

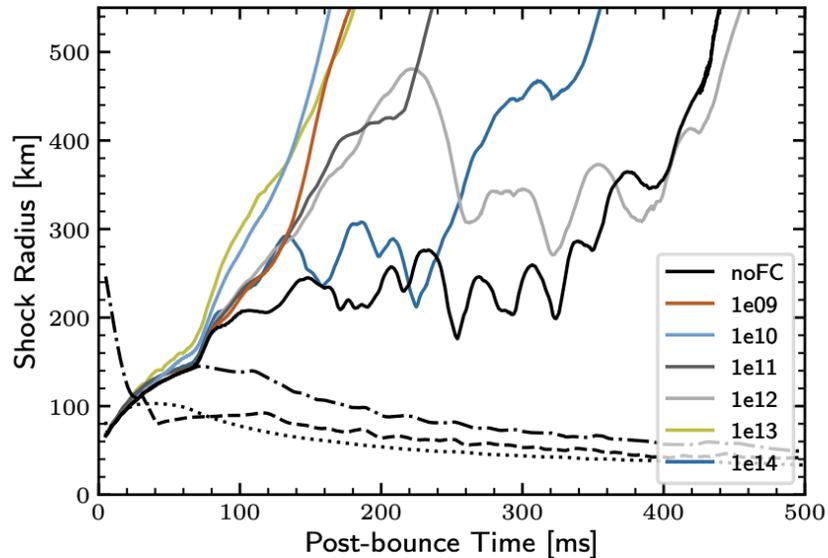
Towards Many-Body Models of Neutrino Flavor Instability

Zoha Laraib
Sherwood Richers

Neutrino flavor induces uncertainty

Hydrodynamics

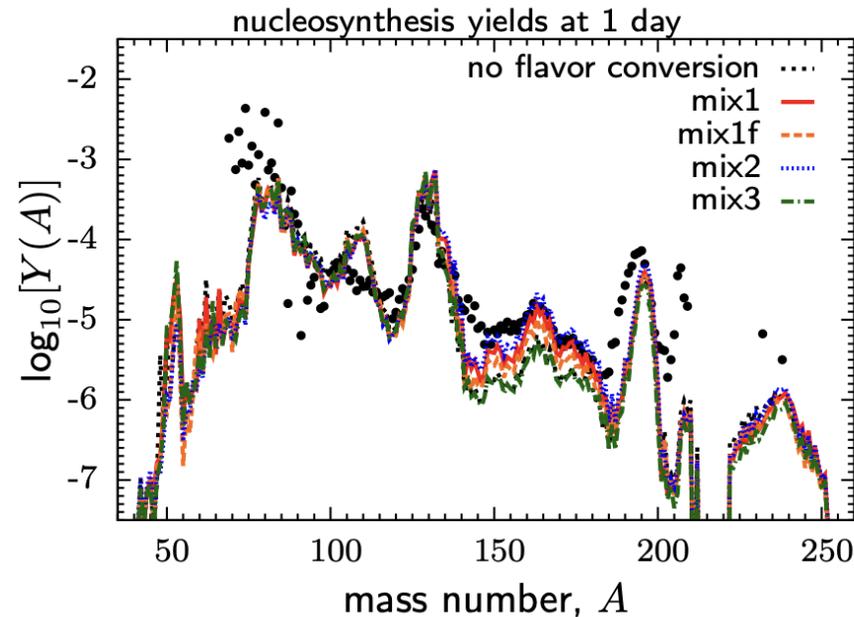
Supernova Shock
NSM mass ejection



Ehring et al. (2023)
PRL 131(6):061401

Nucleosynthesis

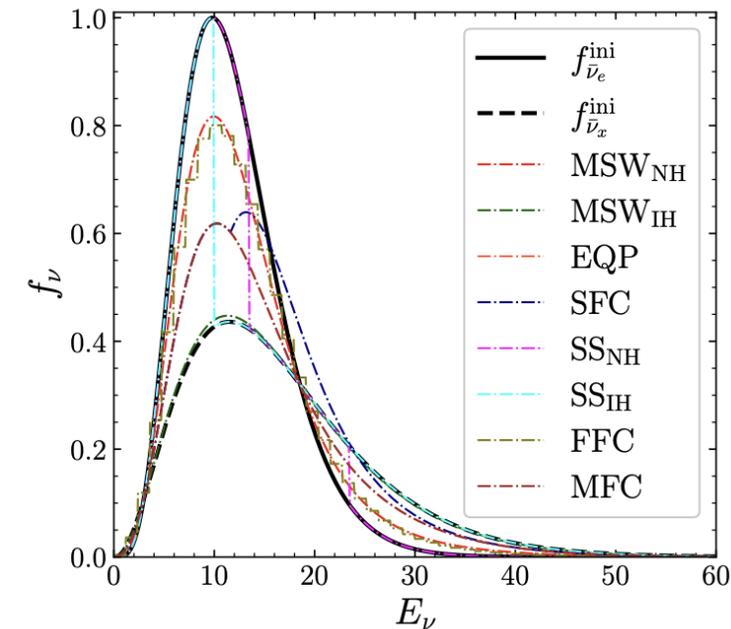
Light curve
Chemical evolution



Just et al. (2022)
PRD 105(8):083024

Neutrino Signal

(Supernova only)

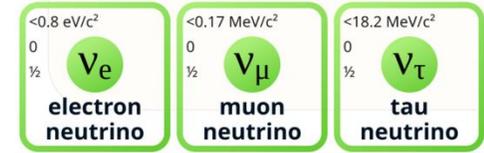


Abbar & Volpe (2024)
2401.10851

Electron Neutrinos are Special

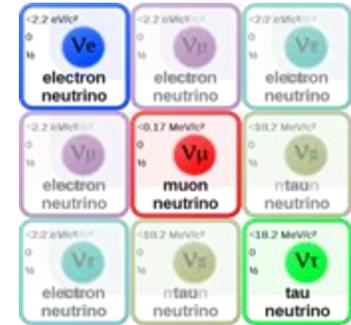
$$\frac{\partial f_a}{\partial t} + c\Omega \cdot \nabla f_a = C_a$$

Classical



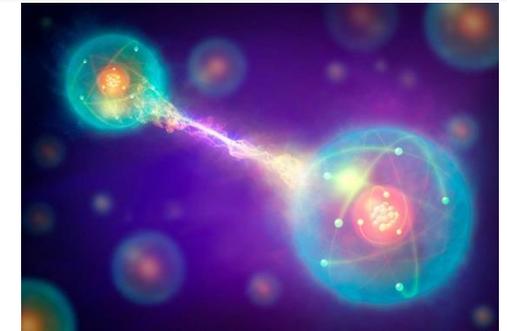
$$\frac{\partial f_{ab}}{\partial t} + c\Omega \cdot \nabla f_{ab} = C_{ab} - \frac{i}{\hbar} [\mathcal{H}, f]_{ab}$$

Quantum Mean Field



$$\frac{\partial \psi_{a_1 a_2 \dots a_n}}{\partial t} = -\frac{i}{\hbar} \mathcal{H} \psi_{a_1 a_2 \dots a_n}$$

Quantum Many Body



Getty Images

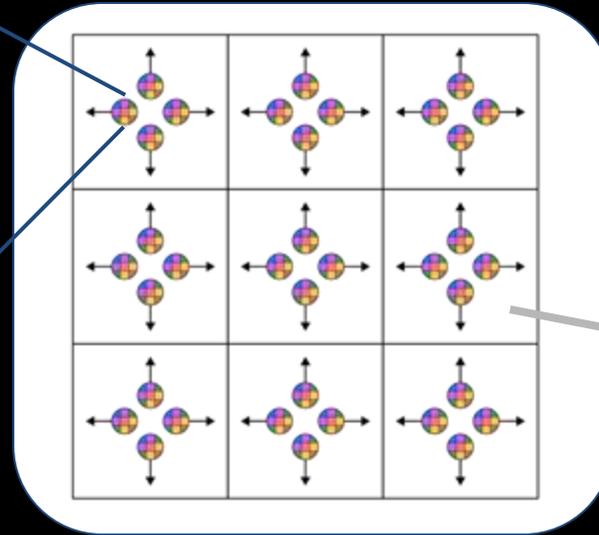
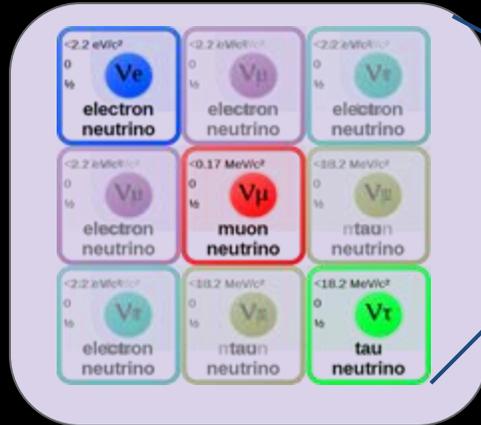
Need accurate neutrino transport to extract physics from observed neutrinos, gravitational waves, and light.



AMReX-based Flavor Simulation



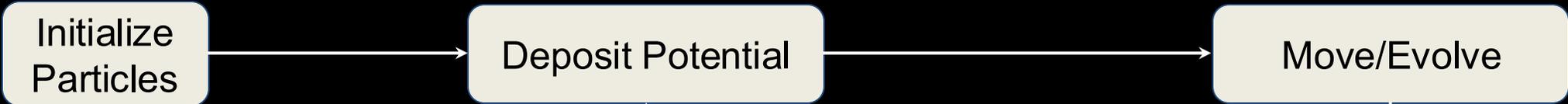
Don Willcox
(LBNL)



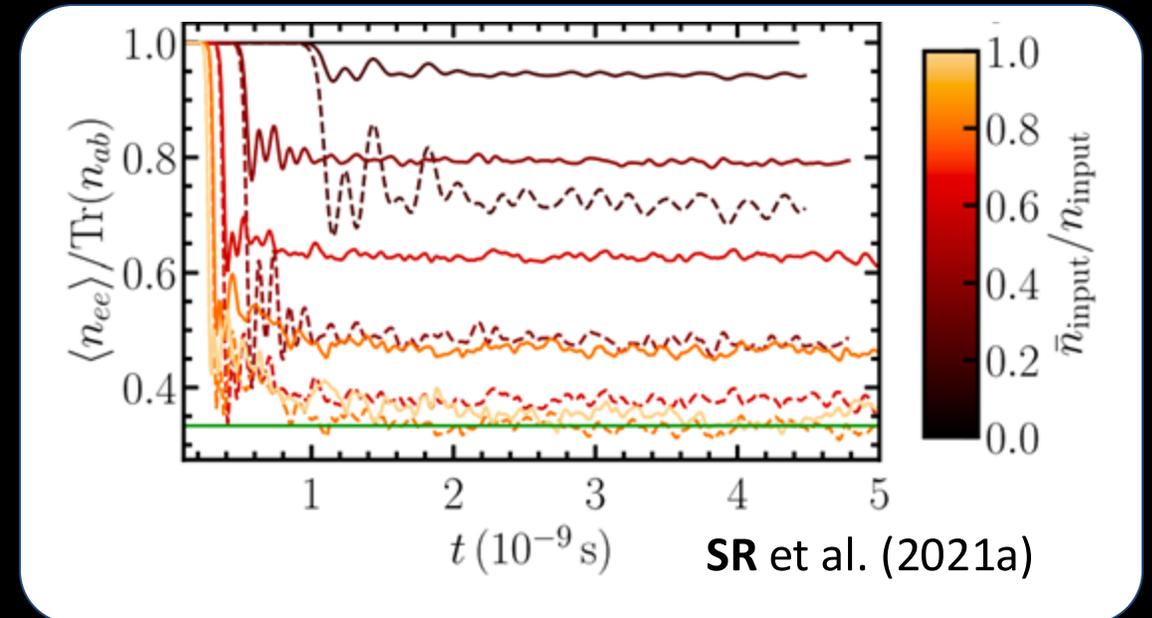
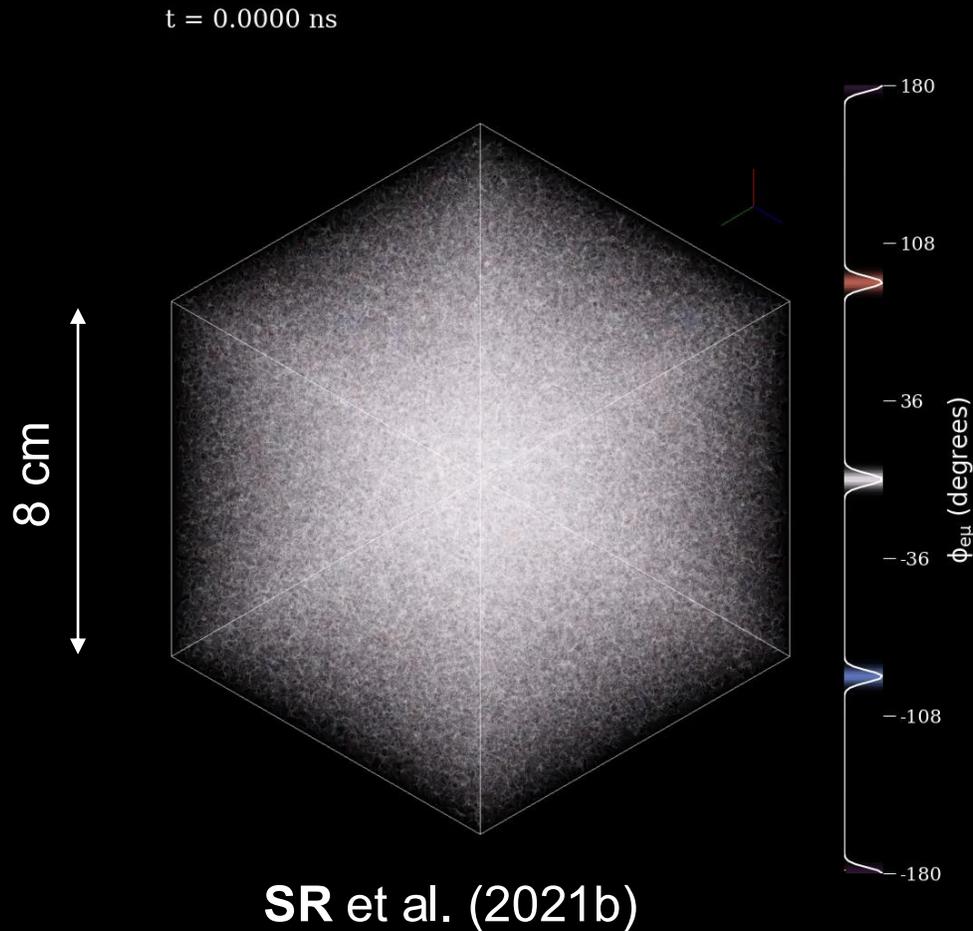
1500 particles
per cell

$$H_{\text{neutrino}} = \sqrt{2}G_F\hbar^3 \int d^3\nu' (1 - \cos\theta)(f' - \bar{f}')$$

$$\frac{d\mathcal{F}}{d\lambda} + \text{force} + \text{drift} = -p^\mu u_\mu \left(c - \frac{i}{\hbar c} [\mathcal{H}, \mathcal{F}] \right)$$

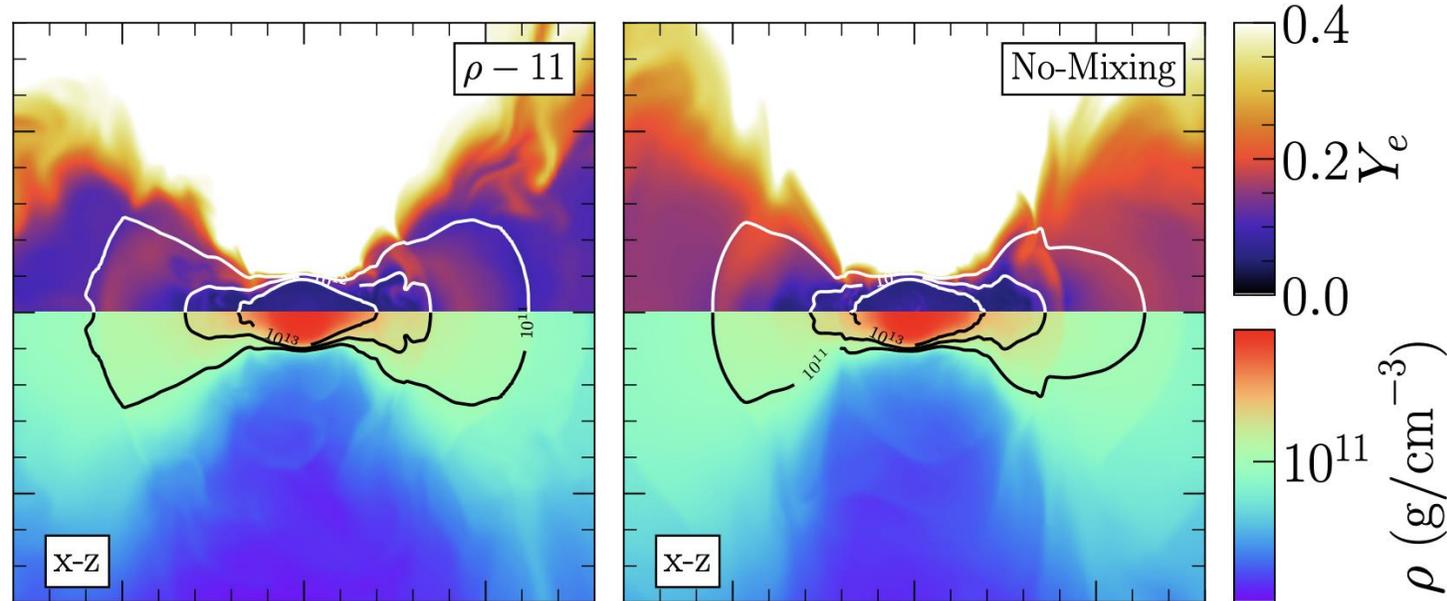


What does the FFI look like?

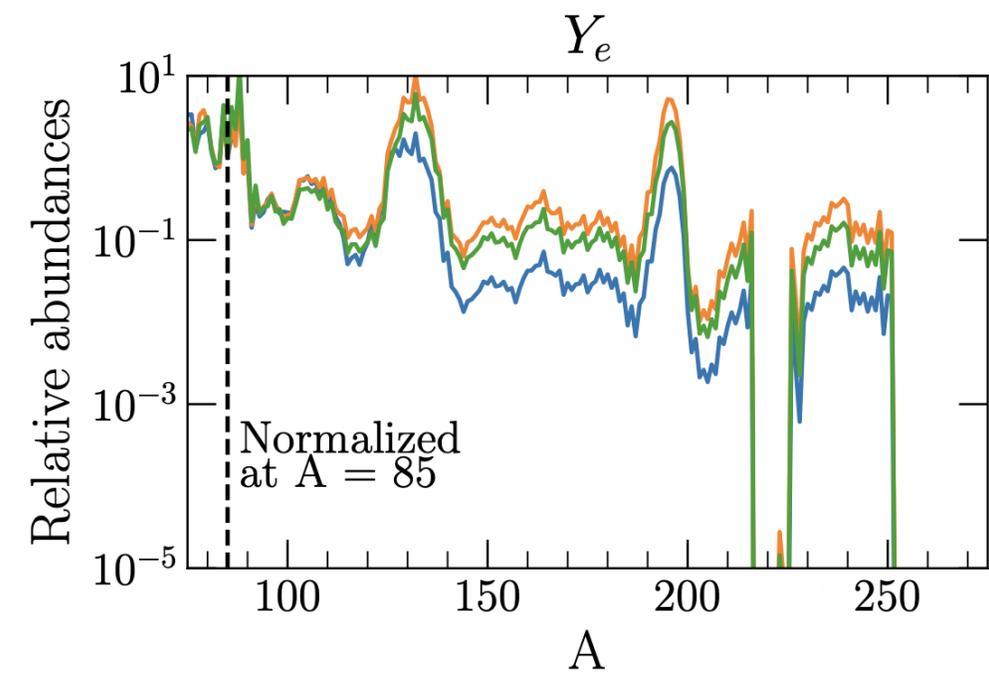
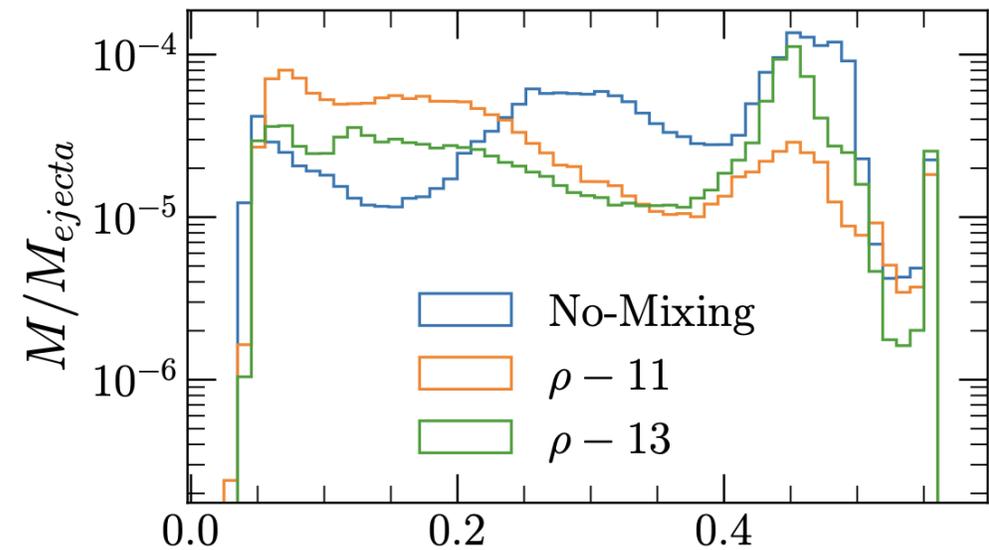


Amount of flavor transformation depends on the angular distribution.

Flavor transformation modifies nucleosynthesis



Qiu, Radice, SR, Bhattacharyya 2503.11758



Many-Body Neutrino Hamiltonian

$$H_{\nu\nu} = \frac{G_F}{\sqrt{2}} \sum_{\alpha, \alpha', \beta, \beta'} \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{d\mathbf{q}'}{(2\pi)^3} \frac{d\mathbf{p}}{(2\pi)^3} \frac{d\mathbf{p}'}{(2\pi)^3} (2\pi)^3 \delta(\mathbf{p} + \mathbf{q} - \mathbf{p}' - \mathbf{q}') \\ \times \left(a_{\alpha'}^\dagger(\mathbf{p}') a_\alpha(\mathbf{p}) a_{\beta'}^\dagger(\mathbf{q}') a_\beta(\mathbf{q}) \frac{(\delta_{\alpha'\alpha} \delta_{\beta'\beta} + \delta_{\alpha'\beta} \delta_{\beta'\alpha})}{2} g(\mathbf{p}', \mathbf{p}, \mathbf{q}', \mathbf{q}) + \dots \right),$$

$$g(\mathbf{p}', \mathbf{p}, \mathbf{q}', \mathbf{q}) \equiv \bar{u}(\mathbf{p}', -) \gamma_\mu P_L u(\mathbf{p}, -) \bar{u}(\mathbf{q}', -) \gamma^\mu P_L u(\mathbf{q}, -) = f^\dagger(\mathbf{p}', \mathbf{q}') f(\mathbf{p}, \mathbf{q})$$

$$f(\mathbf{p}, \mathbf{q}) = \sqrt{2} \left(e^{-i\phi_{\mathbf{p}}} \sin\left(\frac{\theta_{\mathbf{p}}}{2}\right) \cos\left(\frac{\theta_{\mathbf{q}}}{2}\right) - e^{-i\phi_{\mathbf{q}}} \cos\left(\frac{\theta_{\mathbf{p}}}{2}\right) \sin\left(\frac{\theta_{\mathbf{q}}}{2}\right) \right).$$

Forward
Scattering

$$\frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p}, \mathbf{q}} (1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{q}}) \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

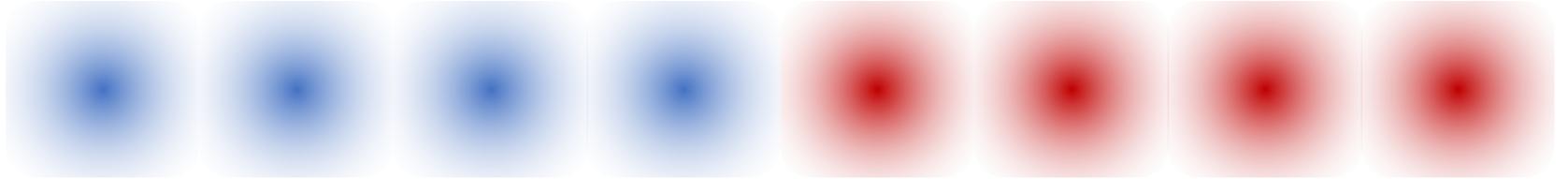
Pantaleone (1992)

Balantekin & Pehlivan (2006)

Cirigliano, Sen, Yamauchi (2024)

Hybrid Many-Body Simulation

$$\frac{\partial \psi}{\partial t} = -iH[X(t)]\psi$$

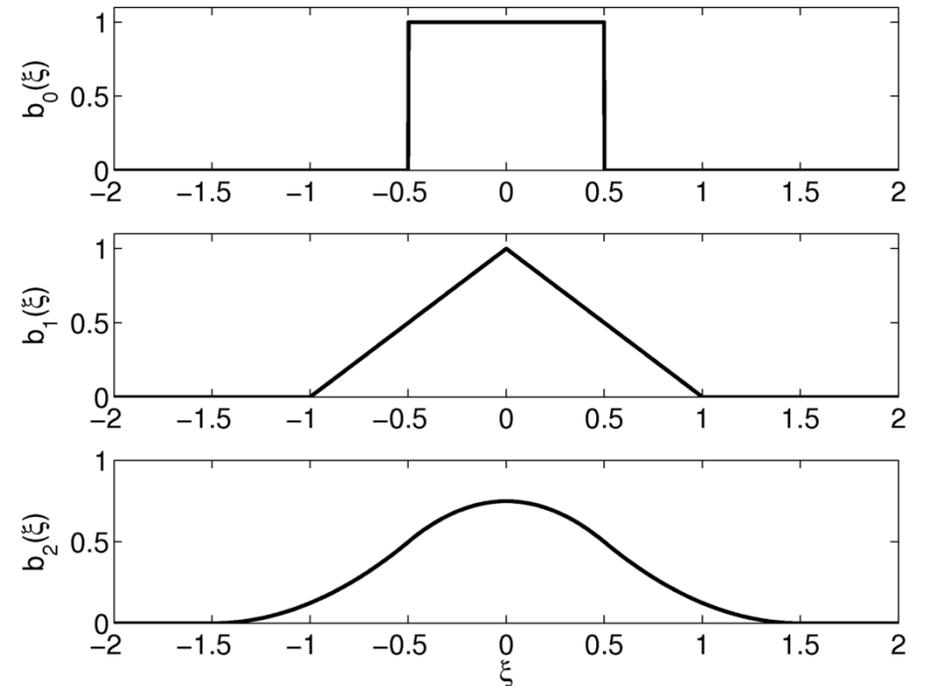


$$H[X(t)] = \frac{\sqrt{2}G_F}{2} \sum_{i < j} \frac{N_i + N_j}{w^3} \vec{\sigma}_i \cdot \vec{\sigma}_j J_{ij} S(\xi_{ij})$$

Forward-scattering term

Angular Factor

$$J_{ij} = (1 - \hat{p}_i \cdot \hat{p}_j)$$



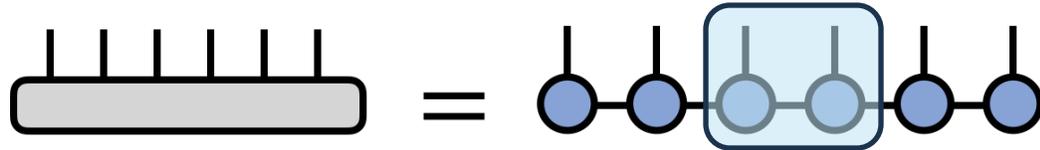
$$\xi_{ij} = (x_i - x_j)/w$$

Tensor Networks for Many-Body Physics

$$\begin{aligned}
 |\psi\rangle &= \sum_{\sigma_1, \dots, \sigma_{N_{\text{sites}}}} \left[\begin{array}{c} \sigma_1 \quad \sigma_2 \quad \dots \quad \sigma_{N_{\text{sites}}} \\ \text{---} \\ C_{\sigma_1, \dots, \sigma_{N_{\text{sites}}}} \end{array} \right] |\sigma_1, \dots, \sigma_{N_{\text{sites}}}\rangle \\
 &= \sum_{\sigma_1, \dots, \sigma_{N_{\text{sites}}}} \sum_{\alpha_1, \dots, \alpha_{N_{\text{sites}}}} \left[\begin{array}{c} \sigma_1 \quad \sigma_2 \quad \dots \quad \sigma_{N_{\text{sites}}} \\ \text{---} \\ A_{\alpha_1}^{\sigma_1} \quad A_{\alpha_1 \alpha_2}^{\sigma_2} \quad \dots \quad A_{\alpha_{N_{\text{sites}}}}^{\sigma_{N_{\text{sites}}}} \end{array} \right] |\sigma_1, \dots, \sigma_{N_{\text{sites}}}\rangle
 \end{aligned}$$

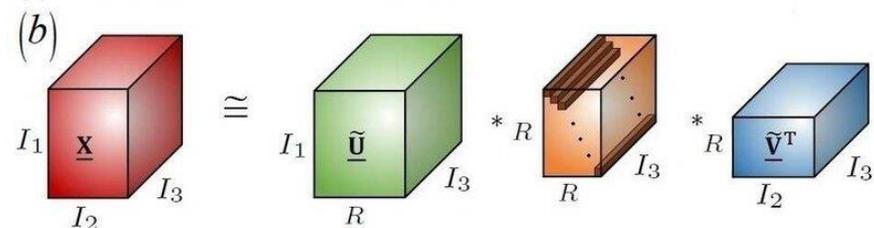
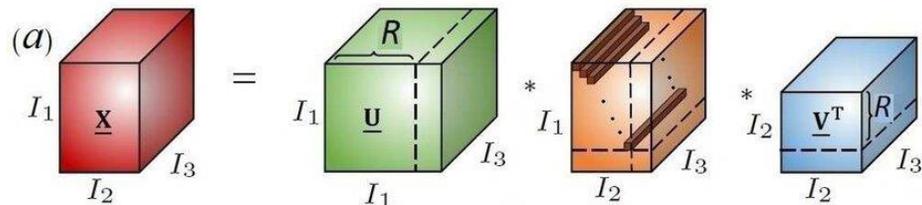
Why would you do that?

Efficient Approximation



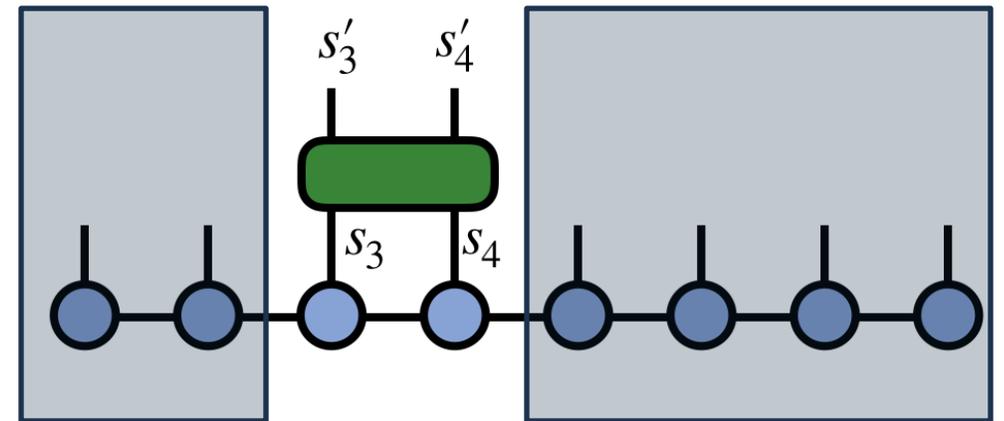
2^N parameters

$2Nm^2$ parameters

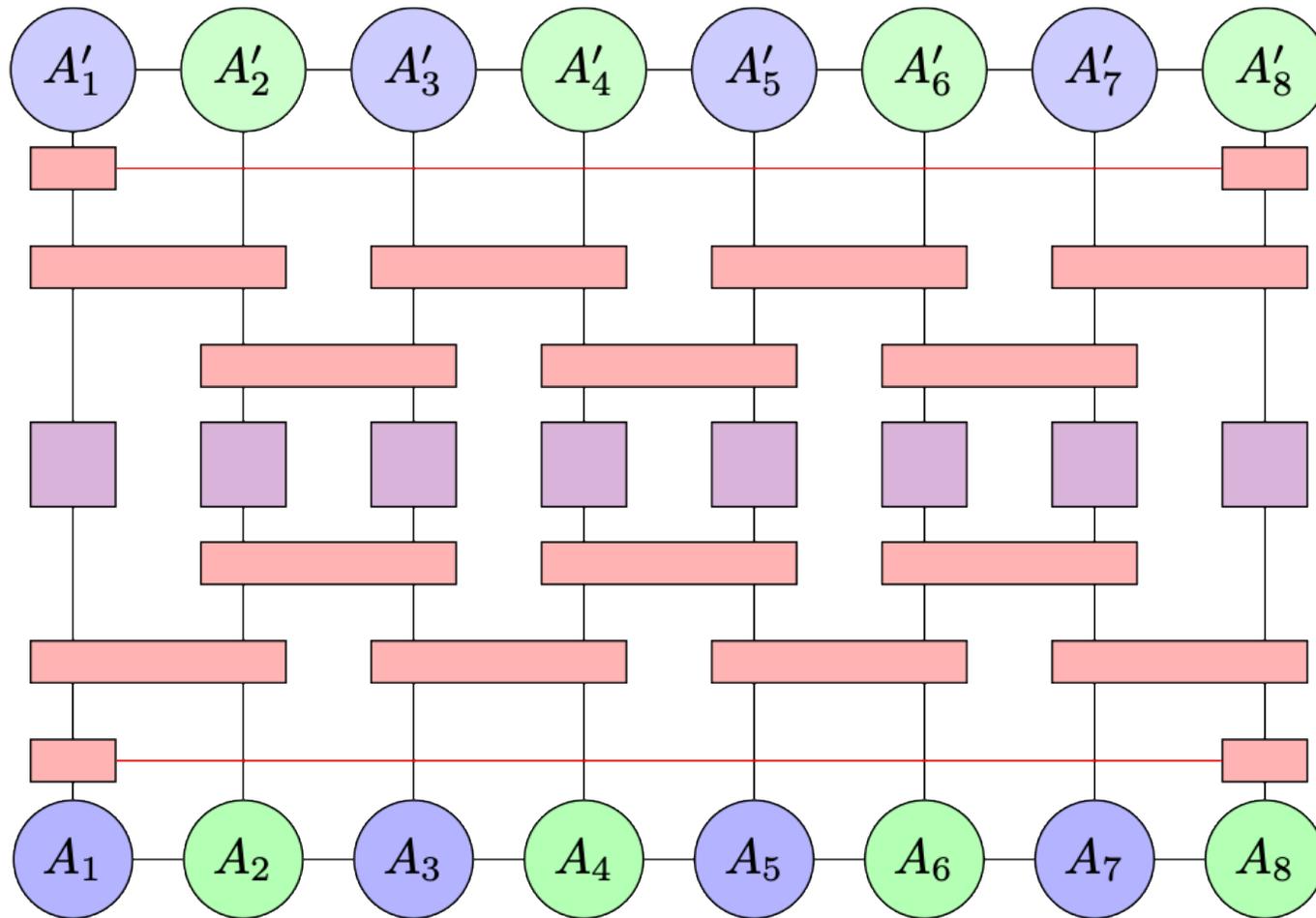


Ahmadi-Asl et al. (2023) arXiv.2305.05030.

Local Interactions



Full Timestep



$|\psi(t + \delta)\rangle$



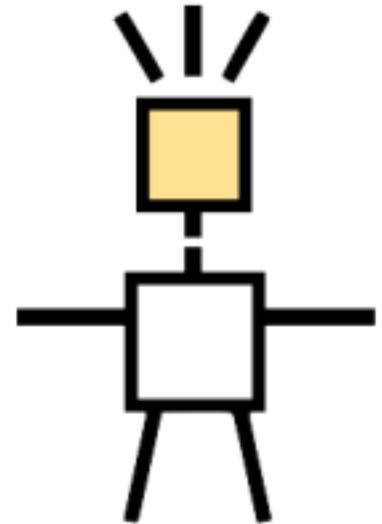
$\exp(iH_{vv}\delta/2)$

$\exp(iH_{vac}\delta)$



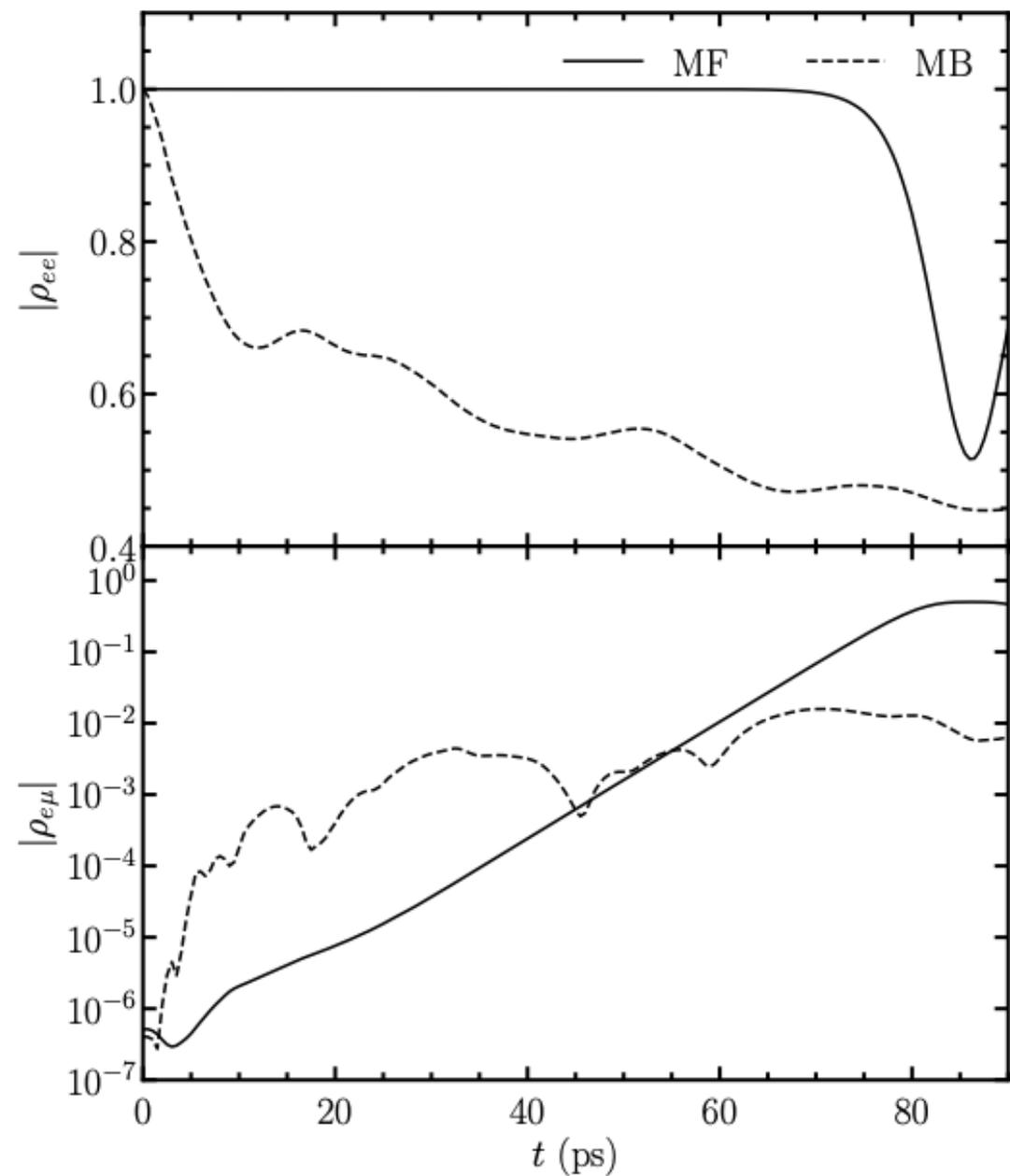
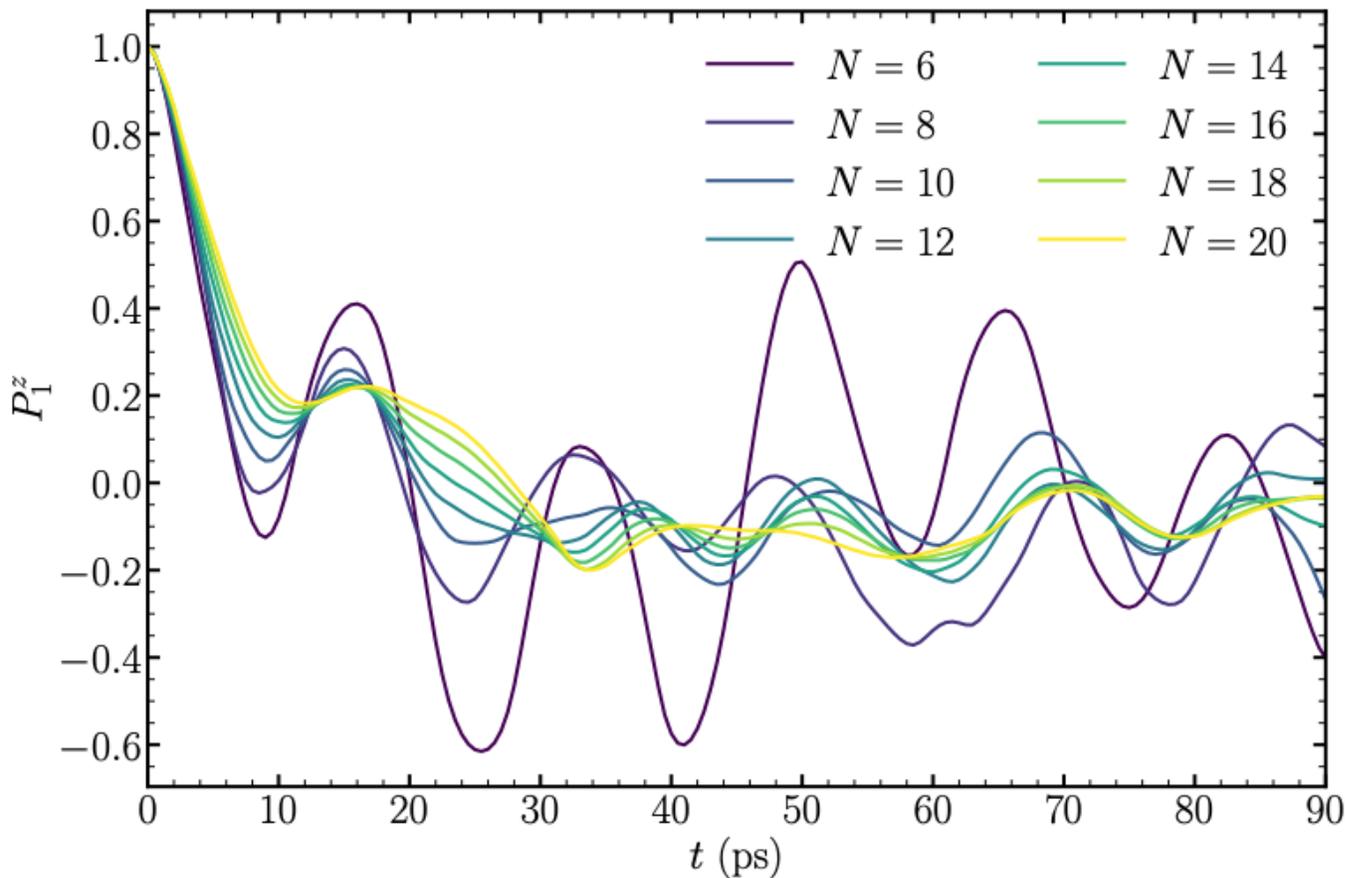
$\exp(iH_{vv}\delta/2)$

$|\psi(t)\rangle$

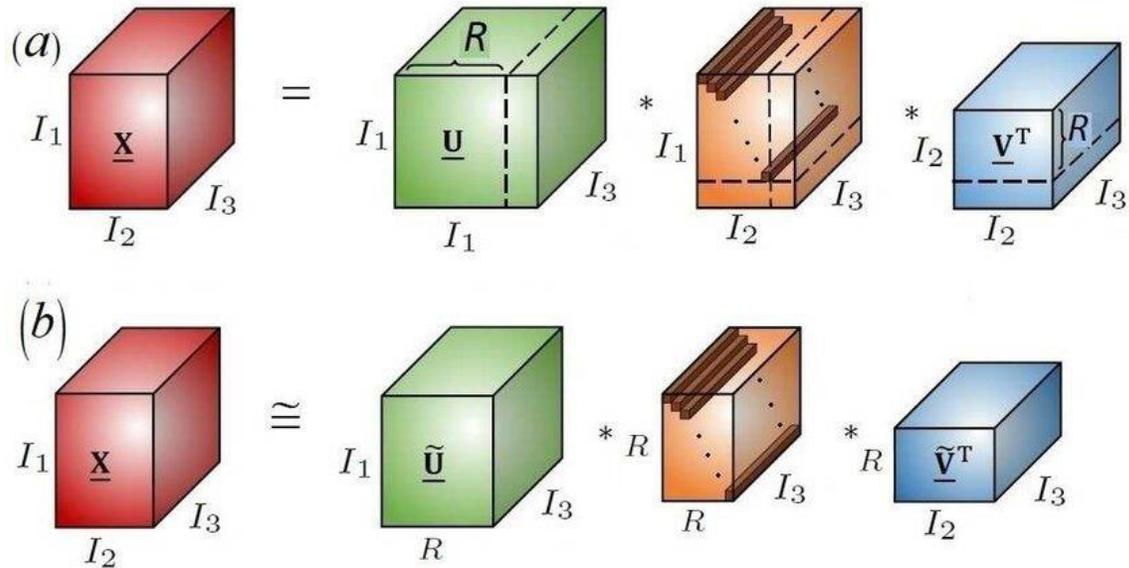


ITensors.jl
ITensorMPS.jl

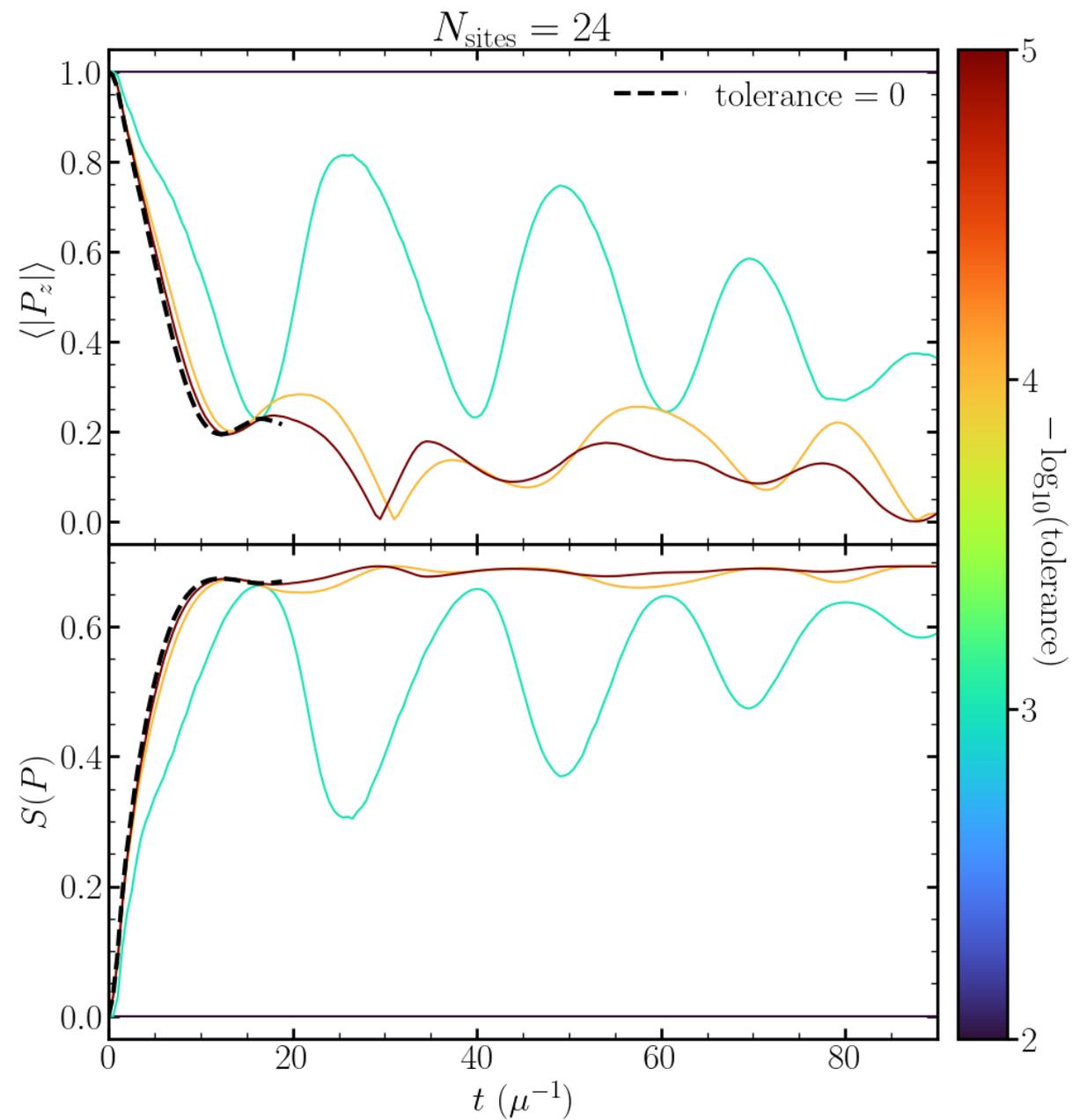
Is flavor instability a lie?



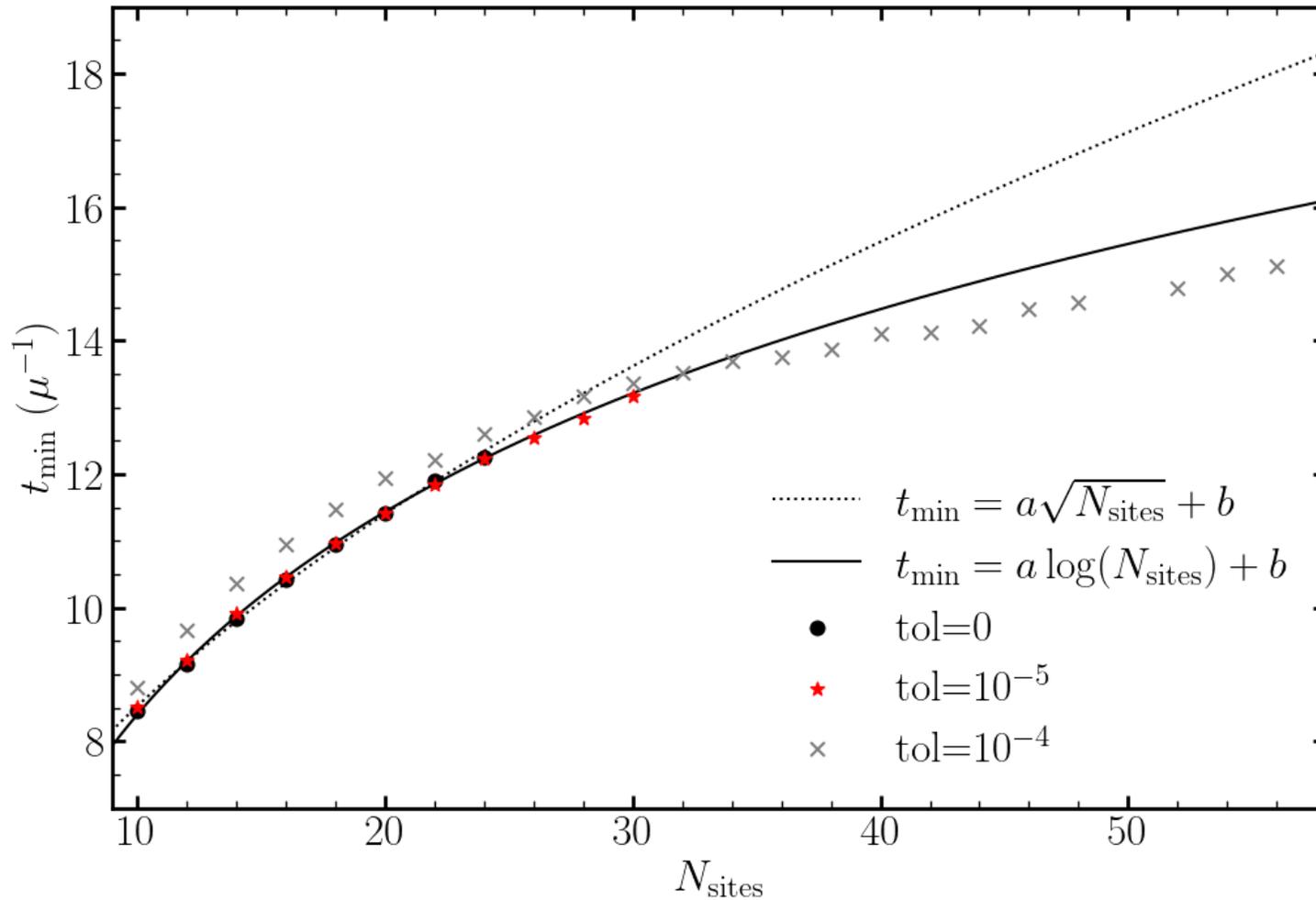
Enabling large simulations



Ahmadi-Asl et al. (2023) arXiv.2305.05030.



Scaling to large N



Data Set	$\epsilon_{RMS} (\mu^{-1})$ log	$\epsilon_{RMS} (\mu^{-1})$ sqrt
tol=0	0.036	0.050
tol=10 ⁻⁵	0.023	0.107

$a = 4.38 \pm 0.05$
 $b = -1.69 \pm 0.14$

Conclusions

Many-body neutrino plasma model seems to predict **fast many-body effects**

Limited by:

- Small system size
- Truncated Hamiltonian