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in Nuclear Science

Gluon Saturation: Status and Future Perspectives

Andrey Tarasov

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Deep inelastic scattering (DIS)

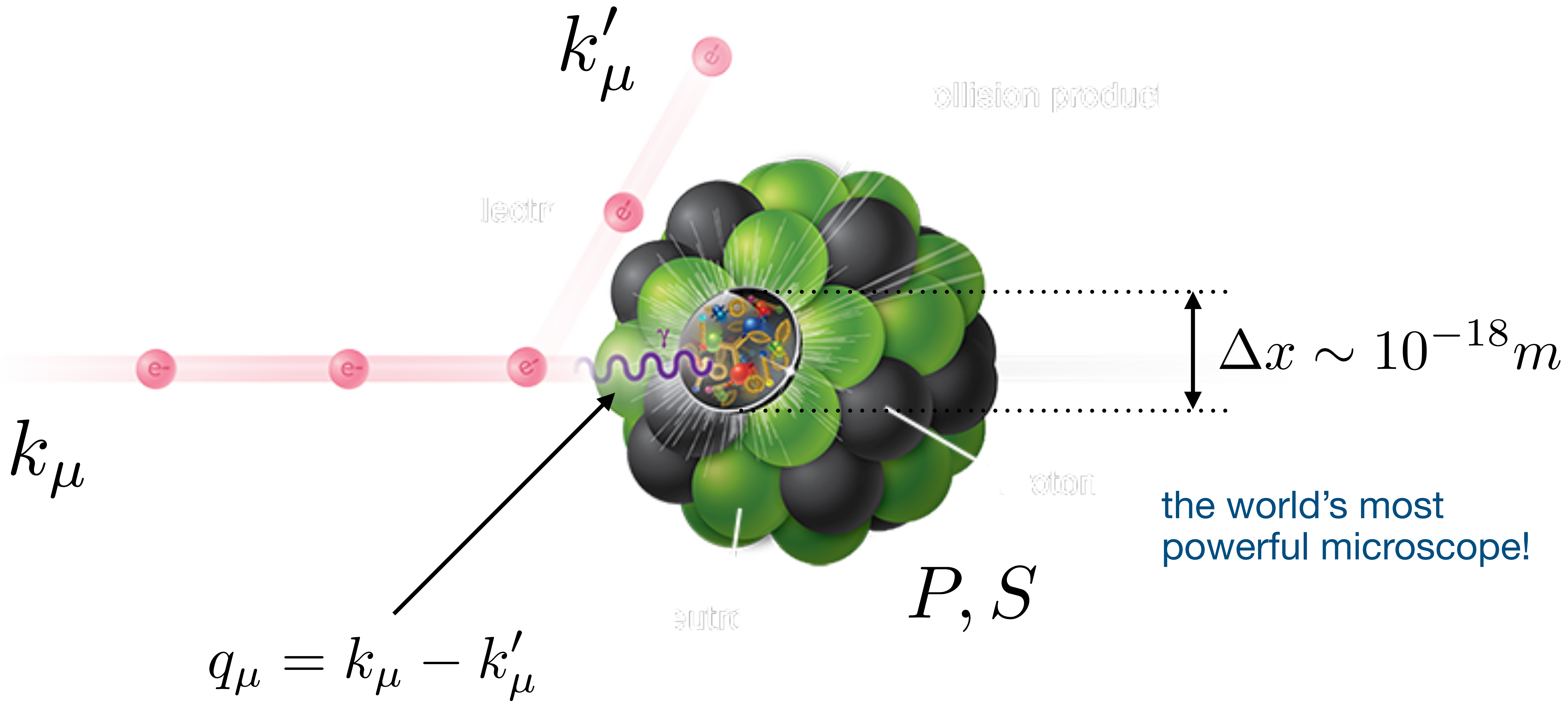
$$e(l) + N(P,S) \to e(l') + X$$

The process is characterized by its virtuality - measure of the resolution power

$$Q^2 = -q^2 \qquad \Delta x \sim \frac{1}{Q}$$

and Bjorken variable - measure of the momentum fraction of a struck parton

$$x_B = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{s + Q^2 - M^2}$$

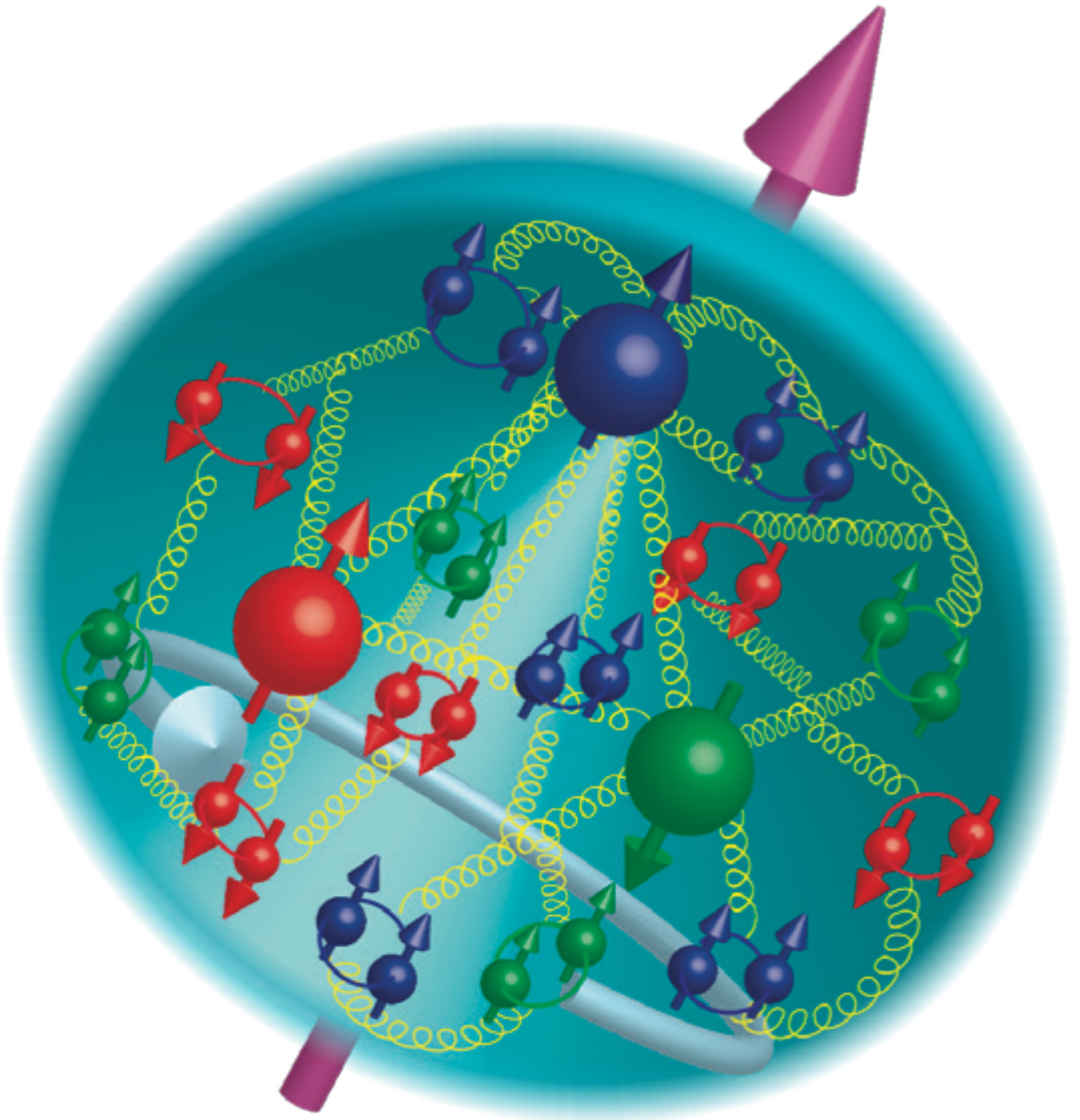


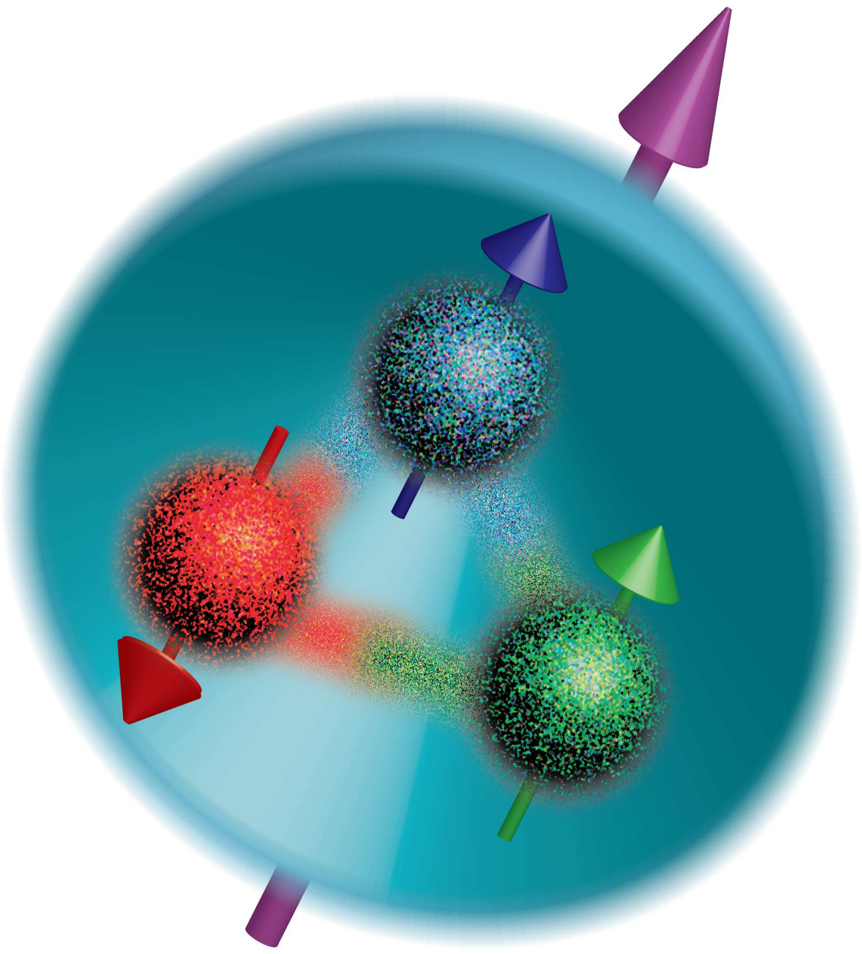
the world's most powerful microscope!

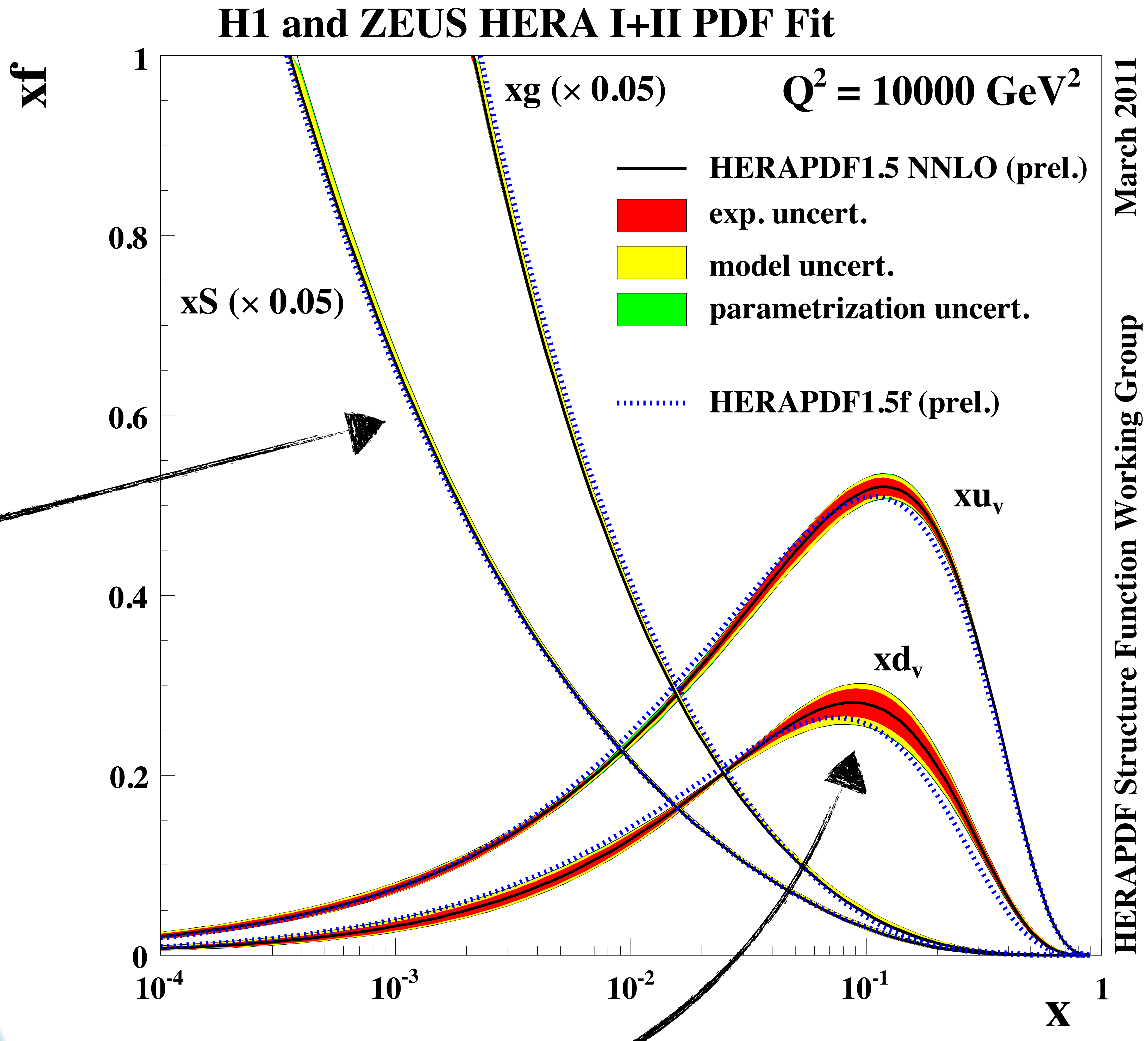
In the Regge limit (high-energy limit) of $s \gg Q^2$ it follows that $x_B \ll 1$

small-x physics = QCD at high energy

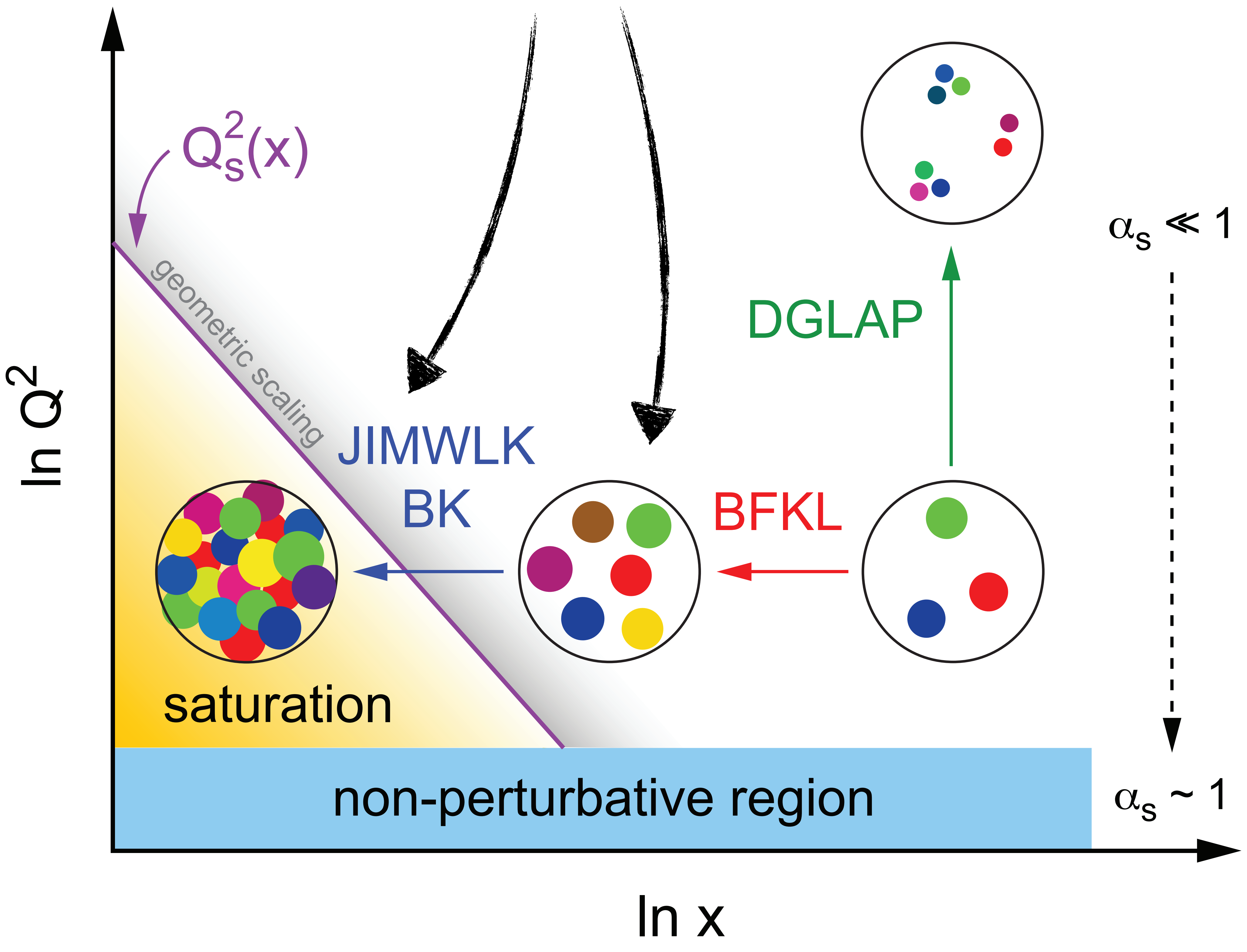
- Hadron is characterized by complex dynamics of parton interactions
- One has to understand the hadron as a complex many-body parton system - dense QCD medium
- Gluons and sea quarks dominate the proton wave function at high energies (small x_B)



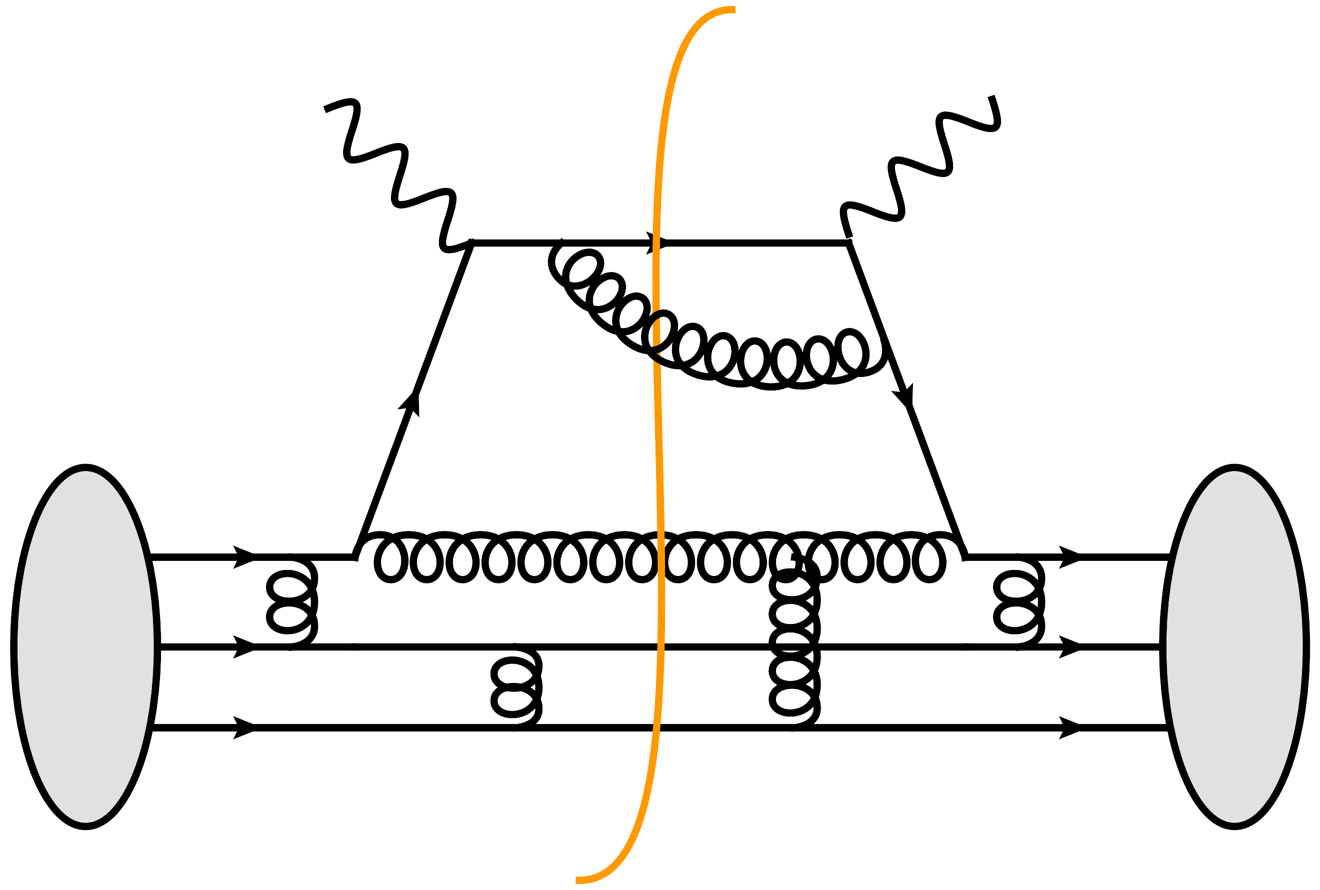




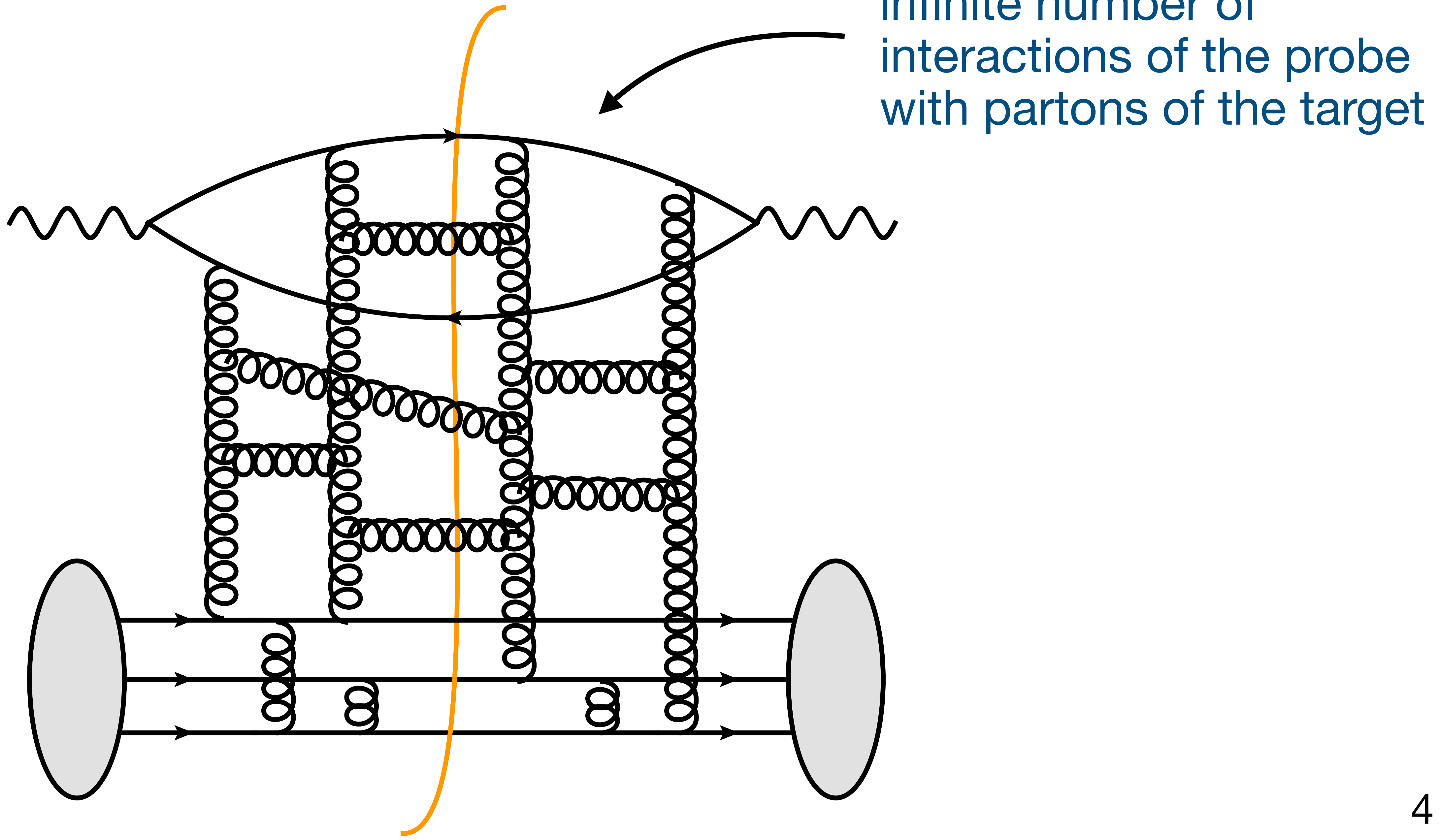
- Boosting the proton uncovers many-body structure



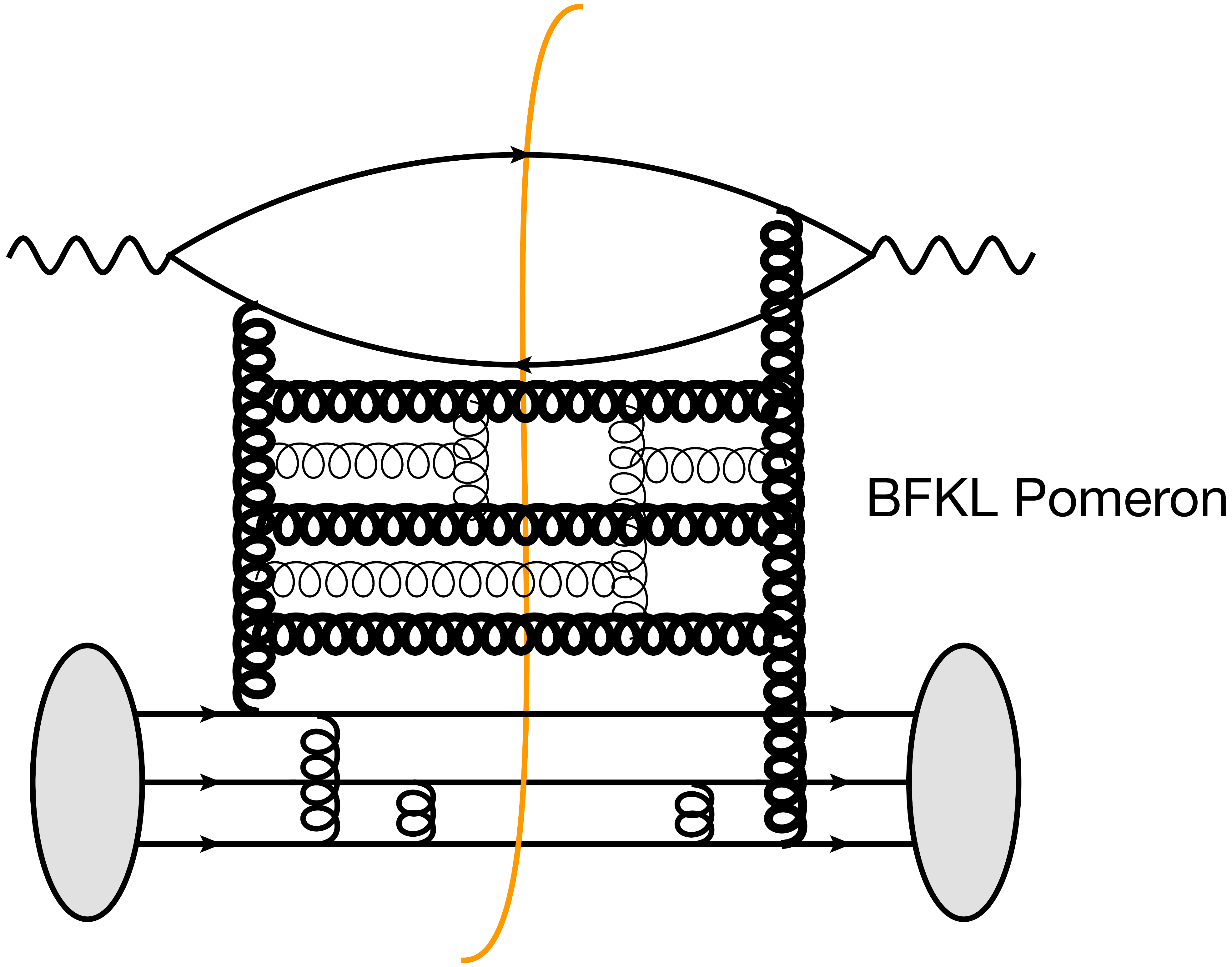
- At large x the process is defined essentially by scattering on individual partons



- The defining characteristic of scattering at small x is many-body parton interactions



- The dominant topology (logarithmically enhances) of Feynman diagrams in the small-x regime is the gluon ladder
- This structure gives rise to the notion of the BFKL Pomeron in QCD



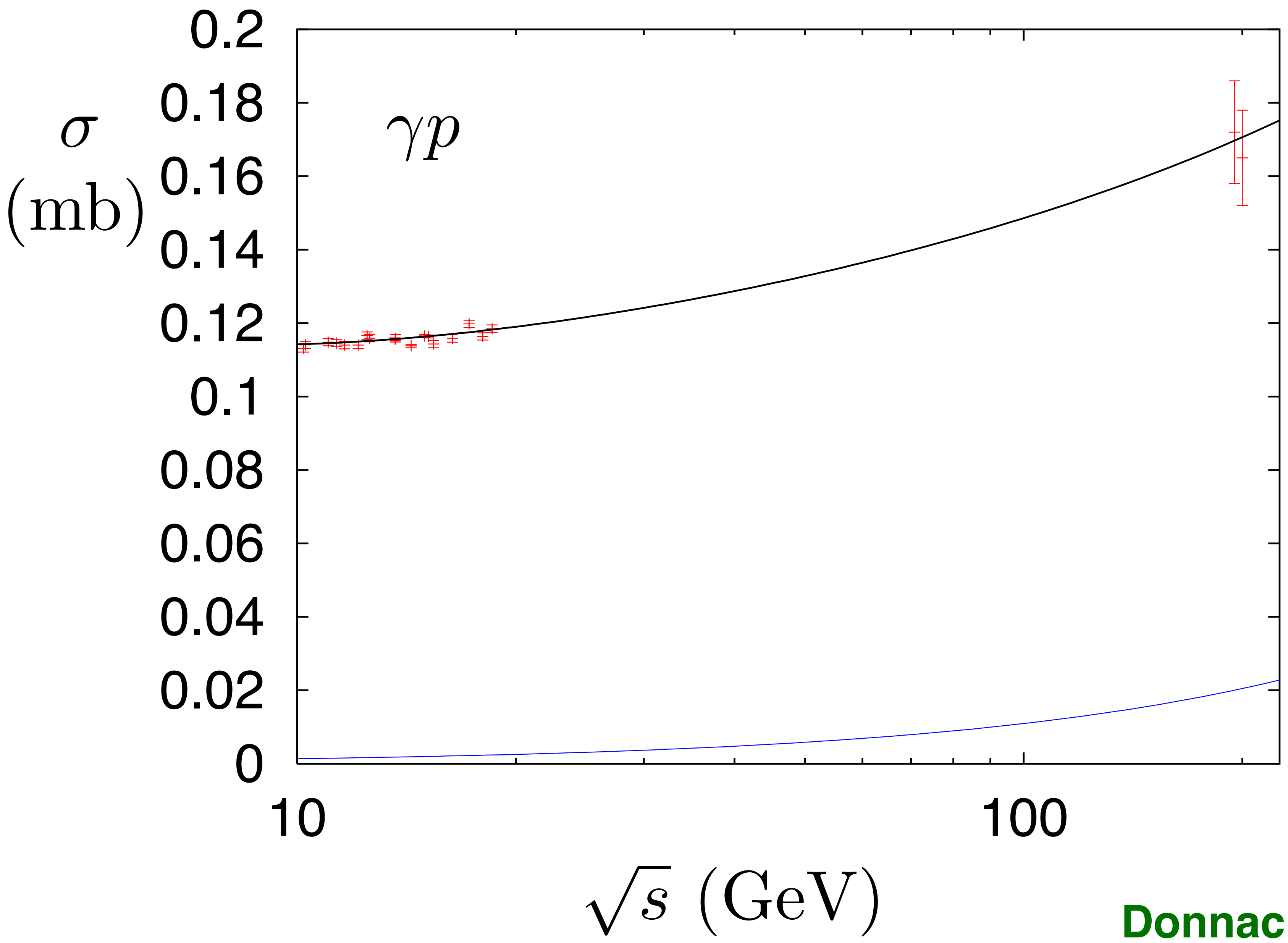
$$\sigma_{tot} \sim s^{\omega_0} \qquad \omega_0 = 4\frac{N_c\alpha_s}{\pi}\ln 2$$

- The resumation of the ladder diagrams leads to the BFKL (Balitsky-Fadin-Kuraev-Lipatov) equation

Balitsky, Fadin, Kuraev, Lipatov (1978)

$$\boxed{\frac{\partial}{\partial \ln 1/x}N(x,k_\perp) = \alpha_s N_c K_{BFKL} \otimes N(x,k_\perp)}$$

- BFKL equation resums powers of $\alpha_s \ln 1/x$

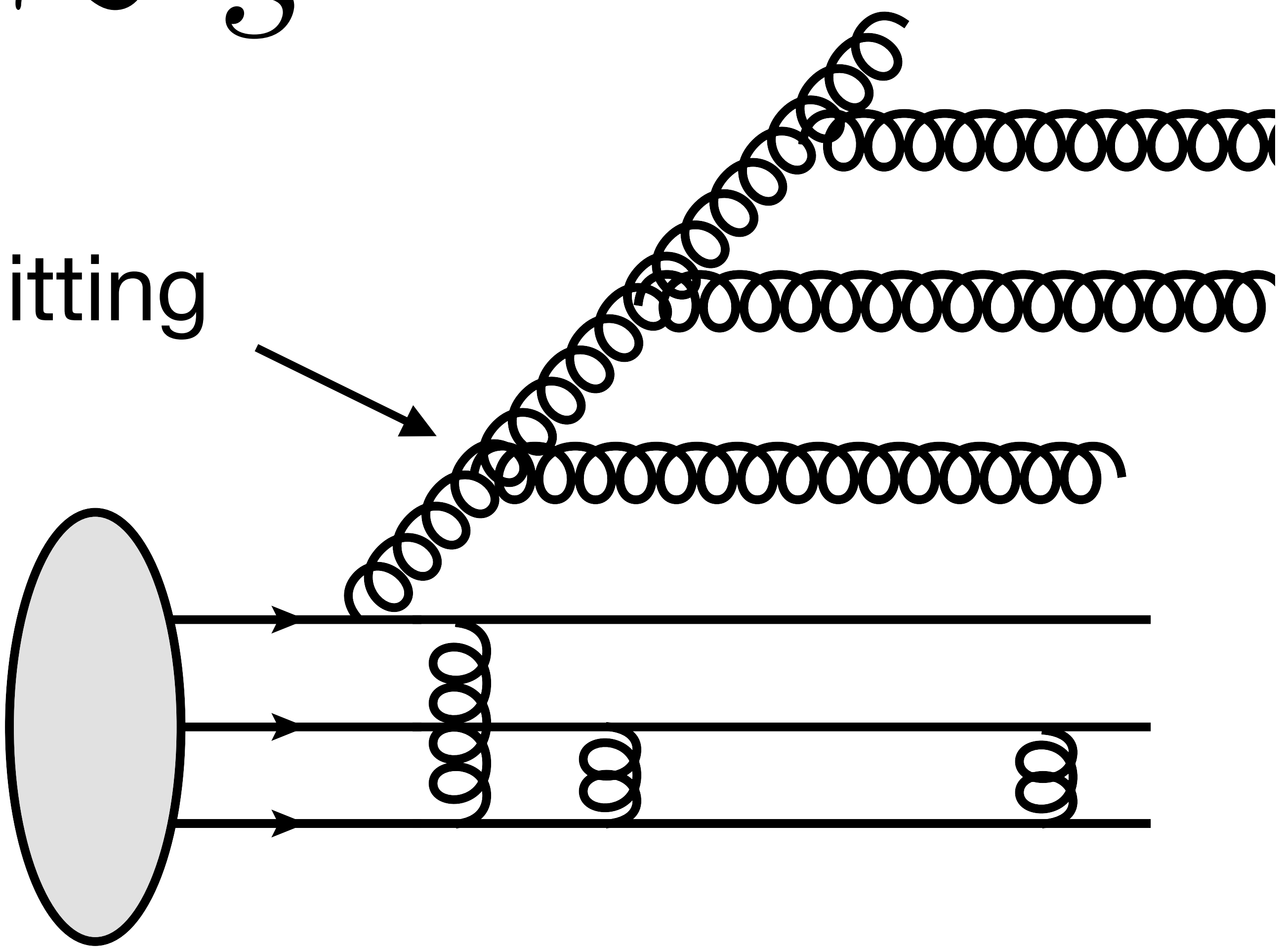


Donnachie, Landshoff (2004)

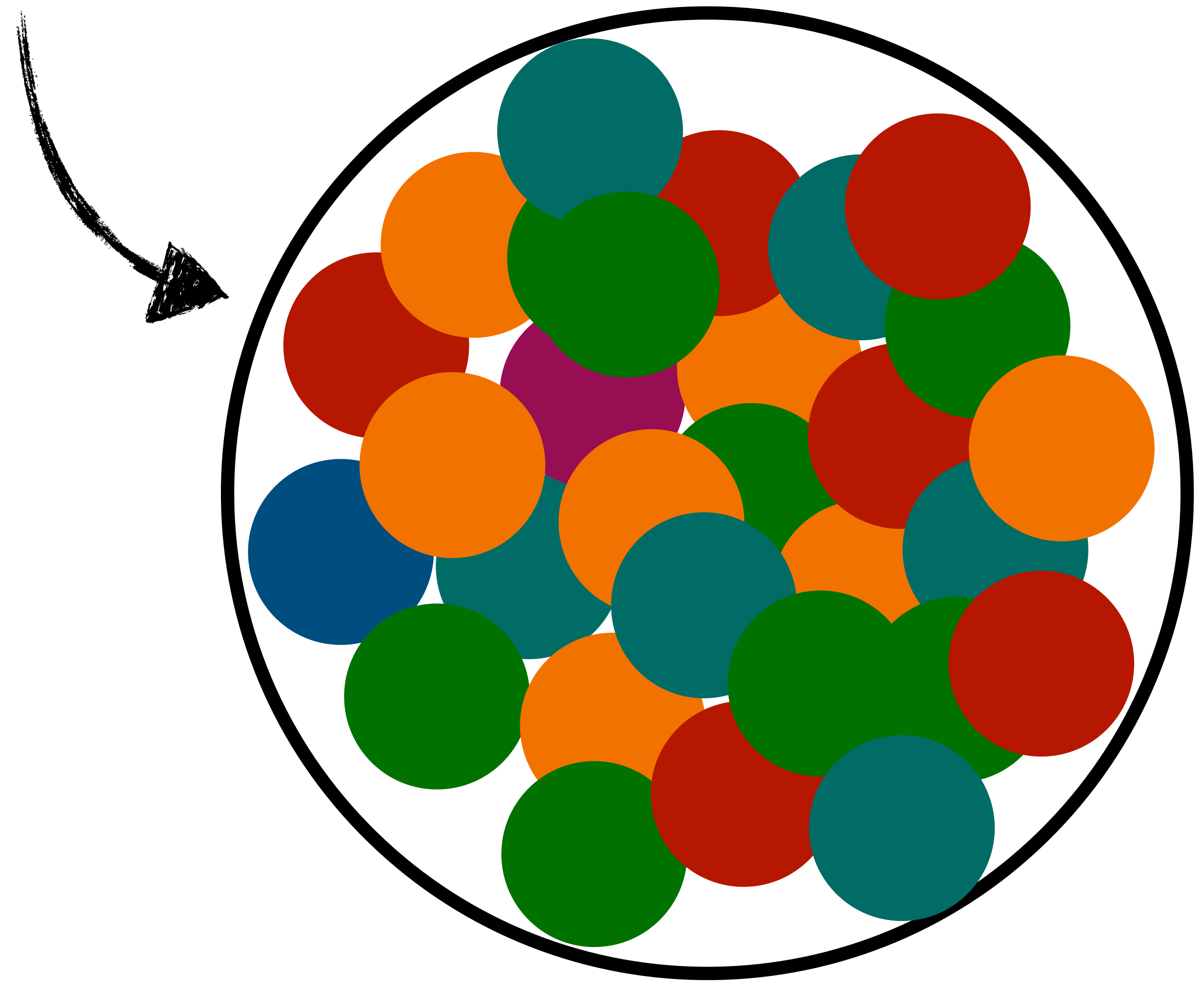
- Total cross-sections across a wide range of energies can be described in terms of Pomeron exchanges (soft and hard)

- Increase of parton density at small-x is driven by the gluon splitting
- Leads to violation of the Froissart bound $\sigma_{tot} < c \ln^2 s$!

$$\sigma_{tot} \sim s^{\omega_0}$$

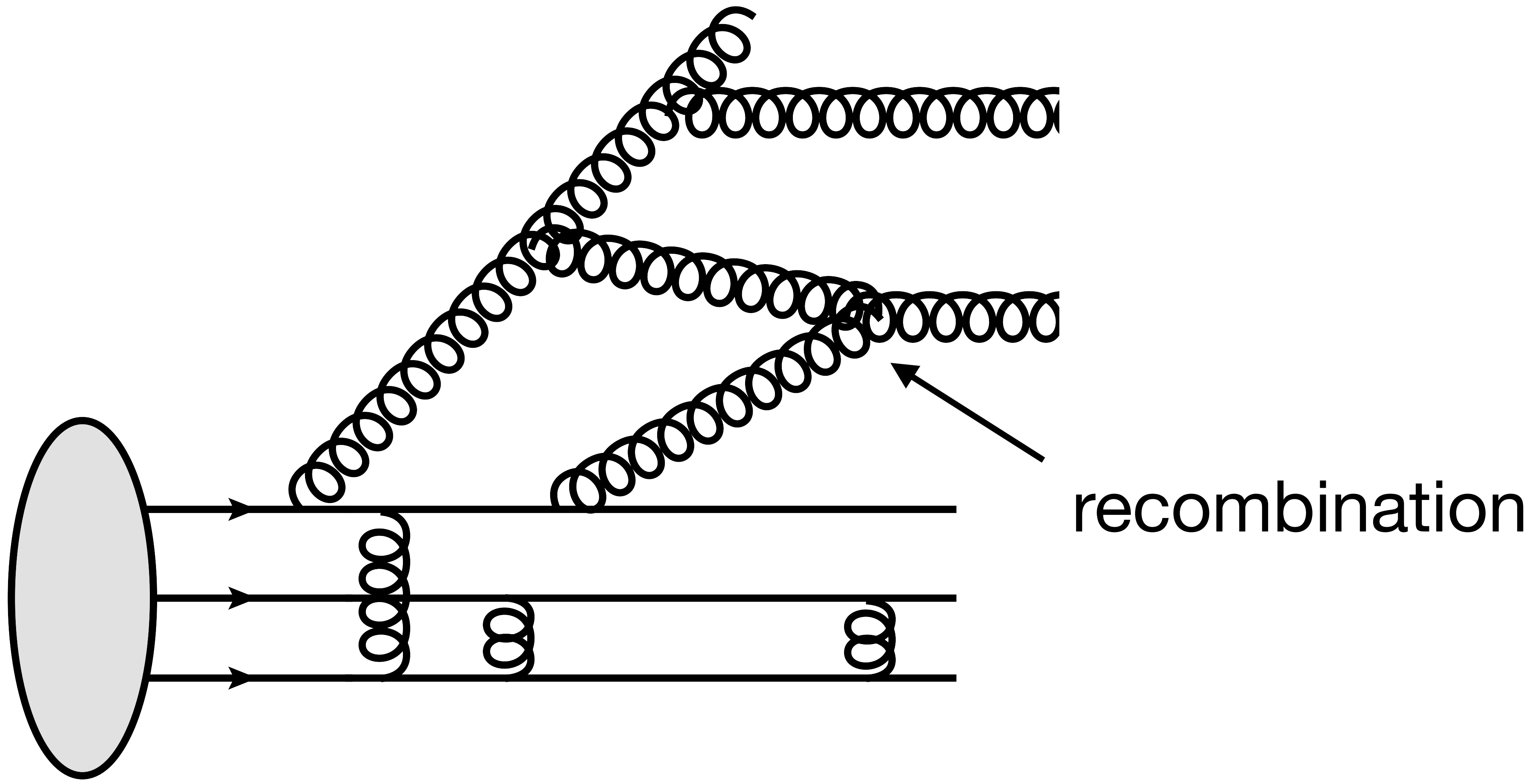


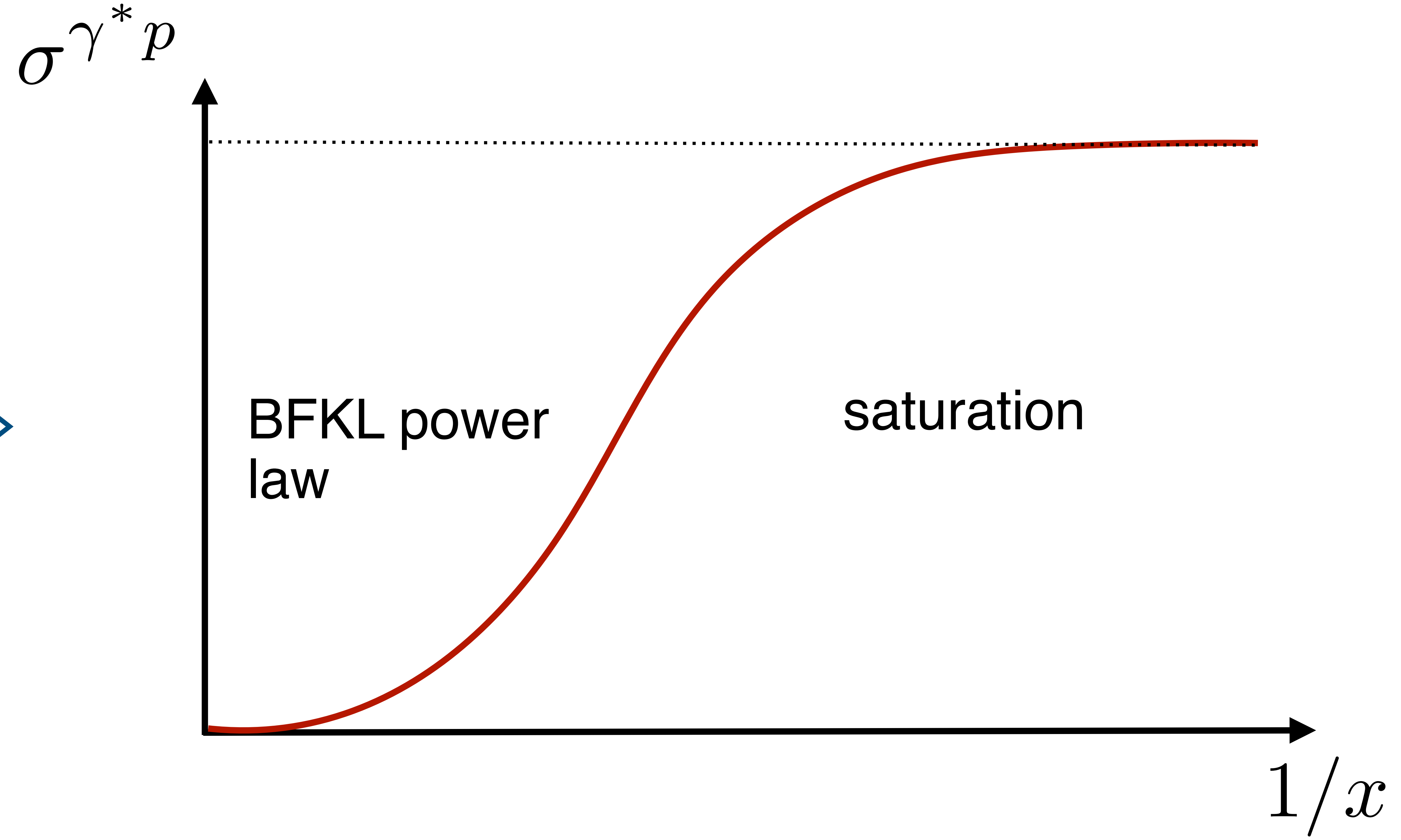
strong field color field $A_\mu \sim 1/g$

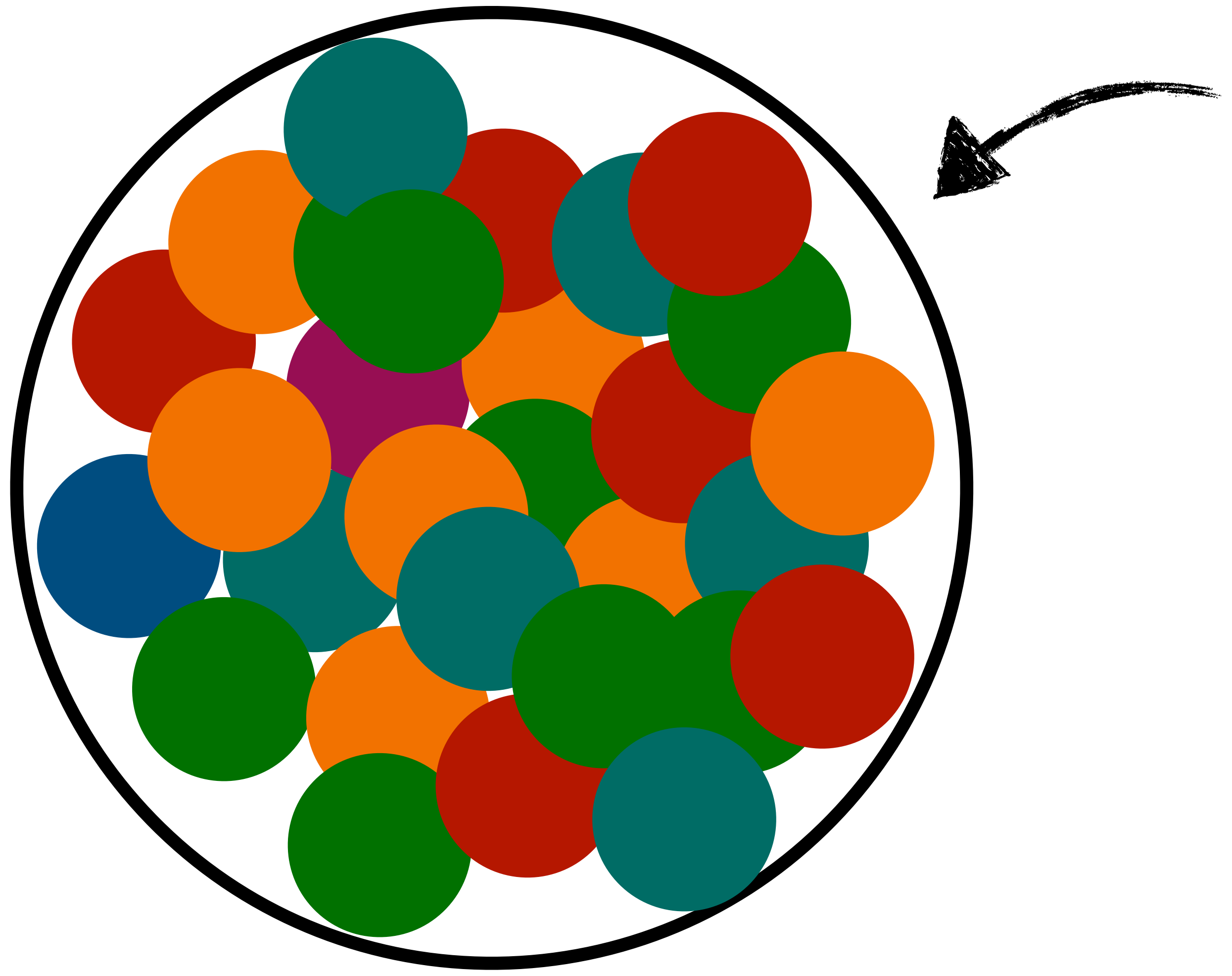


Gluon recombination: taming the growth

Gribov, Levin, Ryskin (1983)
Mueller, Qiu (1986)







strong field $A_\mu \sim 1/g$ in the saturation regime ⇒ “classicalization” of the QCD medium in the saturation regime!

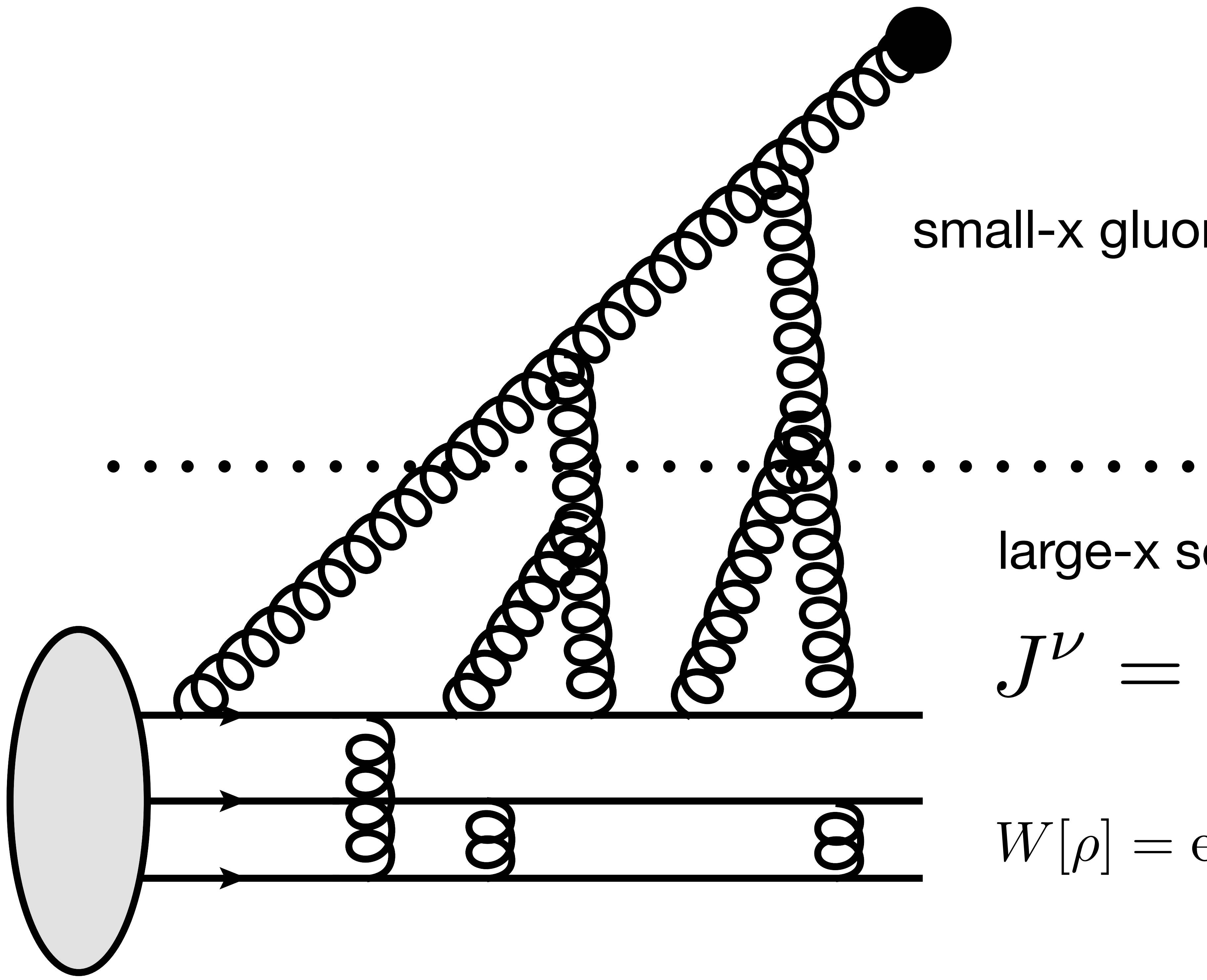
Color Glass Condensate (CGC) EFT

$$A^{cl}_{-}(x) = -\frac{1}{\partial_\perp^2}\rho(x_\perp)\delta(x^-)$$

$$A^{cl}_{+} = A^{cl}_{\perp} = 0$$

↑ shock-wave structure

McLerran, Venugopalan (1994)



small-x gluons

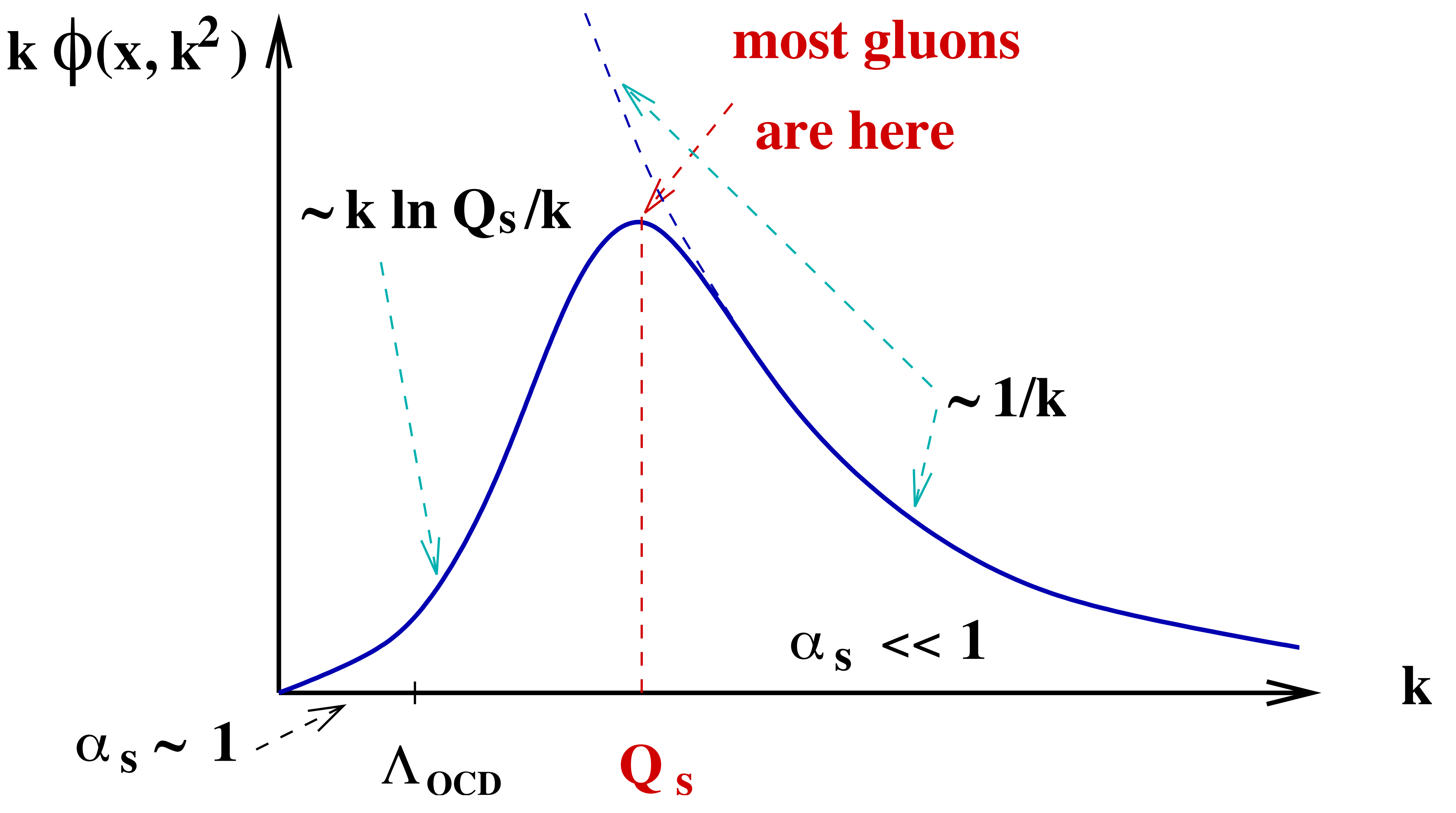
large-x sources

$$J^\nu = \delta^{\nu +}\rho(x^-, x_\perp)$$

$$W[\rho] = \exp\left\{-\int d^2x_\perp \int dx^- \frac{\rho^2(x^-, x_\perp)}{\mu^2(x^-, x_\perp)}\right\}$$

The fields can be found as solution of classical equation of motion

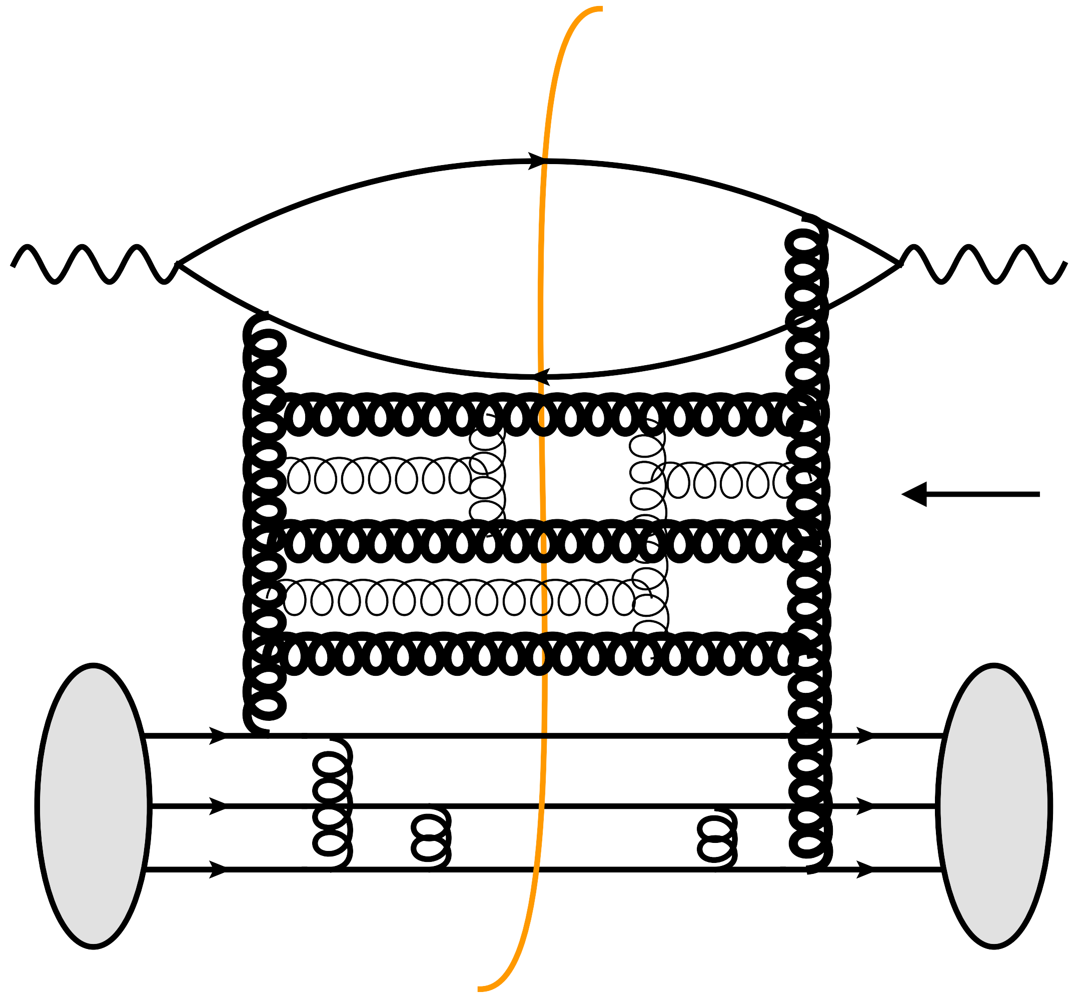
$$\mathcal{D}_\mu F^{\mu\nu} = J^\nu$$



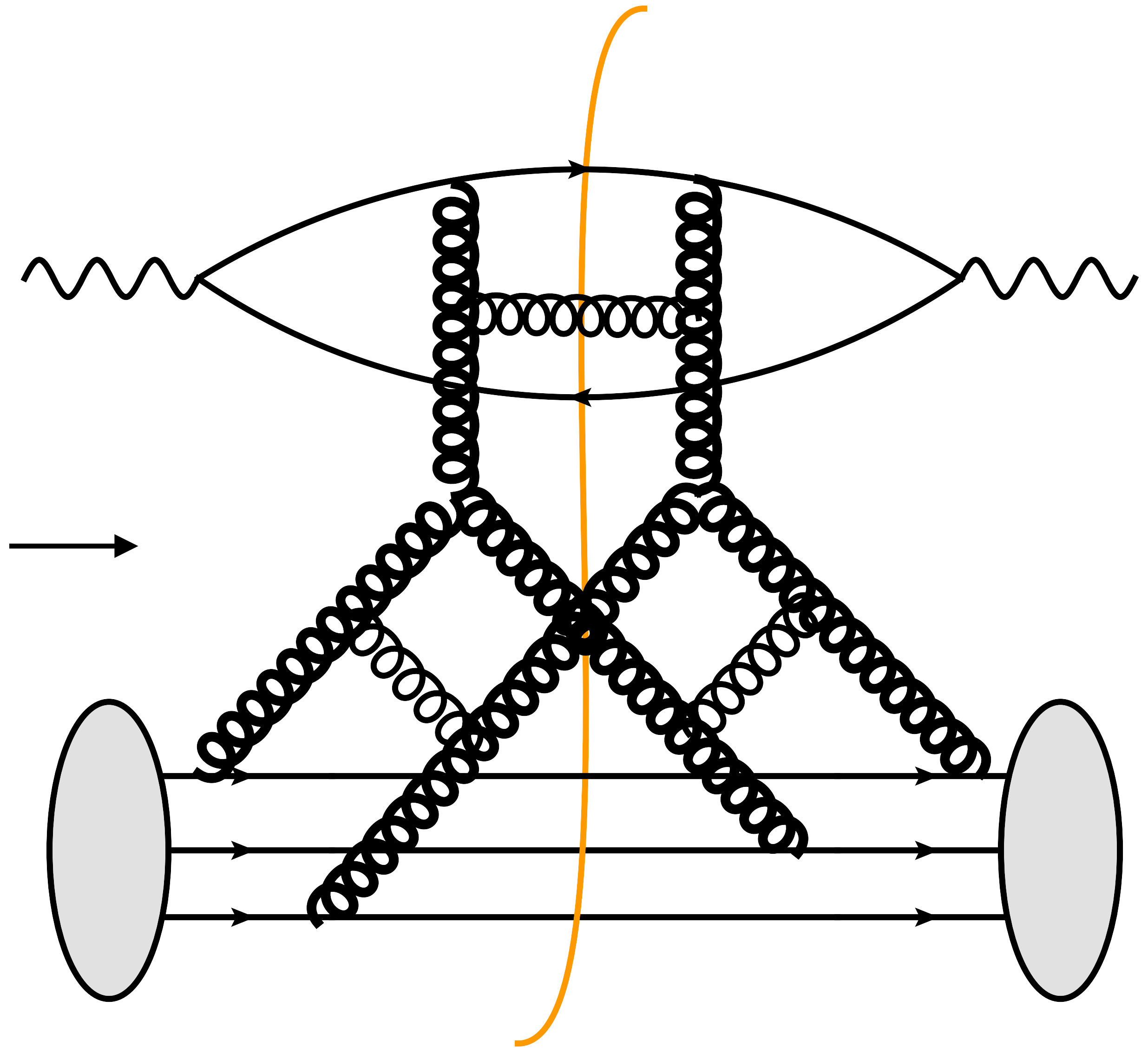
Mueller (1994)
Jalilian-Marian, Kovchegov (2005)

- The QCD medium develops a saturation scale (transverse scale) Q_S

- Beyond the classical approximation: resummation of quantum loops:



BFKL pomeron



“fan” BFKL pomeron diagrams

Balitsky (1996), Kovchegov (1999)

$$\frac{\partial}{\partial \ln 1/x} N(x, k_\perp) = \alpha_s N_c K_{BFKL} \otimes N(x, k_\perp)$$

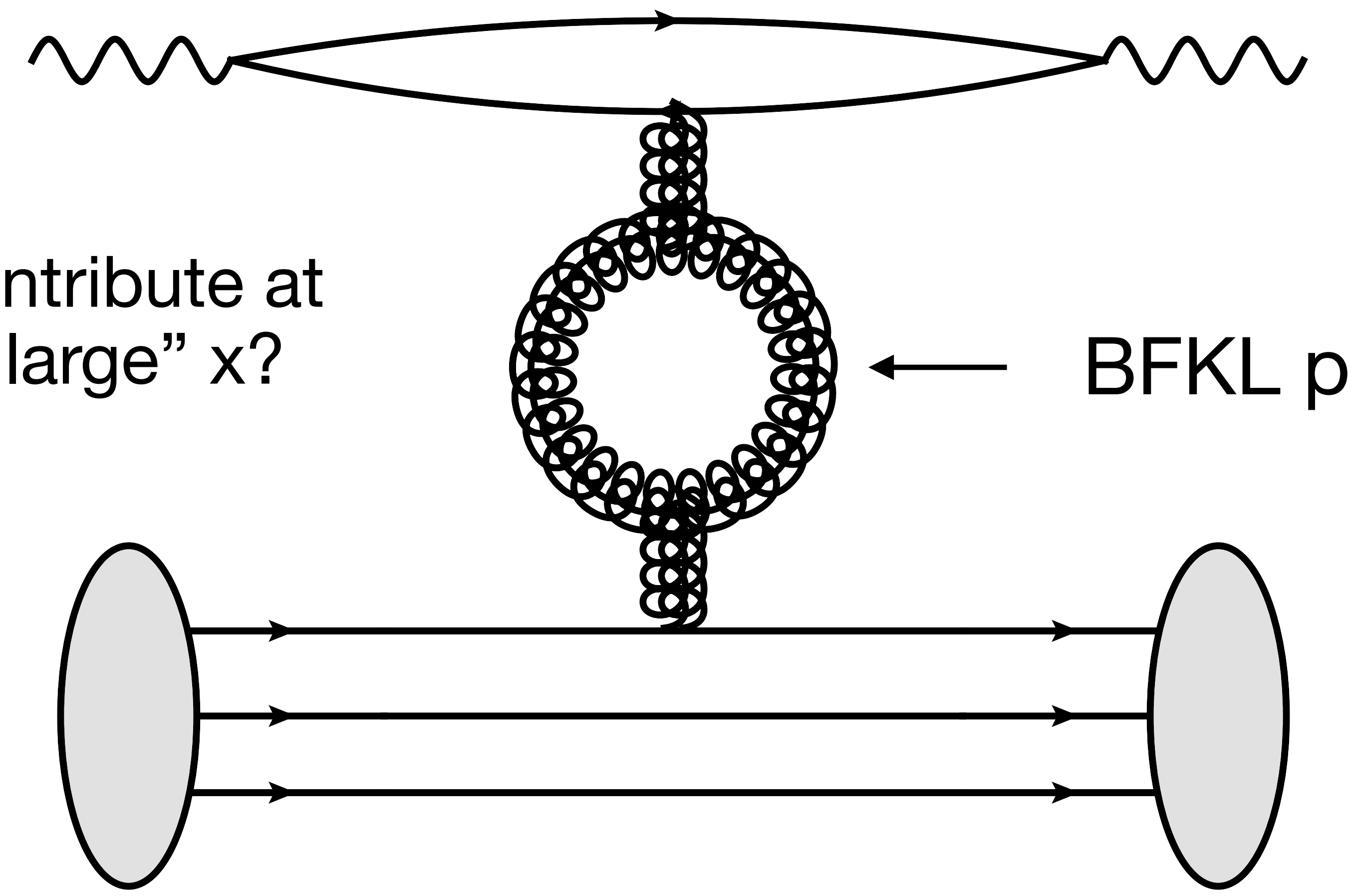
$$\boxed{\frac{\partial}{\partial \ln 1/x} N(x, k_\perp) = \alpha_s N_c K_{BFKL} \otimes N(x, k_\perp) - \alpha_s N_c N^2(x, k_\perp)}$$

Non-linear evolution in the saturation regime!

$$\lim_{x \to 0} N(x, k_\perp) = 1$$

$$Q_s^2(x, A) \propto A^{1/3} \left(\frac{1}{x}\right)^{\lambda}$$

- Pomeron loops:



start to contribute at relatively “large” x?

BFKL pomeron loop

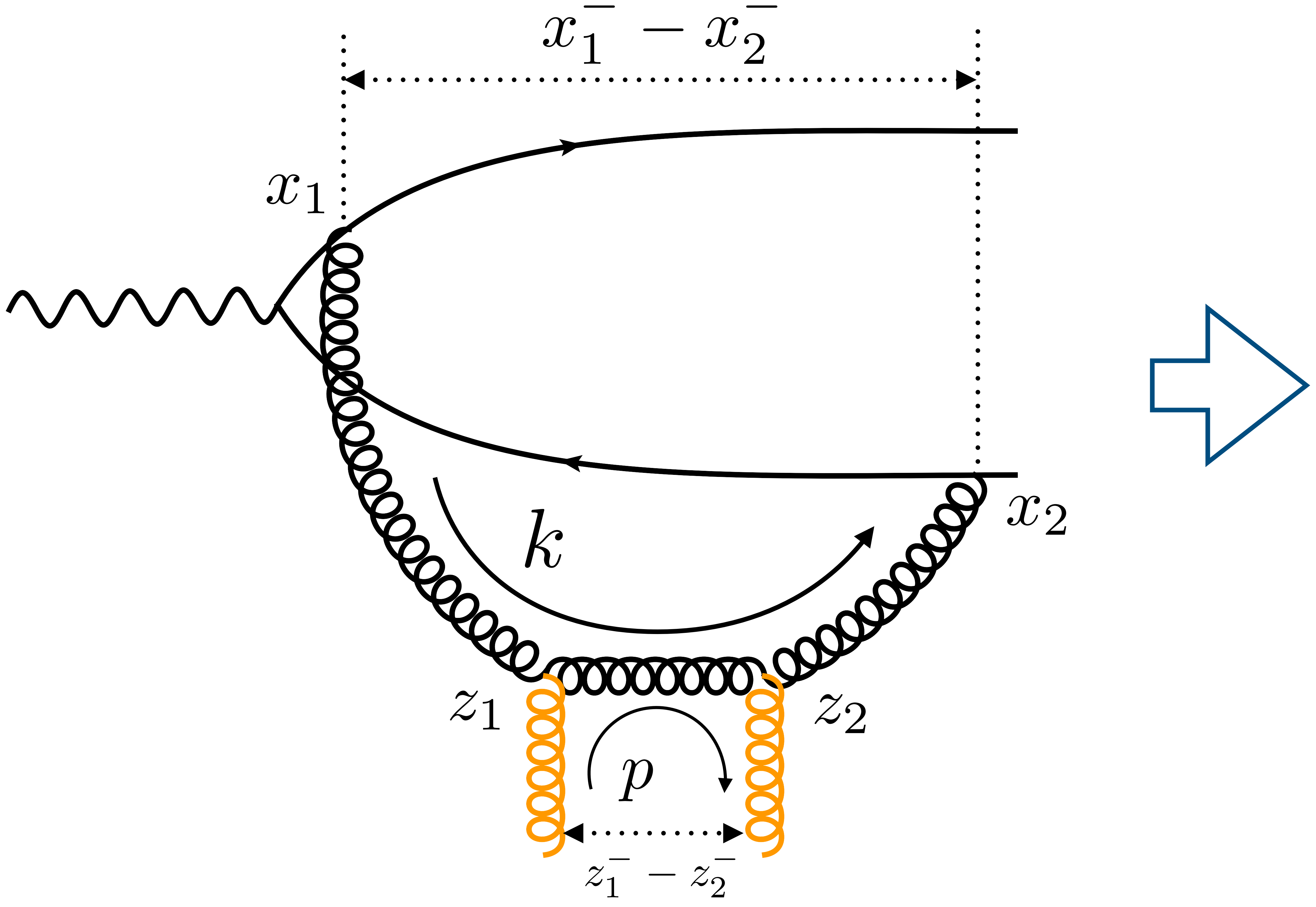
Levin (2025), Braun, Tarasov (2012)

- The saturation scale growth with decreasing x
- The perturbative calculation is valid $Q_s^2 \gg \Lambda_{QCD}^2$
- The presence of the saturation scale allows to directly calculate experimental observables
- The non-linear dynamics can be probed in experiments

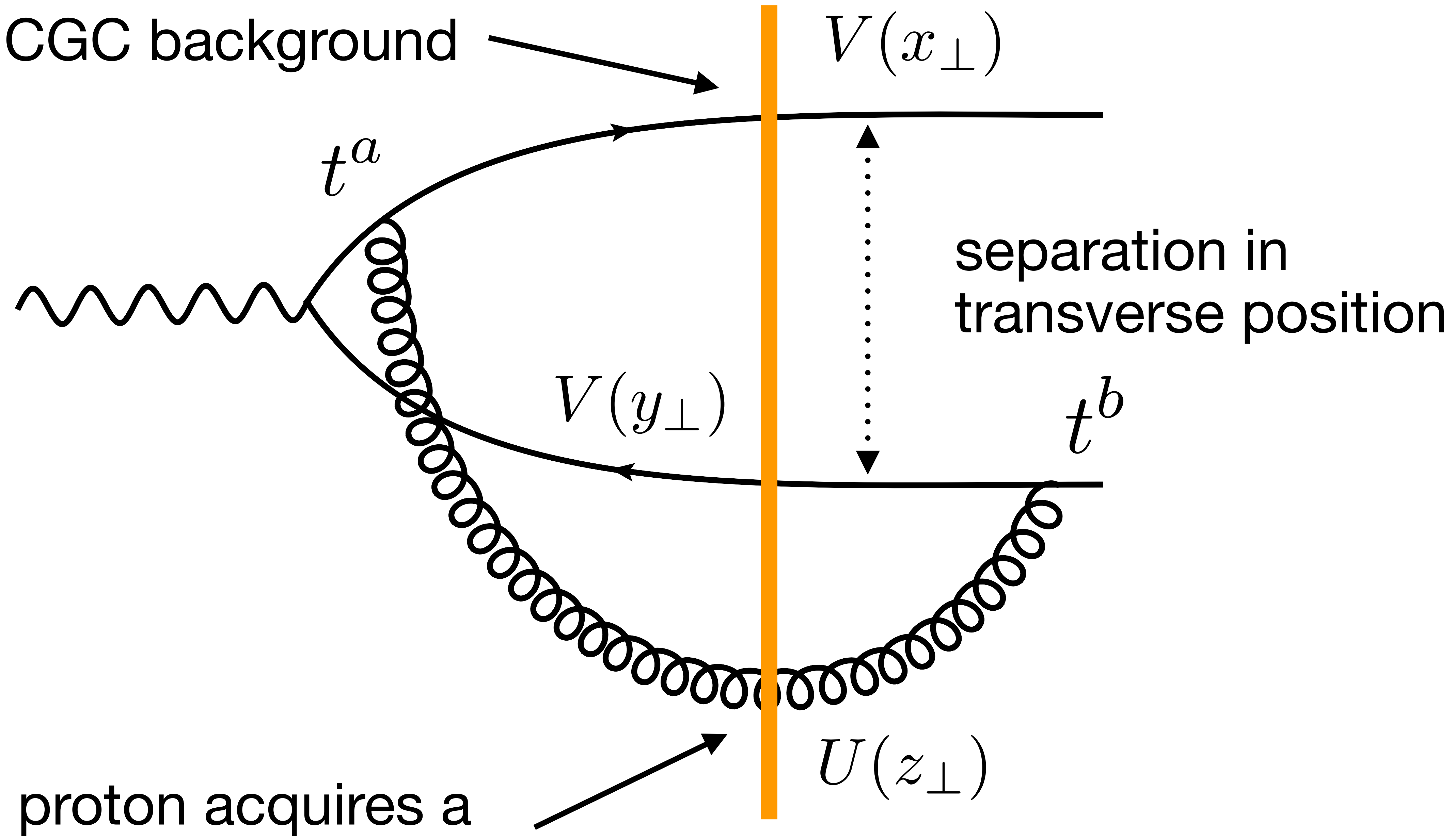
- Kinematics of the saturation: strict ordering in longitudinal momenta ⇒ shock-wave approximation

$$k^- \gg p^- \gg \dots$$
$$k^+ \ll p^+ \ll \dots$$
$$k_\perp \sim p_\perp \sim \dots$$

$$\Rightarrow \quad x_1^- - x_2^- \gg z_1^- - z_2^-$$



instantaneous interaction with the CGC background



separation in transverse position

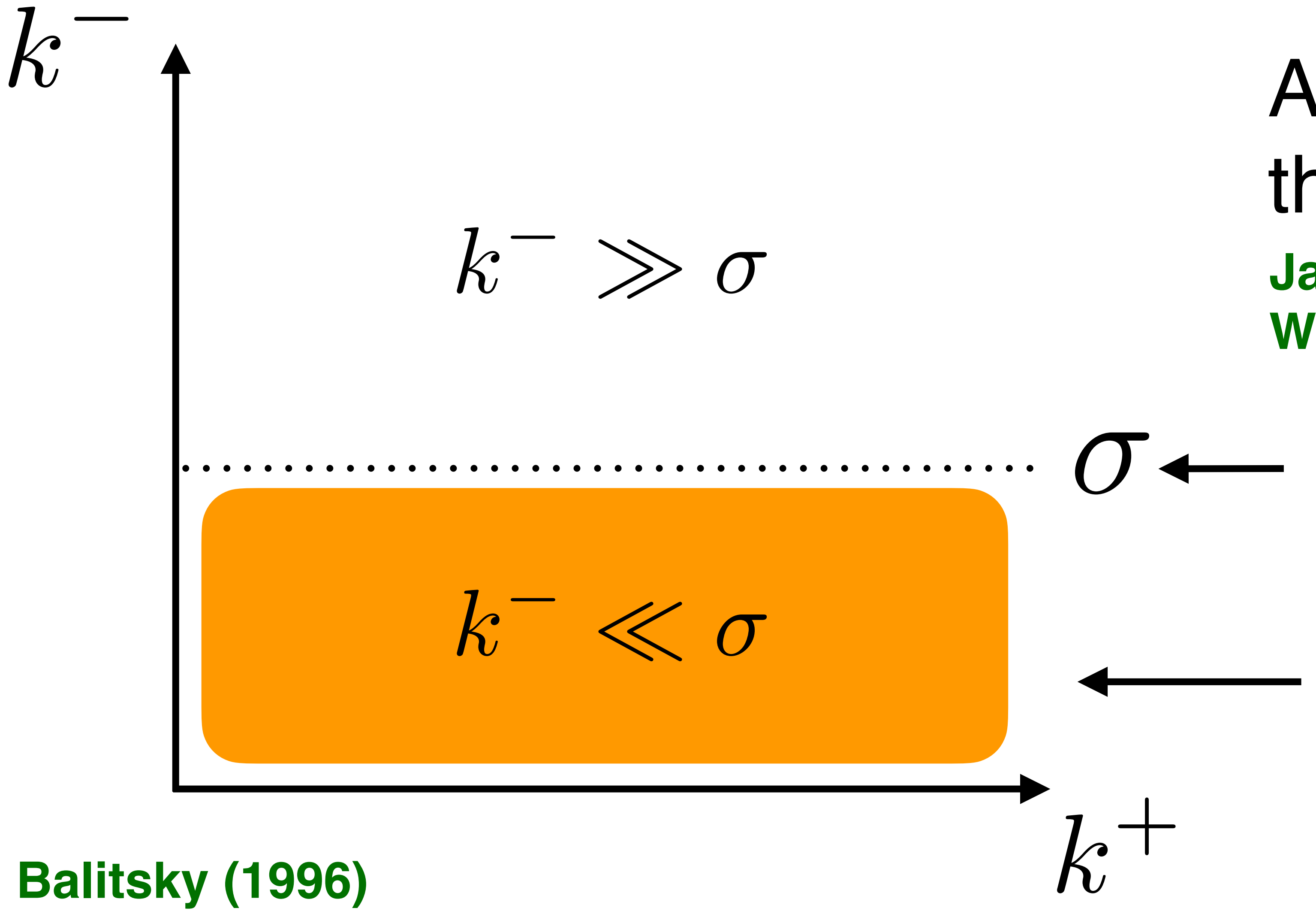
The proton acquires a scalar phase: Wilson line

$$U(z_\perp) = \exp\left(ig\int_{-\infty}^{\infty} dz^- A_-(z^-, z_\perp)\right)$$

- Interaction of the probe with the CGC background is defined by a product of Wilson line separated in the transverse position → dipole operators

$$\mathrm{tr}\{t^a V(x_\perp) t^b V(y_\perp)\} U^{ab}(z_\perp)$$

- Separation between the hard mode and the background field (CGC) is formalized in the high-energy rapidity **factorization** approach



k^-

$k^- \gg \sigma$

σ ← factorization scale

$k^- \ll \sigma$

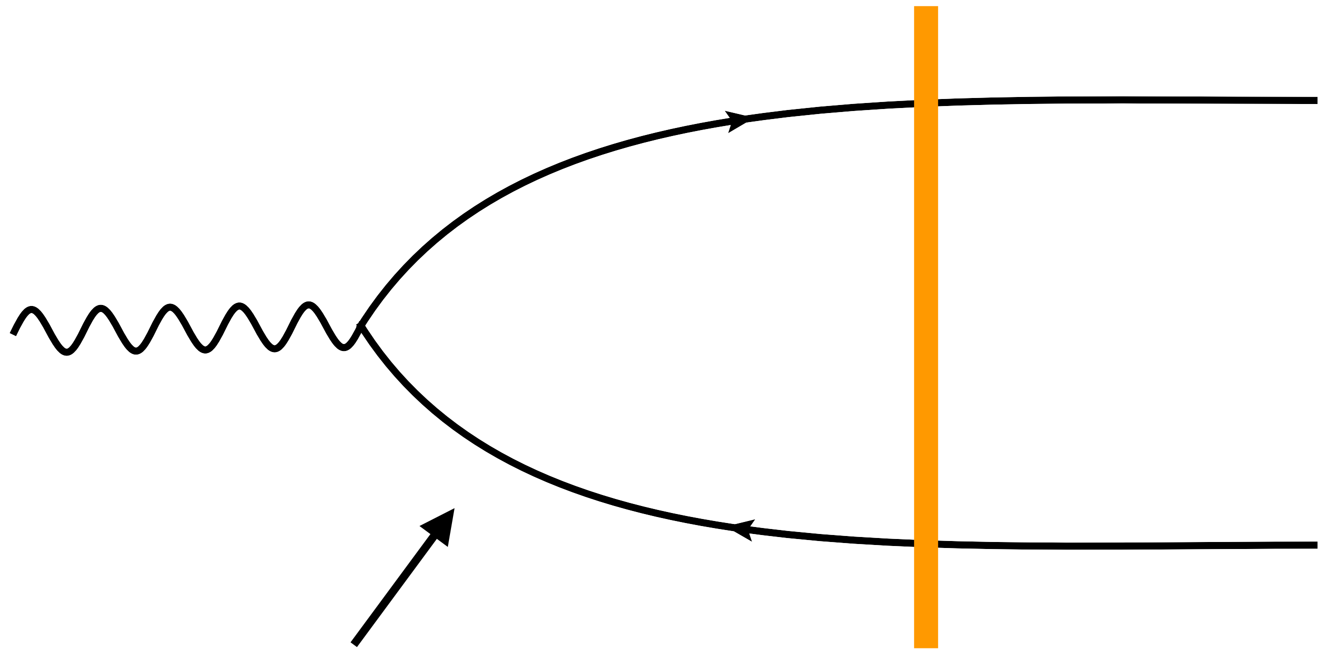
← semi-classical fields

k^+

Balitsky (1996)

Alternative formulation in the JIMWLK framework

Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (1997-2002)



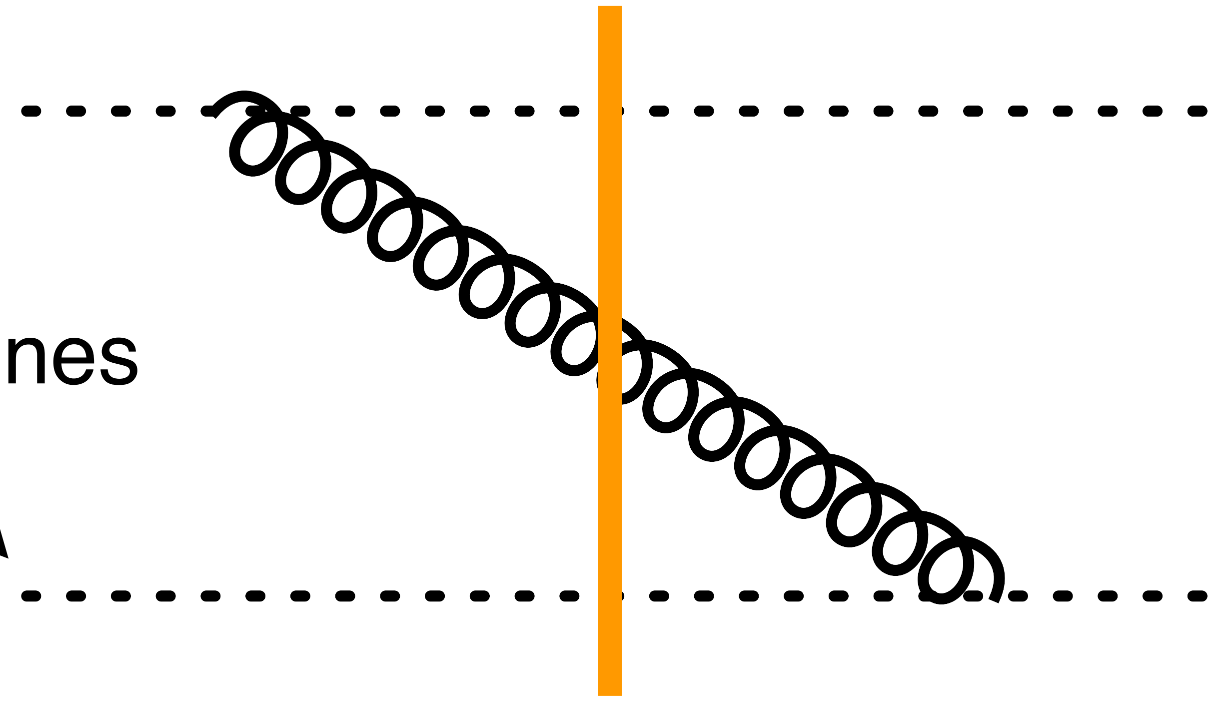
impact factor

$$\sigma \propto \int \frac{d^2 p_\perp}{4\pi^2} I(p_\perp, q_\perp) \langle P | \mathrm{tr}\ V(p_\perp) V^\dagger(q_\perp - p_\perp) | P \rangle$$

dipole operator describes interaction with a target. Can be calculated in the MV model

- The matrix elements of the dipole operators at initial scale can be calculated using MV model for the classical background
- In the small-x framework one can calculate **predictions** for experimental observables

- Loop corrections to dipole operators
  ⇒ evolution properties (BFKL/BK)

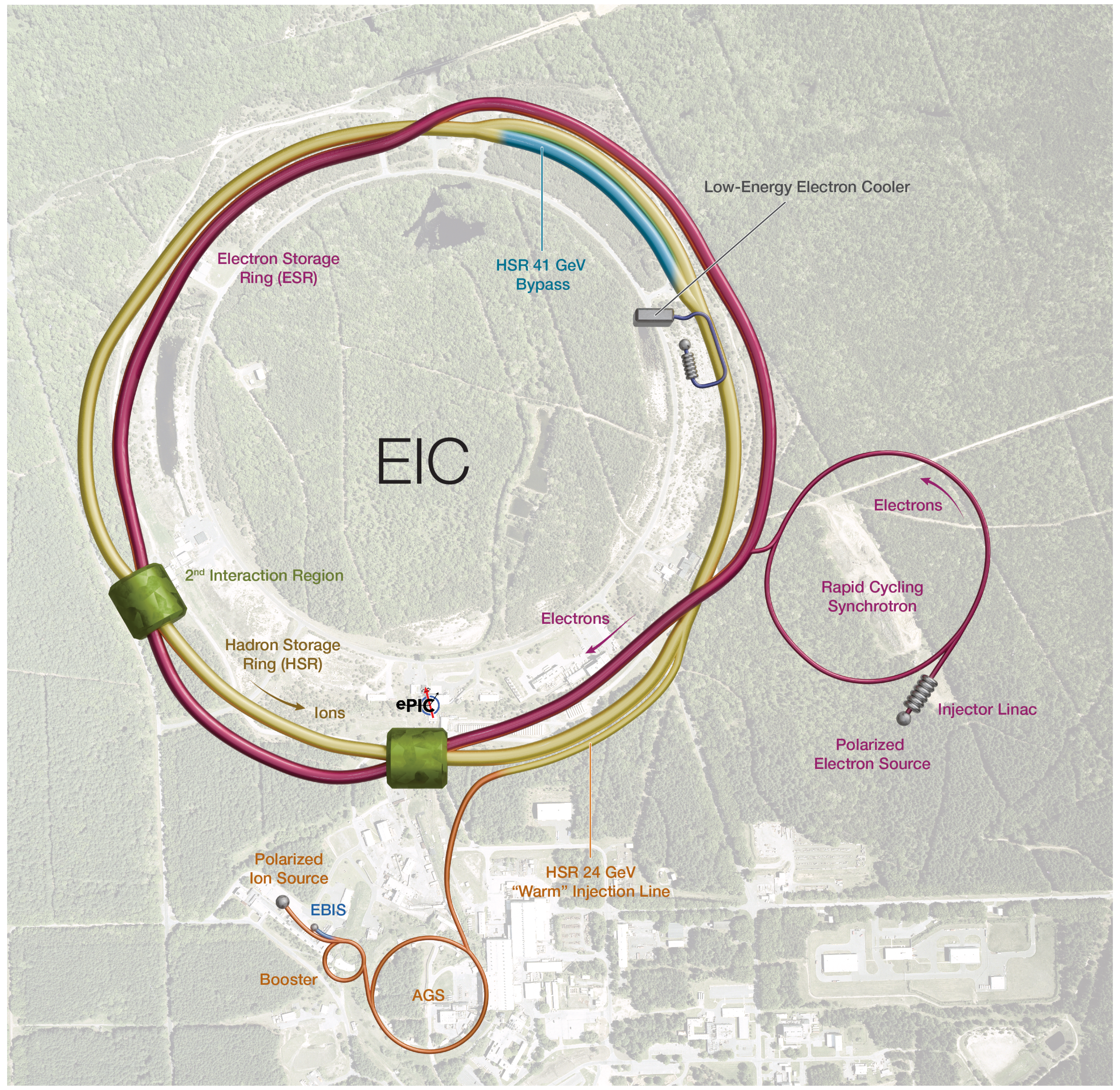


Wilson lines

leads to hierarchy of dipole operators

$$\int_{\sigma'}^{\sigma} \frac{dk^-}{k^-}$$

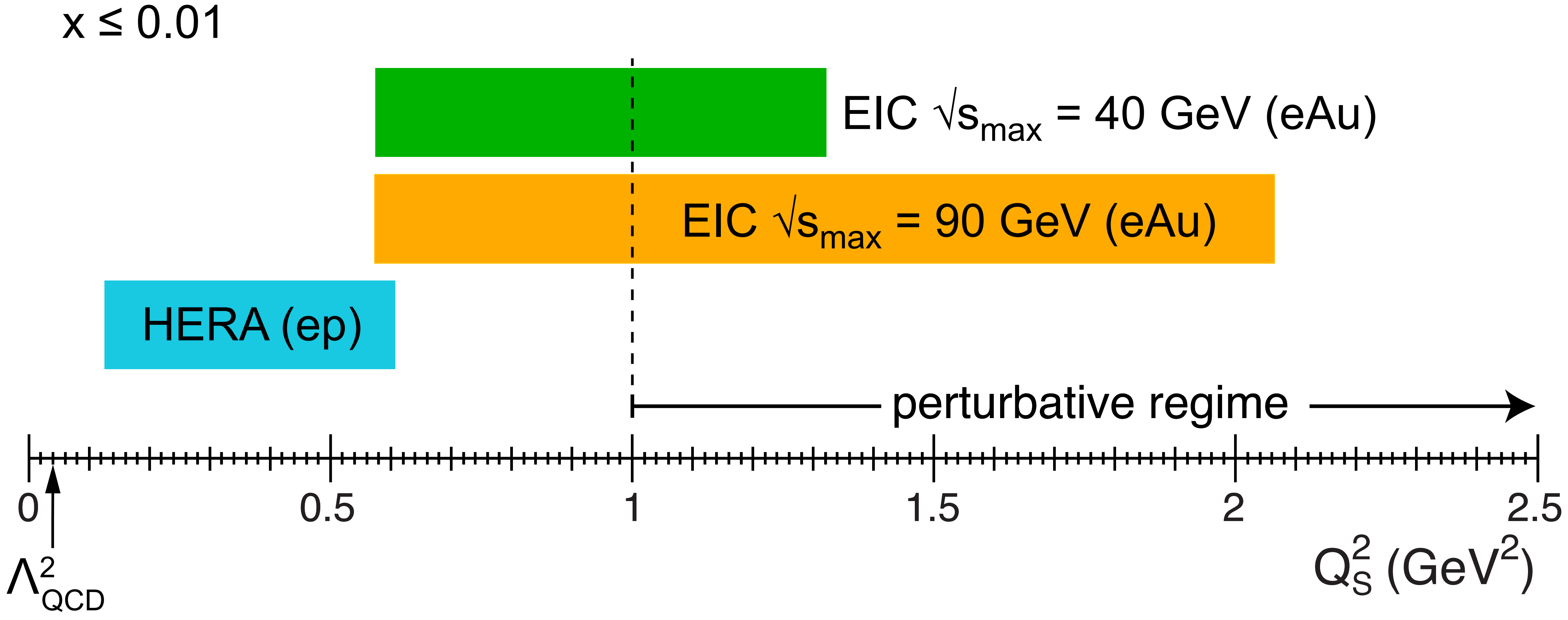
rapidity divergence
⇒ small-x evolution



EIC will be instrumental in understanding “how the characteristic properties of the proton, such as mass and spin, arise from the interactions between quarks and gluons, and *how new phenomena and properties emerge in extremely dense gluonic, nuclear environments*.”

The 2023 Long Range Plan for Nuclear Science

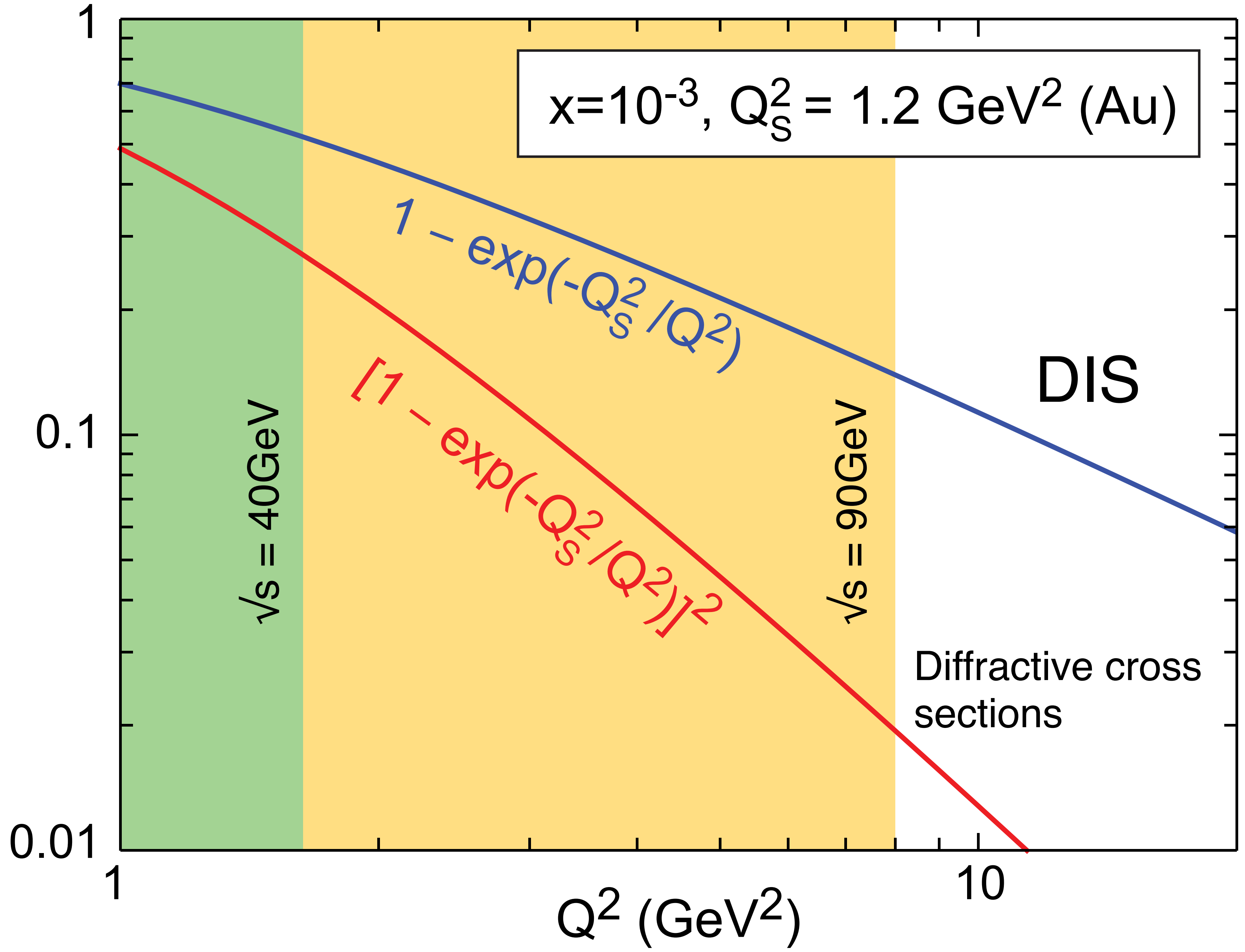
- Search of the signatures of saturation: comparing ***predictions*** of the small-x calculations with experimental observables



E. Aschenauer et al. (2017)

- Up to ~ 140 GeV center of mass energy

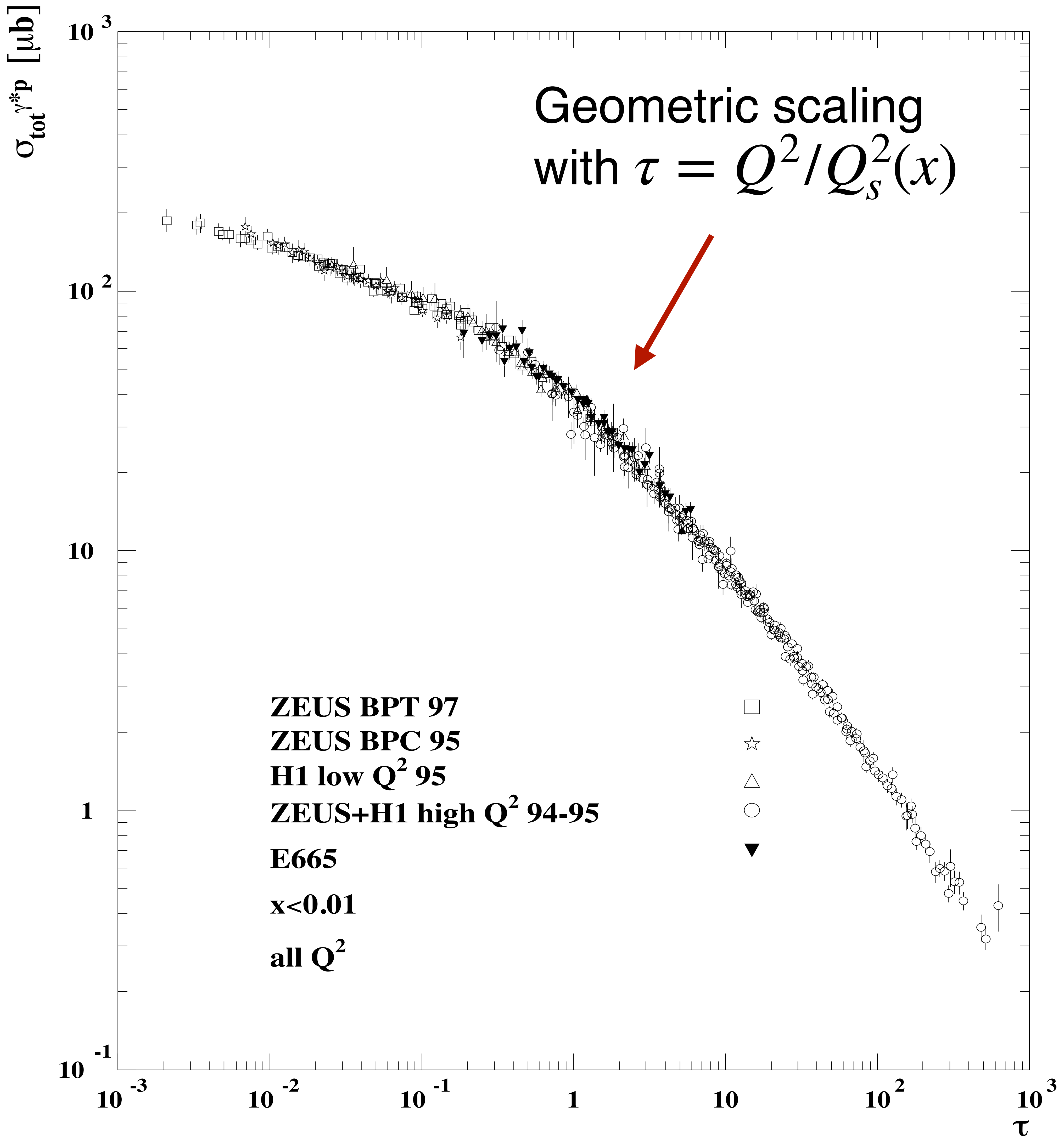
dipole amplitude and its square



- Different from the large-x approach where the predictive power is usually estimated from the quality of the fit

DIS structure functions

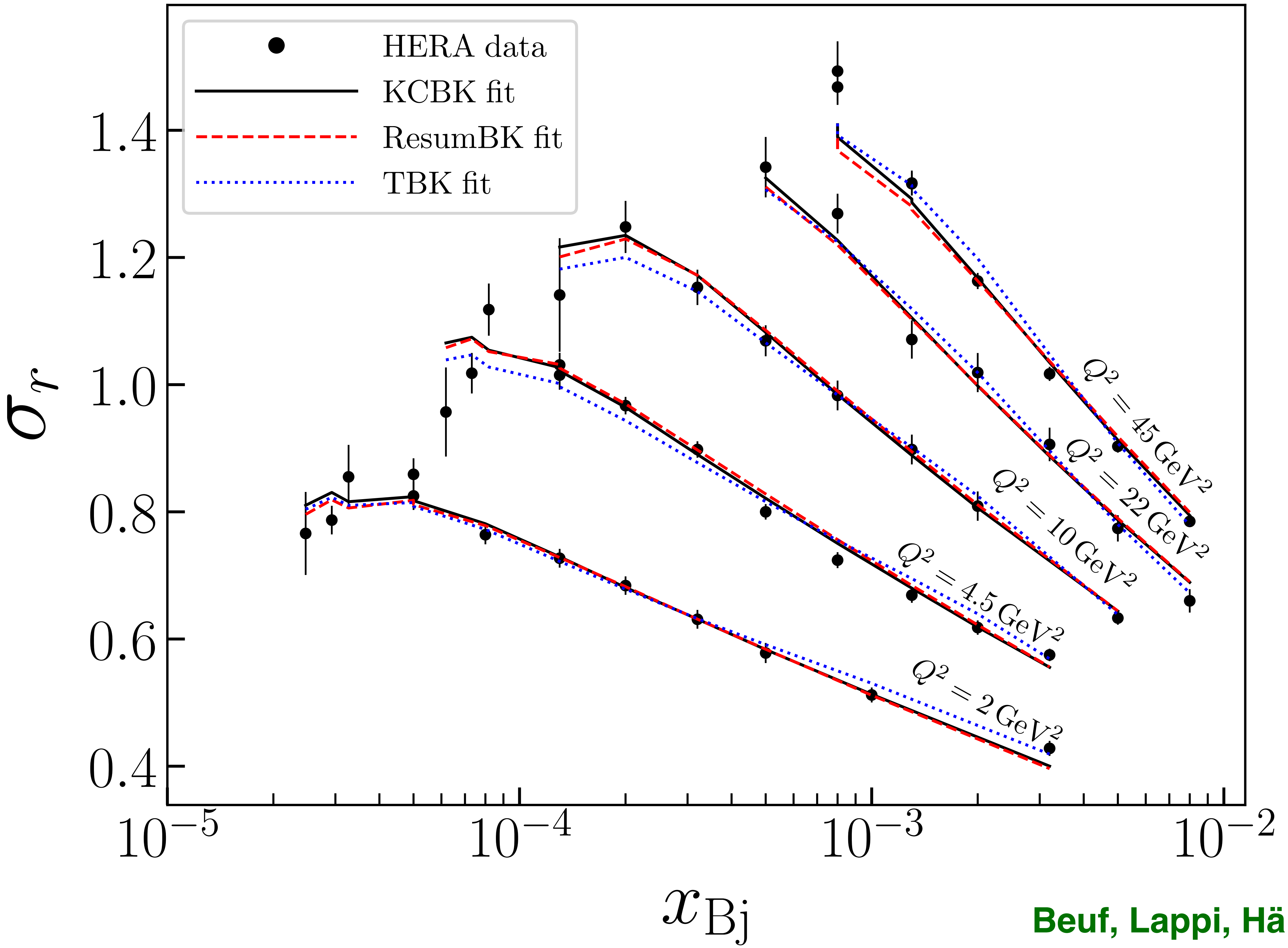
$$\sigma_r(x,y,Q^2) = F_2(x,Q^2) - \frac{y^2}{1+(1-y)^2} F_L(x,Q^2)$$



Geometric scaling with $\tau = Q^2/Q_s^2(x)$

Stasto, Golec-Biernat, Kwiecinski (2000)
Iancu, Itakura, McLerran (2002)

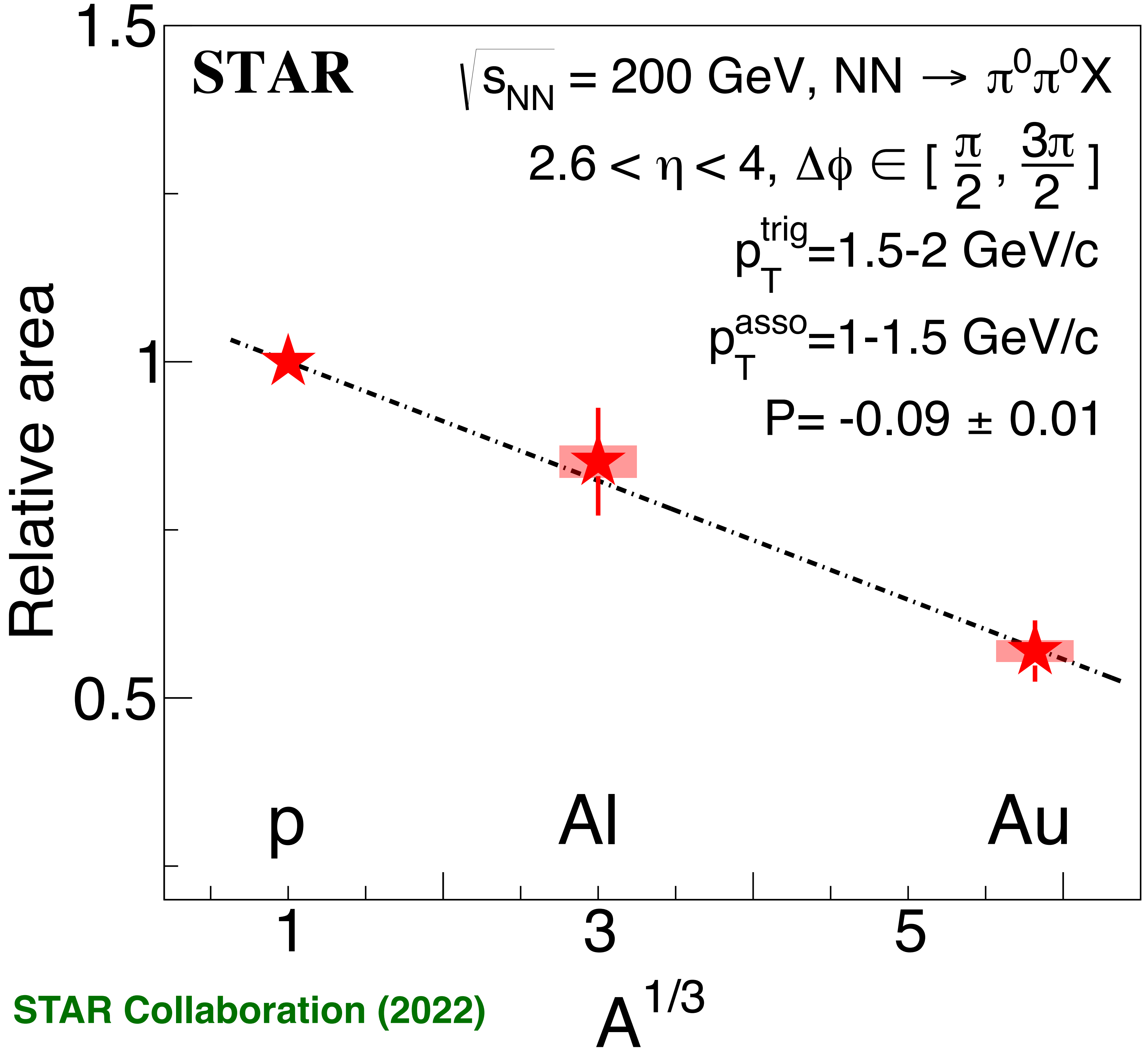
- CGC predictions at the NLO order:



Beuf, Lappi, Hänninen, Mäntysaari (2020)

- Compare with structure functions at EIC
- Analysis of the scaling for different nuclei. Dependence on the nuclear size?

Two particle correlations

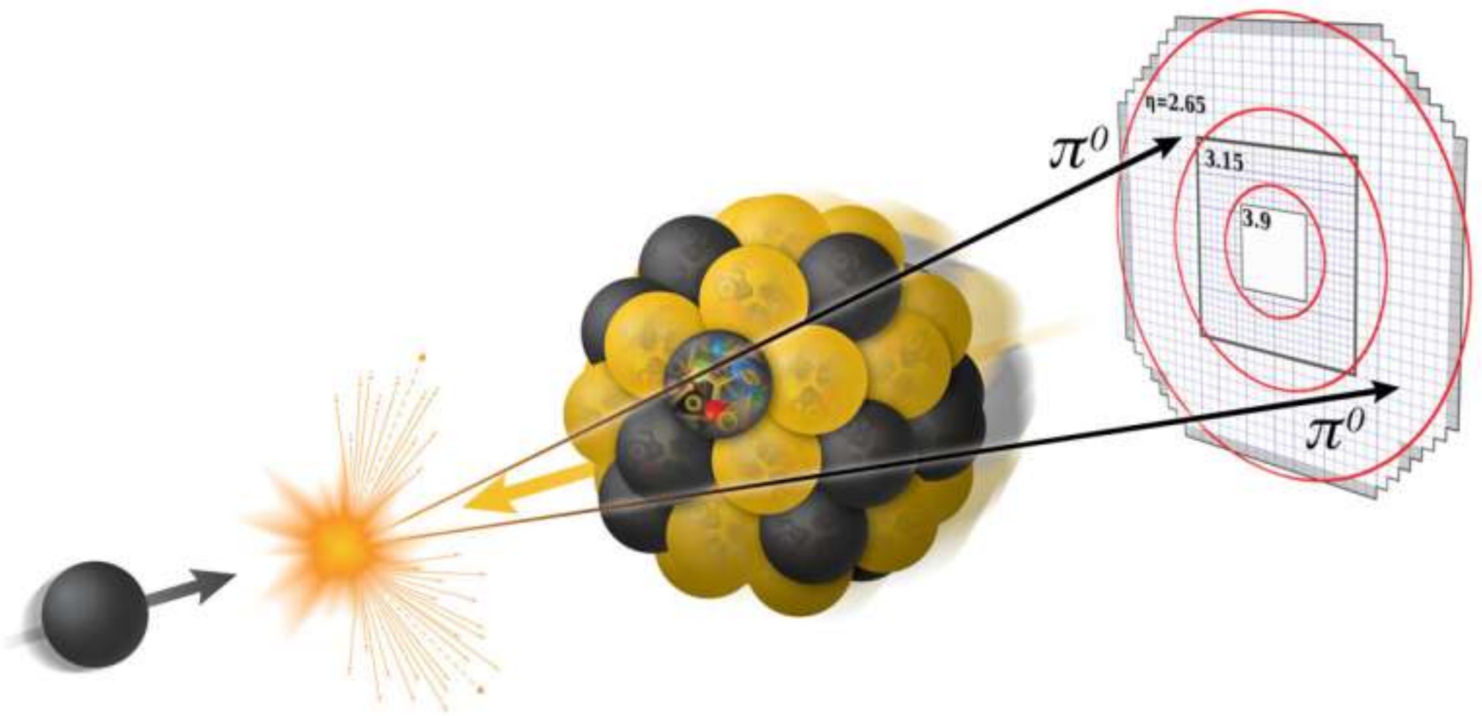


STAR Collaboration (2022)

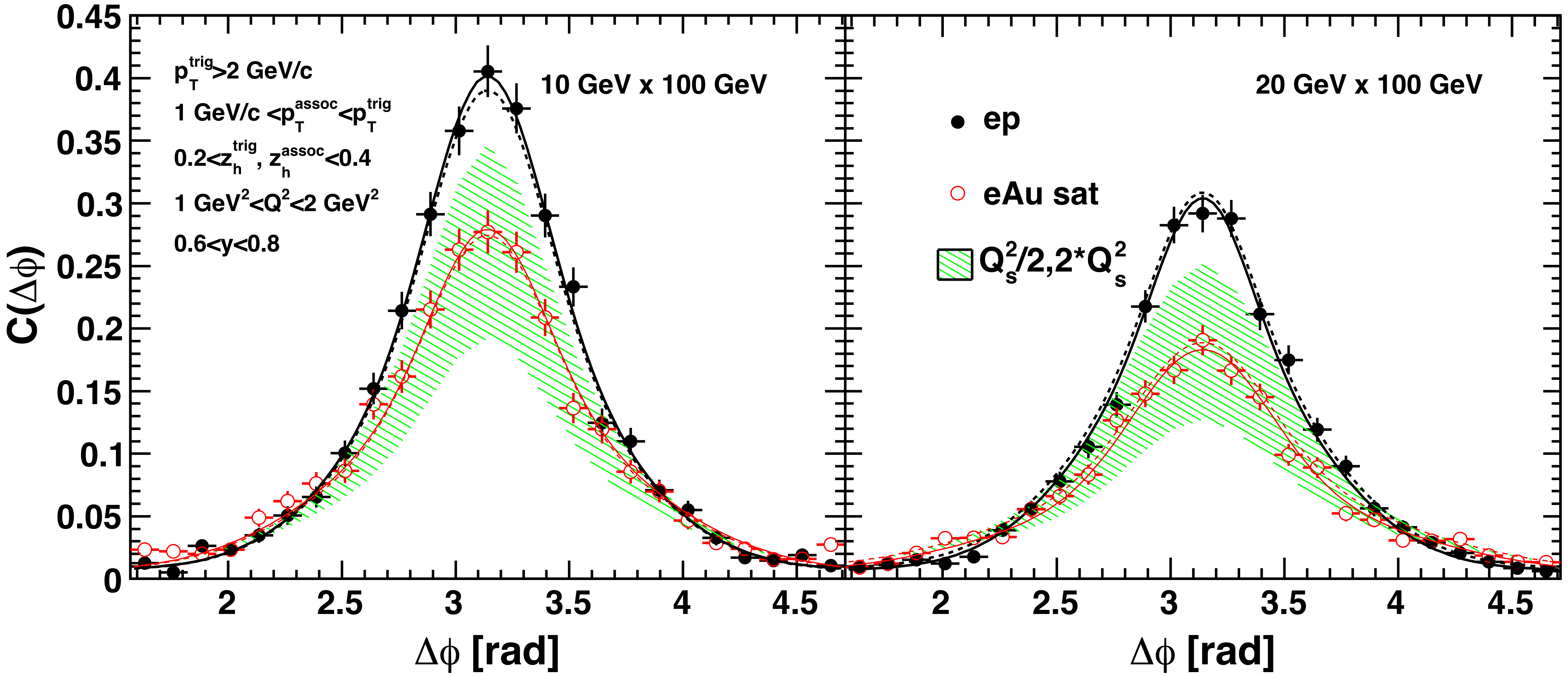
- Experimental observation of the $Q_s^2 \propto A^{1/3}$ dependence

$$Q_s^2(x, A) \propto \left(\frac{A}{x}\right)^{1/3}$$

- Signs of saturation emerge from particle collisions at RHIC
- Suppression of the correlation function for heavy nuclei can be explained as the growth of the saturation scale Q_S^2

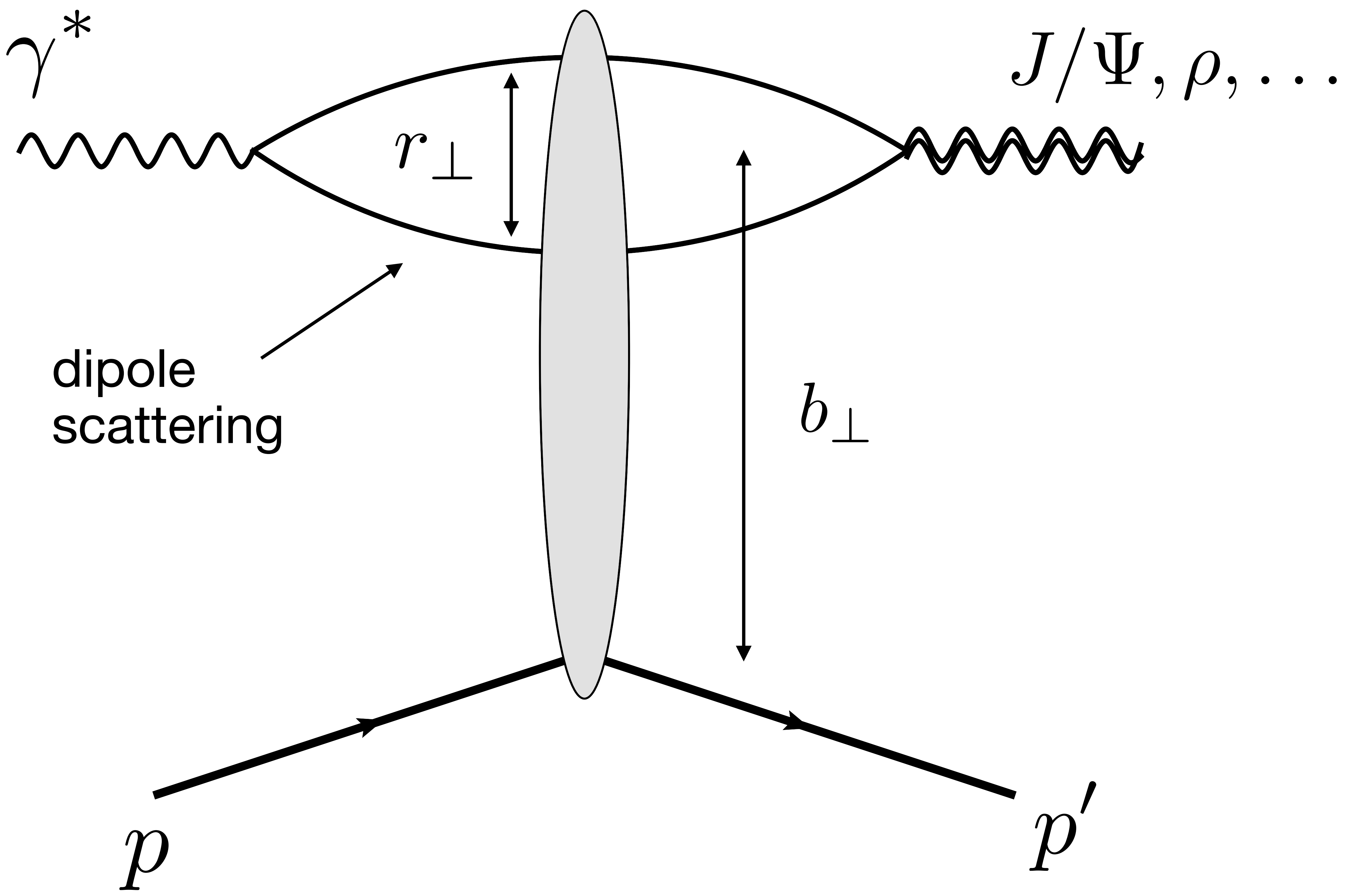


- PYTHIA simulations with a saturation based model



Zheng, Aschenauer, Lee, Xiao (2014)

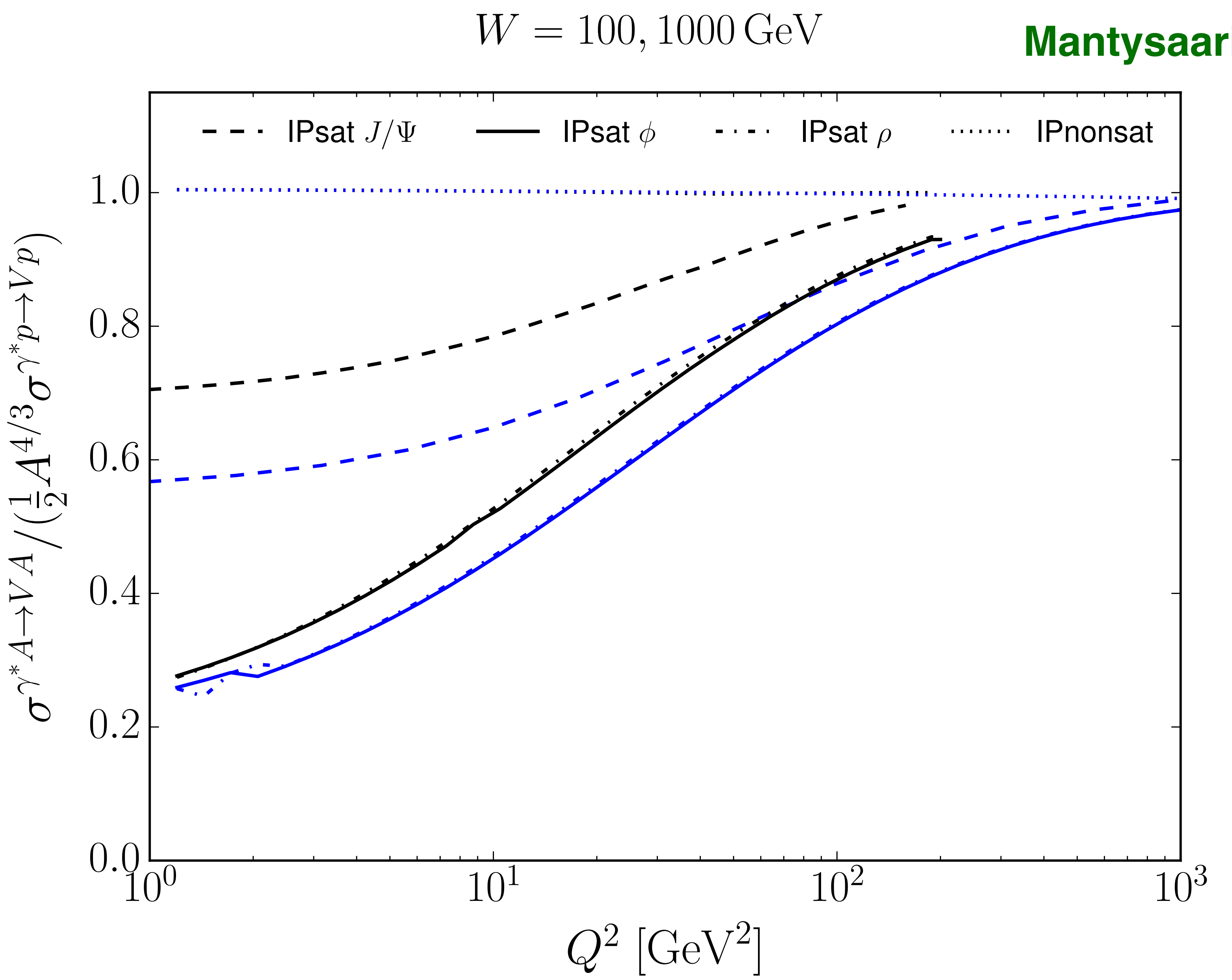
Diffractive vector meson production



- The process is defined by scattering of a color dipole on a target

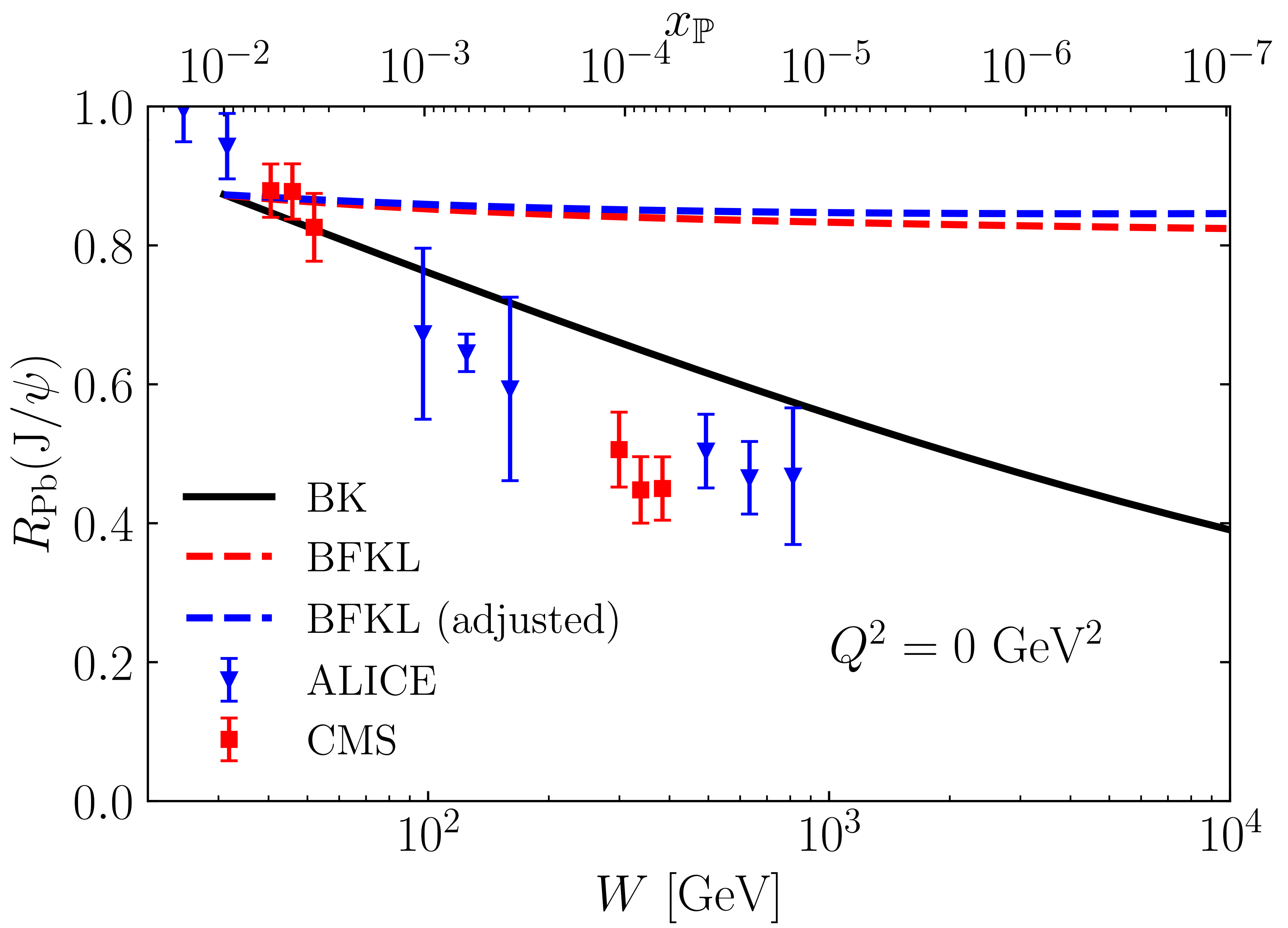
$$\sigma^{\gamma^* p \to V p} \sim |N(r_\perp)|^2$$

Mantysaari, Zurita (2018)



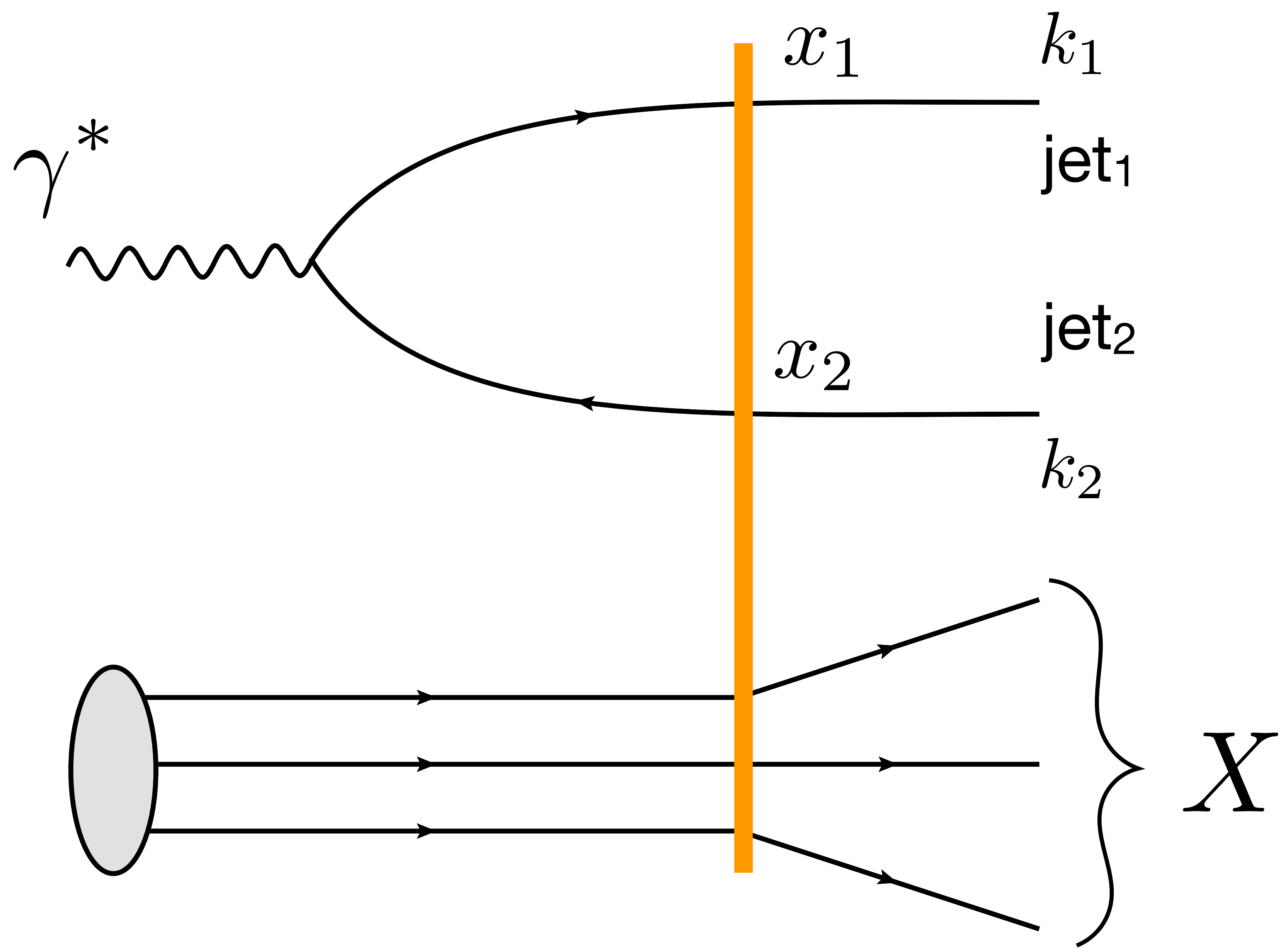
- Exclusive Heavy Vector Meson Photoproduction

Penttala, Royon (2024)



photoproduction data for Pb
shows preference for the existence
of gluon saturation

Dijet production



Dominguez, Marquet, Xiao, Yuan (2011)
Metz, Zhou (2011)

$$\frac{d\sigma^{\gamma^* A\to q\bar{q}X}}{d^3k_1 d^3k_2} \propto \prod_{i=1,2}\int d^2x_{i\perp}\int d^2y_{i\perp}\, e^{-ik_{i\perp}(x_{i\perp}-y_{i\perp})}$$

$$\times \sum_{\gamma\alpha\beta}\psi^{T,L\gamma}_{\alpha\beta}(x_{1\perp}-x_{2\perp})\psi^{T,L\gamma*}_{\alpha\beta}(y_{1\perp}-y_{2\perp})$$

quadrupole operators. Four Wilson lines at different locations!

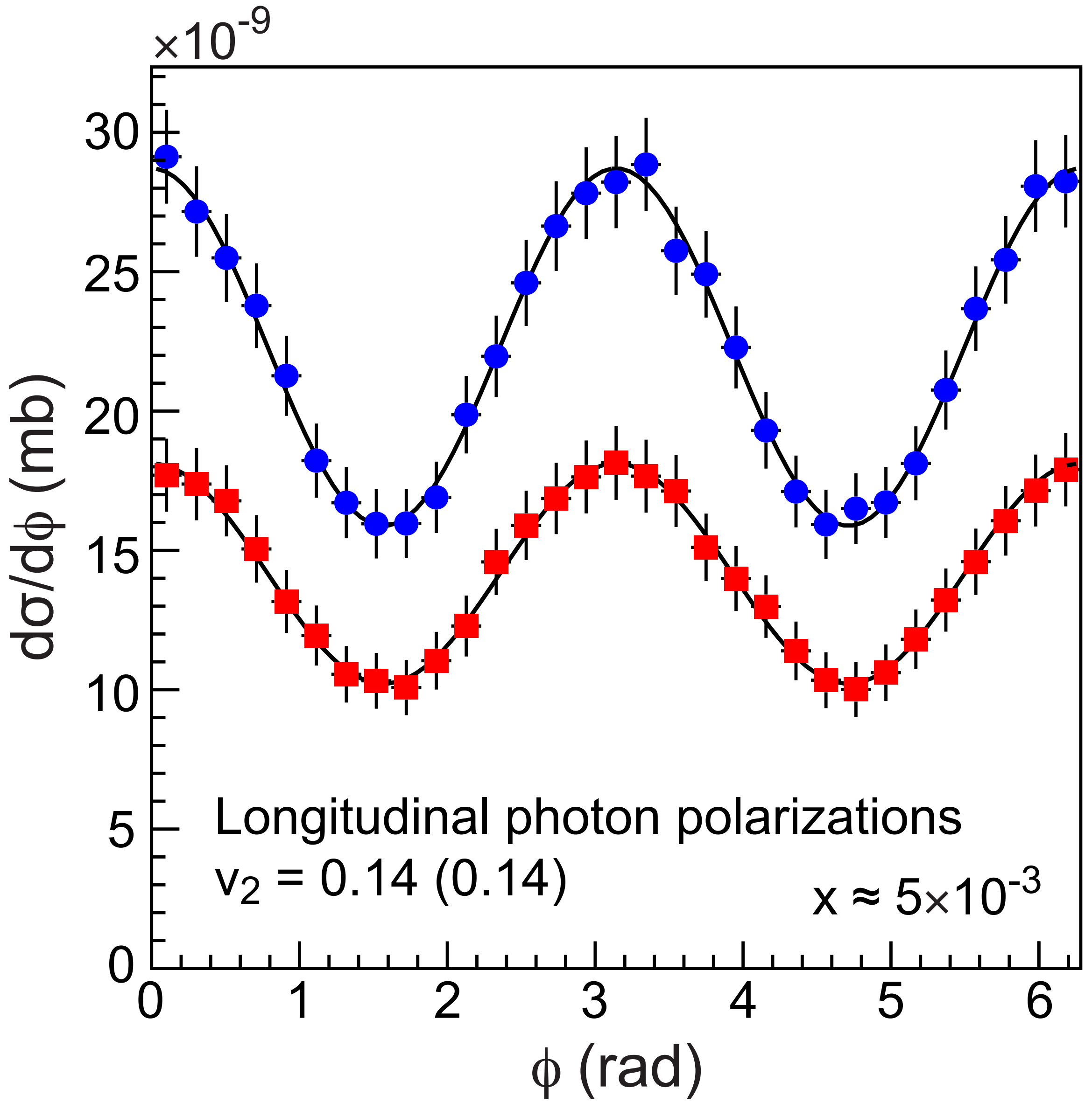
$$\times\Big[1+\frac{1}{N_c}\Big(\langle TrU(x_{1\perp})U^\dagger(y_{1\perp})U(y_{2\perp})U^\dagger(x_{2\perp})\rangle$$

$$-\langle TrU(x_{1\perp})U^\dagger(x_{2\perp})\rangle - \langle TrU(y_{1\perp})U^\dagger(y_{2\perp})\rangle\Big)\Big]$$

dipole operators

dipole operators

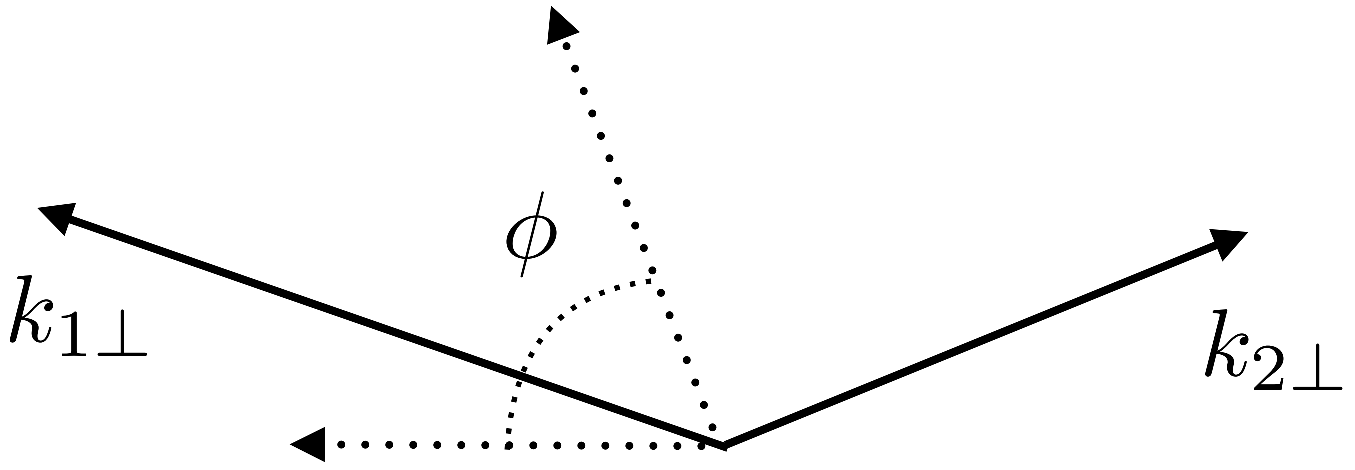
- non-trivial angular dependence



Dumitru, Skokov, Ullrich (2019)

- New types of operators! Typical for small-x calculations beyond the leading order of for more complicated scattering reactions
- Direct access to multi-point correlators
- Some simplifications in the back-to-back configuration

$$q_\perp = k_{1\perp} + k_{2\perp}$$



$$P_\perp = (k_{1\perp} - k_{2\perp})/2$$

$$\langle TrU(x_{1\perp})U^\dagger(y_{1\perp})U(y_{2\perp})U^\dagger(x_{2\perp})\rangle$$

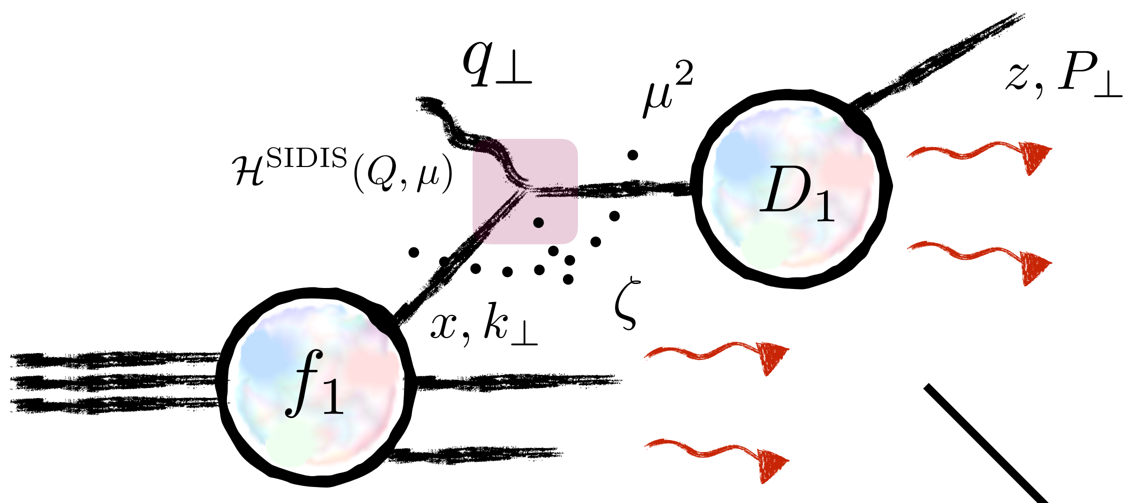
$$\Rightarrow xG^{ij}_{WW}(x,q_\perp) \propto \int\frac{d^2x_\perp}{(2\pi)^2}\int\frac{d^2y_\perp}{(2\pi)^2}e^{-iq_\perp(x_\perp-y_\perp)}$$

$$\times\boxed{\langle U^\dagger(x_\perp)\partial_i U(x_\perp)\rangle U^\dagger(y_\perp)\partial_j U(y_\perp)\rangle}$$

dipole operator related to the TMD gluon operator

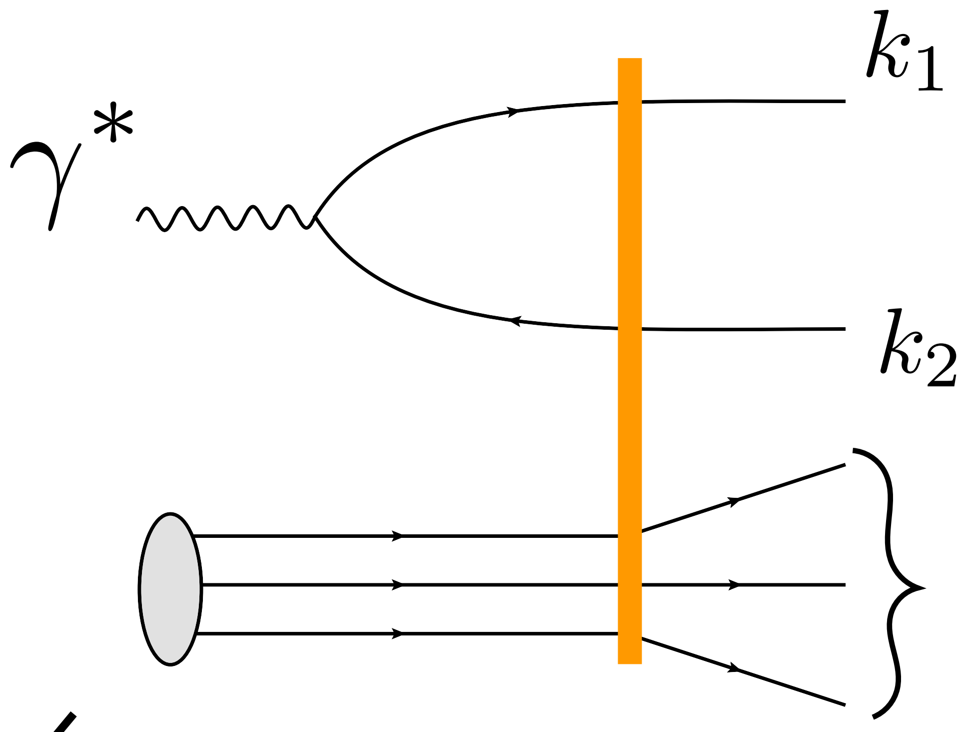
Understanding the structure of operators at small-x

- Operators appearing in different types of factorizations are related to each other
- We can **match** operators appearing in different factorization scheme
- We want to understand these relations both from phenomenological (improve extraction of operators) and theoretical (what are universal properties describing the QCD medium) points of view. Is there universality of QCD operators?



TMD factorization approach for analysis of scattering with production of a final state of small transverse, e.g. semi-inclusive DIS

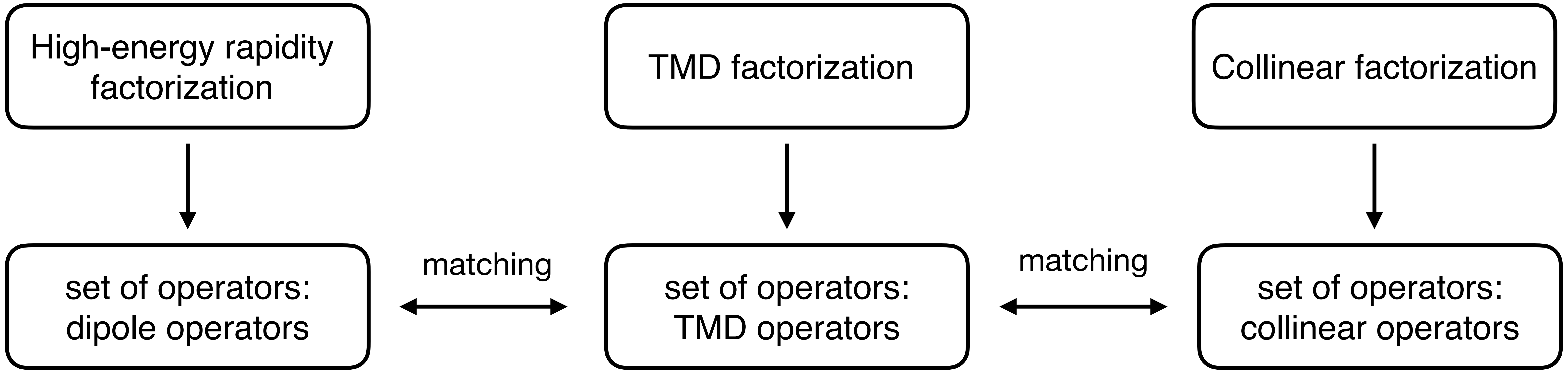
gluon TMD operator



dipole operator

$$\int_{-\infty}^{\infty} dz^- e^{-ix_B P^+ z^-} \langle p|F_{-i}(z^-,b_\perp)[z^-,\infty]_b[\infty,0^-]_0 F_{-j}(0^-,0_\perp)|p\rangle$$

$$= -\frac{1}{N_c L^-}\langle p|tr\{\partial_i U(b_\perp)U^\dagger(b_\perp)\partial_j U(0_\perp)U^\dagger(0_\perp)\}|p\rangle + \dots$$



High-energy rapidity factorization

set of operators: dipole operators

matching

TMD factorization

set of operators: TMD operators

matching

Collinear factorization

set of operators: collinear operators

- The matching can be efficiently done using the background field method
- In the small-x framework this corresponds to going beyond the eikonal picture of scattering and calculating the sub-eikonal corrections

Small-x physics at EIC: calculation of sub-ekonal corrections

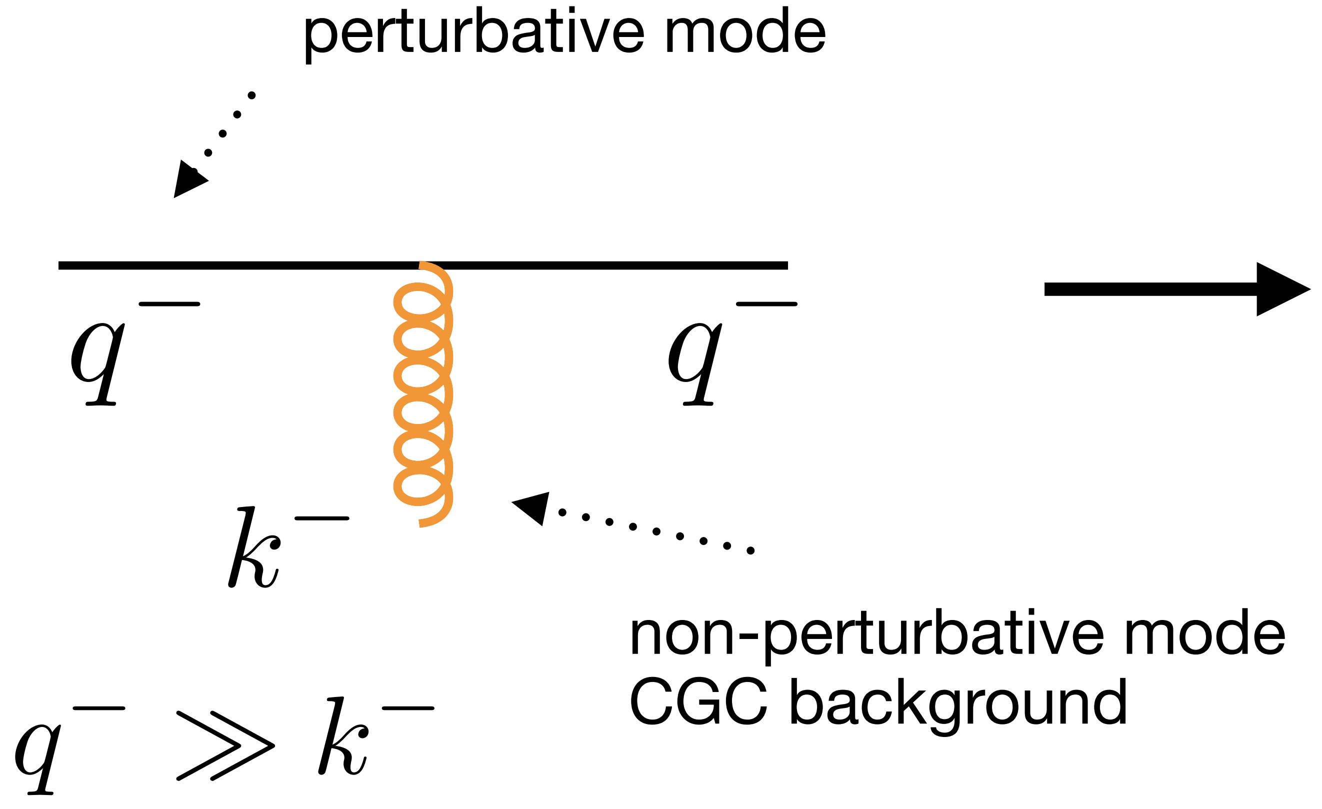
- Sub-eikonal corrections describe corrections to the shock-wave picture of scattering
- But for some observables the sub-eikonal terms provide the leading order contribution
- This corrections corresponds to matching with TMD operators

$$d\sigma = \mathrm{LO} + \mathrm{SubEik}$$

$1/s$ corrections

Altinoluk, Armesto, Beuf, Martínez, Salgado, 2014; Balitsky, Tarasov, 2015; Chirilli, 2019; Jalilian-Marian, 2019; Altinoluk and Beuf, 2022

In the leading order (LO) shock-wave picture of scattering the interaction between factorized modes is instantaneous

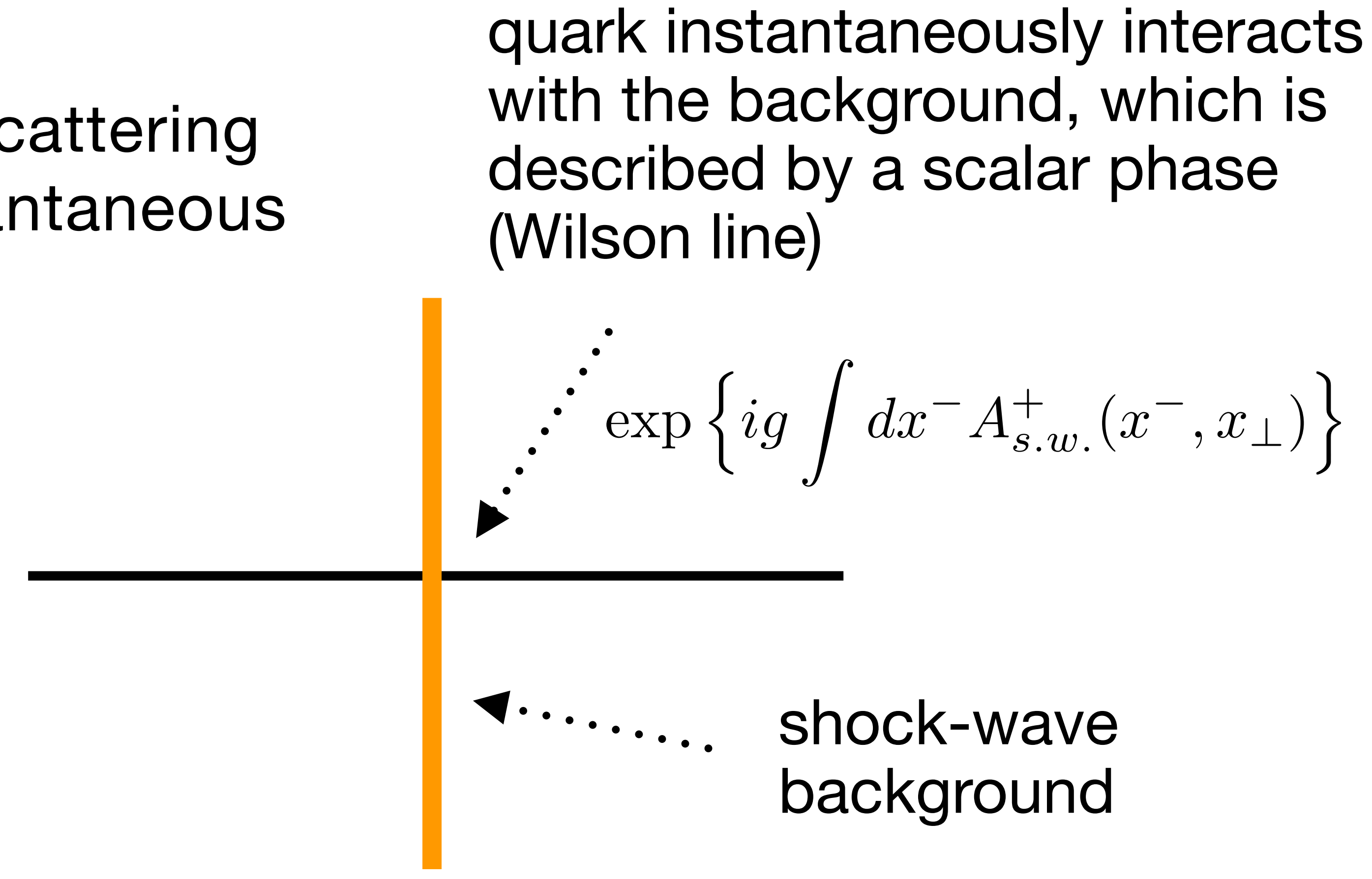


perturbative mode

q^- q^- k^-

non-perturbative mode
CGC background

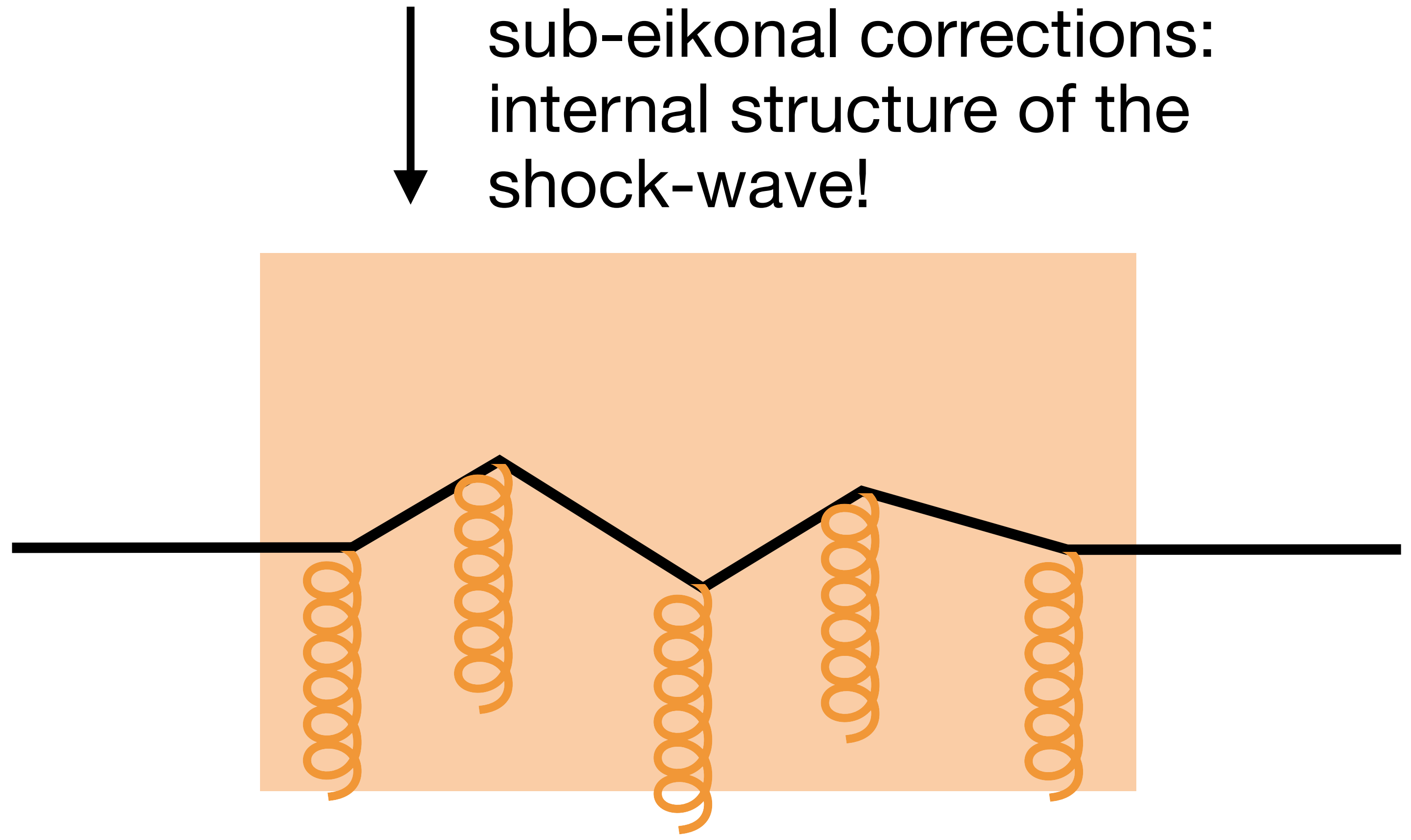
$q^- \gg k^-$



quark instantaneously interacts with the background, which is described by a scalar phase (Wilson line)

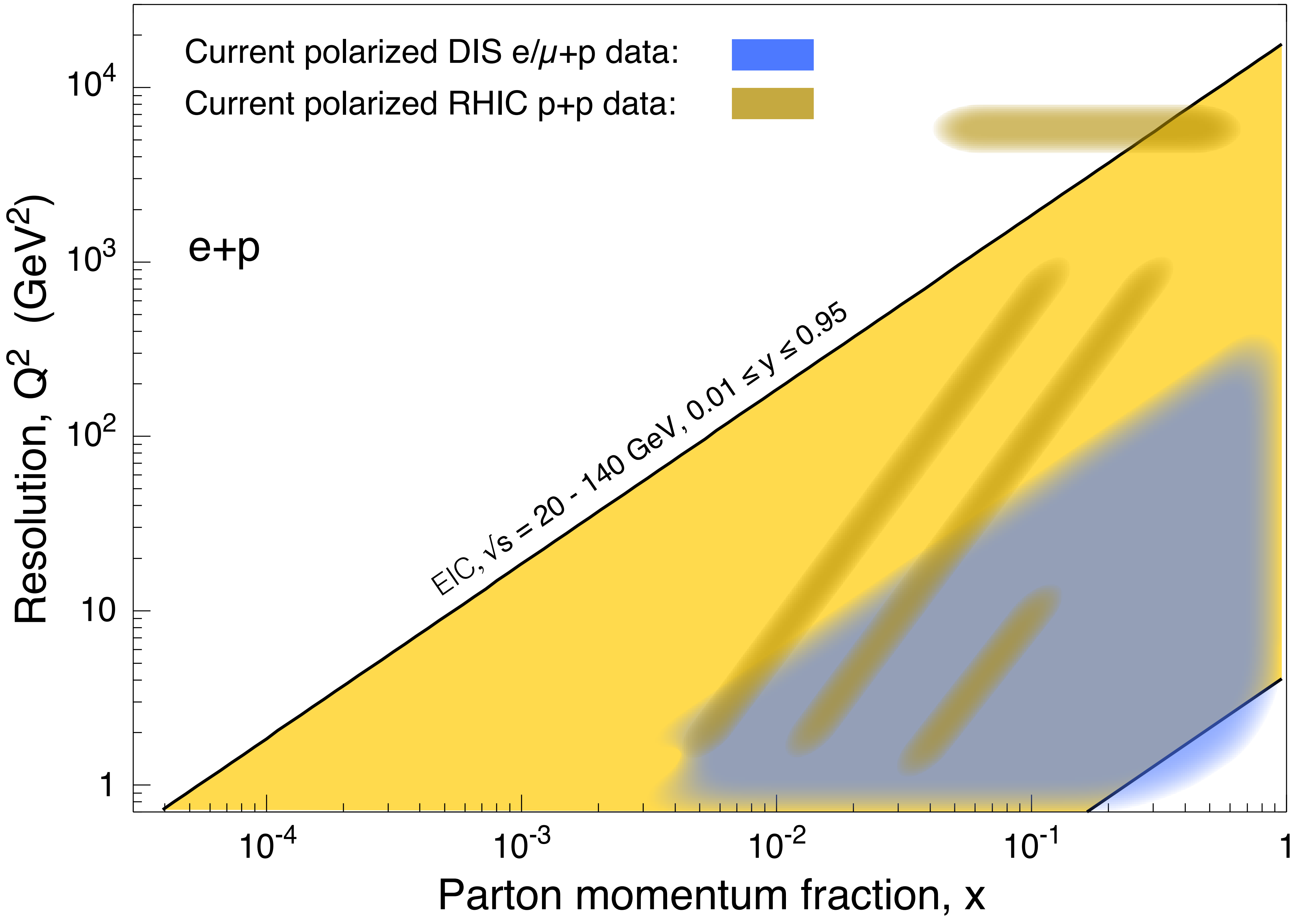
$\exp\left\{ig\int dx^- A^+_{s.w.}(x^-,x_\perp)\right\}$

shock-wave background



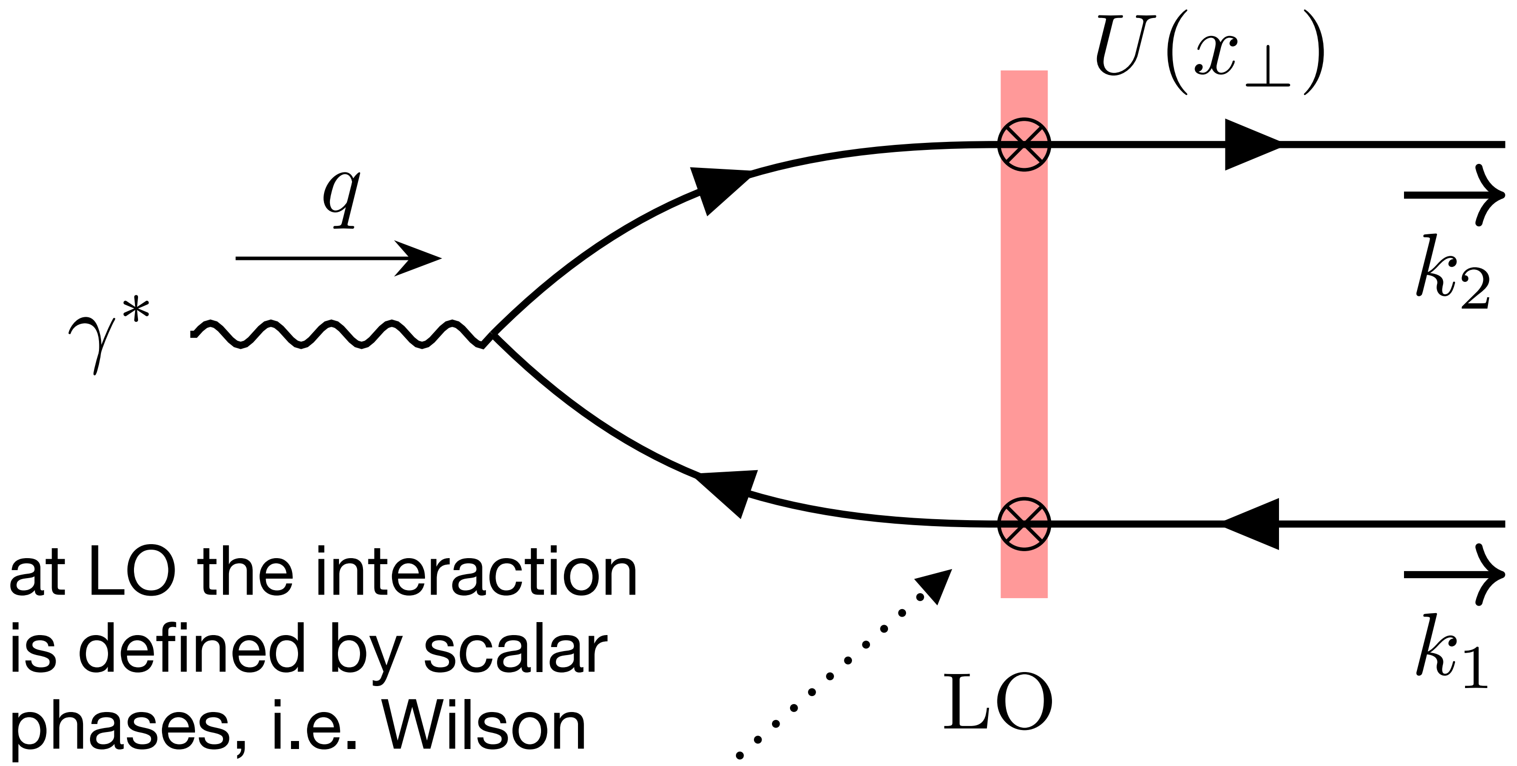
sub-eikonal corrections: internal structure of the shock-wave!

- How does the shock-wave decay? ⇒ large-x effects



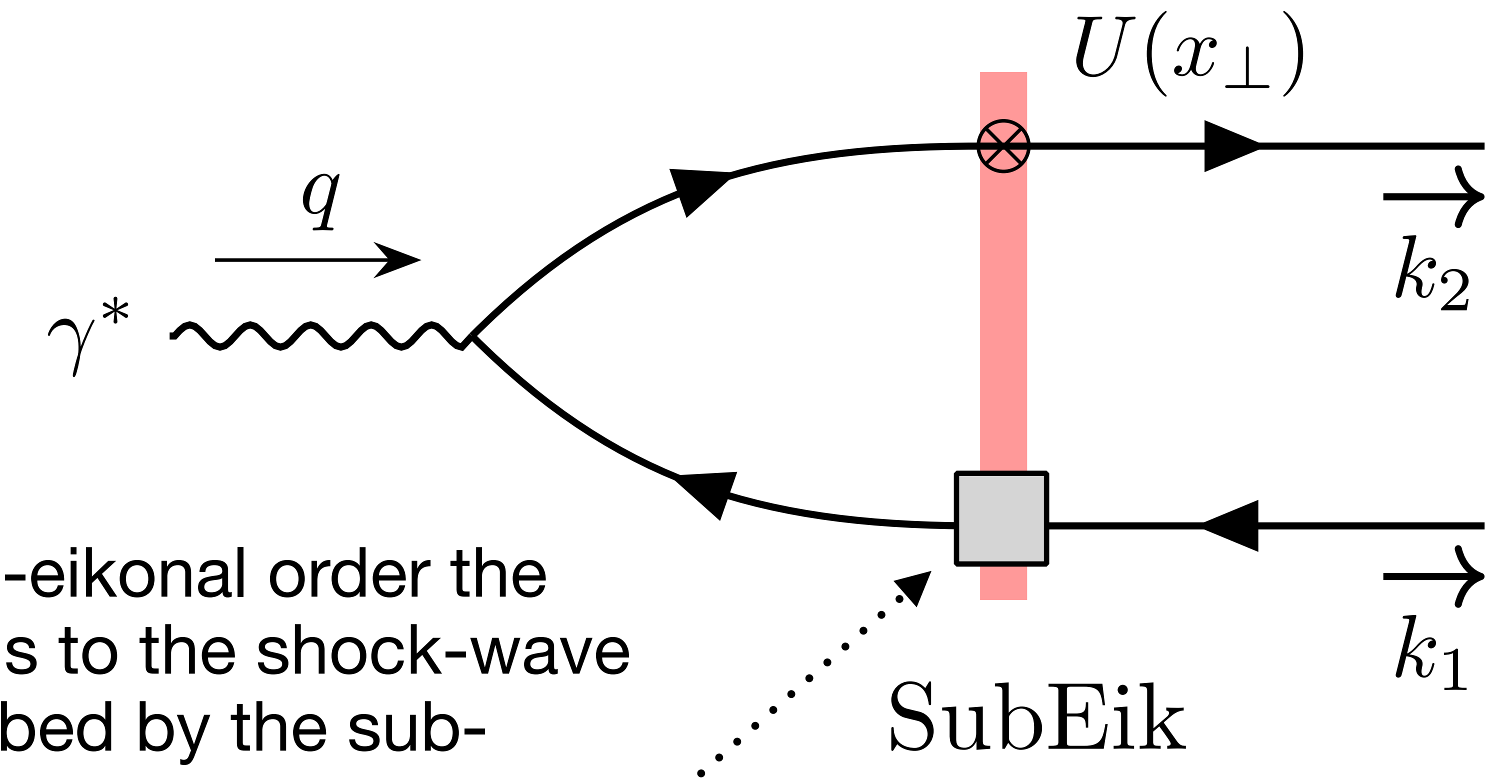
- Sub-eikonal effects dominate the region of moderate x (decay of the shock-wave)
- Sub-eikonal corrections are crucial for the search of saturation at EIC

The scattering at the sub-eikonal order



at LO the interaction is defined by scalar phases, i.e. Wilson line operators

$$U(y_\perp) \equiv \exp\left\{ig\int_{-\infty}^{\infty} dz^- A^+_{s.w.}(z^-, y_\perp)\right\}$$

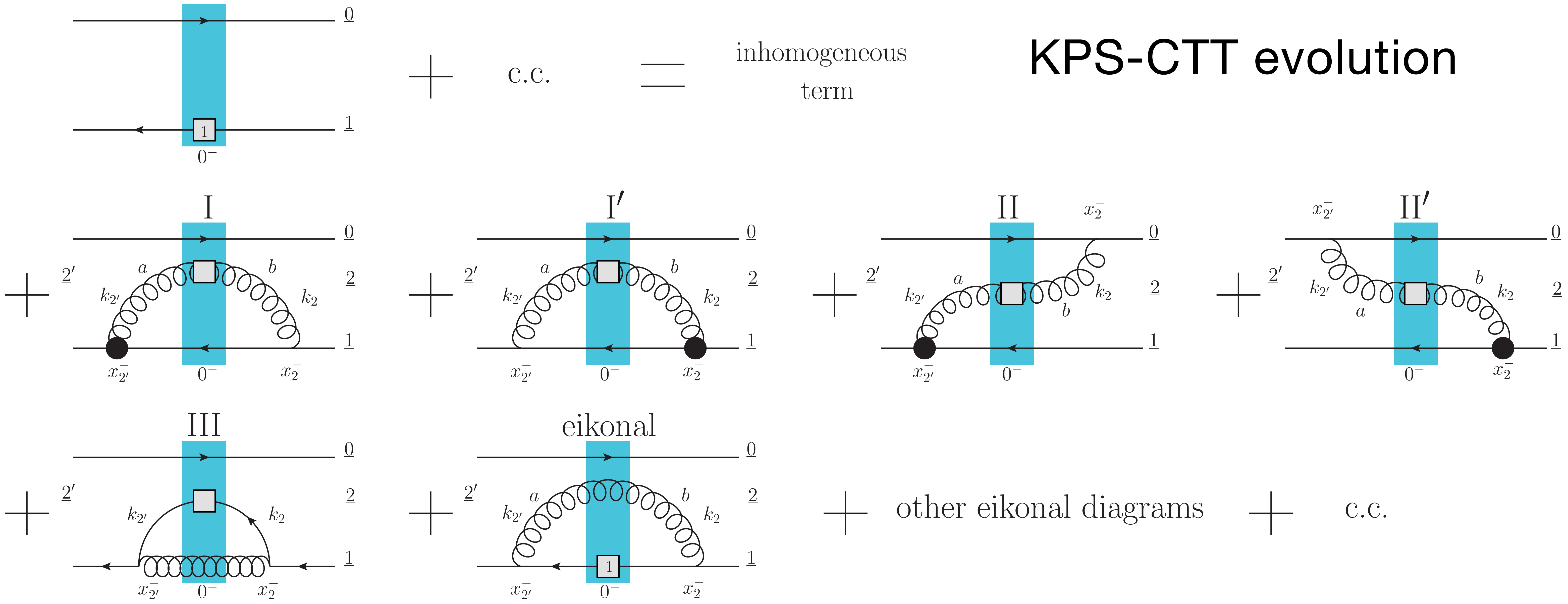


at the sub-eikonal order the corrections to the shock-wave are described by the sub-eikonal operators

$$U^{\mathrm{sub-eik.}}(y_\perp) = ig\int_{-\infty}^{\infty} dz^- U_{[\infty,z^-]}(y_\perp) F_{12}(z^-, y_\perp) U_{[z^-,-\infty]}(y_\perp)$$

- The *scalar* phase at LO cannot describe spin effects ⇒ spin effects at small-x appear at sub-eikonal order
- Evolution properties of sub-eikonal operators are known ⇒ can construct predictions for observables

Kovchegov, Pitonyak, Sievert, 2016; Cougoulic, Kovchegov, Tarasov, Tawabutr, 2022; Borden, Kovchegov, Li, 2023

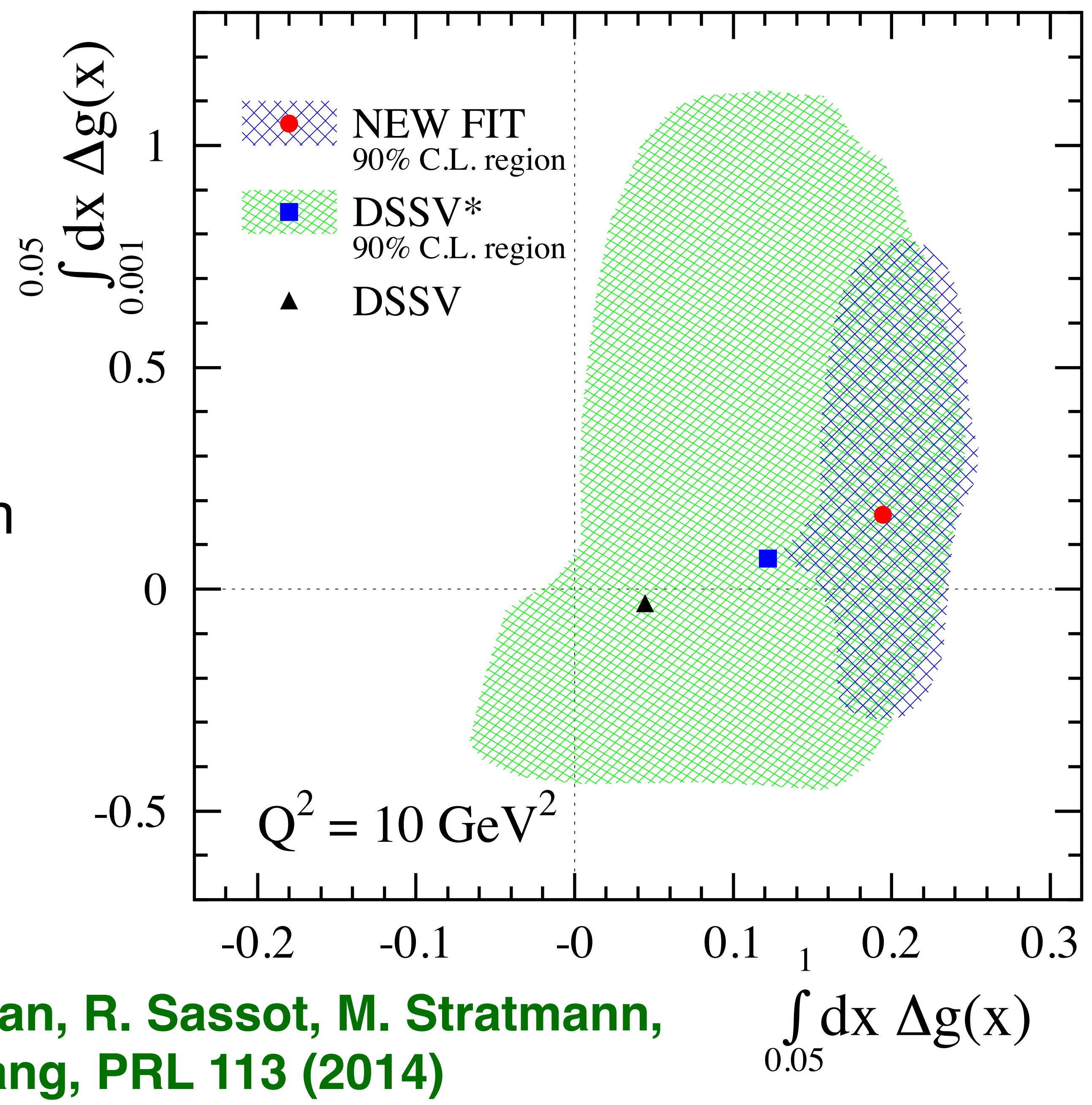


KPS-CTT evolution

How does the spin of the nucleon arise?

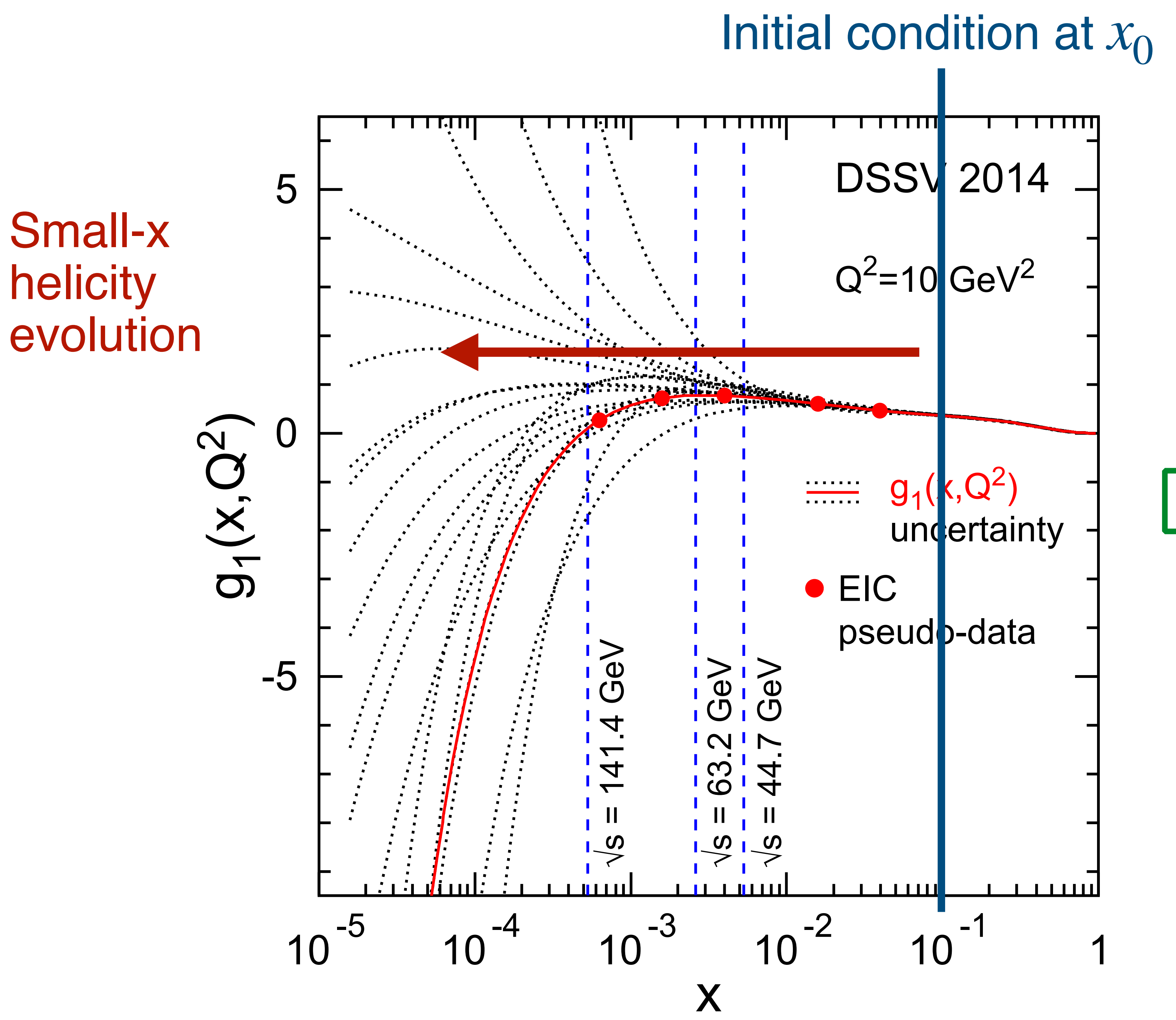
$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

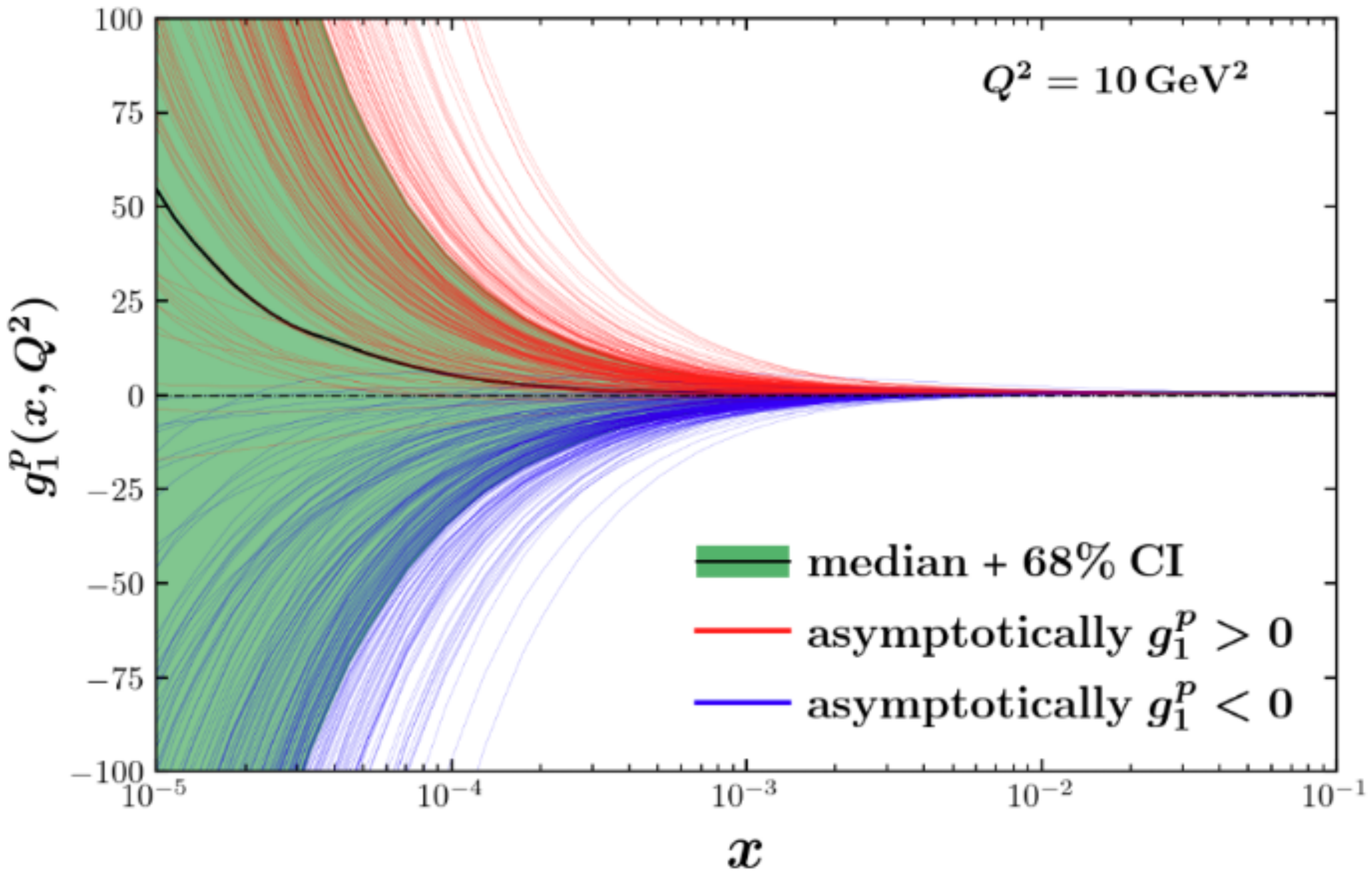
large uncertainties from the small-x region



D. De Florian, R. Sassot, M. Stratmann, W. Vogelsang, PRL 113 (2014)

- In the small x region that cannot be probed experimentally, DGLAP-based predictions acquire a broad uncertainty band
- The benefit of small-x evolution is that it makes a genuine prediction for PDFs at small x given some initial conditions at a higher values of x





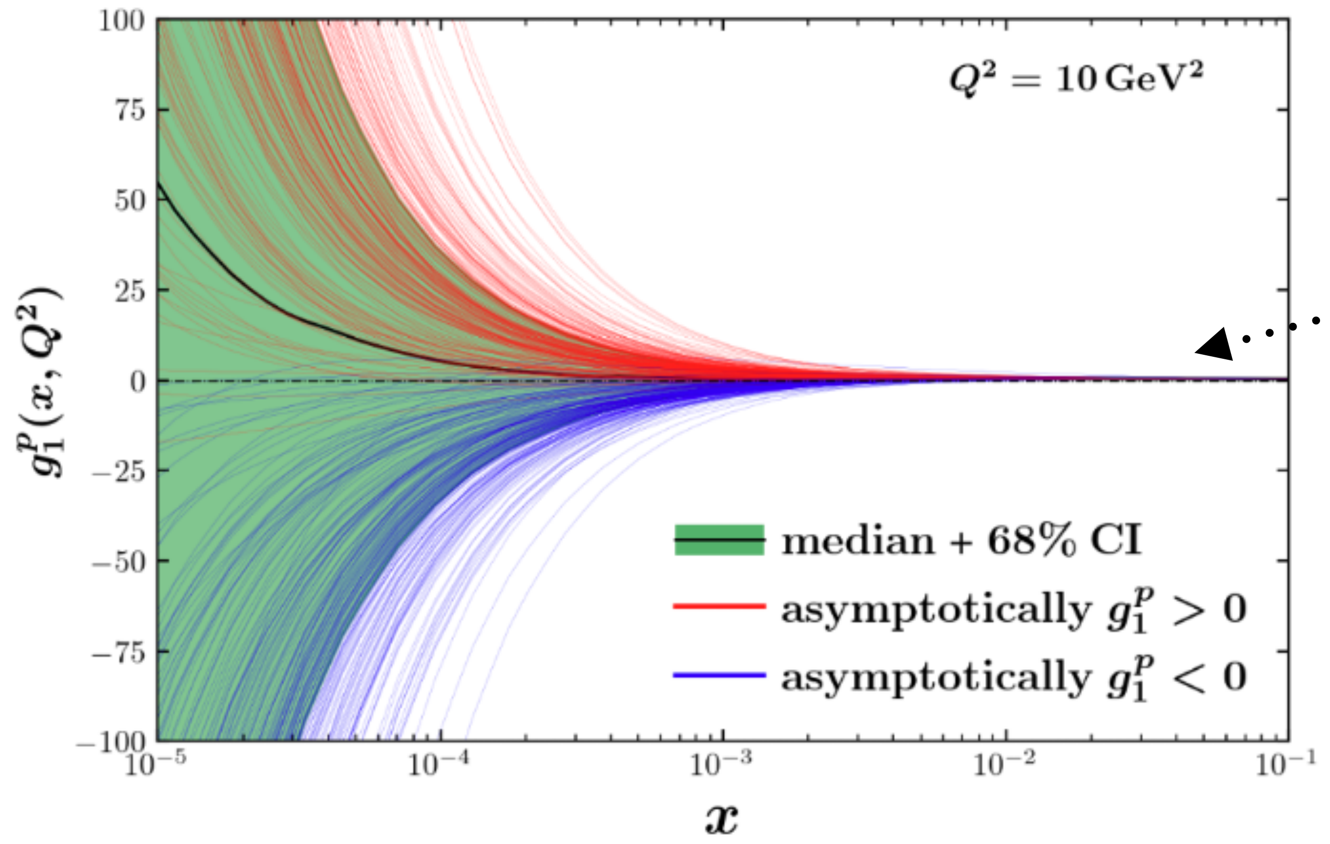
Baldonado et al. (2025)

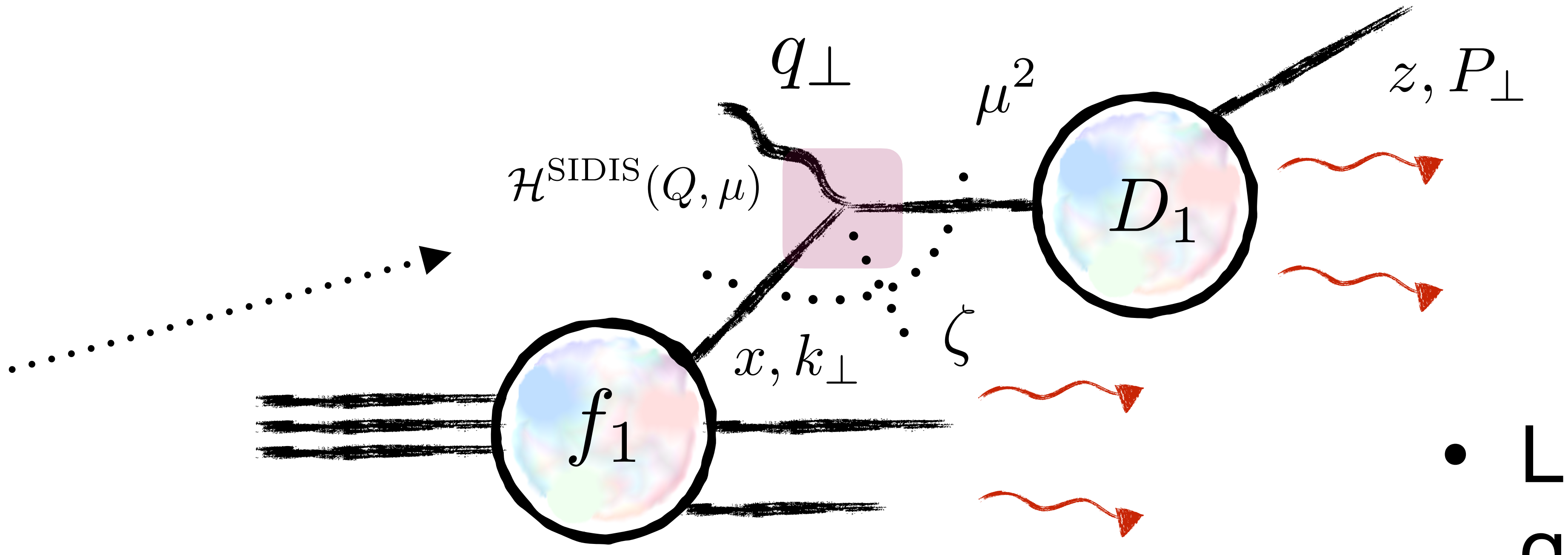
All curves are defined by the same intercept

$$g_1(x,Q^2) \sim \left(\frac{1}{x}\right)^{\alpha_h}$$

The uncertainties are driven by the initial conditions ⇒ TMD physics can provide appropriate initial condition at large-x for the small-x evolution ⇒ **matching with TMDPDFs**

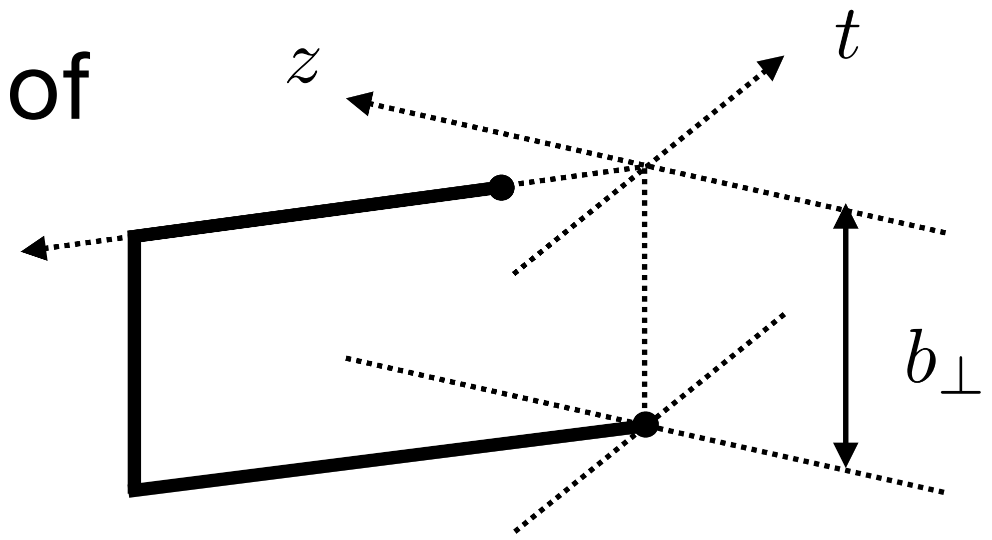
Initial conditions for the small-x evolution





- Can we reconstruct initial conditions for the small-x evolution through matching with TMDPDFs extracted at large-x

- Lattice calculations of quasi-TMDPDFs



- Currently in the TMD phenomenology the TMDPDFs are reconstructed through the matching with the collinear PDFs:

TMD distribution for the small $q_\perp$ region

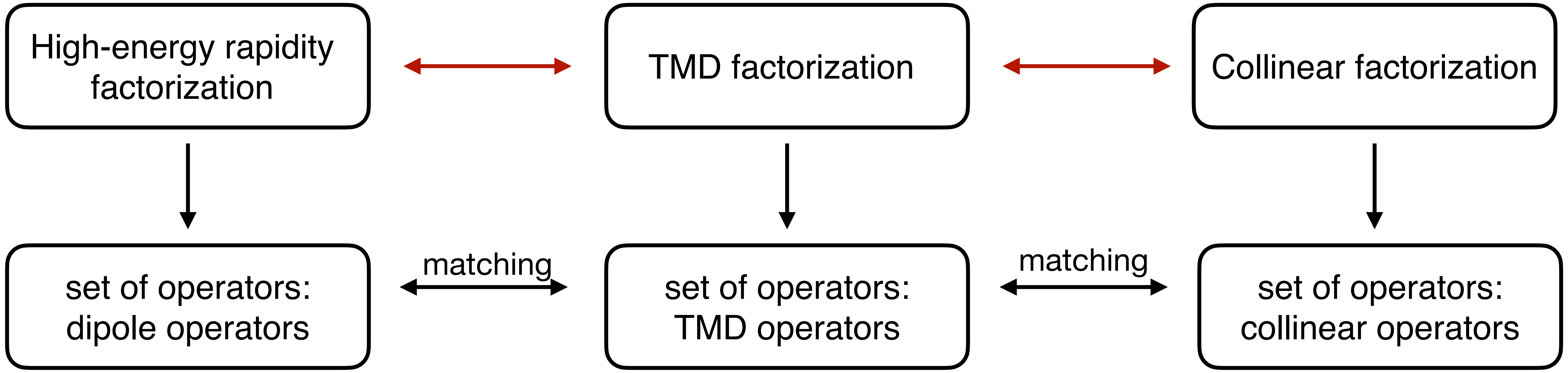
Collins (2011)
TMD Handbook (2024)

$$\hat{f}_1^a(x, b_\perp^2; \mu_f, \zeta_f)$$

$$= \underbrace{[C \otimes f_1](x, b_*; \mu_{b_*}, \mu_{b_*}^2)}_{\text{Collinear PDFs with DGLAP logarithms}} \underbrace{\exp\left\{\int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu}\gamma(\mu, \zeta_f)\right\}\left(\frac{\zeta_f}{\mu_{b_*}^2}\right)^{K(b_*, \mu_{b_*})/2}}_{\text{CSS evolution}} \underbrace{f_{1\mathrm{NP}}(x, b_\perp^2; \zeta_f, Q_0)}_{\text{Phenomenological function encompassing all-collinear twist content of the distribution}}$$

- We need to extract TMD in a way that is consistent with the small-x evolution

- **We want to understand how different factorization schemes are related to each other**


High-energy rapidity factorization

TMD factorization

Collinear factorization

set of operators: dipole operators

matching

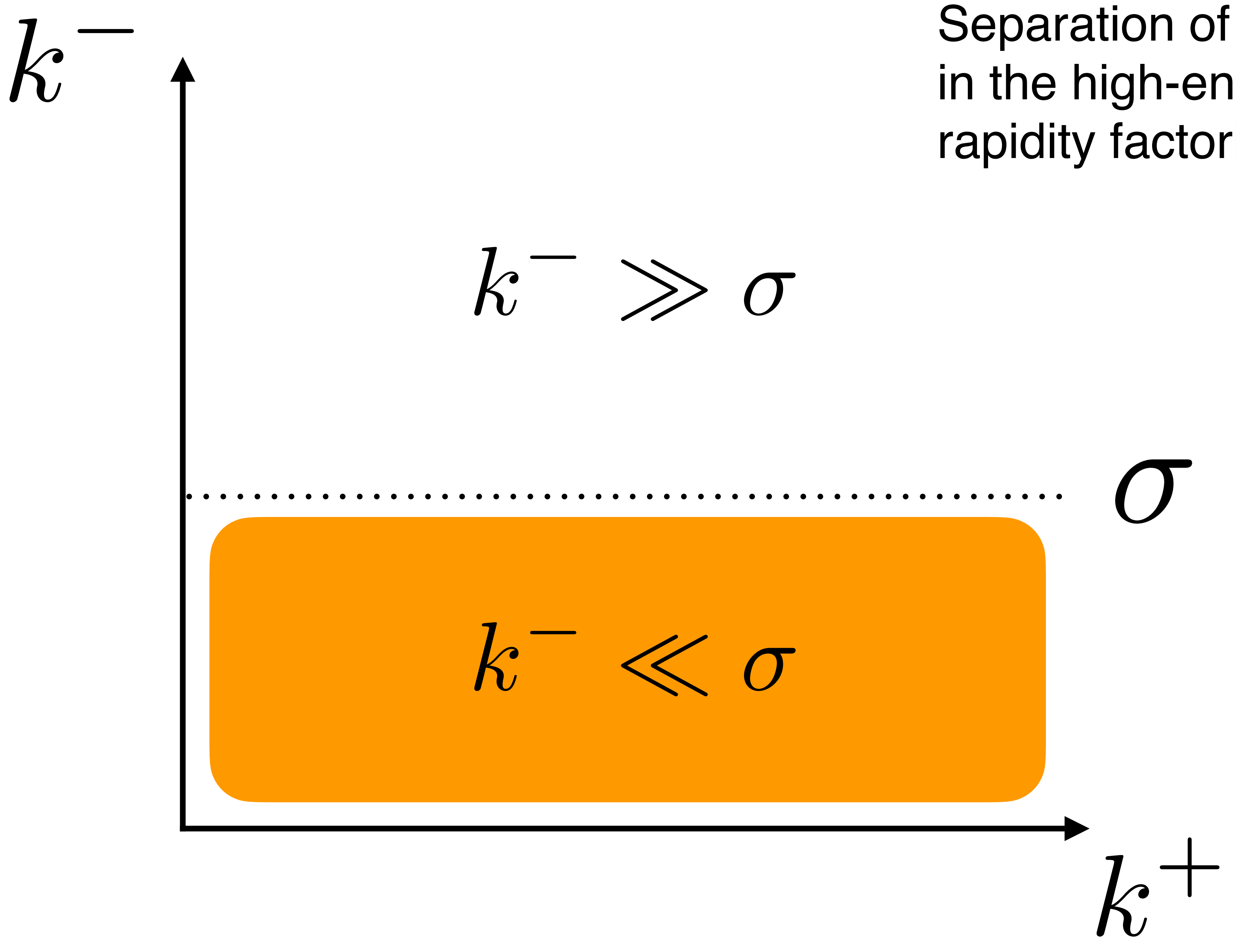
set of operators: TMD operators

matching

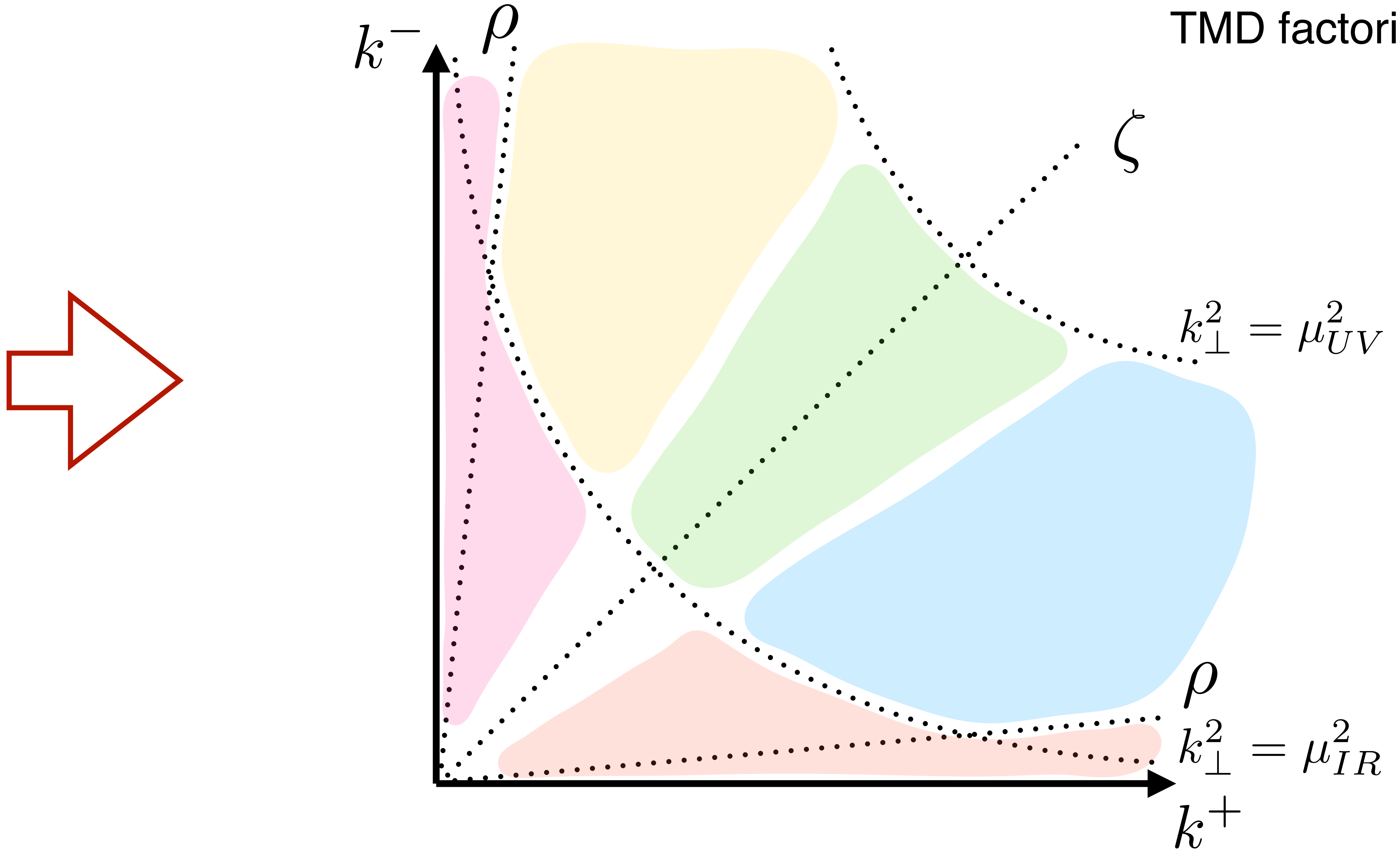
set of operators: collinear operators

- The set of operators in only defined within a particular factorization scheme

Mukherjee, Skokov, Tarasov, Tiwari (2023)
Duan, Kovner, Lublinsky (2024)
Caucal, Iancu (2024)



Separation of modes in the high-energy rapidity factorization



Separation of modes in the TMD factorization

Virtual diagrams for the gluon TMD operator

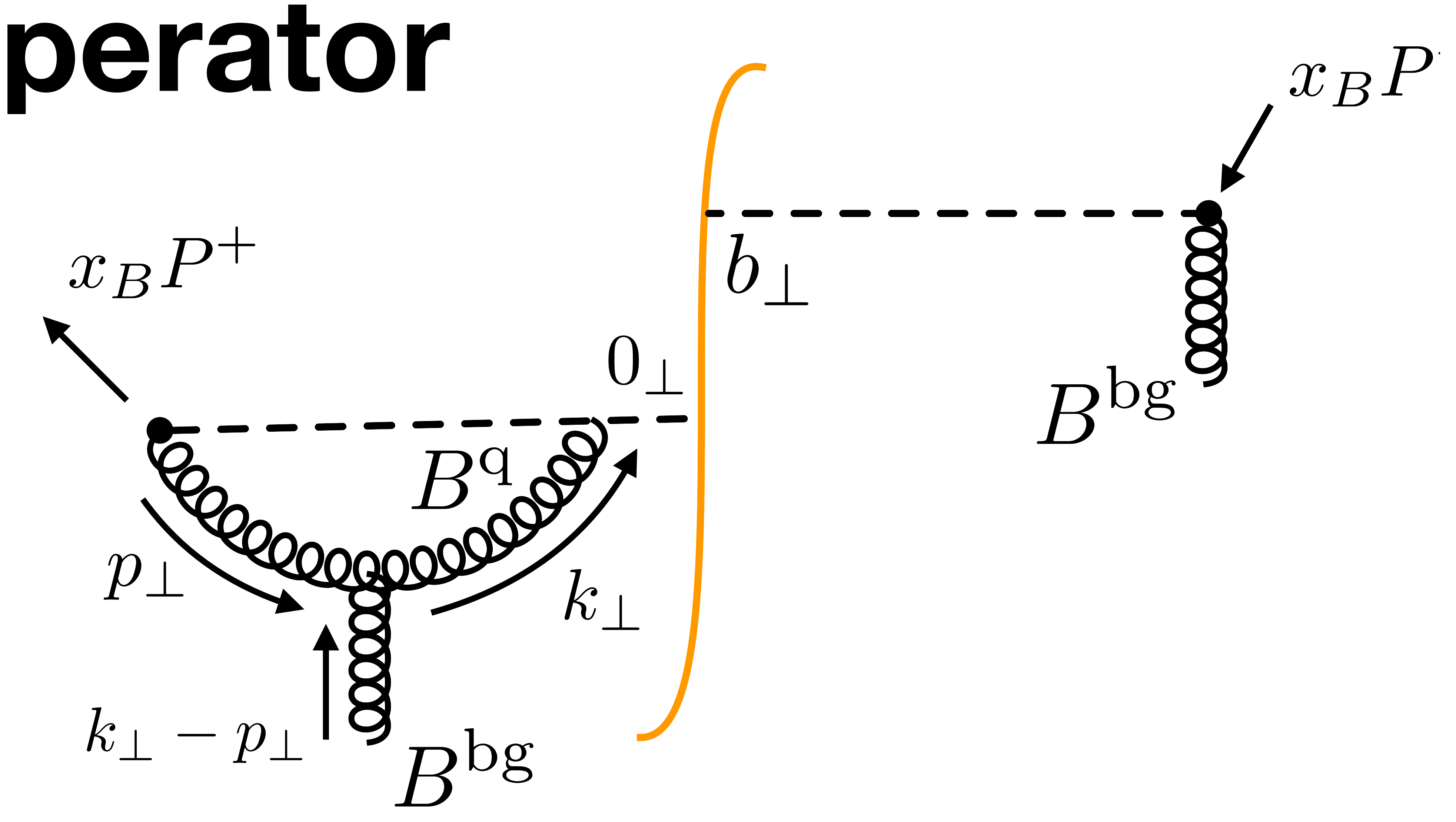
- Virtual emission in the gluon TMDPDFs

logarithmic dependence on ρ; “virtual” part of the BFKL evolution kernel

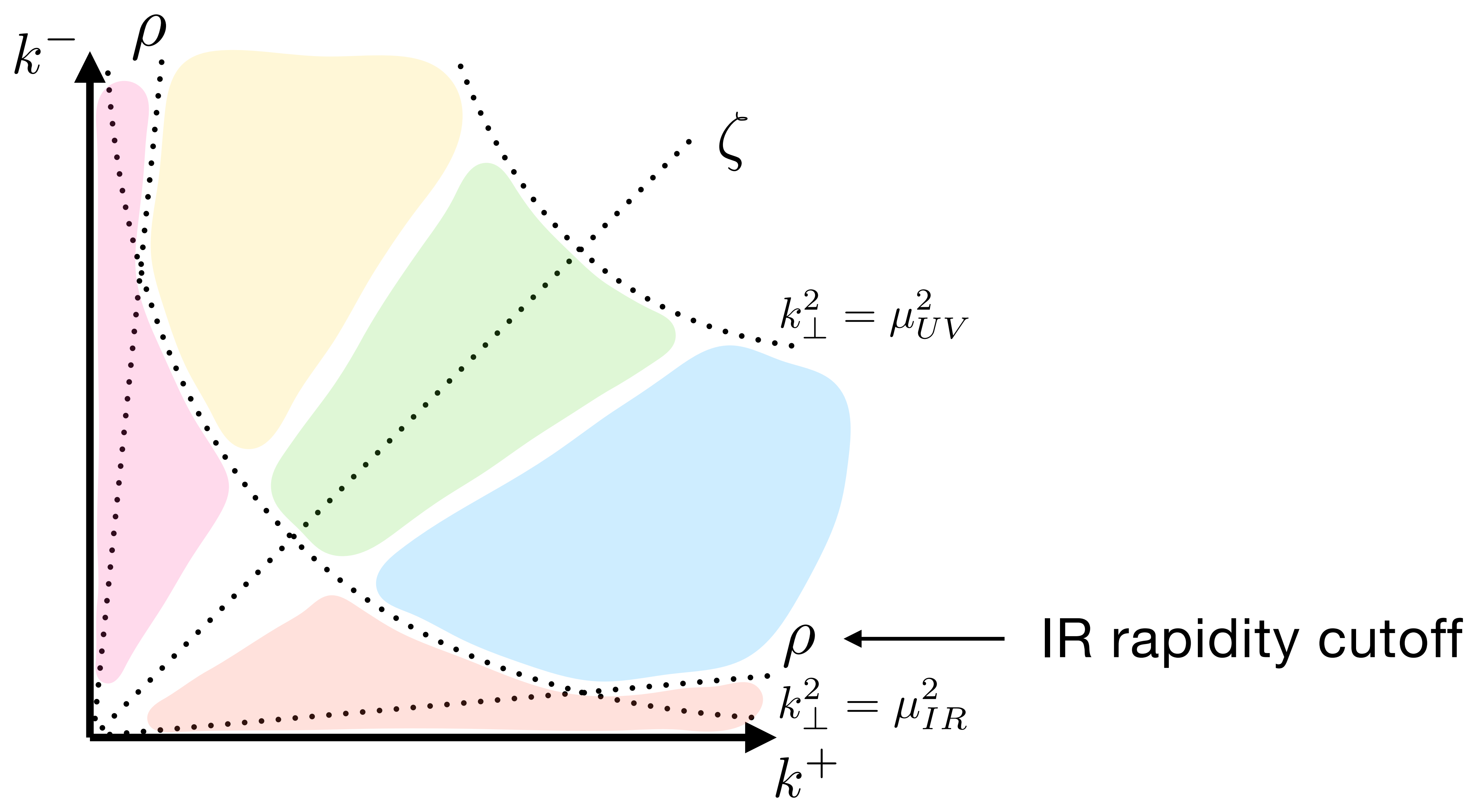
The result doesn’t depend on the value of x

$$\mathcal{B}_{ij}^{\mathrm{q}(1)+\mathrm{bg;virt}}(x_B,b_\perp) = -\frac{\alpha_s N_c}{2\pi}\left(\frac{1}{\epsilon_{\mathrm{IR}}^2}+\frac{1}{\epsilon_{\mathrm{IR}}}\left(\frac{1}{\xi}+\ln(\frac{\rho}{x_BP^+})\right)-\frac{\pi^2}{12}\right)$$

$$\times\int d^2z_\perp\int \bar{d}^2p_\perp e^{ip_\perp(b-z)_\perp}\left(\frac{\mu_{\mathrm{IR}}^2}{p_\perp^2}\right)^{\epsilon_{\mathrm{IR}}}\frac{g_{il}p_jp_m+p_ip_lg_{mj}}{p_\perp^2}\mathcal{B}^{\mathrm{bg}}_{lm}(x_B,z_\perp)$$



Mukherjee, Skokov, Tarasov, Tiwari (2023)



- The gluon TMDPDFs at **large** x_B contain logarithms of the BFKL type!
- Deep relation between the small-x formalism and the TMD factorization approach

Small-x physics at EIC: calculation of NLO corrections

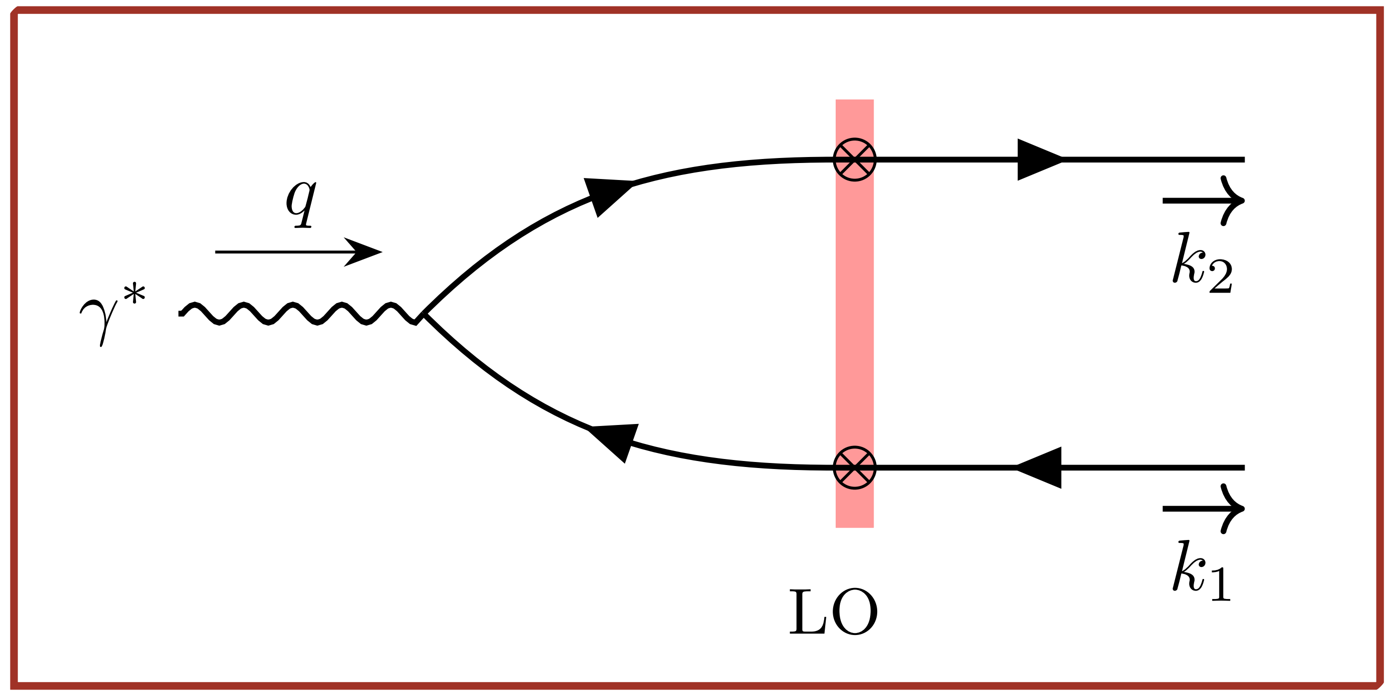
- Understanding the matching between different factorization schemes is crucial for the NLO calculations at small-x which are sensitive to contribution of the collinear and Sudakov logarithms

$$d\sigma = \text{LO} + \text{NLO}$$

α_s corrections

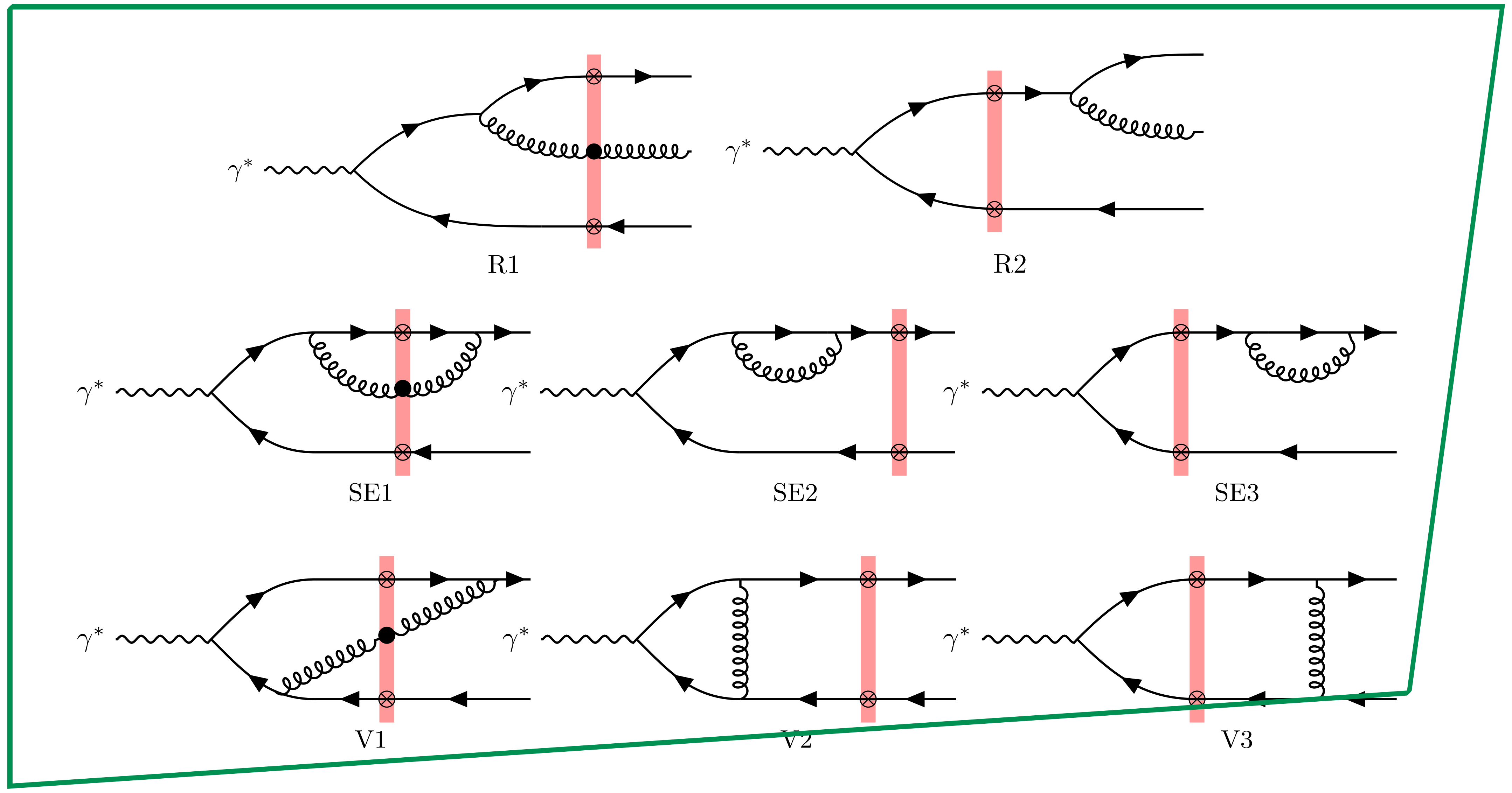
Balitsky, Chirilli, 2008; Caron-Huot, Herranen, 2016; Chirilli, Xiao, Yuan, 2012; Boussarie, Grabovsky, Ivanov, Szymanowski, Wallon 2016; Iancu, Mulian 2021, Kovchegov, Weigert 2007; Kovner, Lublinsky, Zhao (2024)

Precision small-x physics at EIC



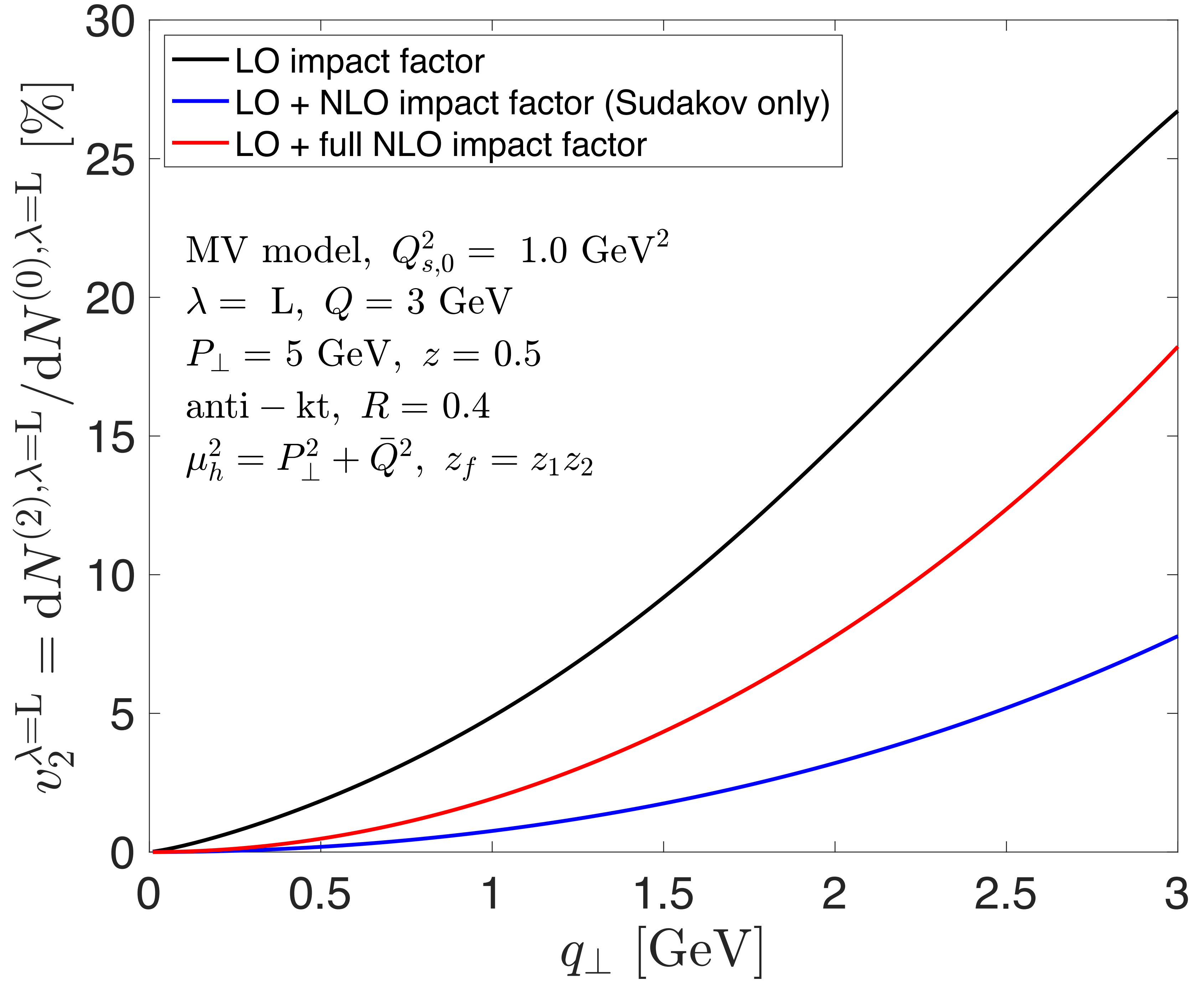
leading order (LO) contribution to the dijet production

Still many open questions: different schemes of resummation of DGLAP and Sudakov logs, instability of solutions



NLO corrections in the shock-wave picture of scattering

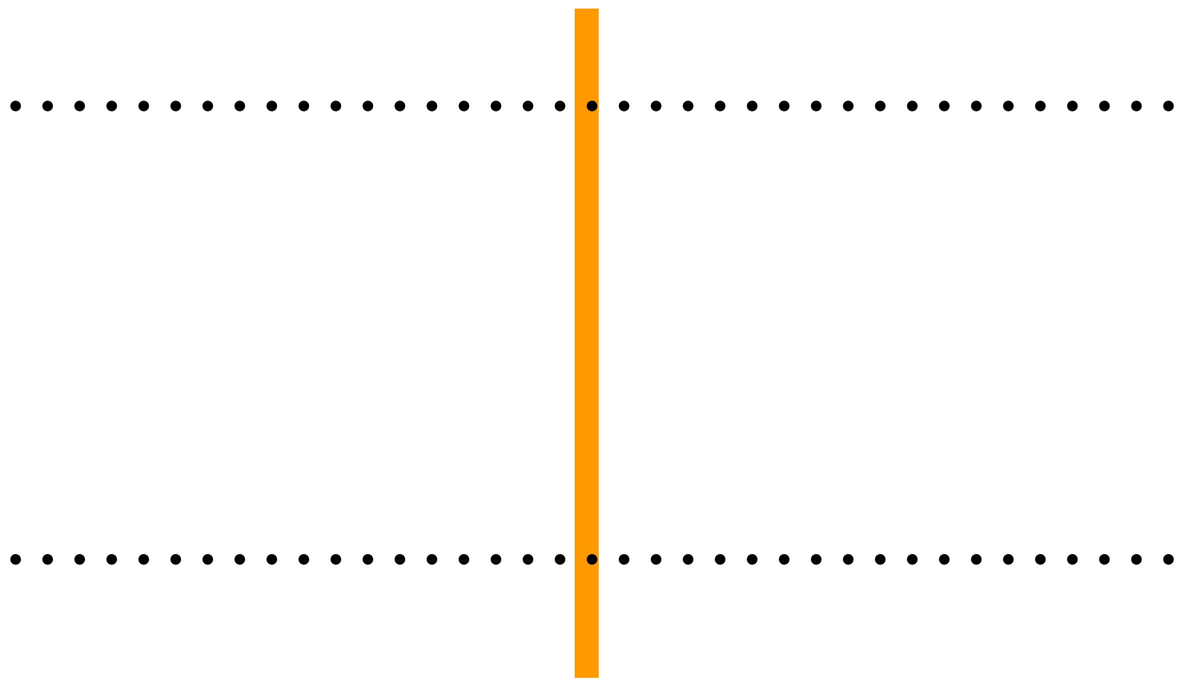
+ NLO evolution, running coupling corrections



Caucal, Salazar, Schenke, Stebel, Venugopalan (2023)

Extension of the MV model

- The MV model provides a semi-classical solution for the dipole amplitudes



$$\langle TrU(x_\perp)U^\dagger(y_\perp)\rangle$$

color dipole

- MV model for the background field (shock-wave)

shock-wave structure

$$A^{cl}_-(x) = -\frac{1}{\partial^2_\perp}\rho(x_\perp)\delta(x^-)$$

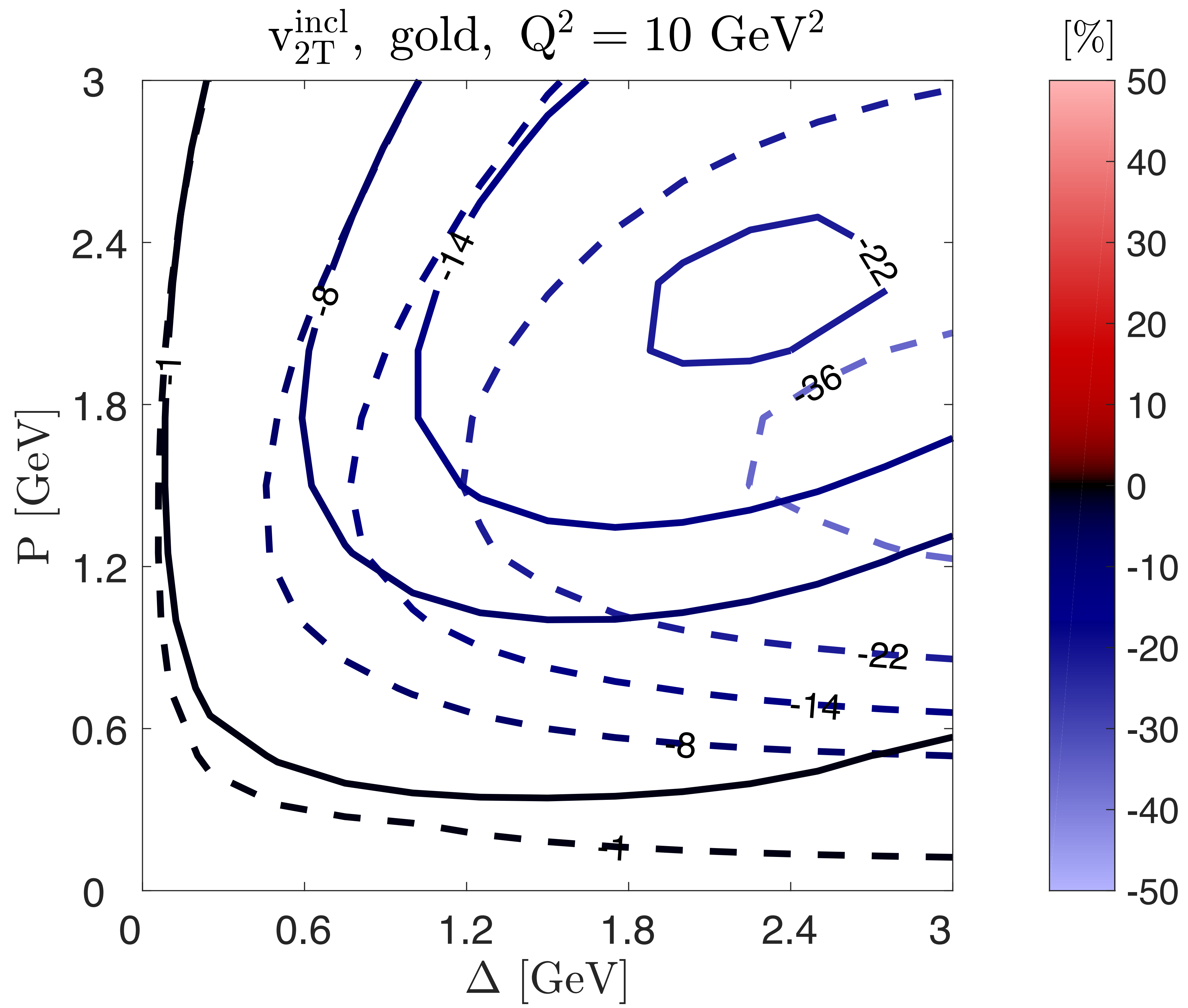
$$A^{cl}_+ = A^{cl}_\perp = 0$$

- MV doesn't work for sub-eikonal and spin

- In many problems more complicated operators appear

Mantysaari, Mueller, Salazar, Schenke (2020)

$$\langle Tr\partial_i U(b_\perp)U^\dagger(b_\perp)\partial_j U(0_\perp)U^\dagger(0_\perp)\rangle$$



- Can we calculate more complicated operators, e.g. with subeikonal effects etc. ⇒ extension of the MV model

- The general model should satisfy the high-energy scaling ($\lambda \to \infty$)

Li (2025), Kovchegov, Cougoulic (2020)

$$A_-(x^+, x^-, x_\perp) \sim \lambda\tilde{A}_-(\lambda^{-1}x^+, \lambda x^-, x_\perp)$$

$$A_i(x^+, x^-, x_\perp) \sim \tilde{A}_i(\lambda^{-1}x^+, \lambda x^-, x_\perp)$$

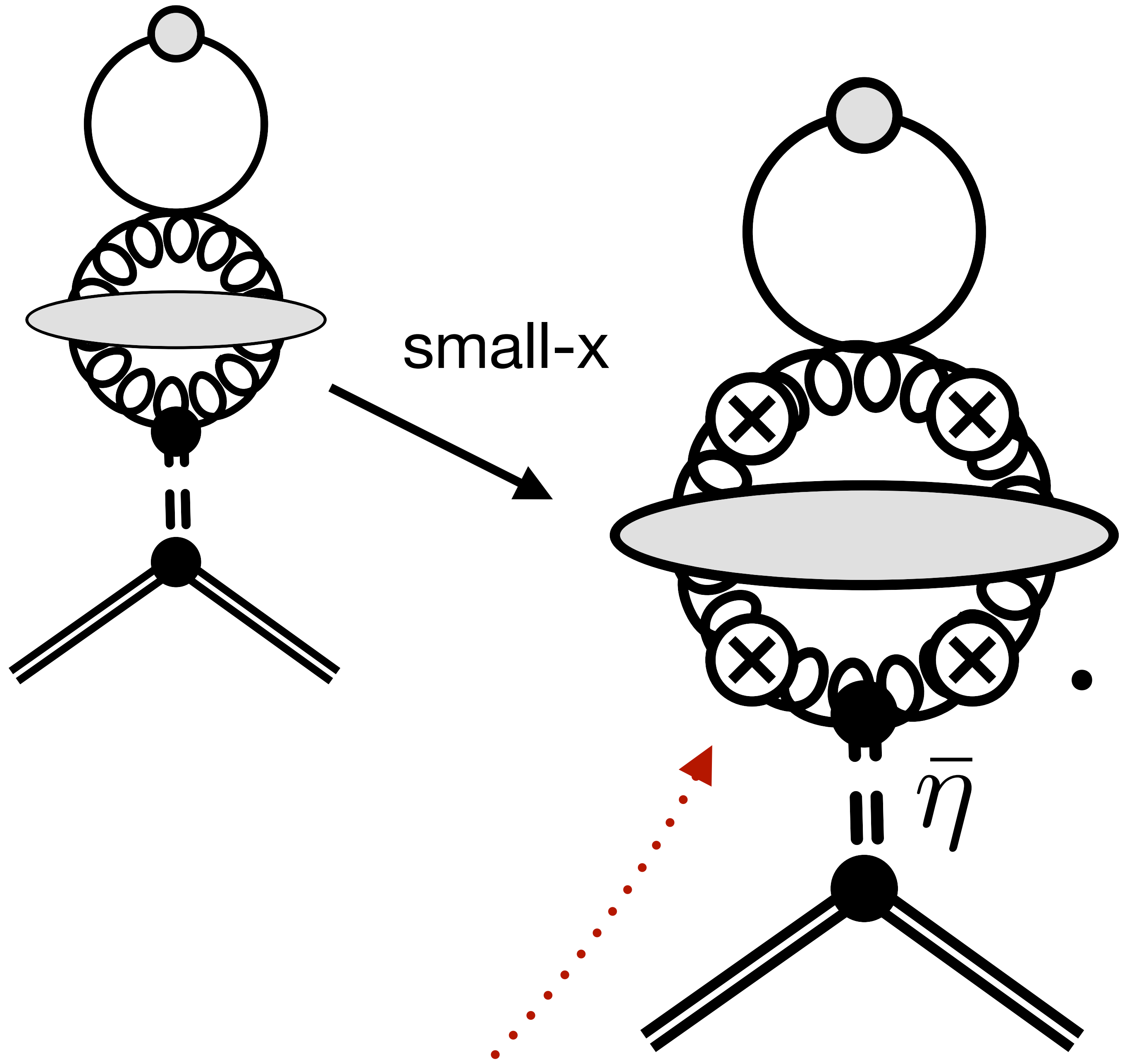
$$A_+(x^+, x^-, x_\perp) \sim \lambda^{-1}\tilde{A}_+(\lambda^{-1}x^+, \lambda x^-, x_\perp)$$

Verlinde, Verlinde (1993)

Sphaleron transitions in DIS structure functions at small-x

Tarasov, Venugopalan (2020)

Get access to topological properties of the QCD vacuum in DIS experiments → $g_1(x_B)$



$$\int_0^1 dx g_1^{p,n}(x,Q^2) = \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right)\left[\pm\frac{1}{12}A_3 + \frac{1}{36}A_8 + \frac{1}{9}\Delta\Sigma(Q^2)\right] + \mathcal{O}(\alpha_s^2)$$

$$\Delta\Sigma|_{m=0} = \frac{N_f}{M_N}\sqrt{\chi'_{\mathrm{QCD}}(0)}\, g_{\bar{\eta}NN}$$

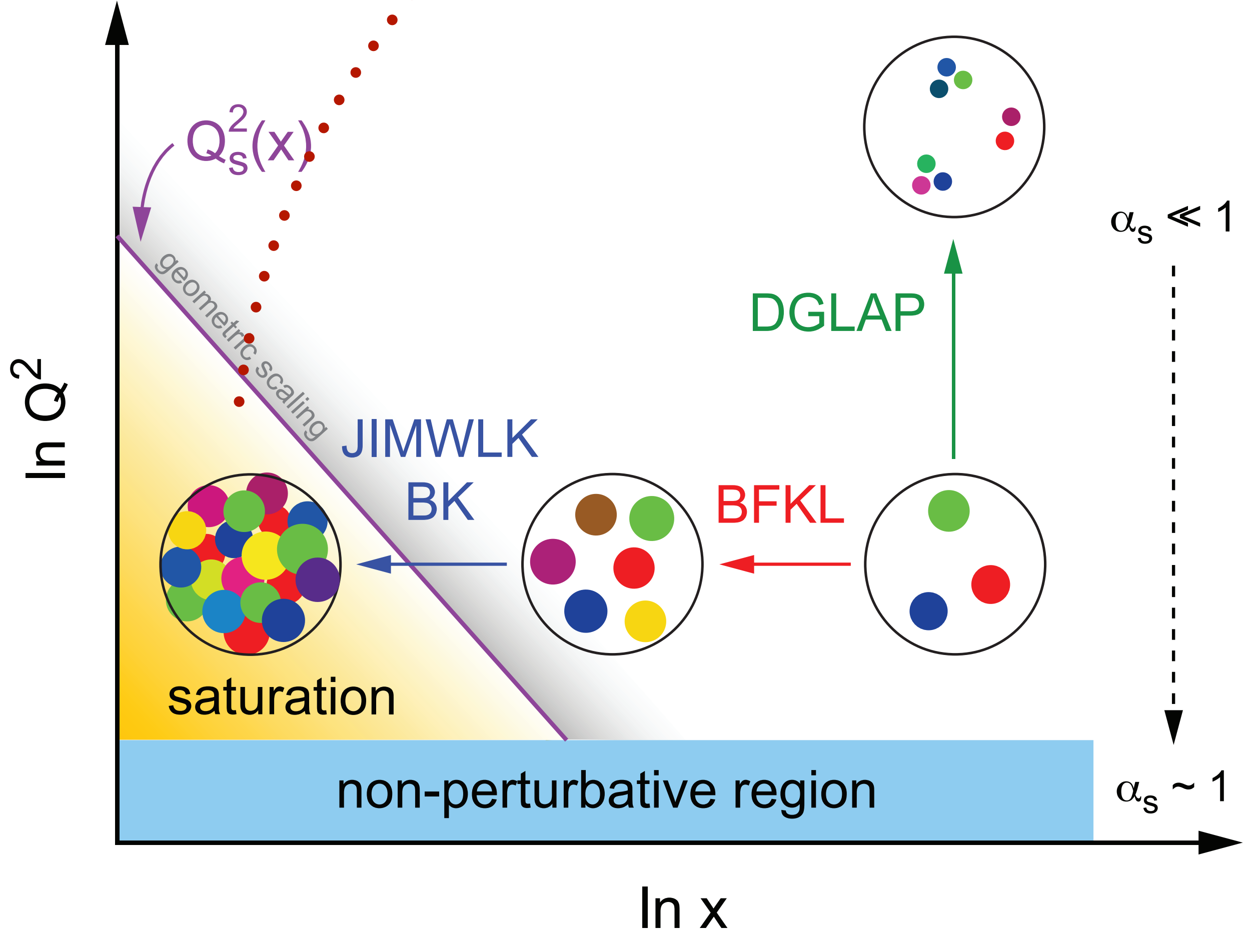
Shore, Veneziano (1992)

- The g_1 structure function is sensitive to the physics of the QCD anomaly, i.e. mass generation of η', Wess-Zumino-Witten coupling. Interplay with CGC at small-x?

The sphaleron-like transitions induced by interactions with the small-x background fields introduce a “drag force” on $\bar{\eta}$ “axion” propagation proportional to sphaleron transition rate:

“drag force” effect

$$\frac{\partial^2\eta'}{\partial t^2} = \boxed{-\gamma\frac{\partial\eta'}{\partial t}} - m_{\eta'}^2\eta' \quad \Rightarrow \quad g_1^{\mathrm{Regge}}(x_B,Q^2) \propto \frac{Q_S^2 m_{\eta'}^2}{F_{\bar{\eta}}^3 M_N}\exp\left(-4\,n_f C\,\frac{Q_S^2}{F_{\bar{\eta}}^2}\right)$$



While at large x_B the gluon field is dominated by the instanton configurations, at small x_B the CGC background ($Q_s^2 > m_{\eta'}^2$) can induce over-the-barrier transitions.

Over-the-barrier sphaleron transitions between different topological sectors of the QCD vacuum ⇒ can be detected in DIS at small x

Thank you for your attention!