On the sensitivity of nuclear clocks to new physics

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[Caputo, Gazit, Hammer, Kopp, GP, Perez, Springmann arXiv:2407.17526 (hep-ph), to apper in Phys. Rev. C.]

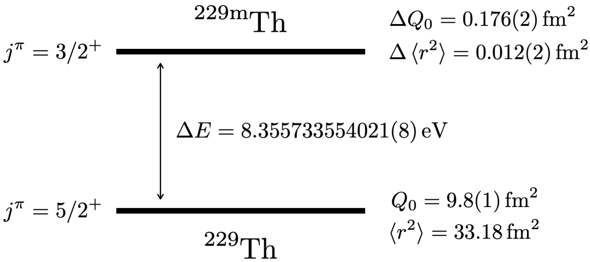
Outline

- Nuclear clock and new physics
- Estimating nuclear clock's new physics sensitivity: classical model
- Estimating nuclear clock's new physics sensitivity: quantum model
- Conclusions

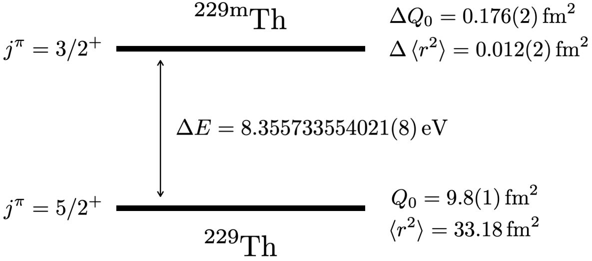
Nuclear clock and new physics

Nuclear clock

- Atomic clock such as $^{171}\mathrm{Yb}^+$ has a $\sim 10^{-18}$ uncertainty
  [Huntemann et al. PRL **116** (2016) 063001]
- Can we do better than an atomic clock? Yes!
  Nuclear clock can reach $\sim 10^{-19}$ uncertainty
  [Campbell et al. PRL **108** (2012) 120802]
- Beyond precision goals, this is a *new* kind of technology
- The candidate is $^{229}\mathrm{Th}$ that has two states (isomers) with $\Delta E \sim 8$ eV



Nuclear clock



- Laser excitation of this transition was reported last year
  [Tiedau et al. PRL **132** (2024) 182501]
  [Elwell et al. PRL **133** (2024) 013201]
  [Zhang et al. Nature **633** (2024) 63-70]

- Fun fact: Need to use *exact* values of $\hbar$ and e for such precision

- Such a small ΔE is assumed to be an *accidental* cancelation between electromagnetic (ΔE_{em}) and nuclear (ΔE_{nuc}) nuclear energies

Nuclear clock

- Such a small ΔE is assumed to be an *accidental* cancelation between electromagnetic ($\Delta E_{\rm em}$) and nuclear ($\Delta E_{\rm nuc}$) nuclear energies

- How can nuclear clock help with searching for new physics?

- Assume $\alpha_{\rm em}$ is changing with time

$$\frac{\delta(\Delta E)}{\Delta E} = \frac{1}{\Delta E}\left(\frac{\partial \Delta E_{\rm em}}{\partial \alpha_{\rm em}}\delta\alpha_{\rm em} + \frac{\partial \Delta E_{\rm nuc}}{\partial \alpha_{\rm em}}\delta\alpha_{\rm em}\right) = \frac{1}{\Delta E}\frac{\partial \Delta E_{\rm em}}{\partial \alpha_{\rm em}}\alpha_{\rm em}\frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}}$$

- Since $\Delta E_{\rm em}$ is linear in $\alpha_{\rm em}$ (up to $\alpha_{\rm em}^2$ corrections)

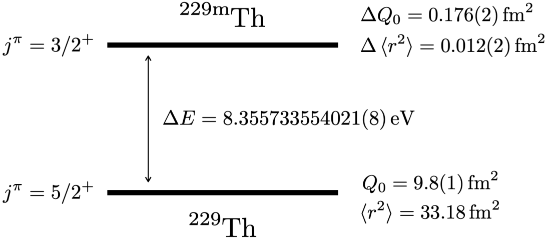
$$\frac{\delta(\Delta E)}{\Delta E} = \frac{1}{\Delta E}\frac{\partial \Delta E_{\rm em}}{\partial \alpha_{\rm em}}\alpha_{\rm em}\frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} \simeq \frac{\Delta E_{\rm em}}{\Delta E}\frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} \approx \frac{\text{MeV}}{10\ \text{eV}}\frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} \approx 10^5\frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}}$$

- Nuclear clock has enhanced sensitivity to $\alpha_{\rm em}$ time variation arising, e.g., from dark matter

Nuclear clock

$$\frac{\delta(\Delta E)}{\Delta E} \simeq \frac{\Delta E_{\rm em}}{\Delta E}\frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}} \equiv K\frac{\delta\alpha_{\rm em}}{\alpha_{\rm em}}$$

- Nuclear clock has enhanced sensitivity to $\alpha_{\rm em}$ time variation
- We need to estimate $\Delta E_{\rm em}$ to find the enhancement factor K
- What do we know about the EM properties of the nucleus?



- Apart from the charge radius $\langle r^2\rangle \equiv \int d^3\mathbf{r}\, r^2\rho(r,\theta)/e\,Z$, the quadrupole moment $Q_0 \equiv \int d^3\mathbf{r}\, r^2\rho(r,\theta)\left[3\cos^2(\theta)-1\right]/e$ can help estimate $\Delta E_{\rm em}$

Nuclear clock

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- We estimated $\Delta E_{\rm em}$ in [Caputo, Gazit, Hammer, Kopp, GP, Perez, Springmann arXiv:2407.17526 (hep-ph)]

  In particular we asked can $K = \Delta E_{\rm em}/\Delta E$ be zero?

- Used two methods

- A classical "geometric" model
- A quantum "halo" model

Estimating nuclear clock's sensitivity: classical model

Estimating $\Delta E_{\rm em}$

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  In particular we asked can $K = \Delta E_{\rm em}/\Delta E$ be zero?

- First method: classical "geometric" model
- Assume Woods–Saxon like distribution

$$\rho(r,\theta) = \frac{\rho_0}{1+\exp\left(\frac{r-R(\theta)}{z}\right)},$$

  where z is the "surface thickness" of the nucleus and

$$R(\theta) = R_0\left[1+\beta_2 Y_{20}(\theta)+\beta_3 Y_{30}(\theta)+\beta_4 Y_{40}(\theta)+\dots\right],$$

  R_0 is fixed by the charge radius, β_2 by the quadrupole moment

- The energy is

$$E_{\rm em} \simeq E_{\rm C}[\langle r^2\rangle, Q_0, z, \beta_3, \beta_4] = \frac{1}{2}\int d^3\mathbf{r}\, d^3\mathbf{r}'\, \frac{\rho(r,\theta)\,\rho(r',\theta')}{|\mathbf{r}-\mathbf{r}'|}$$

Classical "geometric" model

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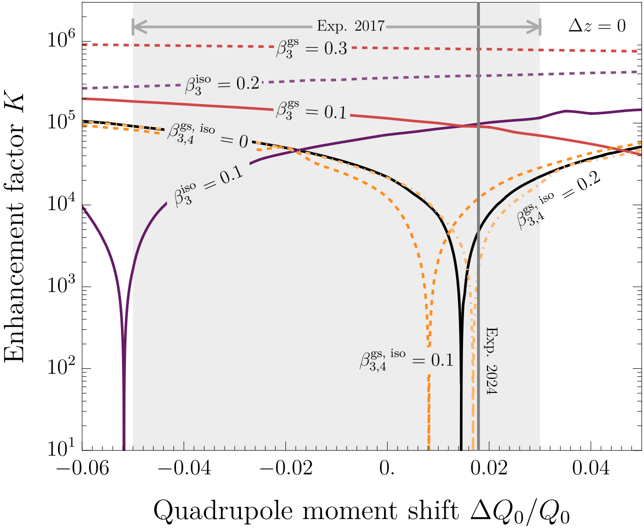
- Previous study assumed constant nuclear volume correlating R_0 &β_2 [Fadeev, Berengut, Flambaum, PRA **102** 052833 (2020)]
- We do not make such assumptions, add β_4 and "scan" over the values of β_3 and β_4 to find the energy

$$E_{\text{em}} \simeq E_{\rm C}[\langle r^2\rangle, Q_0, z, \beta_3, \beta_4] = \frac{1}{2}\int d^3\mathbf{r}\, d^3\mathbf{r}'\, \frac{\rho(r,\theta)\,\rho(r',\theta')}{|\mathbf{r}-\mathbf{r}'|}$$

Estimating $\Delta E_{\rm em}$ using a classical geometric model

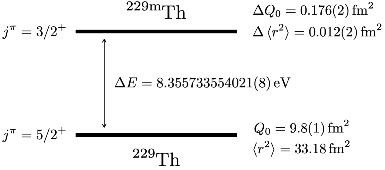
- Using the classical geometric model



- $K = 0$ is unlikley

Estimating nuclear clock's sensitivity: quantum model

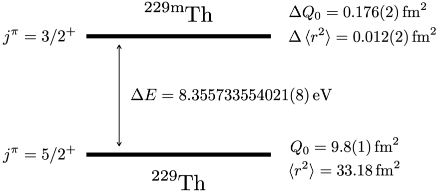
Halo picture of ^{229}Th



- Consider ground state ($J = 5/2$) and the isomer ($J = 3/2$) as $L = 2$ "halo" neutron around a spin-0 ^{228}Th "core"
  The difference between the two states is the neutron spin
- In principle for halo nuclei can use halo EFT
  [Hammer, Ji, Phillips, J. Phys. G **44**, 103002 (2017)]
- Two problems

1) Scales not well-separated: Neutron separation energy is 5.2 MeV for ^{229}Th vs. 7.1 MeV for ^{228}Th
   Compare to ^{19}C$\approx$ ^{18}C+halo neutron. Neutron separation energy is 0.5 MeV for ^{19}C vs. 4 MeV for ^{18}C
2) $L = 2$ halo EFT less predictive
   leading-order dependence of most observables on counter-terms

Estimating $\Delta E_{\rm em}$ using a halo model



- Consider ground state ($J = 5/2$) and the isomer ($J = 3/2$) as $L = 2$ "halo" neutron around a spin-0 ^{228}Th "core"
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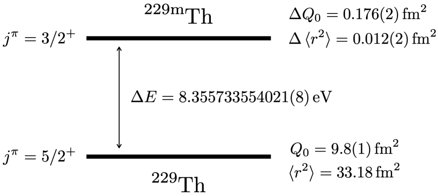
- While EFT description is difficult it inspires a "halo model"
  Same charge densities for the states apart from spin–orbit interaction

- To test the model we calculate
- $\Delta\langle r^2\rangle_{\rm SO} = 0.0047\ \rm fm^2$ compared to $\Delta\langle r^2\rangle = 0.012(2)\ \rm fm^2$
- $\Delta\langle Q_0\rangle_{\rm SO} = 0.185\ \rm fm^2$ compared to $\Delta\langle Q_0\rangle = 0.176(2)\ \rm fm^2$
  Both are *insensitive* to neutron wave-function near the origin ("UV")

Estimating $\Delta E_{\rm em}$ using a halo model



- Consider ground state ($J = 5/2$) and the isomer ($J = 3/2$) as $L = 2$ "halo" neutron around a spin-0 ^{228}Th "core"
  The difference between the two states is the neutron spin

- While EFT description is difficult it inspires a "halo model"
  Same charge densities for the states apart from spin–orbit interaction

- Calculating the energy difference

$$\Delta E_{\rm SO} \approx 144\,{\rm keV} \times \mathcal{O}(1) \text{ factor}$$

  $\mathcal{O}(1)$ factor arises from neutron wave-function near the origin
- $K = \Delta E_{\rm SO}/\Delta E \approx 10^4$ from halo model

Conclusions

Conclusions

- Nuclear clock will be a a *new* kind of technology
  Laser excitation of the required transition was reported last year

- Nuclear clock has enhanced sensitivity to $\alpha_{\rm em}$ time variation arising, e.g., from dark matter. Sensitivity depends on $K = \Delta E_{\rm em}/\Delta E$

- We estimated $\Delta E_{\rm em}$ in [Caputo, Gazit, Hammer, Kopp, GP, Perez, Springmann arXiv:2407.17526 (hep-ph)]

- Used two methods
  - A classical "geometric" model: $K = 0$ unlikely
  - A quantum "halo" model: $K \approx 10^4$

- Nuclear clock very likely sensitive to new physics

- This is just the beginning. Field is advancing fast. More work to do!

Backup

Halo model calculation details

- $E_{\rm em} \sim$ MeV $\ll$ neutron mass $\Rightarrow$ use non-relativistic expansion to find the spin-dependent part of charge density $\rho_{\rm SO}$
- $\rho_{\rm SO}$ at lowest order in non-relativistic expansion [Manohar PRD **56** 230 (1997)]

$$\rho_{\rm SO} = \frac{\mu_n}{2m_n^2} i\boldsymbol{\sigma}\cdot\left(\boldsymbol{p}'\times\boldsymbol{p}\right) \to \frac{\mu_n}{2m_n^2} i\boldsymbol{\sigma}\cdot(\boldsymbol{\nabla}\boldsymbol{\Phi}^\dagger)\times(\boldsymbol{\nabla}\boldsymbol{\Phi})$$

[Ong, Berengut, Flambaum PRC **82** 014320 (2010)]

Φ is neutron wave function and μ_n neutrino magnetic moment

- Both
- charge radius squared $\langle r^2\rangle \equiv \int d^3\mathbf{r}\, r^2\rho(r,\theta)/e\,Z$, [Ong, Berengut, Flambaum PRC **82** 014320 (2010)]
- quadrupole moment $Q_0 \equiv \int d^3\mathbf{r}\, r^2\rho(r,\theta)\left[3\cos^2(\theta)-1\right]/e$ [Caputo, Gazit, Hammer, Kopp, GP, Perez, Springmann arXiv:2407.17526 (hep-ph)]

are related to the normalization of Φ and don't require its form

- $\Delta\langle r^2\rangle_{\rm SO} = 0.0047$ fm^2 compared to $\Delta\langle r^2\rangle_{\rm Exp} = 0.012(2)$ fm^2
- $\Delta\langle Q_0\rangle_{\rm SO} = 0.185$ fm^2 compared to $\Delta\langle Q_0\rangle_{\rm Exp} = 0.176(2)$ fm^2

Halo model calculation details

- The energy is [Sakurai, Modern Quantum Mechanics (2nd Edition)]

$$E_{\rm SO} = \frac{e\,\mu_n}{2m_n^2}\frac{1}{2}\left[j(j+1) - \ell(\ell+1) - \frac{3}{4}\right]\int_0^\infty \left[\frac{[u(r)]^2}{r}\frac{dV_{\rm C}}{dr}\right] dr$$

  - $\ell = 2$ and $j = 5/2$ (ground state) or $3/2$ (isomer)
  - $V_{\rm C}(r)$ Coulomb potential of Th^{228} core
- $u(r)$ is the "excess" neutron's radial d-wave function:

$$u(r) = A(r)e^{-\gamma r}\left(1 + \frac{3}{\gamma r} + \frac{3}{(\gamma r)^2}\right)$$

[Zelevinsky, Volya, Physics of Atomic Nuclei, Wiley-VCH, 2017]

  - $\gamma \equiv \sqrt{2m_n E_B}$, $E_B = 5.2$ MeV is the binding energy
  - $A(r)$ is determined by short-range physics
- Calculating the energy difference

$$\Delta E_{\rm SO} \approx 144\,\text{keV} \times \mathcal{O}(1) \text{ factor}$$

$\mathcal{O}(1)$ factor arises from neutron wave-function near the origin