Experimental Results on Di-Hadron Production

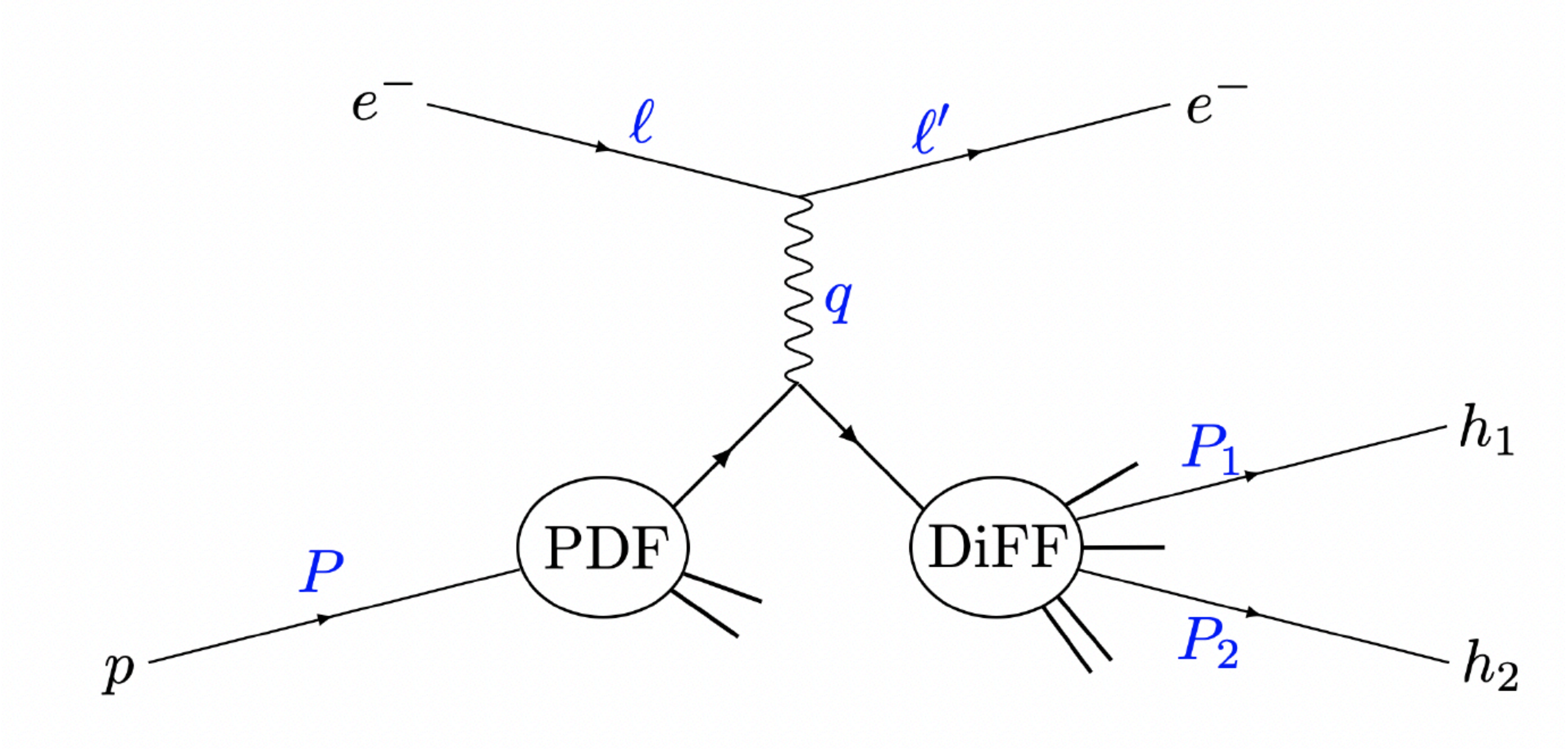
Nilanga Wickramaarachchi

Duke

Research supported by DOE
Office of Science

06/10/2025

Dihadron SIDIS



[1]

- In the dihadron SIDIS we detect two hadrons in the final state

$$e^- p \to e^- h_1 h_2 X$$

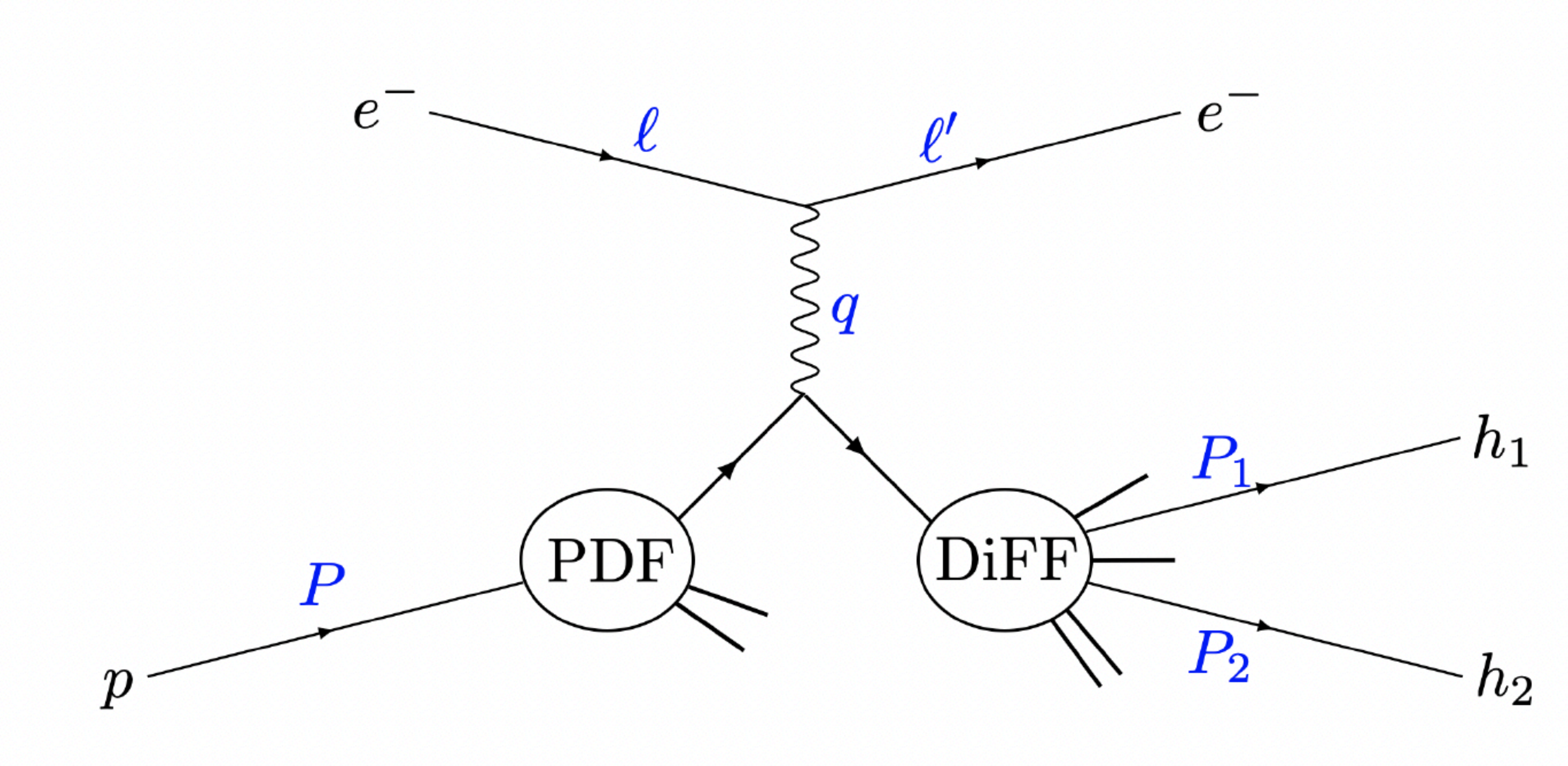
- Dihadron fragmentation function (DIFF) provide information on how the struck quark produces two hadrons

- Relative angular momentum between two hadrons provide more information on hadronization compared to single hadron case

  - Quark transverse momentum - angular momentum correlations

  - Access more structure functions

Dihadron kinematics



- Dihadron momentum is defined as sum of momenta of two hadrons

$P_h = P_1 + P_2$ $\qquad$ M_h - dihadron mass

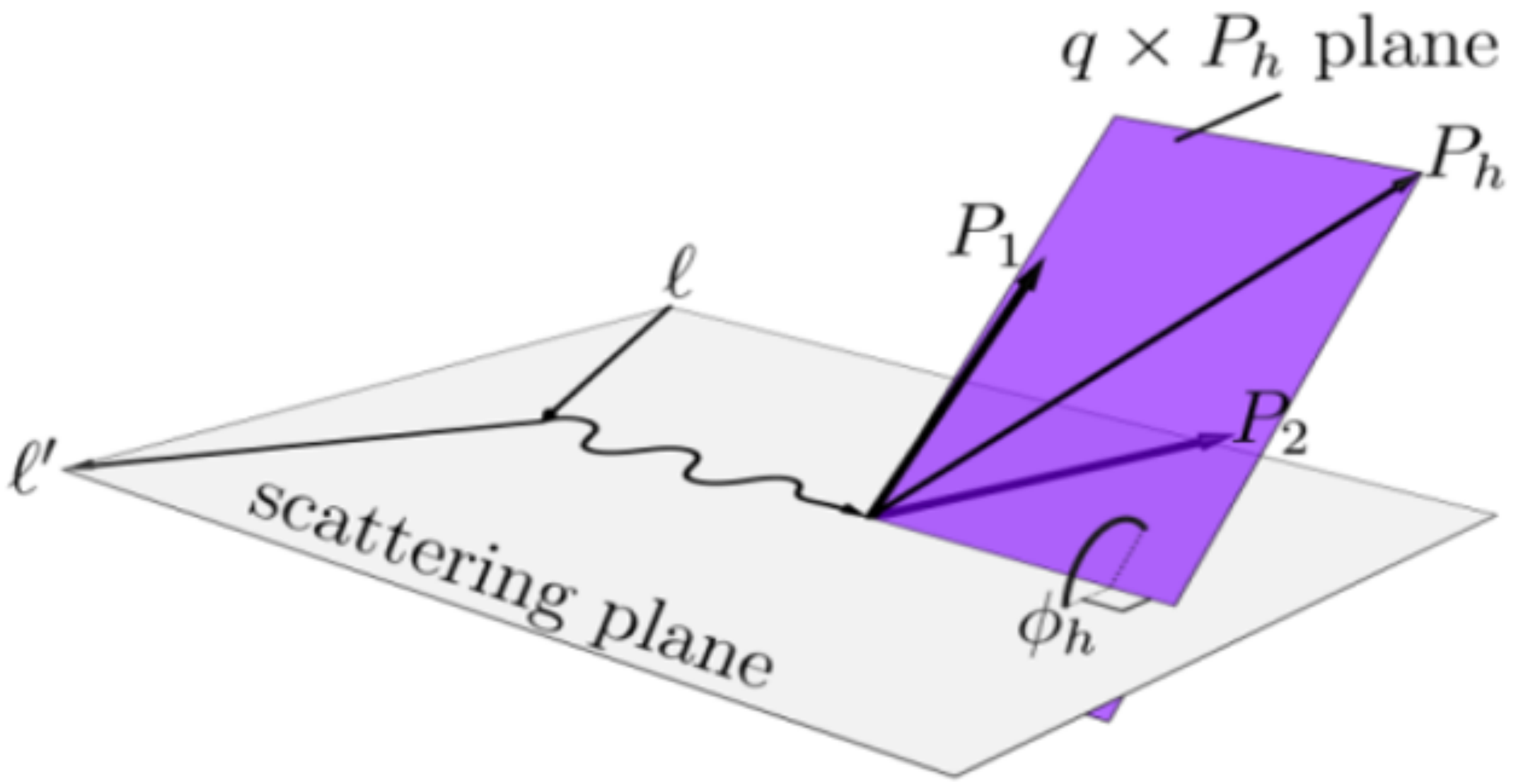
- Fractional energy of dihadron

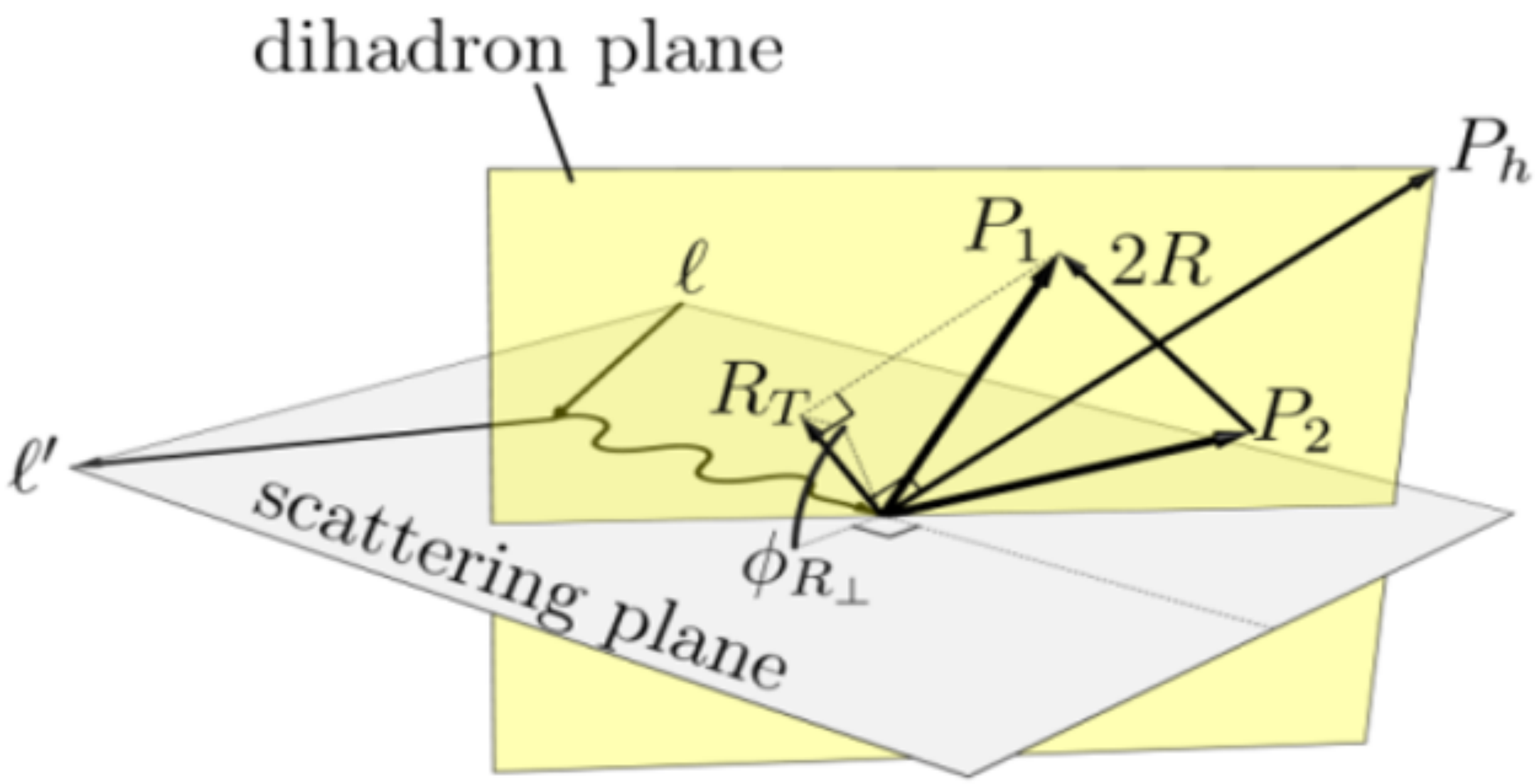
$$z = \frac{P \cdot P_h}{P \cdot q}$$

- Feynman-x variable used to consider current fragmentation region

$$x_F = \frac{2\,P_h \cdot q}{|q|\,W}$$

Dihadron kinematics



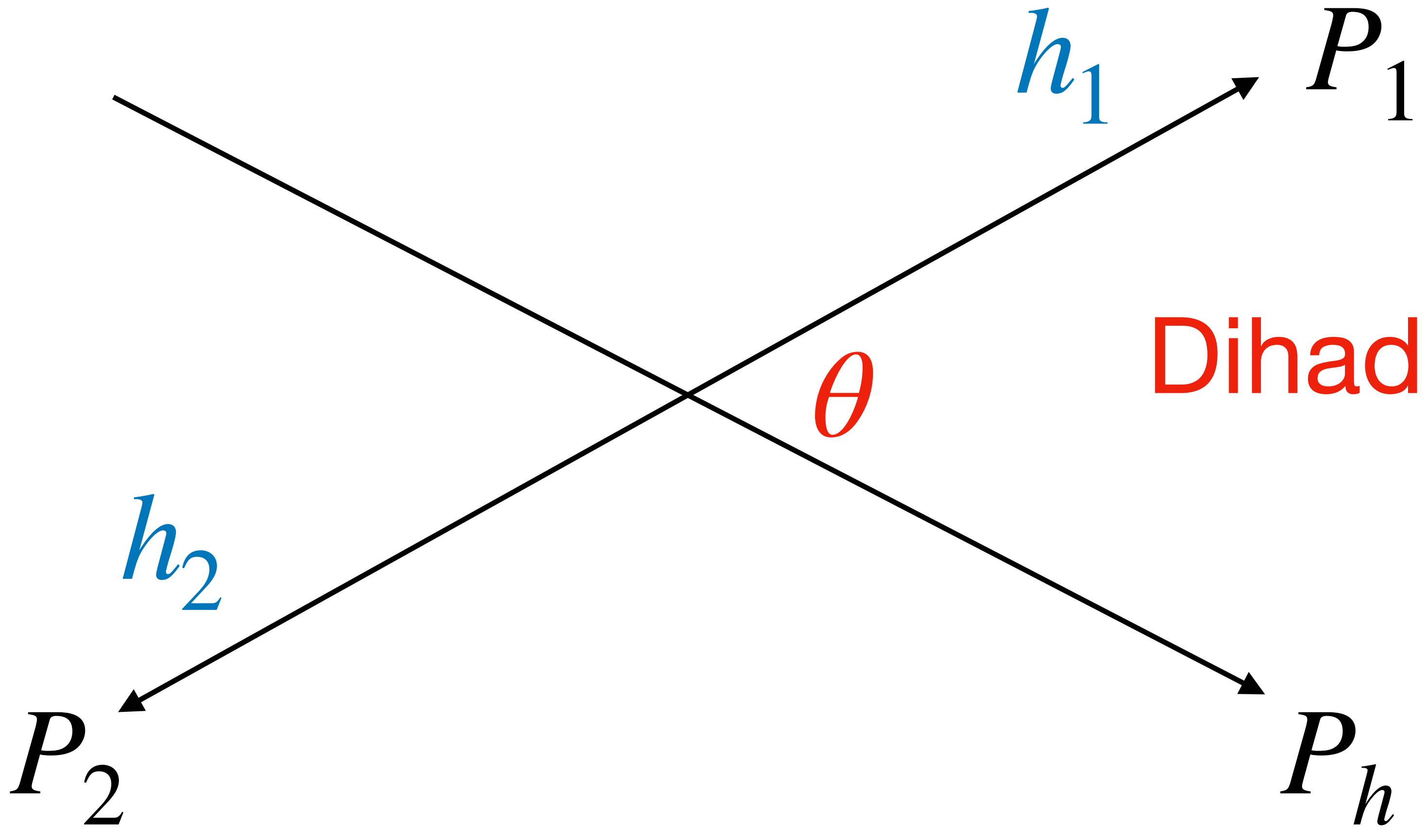


[1]

$$\phi_h = \frac{(\vec{q}\times\vec{l})\cdot\vec{P_h}}{|(\vec{q}\times\vec{l})\cdot\vec{P_h}|}\, arccos\frac{(\vec{q}\times\vec{l})\cdot(\vec{q}\times\vec{P_h})}{|\vec{q}\times\vec{l}\,||\vec{q}\times\vec{P_h}|}$$

$$\phi_R = \frac{(\vec{q}\times\vec{l})\cdot\vec{R_T}}{|(\vec{q}\times\vec{l})\cdot\vec{R_T}|}\, arccos\frac{(\vec{q}\times\vec{l})\cdot(\vec{q}\times\vec{R_T})}{|\vec{q}\times\vec{l}\,||\vec{q}\times\vec{R_T}|}$$

$$\vec{R}_T = \frac{z_2\vec{P_1}^{\perp} - z_1\vec{P_2}^{\perp}}{z}$$



Dihadron COM frame

Provides additional degrees of freedom
ϕ_R, θ and M_h compared to single hadron SIDIS

Transverse momentum dependent PDFs

- Depends on x and transverse momentum p_T^2 of struck quark
- Twist-t means contribution to cross section suppressed by $\left(\frac{M}{Q}\right)^{t-2}$

		Quark		
		U	L	T
Nucleon	U	Unpolarized $f_1 =$		Boer-Mulders $h_1^\perp = \quad - \quad$
	L		Helicity $g_{1L} = \quad - \quad$	Worm-gear 1 $h_{1L}^\perp = \quad - \quad$
	T	Sivers $f_{1T}^\perp = \quad - \quad$	Worm-gear 2 $g_{1T}^\perp = \quad - \quad$	Transversity $h_1 = \quad - \quad$ Pretzelosity $h_{1T}^\perp = \quad - \quad$

Twist-2

U - unpolarized
L - longitudinally polarized
T - transversely polarized

[2]

		Quark		
		U	L	T
Nucleon	U	$f^\perp$	$g^\perp$	$h,\ e$
	L	$f_L^\perp$	$g_L^\perp$	$h_L,\ e_L$
	T	$f_T,\ f_T^\perp$	$g_T,\ g_T^\perp$	$h_T,\ e_T,\ h_T^\perp,\ e_T^\perp$

Twist-3

sensitive to quark-gluon correlations

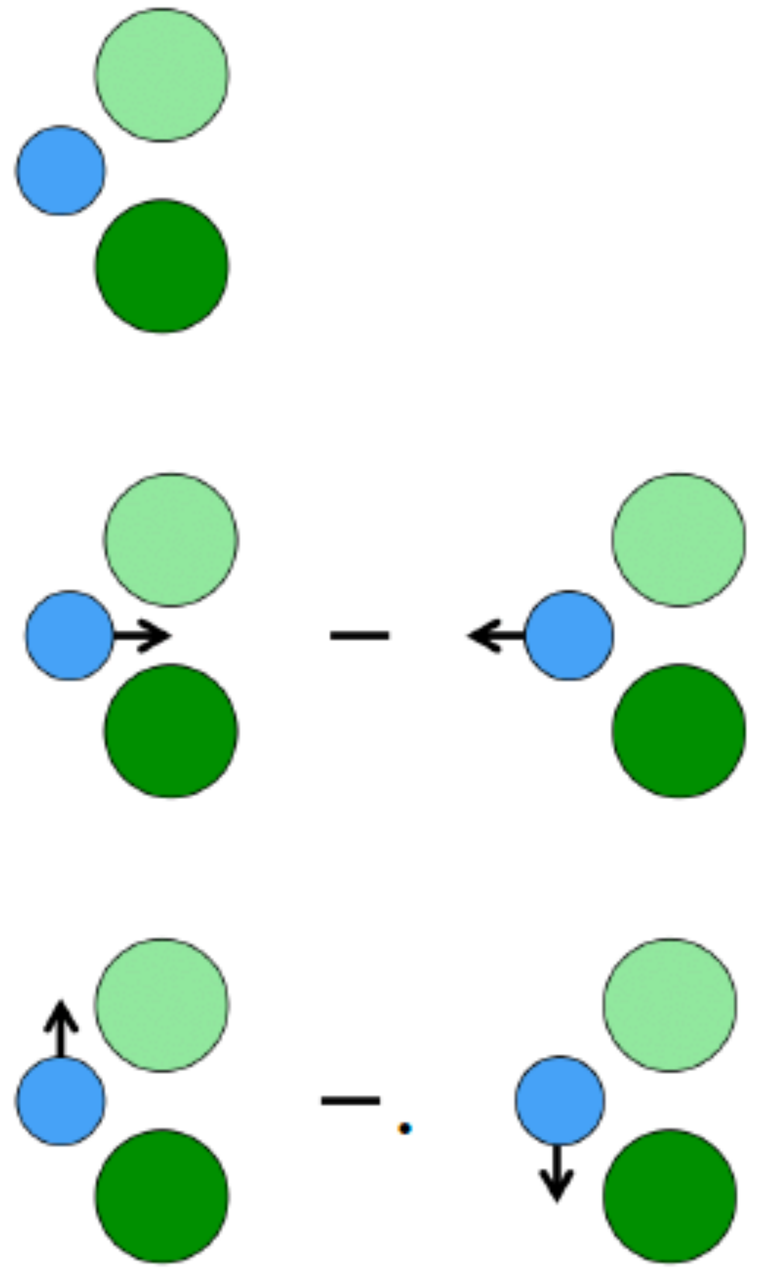
Dihadron fragmentation functions

- Depends on z, M_h and transverse momentum k_T^2 of fragmenting quark $(k = p + q)$
- Probability difference of quark with a certain polarization to fragment into unpolarized hadrons

Unpolarized $D_1 =$

Longitudinally polarized $G_1^\perp =$ −

Transversely polarized $\overline{H}_1^\angle, H_1^\perp =$ −.



[2]

Twist-2

$\tilde{D}^\perp$

$\tilde{E}$

$\tilde{G}^\perp$

$\tilde{H}$

Twist-3

Dihadron SIDIS cross section

- Cross section components for certain polarization states of beam and target

$$d\sigma_{LU} = \frac{\alpha^2}{4\pi xyQ^2}\left(1+\frac{\gamma^2}{2x}\right)\lambda_e \sum_{l=0}^{l_{max}} \left\{ C(x,y)\sum_{m=1}^{l}\left[P_{l,m}\sin(m(\phi_h-\phi_{R_\perp}))\, 2\left(F_{LU,T}^{P_{l,m}\sin(m(\phi_h-\phi_{R_\perp}))} + \epsilon F_{LU,L}^{P_{l,m}\sin(m(\phi_h-\phi_{R_\perp}))}\right)\right.\right.$$

$$\left.+W(x,y)\sum_{m=-l}^{l} P_{l,m}\sin((1-m)\,\phi_h + m\phi_{R_\perp})\, F_{LU}^{P_{l,m}\sin((1-m)\phi_h+m\phi_{R_\perp})}\right\}$$

- $F_{XY}^{m(\phi_h,\phi_R)}$'s are structure functions which can be written as convolutions of TMD PDF and DIFF

- $P_{l,m}$'s are Legendre polynomials depend on $\cos\,\theta$

- Limit l_{max} = 2 (dihadron invariant mass in CLAS12 limited to around 1 GeV)

- Consider integration over θ

- Cross section is related to azimuthal modulations of angles ϕ_h and ϕ_R

Dihadron SIDIS cross section

$$d\sigma_{UL} = \frac{\alpha^2}{4\pi x y Q^2}\left(1+\frac{\gamma^2}{2x}\right) S_L \Bigg\{ A(x,y) \sum_{\ell=1}^{\ell_{\max}} \sum_{m=1}^{\ell} P_{\ell,m} \sin(-m\phi_h + m\phi_{R_\perp}) F_{UL}^{P_{\ell,m}\sin(-m\phi_h+m\phi_{R_\perp})}$$
$$+ B(x,y) \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} P_{\ell,m} \sin((2-m)\phi_h + m\phi_{R_\perp}) F_{UL}^{P_{\ell,m}\sin((2-m)\phi_h+m\phi_{R_\perp})}$$
$$+ V(x,y) \sum_{\ell=0}^{\ell_{\max}} \sum_{m=-\ell}^{\ell} P_{\ell,m} \sin((1-m)\phi_h + m\phi_{R_\perp}) F_{UL}^{P_{\ell,m}\sin((1-m)\phi_h+m\phi_{R_\perp})} \Bigg\}.$$

[3]

$$d\sigma_{LL} = \frac{\alpha^2}{4\pi x y Q^2}\left(1+\frac{\gamma^2}{2x}\right) \lambda_e S_L$$
$$\times \sum_{\ell=0}^{\ell_{\max}} \Bigg\{ C(x,y) \sum_{m=0}^{\ell} 2^{2-\delta_{m0}} P_{\ell,m} \cos(m(\phi_h - \phi_{R_\perp})) F_{LL}^{P_{\ell,m}\cos(m(\phi_h-\phi_{R_\perp}))}$$
$$+ W(x,y) \sum_{m=-\ell}^{\ell} P_{\ell,m} \cos((1-m)\phi_h + m\phi_{R_\perp}) F_{LL}^{P_{\ell,m}\cos((1-m)\phi_h+m\phi_{R_\perp})} \Bigg\}.$$

[3]

Structure functions, PDFs and DIFFs

Structure function and modulation	PDF and DIFF	Depolarization factor
$F_{LU}^{\sin(\phi_R)}$	$eH_1^{\perp}$	W
$F_{LU}^{\sin(\phi_h-\phi_R)}$	$f_1 G_1$	C
$F_{UL}^{\sin(\phi_R)}$	$h_L H_1^{\perp}$	V
$F_{UL}^{\sin(2\phi_R)}$	$h_{1L}^{\perp} H_1^{\perp}$	B
$F_{UL}^{\sin(-\phi_h+\phi_R)}$	$g_{1L} G_1$	A
F_{LL}^{const}	$g_{1L} D_1$	C
$F_{LL}^{\cos(\phi_R)}$	$g_{1L} \tilde{D}$	W
F_{UU}	$f_1 D_1$	A

$$A(\epsilon, y) = \frac{y^2}{2(1-\epsilon)}$$

$$V(\epsilon, y) = \frac{y^2}{2(1-\epsilon)} \sqrt{2\epsilon(1+\epsilon)}$$

$$B(\epsilon, y) = \frac{y^2}{2(1-\epsilon)} \epsilon$$

$$W(\epsilon, y) = \frac{y^2}{2(1-\epsilon)} \sqrt{2\epsilon(1-\epsilon)}$$

$$C(\epsilon, y) = \frac{y^2}{2(1-\epsilon)} \sqrt{1-\epsilon^2}$$

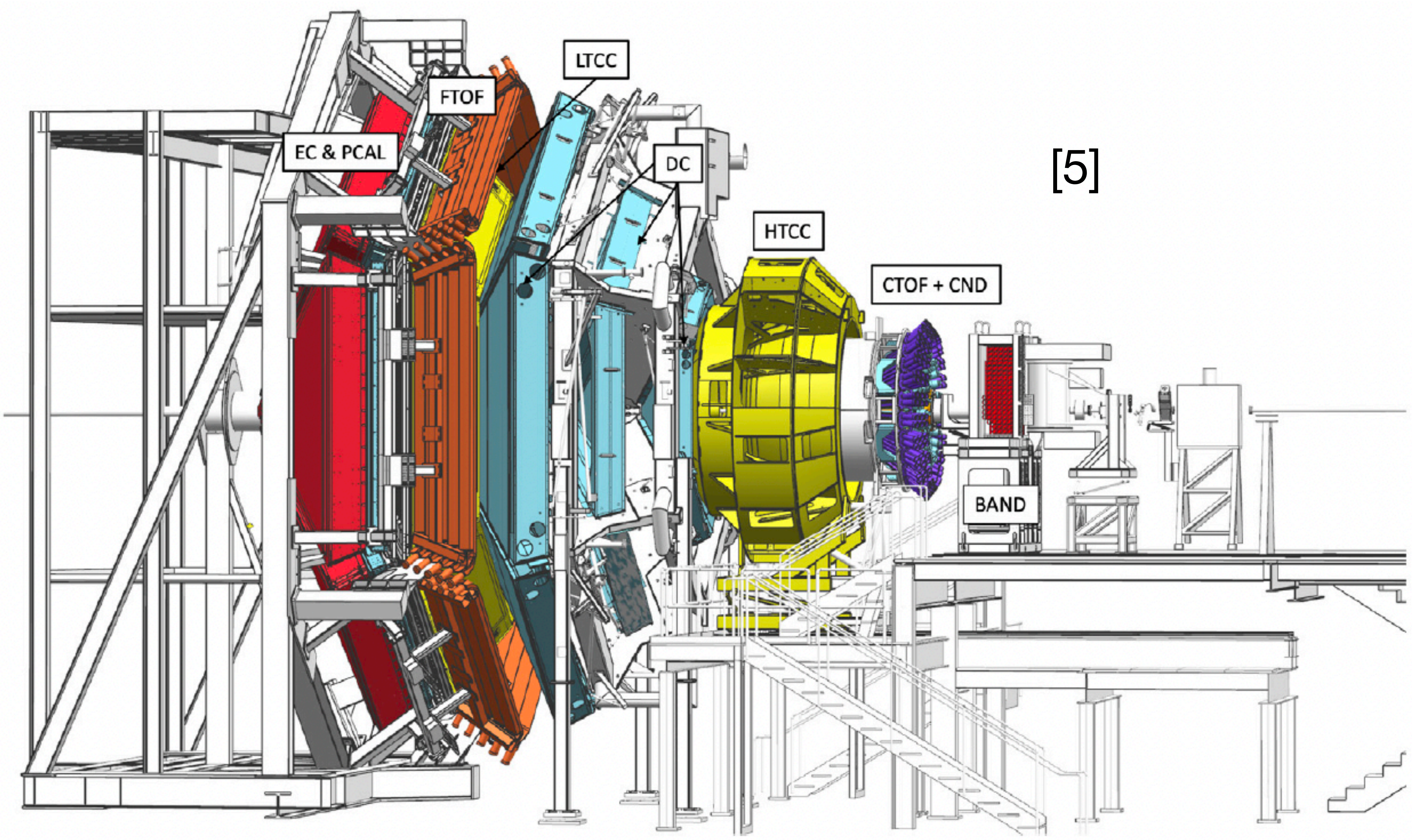
[4]

$$\epsilon = \frac{1 - y - \gamma^2 y^2/4}{1 - y + y^2/2 + \gamma^2 y^2/4}$$

$$\gamma = \frac{2Mx}{Q}$$

CLAS12 detector

- In Hall B at Jefferson Lab



[5]

Forward Detector

- HTCC
- Drift Chamber
- LTCC / RICH
- FTOF
- Forward Tagger
- Calorimeters

Central Detector

- Central Vertex Tracker
- CTOF
- Central Neutron Detector
- Back Angle Neutron Detector

- Torus magnet and a solenoid magnet
- Large coverage in both azimuthal and polar angles for charged and neutral particles
- Particle identification by Cherenkov counters and time-of-flight detectors
- Luminosity 10^{35} $cm^{-2}s^{-1}$

Unpolarized target data from CLAS12

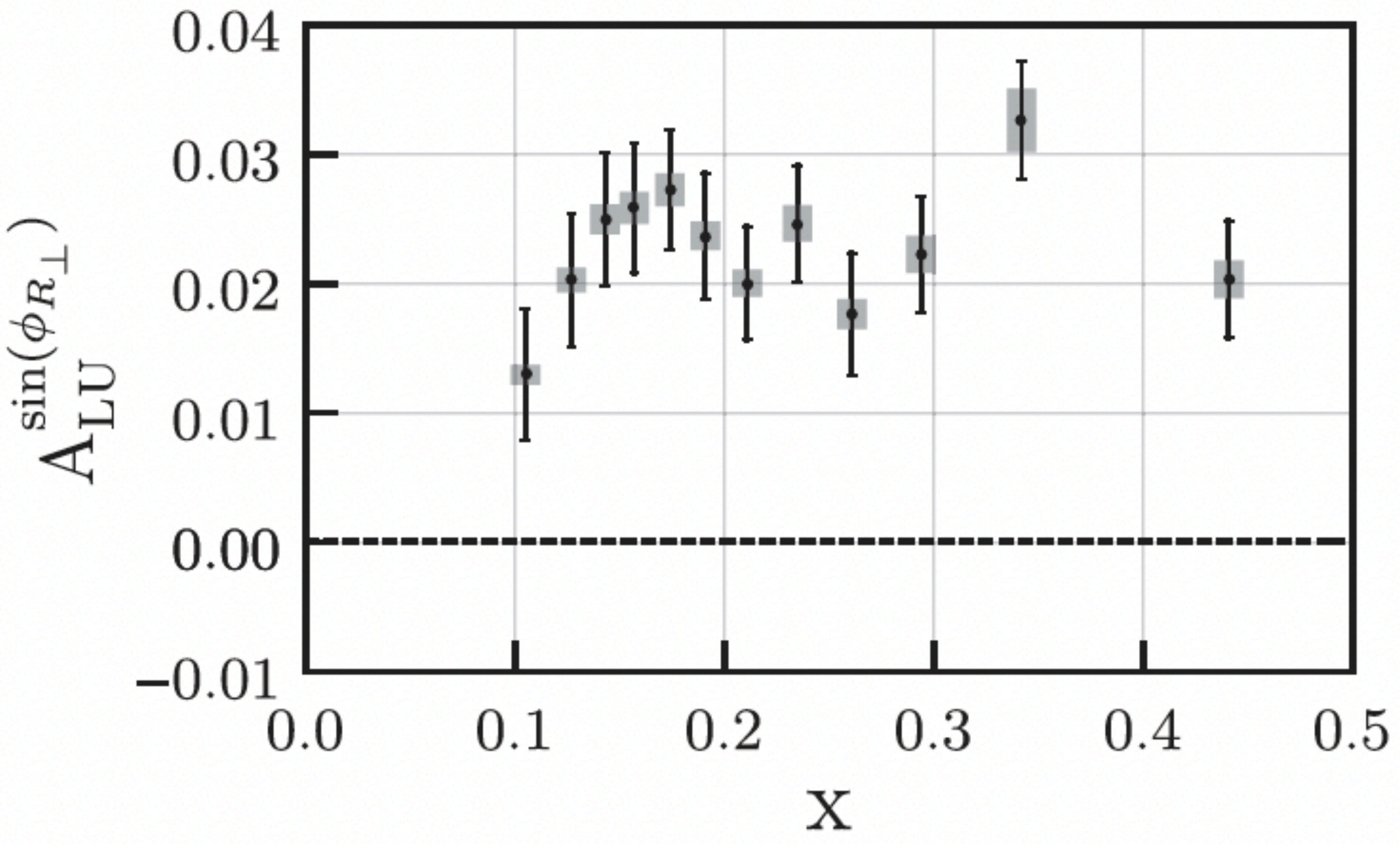
- Longitudinally polarized 10.6 GeV electron beam
- Unpolarized LH_2 target
  - Access beam spin asymmetries
    - $e^- p \to e^- \pi^+ \pi^- X$
    - $e^- p \to e^- \pi^+ \pi^0 X$
    - $e^- p \to e^- \pi^- \pi^0 X$
    - $e^- p \to e^- K^+ \pi^- X$
    - $e^- p \to e^- \pi^+ K^- X$
    - $e^- p \to e^- K^+ K^- X$

$$A_{LU} = \frac{d\sigma_+ - d\sigma_-}{d\sigma_+ + d\sigma_-} \propto \frac{F_{LU}}{F_{UU}}$$

- Unpolarized LD_2 target
  - Compare beam spin asymmetry for proton and deuteron targets
    - $e^- p \to e^- \pi^+ \pi^- X$

$$e^- p \rightarrow e^- \pi^+ \pi^- X$$

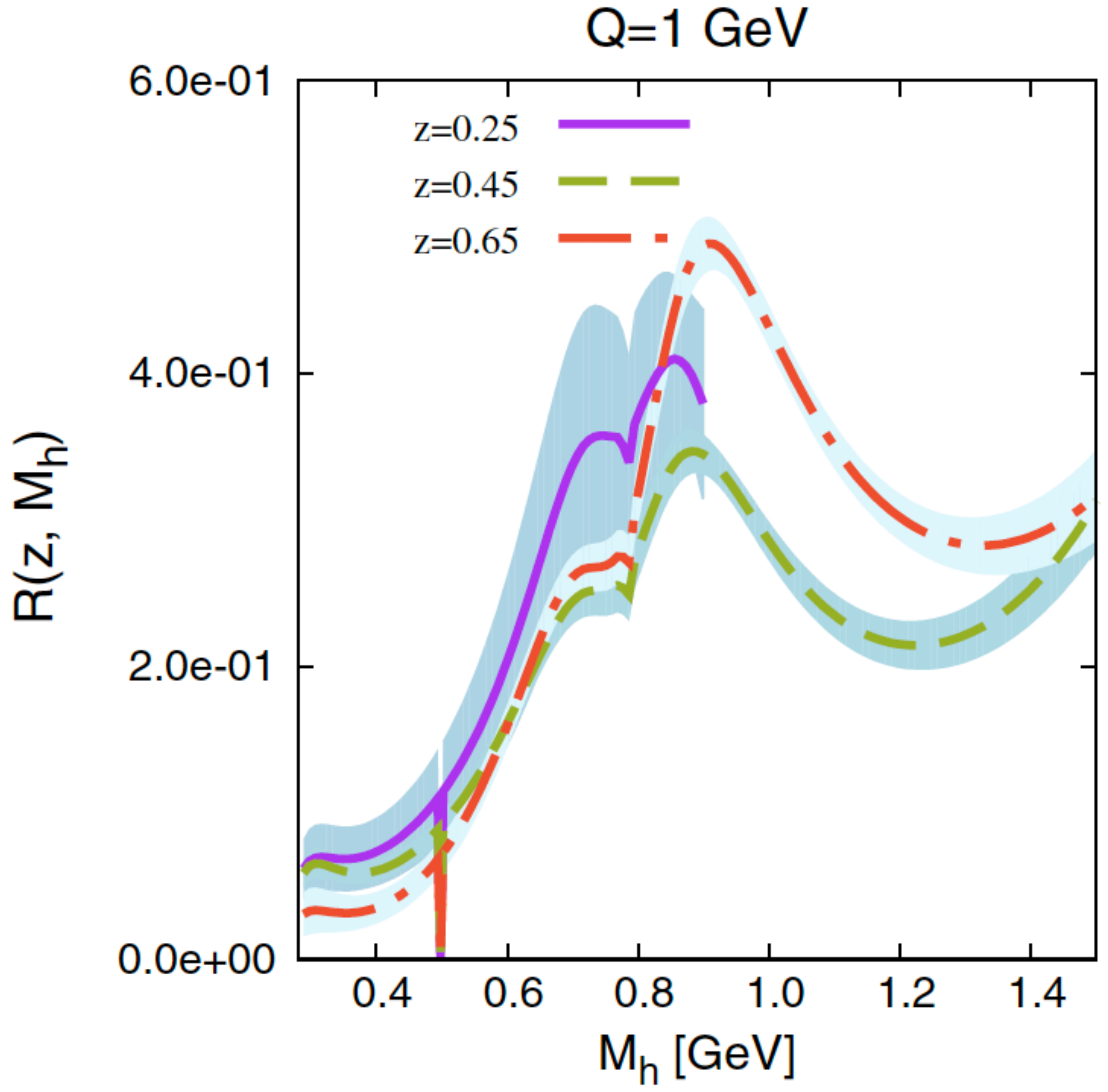
PRL 126, 152501 (2021)

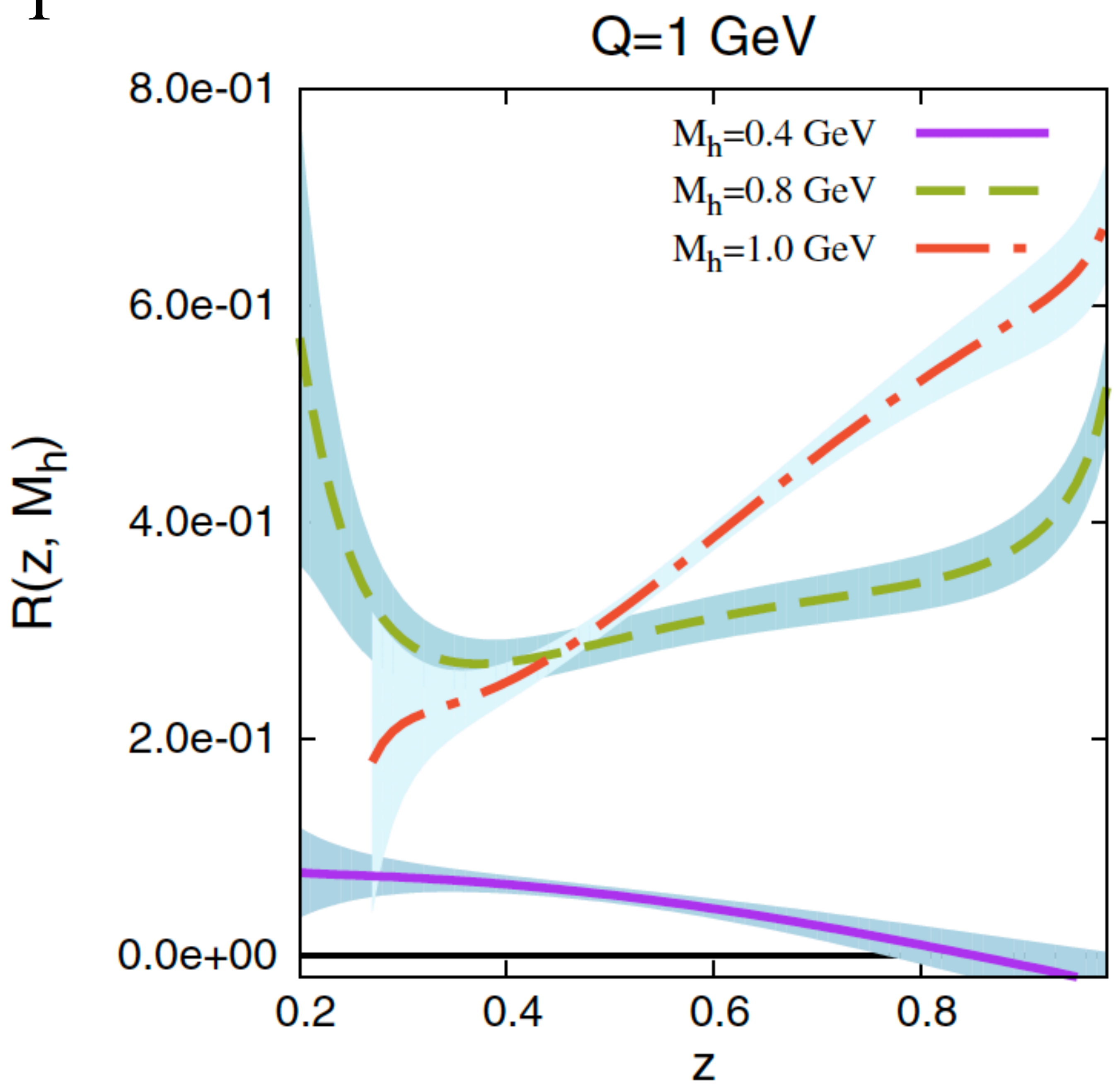


$$A_{LU}^{\sin(\phi_{R_\perp})} \propto e(x)\, H_1^{\sphericalangle}$$

$$R = \frac{|\vec{R}|}{M_h} \frac{H_{1,sp}^{\sphericalangle u}}{D_1^u}$$

PRD 85, 114023 (2012)



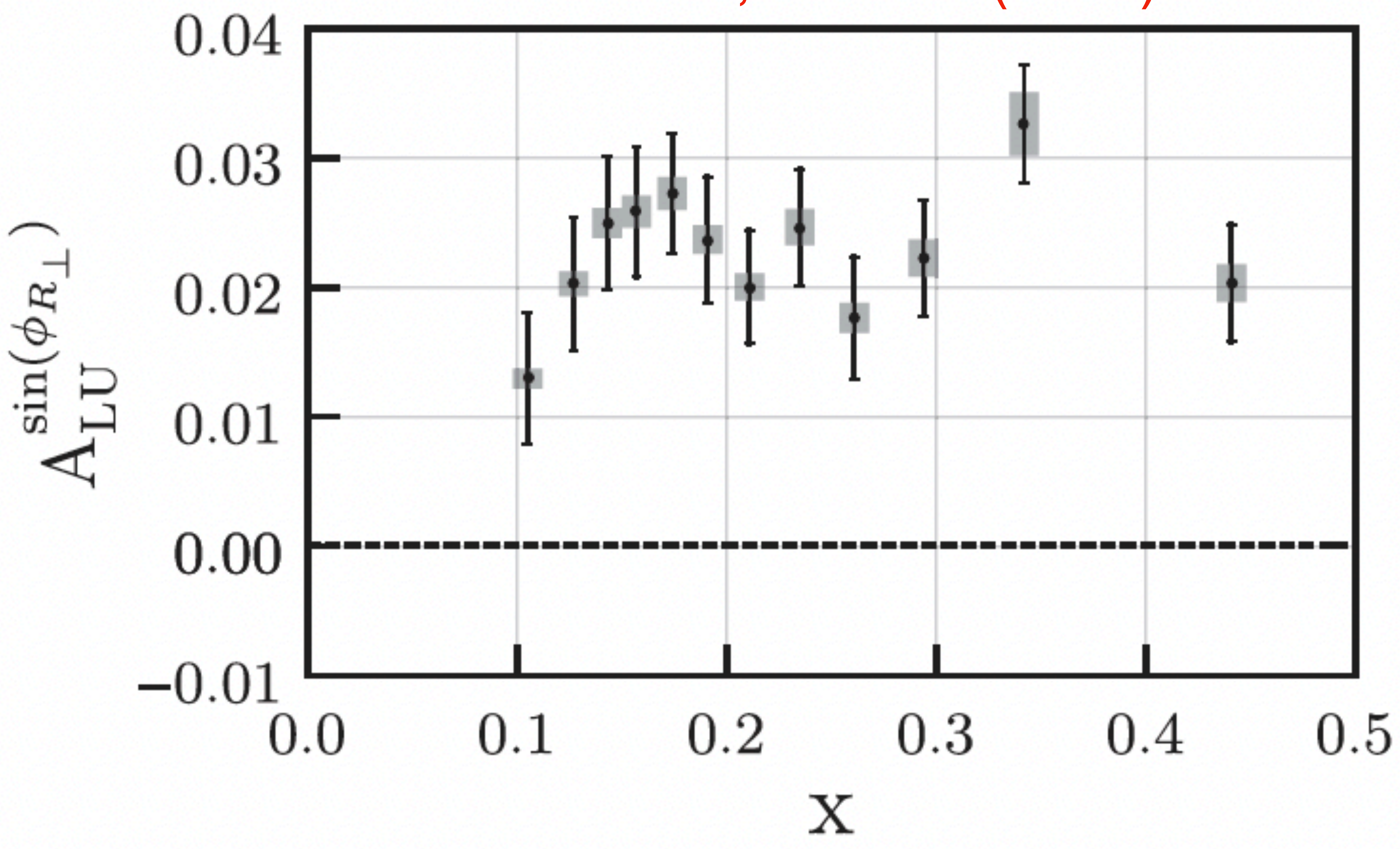


$H_1^{\sphericalangle}$ extracted from Belle e^+e^- data (PRL 107, 072004 (2011))

- First x moment of e(x) is related to the pion-nucleon σ term, representing the contribution to the nucleon mass from the finite quark masses
- Third x moment is proportional to the transverse force experienced by a transversely polarized quark in an unpolarized nucleon immediately after scattering

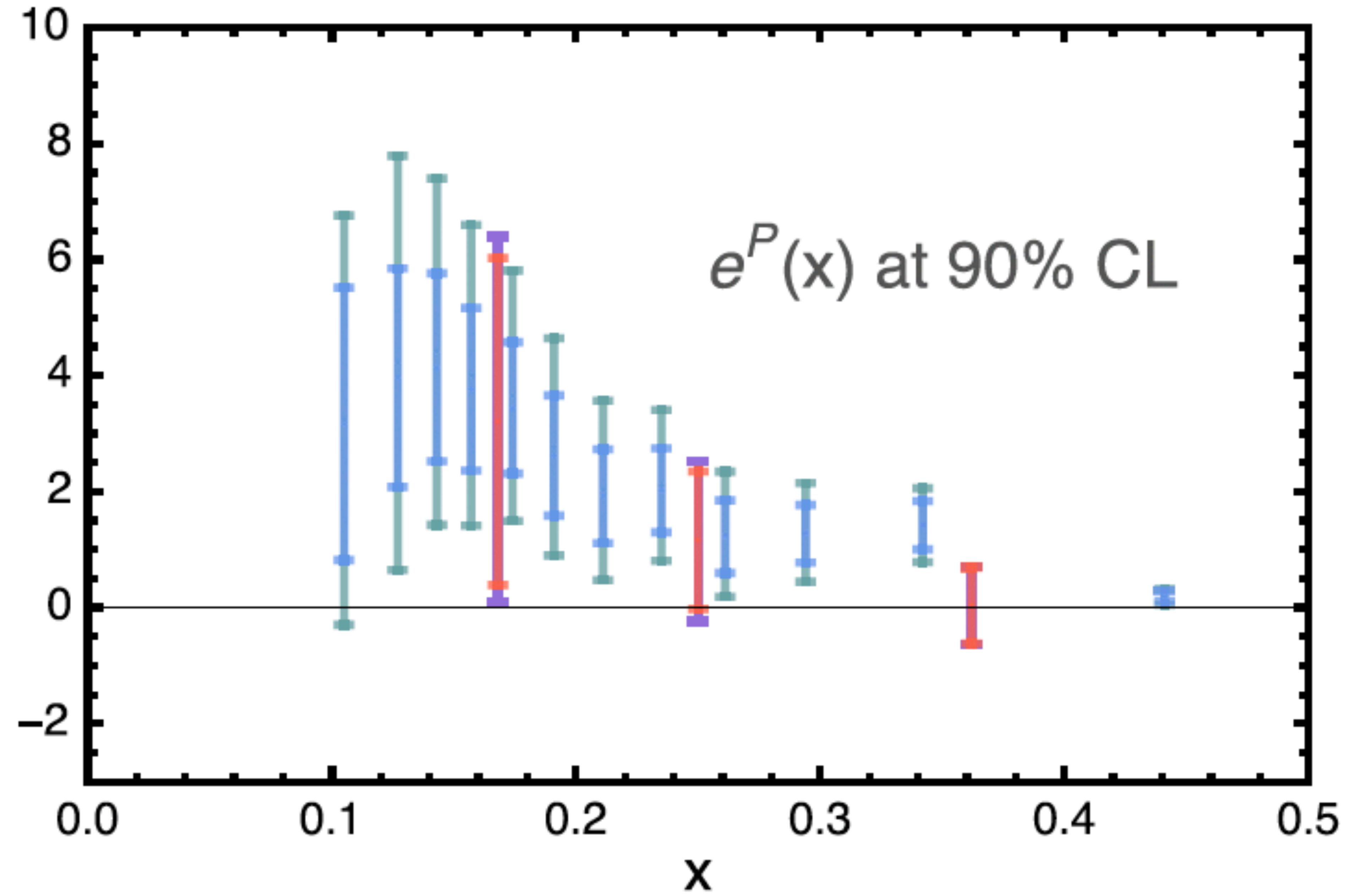
$e^- p \to e^- \pi^+ \pi^- X$

PRL 126, 152501 (2021)



$A_{LU}^{\sin(\phi_{R_\perp})} \propto e(x)\, H_1^{\sphericalangle}$

PRD 106, 014027 (2022)



$e^P(x)$ extracted from CLAS and CLAS12 data

Two cases considered

1. Wandzura-Wilczek (WW) approximation (inner bars)

2. Beyond WW approximation (outer bars)

- Results for $A_{LU}^{\sin(\phi_{R_\perp})}$ from CLAS12 and $H_1^{\sphericalangle}$ results from Belle used to extract twist-3 PDF $e(x)$

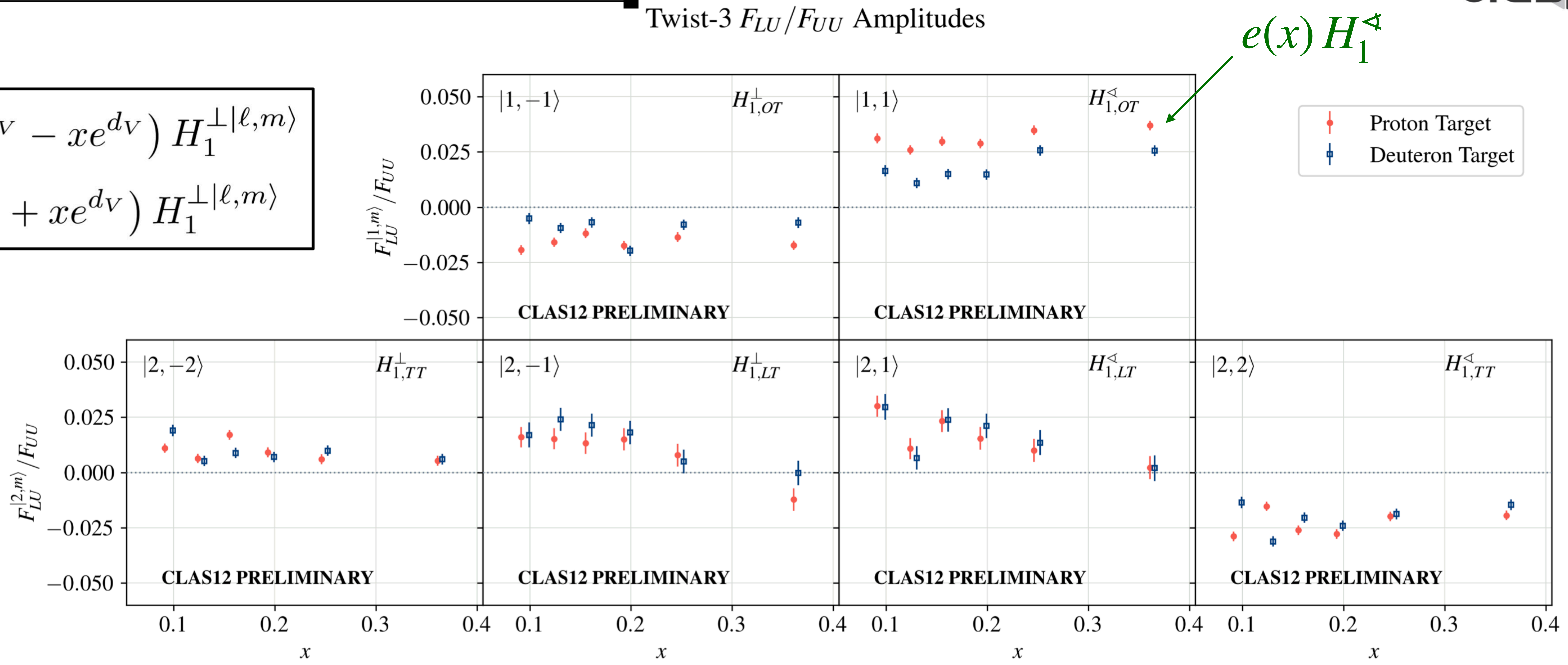
$e^- p \to e^- \pi^+ \pi^- X$

C. Dilks

x Bins

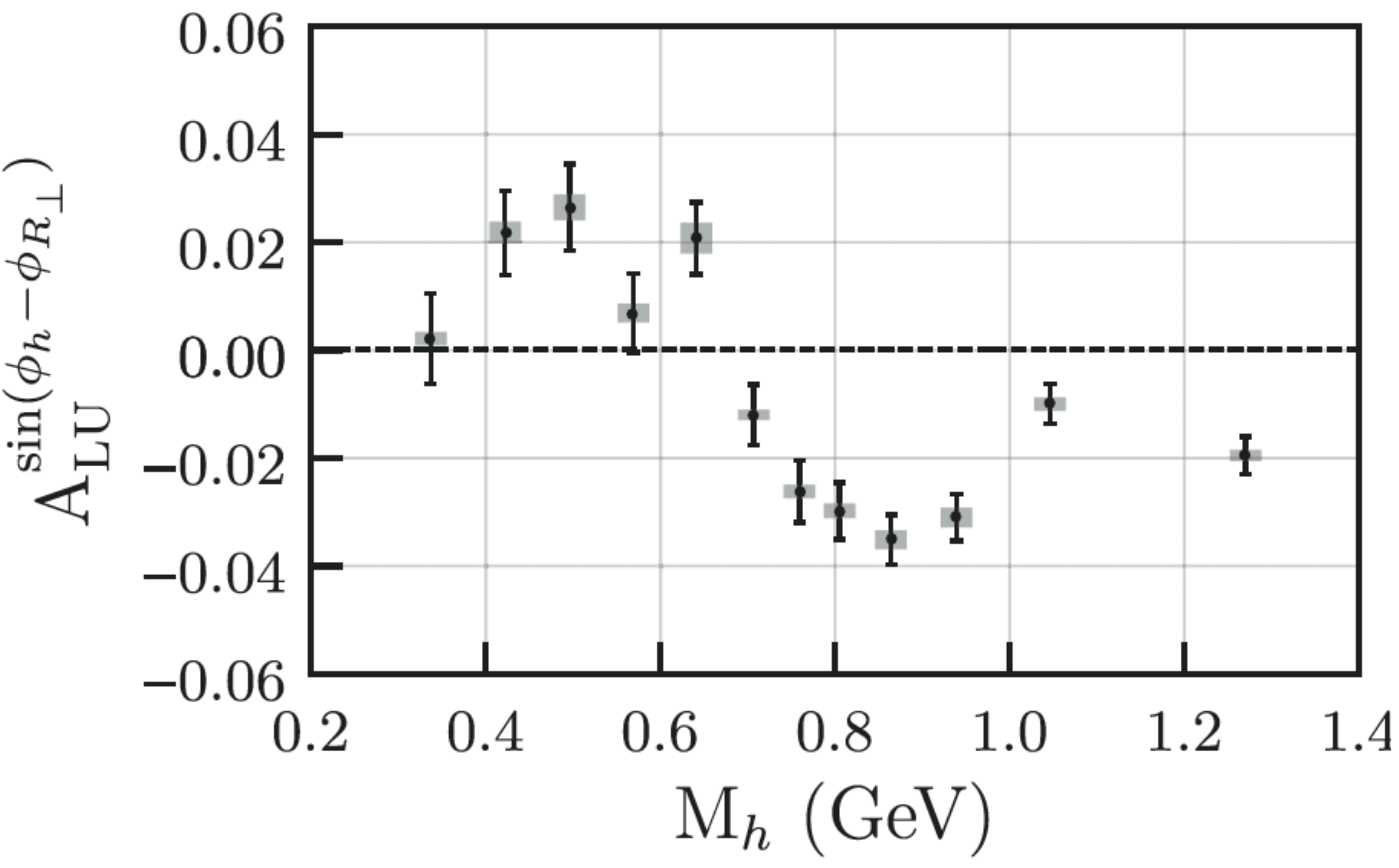
$$A_{LU,\mathbf{p}}^{|\ell,m\rangle} \propto \left(4xe^{u_V} - xe^{d_V}\right) H_1^{\perp|\ell,m\rangle}$$

$$A_{LU,\mathbf{d}}^{|\ell,m\rangle} \propto \left(xe^{u_V} + xe^{d_V}\right) H_1^{\perp|\ell,m\rangle}$$

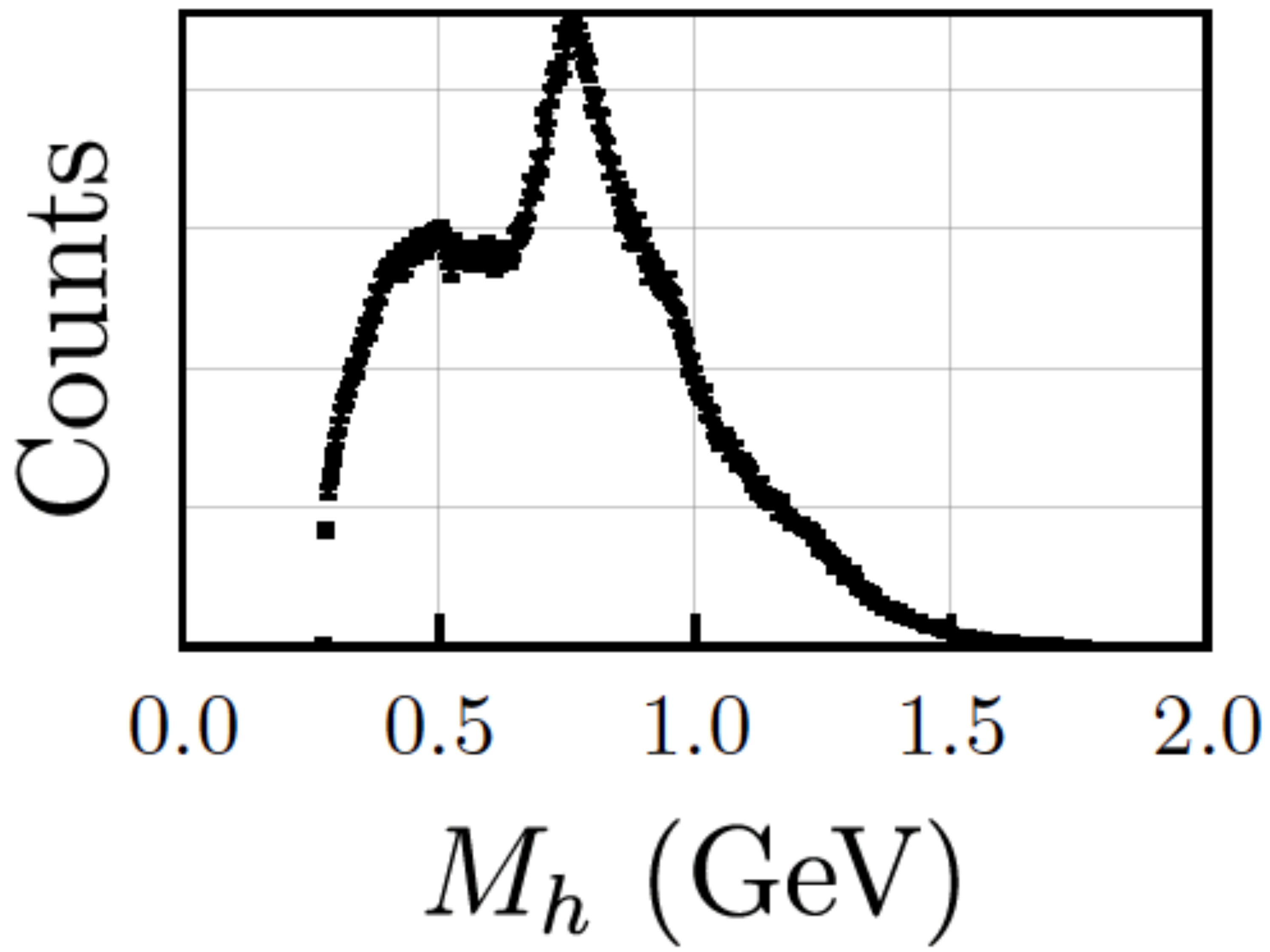


- Consistent results for LH_2 and LD_2 targets except for |1,1> partial wave
- Allows flavor decomposition of twist-3 PDF e(x)

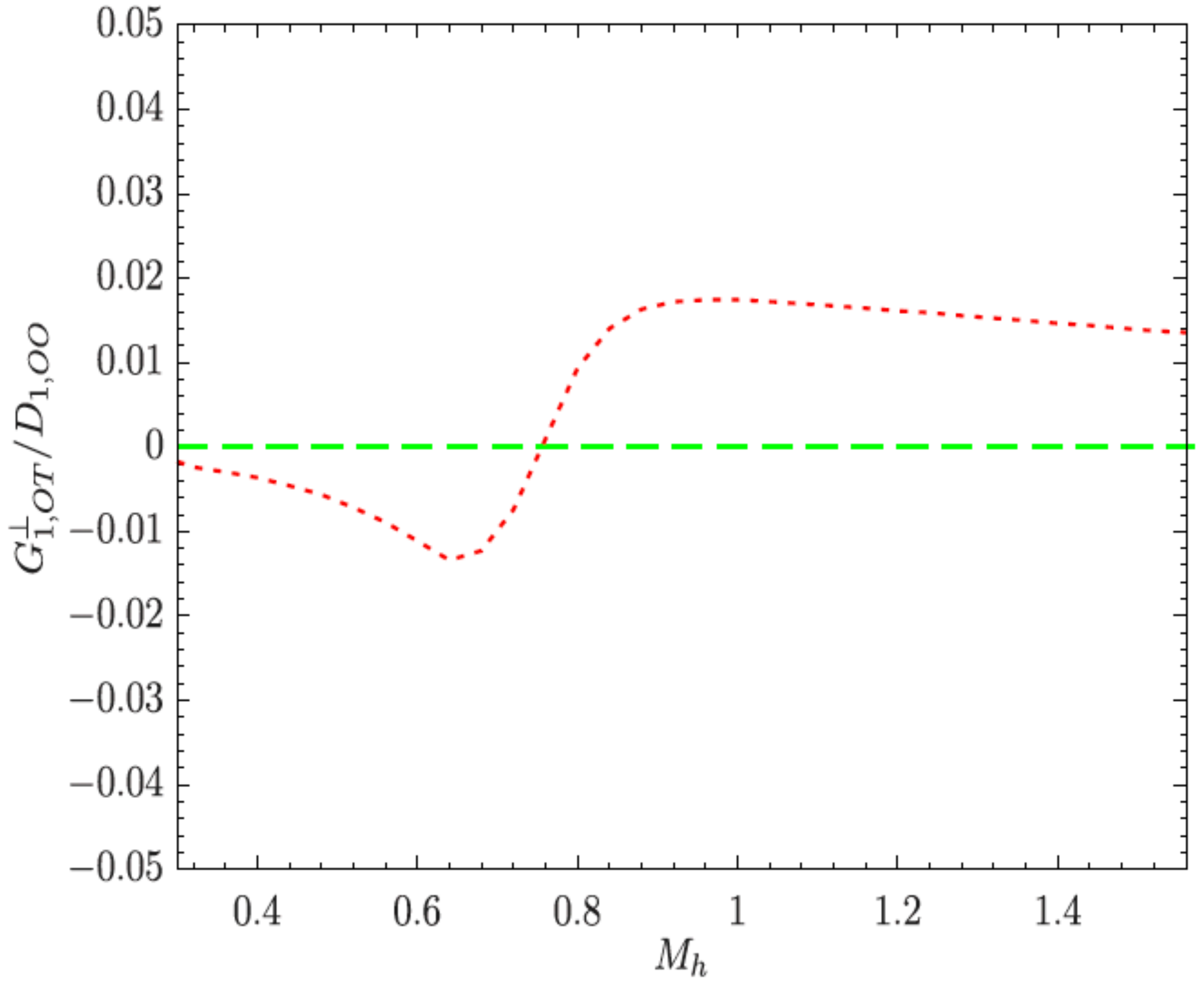
$e^- p \to e^- \pi^+ \pi^- X$



PRL 126, 152501 (2021)



$$A_{LU}^{\sin(\phi_h - \phi_{R_\perp})} \propto f(x) \otimes G_1^\perp$$



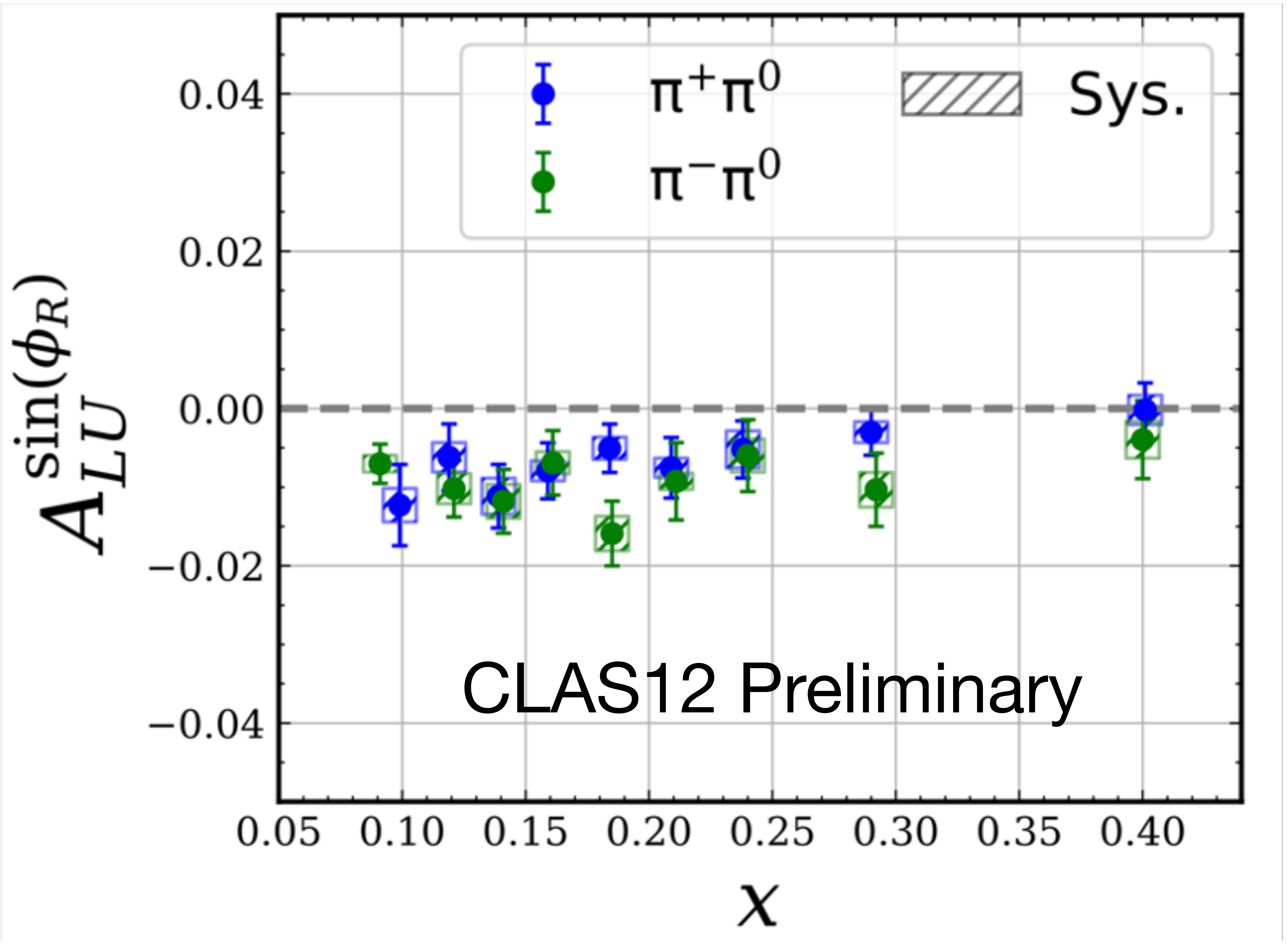
Spectator model prediction

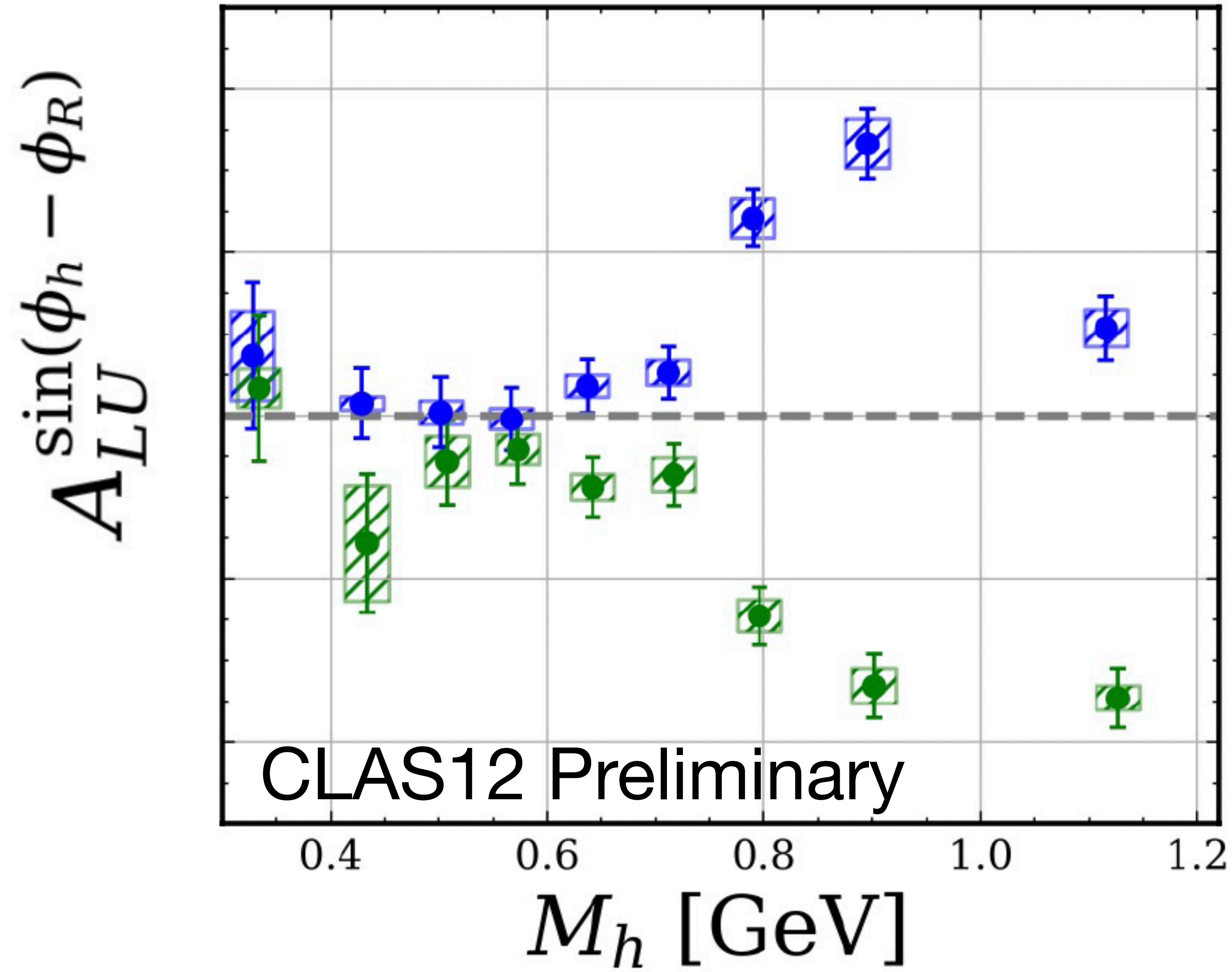
PRD 101, 054020 (2020)

- First measurement sensitive to DIFF $G_1^\perp$
- Describes dependence of two-pion production on the helicity of fragmenting quark
- Sign change seen around ρ mass which also predicted from theory

$e^-p \to e^-\pi^+\pi^0 X, e^-p \to e^-\pi^-\pi^0 X$

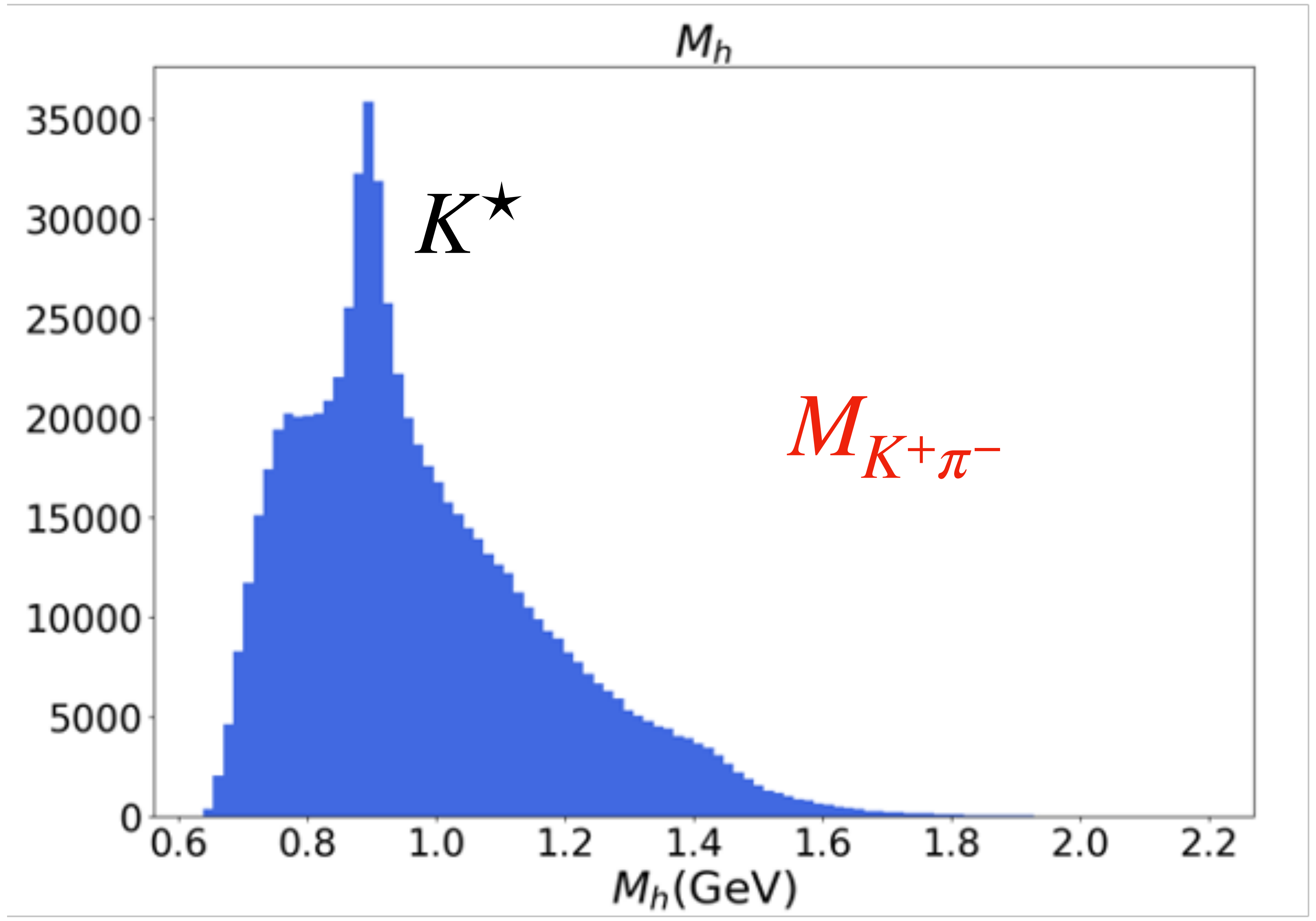
G. Matousek (Duke)

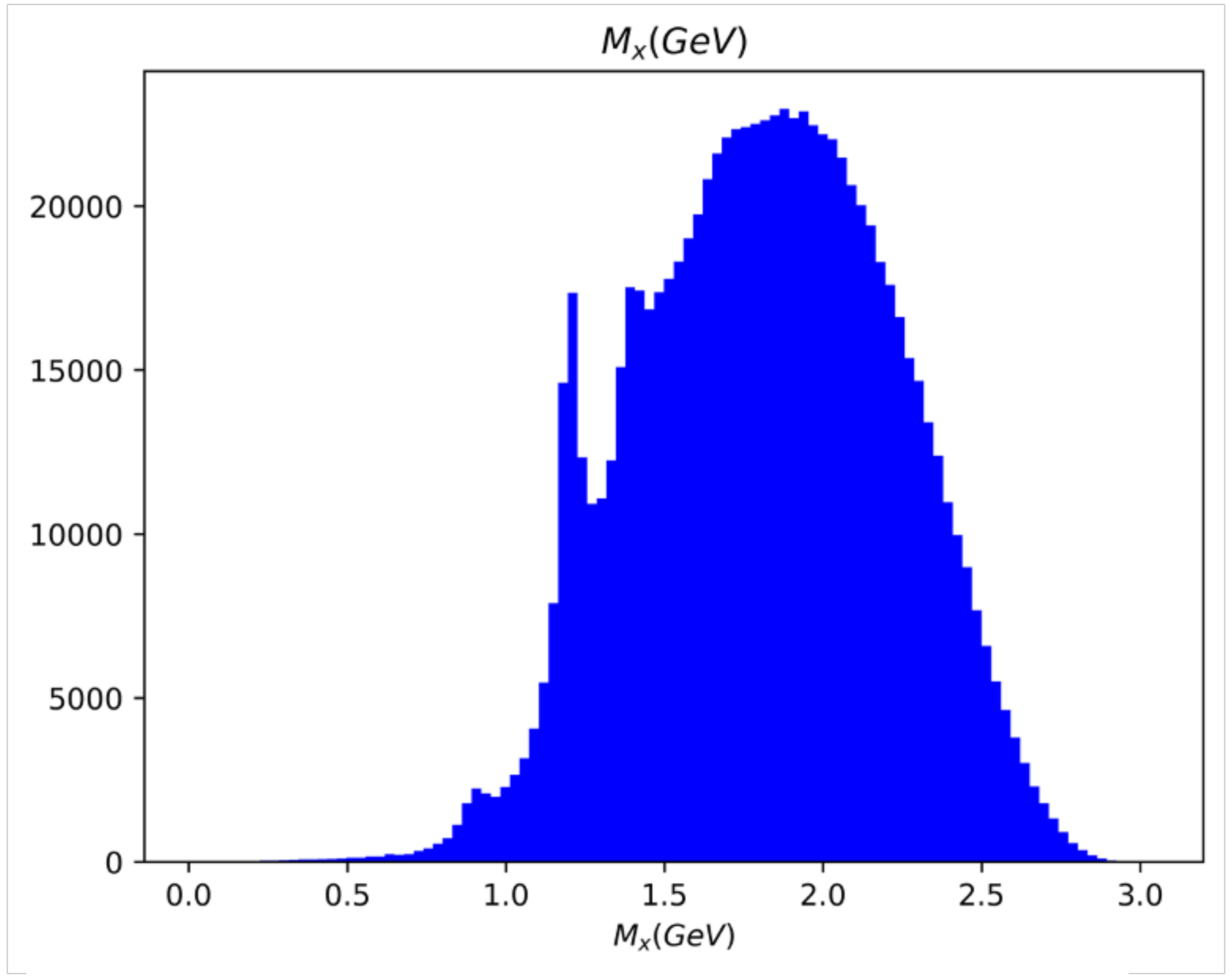




- $\sin(\phi_h - \phi_R)$ modulation shows isospin symmetry of $G_1^\perp$
- Enhancement near ρ mass region

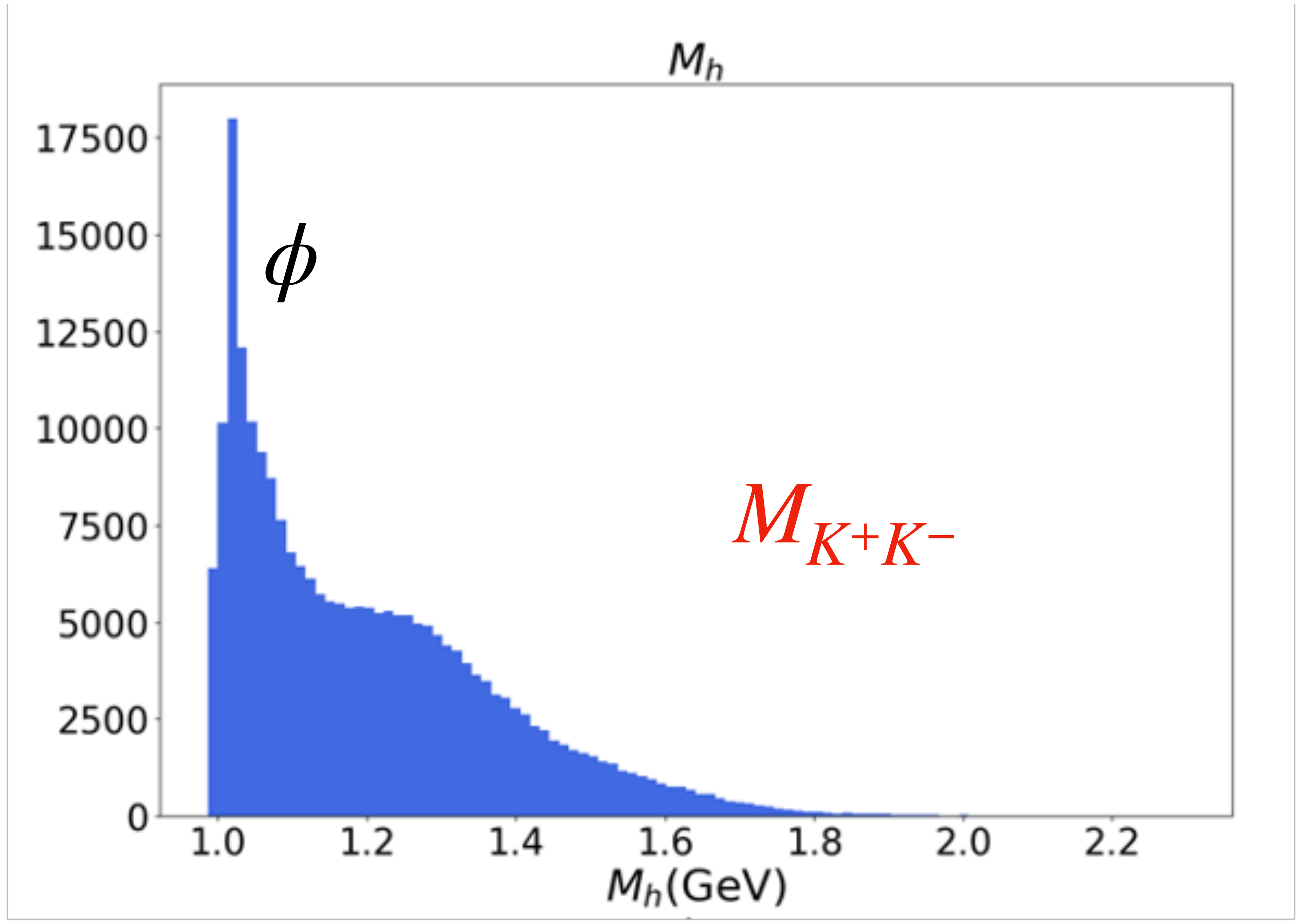
Data from kaon channels

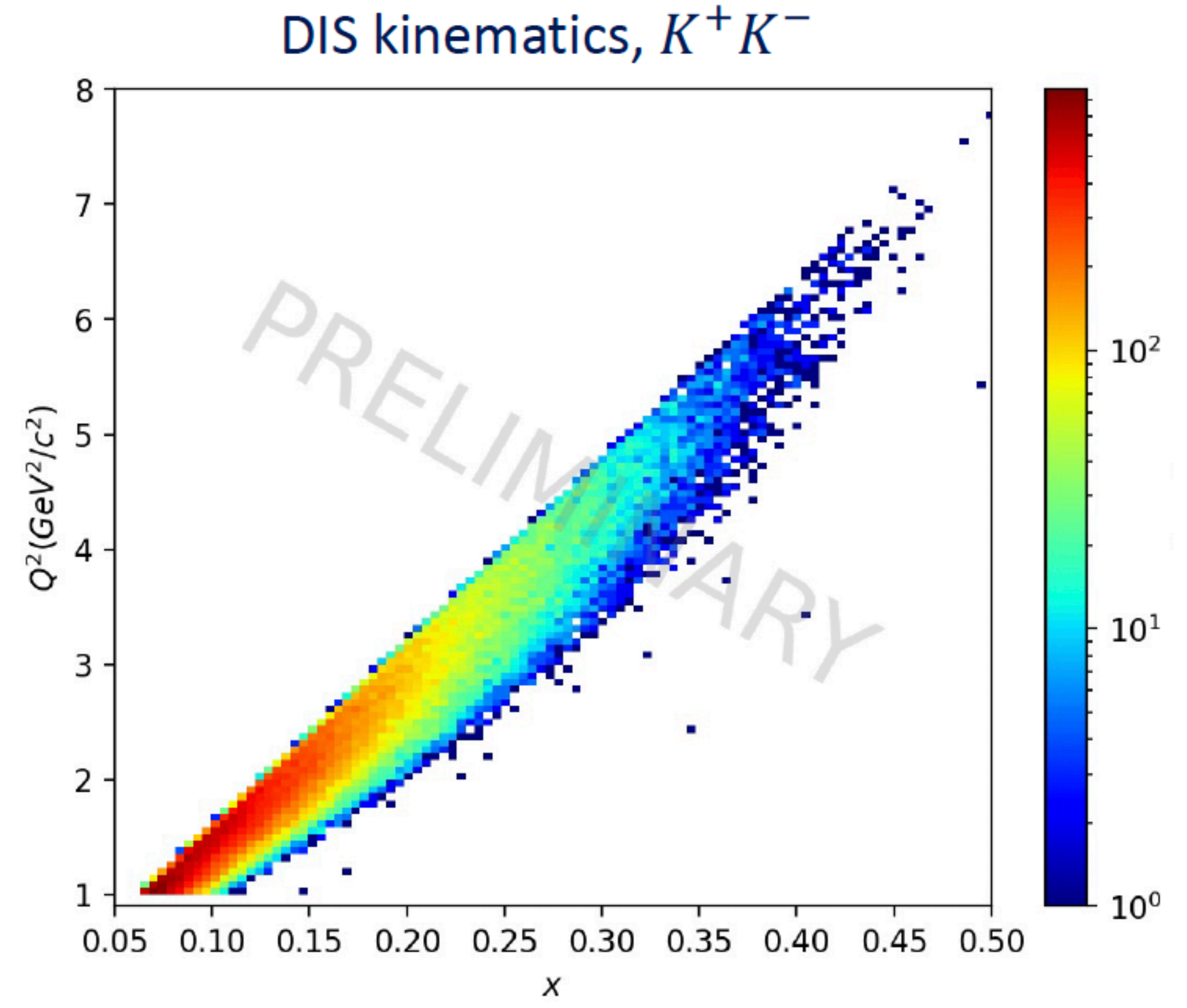




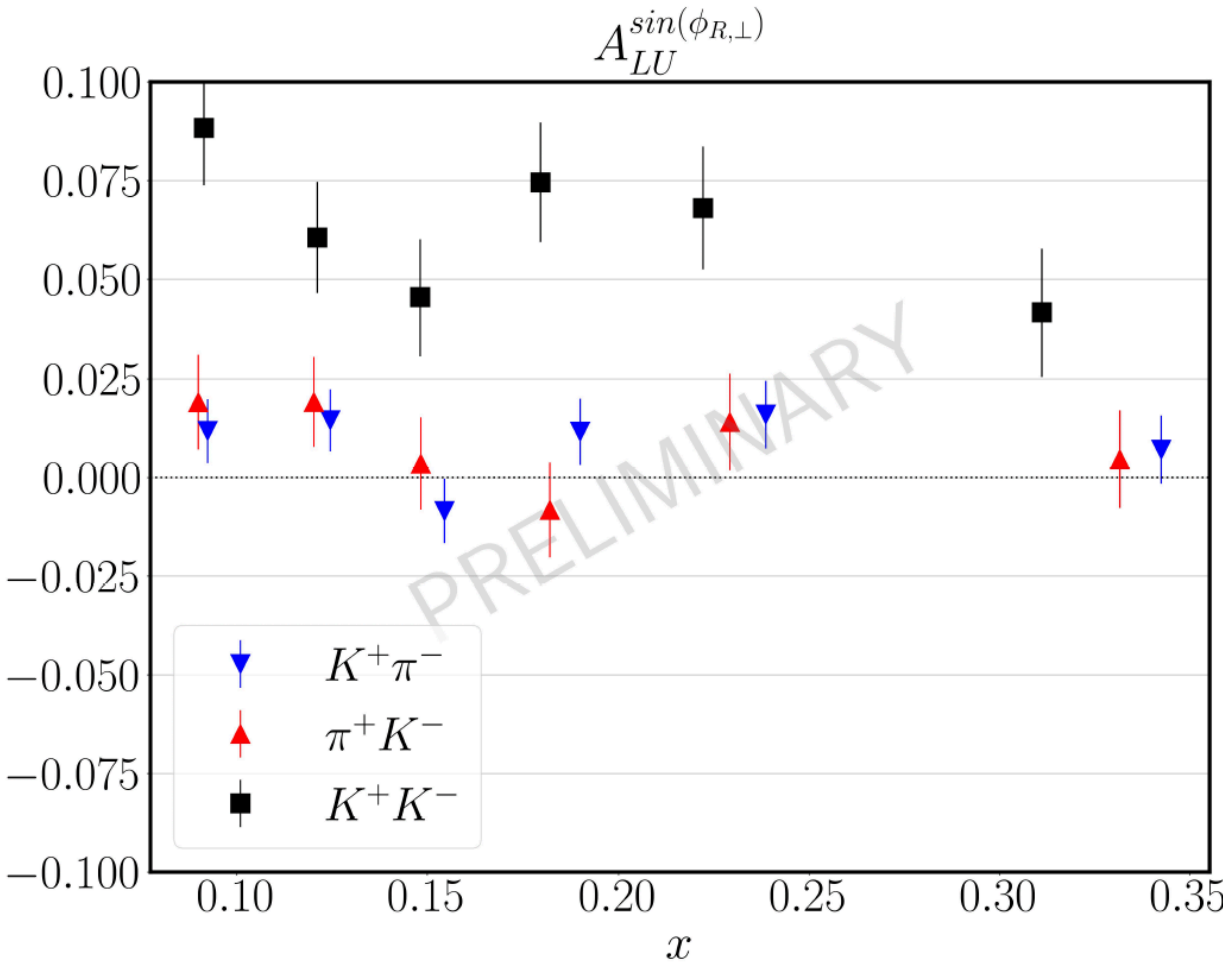
C. Pecar (Duke)

Missing mass

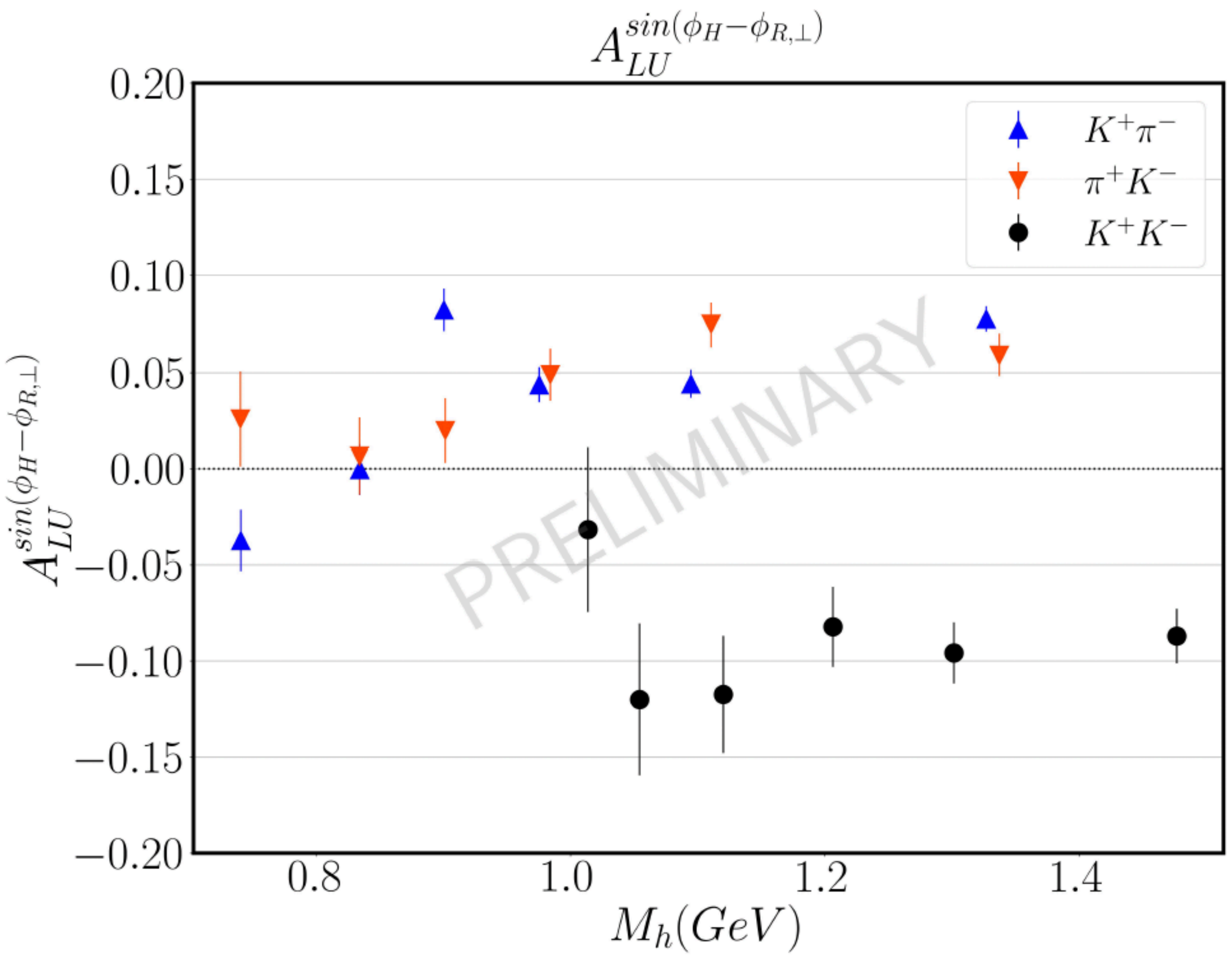




Q^2 vs x



$A_{LU}^{\sin(\phi_{R_\perp})} \propto e(x)\, H_1^{\sphericalangle}$



$A_{LU}^{\sin(\phi_h-\phi_{R_\perp})} \propto f(x) \otimes G_1^{\perp}$

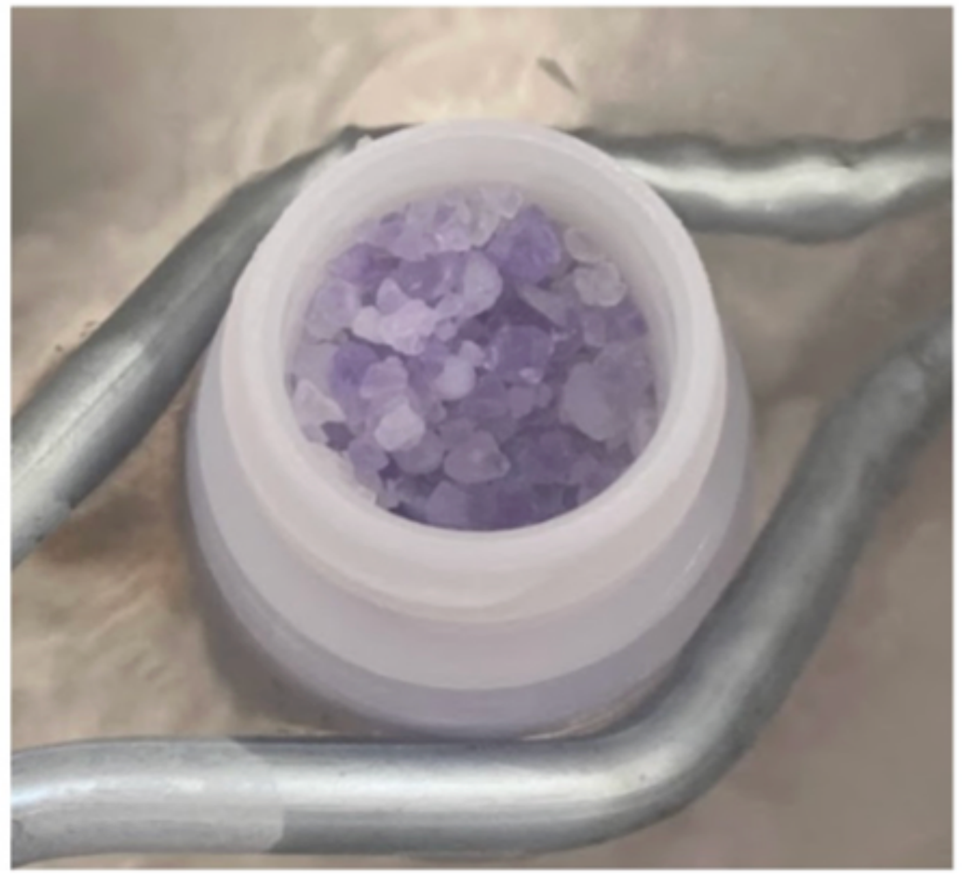
C. Pecar (Duke)

- Larger modulation of $A_{LU}^{\sin(\phi_{R_\perp})}$ seen for K^+K^- channel
- Uncertainties shown are only statistical
- Access to flavor dependence and $K^\star$, ϕ vector meson contributions to asymmetries

Longitudinally polarized target data from CLAS12

- Longitudinally polarized 10.6 GeV electron beam

- Longitudinally polarized hydrogen in solid ammonia (NH_3) target
  - Target polarization ≈ 85%
  - Dilution factor ≈ 3/17
  - Access beam, target and double spin asymmetries (A_{LU}, A_{UL}, A_{LL})
    - $e^- p \to e^- \pi^+ \pi^- X$



Likelihood PDF

- Use a "combined fit" to extract $A_{LU}^{\psi_i}, A_{UL}^{\psi_i}$ and $A_{LL}^{\psi_i}$ simultaneously
- Probability distribution looks something like

$$weight \,*\, \left(1 + P_b h_b \frac{W(\epsilon,y)}{A(\epsilon,y)} \frac{F_{LU}^{sin\,\phi_R}}{F_{UU}^{const}} sin\,\phi_R + \ldots\right.$$

$$+ P_t h_t f \frac{V(\epsilon,y)}{A(\epsilon,y)} \frac{F_{UL}^{sin\,\phi_R}}{F_{UU}^{const}} sin\,\phi_R + \ldots$$

$$\left. + P_b h_b P_t h_t f \frac{C(\epsilon,y)}{A(\epsilon,y)} \frac{F_{LL}^{const}}{F_{UU}^{const}} + P_b h_b P_t h_t f \frac{W(\epsilon,y)}{A(\epsilon,y)} \frac{F_{LL}^{cos\,\phi_h}}{F_{UU}^{const}} cos\,\phi_h + P_b h_b P_t h_t f \frac{W(\epsilon,y)}{A(\epsilon,y)} \frac{F_{LL}^{cos\,\phi_R}}{F_{UU}^{const}} cos\,\phi_R + \ldots\right)$$

$P_b = 0.8$

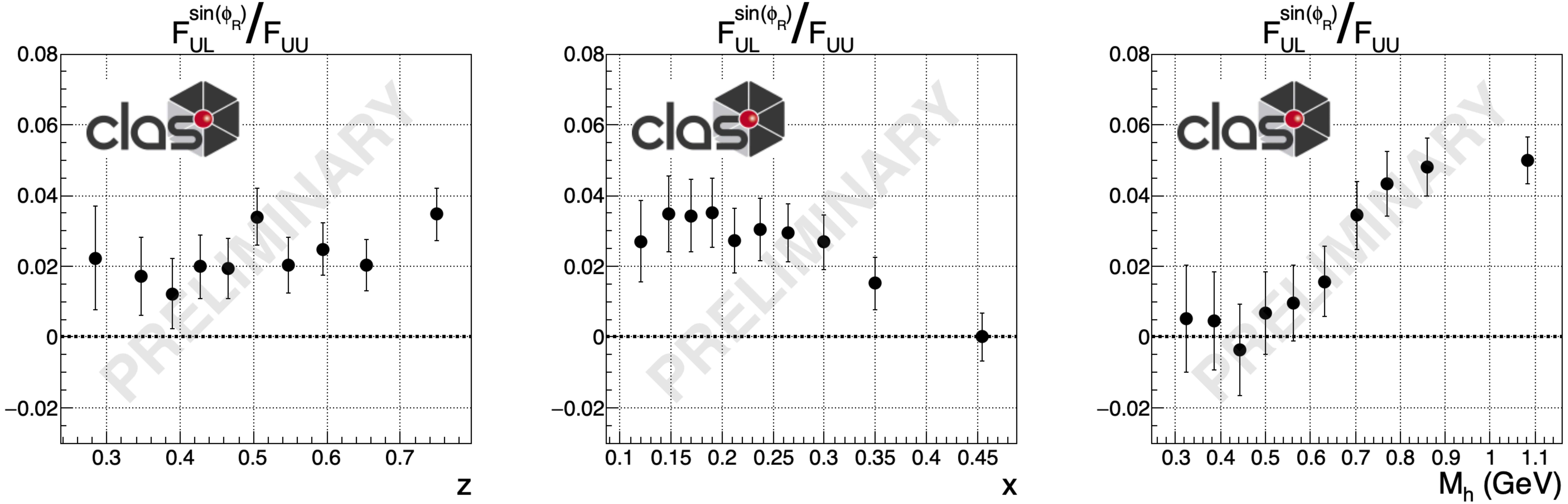
A, C, V, W - depolarization factors

weight - to account for imbalance between accumulated charges for +,- helicity states of beam, target

- Total of 27 modulation amplitudes

(fit parameters give structure functions ratio $\frac{F_{XY}^{\psi_i}}{F_{UU}^{const}}$)

$F_{UL}^{\sin(\phi_R)}/F_{UU}$

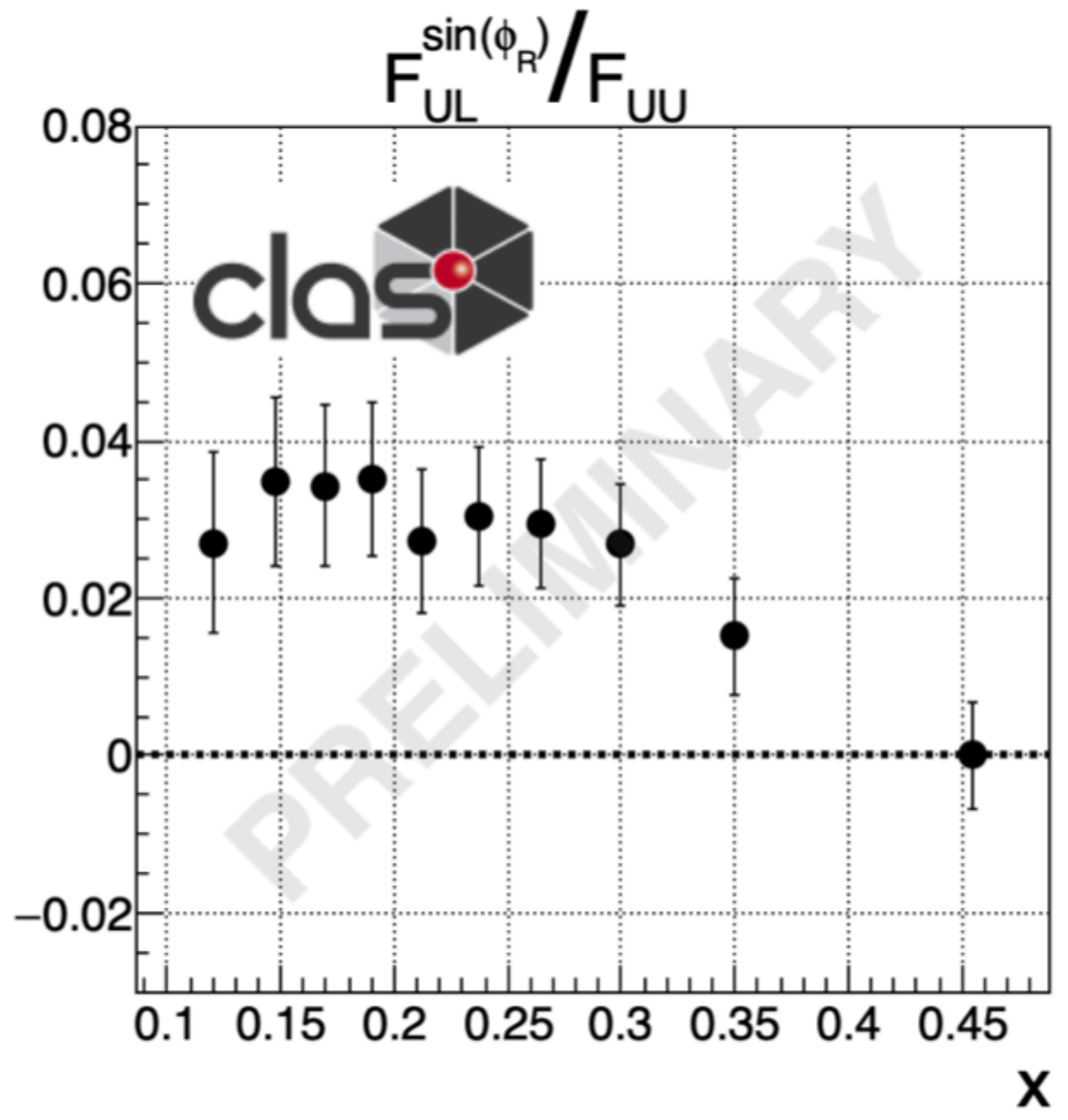


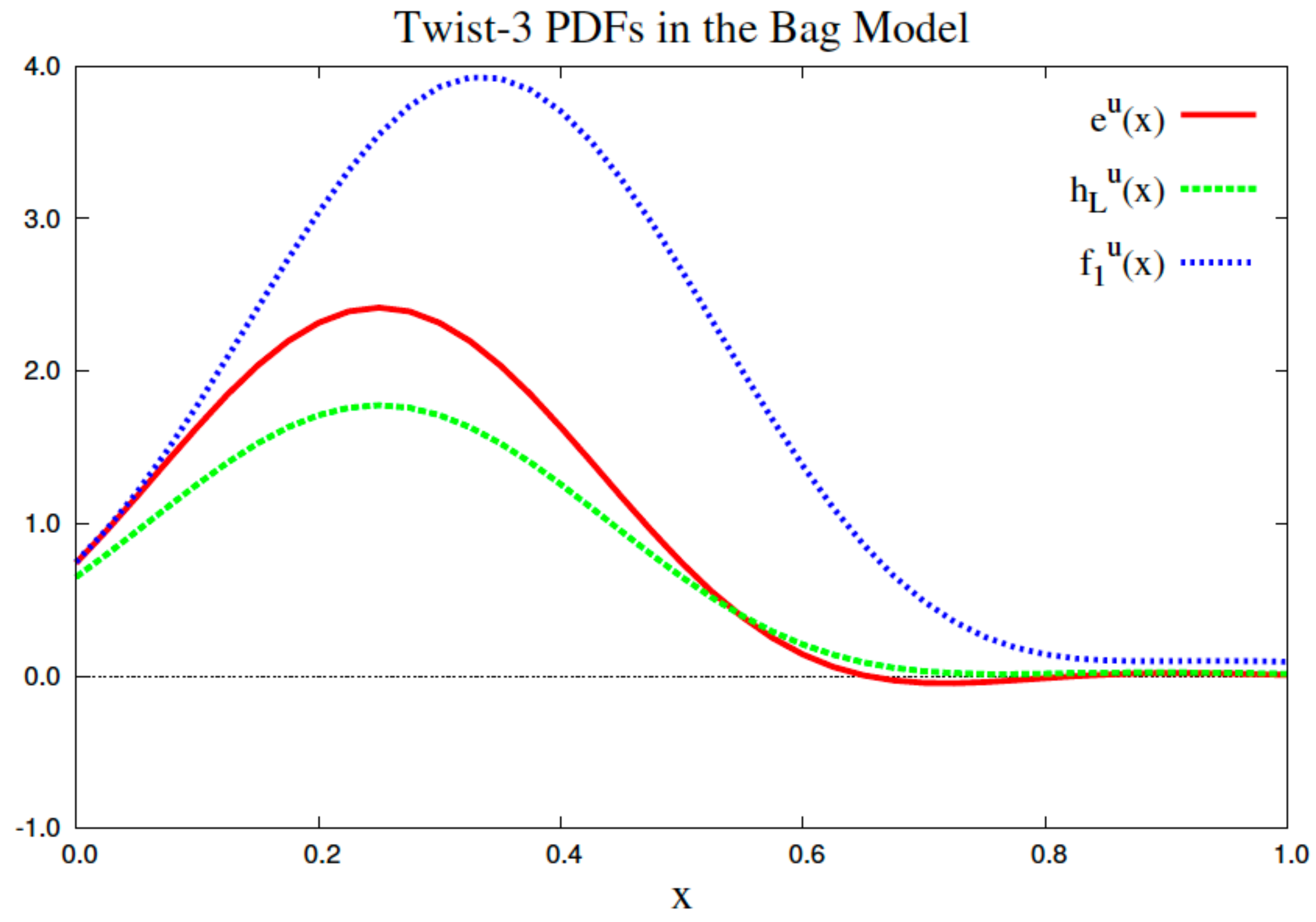
- Non-zero and positive amplitude < 0.06 wrt z, x and Mh

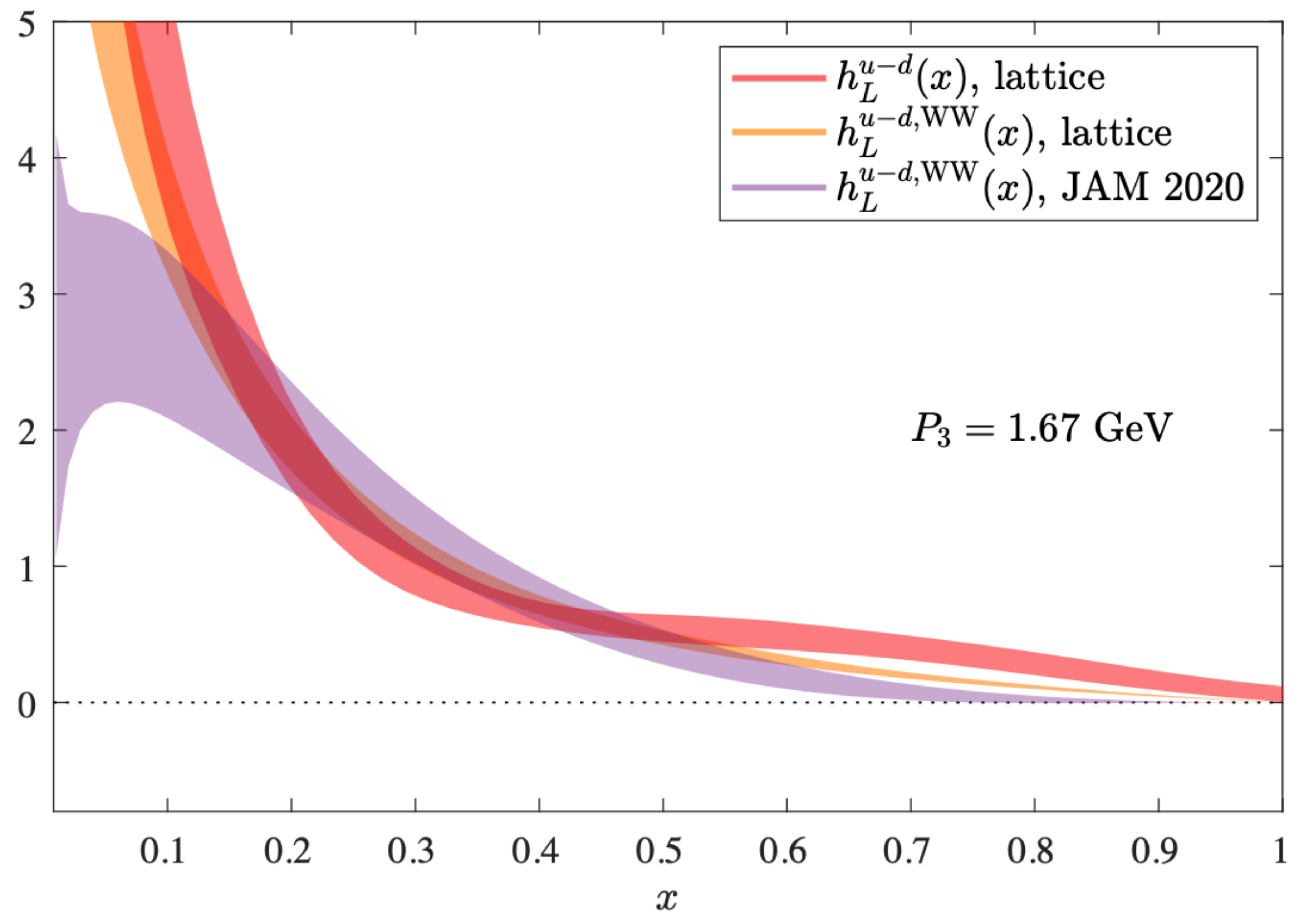
Sensitive to $h_L H_1^{\perp}$

- No published results on h_L
  - x^3 moment of h_L describes the average longitudinal gradient of the transverse force that acts on a transversely polarized quark [6]
  - Sign of gradient will help to study correlations between nucleon spin and its color magnetic field

Nucl. Phys. B375, 527 (1992)



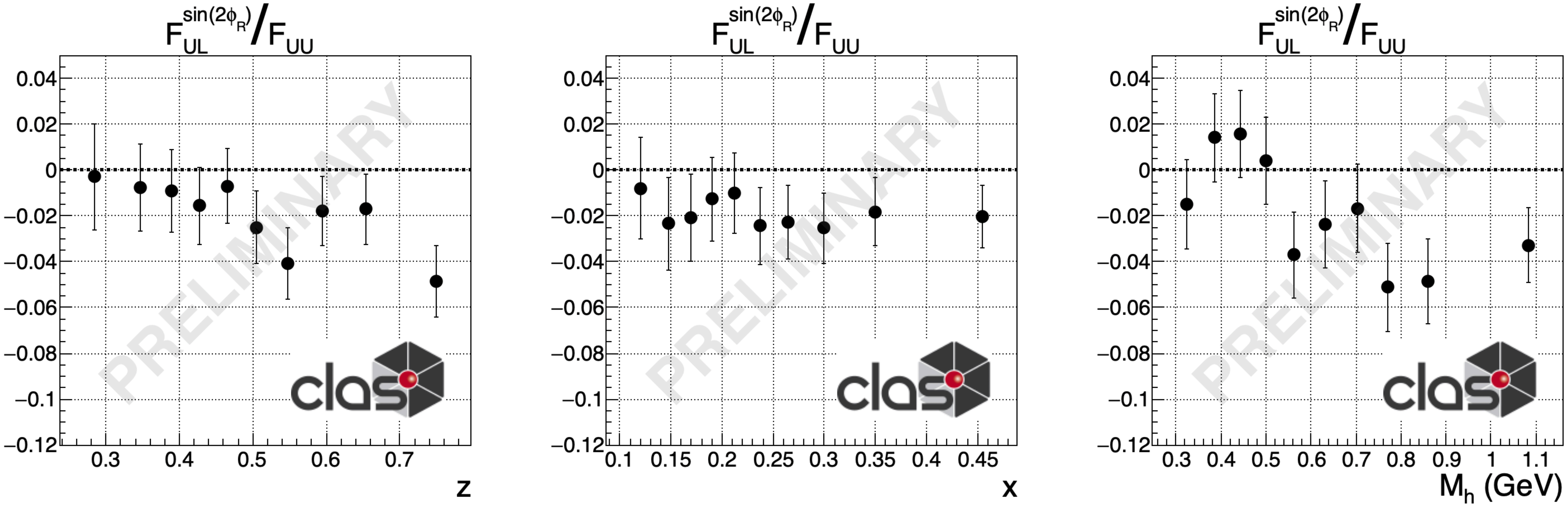




CLAS12 results will allow extraction of h_L from data

arXiv:2111.01056

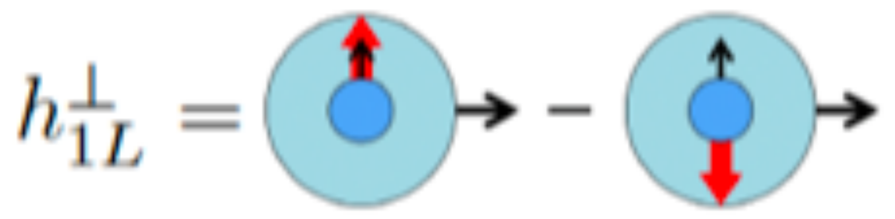
$F_{UL}^{\sin(2\phi_R)} / F_{UU}$



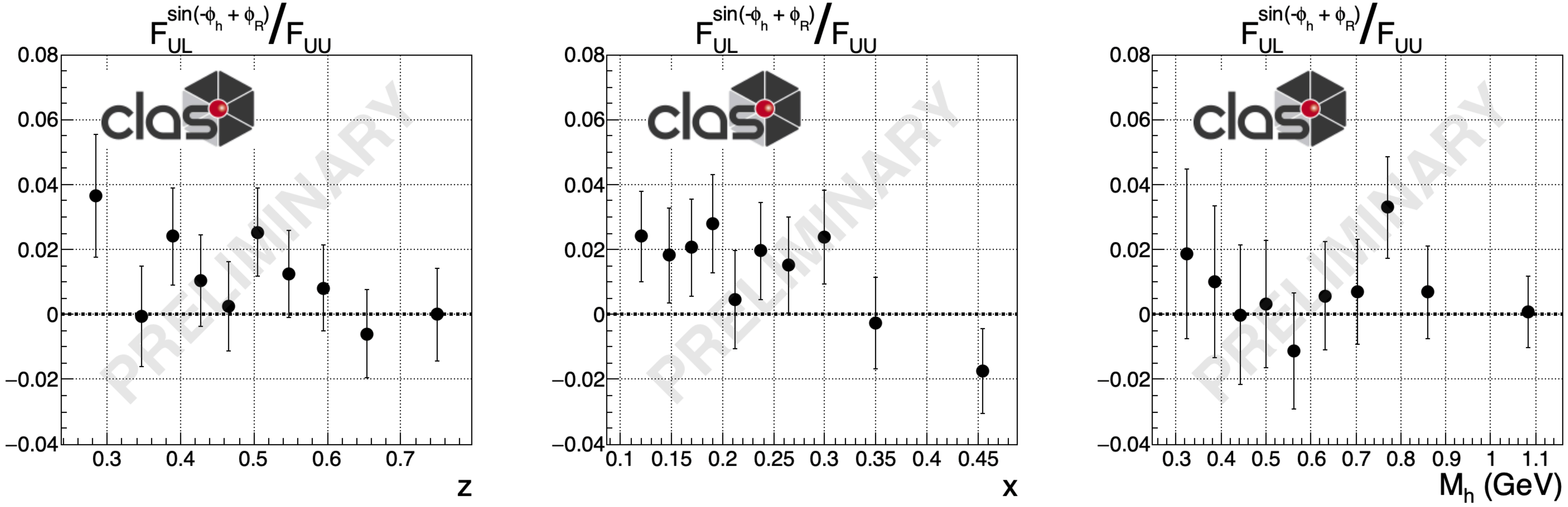
- Amplitude mostly negative

Sensitive to $h_{1L}^{\perp} H_1^{\perp}$

Worm-gear 1



$F_{UL}^{\sin(-\phi_h+\phi_R)}/F_{UU}$

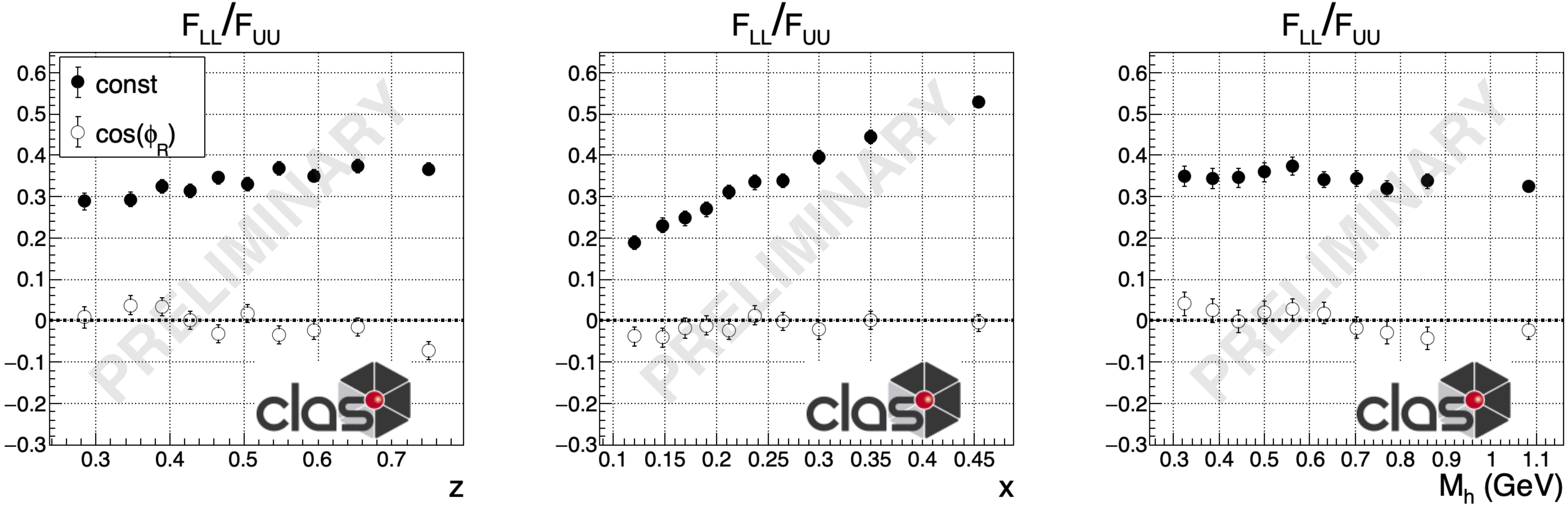


- Sensitive to $g_{1L}G_1^{\perp}$

arXiv: 1702.07317
PRD 101, 054020 (2020)

- COMPASS measurement from unpolarized muons off longitudinally polarized protons
- Asymmetry close to zero
- Spectator model prediction agrees with data

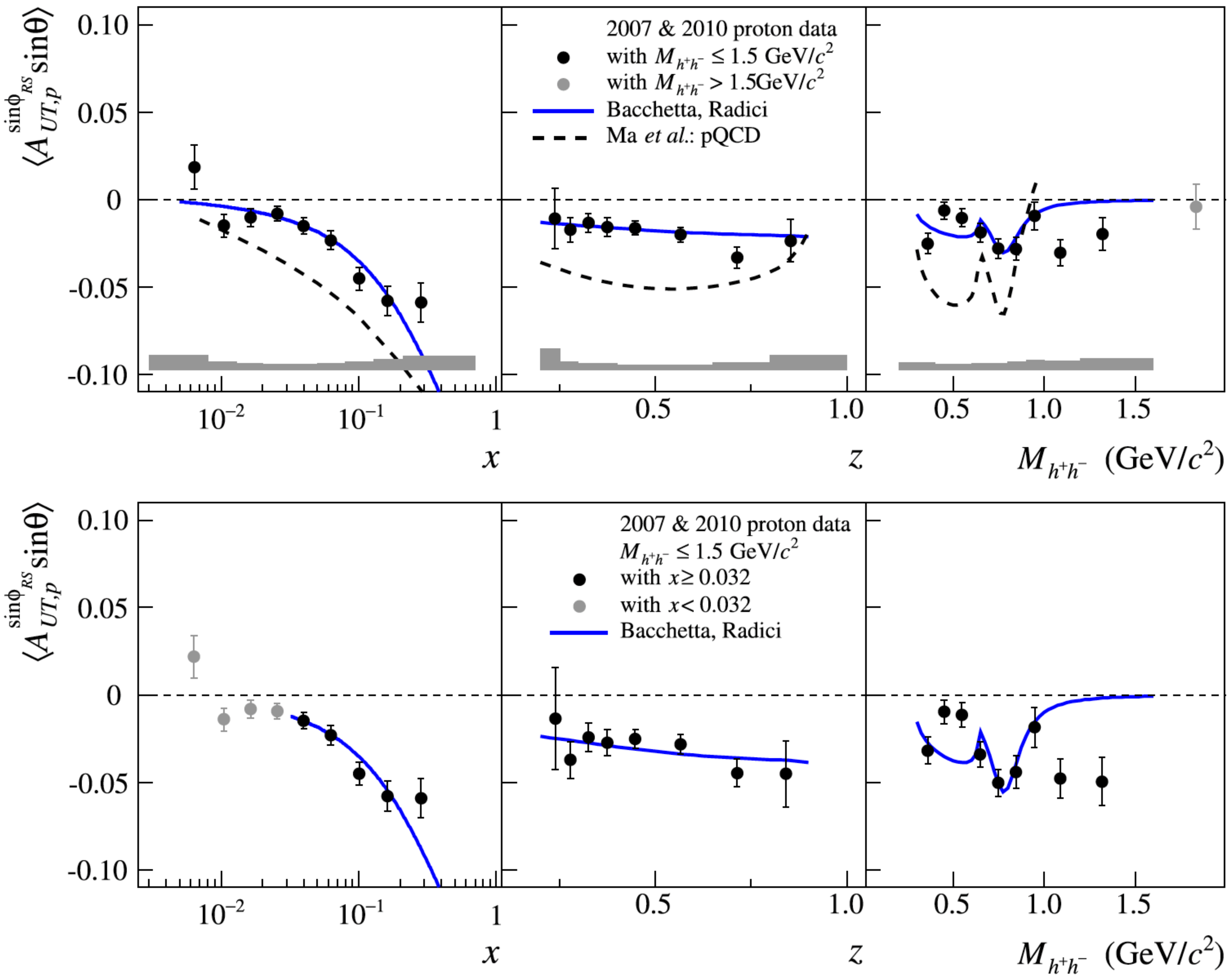
F_{LL}/F_{UU}



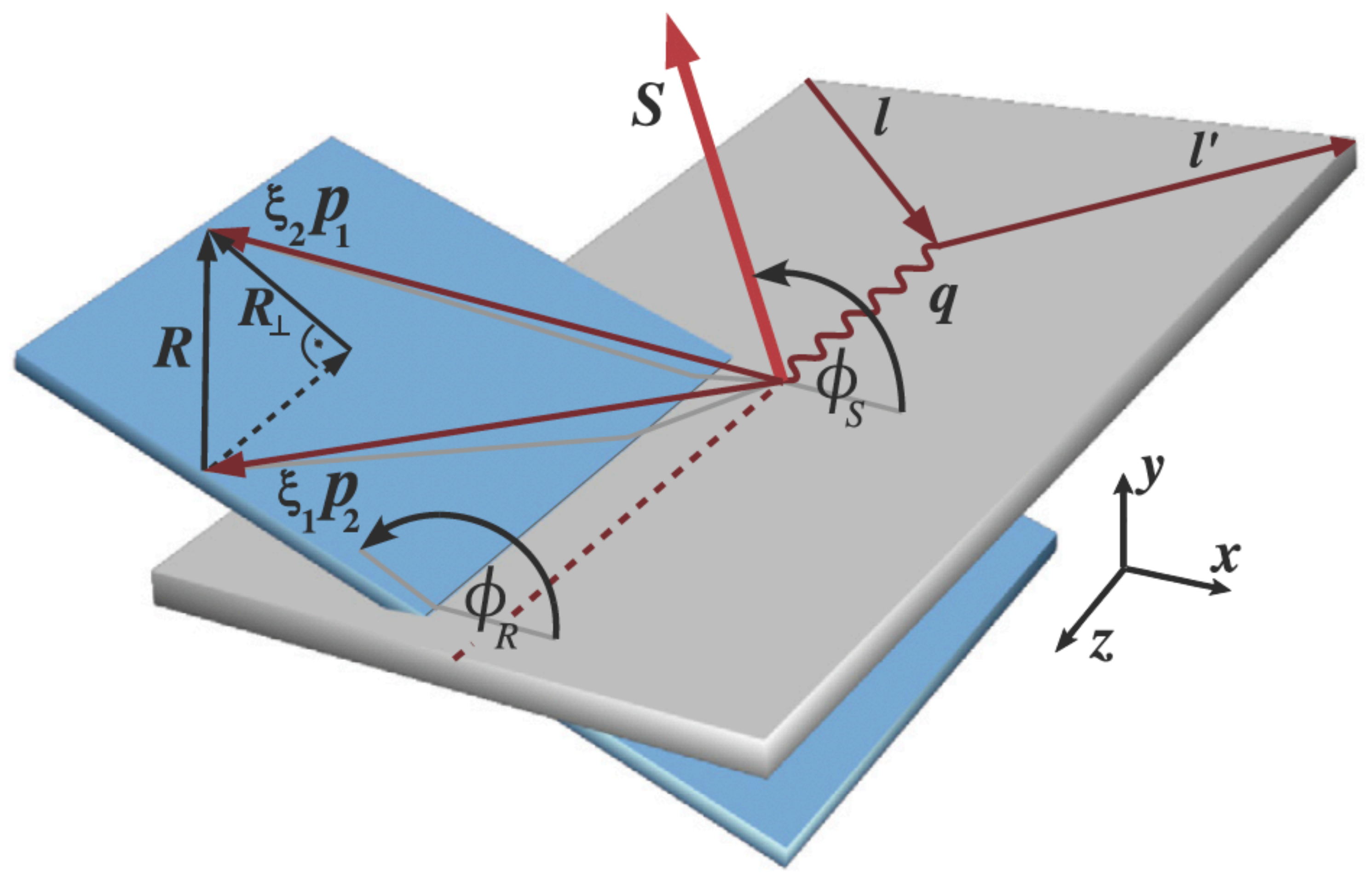
$$\frac{F_{LL}^{const}}{F_{LL}^{\cos(\phi_R)}} \text{ Sensitive to } \frac{g_{1L}D_1}{g_{1L}\tilde{D}}$$

- Suggests $\tilde{D}$ is roughly one order of magnitude smaller than D_1

Transversely polarized target data from COMPASS



Scattering 160 GeV/c μ^+off a solid NH_3 target



$$\phi_{RS} = \phi_R + \phi_S - \pi$$

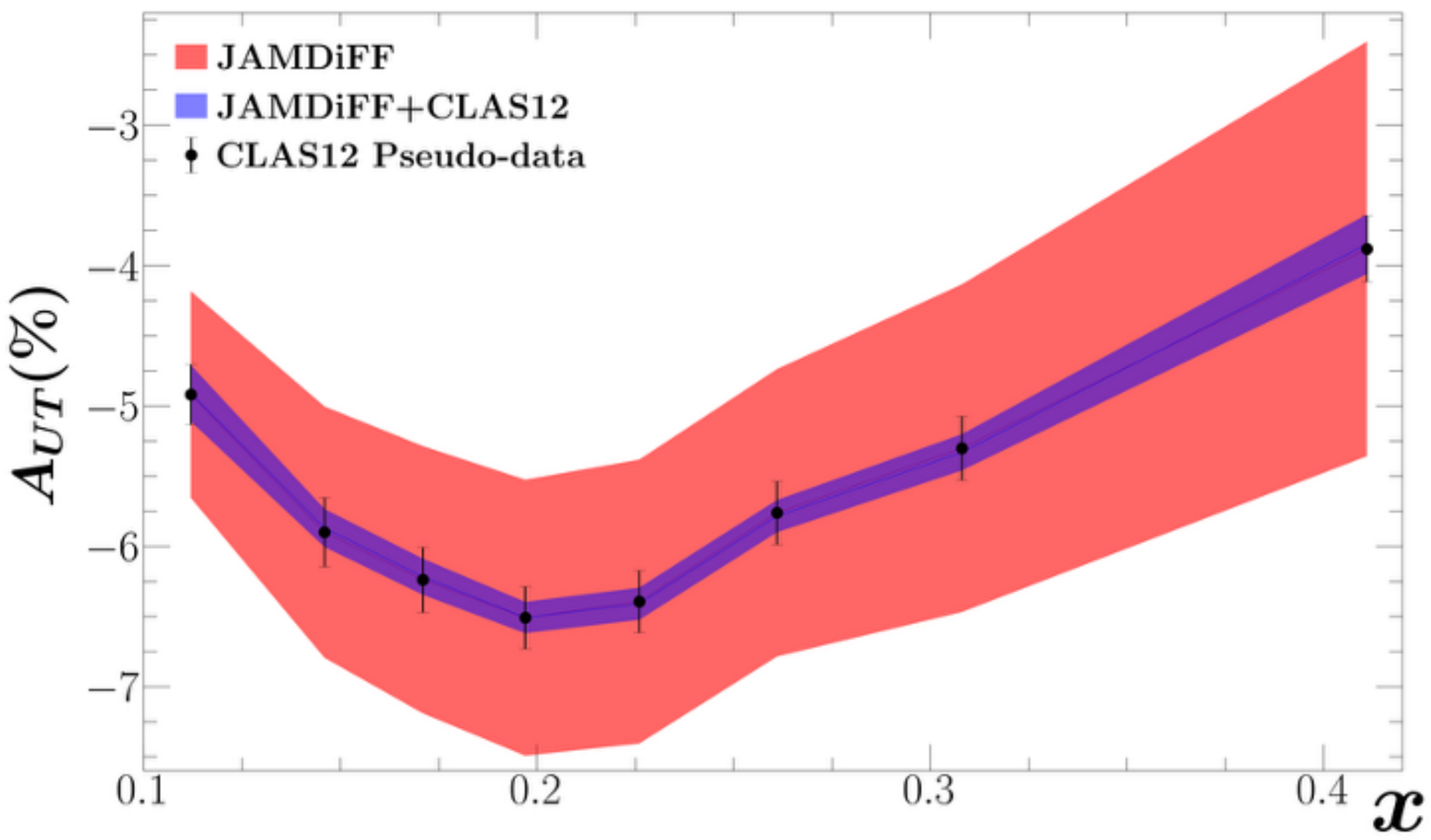
Phys. Lett. B 736 (2014) 124-131

– – – Phys. Rev. D 77 014035 (2008)

Improve extraction of h_1

Transversely polarized target at CLAS12

- Planned to start taking data in few years
- Transversely polarized solid NH_3 target and recoil detector
- Access transversity distribution (h_1) via dihadron production (will allow accessing x > 0.3 valence region)
  - First moment in x of h_1 gives the tensor charge



Yorgo Sawaya (POETIC 2025)

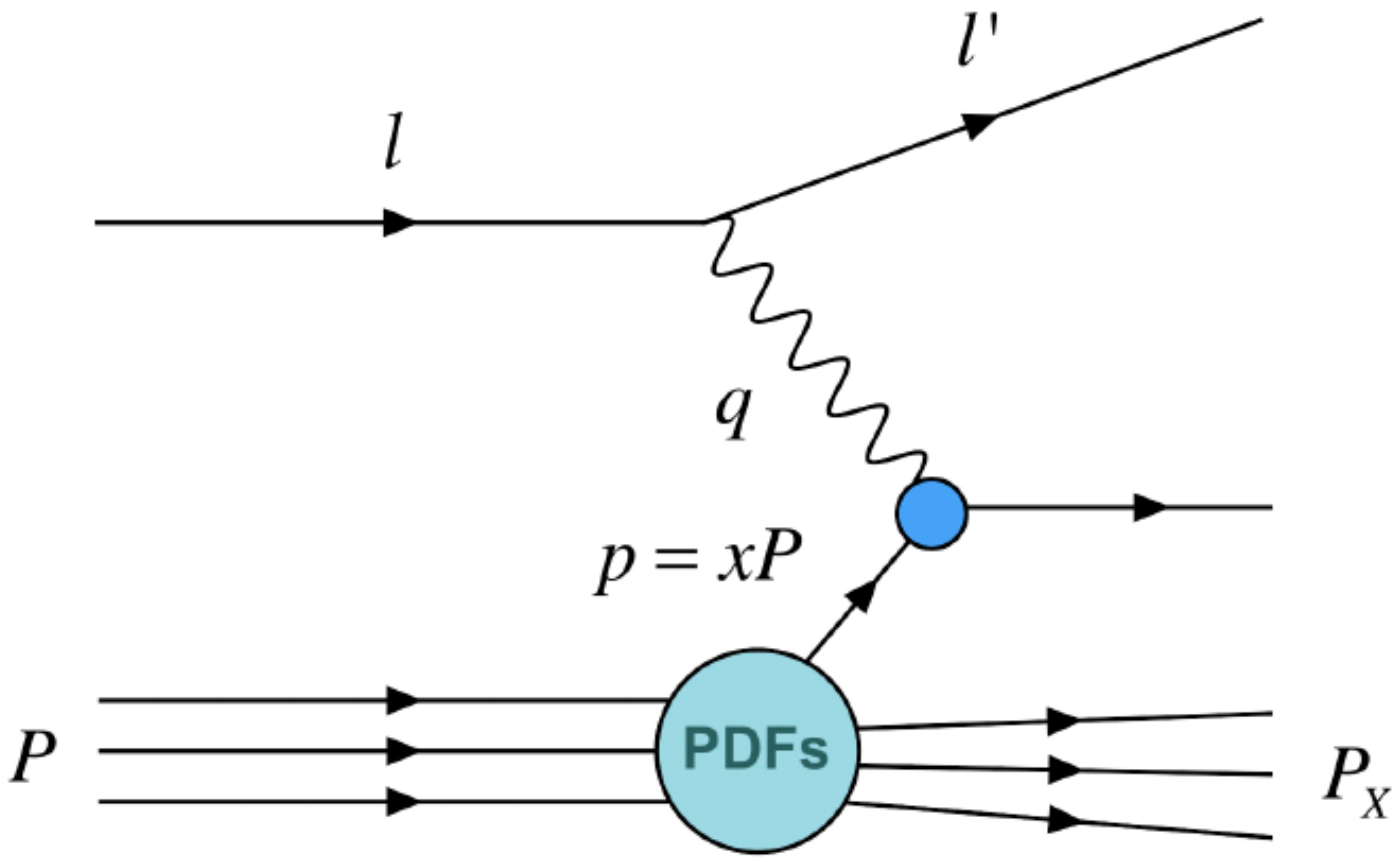
Summary

- Dihadron production in SIDIS provide more access to study nucleon structure and hadronization compared to single hadron production
- Beam, target and double spin asymmetry results from CLAS12 available with high statistics
  - Twist-3 PDF e(x) extracted for the first time
  - Opportunity to extract twist-3 PDF $h_L(x)$ from target spin asymmetry results
  - Flavor decomposition of PDFs with deuteron data and kaon final states
- Future results with transversely polarized target of CLAS12 will improve knowledge of transversity h_1 in the valence region

Backup - Summary of DIS kinematics

- Four momentum of virtual photon $\Longrightarrow \quad q = l - l'$
- Virtuality of the virtual photon $\Longrightarrow \quad Q^2 = -q^2$
- Energy of the virtual photon $\Longrightarrow \quad \nu = \dfrac{P \cdot q}{M}$

  (M – Mass of proton)



[2]

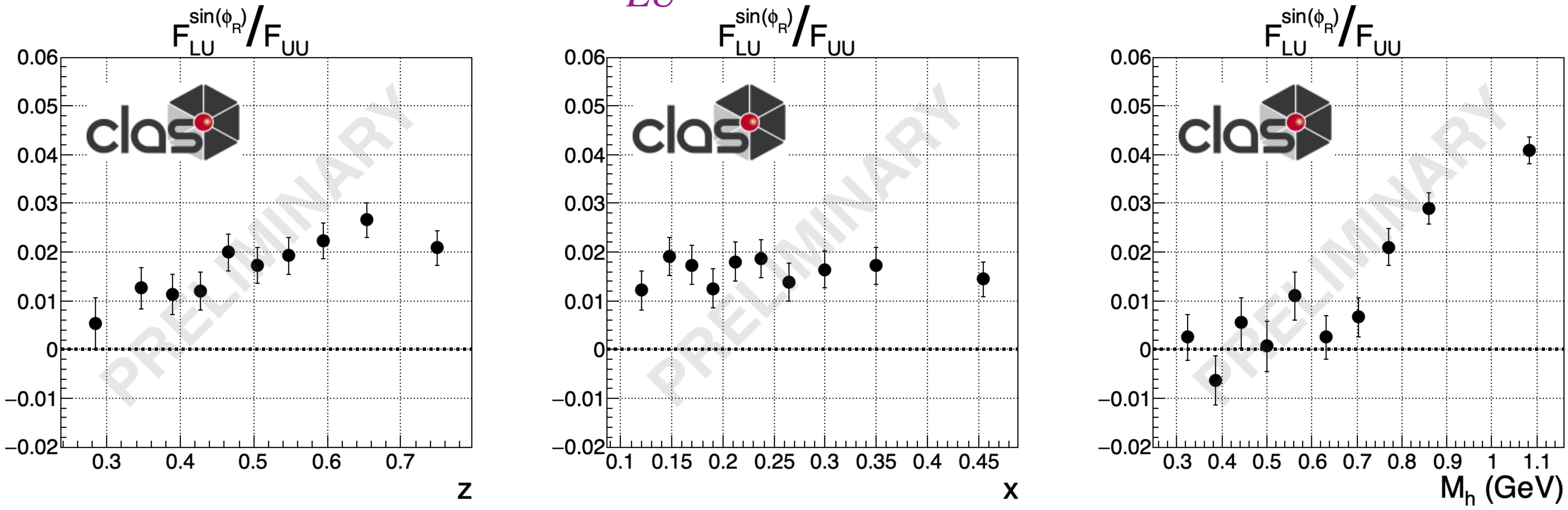
- Squared invariant mass of photon-proton system

$$\Longrightarrow \quad W^2 = (P+q)^2 = M^2 + 2M\nu - Q^2$$

- Momentum fraction of the proton carried by struck quark $\Longrightarrow \quad x = \dfrac{Q^2}{2\,P \cdot q}$
- Fraction of incoming electron energy transferred to the target proton

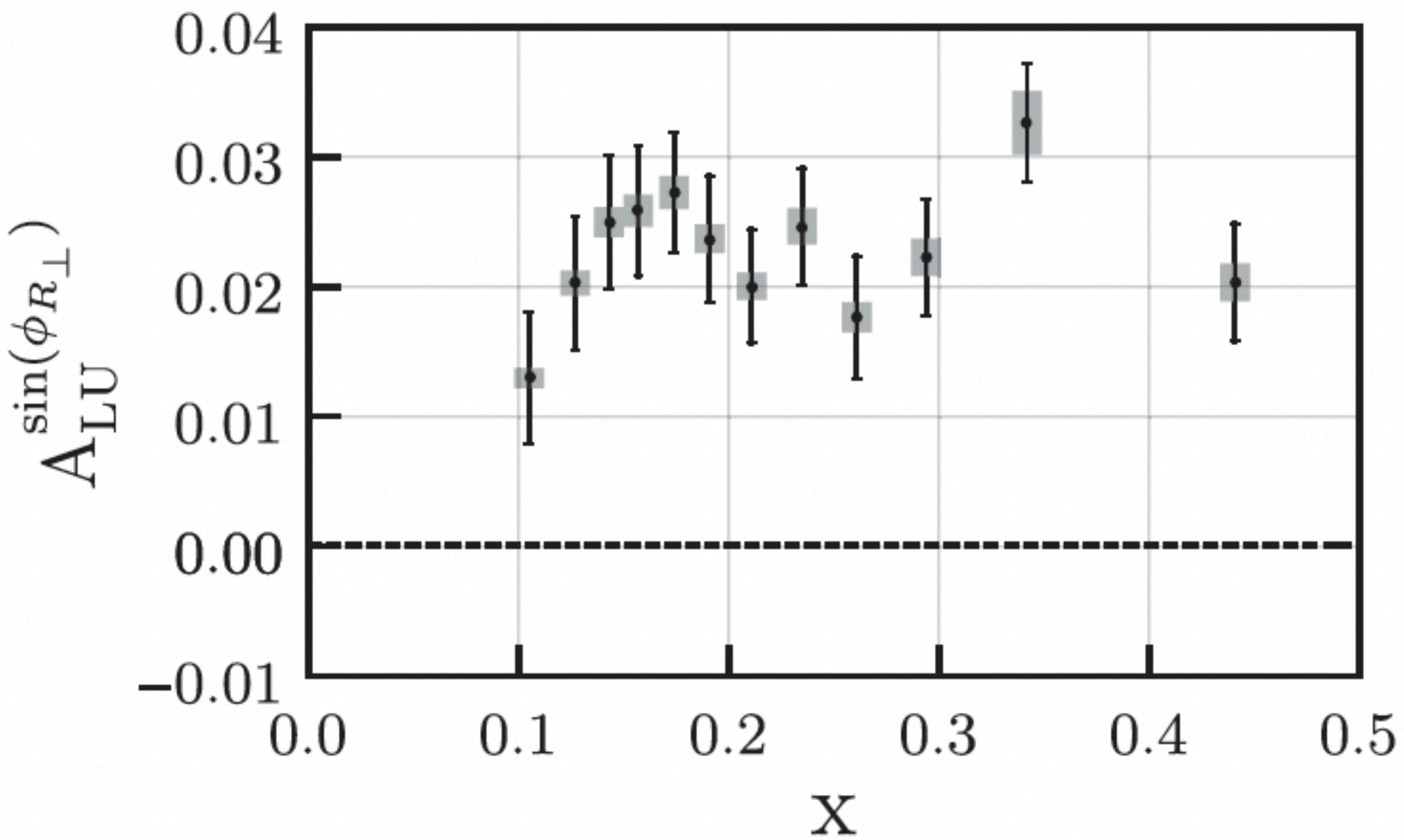
$$\Longrightarrow \quad y = \frac{P \cdot q}{P \cdot l}$$

$F_{LU}^{\sin(\phi_R)}/F_{UU}$



- Flat wrt x
- Lower amplitude from NH_3 data compared to LH_2 data
- Similar behavior with LH_2/LD_2

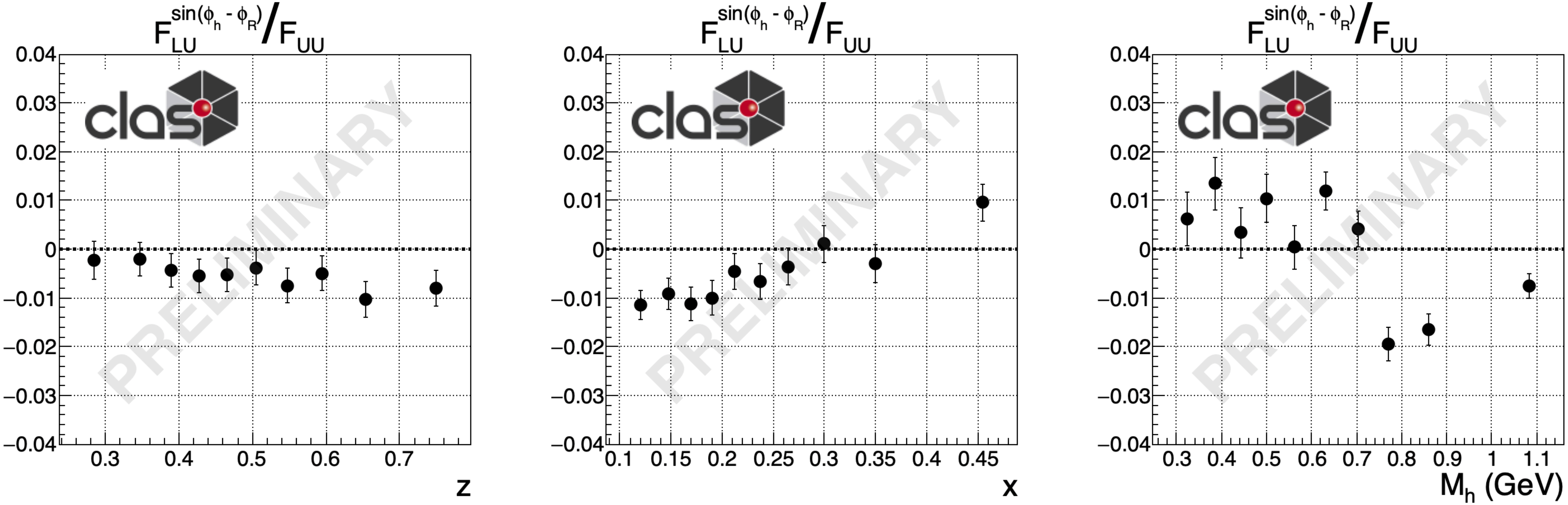
Sensitive to $eH_1^{\sphericalangle}$



PRL 126, 152501 (2021)

Unpolarized LH_2 target

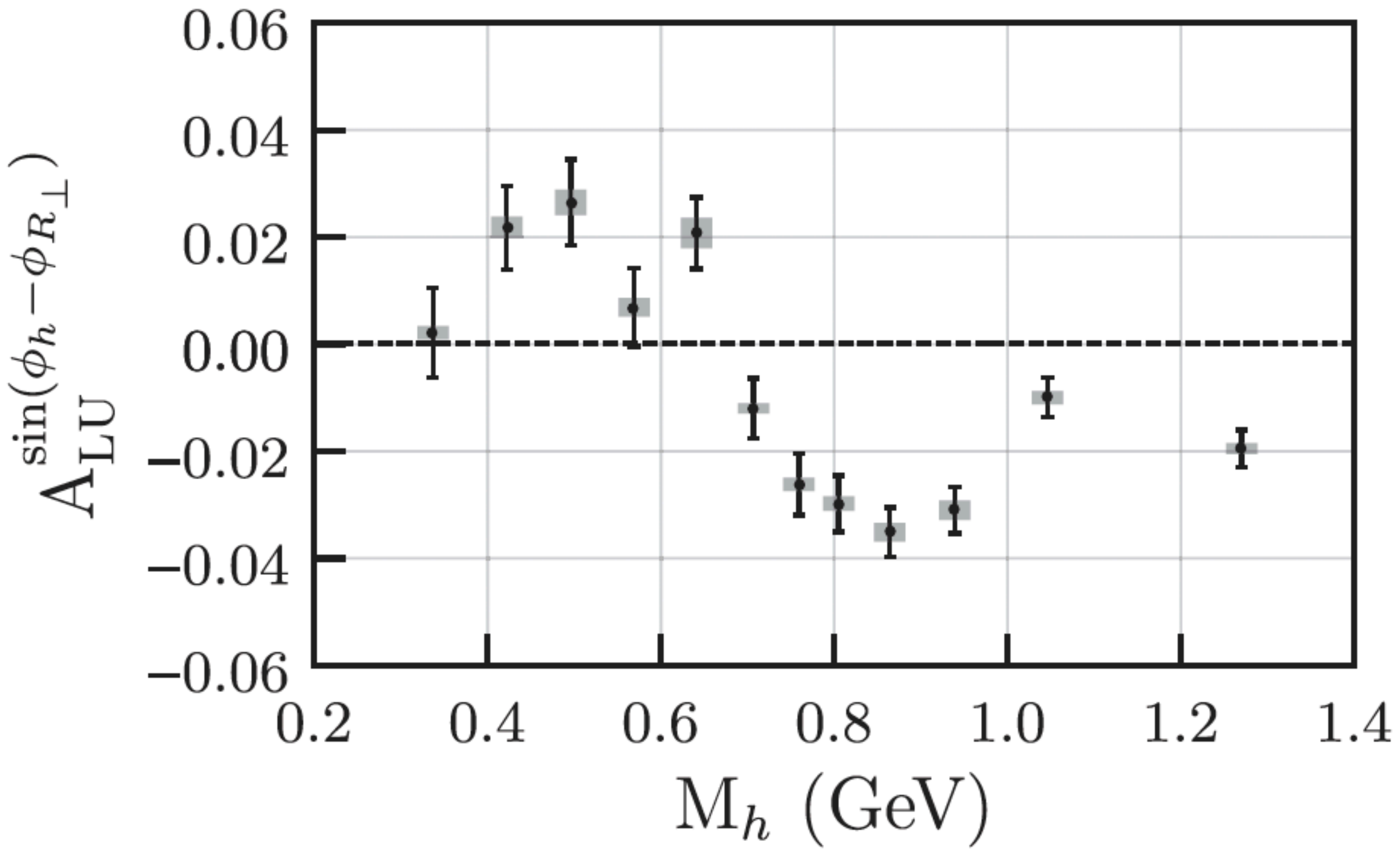
$F_{LU}^{\sin(\phi_h-\phi_R)}/F_{UU}$



- Sign change around ρ mass
  (Shape consistent with results from unpolarized target data)

Sensitive to $f_1 G_1$

PRL 126, 152501 (2021)



References

[1] T. Hayward, Ph.D. thesis - “Dihadron beam spin asymmetries on an unpolarized hydrogen target with CLAS12” (2021)

[2] S. Sirtl, Ph.D. thesis - “Azimuthal asymmetries in semi-inclusive deep-inelastic hadron muoproduction on longitudinally polarized protons” (2016)

[3] S. Gliske, A. Bacchetta, and M. Radici, Phys. Rev. D90, 114027 (2014)

[4] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders, and M. Schlegel, JHEP 02, 093 (2007)

[5] V. D. Burkert et al. (CLAS Collaboration), Nucl. Instrum.Methods Phys. Res., Sect. A 959, 163419 (2020)

[6] M. Abdallah and M. Burkardt, Phys. Rev. D 94, 094040 (2016)