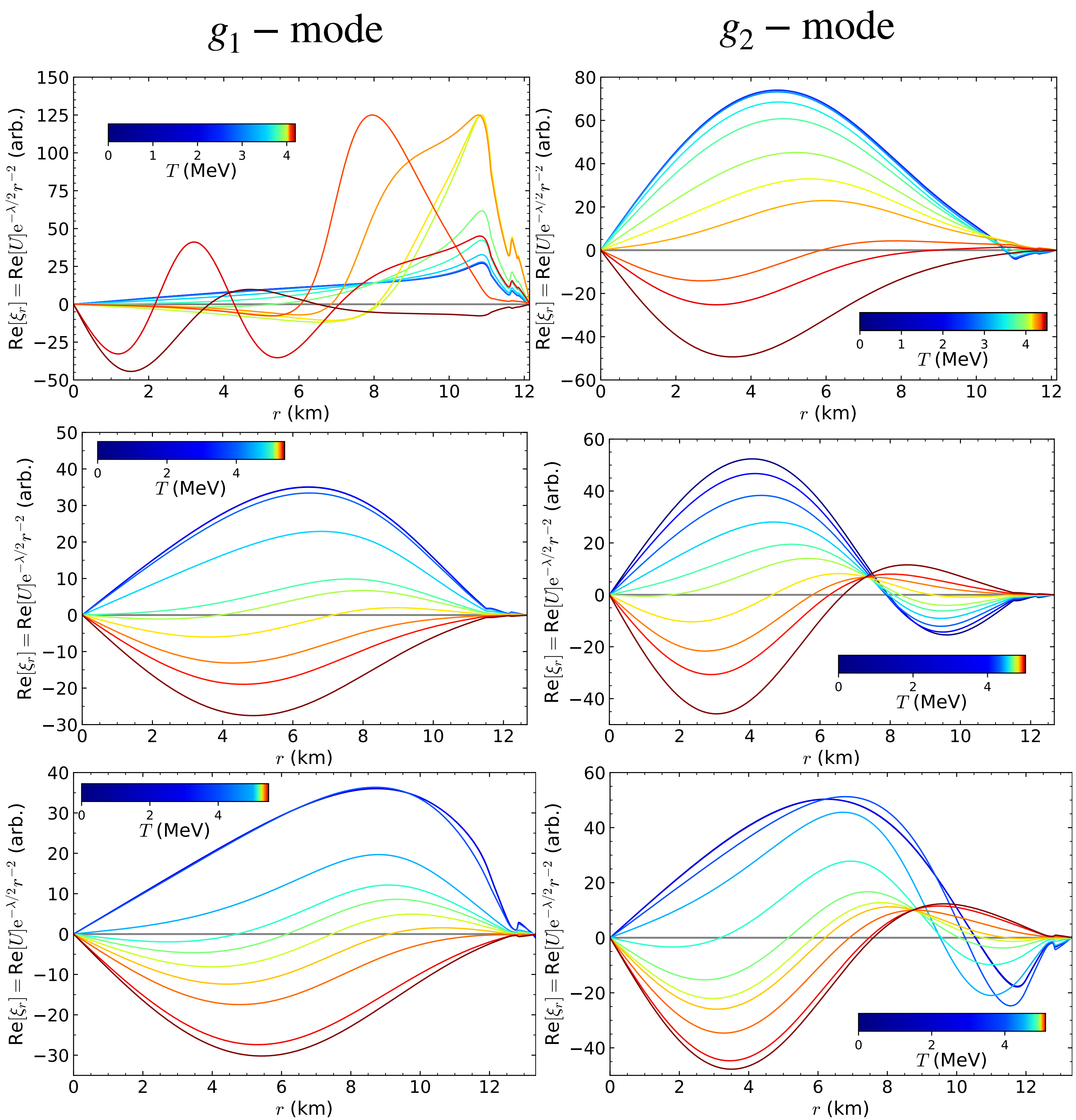




- No impact to f-mode. f-mode is semi divergence-free.
- g-mode gets suppressed at larger T. Avoided crossing between g-modes.

Fluid perturbation

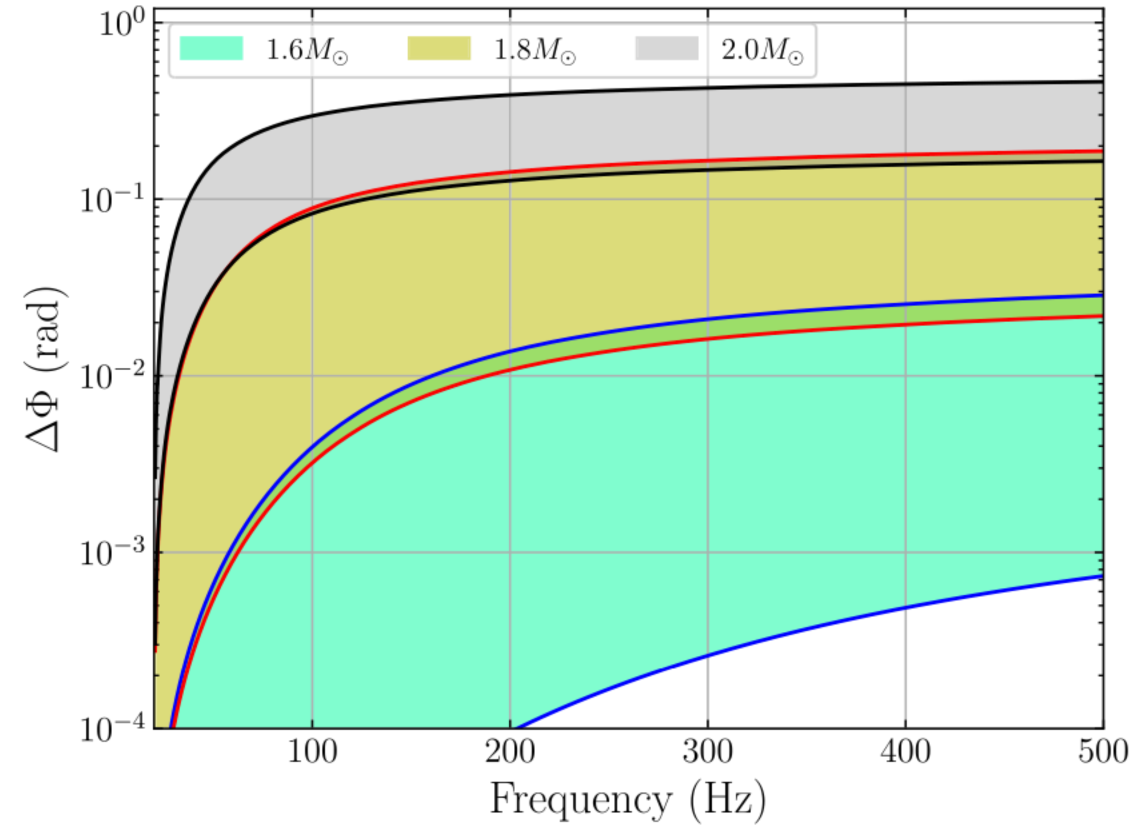
- QMC-RMF3
- IUFSU
- IOPB-I



Where this matters:

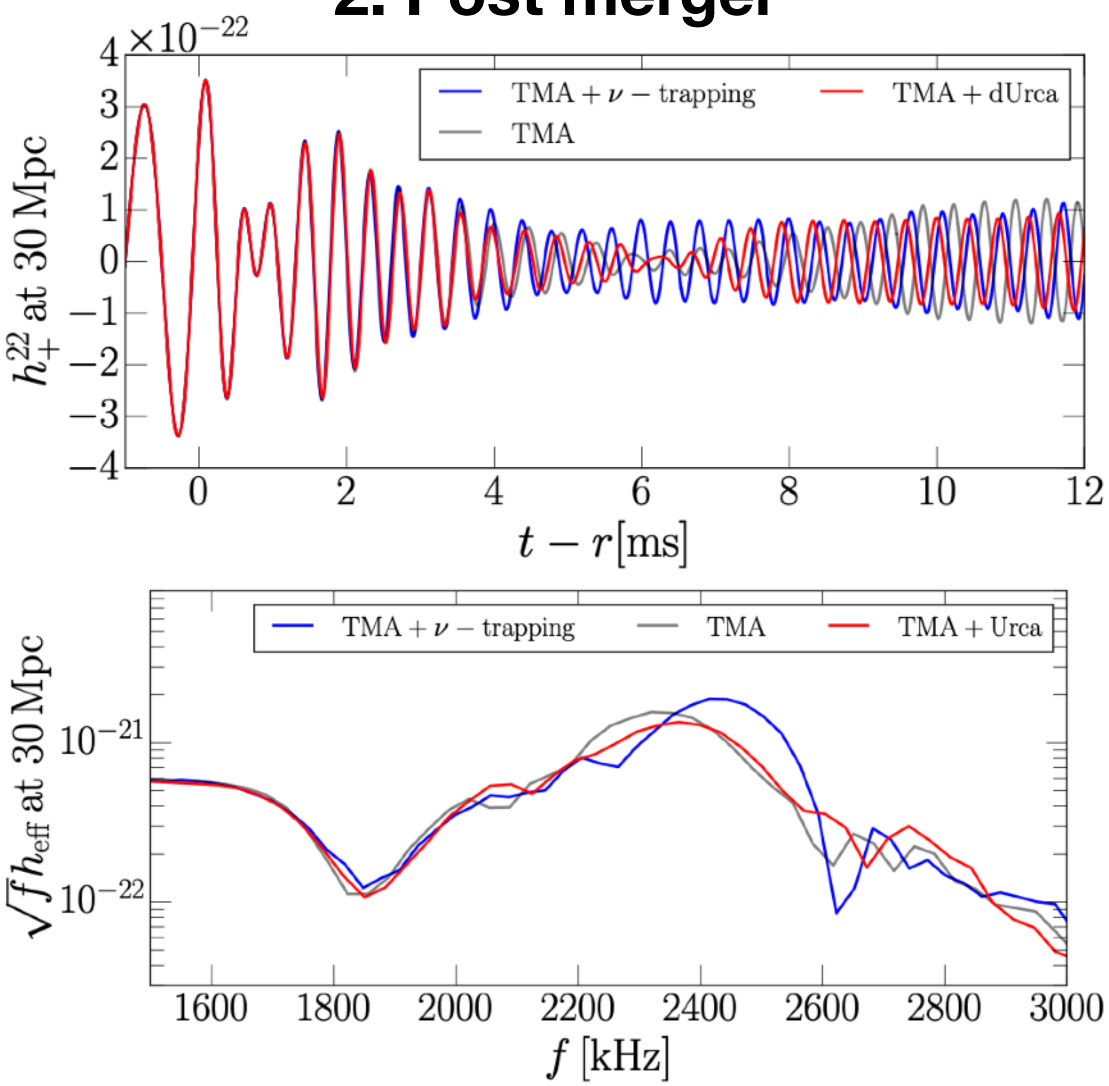
- $\zeta = \frac{\varepsilon + p}{\omega} \mathrm{Im}\left[c_{dy}^2\right]$

1. Merger spiraling



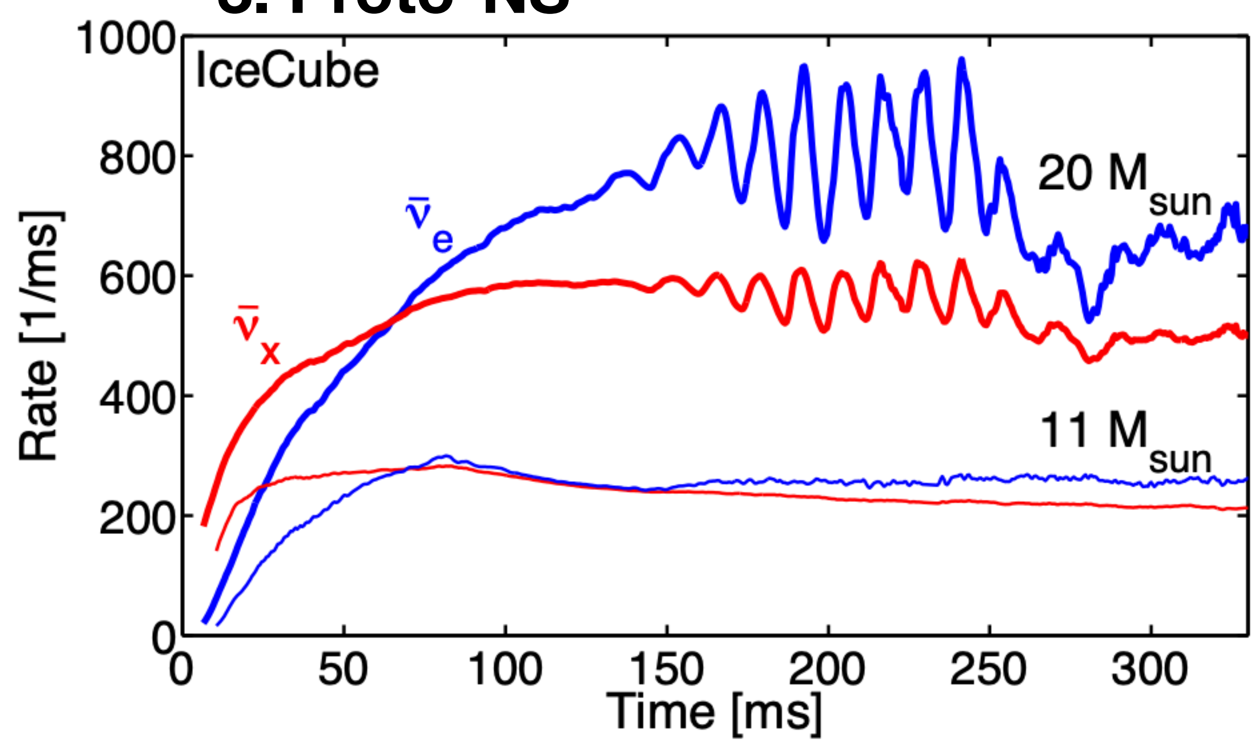
Ghosh, Pradhan and Chatterjee 2306.14737

2. Post merger



Most, Haber, Harris, Zhang, Alford, and Noronha 2207.00442

3. Proto-NS



Tamborra, Hanke, Muller, Janka and Raffelt 1307.7936

To-do Next

- Beyond the Fermi surface approximation
- More realistic composition & entropy profile.
- Consider the Neutrino-trapped scenario.
- Gravitational waveform with tidal excitation in NS spiraling.
- Viscosity due to other degrees of freedom: muons, pions, hyperons, deconfined quarks, and perhaps dark matter ...

Presenter: Tianqi Zhao

Collaborators: Peter Rau, Alexander Haber, Steven Harris, Constantinos Constantinou, Sophia Han, Prashanth Jaikumar, Sanjay Reddy, Madappa Prakash, James Lattimer

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