Measurement of the muon's anomalous spin precession frequency in the Fermilab Muon g-2 Experiment

Scott Israel

CIPANP 2025, Madison, WI

June 11th, 2025

In partnership with:

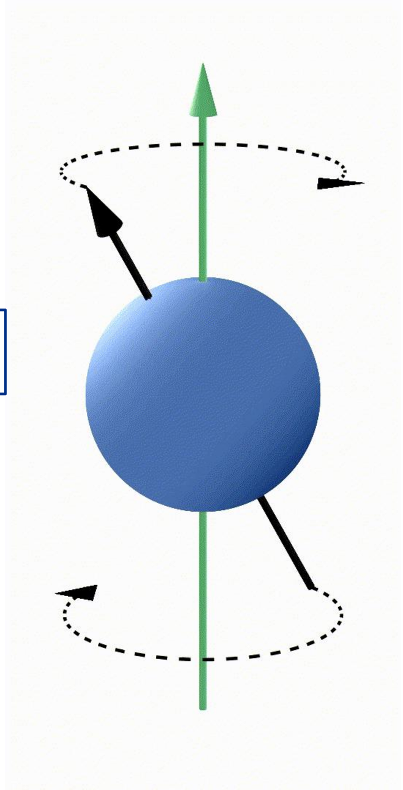
What is Fermilab Muon g-2 about?

- **Measuring the muon's magnetic anomaly to <140 parts-per-billion precision!**
- **Magnetic moment:** how strongly a particle reacts when placed in an external magnetic field
- **Magnetic anomaly:** a_μ
- Muon's magnetic anomaly – tension between theory and experiment! Go measure!

$$\boldsymbol{\mu} = g\frac{e}{2m}\boldsymbol{S}$$

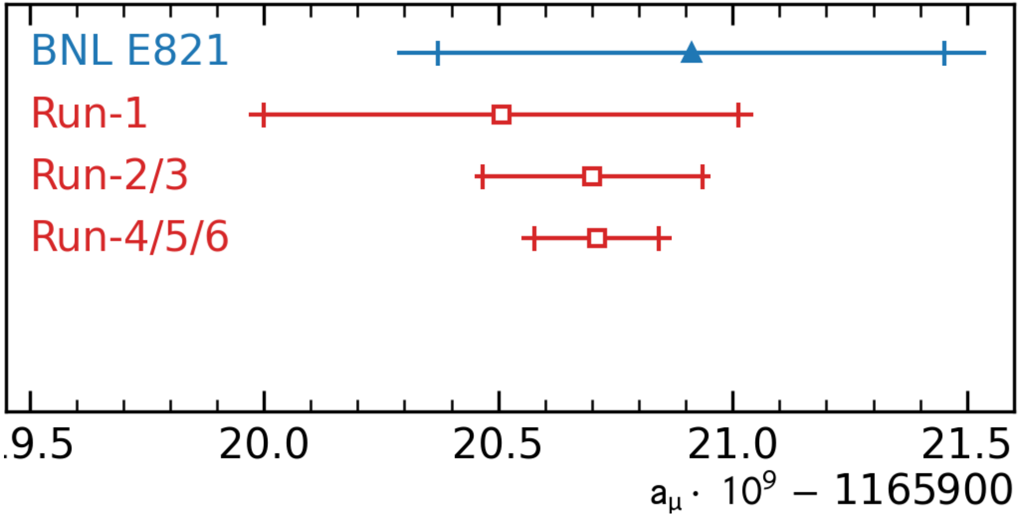
Gyromagnetic ratio

$$a_\mu = \frac{\boxed{g-2}}{2}$$



New results! Fermilab Muon g-2 magnetic anomaly

Runs-4/5/6 result announced June 3rd, 2025, at Fermilab



- Excellent agreement with Runs-2/3 results
- Uncertainty further reduced – met experimental uncertainty goal!
- $a_\mu(\text{Exp; Run 4,5,6}) = \mathbf{0.00116592071 0(162)}[139\text{ ppb}]$

New results! Fermilab Muon g-2 magnetic anomaly

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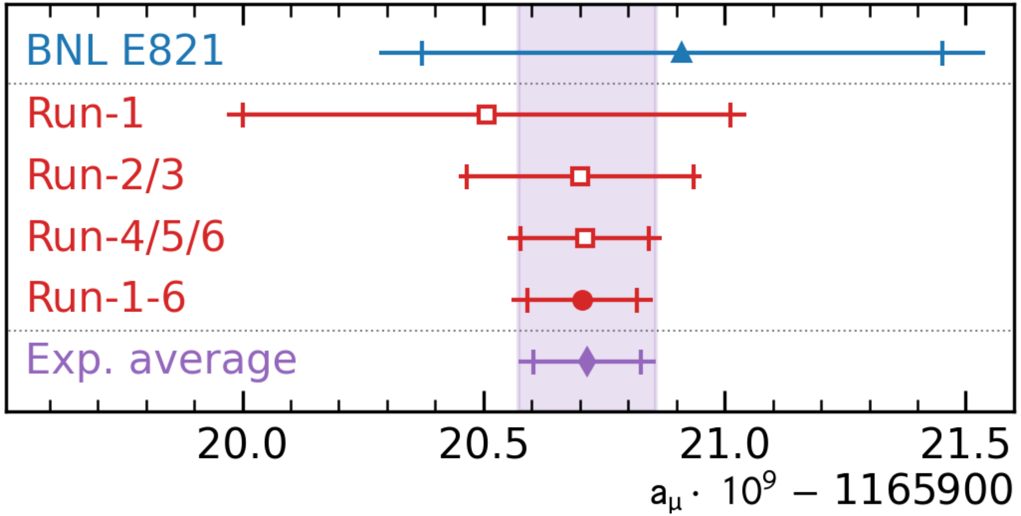


- Final combination with Runs-1/2/3
- Uncertainty further reduced – exceeded experimental uncertainty goal!
- $a_\mu(\text{Exp; Run } 1-6) = \mathbf{0.001165920705(148)}[127\text{ ppb}]$
- $a_\mu(\text{Exp; All}) = \mathbf{0.001165920715(145)}[124\text{ ppb}]$

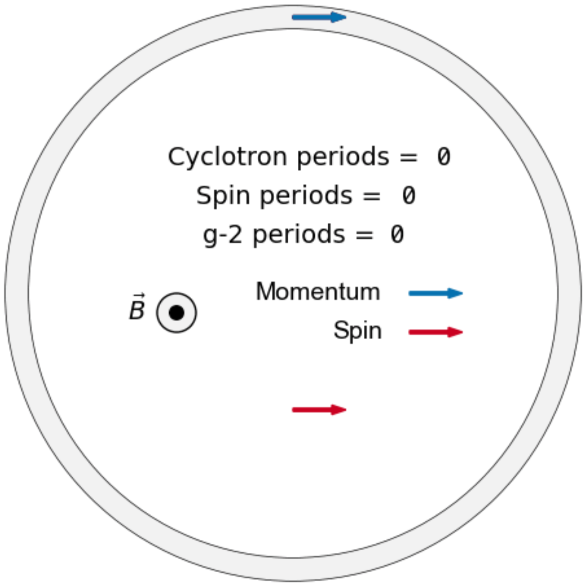
Via Sean Foster

Experimental principle

- Storing anti-polarized positive muons in a *magnetic storage ring* using electrostatic quadrupoles
  - Momentum ***rotates*** w.r.t. storage ring: ω_c
  - Spin ***precesses*** w.r.t. ring because $g > 2$: ω_s
- Difference in frequencies proportional to magnetic moment

$$\omega_s - \omega_c = \omega_a \propto \left(\frac{g-2}{2}\right)\frac{eB}{m} = \frac{a_\mu eB}{m}$$

We measure ω_a, B



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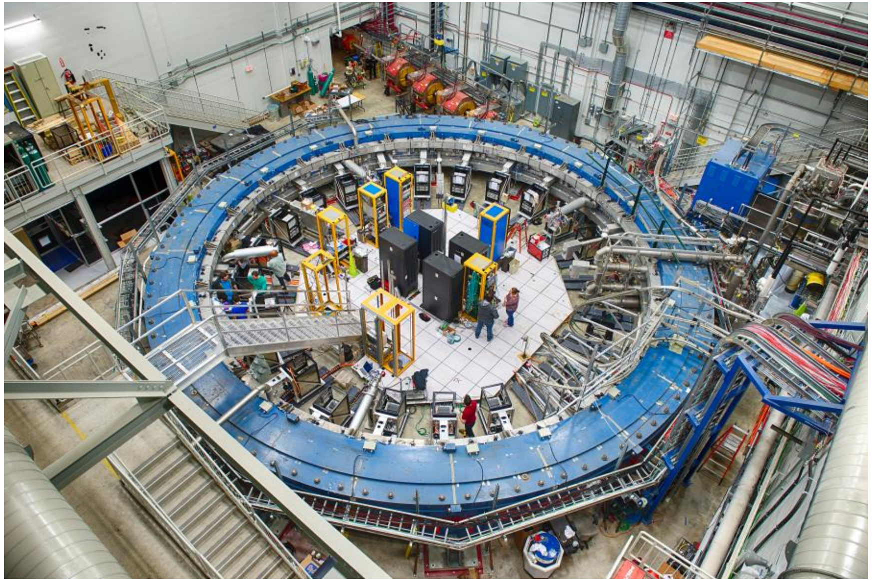
This talk!

Next talk by David Kessler



Via Reidar Hahn

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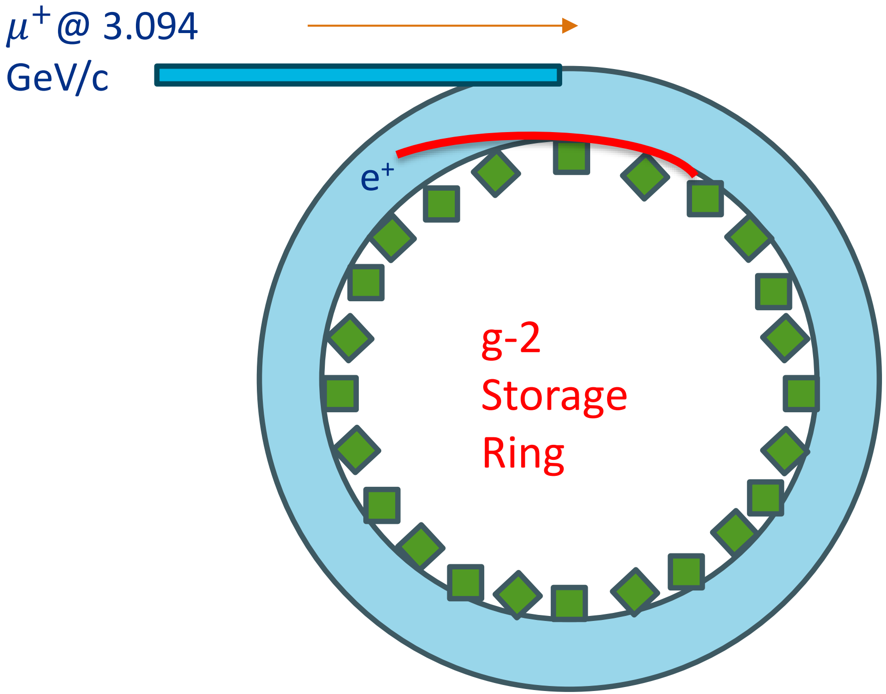
Count your positrons! Not muons?

- We don't measure muons directly; measure decay positrons
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$
- Positrons travel inward towards inside wall of storage ring
- Let's put detectors there! Obvious choice, calorimeters!



Fermilab

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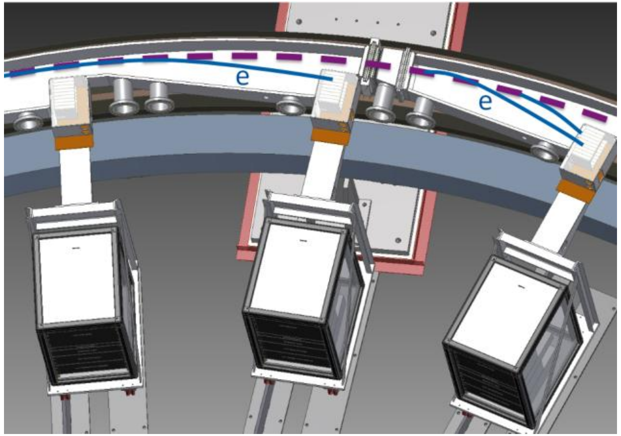
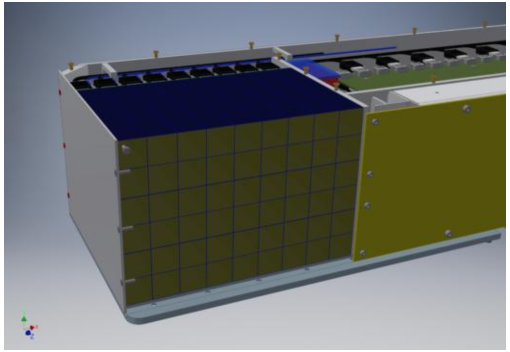
$$\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$$

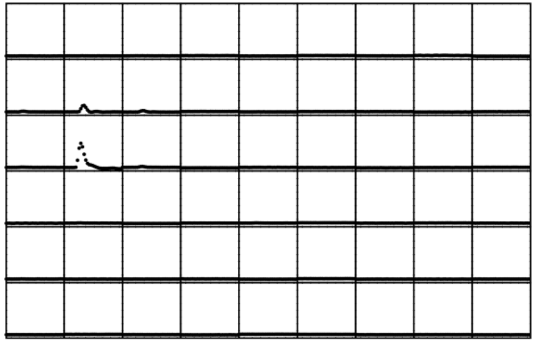
- Positrons travel inward towards inside wall of storage ring
  - $r = \gamma m v / qB$ and $m_e = m_\mu / 200$
- Let's put detectors there! Obvious choice, calorimeters!

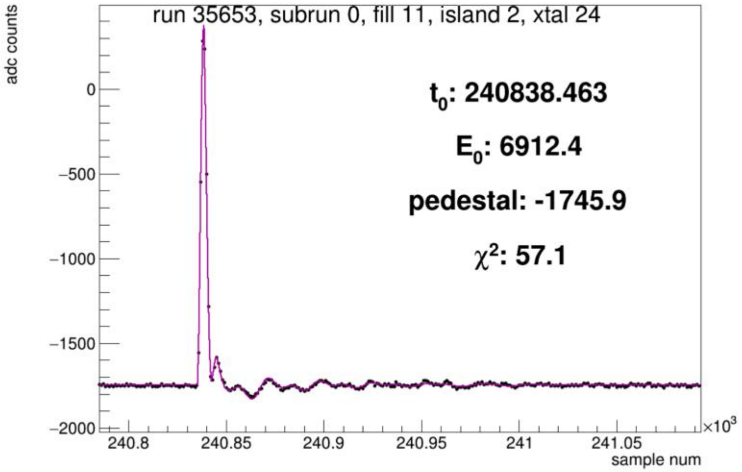


Reconstruction procedure

- 24 calorimeters: **6x9 grid** of **PbF_2 crystals**
- Decay positrons produce **Cherenkov light**
- Light collected by **silicon photomultipliers**
- Signal is **digitized** and then fitted with empirical **template functions**
- Extract **time** and **energy** from fit - (t, E)

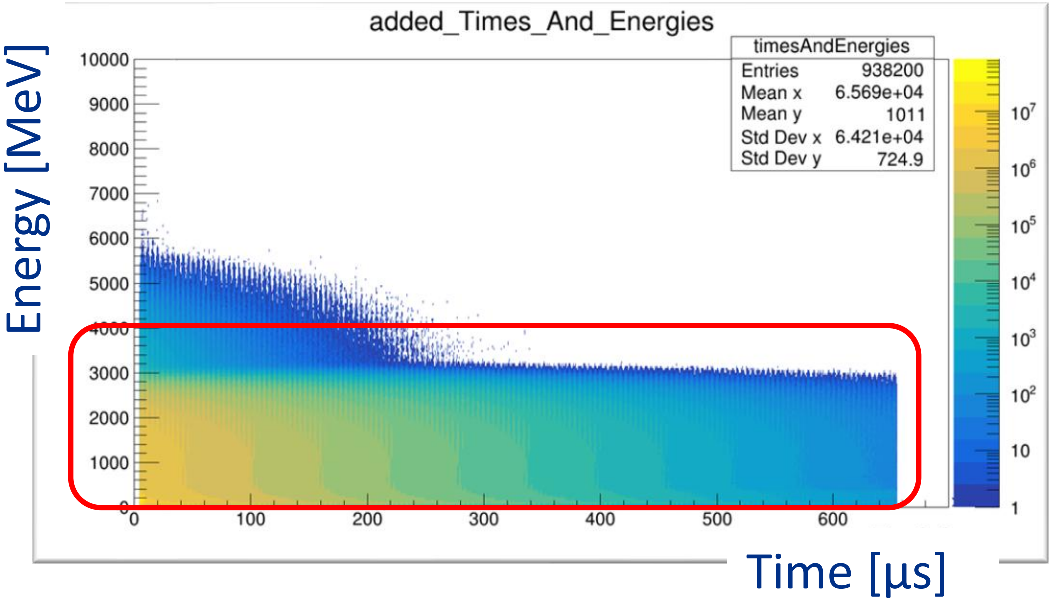






Some collected data!

- 2D histogram of each recorded energy and its time
- g-2 oscillations visible!
- Muon decay visible!
- **Extract precession frequency from time spectrum!**



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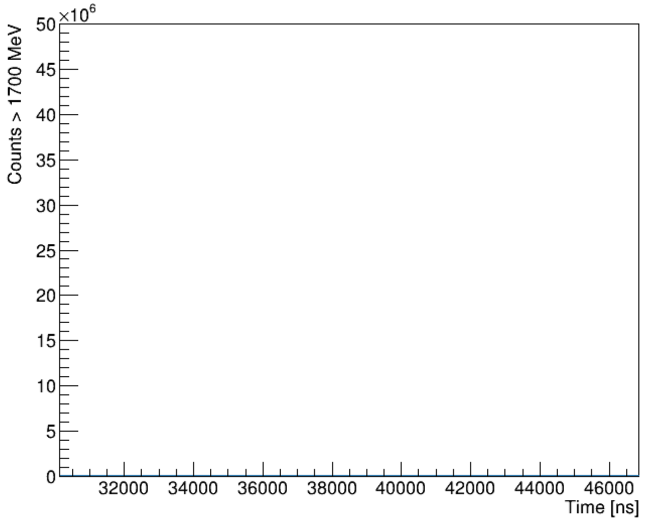
Histogram construction

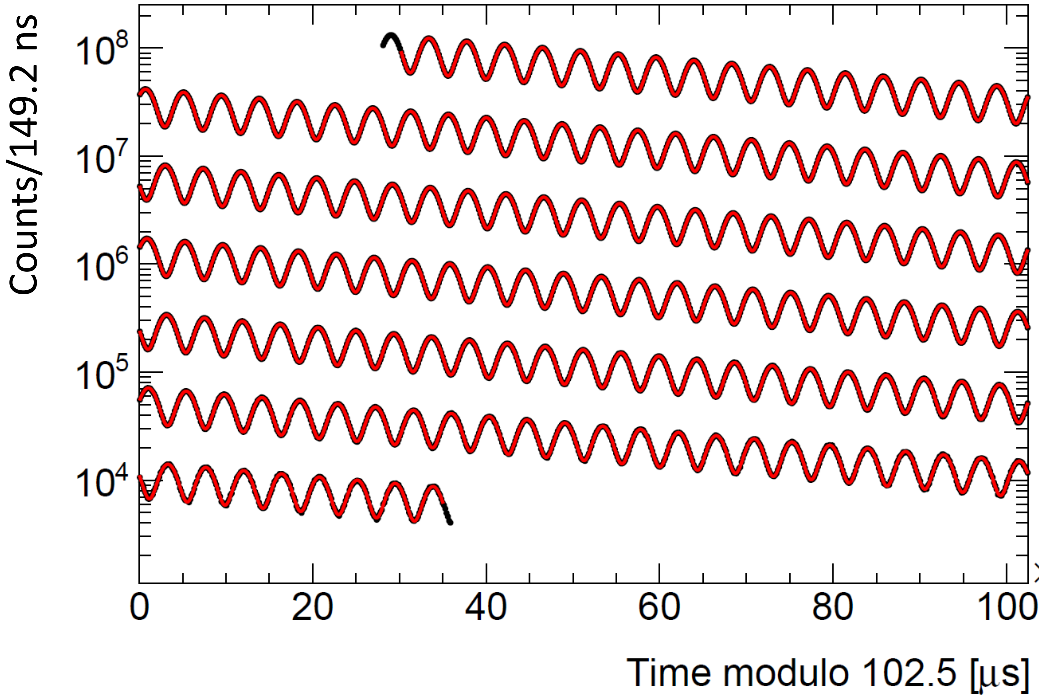
Decay positron energy spectrum vs time



Number of decay positrons with E > 1700 MeV vs time

- e^+ energy spectrum; in weak decay , the number of highest energy e+s oscillates – via **parity violation**!
- Muon **spin rotates** – points towards and away from detectors! – correlated to emission direction
- Maximize oscillation with energy threshold – reduces **statistical uncertainty** on ω_a

Plot the Wiggle!



$$f(t) = N_0 e^{-\frac{t}{\tau_\mu}}(1 + A\cos(\boldsymbol{\omega_a} t - \phi))$$

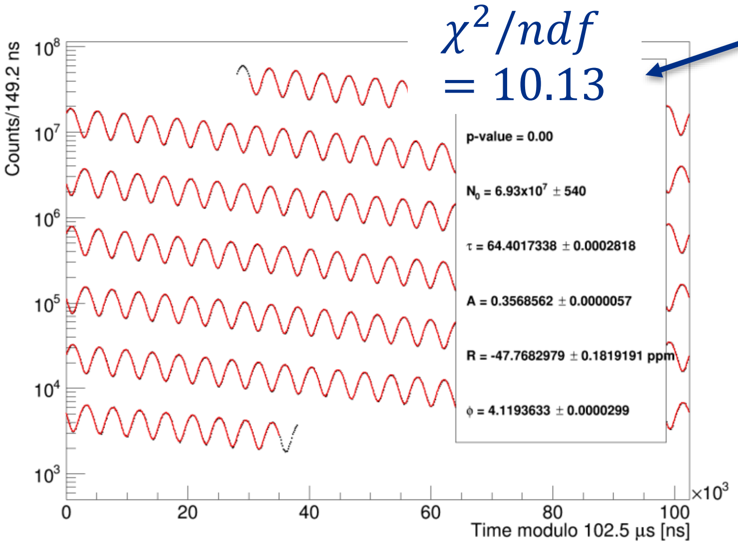
- Basic 5-parameter fit
- Captures two physical processes
  - Muon decay
  - g-2 oscillation
- Are we done?

NO

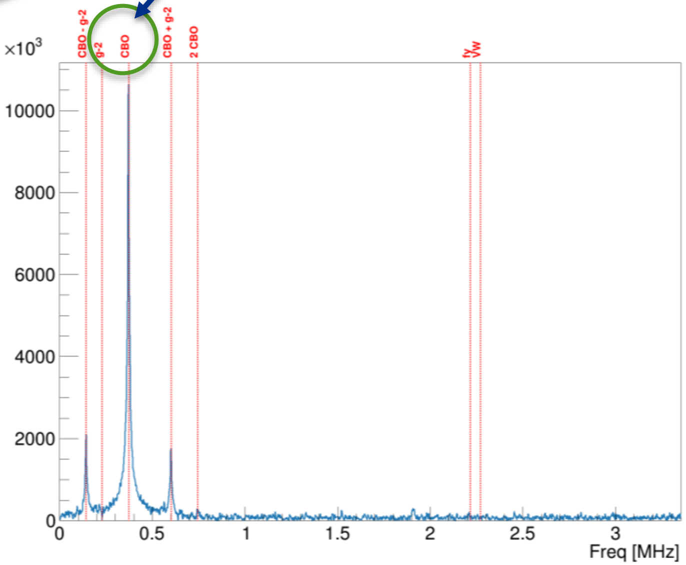
The decay positron time spectrum (the wiggle plot)

How good is the fit?

Problematic!



Simple fit – only muon decay and g-2 oscillations accounted for, fit can be improved



FFT of residuals highlights unaccounted for beam dynamics

Full fit and multiple analyses

- 7 different and independent analysis groups
- Each group has its own fit function
- Each group was blinded with respect to one another
- Equal contribution from each group in final combination
- On right: Example of one group's fit function

$$f_{standard}(t) = N(t)e^{-t/\tau_\mu}\left(1 + A(t)\cos(\omega_a t - \phi(t))\right)$$

Blinding

$$\omega_a = 2\pi \cdot 0.2291\text{MHz}(1 + \mathbf{R} \times 10^{-6})$$

$$N(t) = \boldsymbol{N_0} \cdot \Lambda(t) \cdot N_{cbo}(t) \cdot N_{2-cbo}(t) \cdot f_y(t)$$

$$A(t) = \mathbf{A} \cdot A_{cbo}(t)$$

$$\phi(t) = \boldsymbol{\phi_0} + \phi_{cbo}(t)$$

Muon Losses

$$\Lambda(t) = 1 - \left(\frac{\boldsymbol{\kappa_{loss}}}{1+\boldsymbol{\kappa_{loss}}\epsilon}\right)\int_{t_1}^{t} dt' \; \bar{l}(t')e^{t'/\tau}$$

$$\bar{l}(t) = \frac{l(t)}{e^{t_1/\tau}\int_{t_1}^{\infty} dt' l(t')}$$

$$\epsilon = \int_{t_-}^{t_1} dt'\bar{l}(t')e^{t'/\tau}$$

Beam Dynamics

$$N_{cbo}(t) = 1 + \boldsymbol{A_{cbo-N}}E(t)\cos(\omega_{cbo}(t)t - \boldsymbol{\phi_{cbo-N}})$$

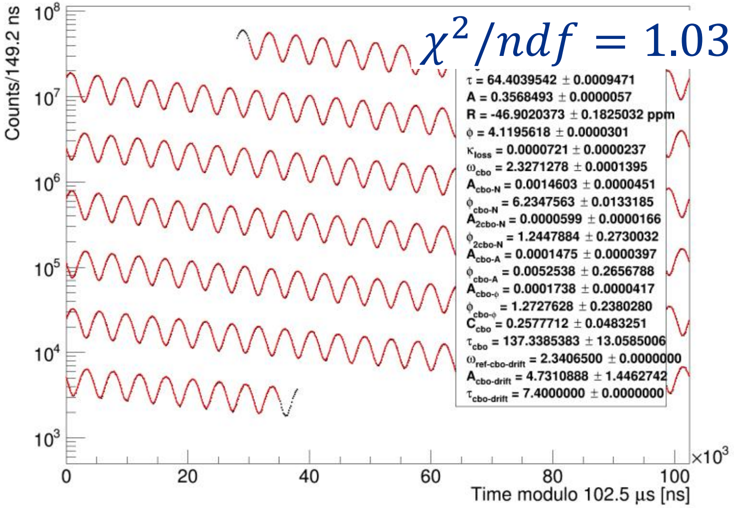
$$N_{2-cbo}(t) = 1 + \boldsymbol{A_{2-cbo-N}}e^{-2t/\boldsymbol{\tau_{cbo}}}\cos(2\omega_{cbo}(t)t - \boldsymbol{\phi_{2-cbo-N}})$$

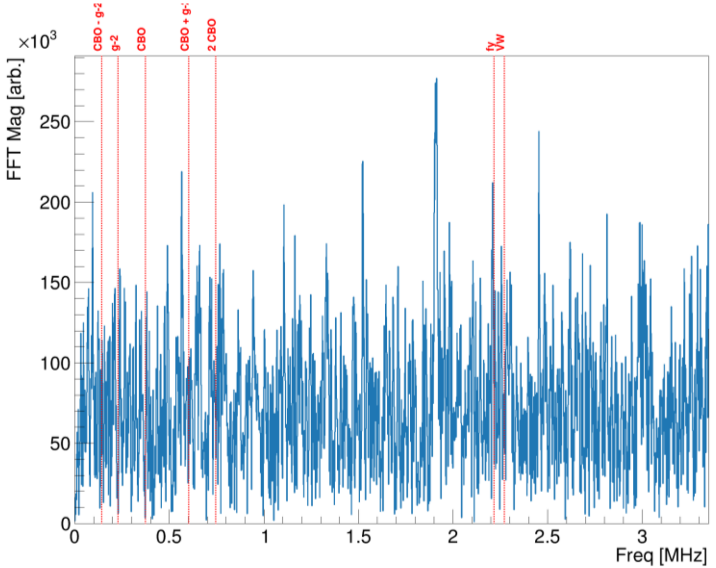
$$\phi_{cbo}(t) = 1 + \boldsymbol{A_{cbo-\phi}}e^{-t/\boldsymbol{\tau_{cbo}}}\cos(\omega_{cbo}(t)t - \boldsymbol{\phi_{cbo-\phi}})$$

$$A_{cbo}(t) = 1 + \boldsymbol{A_{cbo-A}}e^{-t/\boldsymbol{\tau_{cbo}}}\cos(\omega_{cbo}(t)t - \boldsymbol{\phi_{cbo-A}})$$

$$\omega_{cbo}(t) = \boldsymbol{\omega_{cbo}}\left(1 + \frac{\boldsymbol{A_{cbo-drift}}e^{-t/\tau_{cbo-drift}}}{\omega_{ref-freq}t}\right)$$

How good is the fit?





Full fit – muon decay, g-2 oscillations, muon losses, beam dynamics accounted for

Much better residuals – no strong stand-outs for beam dynamics / unaccounted for physical effects

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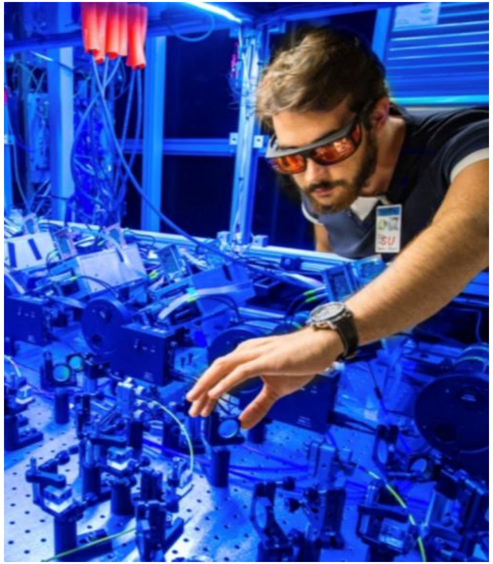
Account for additional effects?

Detector Gain

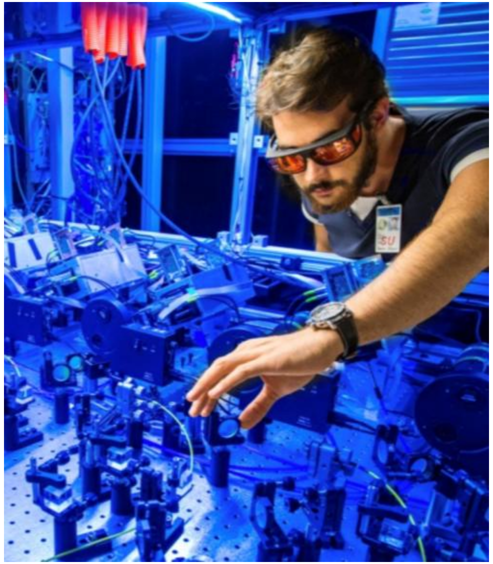
Coherent Betatron motion

Gain Corrections!



- Use a laser calibration system for gain monitoring and stability
- Four gain corrections required!
  - Short-term double pulse correction
  - Intermediate-term double pulse correction
  - In-fill gain correction
  - Out-of-fill gain correction: change in gain from change in temperature - $\mathcal{O}(\mathrm{h})$

Gain Corrections!



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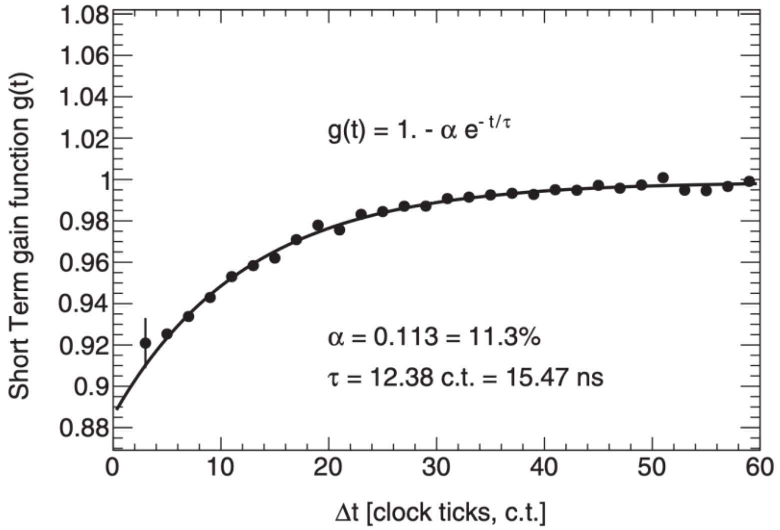
Bias ω_a!

Short-Term Double Pulse Correction

- Change in gain from consecutive hits
- Scale: $\mathcal{O}(\mathrm{ns})$
- Sensitive to pileup
- Accounted for in reconstruction chain
- Send two laser pulses at SiPM
  - One sags the gain
  - Second probes size of gain
- **Size: <2 ppb on $\boldsymbol{\omega_a}$**

Short-term double pulse



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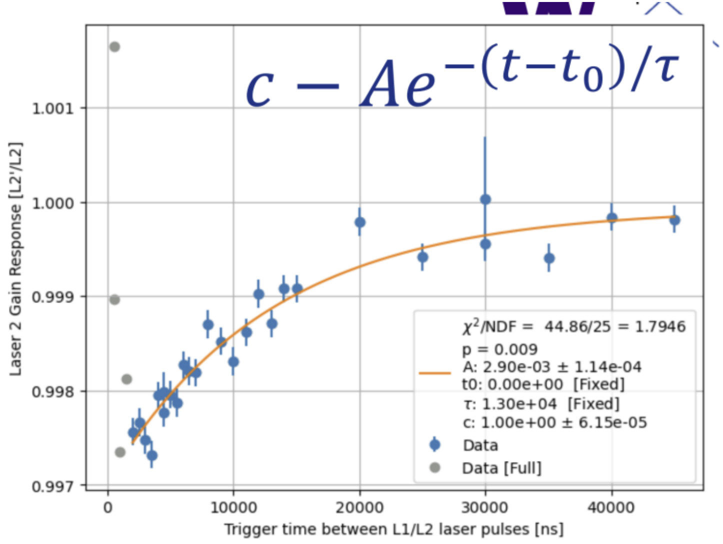
Intermediate-Term Double Pulse Correction

- Change in gain from hit rate
- Scale: $\mathcal{O}(\mu s)$
- Applied as a histogram level correction immediately before the fitting stage, post-reconstruction
- Strongly contributes to a “slow effect”, low frequency residual if not properly accounted for
- **Size: 5-20 ppb on $\boldsymbol{\omega_a}$**

Intermediate-term double pulse

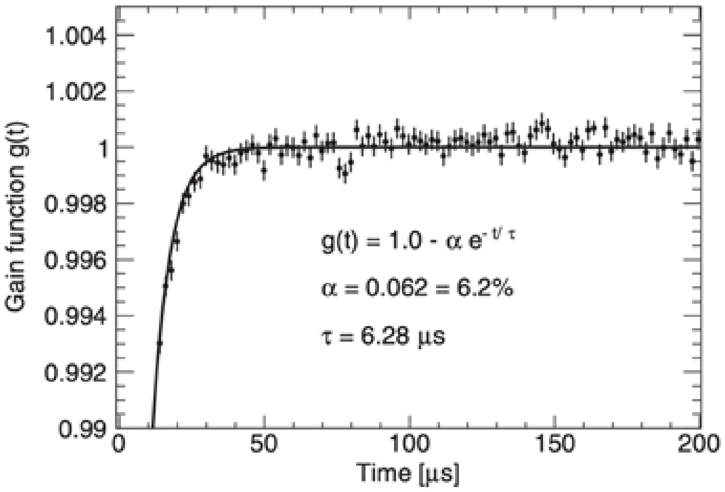


Via Josh LaBounty

In-Fill Gain Correction

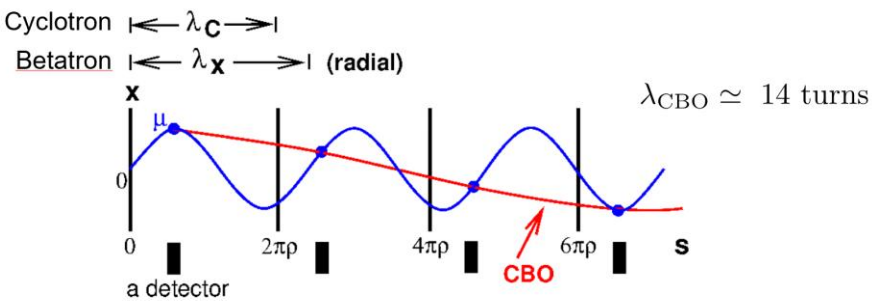
- Change in gain (gain sag) from the beam "flash"
- Scale: $\mathcal{O}(\mu s)$
- Accounted for in the reconstruction chain
- Send in-fill laser shots at the same time as the muon beam in some of the fills
  - Measure the laser energy detected by the SiPMs
- **Size: <5 ppb on ω_a**

In-fill Gain



Archnemesis: The Coherent Betatron Oscillations

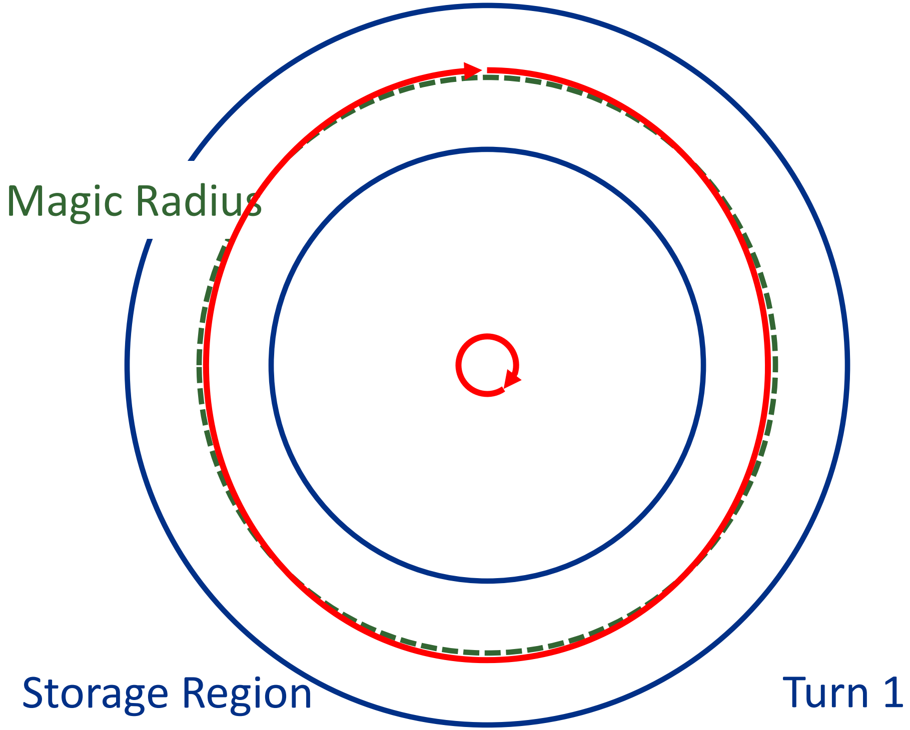
- Observed signal due to radial betatron oscillations of the beam
- **Dependence**: injection process, strength and phase of the kicker pulse
- CBO frequency around 370 kHz (2.7 μs period) → decoheres after 200 μs on average
- Affects e+ acceptance
  - Acceptance vs R and azimuth to detector



$$\omega_{CBO} = \omega_C - \omega_x$$

Sample radial CBO at the cyclotron frequency; each detector is only at a single location around the storage ring

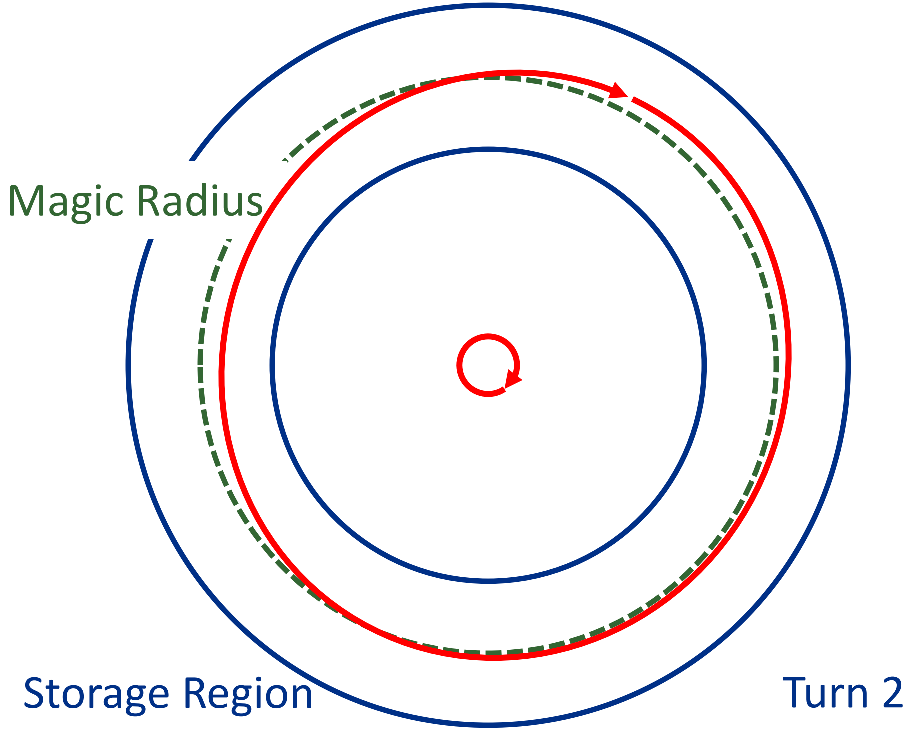
Beam oscillations modeling: Coherent Betatron Oscillations



Via James Mott

- The center of the orbit precesses at betatron frequency ω_x
- Coherent movement of muon orbit
- Effect decoheres with time – distribution of cyclotron frequencies among injected muons

Beam oscillations modeling: CBO



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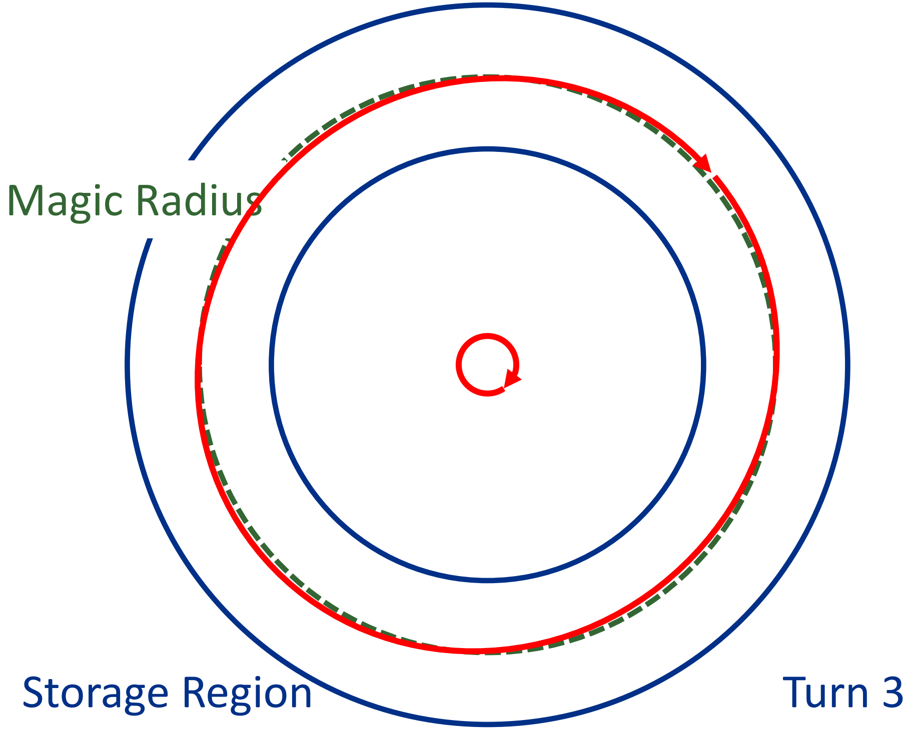

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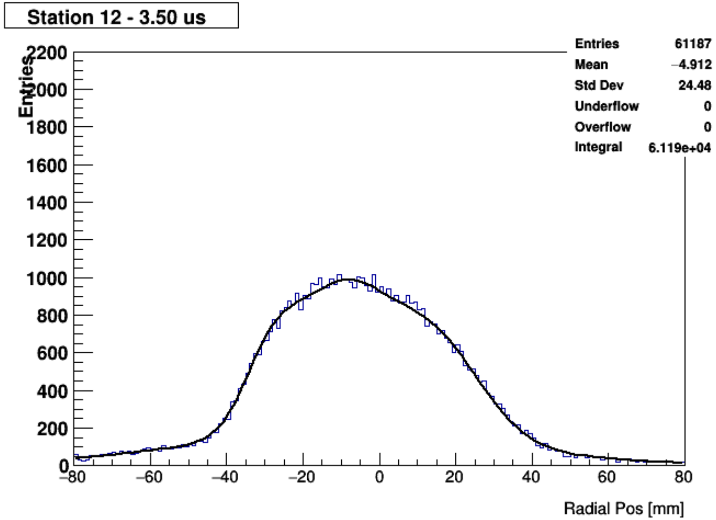
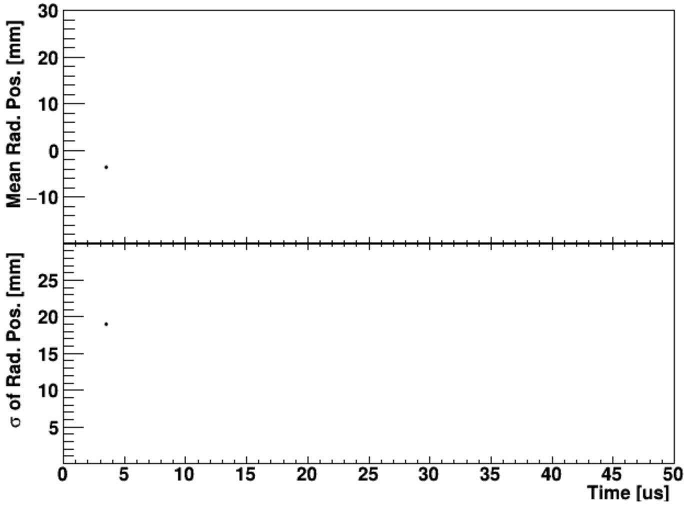
CBO Visualized





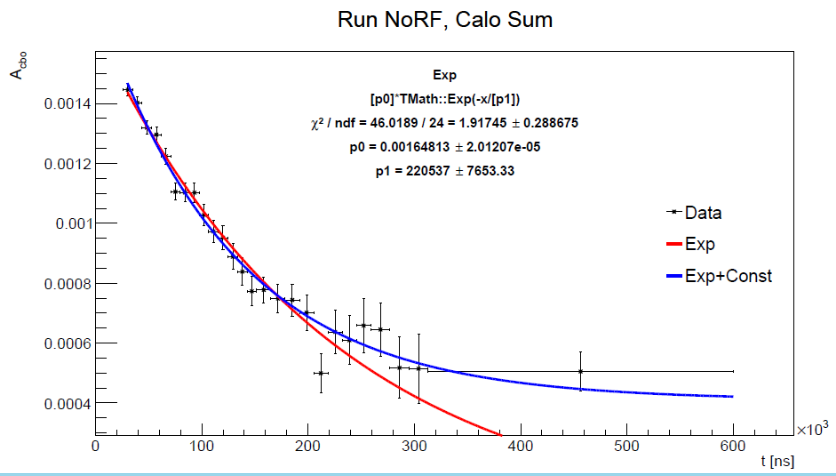
- **Affects the acceptance of the calorimeters!**
- **Calorimeters have finite transverse coverage – finite acceptance**
- **Each detector can only sample muon beam at a particular phase of the cyclotron period**

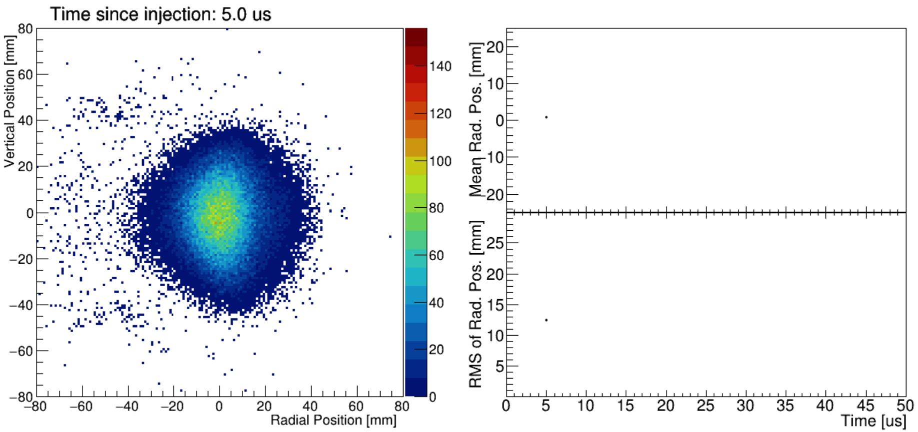
Handling the CBO?

- Expand the fit model!

$$N \text{ particles} \rightarrow N_0 \cdot N_{CBO}(t)$$
$$N_{CBO}(t) = 1 + (\text{CBO envelope}) A_{cbo} \cos(\omega_{cbo}(t)t - \phi_{cbo}(t))$$

- Evaluate systematic uncertainties from fit model choices!
- Can’t ignore CBO; $\boldsymbol{\omega_a}$ **differs by ~800 ppb**! ~80 for Run 5 & 6





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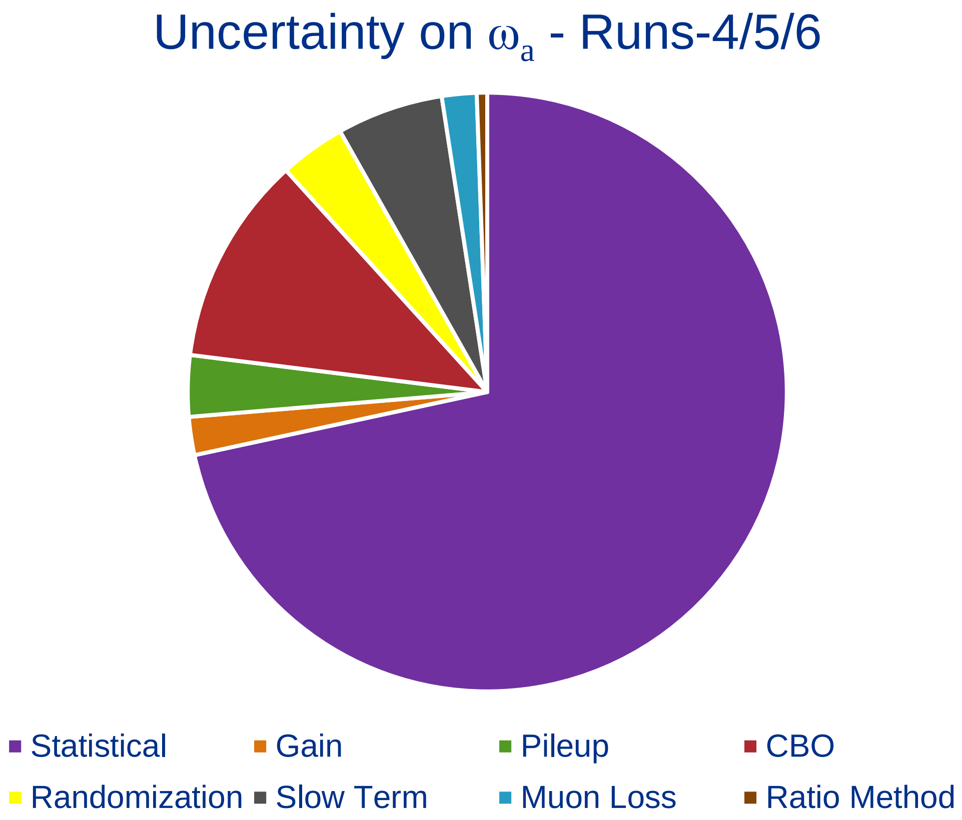
Total ω_a Uncertainties Runs-4/5/6

- Statistical uncertainty: 114.8 [ppb]
- Systematic uncertainty: 30 [ppb]
- Total uncertainty ω_a: 118.6 [ppb]

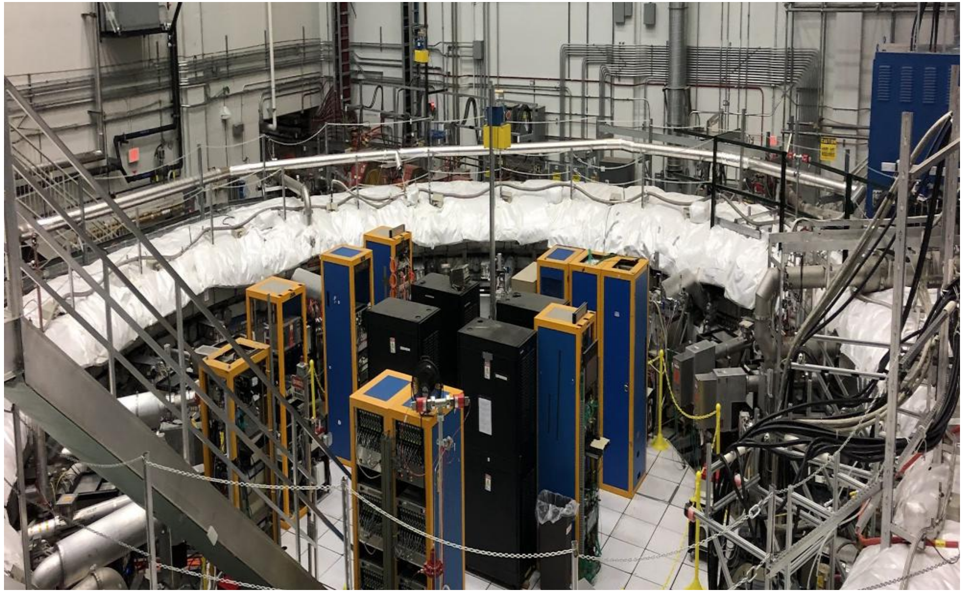
Still statistics dominated!



Many other effects and parts of the analysis!

- Muon losses
- Pileup
- Corrections to ω_a
- Inputs from the tracker and other detectors
- Other storage ring components: kicker, inflector
- Systematic uncertainties

Thank you for listening!

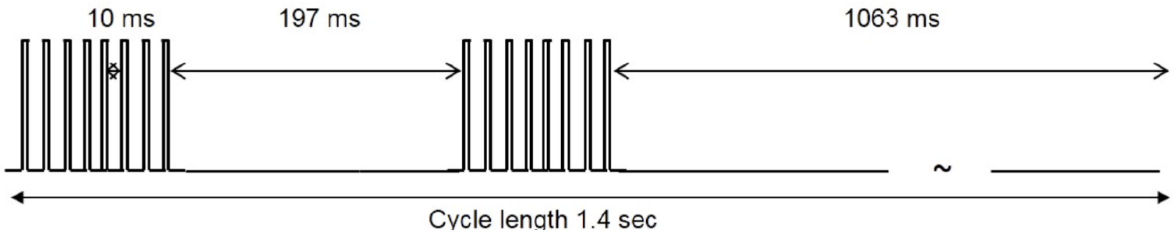


The g-2 ring, photo taken by me (left), selfie taken by me (right)

Backup

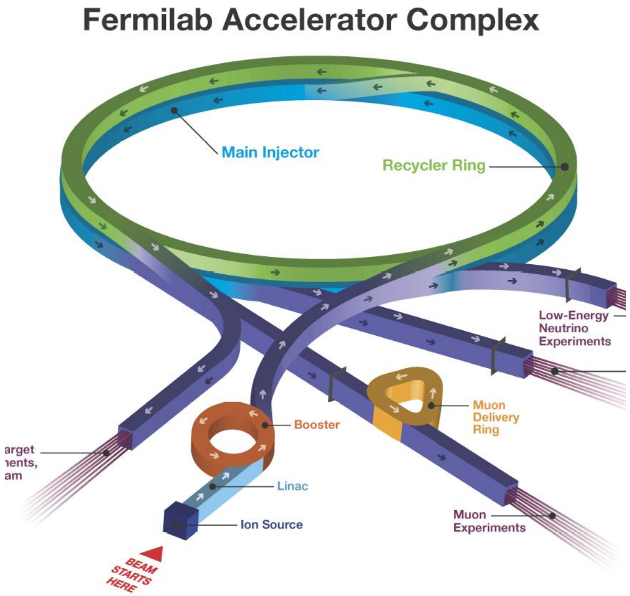
Back to the beginning

- Fermilab accelerator complex produces
  - 1 batch = 4×10^{12} protons at 8 GeV/c
  - 1 bunch = 1×10^{12} protons at 8 GeV/c
- Every 1.4s (accelerator super cycle) – 11.4 Hz rate w/ 16 bunches
  - Two sets of 8 bunches, each bunch 120 ns wide, 10 ms between each bunch, and 197 ms between first set of 8 bunches and the second set of 8 bunches
  - Corresponds to 11.4×10^{12} protons/s



Pion and muon production

- Inconel target:
  - $1 \times 10^{-5}\ \pi^+/POT$
  - Produces 3.1 GeV/c beam, with 10% momentum spread
- Muon beam
  - Can extract 3.094 GeV/c muons from decay of pions
  - Not entirely pure; contains muons, pions, protons, positrons
  - **Protons** lag ~200 ns (higher mass) and are ejected out of beam
  - **Pions** are circulated until decay
  - **Positrons** lost via synchrotron radiation within $4\ \mu s$ after beam injection
  - Muon beam is ~95% polarized

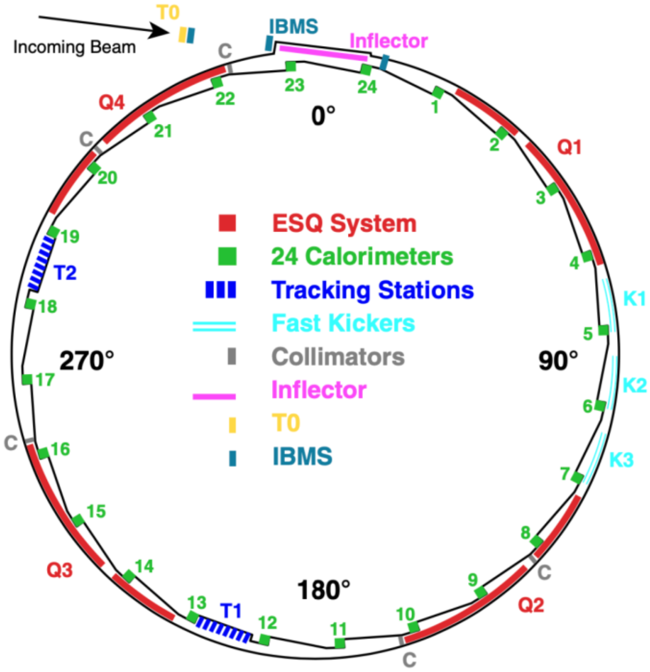


Muon beam injection

- Beam state at injection
  - 3.094 GeV/c with 2% momentum spread, about 120ns wide
  - Injection efficiency $\sim 2 \times 10^{-7} \mu/POT$
  - Observed for $700\ \mu s$ post injection
- Stored muons have max beam radius of $45 \pm 0.5\%$ mm (inner radius of collimators), orbit radius of ~ 7.112m
- 1 bunch corresponds to 1 fill
  - 1 fill typically contains $\boldsymbol{\mathcal{O}(10^4)}$ **muons** when in the storage ring
  - Corresponds to ~500 detected positrons; has *decreased* to ~350 detected positrons in later runs

In the ring

- Injected muons
- Boost factor $\gamma = 29.3$
- Boosted lifetime $64.44\ \mu s$
- Average 400 rotations around the ring before decay
- Storage ring average magnetic field $B = 1.451\ T$

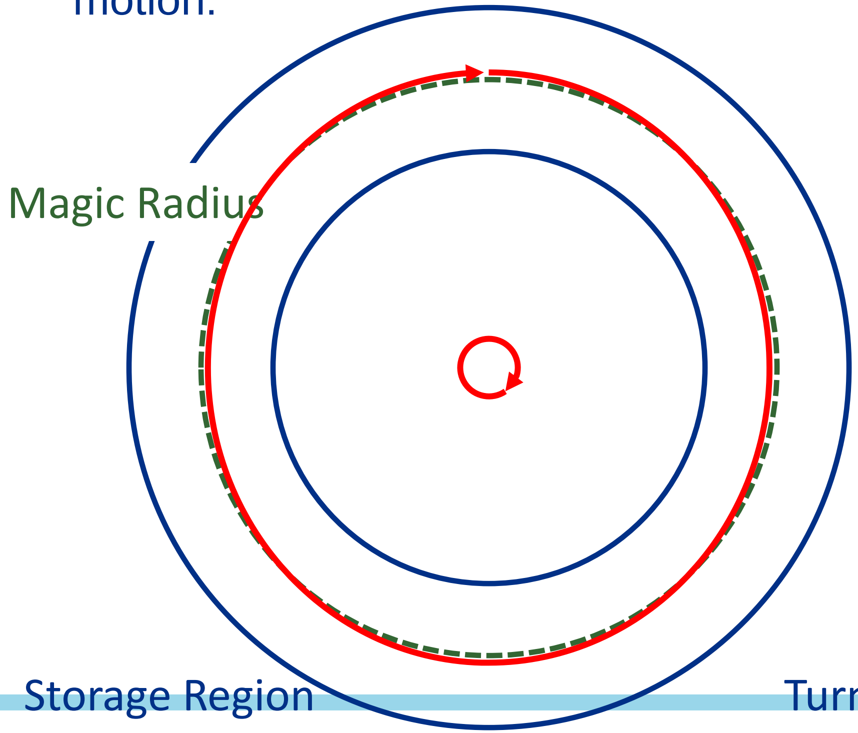


Beam Dynamics

Muon Beam Frequencies					
Name	Symbol	Expression	Frequency [MHz]	Frequency [rad/μs]	Period [ns]
$g-2$	f_a	$a_\mu Be/2\pi mc$	0.229	1.439	4365
cyclotron	f_c	$v/2\pi R_0$	6.702	42.113	149.2
horizontal betatron	f_{xBO}	$\sqrt{1-n} f_c$	6.330	39.772	158.0
vertical betatron	f_{yBO}	$\sqrt{n} f_c$	2.203	13.845	453.8
coherent betatron	f_{cbo}	$f_c - f_{xBO}$	0.373	2.341	2684
vertical waist	f_{VW}	$f_c - 2f_{yBO}$	2.295	14.423	436.6

Betatron Oscillations: Radial Case with Quads

- Quad fields keep beam out longer at high/low radii which distorts motion:



Centre point of orbit precesses at betatron frequency

Makes oscillation march around the ring with period slightly longer than cyclotron period

Turn 1

Betatron Oscillations: Radial Case with Quads

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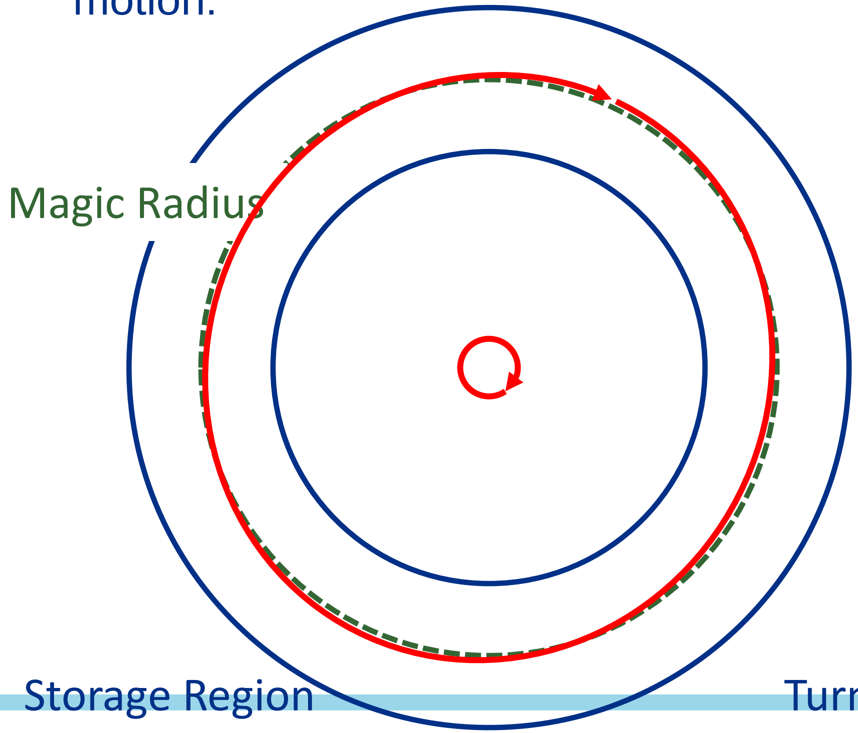

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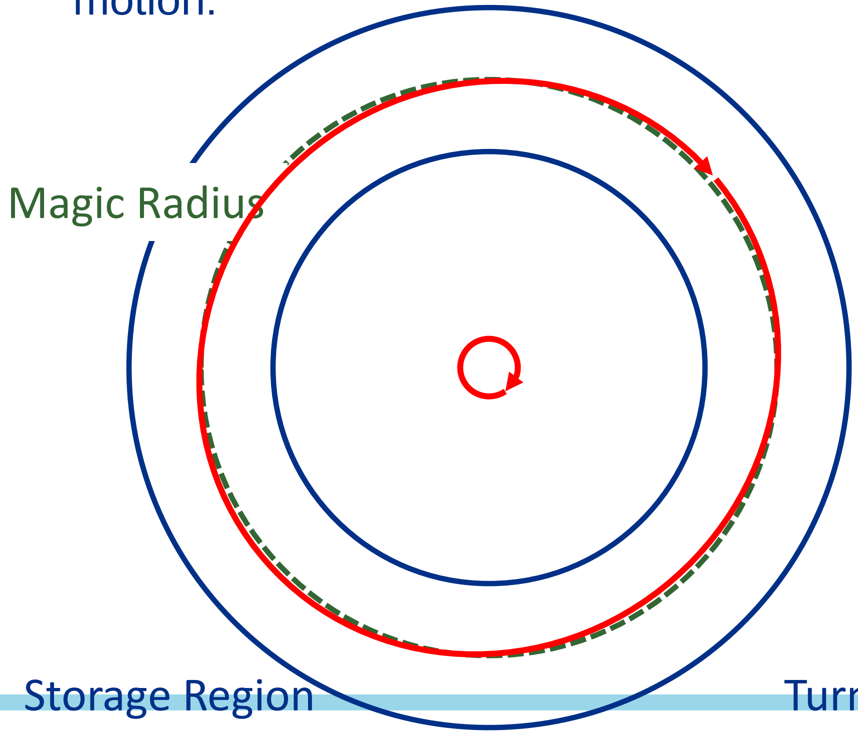

Centre point of orbit precesses at betatron frequency

Makes oscillation march around the ring with period slightly longer than cyclotron period