Unpolarized gluon PDF of the proton from lattice QCD at the continuum limit

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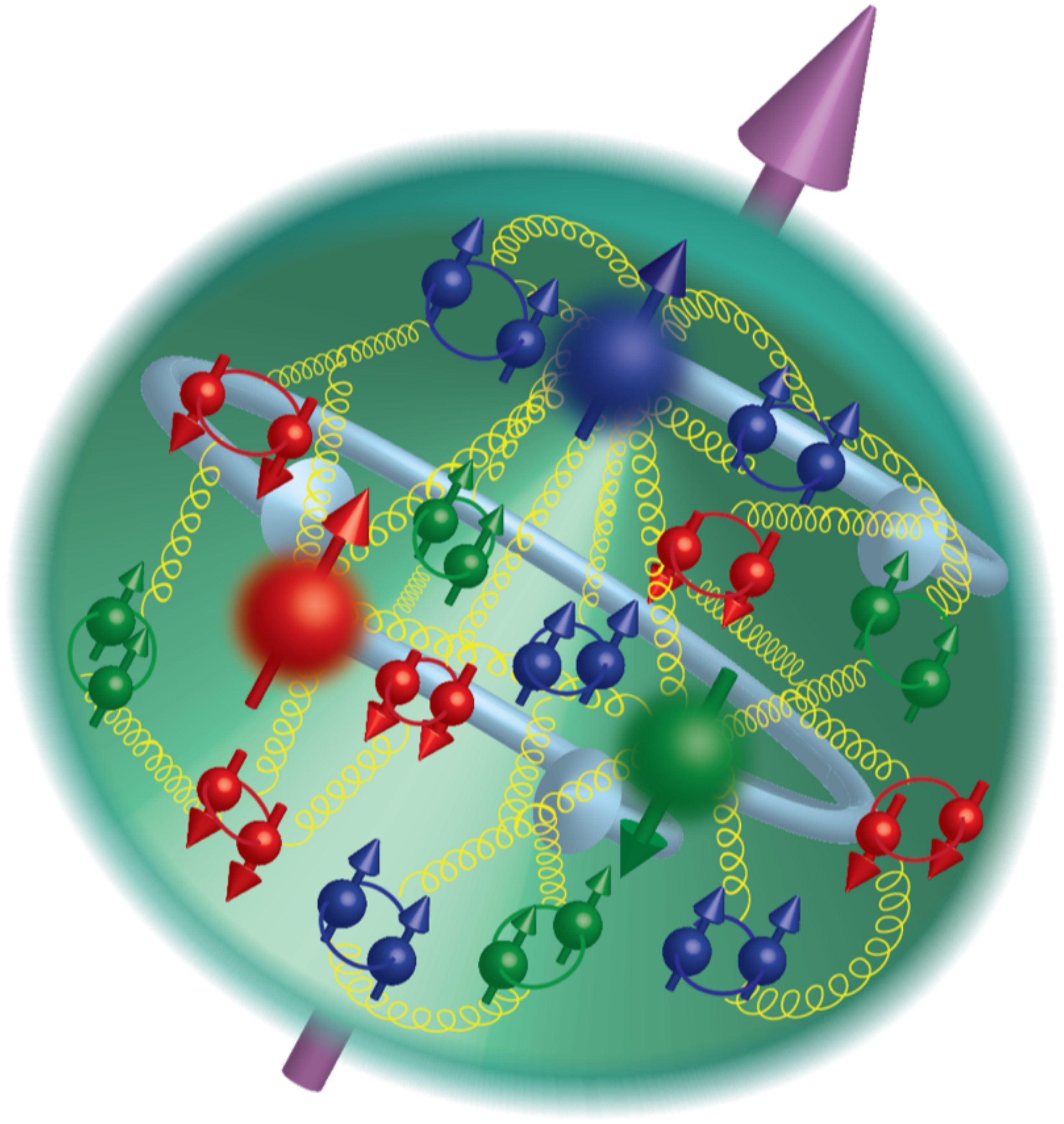
Argonne NATIONAL LABORATORY

Outline

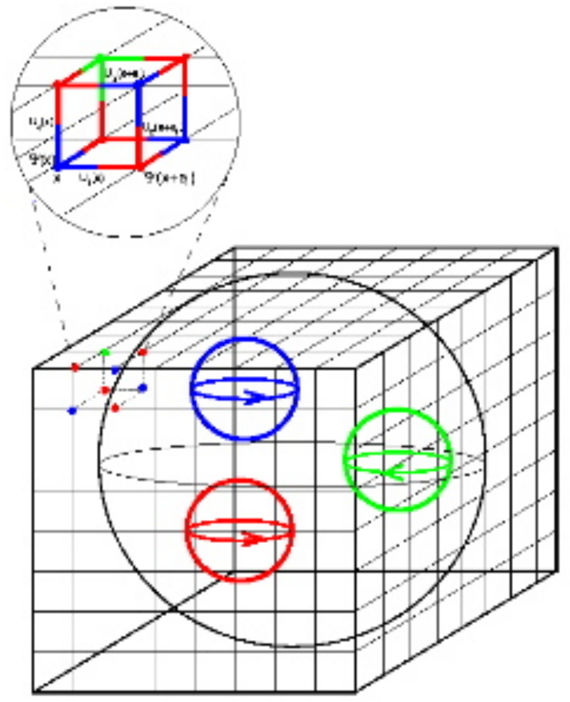
- Introduction
- Gluon x-dependence from lattice
  - Unpolarized gluon PDF of the proton
  - Moments of the gluon PDF
- Future Work



Motivation

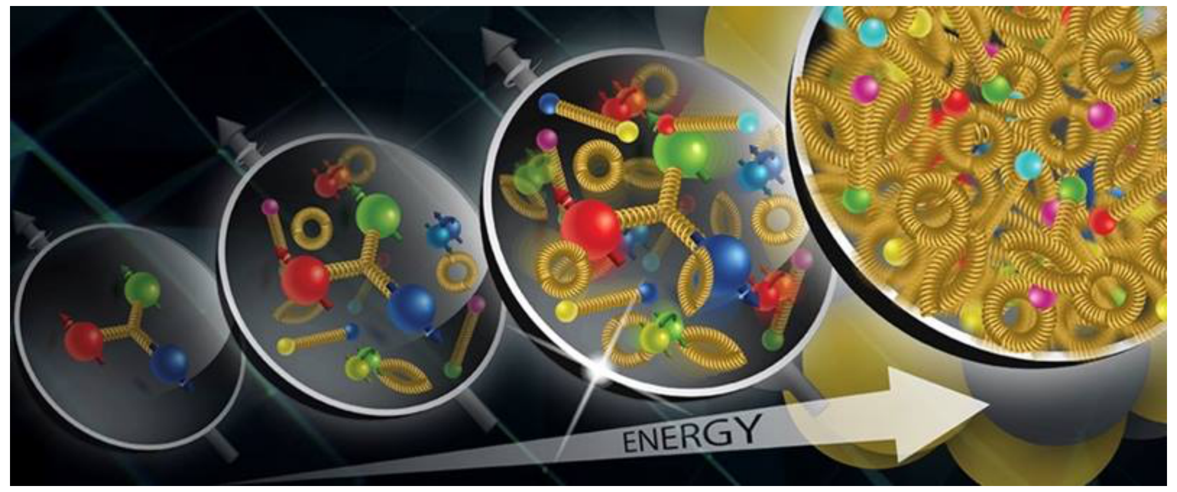
- Gluons carry a significant fraction of the the proton's momentum
  - Account for a large portion of mass-energy
- PDFs provide insight on momentum at different resolution scales
- Mellin moments provide additional insight to proton properties
- Precision PDFs are required for controlling uncertainties in collider processes
- Gluon behavior in extreme x regions of particular interest for hadron structure

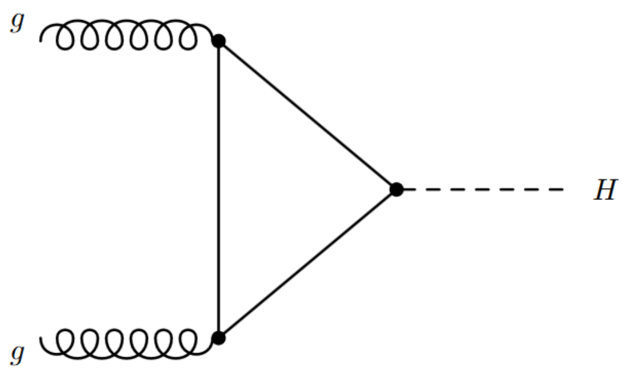
- Gluon PDFs not easily accessible from experiment (no direct probe, don't contribute at leading order)
- Extractions carry model dependence
- Kinematics limitations restrict precision at extreme x



<u>Lattice</u>

- First-principles approach
- Various methodologies for extracting information on x-dependence





Studying x-dependence on the Lattice

How do we access x-dependent quantities?

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Differential local operators

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Mellin Moments

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Differential local operators

Matrix elements of nonlocal operators

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pseudo-PDF

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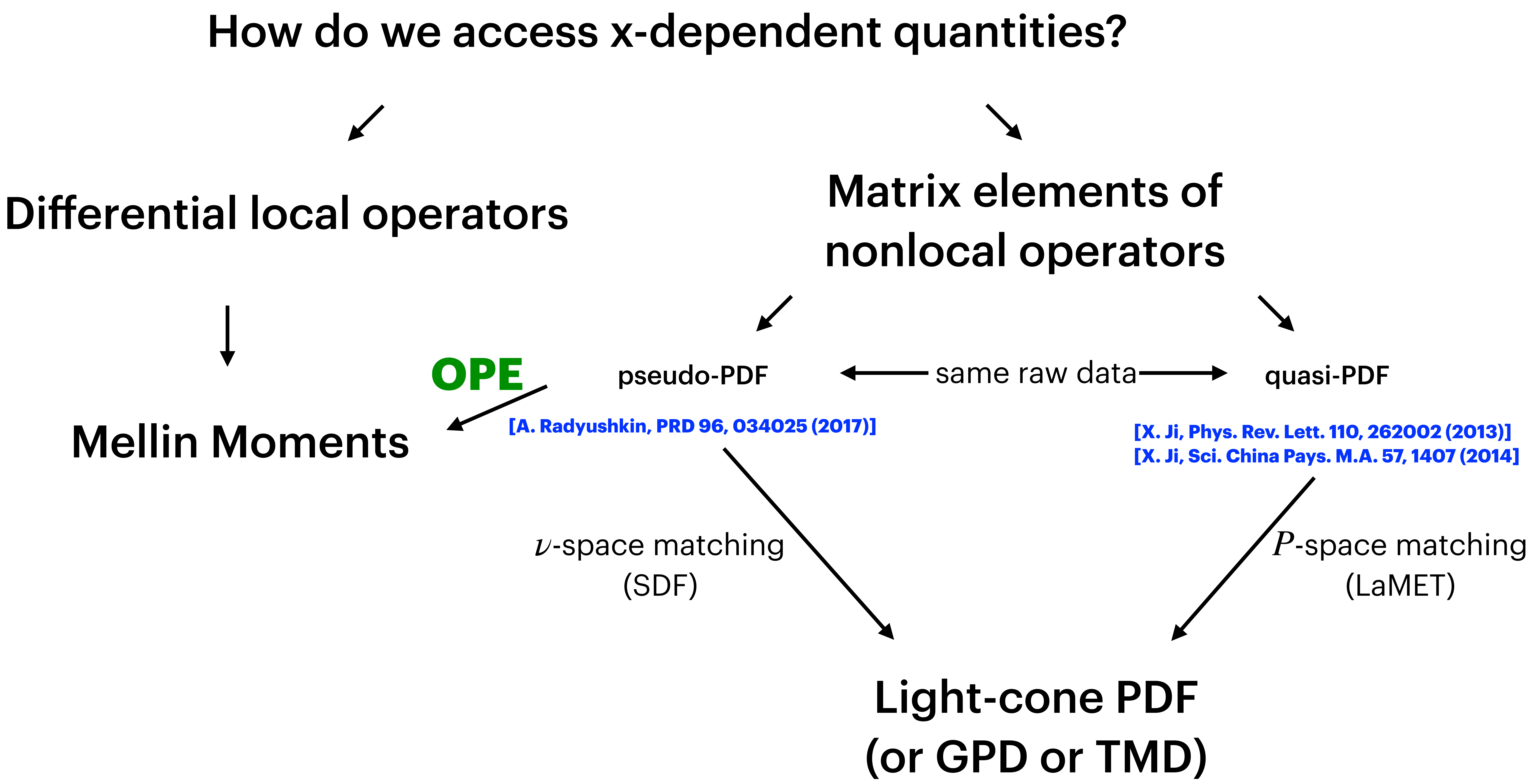
ν-space matching (SDF)

P-space matching (LaMET)

Light-cone PDF (or GPD or TMD)

Studying x-dependence on the Lattice



Approaches to studying x-dependence

PDF Extraction via pseudo-distributions

- Invert matching equation

$$Q_{gq}(\nu, z^2, \mu^2) = \mathfrak{M}_g(\nu, z^2)\,\langle x\rangle_g^{\mu} + \frac{\alpha_s N_c}{2\pi}\int_0^1 du\, \mathfrak{M}_g(u\nu, z^2)\,\langle x\rangle_g^{\mu}\left\{ \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)\mathfrak{B}_{gg}(u) + L(u)\right\}$$
$$+\frac{\alpha_s C_F}{2\pi}\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)\int_0^1 du\,\left(\mathfrak{M}_S(u\nu, \mu^2) - \mathfrak{M}_S(0,\mu^2)\right)\,\mathfrak{B}_{gq}(u)$$

- Reduced-ITD matched to light-cone counterpart

- ITD related to PDF by Fourier transform

$$Q_{g/gq}(\nu, \mu^2) = \int_0^1 dx\, \cos(\nu x) x g(x, \mu^2) \quad \text{where} \quad xg(x) = N x^a (1-x)^b$$

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Mellin Moments

- Expand the matching in terms of the PDF moments through the Operator Product Expansion

$$\mathfrak{M}_g(\nu, z^2) I_g(0,\mu^2) = I_g(\nu, \mu^2) - \frac{\alpha_s N_c}{2\pi}\int_0^1 du\; I_g(u\nu, \mu^2)\left\{ \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)\mathfrak{B}_{gg}(u) + L(u)\right\}$$

$$-\frac{\alpha_s C_F}{2\pi}\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)\int_0^1 dw\;\left(I_S(w\nu, \mu^2) - I_S(0,\mu^2)\right)\;\mathfrak{B}_{gq}(u)$$

where

$$I_g(\nu, \mu^2) = 1/2\int_{-1}^1 dx\, e^{ix\nu} x f_g(x, \mu^2) = \int_0^1 dx \sum_{n=0}^{\infty}\frac{(ix\nu)^n}{n!} x f_g(x)$$

- Yields expansion in terms of the PDF moments

$$\mathfrak{M}_g(\nu, z^2) = 1 - \sum_{n=2}^{\infty}\frac{(i\nu)^n}{n!} a_n(z, \mu)\frac{\langle x^{n+1}\rangle}{\langle x\rangle}$$

- Moments can be fit directly from lattice data

Unpolarized Gluon PDF

Lattice Details

- Three ensembles of twisted-clover fermions and Iwasaki improved gluons at greater than physical pion mass

$$\mathscr{L}_f = \bar{\psi}_2'\left(\gamma^\mu \partial_\mu + m_q' + i\mu\gamma_5\tau_3\right)\psi_2'$$
$$m_q' = m_q \cos(\alpha), \quad \mu = m_q \sin(\alpha)$$

Ensemble	β	a (fm)	$L^3 \times T$	N_f	m_π (MeV)	L (fm)
cA211.30.32	1.726	0.094	$32^3 \times 64$	$2+1+1$	260	3
cB211.25.32	1.778	0.079	$32^3 \times 64$	$2+1+1$	250	2.5
cD211.17.48	1.900	0.057	$48^3 \times 96$	$2+1+1$	210	2.75

- Gluonic quantities very sensitive (purely disconnected contributions), requires much higher statistics than quark case
- Momentum smearing applied to $P > \dfrac{2\pi}{L}$

Ensemble	P (GeV)	N_{conf}	N_{src}	N_{dir}	N_{meas}
cA211.30.32	0, 0.42, 0.83, 1.25, 1.67	1,134	200	6	1,360,800
cB211.25.32	0, 0.49, 0.98, 1.47, 1.96	901	400	6	2,162,400
cD211.17.48	0, 0.45, 0.90, 1.36, 1.81	1,167	400	6	2,800,800

Theoretical Setup

- Light-cone matching carried out using pseudo-distribution approach
- Several choices of operator: $\mathcal{O}_4 \equiv \frac{1}{2}\sum_i F_{it}(x+z\hat{z})W(x+z\hat{z},x)F_{it}(x)W(x,x+z\hat{z}) - \sum_{i<j} F_{ij}(x+n\hat{k})W(x+z\hat{z},x)F_{ij}(x)W(x,x+z\hat{z}), \quad i \neq t \neq z$
  - Our choice avoids power-divergent mixing (on the lattice) under renormalization
  - Requires subtraction of vacuum contribution
- Mixing with quark singlet unavoidable, addressing requires calculation of unpolarized quark contributions

$$M_f(z,P) = \langle N(P)|\overline{\psi}_f(z)\gamma^0 W(z)\psi_f(0)|N(P)\rangle \qquad \mathscr{L}(t_{\text{ins}},z) = \sum_{\vec{x}_{\text{ins}}} \text{Tr}\left[\text{D}_\text{q}^{-1}(\text{x}_{\text{ins}};\text{x}_{\text{ins}}+\text{z})\gamma^0 \text{W}(\text{x}_{\text{ins}},\text{x}_{\text{ins}}+\text{z})\right]$$

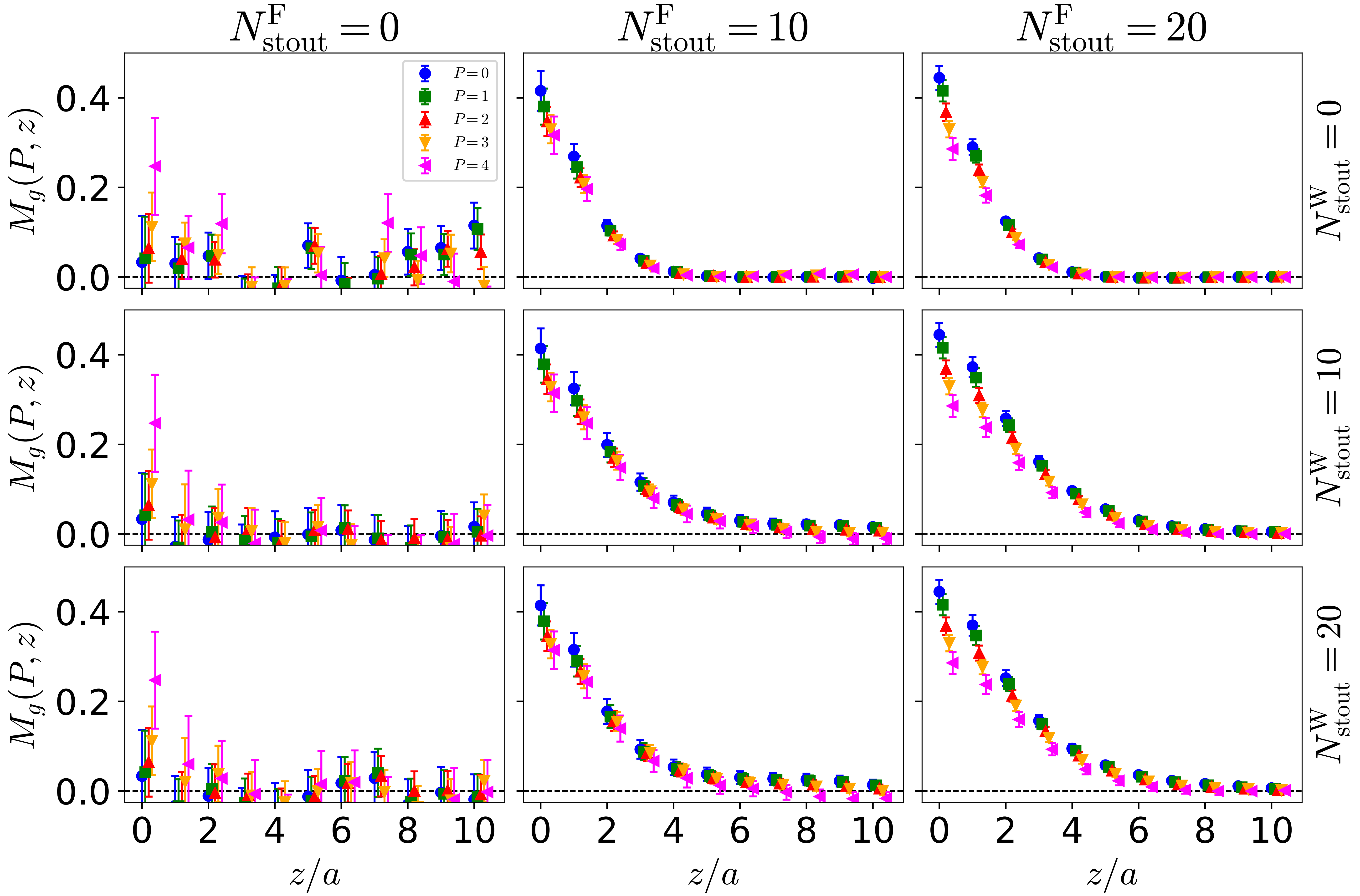
- Reduced Ioffe-time distribution constructed from ratio of ground-state matrix elements

$$\mathfrak{M}_g(\nu,z^2) \equiv \left(\frac{M_g(\nu,z^2)}{M_g(\nu,0)|_{z=0}}\right) \Big/ \left(\frac{M_g(0,z^2)|_{p=0}}{M_g(0,0)|_{p=0,z=0}}\right)$$

- Matching relates reduced-ratio and light-cone Ioffe-time distribution

$$\mathfrak{M}_g(\nu,z^2)I_g(0,\mu^2) = I_g(\nu,\mu^2) - \frac{\alpha_s N_c}{2\pi}\int_0^1 du\, I_g(u\nu,\mu^2)\left\{\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)\mathfrak{B}_{gg}(u) + L(u)\right\}$$
$$-\frac{\alpha_s C_F}{2\pi}\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)\int_0^1 dw\ \left(I_S(w\nu,\mu^2) - I_S(0,\mu^2)\right)\ \mathfrak{B}_{gq}(u)$$

Stout Smearing Testing



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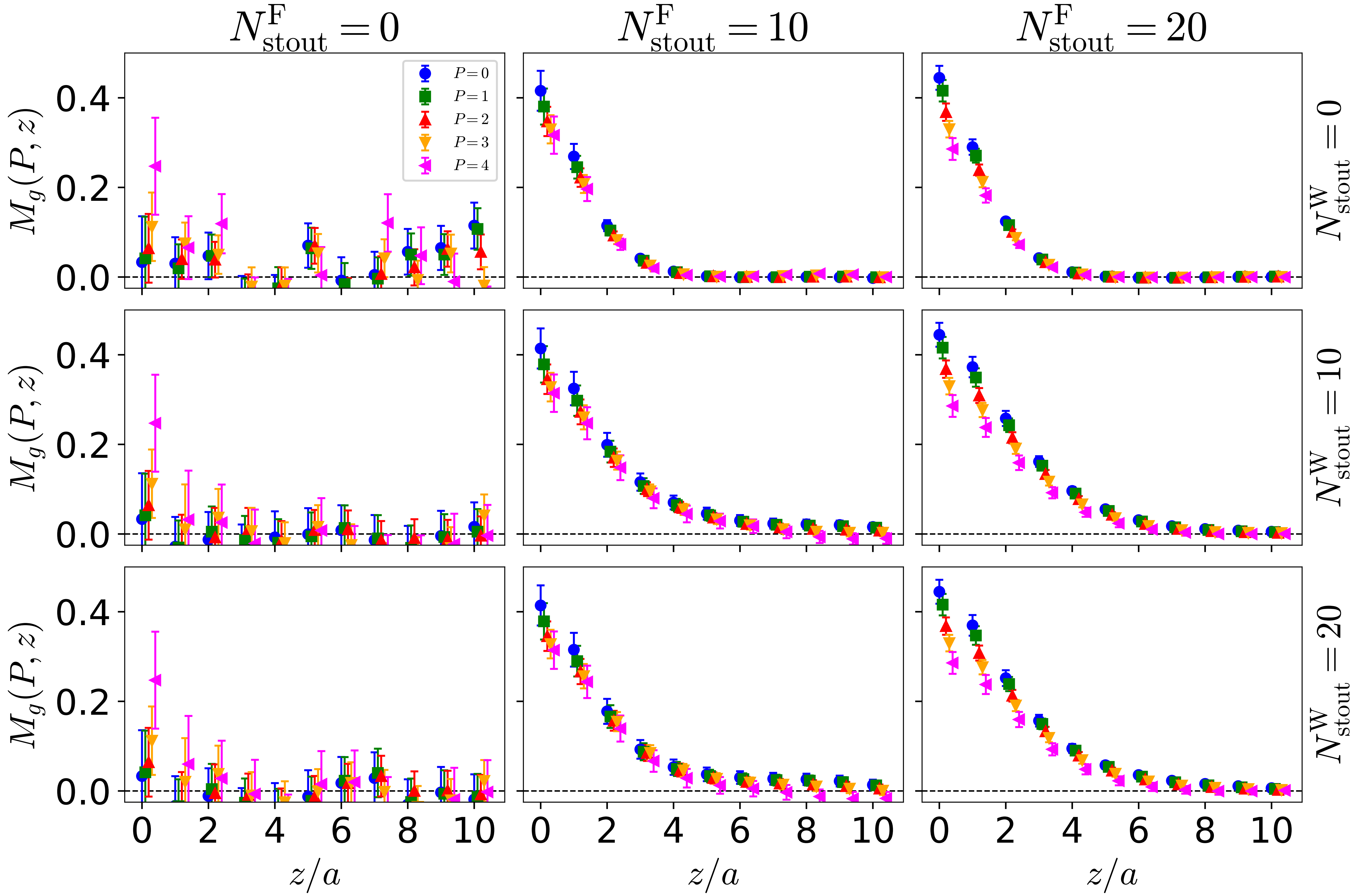

- Smeared field strength tensor and Wilson line independently
- Testing performed in [0, 20] in steps of 5
- Smearing of field strength tensor key to resolve signal

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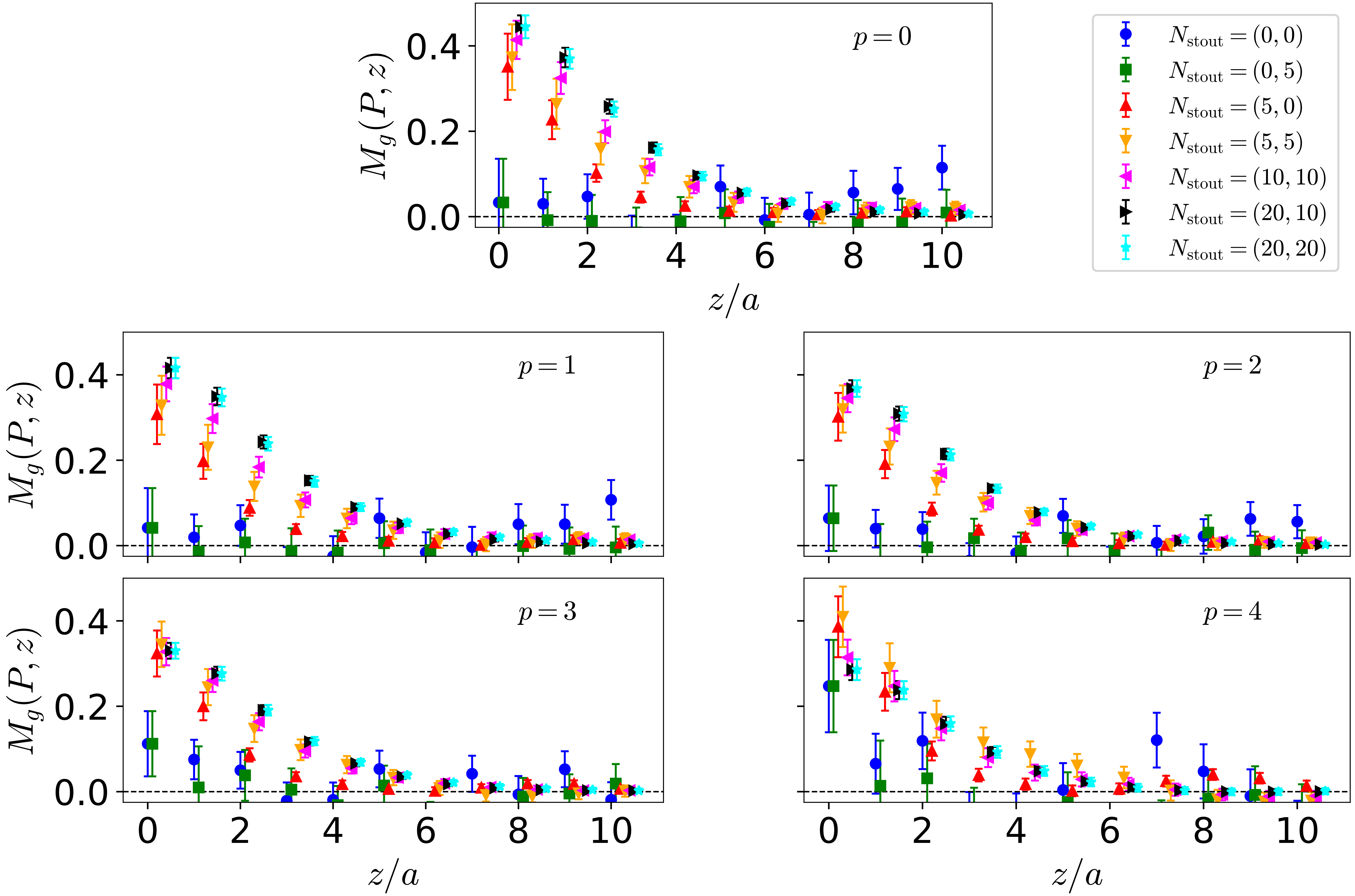

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Reduced-ITD Parametrization

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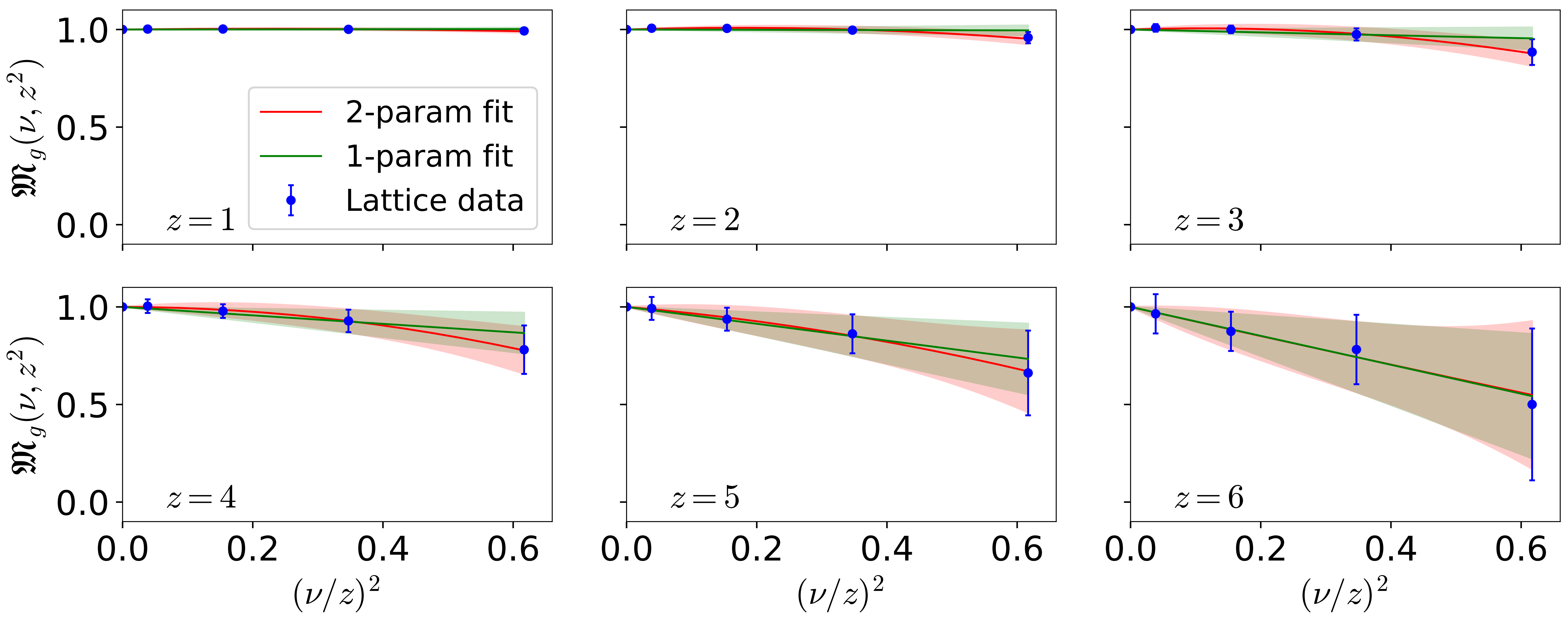

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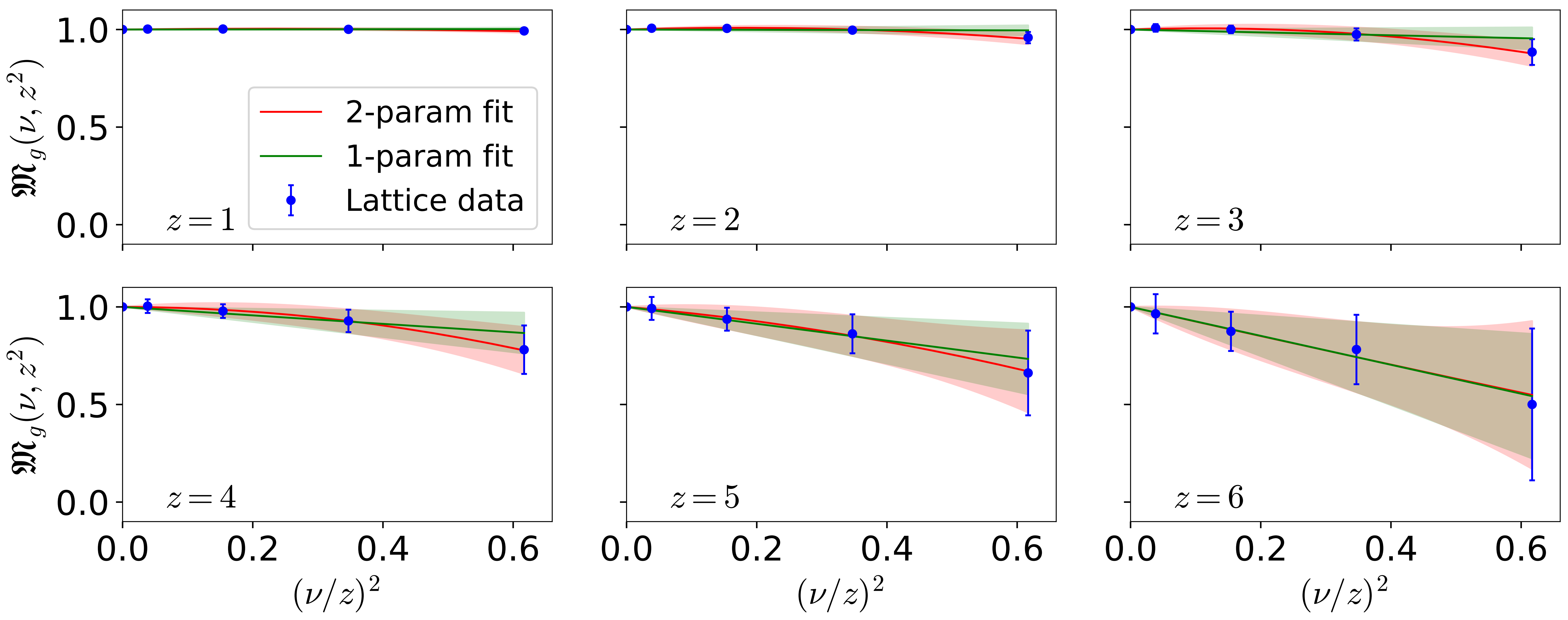
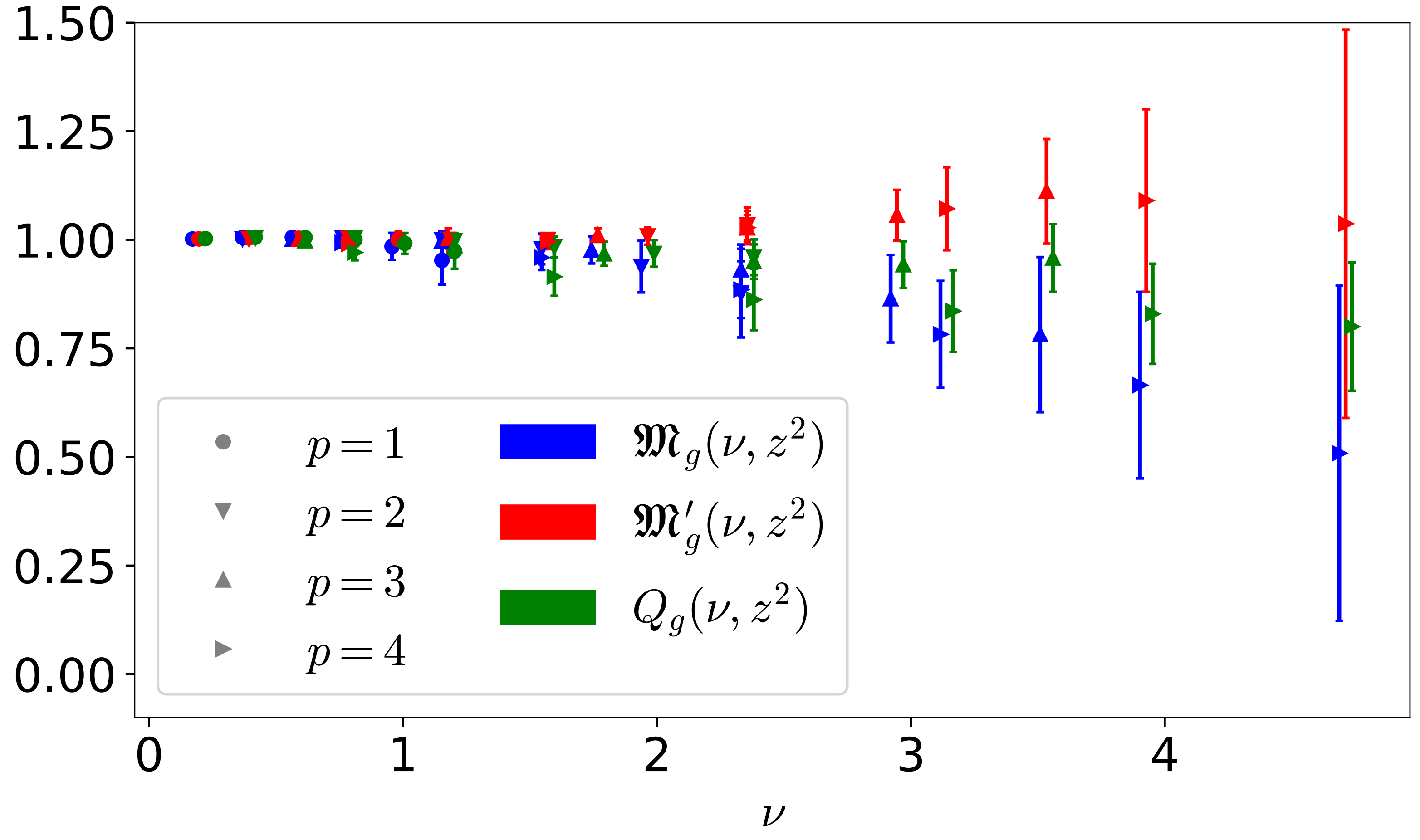
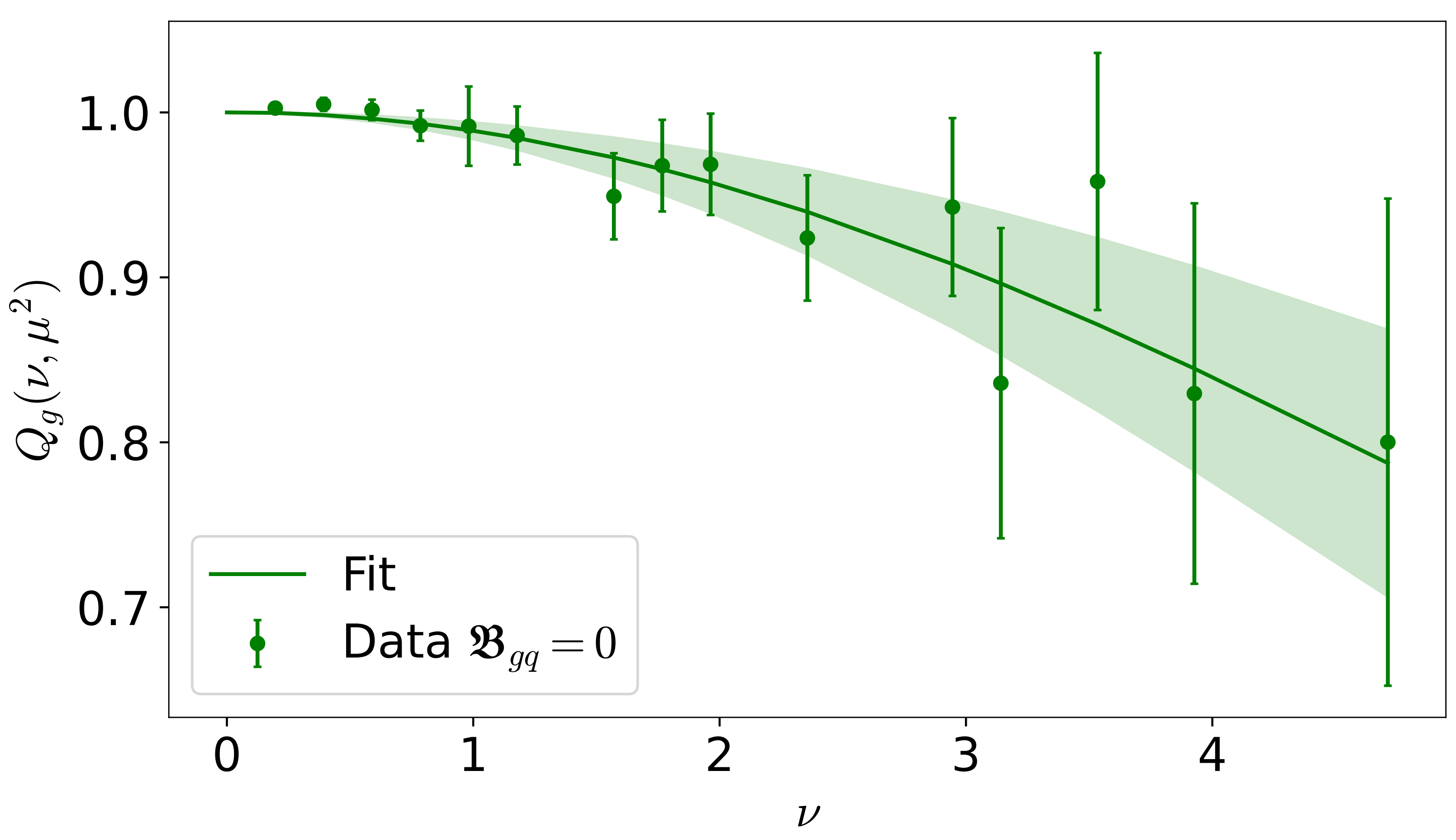


- Matching and evolution integrals require parametrization of discrete lattice data
- Good agreement between 1- and 2-parameter fits

ITD Development





- ITD integral includes evolution to 2 GeV scale and matching to light-cone
- Common Ioffe-times averaged over prior to fit

$$Q_{g/gq}(\nu, \mu^2) = \int_0^1 dx\, \cos(\nu x) x g(x, \mu^2) \quad \text{where} \quad xg(x) = N x^a (1-x)^b$$

$$\chi^2 = \sum_{\nu=0}^{\nu_{\max}} \frac{\left(Q_{g/gq}(\nu, \mu^2) - Q_f(\nu, \mu^2)\right)^2}{\sigma_Q^2(\nu, \mu^2)}$$

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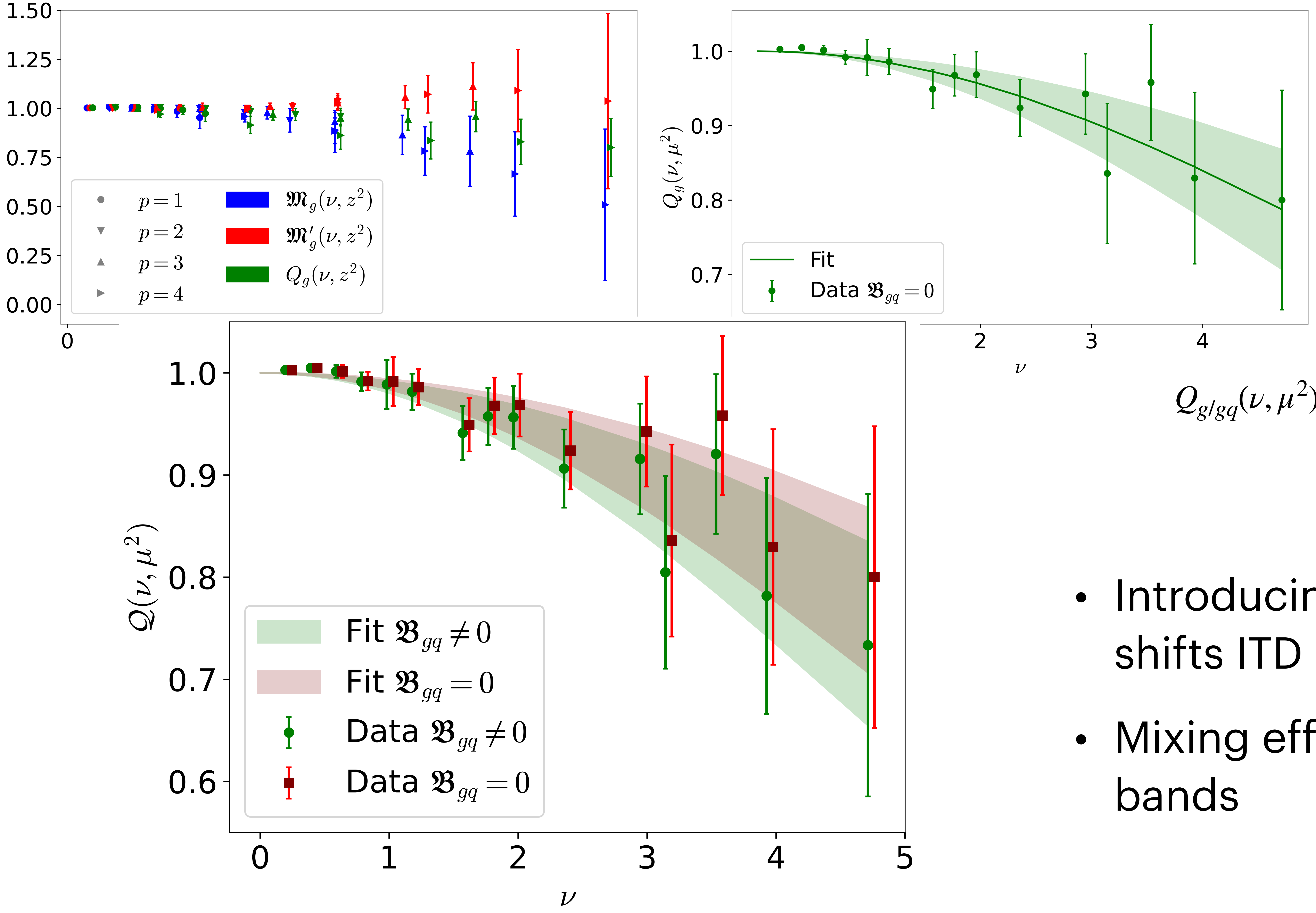

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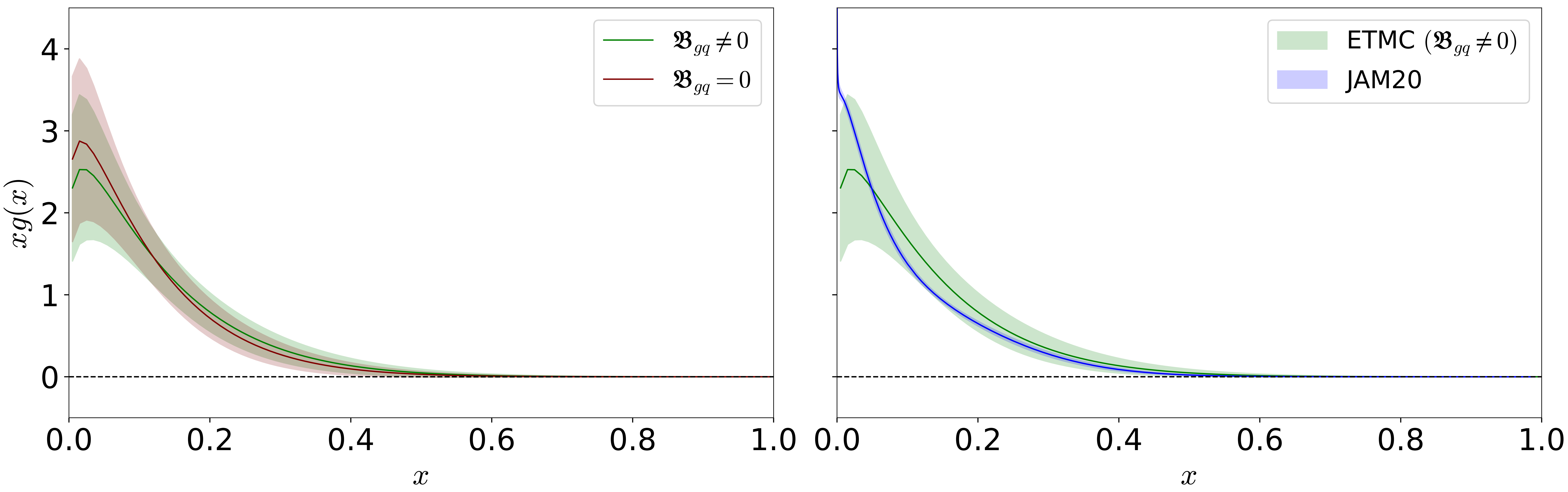
- Introducing singlet contribution shifts ITD slightly downward
- Mixing effects occur within error bands

Unpolarized Gluon PDF

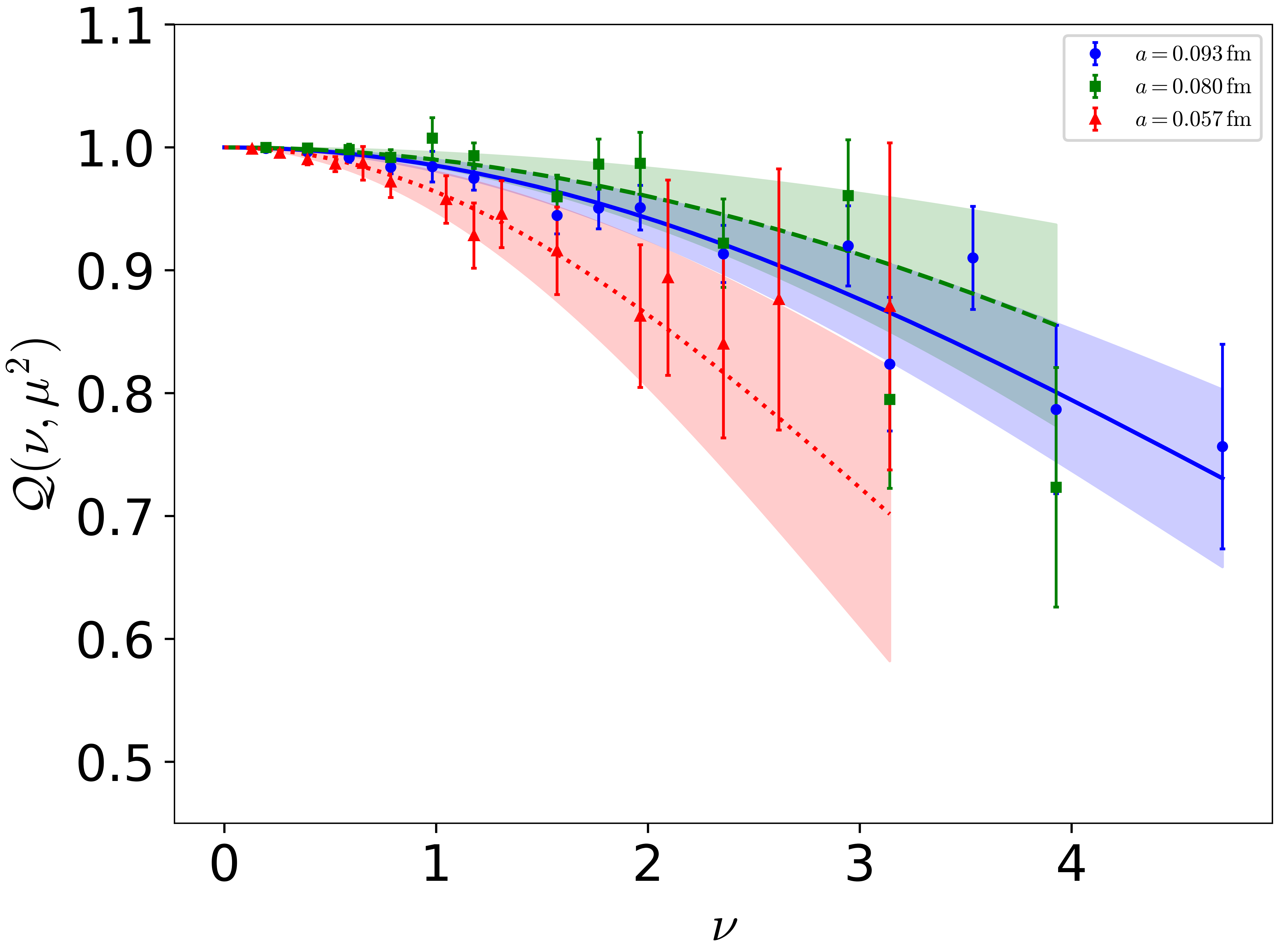


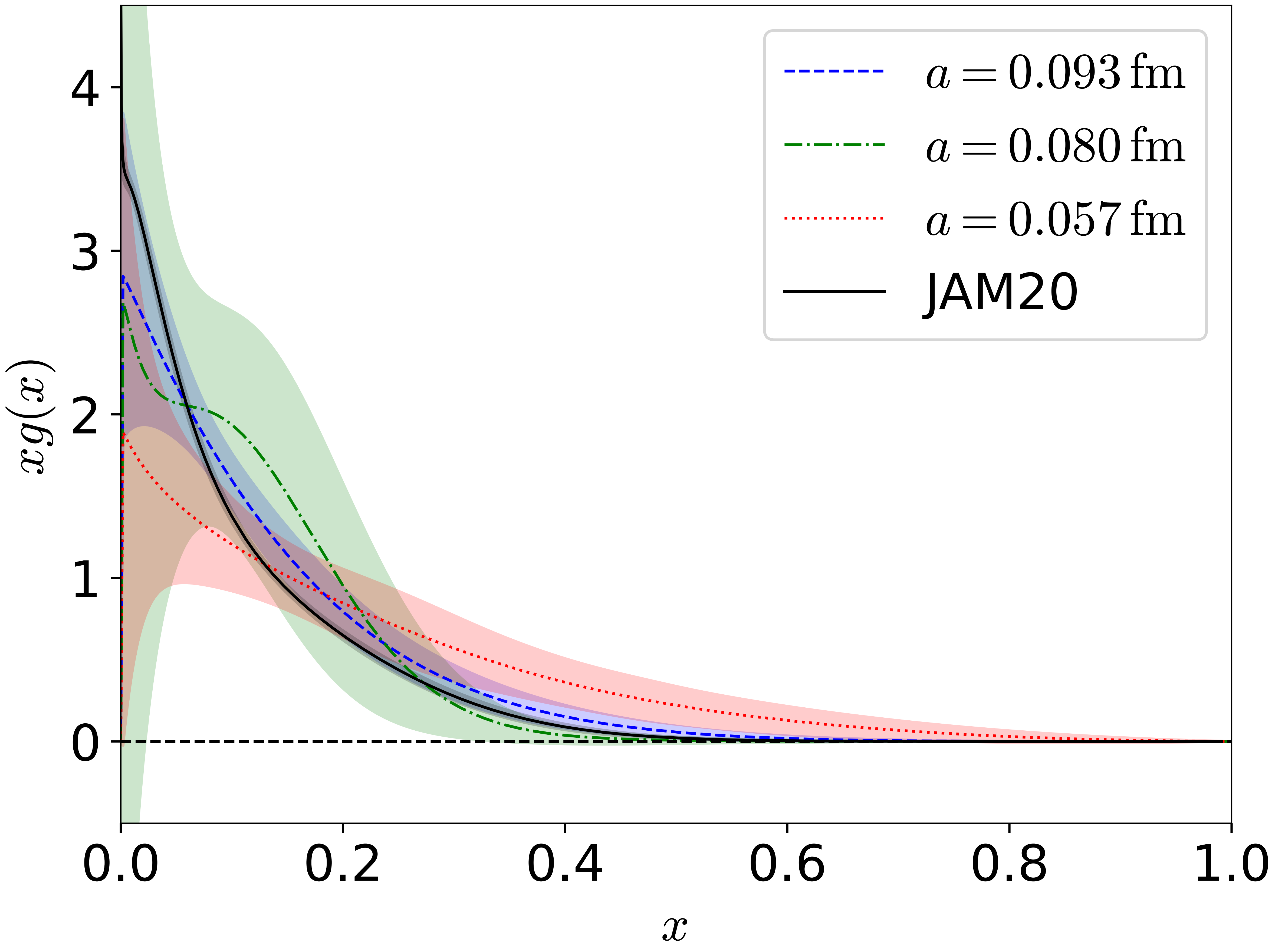
- Effects of the singlet minimal at current precision
- Trend of data matches expectation from global fits

Preliminary Results Follow

Continuum PDF

- Follow the same process as before for our additional ensembles





- All ensembles exhibit general downward trend
- Different ensembles may exhibit different systematics (volume effects, discretization effects)

- Fits much noisier for additional ensembles
- Lower ν_{max} contributes to extended tail

Mellin Moment Ratio

- Reduced-ITD expanded in terms of the PDF moments

$$\mathfrak{M}_g(\nu, z^2) I_g(0,\mu^2) = I_g(\nu,\mu^2) - \frac{\alpha_s N_c}{2\pi}\int_0^1 du\, I_g(u\nu,\mu^2)\left\{\ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right)\mathfrak{B}_{gg}(u) + L(u)\right\}$$

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$$\Downarrow$$

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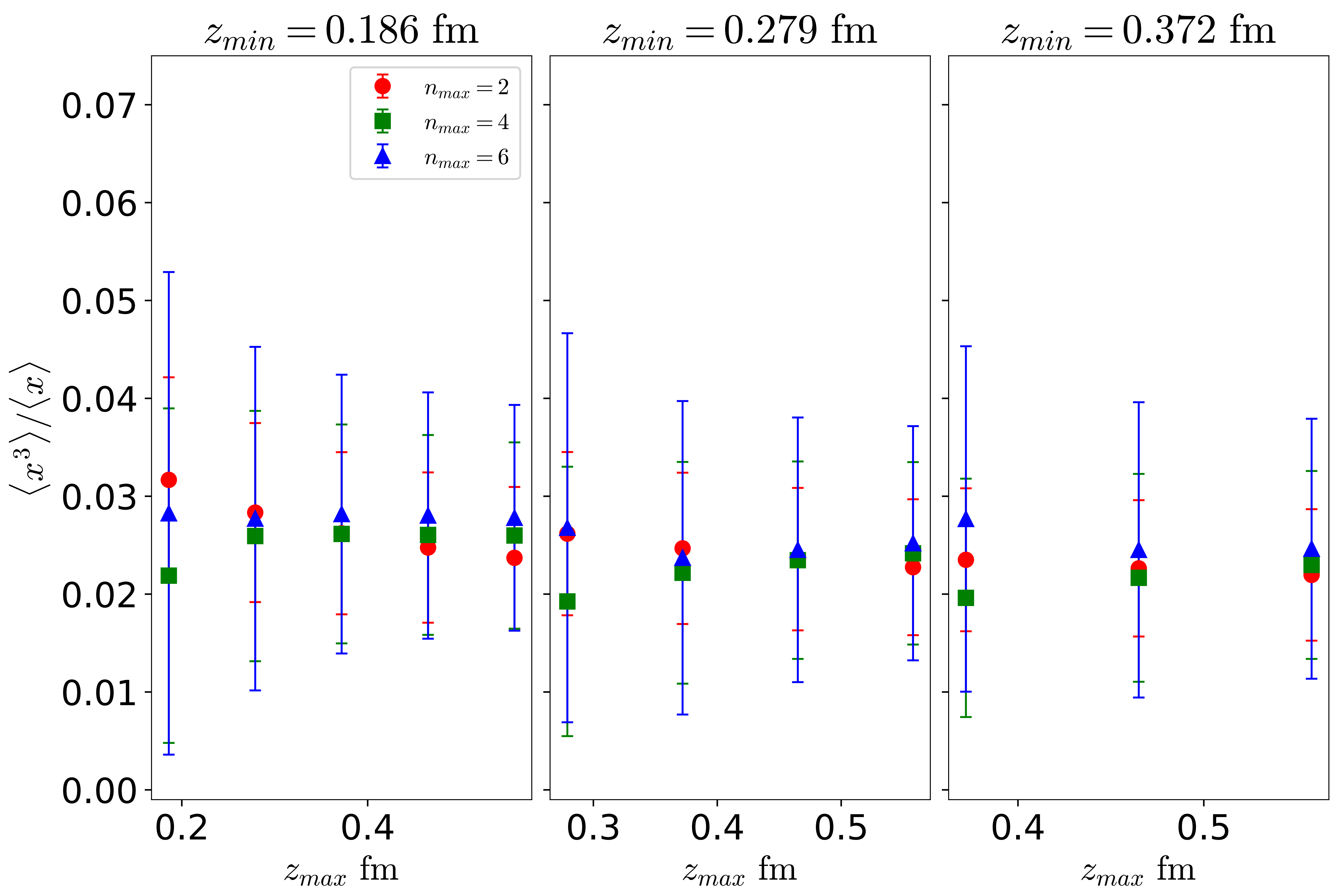
- Explore truncation order, minimum and maximum spatial separation in MEs



Scale Evolution of Mellin Moments

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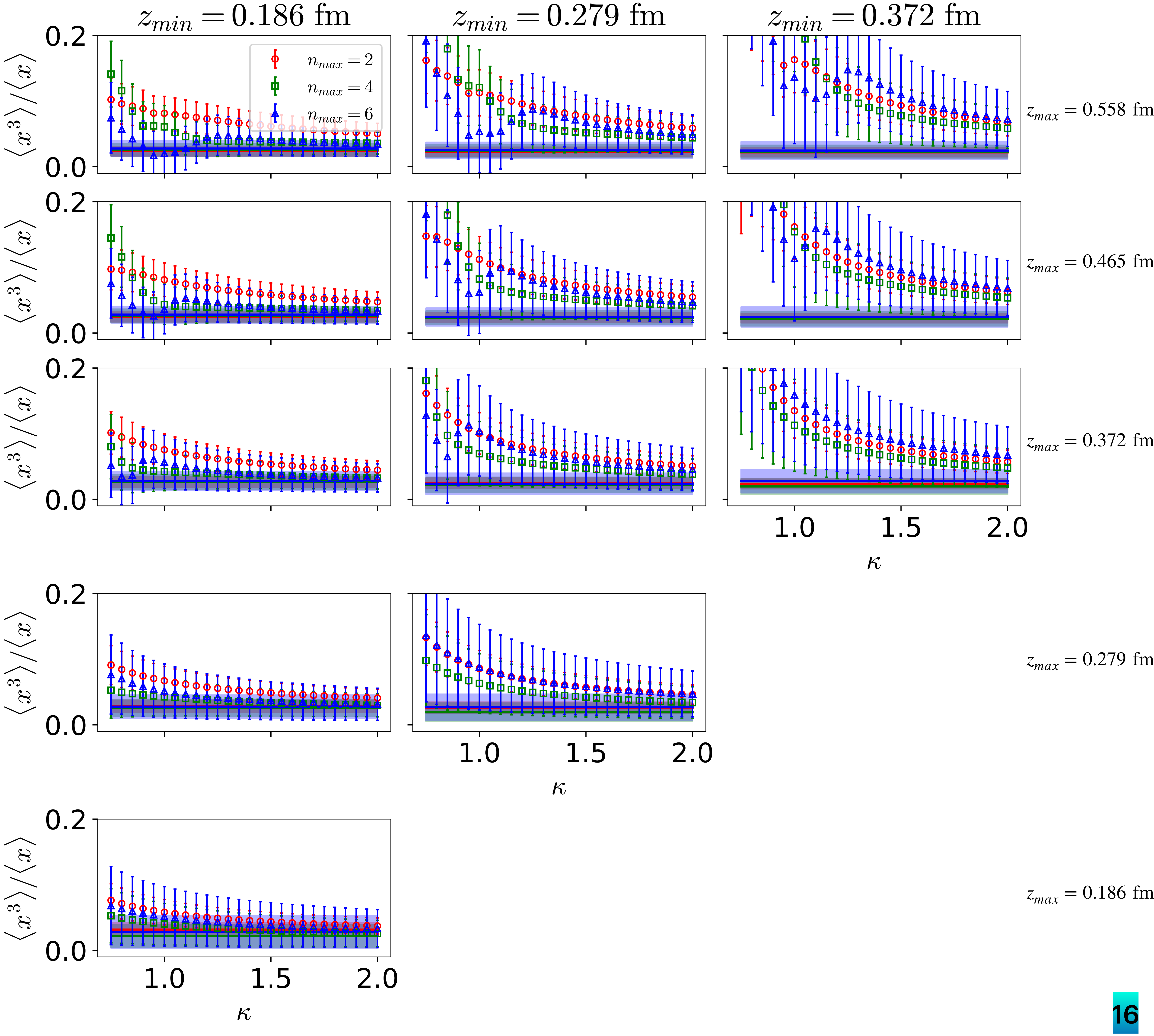
- Examining the evolution from initial scale μ_0 allows for study of truncation and perturbative effects
- DGLAP carried out with two-loop matching



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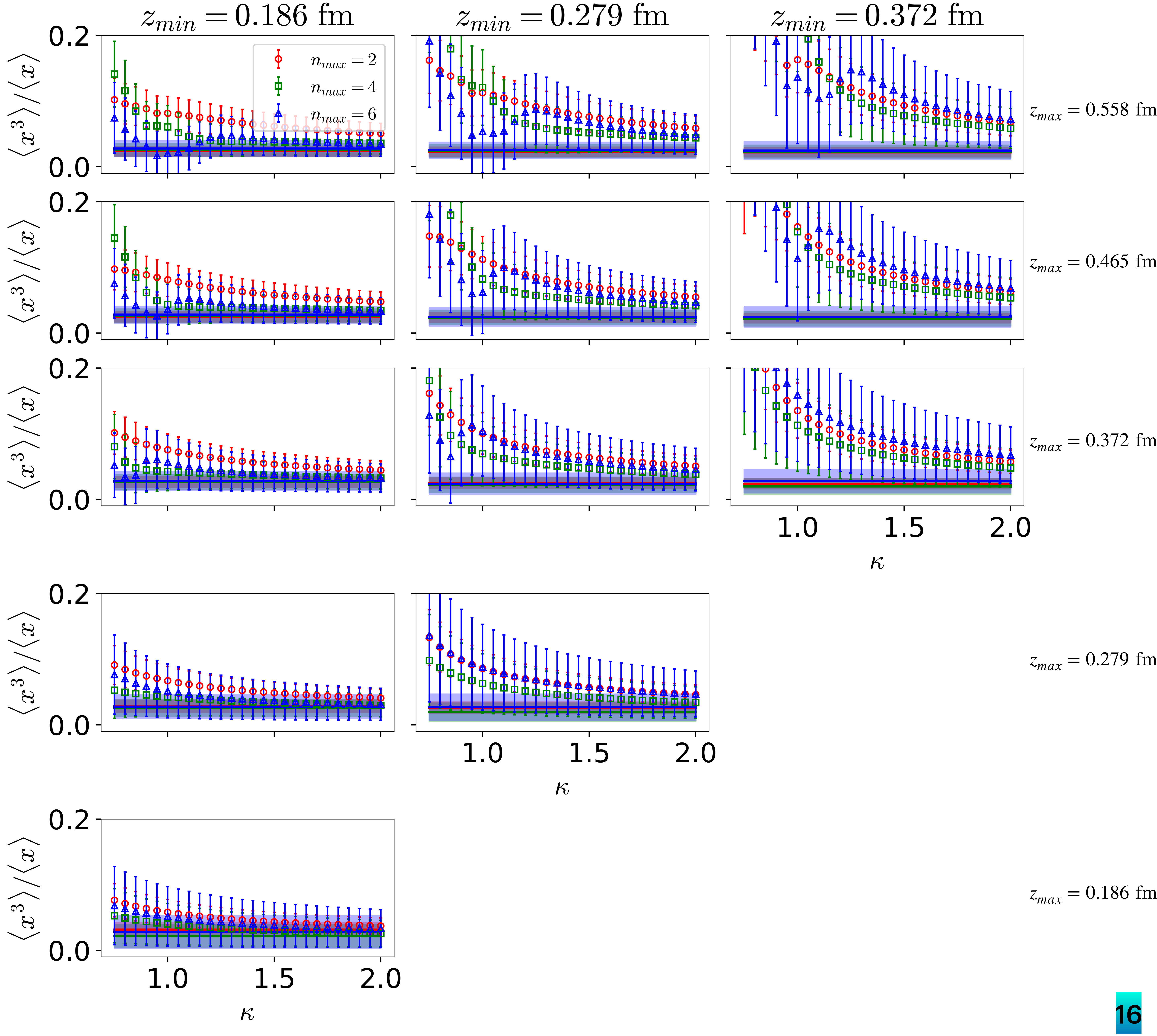

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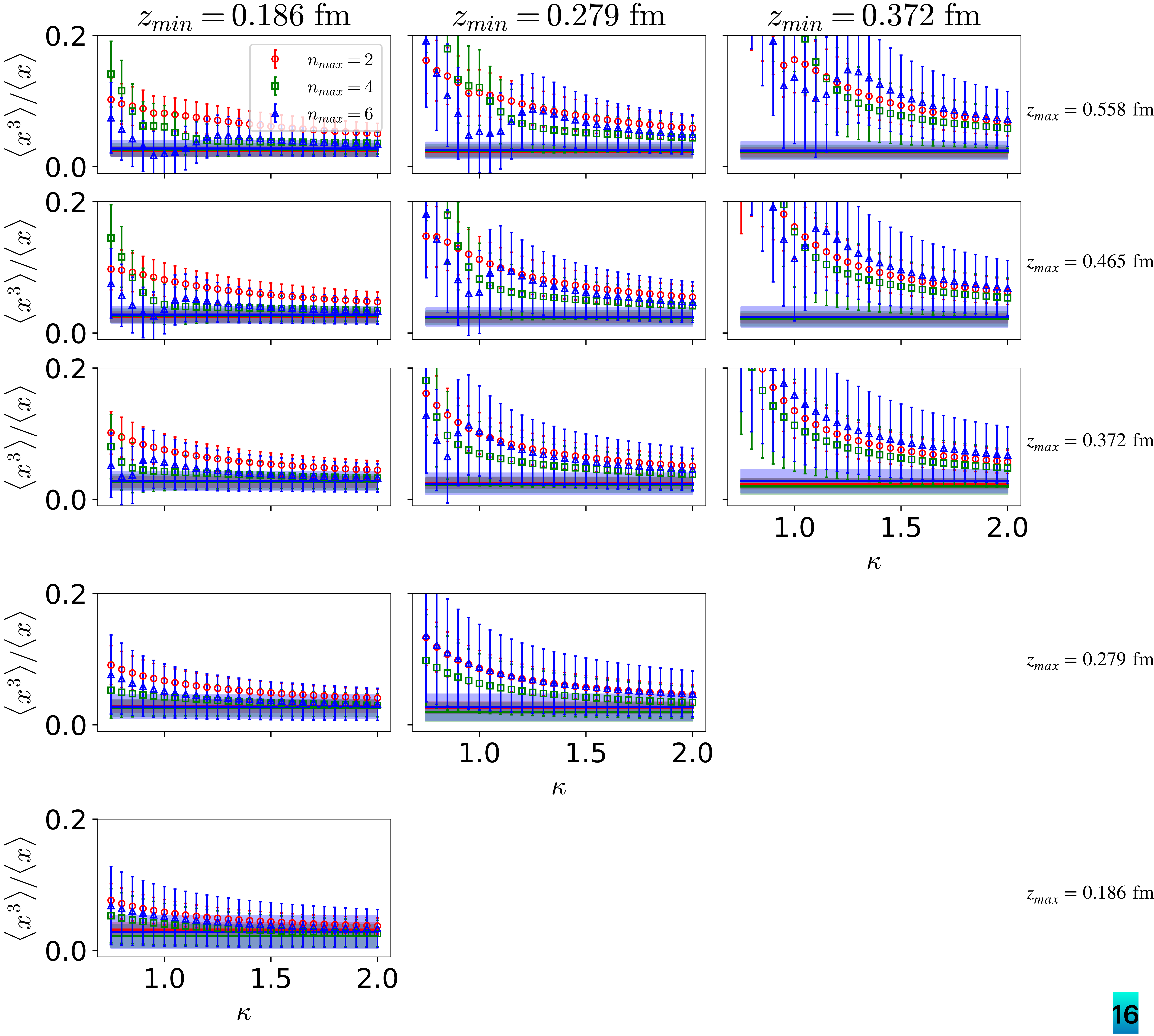
- Examining the evolution from initial scale μ_0 allows for study of truncation and perturbative effects
- DGLAP carried out with two-loop matching
- Compare with JAM data:

$$\langle x \rangle = 0.400(1) \text{ and } \frac{\langle x^3 \rangle}{\langle x \rangle} = 0.0214(3)$$



Future Work on Gluon x-dependence

Unpolarized PDF

- Finalize choices of smearing
- Fourth ensemble nearing completion
- Explore hybrid renormalization and LaMET reconstruction

Mellin Moments

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- Compare with different ratios to cancel linear divergence
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Acknowledgement

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Thank You!