Decoherence
in neutrino oscillation

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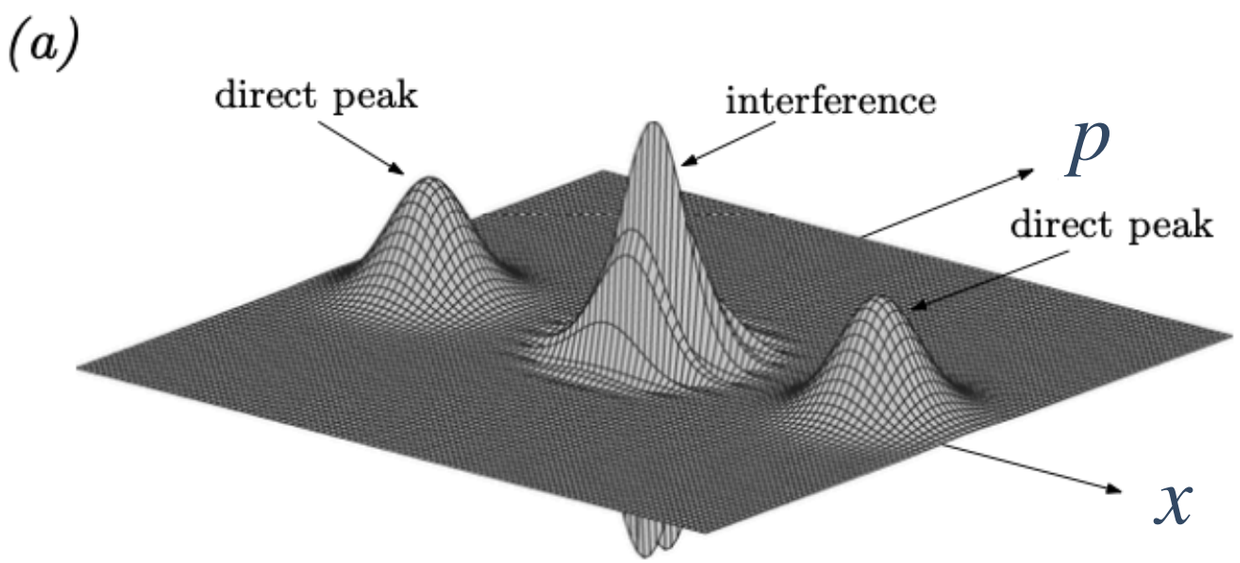
@ CIPANP, Madison, Wisconsin, 6.11.2025

In collaboration with
Manfred Lindner & Werner Rodejohann

Fermilab

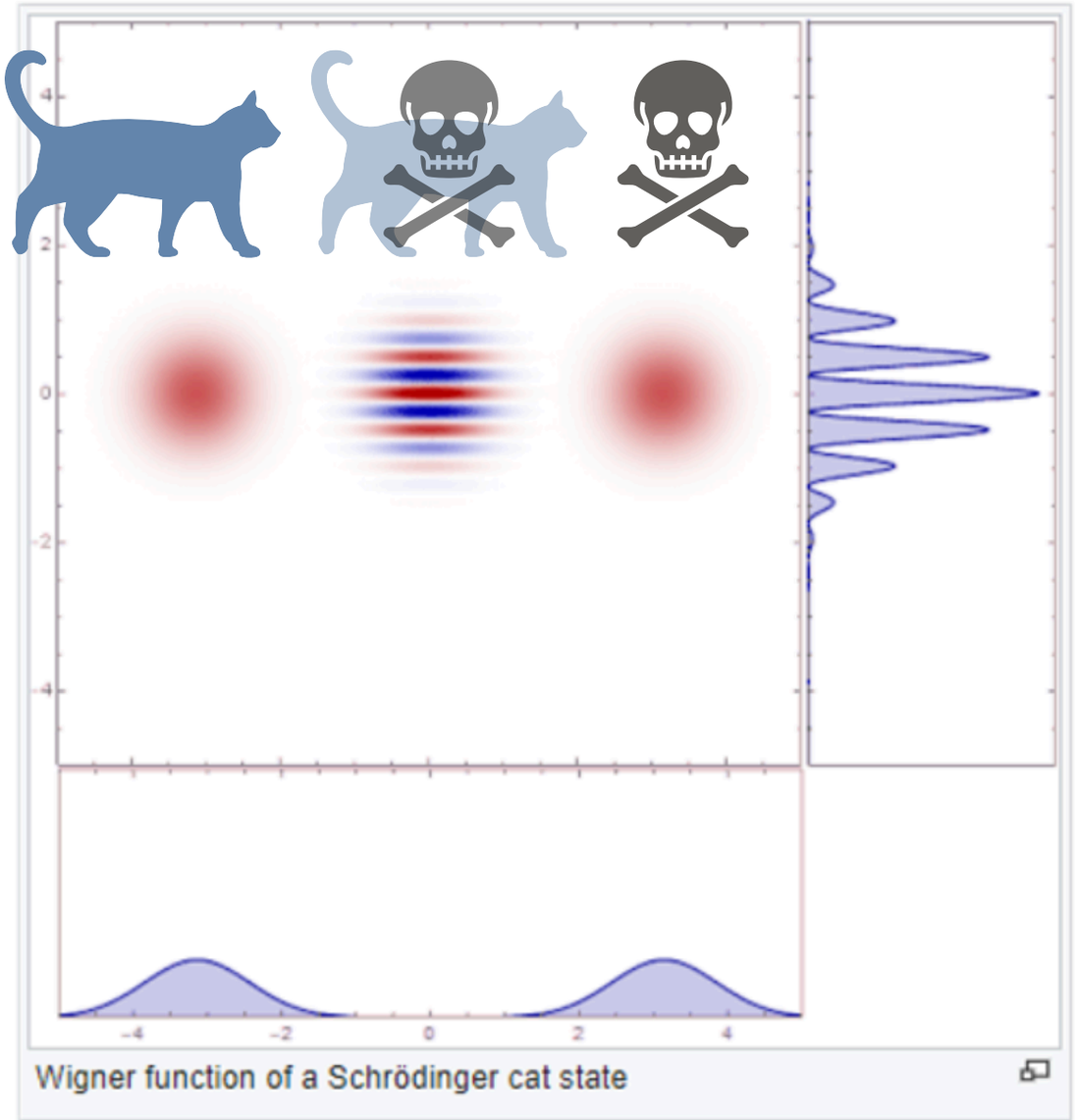
Schrödinger cat state & Wigner quasi-probability distribution

phase space distribution of the Wigner function



["Decoherence" by M Schlosshauer]

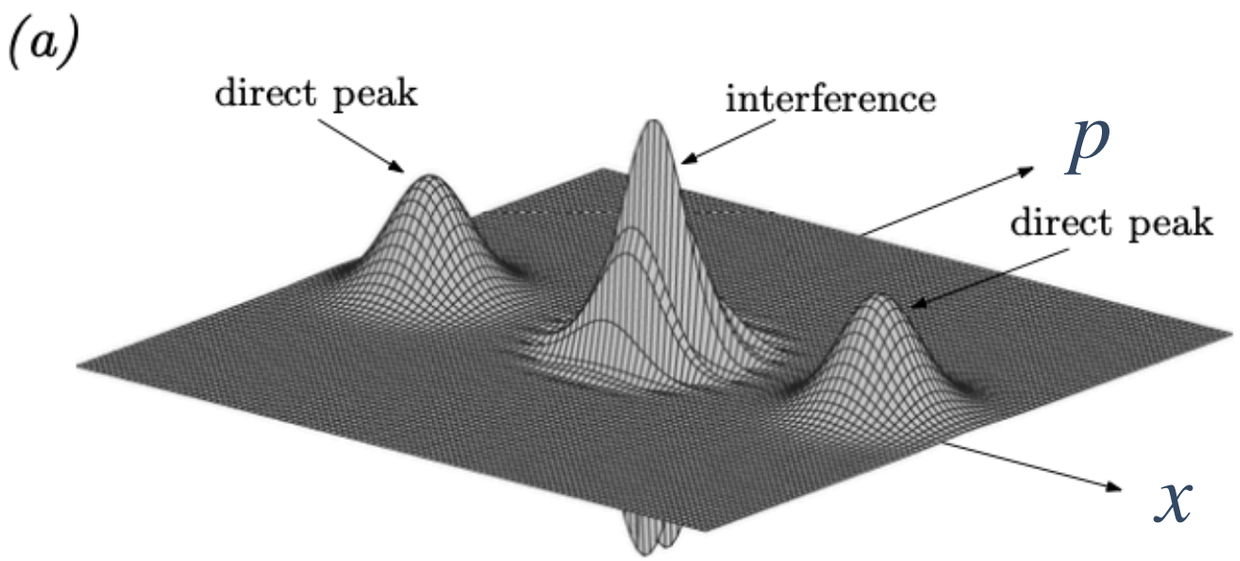
Alive Both Dead



Fermilab SQMS webpage

Schrödinger cat state & Wigner quasi-probability distribution

phase space distribution of the Wigner function



Heisenberg uncertainty principle:

$$\Delta x \Delta p \geq \frac{1}{2}$$

How to describe the state?

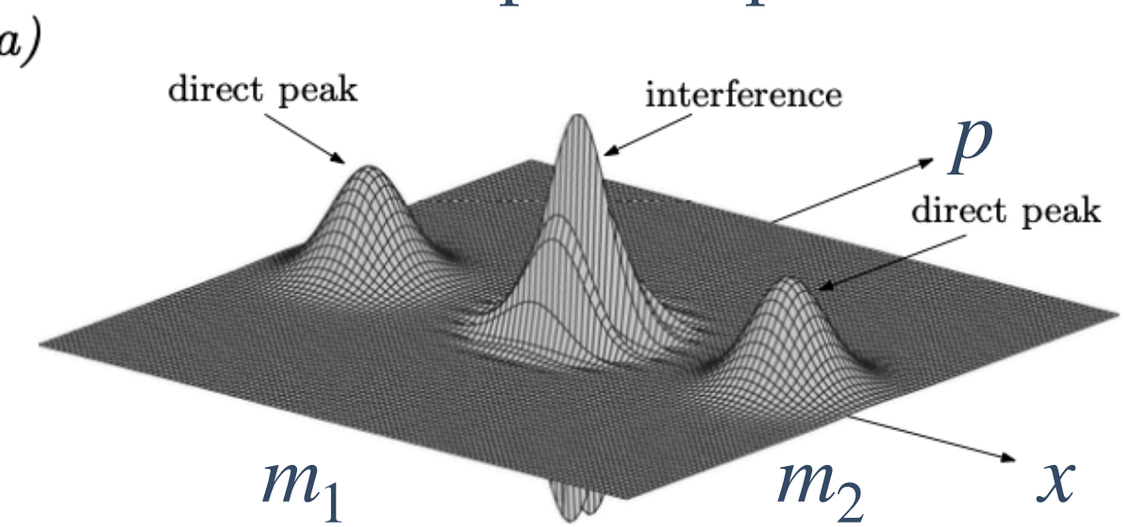
$$\rho(x) \xleftrightarrow{\text{Fourier}} \tilde{\rho}(p)$$

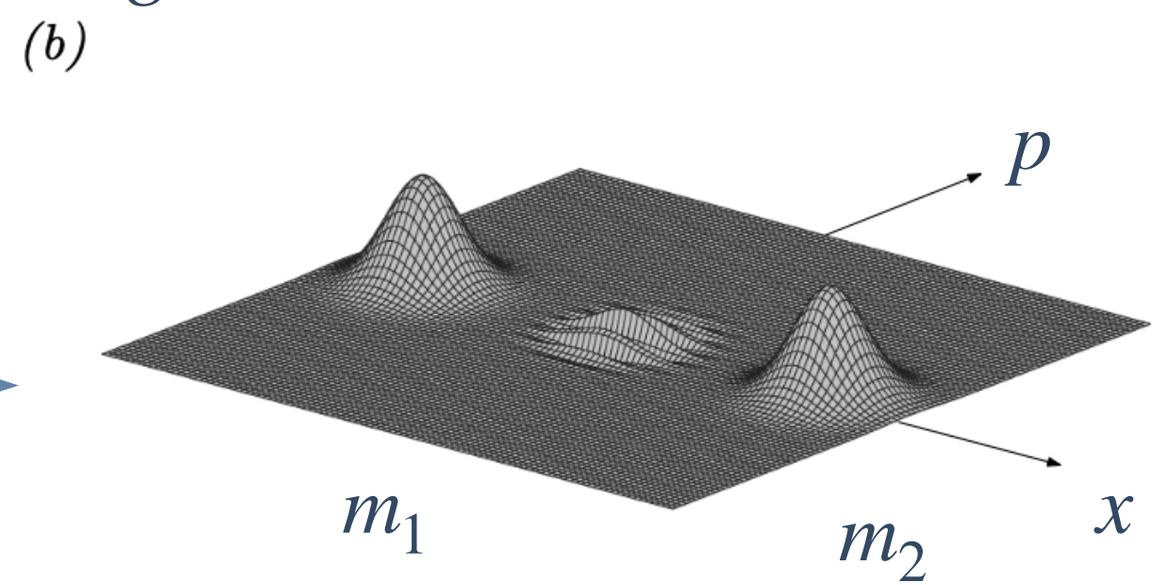
Together?: Wigner function

$$W(x,p) = \frac{1}{2\pi\hbar}\int_{-\infty}^{+\infty} \mathrm{d}y \, \exp\left(\frac{ipy}{\hbar}\right) \rho(x+y/2, x-y/2),$$

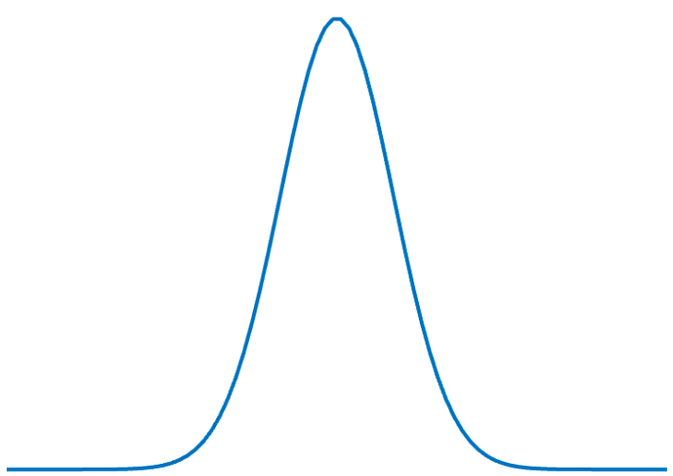
Quantum → Classical: Decoherence

phase space distribution of the Wigher Function





$m_1 \& m_2$



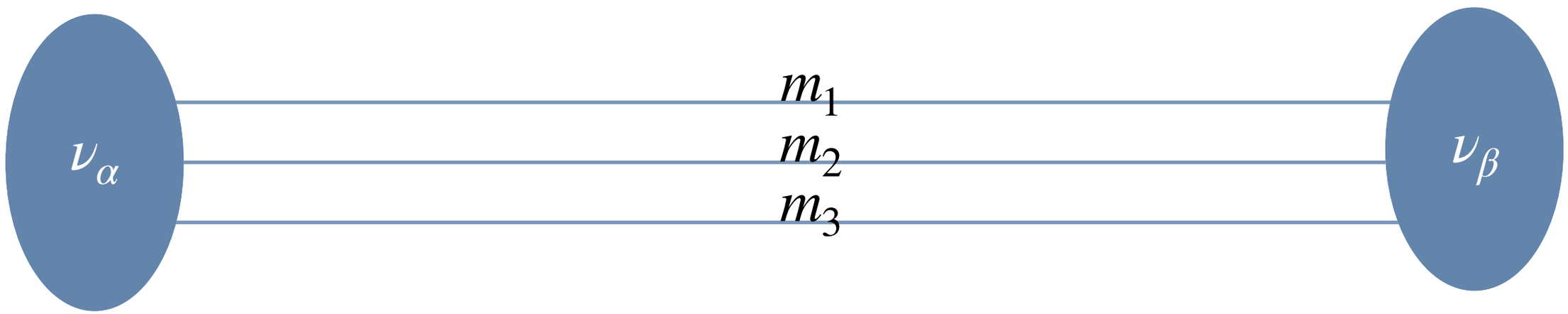
$$\Delta x \Delta p = \frac{1}{2}$$

$m_1 \& m_2$

$$\Delta x \Delta p > \frac{1}{2}$$

Neutrino Oscillation

Uncertainty | Superposition | Uncertainty



B. Kayser (1981), J. Rich (1993), E. Akhmedov, A. Smirnov (2009)

The uncertainties: give and take away coherence

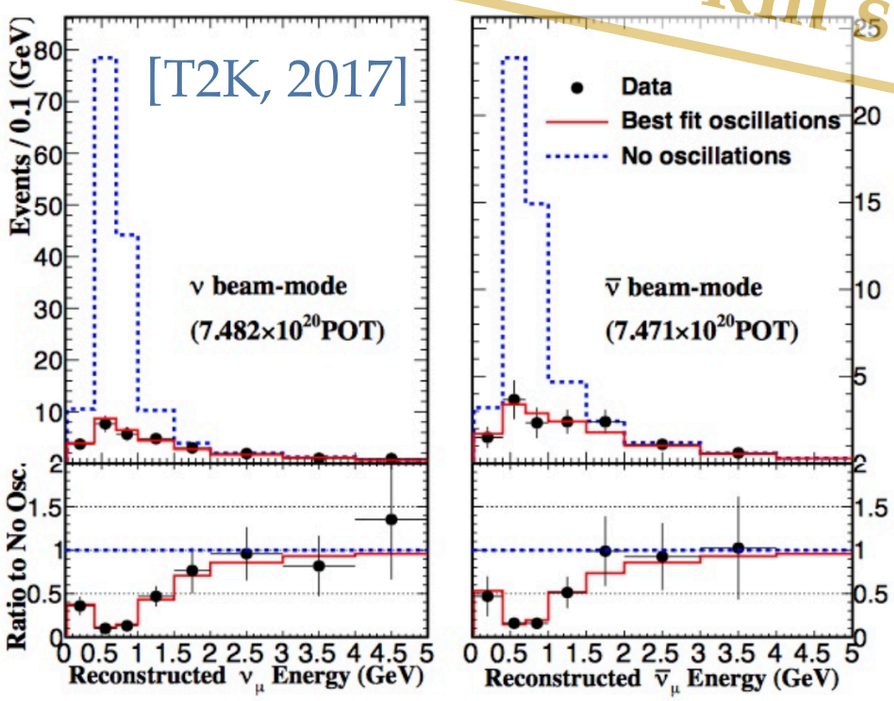
This talk

- Flavor transition probability: $P_{\nu_\alpha \to \nu_\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} \exp\left(-i \frac{\Delta m^2_{jk} L}{2E}\right)$
- Small mass splitting ~100% overlap of each mass eigenstates @ km scale
- Neutrinos interact weakly Closed quantum system

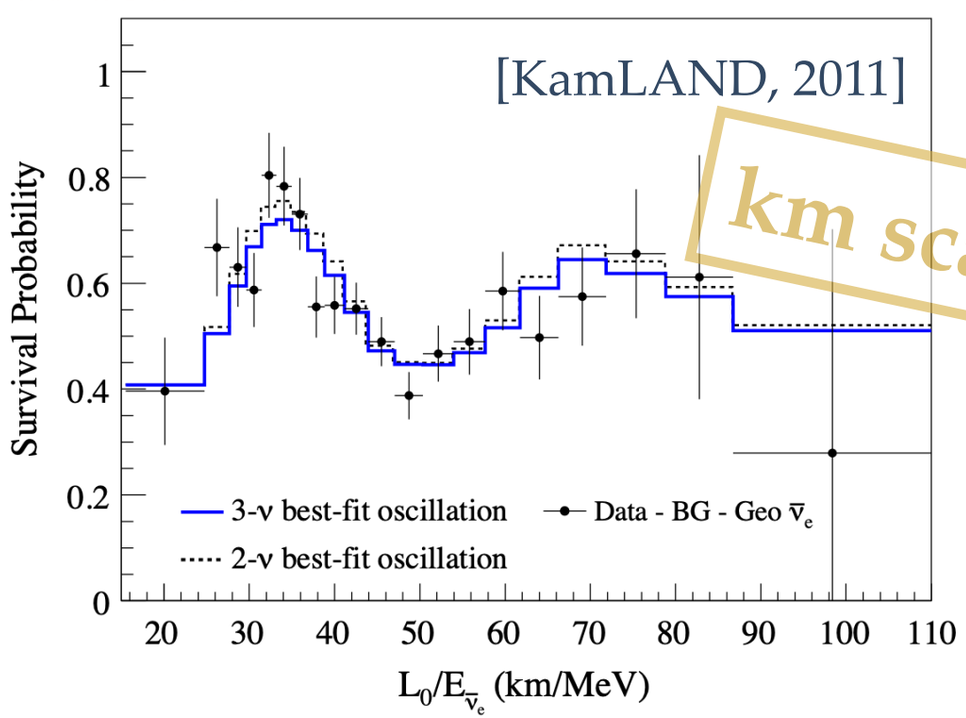
Neutrino Oscillation: *Large* scale *quantum* effect

Accelerator ν

10^{2-3} km scale

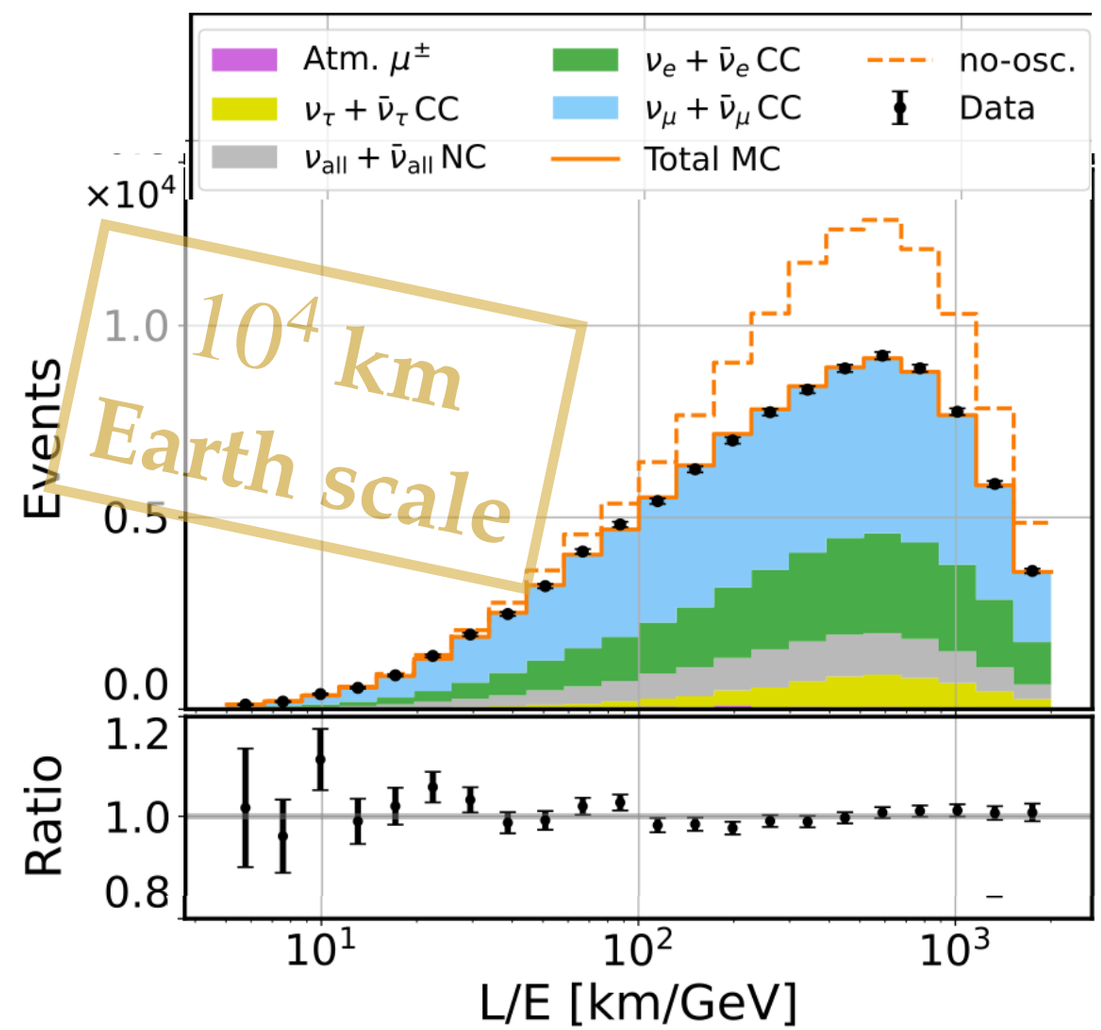


Reactor ν



Atmospheric ν

[IceCube, 2024]



Neutrino Oscillation: *Large* scale *quantum* effect

- Small mass splitting ~100% overlap of each mass eigenstates @ km scale
- Neutrinos interact weakly Closed quantum system

In the precision era of neutrino oscillations, can we go beyond this?

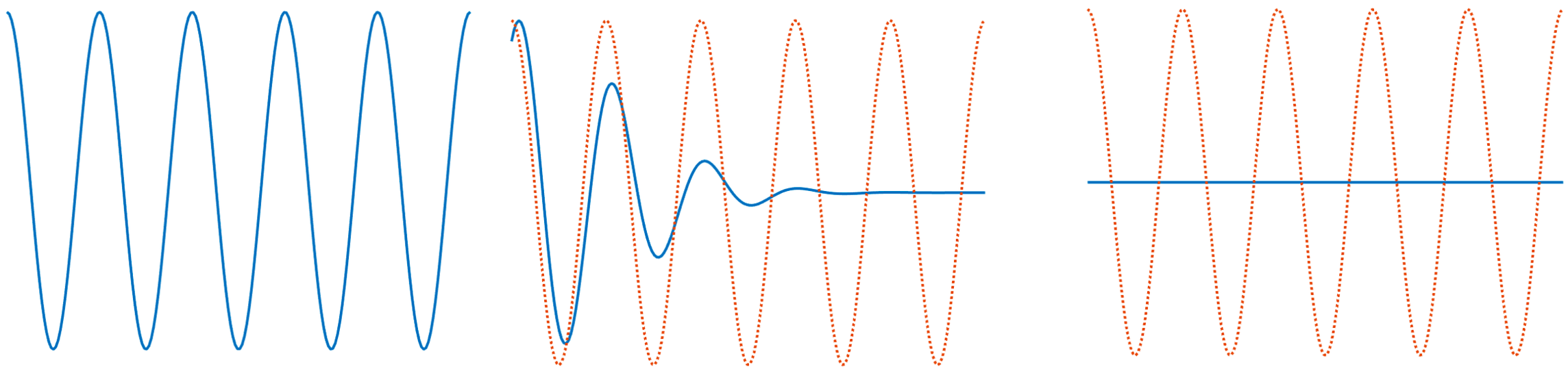
In terms of what?

To see what?

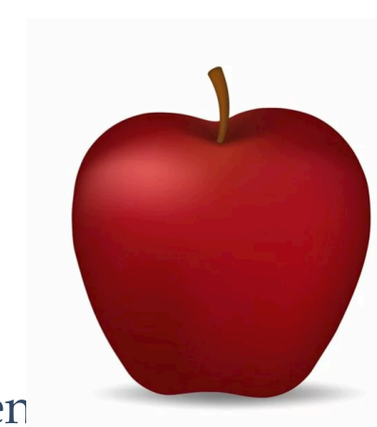
Short answer:
"Quantum information" from Neutrino Decoherence

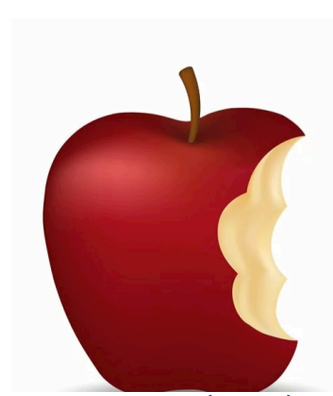
FTP: $P_{3,\alpha\to\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i\frac{\Delta m^2_{jk} L_0}{2E_0}} \phi_{jk}(\vec{\sigma}; L_0, E_0)$ $\quad 0 \leq |\phi_{jk}| \leq 1$

Coherent (Quantum) | Decoherent | Classical



Damping + Phase shift (if ϕ_{jk} has an imaginary part)

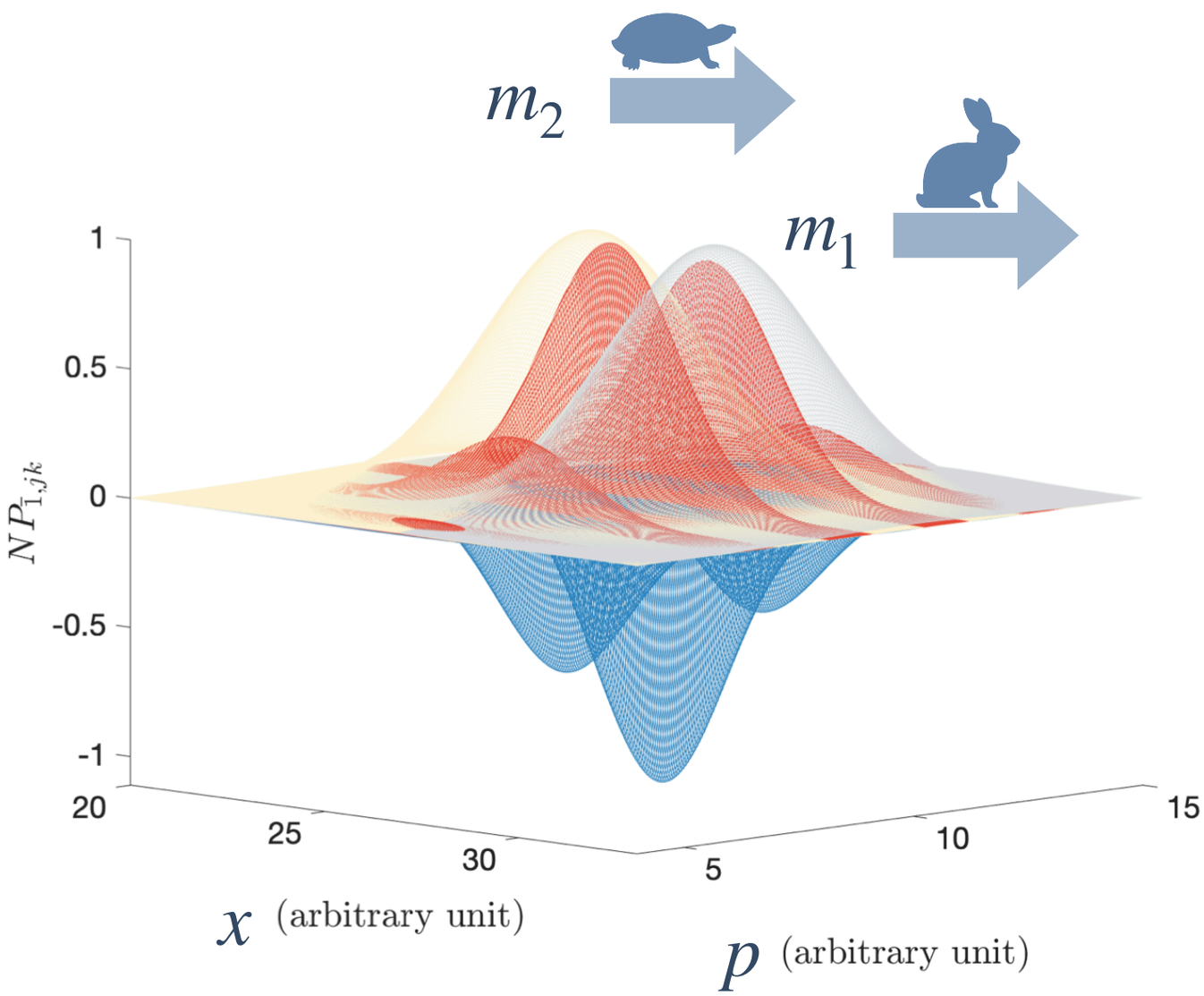




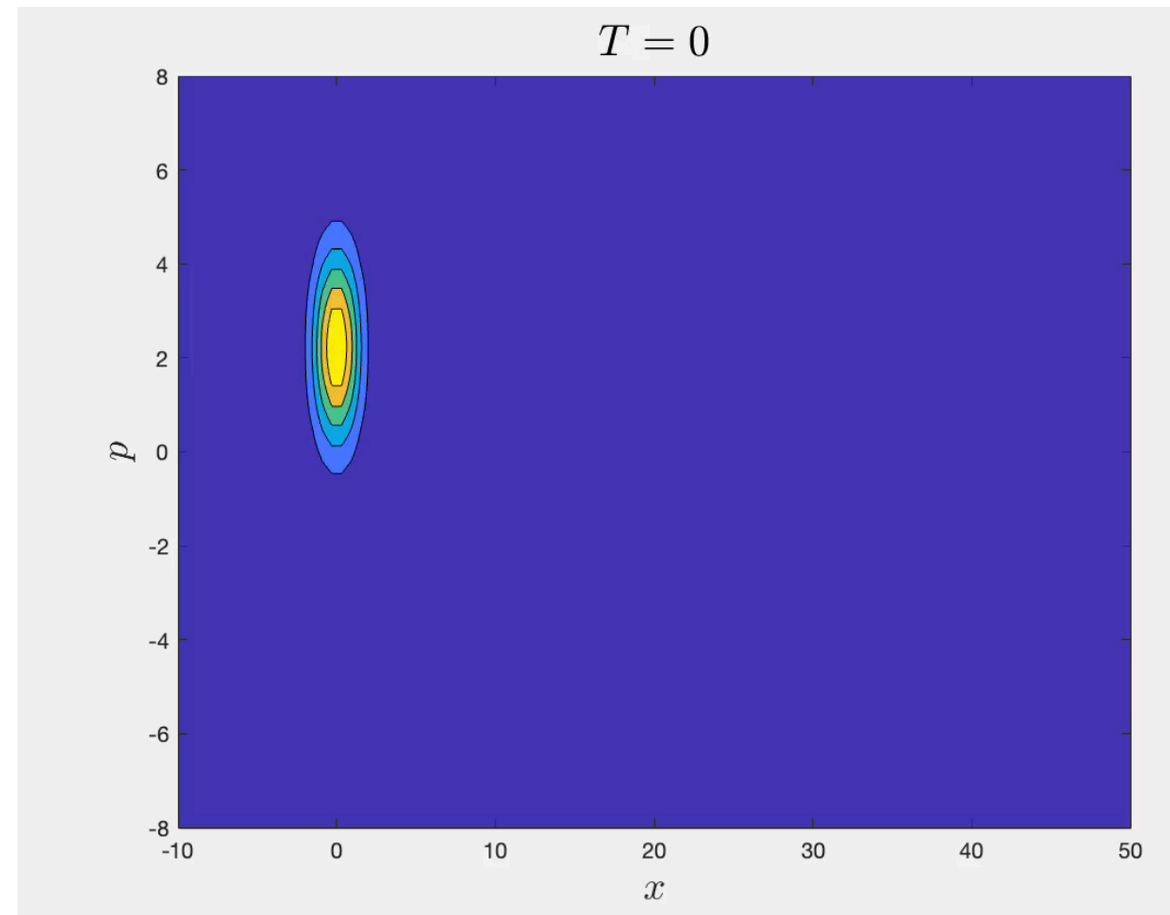
source: https://designbundles.net/vectortatu/3053943-realistic-bitten-apple

Wavepacket Separation

Wigner quasi-probability distribution



$$\left| A(m_1) + A(m_2) \right|^2$$



[TC, M Lindner, W Rodejohann, 2204.10696]

Open quantum system: Decoherence

Where does the width come from?
& how to combine different contribution?

Open quantum system: $\frac{d}{dt}\rho(t) = i\left[\mathcal{H}(t), \rho(t)\right] + \mathscr{D}[\rho]$

Evolution: non-unitary, trace preserving

e.g. $\nu_e + e$: CC

$q^2 = 0$: forward scattering (MSW effect) unitary: $H \to H'$

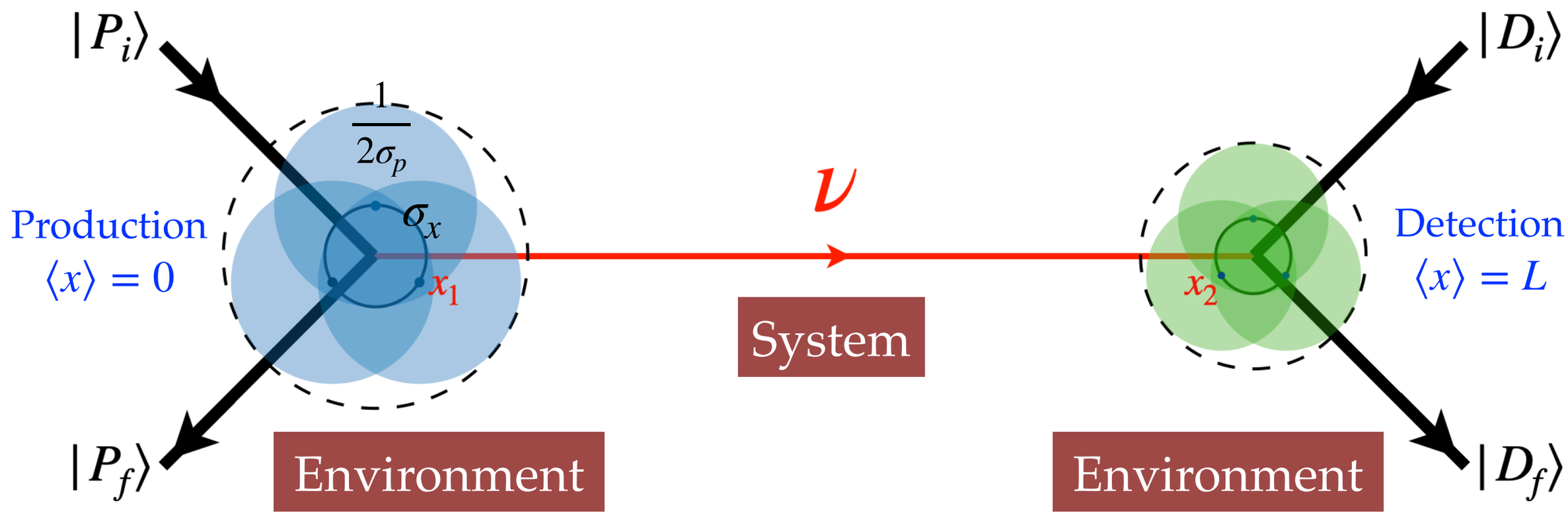
$Q^2 > 0$: attenuation (on the electron side: detection) trace non-preserving

… so none of the above
$(Q^2 = 0, q^2 > 0)$

QFT + open quantum system

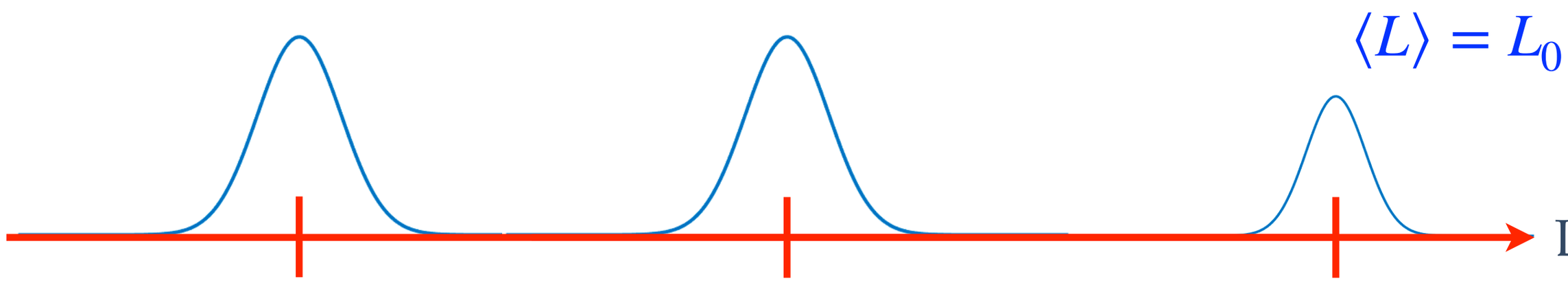
- Intrinsic uncertainties: unavoidable, can only be squeezed
- Quantum superposition: sum over amplitude
- σ_x : (off-shell) neutrino interactions (related to the form factor)
- σ_p : (on-shell) external particle's WP size (mean free path, life time, …)

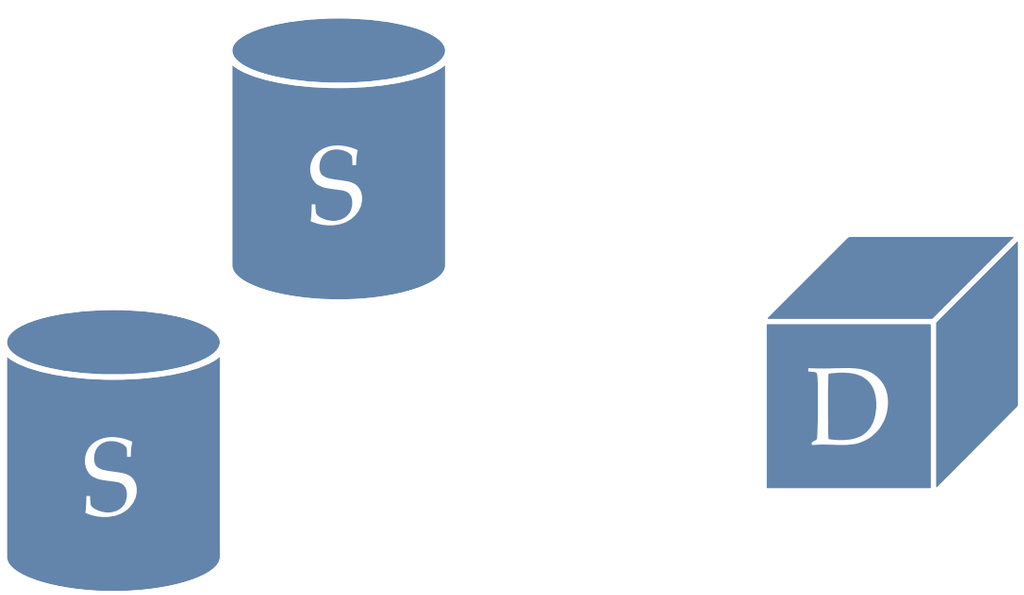
(Just a parameterisation not the total position/momentum uncertainty !)



Classical uncertainties

- Statistical averaging — sum over the probability (amplitude^2)
- Reflects our ignorance — avoidable
- What do we get in the end? Expectation value of the FTP





- σ_L : production profile, background…
- σ_E : energy reconstruction model, energy resolution …

Phase space layer structure

Measurement layer (layer 3)

- Kinematic variables : $\{ T_0 , \mathbf{L}_0 , \mathbf{P}_0 \}$ or $\{T_0 , L , E_0 , \Omega \}$ $\xrightarrow{\text{Consistent emission}}$ $\{L_0 , E_0 , \Omega\}_{T_0=0}$
- Uncertainties : Those from lower layers

$\sigma_L \; \sigma_E$ $\uparrow$ $\mathscr{L}\mathscr{M}\mathscr{O}^2$

Physical layer (layer 2)

- Kinematic variables : $\{T , \mathbf{L} , \mathbf{P}\}$
- Uncertainties : Macroscopic + Those from layer 1

$\sigma_p \; \sigma_x$ $\uparrow$ $\mathscr{L}\mathscr{M}\mathscr{O}^1$

$\uparrow$ $\mathscr{L}\mathscr{M}\mathscr{O}^{\bar{1}}$ $\sigma_{\bar{p}} \; \sigma_{\bar{x}}$

Microscopic layer (layer 1) — Fock space

- Kinematic variables : $\{x , p\} \xrightarrow{\text{on shell}} \{x , \mathbf{p}\}$
- Uncertainties : Microscopic

Microscopic layer (layer 1) — Wigner space

- Kinematic variables : $\{\bar{t} , \bar{\mathbf{x}} , \bar{\mathbf{p}}\}$ (on shell)
- Uncertainties : Microscopic

[TC, M Lindner, W Rodejohann, 2204.10696]

The layer moving operators ($\mathscr{L}\mathscr{M}\mathscr{O}$) contain the uncertainty parameters

General neutrino decoherence

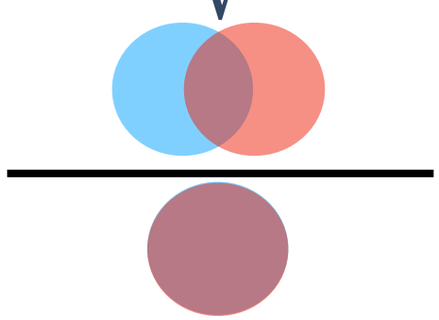
- Operation definition of FTP:

Normalised automatically

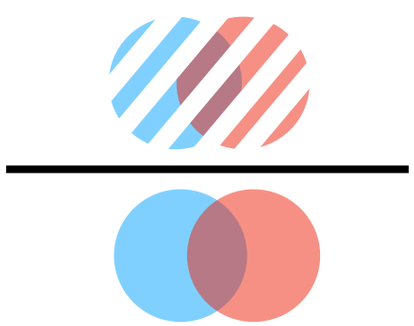
$$P_{3,jk}(X_3) = \frac{\mathscr{L}\mathscr{M}\mathscr{O}_2 \int dX_2\, H(X_2;X_3)\, \Gamma_{2,jk}(X_2;X_3)}{\sqrt{\int dX_2\, H(X_2;X_3)\, \Gamma_{2,jj}(X_2;X_3)}\sqrt{\int dX_2\, H(X_2;X_3)\, \Gamma_{2,kk}(X_2;X_3)}},$$

unnormalised $P_{2,jk}$

$$= \frac{\int dX_2 H\, |\Gamma_{2,jk}|}{\sqrt{\int dX_2\, H\, \Gamma_{2,jj}}\sqrt{\int dX_2\, H\, \Gamma_{2,kk}}} \times \frac{\int dX_2\, H\, \Gamma_{2,jk}}{\int dX_2 H\, |\Gamma_{2,jk}|}$$



$\times$



- PWO effect: $\frac{\int dx\, \Gamma(x;L)}{\int dx\, |\Gamma(x;L)|} = e^{i(\eta(x)|_{x=L}-\beta)}\Phi(L)$ for $\Gamma(x;L) \equiv |\Gamma(x;L)|e^{i\eta(x)}$

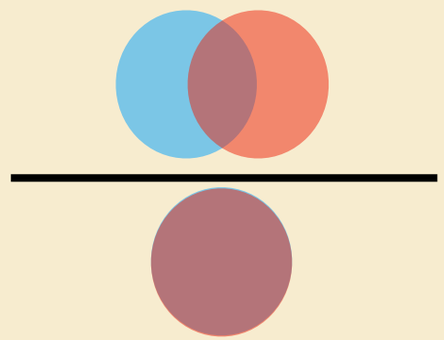
Classification of decoherence

$\phi_{jk} =$ **State Decoherence** $\times$ **Phase Decoherence**

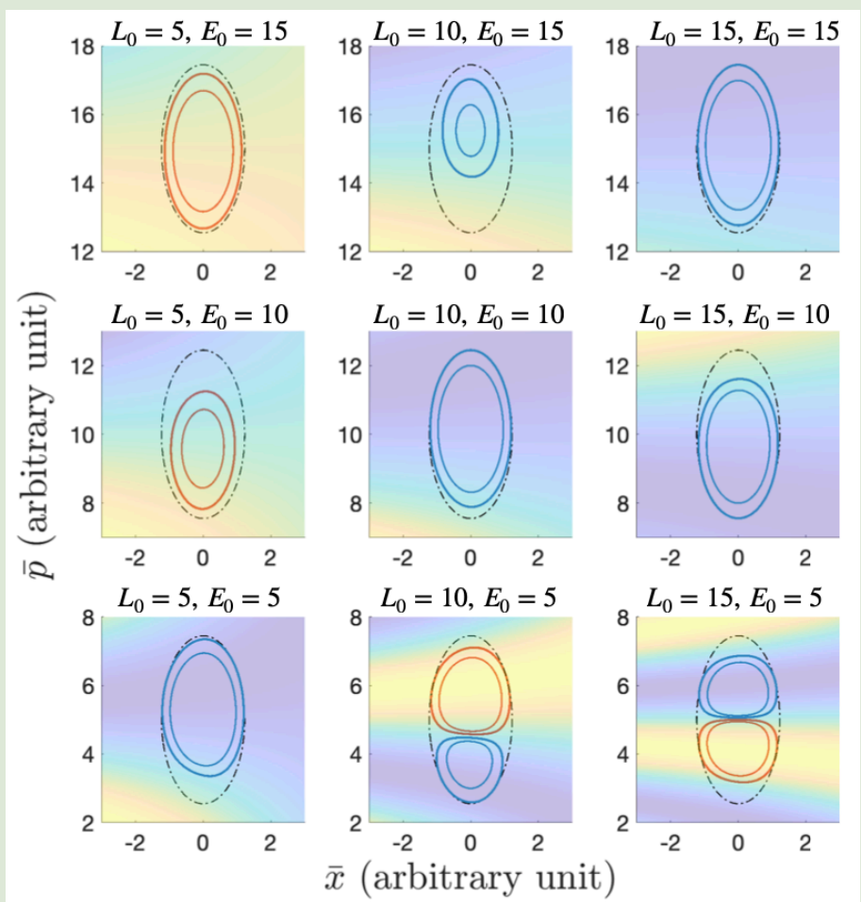
Quantum effects σ_x, σ_p

Dominated by classical effects σ_L, σ_E

Physical Layer



Micro. Layer, Wigner PS



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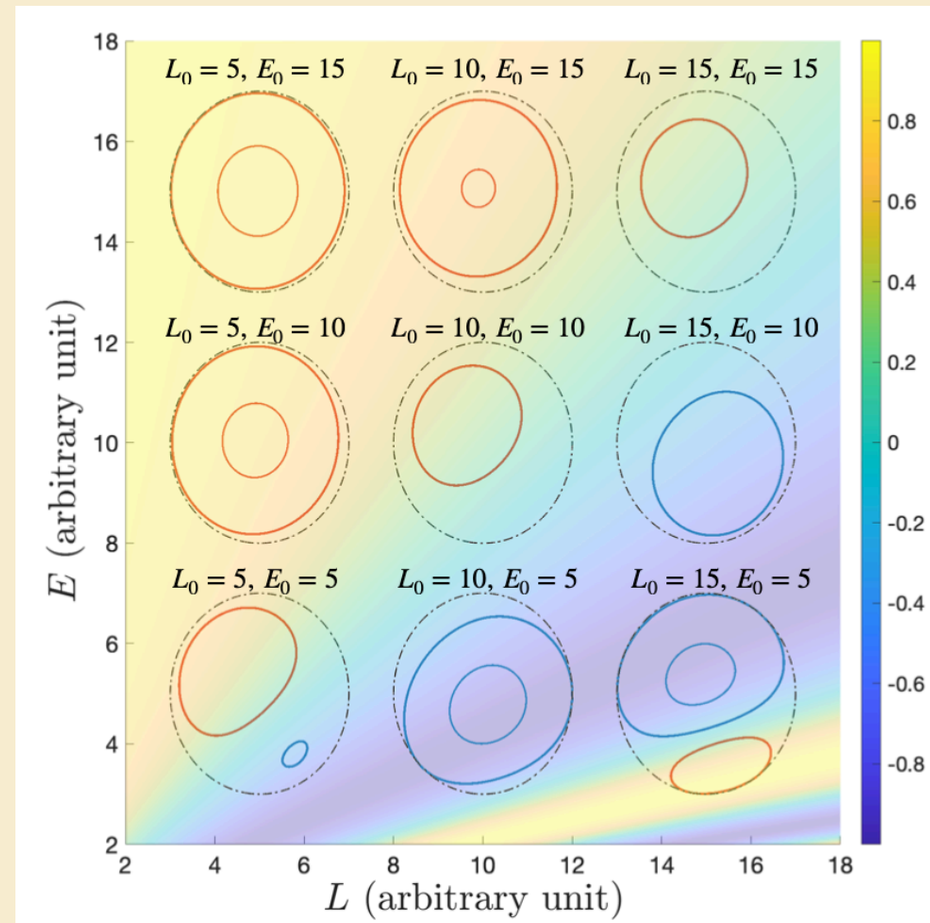


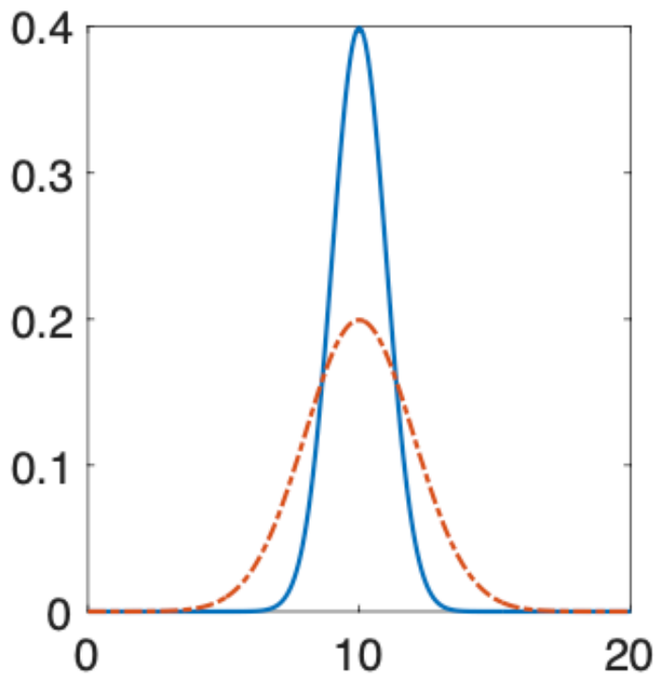
[TC, M Lindner, W Rodejohann, 2204.10696]

- Phase wash-out effect
- Non-symmetric uncertainty distribution: Imaginary part in ϕ_{jk} (phase shift signatures)

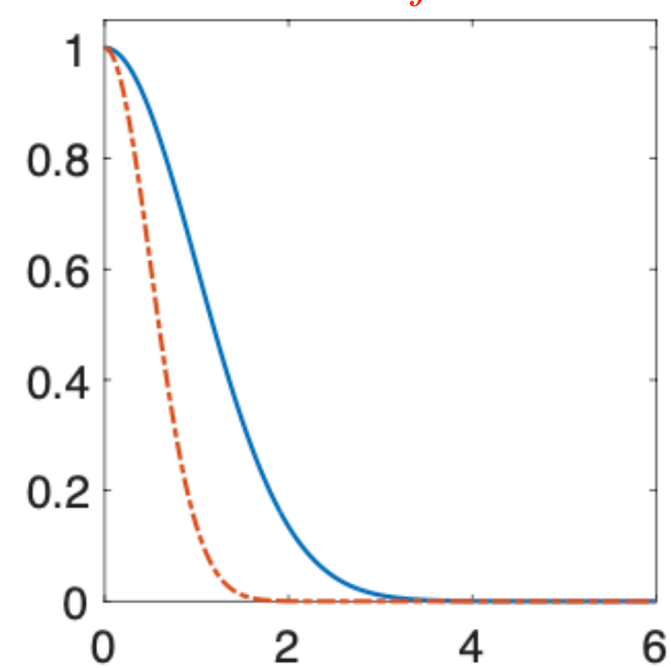
Decoherence signature — damping

$$P_{3,\alpha\to\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i\frac{\Delta m^2_{jk} L_0}{2E_0}} \phi_{jk}(\vec{\sigma}; L_0, E_0) \qquad 0 \le |\phi_{jk}| \le 1$$

PDF



$|\phi_{jk}|$



x-axis:

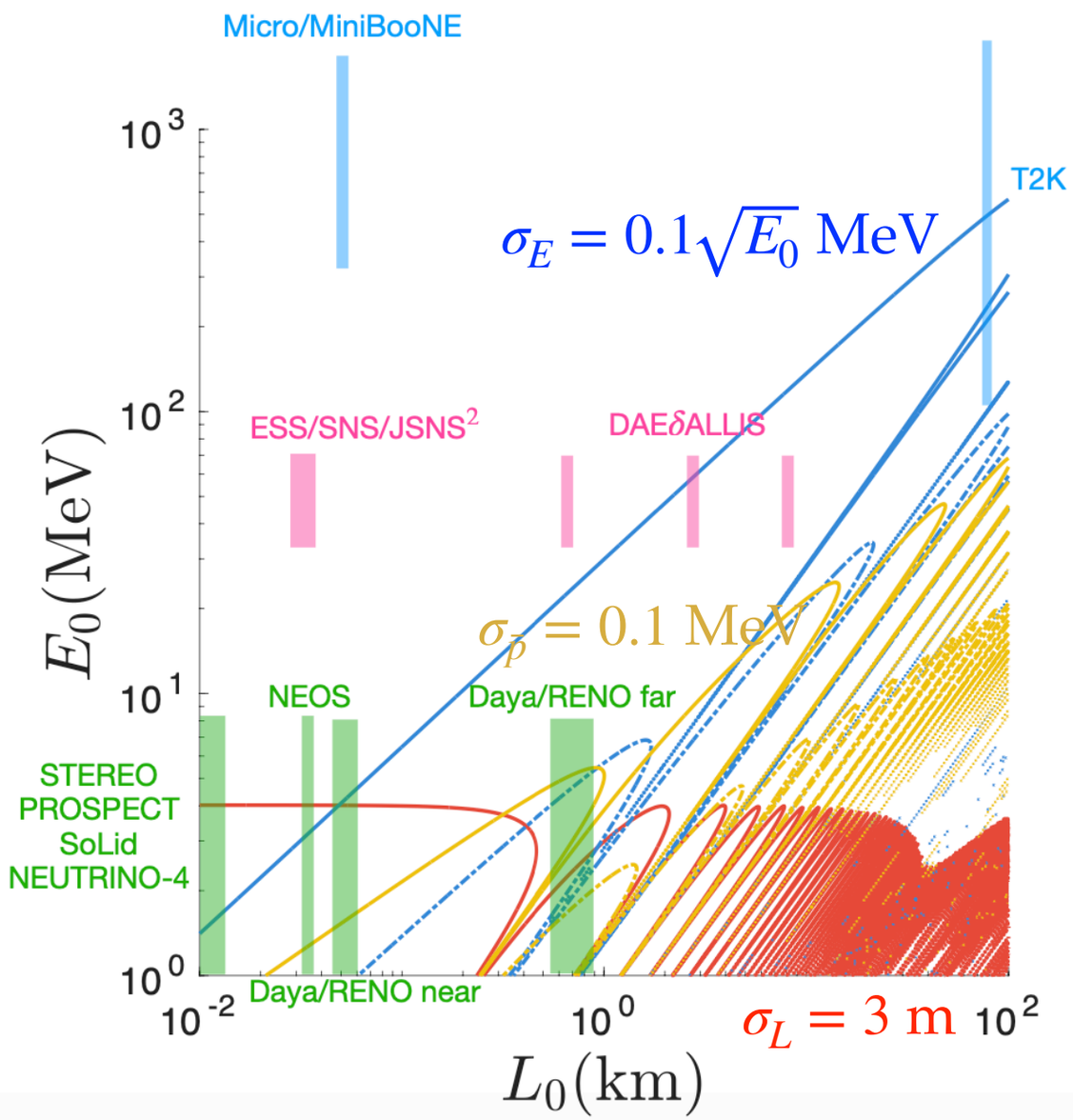
Quantum uncertainty in momentum $\Delta m^2_{jk} L_0/(2E_0^2)$ $\sigma_{\bar{p}} = \dfrac{\sigma_p}{1 + 4\sigma_x^2\sigma_p^2}$

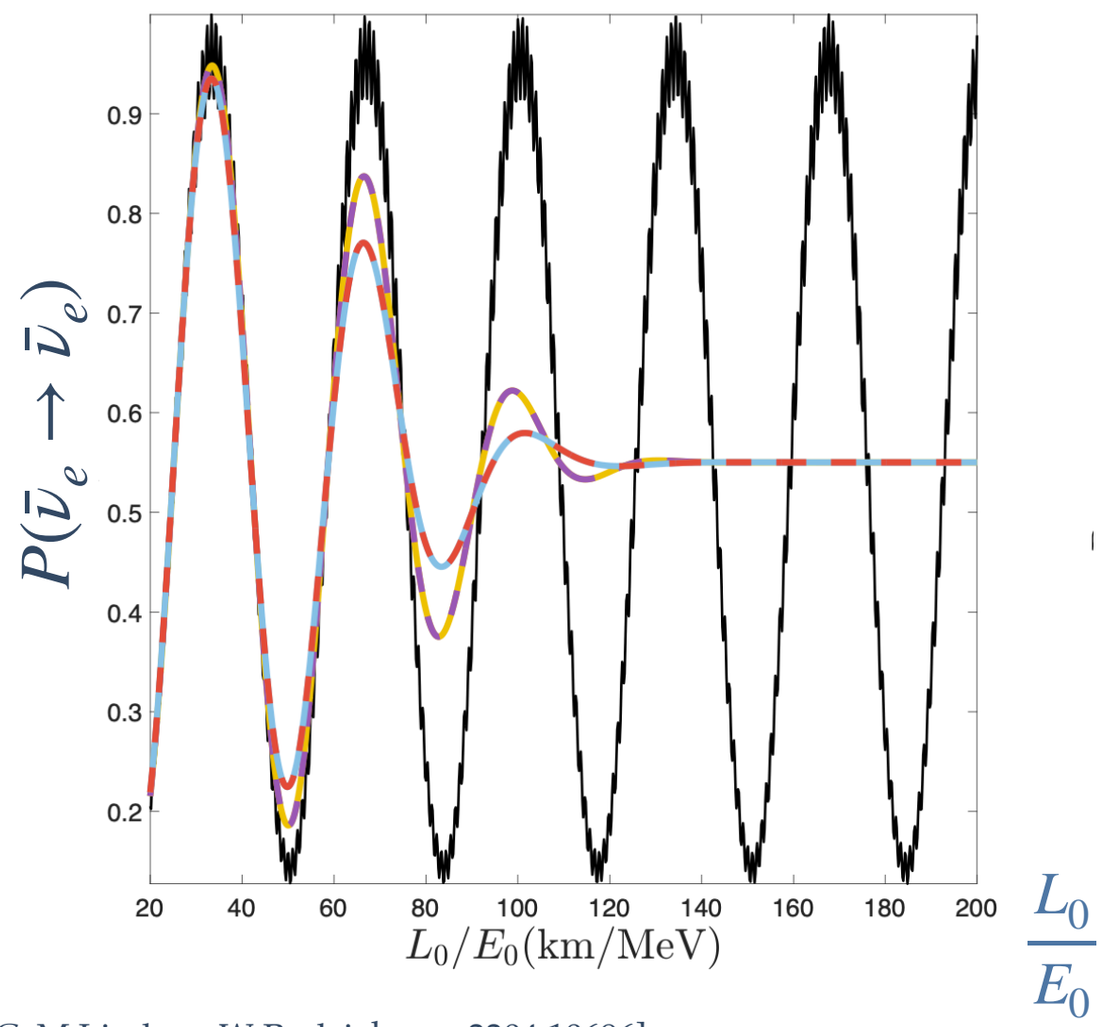
Classical uncertainty in baseline $\Delta m^2_{jk}/(2E_0)$

Classical uncertainty in energy^{-1} $\Delta m^2_{jk} L_0$ $\sigma_E = \sigma_E^o \sqrt{E_0}$

Where to look?

$$|P_{\alpha\beta}(\sigma_n) - P_{\alpha\beta}(\sigma_n = 0)| = 10^{-6}$$





[TC, M Lindner, W Rodejohann, 2204.10696]

Sensitivity for reactor neutrinos

Lower energy → more PWO effect

Where to look? & How far are we?

Current reactor bound $\sigma_{\bar{p}} > 0.47$ MeV

[A. de Gouvea, V. Romeri, C. Ternes (2021)]

see Kyle Leach's talk

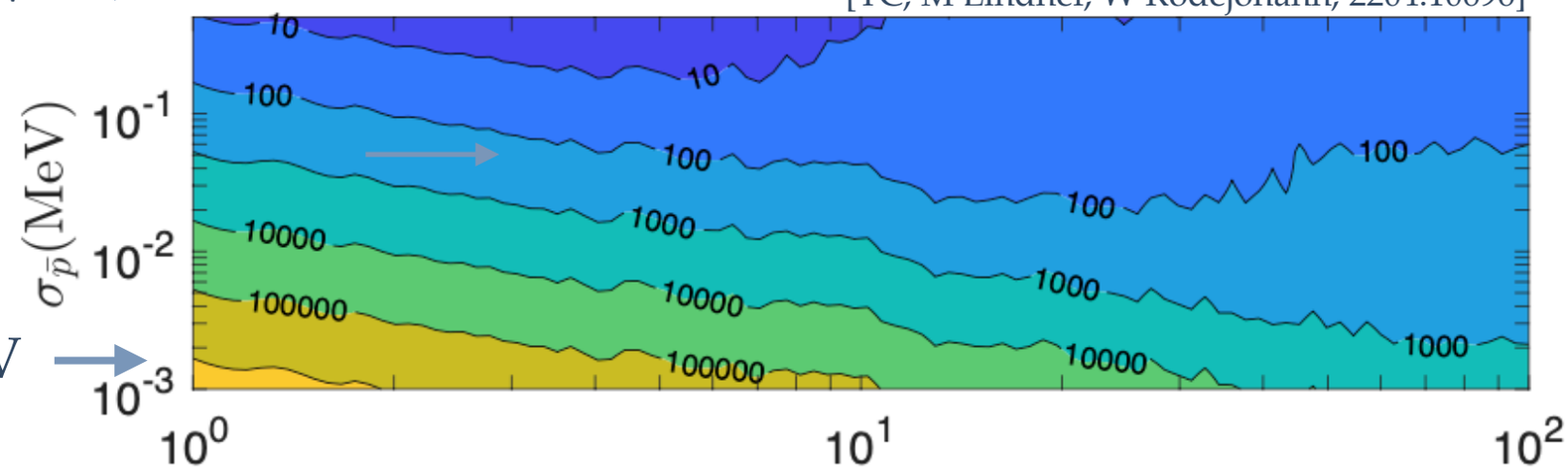
JUNO sensitivity: $\sigma_{\bar{p}} > 3 \times 10^{-3}$ MeV

[JUNO, 2022]

χ^2 analysis

Contour: relative (to RENO) statistic enhancement required for the sensitivity

[TC, M Lindner, W Rodejohann, 2204.10696]



Baseline: L_0 (km)

$$\sigma_{\bar{p}} \simeq \sqrt{\Gamma_N^2 + 1/t_N^2} \sim \mathcal{O}(0.1)\ \text{eV}$$

Isotope lifetime

Collisional broadening

[E.Akhmedov and A. Smirnov (2022)]

[R. Krueger, T. Schwetz (2023)]

$$\sigma_E = 0.03 - 0.06\ \text{MeV}\sqrt{E/\text{MeV}}$$

If: Gaussian & constant WP

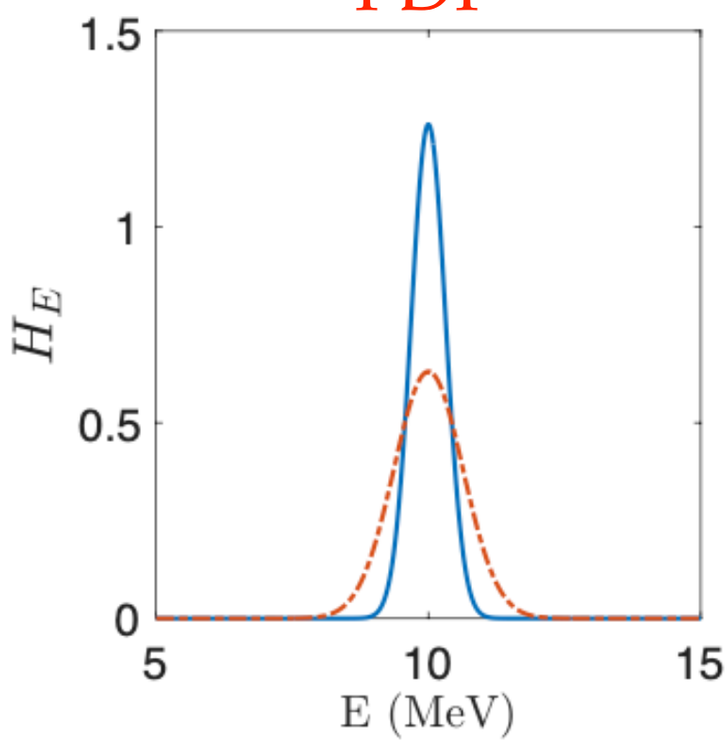
@ one oscillation

→ far from reach

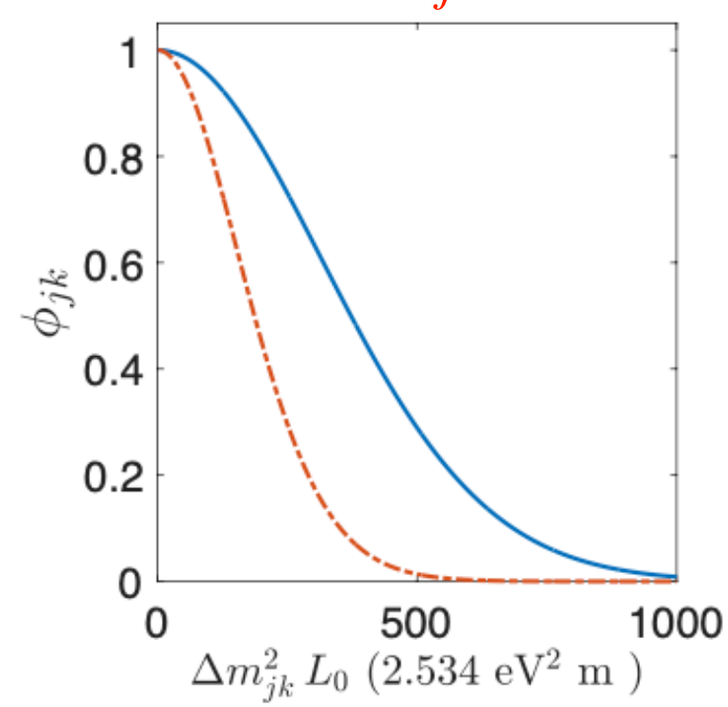
Damping + phase shift from energy resolution
(smearing)

$$P_{3,\alpha\to\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i\frac{\Delta m^2_{jk} L_0}{2E_0}} \phi_{jk}(\vec{\sigma}; L_0, E_0) \qquad \phi_{jk} = |\phi_{jk}| e^{i\beta_{jk}}$$

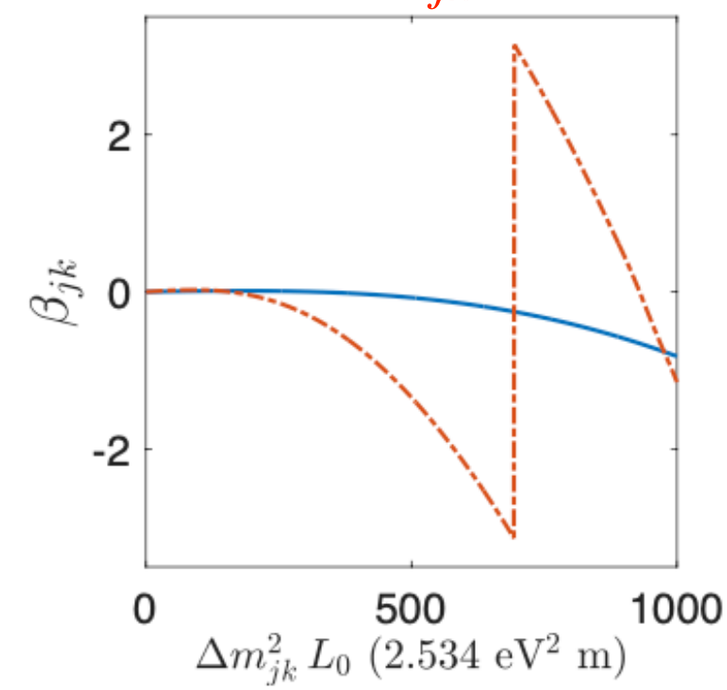
PDF



$|\phi_{jk}|$



β_{jk}



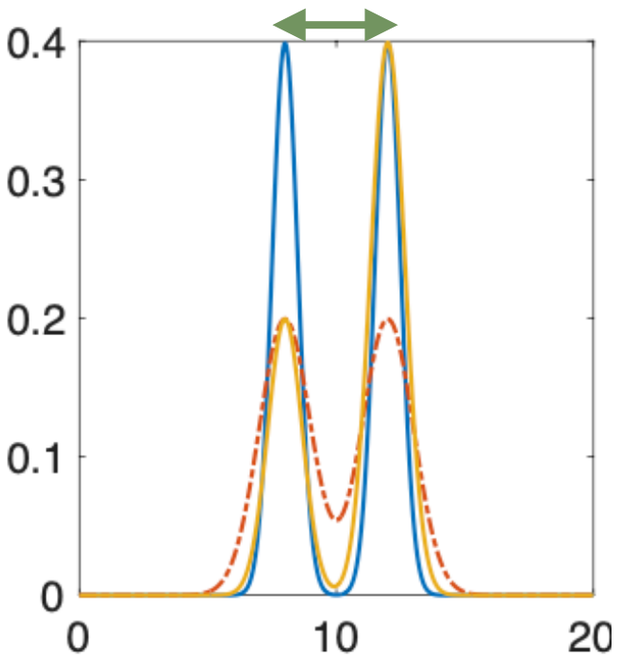
Non-symmetric in 1/E

non-trivial phase shift

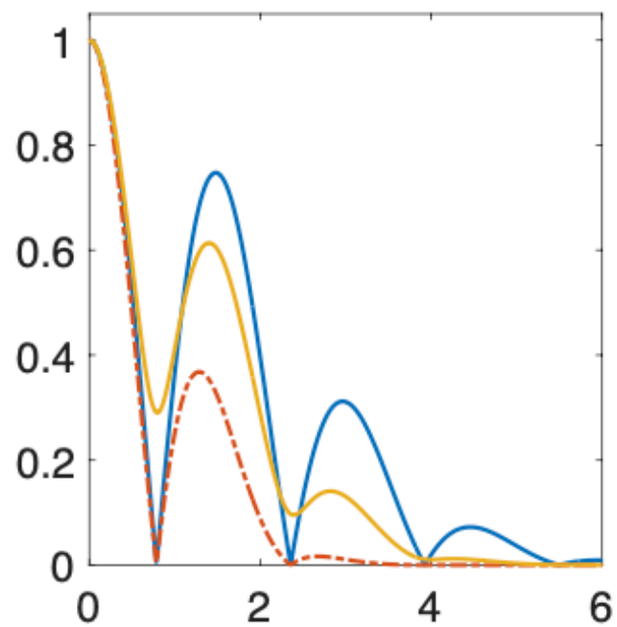
Asymmetric PDFs

$$P_{3,\alpha\to\beta} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i\frac{\Delta m^2_{jk} L_0}{2E_0}} \phi_{jk}(\vec{\sigma}; L_0, E_0) \qquad \phi_{jk} = |\phi_{jk}| e^{i\beta_{jk}}$$

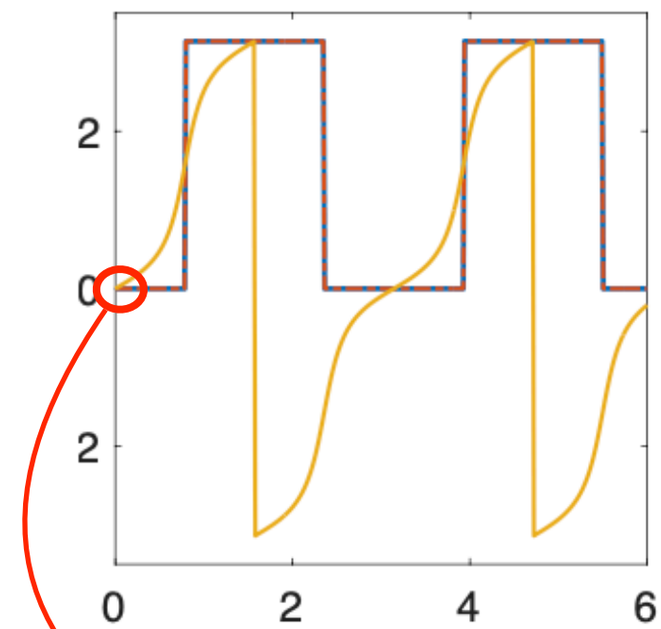
PDF ΔE



$|\phi_{jk}|$



β_{jk}



Quantum uncertainty $\Delta m^2_{jk} L_0/(2E_0^2)$

Classical uncertainty in baseline (Interferometer) $\Delta m^2_{jk}/(2E_0)$

$\beta_{jk} \sim \alpha_{jk} a$ when $\alpha_{jk} << 1$

Asymmetry parameter:
$$a = \frac{1 - rs}{1 + rs}\Delta E$$

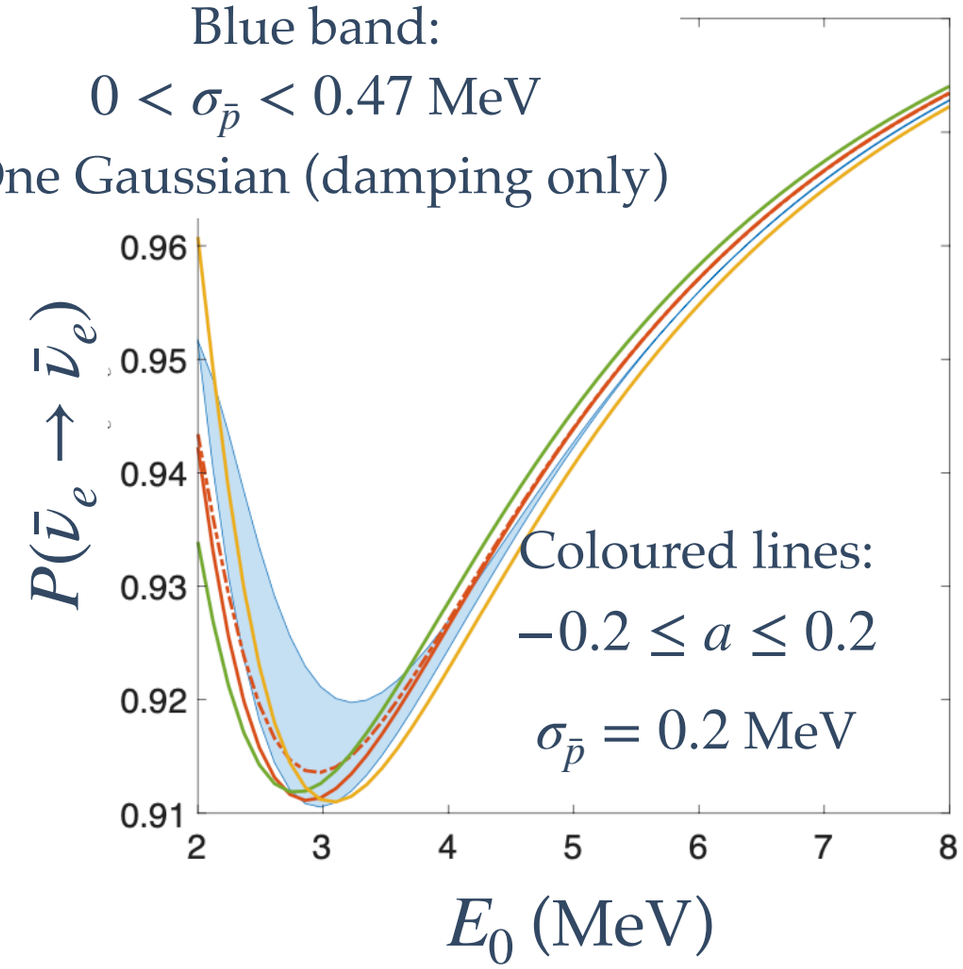
New method?: phase sensitive measurement

Look for *where* an oscillation min is.
(e.g. by moving the detector)

$$\psi_{jk}(L^{jk}_{\rm osc}, E_0) = 2n\pi \to \psi_{jk}(L^{jk}_{\rm min}, E_0) + \beta_{jk}(L^{jk}_{\rm min}, E_0; \vec{\sigma}_n) = 2n\pi,$$

Not sensitive to the damping term

Blue band:
$0 < \sigma_{\bar{p}} < 0.47$ MeV
One Gaussian (damping only)



[TC, M Lindner, W Rodejohann, 2204.10696]

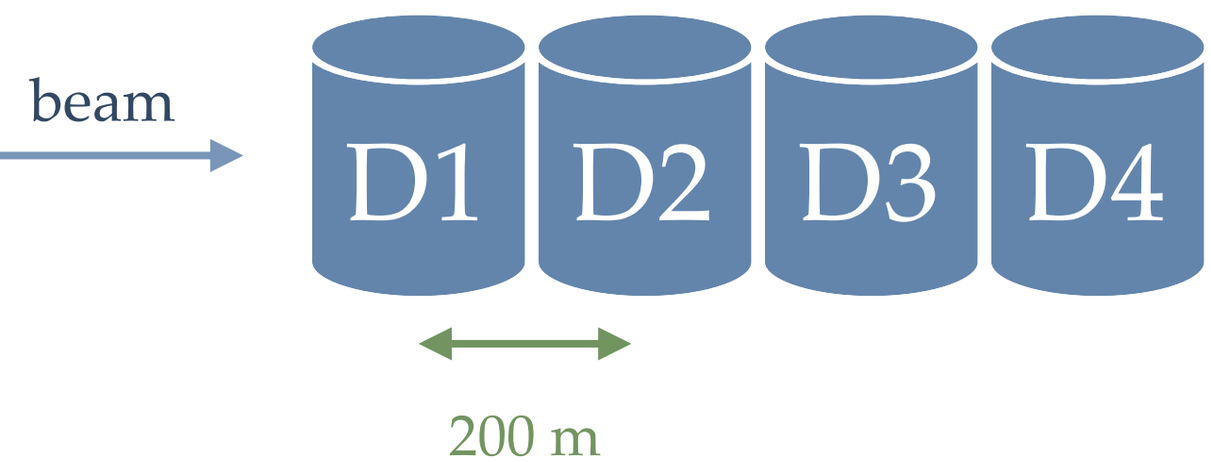
Traditional spectral analysis is not (as) sensitive to the phase shift

New method?: phase sensitive measurement

Look for an oscillation min. (e.g. by moving the detector)

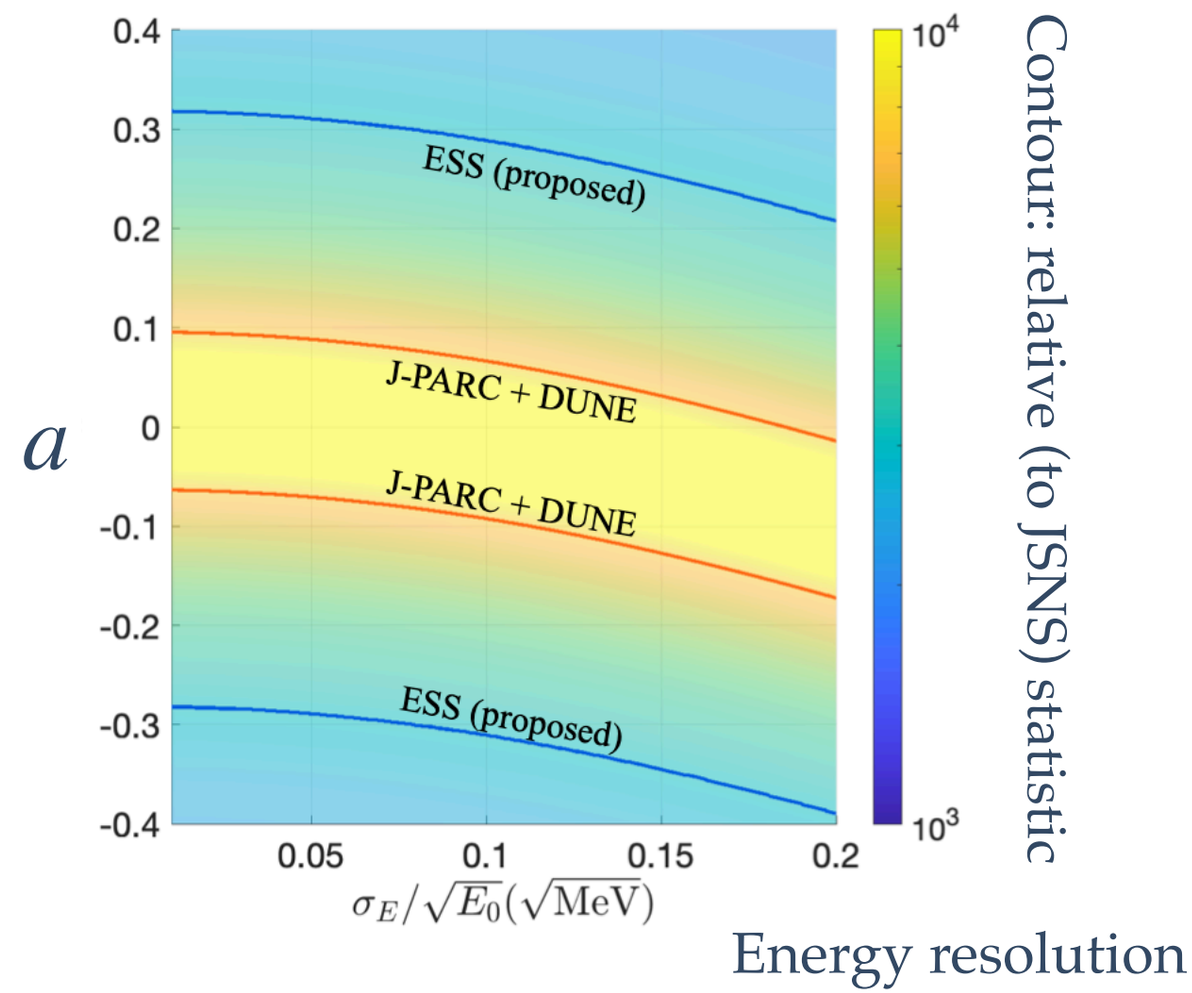
$$\psi_{jk}(L^{jk}_{\rm osc}, E_0) = 2n\pi \to \psi_{jk}(L^{jk}_{\rm min}, E_0) + \beta_{jk}(L^{jk}_{\rm min}, E_0; \vec{\sigma}_n) = 2n\pi,$$

monochromatic πDAR: appearance $\nu_\mu \to \nu_e$



$$F_i(E_0; \vec{\sigma}_n) = \frac{1}{\bar{N}_i} \frac{N_{i+1}(E_0; \vec{\sigma}_n) - N_i(E_0; \vec{\sigma}_n)}{L_{i+1} - L_i}$$

→ find *where* $F = 0$



[TC, M Lindner, W Rodejohann, 2204.10696]

Conclusion

- For the precision frontier — formulate a structure to reconstruct the quantum or classical origins from decoherence pattern

- Find that decoherence can be describe by a phase averaging effects (PWO effects) of the oscillation phase or the phase in the Wigner phase space
  → allows simple numerical calculation & Fourier analysis

The layer structure: QFT+open quantum system, micro. to macro.

Inputs: Diagrams, distributions

Numerical phase averaging(s)

Output: ϕ_{jk} (damping) & β_{ij} (phase shift) tomography (L_0 , E_0)

Thank you !

Backup Sildes

Sensitivity for reactor neutrinos

Lower energy → more PWO effect

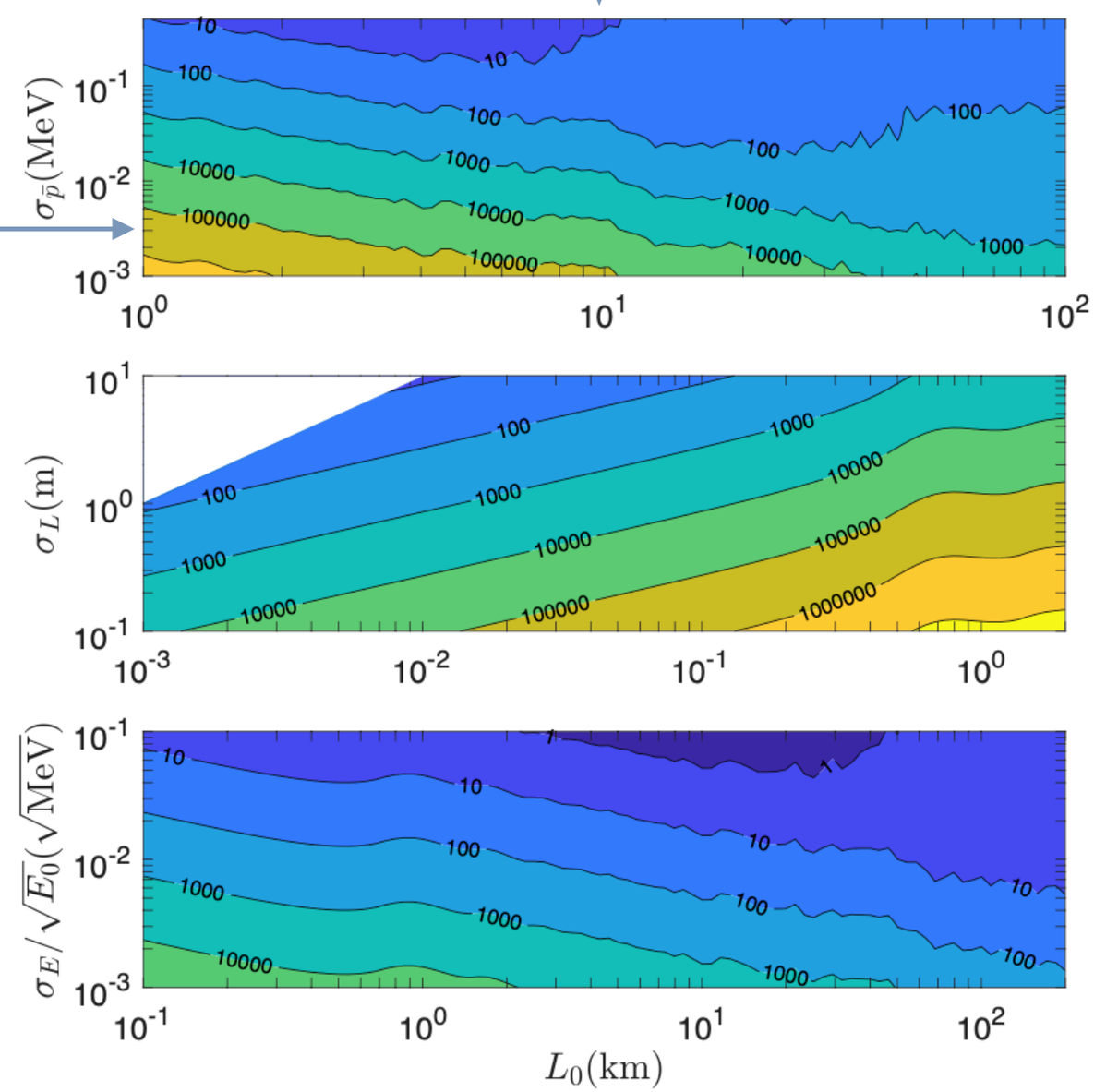
Where to look? & How far are we?

Contour: relative (to RENO) statistic enhancement required for the sensitivity

χ^2 analysis

Current reactor bound (RENO+DYB+KL): $\sigma_{\bar{p}} > 0.47$ MeV
[A. de Gouvea, V. Romeri, C. Ternes (2021)]

JUNO sensitivity: $\sigma_{\bar{p}} > 3 \times 10^{-3}$ MeV
[JUNO collaboration (2022)]



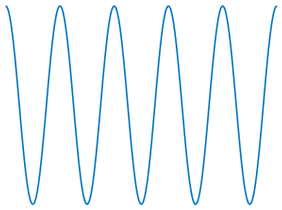
- The usually neglected (quantum) localisation term, contributes manly from classical uncertainties
- ϕ_{jk} does not depend on L_0

→ can go to near source for hight statistics

- Smearing from the energy resolution

→Difficult to discriminate from $\sigma_{\bar{p}}$

Neutrino Oscillation
— simple formalism (coherence)



closed quantum system: $\frac{d}{dt}\rho(t) = i\,[\mathcal{H}(t), \rho(t)]$ Unitary remains

- Formalisms:
  - QM wavefunctions: $A_{\alpha\beta} \supset \sum_j U^*_{\alpha j} U_{\beta j} \psi_{jk}$ → Easy
  - QM density matrices : $\rho(t) = \sum_{jk} U_{j\delta} U^*_{k\delta}\, e^{-i(H_j - H_k)t}\, |\nu_j\rangle\langle\nu_k|$

    → Open quantum system — tailored for decoherence effect
  - QFT: $A_{\alpha\beta} \supset \sum_j U^*_{\alpha j} U_{\beta j} \int \frac{d^4p}{(2\pi)^4} \frac{p + m_j}{p^2 - m_j^2 + i\epsilon} e^{-ipx}$ → Fundamental

Is this enough?

Neutrino Decoherence

- QM Wavepackets $\phi_{jk}(L,E) = \exp\left[-\left(\frac{\Delta m^2_{jk} L\sigma}{2\sqrt{2}E^2}\right)^2 - \left(\frac{\Delta m^2_{jk}}{2\sqrt{2}E\sigma}\right)^2\right]$ (WP seperation)

  [*Phys.Rev.D* 44 (1991) 3635-3640]

- S-Matrix formalism (QFT) & Consider Wavepackets

$$\phi_{jk}(L,E) = \exp\left[-\eta(L,E) - \left(\frac{\Delta m^2_{jk} L\sigma_{eff}}{2\sqrt{2}E^2_{eff}}\right)^2 - \left(\frac{\Delta m^2_{jk}}{2\sqrt{2}E_{eff}\sigma_{eff}}\right)^2\right]$$

(WP seperation)

[hep-ph/0109119], [1001.4815]

- Density Matrix formalism (QM) — Open Quantum System

Lindblad Equation: $\frac{\partial\rho}{\partial t} = -i[H,\rho] - \mathscr{D}[\rho]$ 9×9 entries

$$\phi_{jk}(L,E) = \exp\left[-\left(\frac{\alpha m^2_{jk} L}{E^n}\right)^2\right],$$

e.g.
$n = -2$: matter effect fluctuation
$n = 0,2$: quantum gravity
$n = 1$: neutrino absorption

[2112.14450 as a review]

Layer moving operator

- The layer moving operator:

$$B_{i+1}(x_{i+1}, p_{i+1}) = \mathscr{L}\mathscr{M}\mathscr{O}^i B_i(x_i, p_i) = \int d^4x_i \int d^3p_i \, W_i(x_i, p_i; x_{i+1}, p_{i+1}) B_i(x_i, p_i) \, ,$$

Layer $i+1$ Layer i Weighting function

- width as uncertainties
- Spread through (x_i, p_i)
- Centered at (x_{i+1}, p_{i+1})

- Flavor transition probability:

$$P_3(T_0, \mathbf{L}_0, \mathbf{P}_0) = \mathscr{L}\mathscr{M}\mathscr{O}^2 P_2(T, \mathbf{L}, \mathbf{P})$$ Layer 2 $\rightarrow$ 3

$$P_2(T, \mathbf{L}, \mathbf{P}) = A_2^*(T, \mathbf{L}, \mathbf{P}) A_2(T, \mathbf{L}, \mathbf{P}), \quad A_2(T, \mathbf{L}, \mathbf{P}) = \mathscr{L}\mathscr{M}\mathscr{O}^1 A_1(\mathrm{x}, \mathrm{p})$$

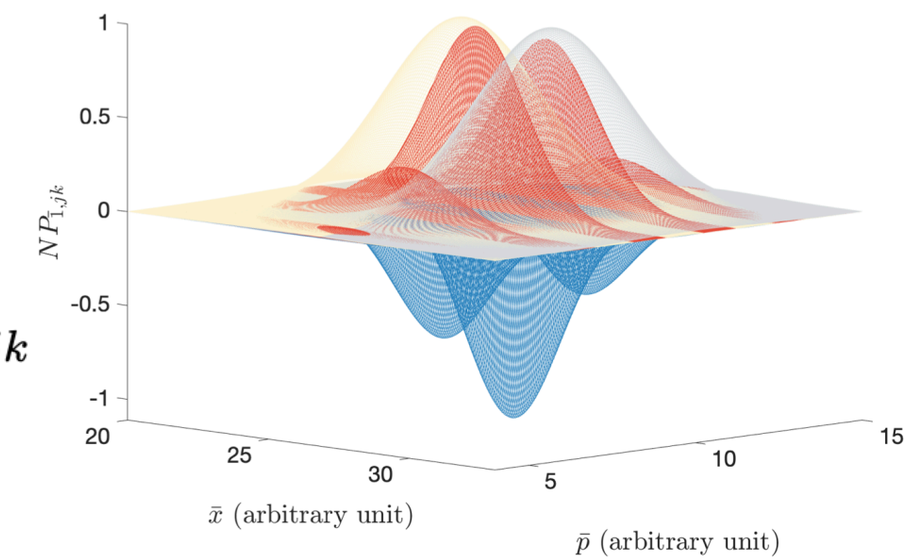
Layer 1 $\rightarrow$ 2

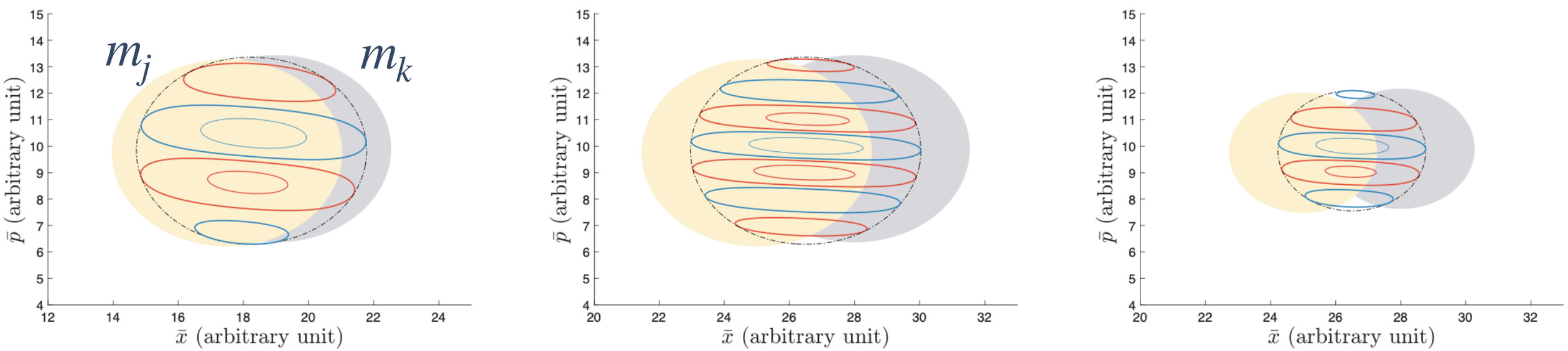
$$P_2(T, \mathbf{L}, \mathbf{P}) = \mathscr{L}\mathscr{M}\mathscr{O}^{\bar{1}} P_{\bar{1}}(\bar{\mathbf{x}}, \bar{\mathbf{p}}) \, .$$ Layer $\bar{1}$ $\rightarrow$ 2

Wigner quasi-probability distribution

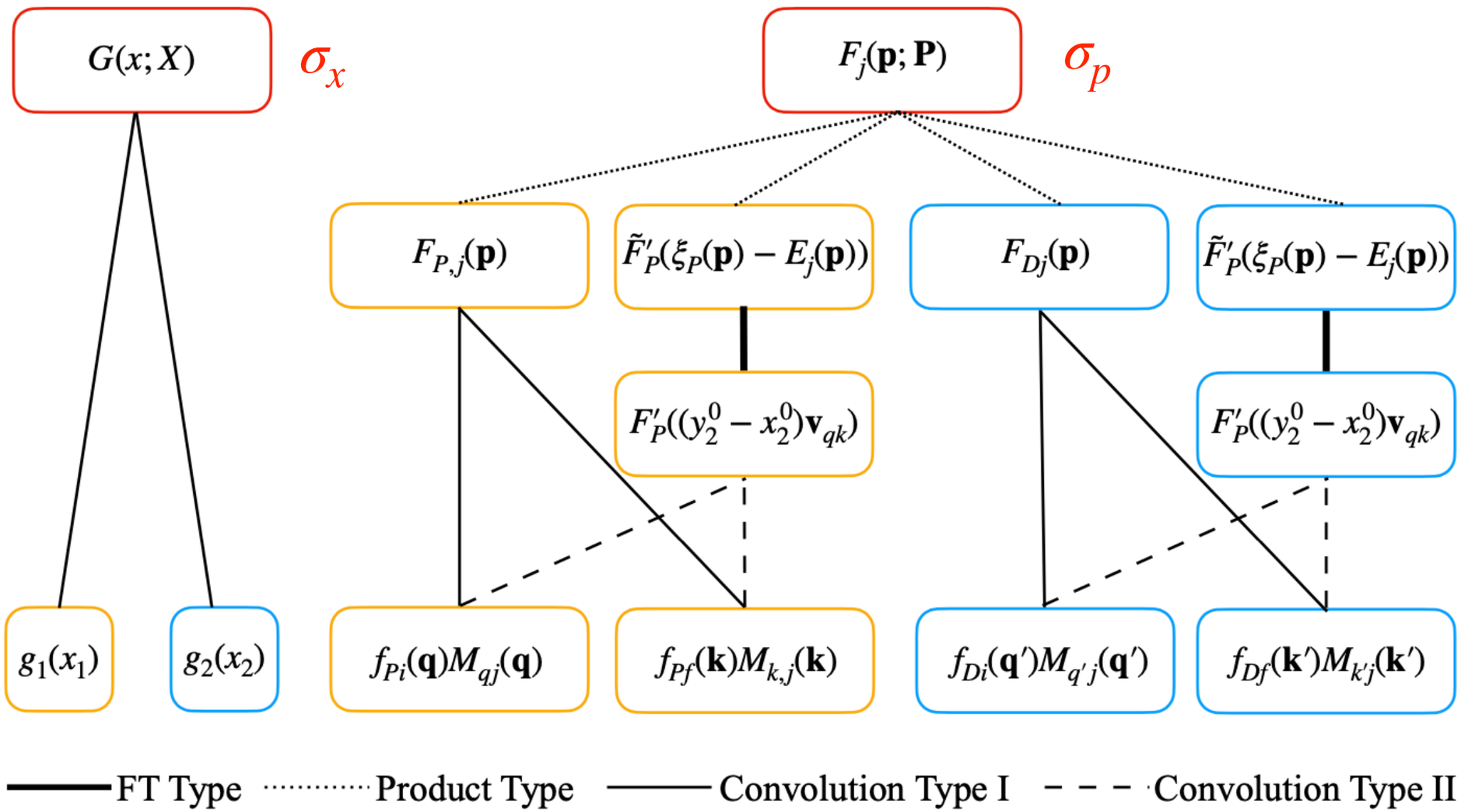
Wigner distribution

$$P_{\bar{1}} = \sum_{j,k} U^*_{\alpha j} U_{\beta j} U_{\alpha k} U^*_{\beta k} e^{-i(E_j - E_k)T + i(\mathbf{v}_j \mathbf{P}_j - \mathbf{v}_k \mathbf{P}_k)T} P_{\bar{1},jk}$$





Quantum uncertainty decomposition: a Fourier structure



Type (notation for σ_h)	Function relation	Width relation	Gaussian case
FT type ($\tilde{\sigma}_f$)	$h = \mathcal{FT}(f)$	NC	$\sigma_h = \frac{1}{2\sigma_f}$
Product type (σ_{fg})	$h = f \times g$	PC, $\sigma_h < \{\sigma_f, \sigma_g\}$	$\frac{1}{\sigma_h^2} = \frac{1}{\sigma_f^2} + \frac{1}{\sigma_g^2}$
Convolution type I (σ_{f*g})	$h = f * g$	PC, $\sigma_h > \{\sigma_f, \sigma_g\}$	$\sigma_h^2 = \sigma_f^2 + \sigma_g^2$

[TC, M Lindner, W Rodejohann, 2204.10696]

Function (width notation), Type	Width relation	Gaussian case
$H(y)$ (σ_H), Convolution type I	PC , $\sigma_H > \{\sigma_f, \sigma_g\}$	$\sigma_H = \sigma_{f*g}$
$I(p)$ (σ_I), Convolution type II	NC	$\sigma_I = 1/\sigma_{fg}$

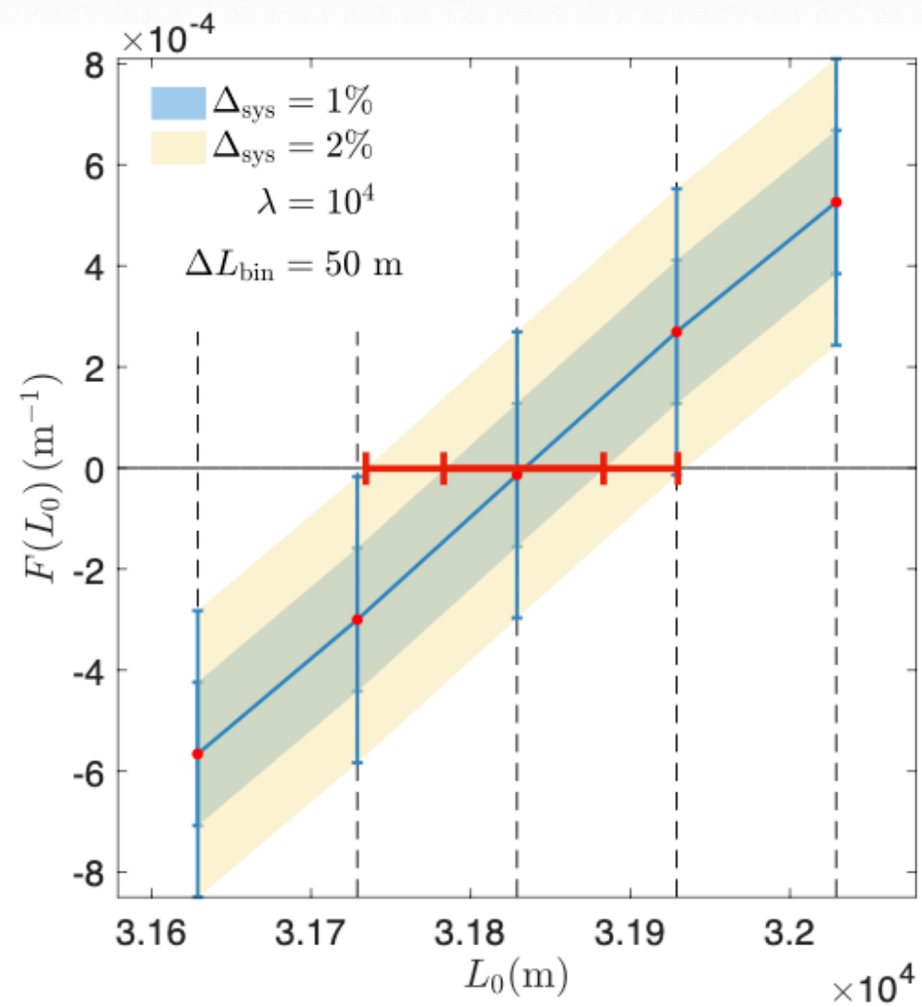
New method?: phase sensitive measurement

Look for an oscillation min. (e.g. by moving the detector)

$$\psi_{jk}(L^{jk}_{\rm osc}, E_0) = 2n\pi \to \psi_{jk}(L^{jk}_{\rm min}, E_0) + \beta_{jk}(L^{jk}_{\rm min}, E_0; \vec{\sigma}_n) = 2n\pi,$$

Binned data: increase both stat. and signal

$$N_i(E_0; \vec{\sigma}_n) = N(E_0) \int_{L_i - \Delta L_{\rm bin}/2}^{L_i + \Delta L_{\rm bin}/2} dL_0 \, \frac{1}{4\pi L_0^2} \, P_{\nu_\alpha \to \nu_\beta}(L_0, E_0; \vec{\sigma}_n),$$

Discrete derivative (min @ $F = 0$)

$$F_i(E_0; \vec{\sigma}_n) = \frac{1}{\bar{N}_i} \frac{N_{i+1}(E_0; \vec{\sigma}_n) - N_i(E_0; \vec{\sigma}_n)}{L_{i+1} - L_i}.$$