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Flavor diagonal charges and isovector form factors of the nucleon

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CIPANP 2025, June 12th

Based on recent works
- [PNDME 25] SP, R. Gupta, T. Bhattacharya, F. He, S. Mondal, H.-W. Lin and B. Yoon, arXiv:2503.07100
- SP, R. Gupta, J. Yoo, et al., in preparation

Physics of flavor diagonal charges

Axial charge $g_A^q = \Delta q$

$$\langle N|\bar{q}\gamma_\mu\gamma_5 q|N\rangle = g_A^q \bar{u}_N \gamma_\mu \gamma_5 u_N,$$

$$g_A^q \equiv \Delta q = \int_0^1 dx(\Delta q(x) + \Delta\bar{q}(x)).$$

First Mellin moment of
$\Delta q(x)$: helicity-dependent, polarized PDF

$g_A^q = \Delta q$ *: Quark contributions to the nucleon spin*

$$\frac{1}{2} = \sum_{u,d,s,\cdots}\left(\frac{1}{2}\Delta q + L_q\right) + J_g$$

X. Ji, PRL 78 (1997) 610
L_q: orbital angular momentum of the quark
J_g: total angular momentum of the gluons

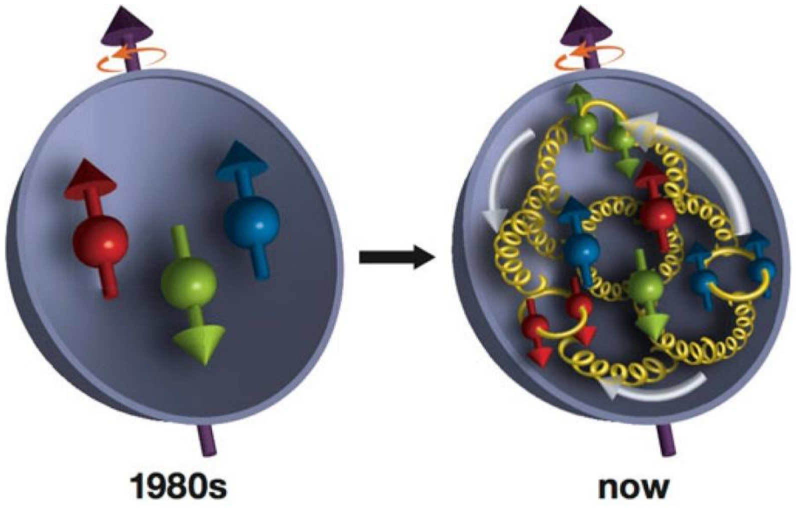


Photo courtesy of Brookhaven National Laboratory

$0.13 \le \sum_q 0.5\Delta q \le 0.18$ COMPASS, Phys. Lett. B753, 18 (2016)

Tensor charge g_T^q

No tree-level standard model interaction, Less well known

$$\langle N|\bar{q}\sigma_{\mu\nu}q|N\rangle = \boldsymbol{g_T^q}\bar{u}_N\sigma_{\mu\nu}u_N, \text{ with } \sigma_{\mu\nu} = i[\gamma_\mu,\gamma_\nu]/2$$

$$\boldsymbol{g_T^q} = \int dx[\delta q(x) - \delta\bar{q}(x)]$$

First Mellin moment of $\delta q(x)$: spin transversely aligned (transversity) PDF

"sensitive prove of new source of CP violation"

- Quark electric dipole moment (qEDM) contributions to the **neutron EDM d_n**

$$|d_n| = |d_u^\gamma \boldsymbol{g_T^u} + d_d^\gamma \boldsymbol{g_T^d} + d_s^\gamma \boldsymbol{g_T^s} + \cdots| < 1.8\times 10^{-26}\, e\cdot\text{cm (90\% C. L.)}$$

C. Abel et al., PRL 124 (2020) 081803

d_q^γ: BSM coupling of CP violating qEDM operators

whereas $|d_n^{CKM}| < 6\times 10^{-32}\, e\cdot\text{cm}$ C.-Y. Seng, PRC 91 (2015) 025502

Scalar charge $\boldsymbol{g_S^q}$

No tree-level standard model interaction, Less well known

- $\langle N|\bar{q}q|N\rangle = \boldsymbol{g_S^q}\bar{u}_N u_N$ *(Direct)*
  coupling of spin-independent Dark Matter (WIMP) interaction
- $\boldsymbol{g_S^q = \frac{\partial M_N}{\partial m_q}}$: *(Feynman-Hellman)*
  rate of change of nucleon mass with quark mass m_q

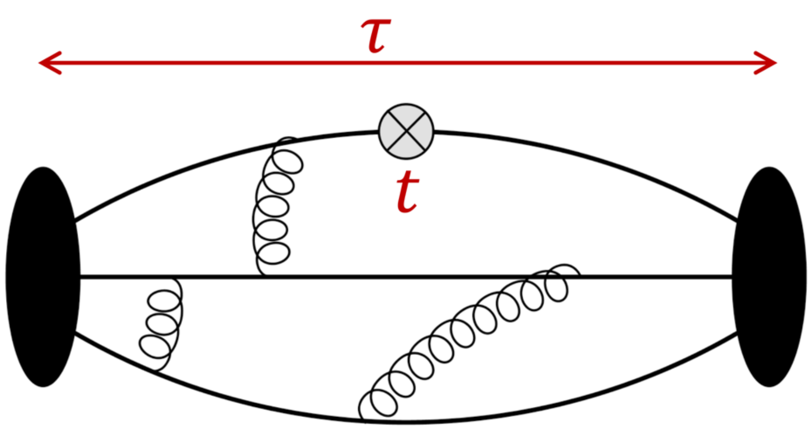
- $\sigma_{\pi N} = m_l \boldsymbol{g_S^{u+d}}$ (Sigma terms) Quark contributions to the nucleon mass
  $\sigma_s = m_s \boldsymbol{g_S^s}$

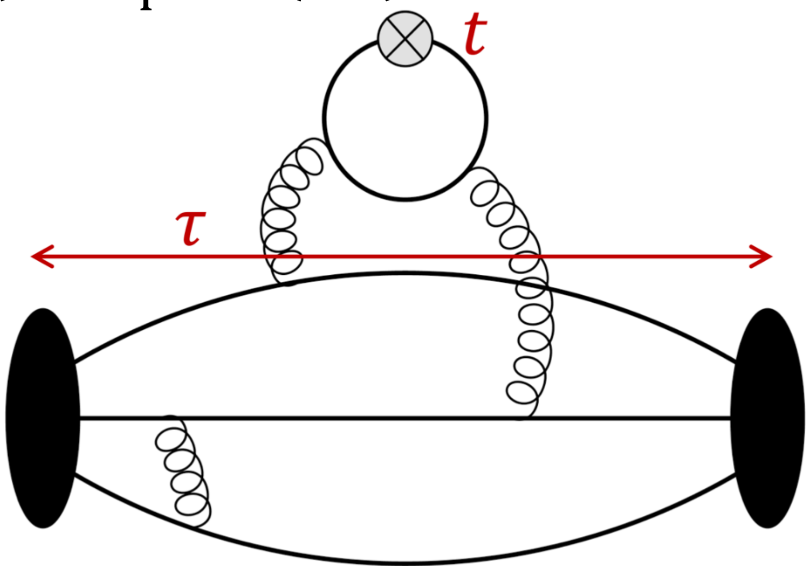
Fundamental parameter of QCD that quantifies the amount of the nucleon mass generated by u&d quarks.

Lattice correlation functions with Connected and disconnected contribution

$$g_\Gamma^{q}|_{\rm bare} = g_\Gamma^{q,\rm conn}|_{\rm bare} + g_\Gamma^{q,\rm disc}|_{\rm bare}$$

$$C_\Gamma^{q}(t,\tau) = C_\Gamma^{q,\rm conn}(t,\tau) + C_\Gamma^{q,\rm disc}(t,\tau)$$





Calculated with covariant Gaussian source smearing, multiple source-sink separation $0.9 \lesssim \tau \lesssim 1.4$, accelerated with coherent sequential inversions and the truncated solver method with bias correction.
PNDME, PRD98, 034503 (2018)

All-to-all quark propagator estimated by stochastic method using Z_4 random sources, accelerated with the truncated solver method with bias correction and hopping parameter expansion.
PNDME, PRD92, 094511 (2015)

MILC N_f=2+1+1 HISQ ensembles

g_A: PRD 98 094512 (2018)
g_T: PRD 98 091501 (2018)

Updates in PNDME 25 since PNDME 18

Ensemble ID	σ	τ/a	$N_{\rm conf}^{\rm 2pt}$	$N_{\rm conf}^{\rm conn}$	$N_{\rm LP}$	$N_{\rm HP}$	$N_{\rm conf}^{l}$	$N_{\rm src}^{l}$	$\frac{N_{\rm LP}^{l}}{N_{\rm HP}^{l}}$	$N_{\rm conf}^{s}$	$N_{\rm src}^{s}$	$\frac{N_{\rm LP}^{s}}{N_{\rm HP}^{s}}$
$a15m310$	4.2	{6,7,8,9}	1917	1917	64	8	1917	2000	50	1917	2000	50
$a12m310$	5.5	{8,10,12,14}	1013	1013	64	8	1013	10000	50	1013	8000	50
$a12m220$	5.5	{8,10,12,14}	959	744	64	4	958(**)	11000	30	870	5000	50
$a09m310$	7.0	{10,12,14,16}	2263	2263	64	4	1017	10000	50	1024	6000	50
$a09m220$	7.0	{10,12,14,16}	964	964	128	8	712	8000	30	847	10000	50
$a09m130$	7.0	{10,12,14,16}	1274	1290	128	4	1270	10000	50	994	10000/4000(†)	50
$a06m310$	12.0	{18,20,22,24}	977	500	128	4	808	12000	50	976	10000/4000(†)	50
New -> $a06m220$	11.0/9.0 (*)	{18,20,22,24}	1010	649	64	4	1001	10000	50	1002	10000	50

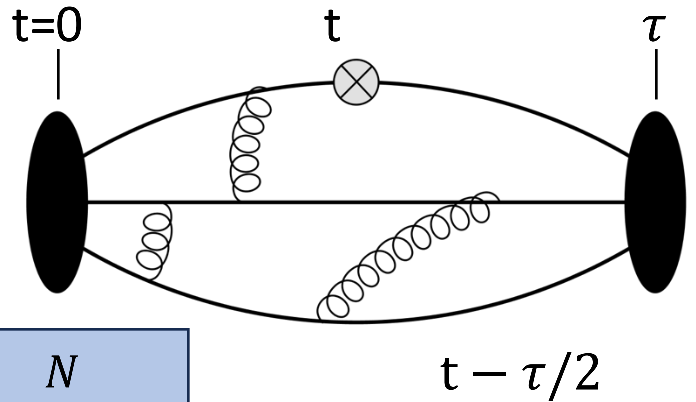
Improved statistics

- Disconnected diagram now calculated at all intermediate $0 \leq t \leq \tau$
- Complete result for the 2+1 flavor mixing matrix $Z_{A,S,T}$ obtained nonperturbatively on the lattice using RI-sMOM scheme (PNDME 18 used $Z_{isoscalar} \approx Z_{isovector}$)
- Improved Excited-state analysis regarding $N\pi/N\pi\pi$ states with conn+disc
- Results for $g_S^{u,d,s}$ are new!

The Ground State Matrix Element is obtained after removing all Excited State Contributions (ESC)

- Towers of excited states spectrum in a finite box
- Excited states (ES) that give significant contribution to a particular nucleon matrix element are not known a priori. → χPT is a very useful guide

<u>**Spectral decomposition of 3-point correlation function**</u> $\boldsymbol{C(t,\tau) \equiv \langle N^p(\tau) O(t) \bar{N}^p(0)\rangle}$:



$$C^{3pt} = \langle 0|\mathcal{O}|0\rangle |A_0|^2 e^{-M_0\tau} \times \Big[1 + \frac{\langle 1|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_1|^2}{|A_0|^2} e^{-\Delta M_1 \tau} + \frac{\langle 2|\mathcal{O}|2\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_2|^2}{|A_0|^2} e^{-(\Delta M_2+\Delta M_1)\tau} + \frac{\langle 0|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle}\frac{|A_1|}{|A_0|} e^{-\Delta M_1 \frac{\tau}{2}} \times 2\cosh\Big(\Delta M_1 (t-\frac{\tau}{2})\Big) + (\cdots)$$

ESC!

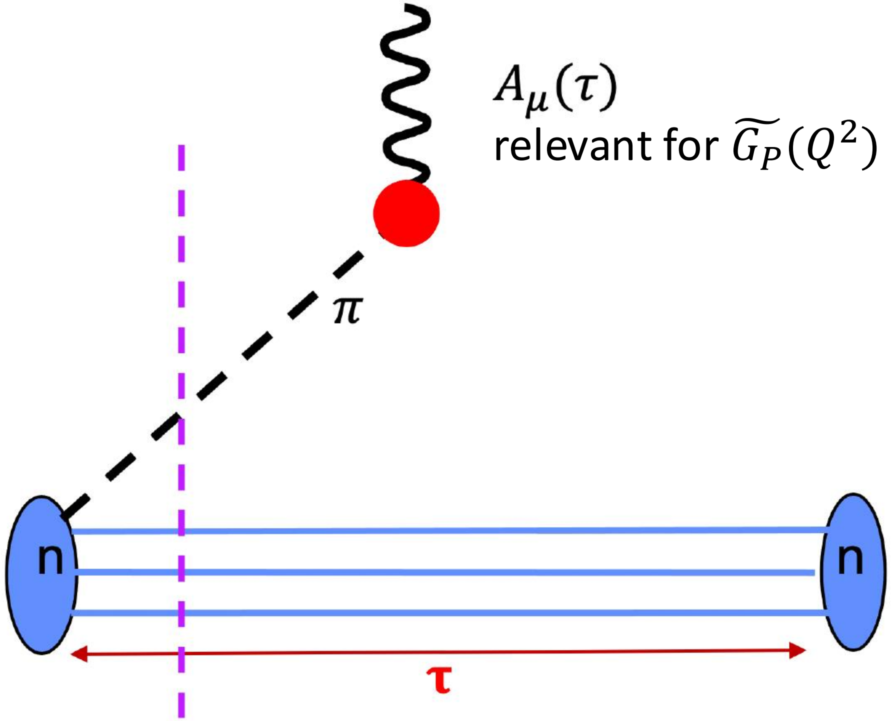
Ground-state matrix element
→Nucleon charge or Form Factor

N
$N\pi/N\pi\pi$
$N^*(1440)$
$N^*(1710)$
(...)

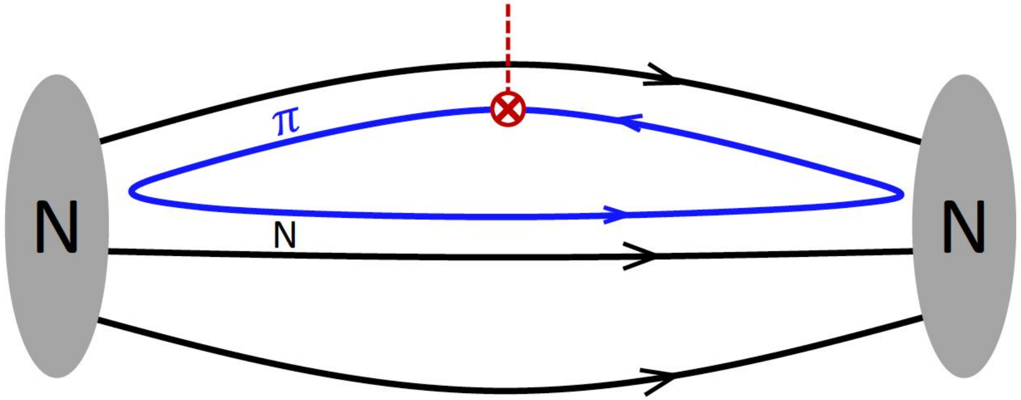
$N\pi/N\pi\pi$ states coupling of the local nucleon operator is suppressed by $1/L^3$

$N\pi$-Excited state contribution can be enhanced

- In nucleon 2pt functions, $1/L^3$-suppressed coupling to $N\pi$-state makes it challenging to extract its spectrum
- In certain 3pt functions (matrix elements) , ChPT suggests $N\pi$-state contributions can be enhanced and compensate $1/L^3$-suppression



Scalar operator
relevant for $g_S^{u,d}$ and $\sigma_{\pi N} = m_{ud} g_S^{u+d}$



ESC from $\boldsymbol{N\pi}$ and $\boldsymbol{N\pi\pi}$

- We carry out two types of analyses:

1. The "**standard**" fit to $C_{2\text{pt}}(\tau)$ uses wide priors for all the excited-state amplitudes, A_i, and masses, M_i, to stabilize the fits.

$\boldsymbol{M_1 \approx 1.6\ GeV}$

2. The "$\boldsymbol{N\pi}$" fit in which a narrow prior is used for M_1 with the central value given by the non-interacting energy of the lowest allowed $N\pi$ or $N\pi\pi$ state (degenerated) on the lattice

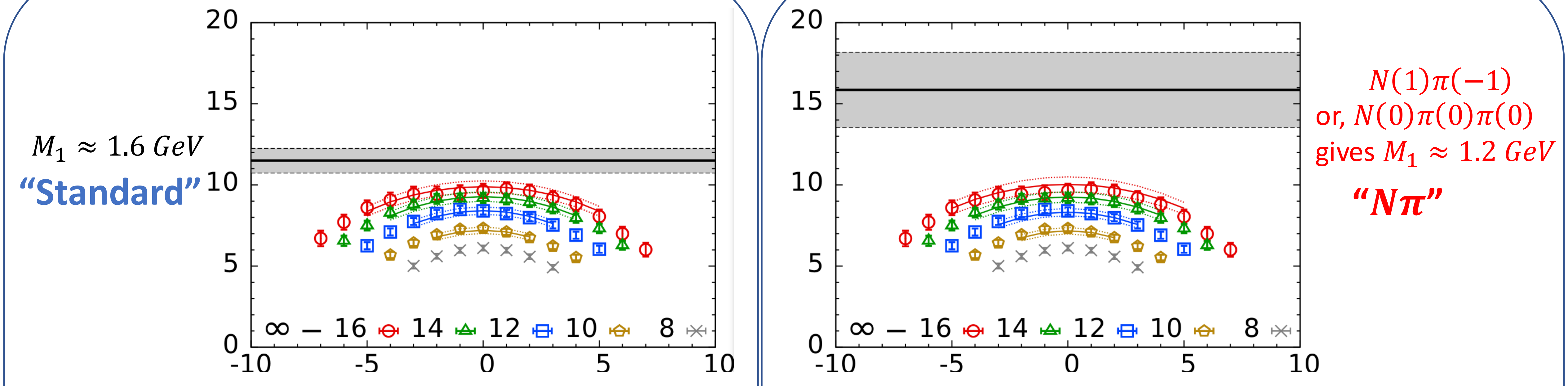
$N(1)\pi(-1)$
or, $N(0)\pi(0)\pi(0)$
gives $\boldsymbol{M_1 \approx 1.2\ GeV}$

- For g_Γ^s, the leading multi-hadron ES is expected to be ΣK → "standard" analysis

g_S^{u+d}: Excited state effect

$a \approx 0.09 fm$
$M_\pi \approx 135 MeV$

PNDME, PRL 127 (2021) 242002



$\sigma_{\pi N} = m_l g_S^{u+d} \sim 40$ MeV, $\frac{\chi^2}{dof} = 1.1$

$\sigma_{\pi N} = m_l g_S^{u+d} \sim 60$ MeV, $\frac{\chi^2}{dof} = 1.2$

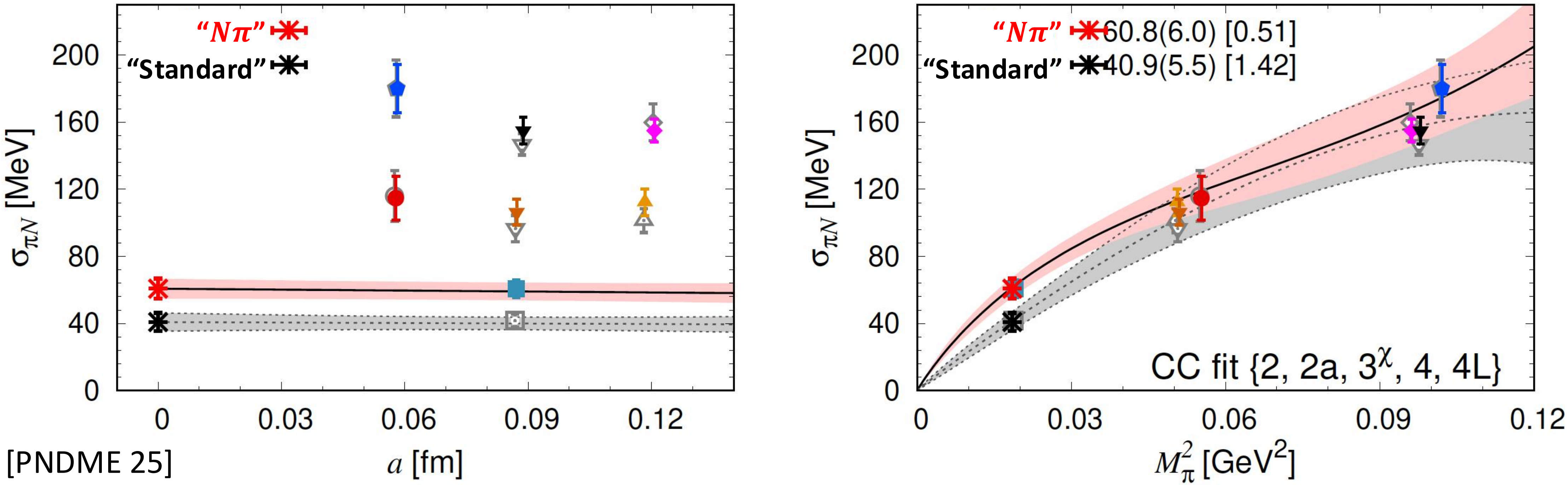
- Scalar is sensitive to $N\pi$ state
- Output is close to the phenomenological determination

(scale independent)

Chiral-continuum extrapolation to $\sigma_{\pi N} = m_l^{\text{bare}} g_S^{u+d,\text{bare}}$

$$\sigma_{\pi N} = (d_2 + d_2^a a) M_\pi^2 + d_3 M_\pi^3 + d_4 M_\pi^4 + d_{4L} M_\pi^4 \log \frac{M_\pi^2}{M_N^2}$$

N^2LO χPT expression



[PNDME 25]

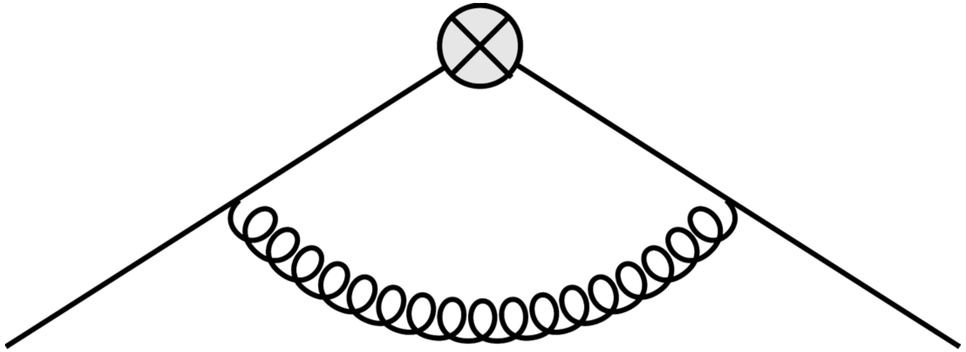
2+1 flavor mixing in RI-sMOM scheme

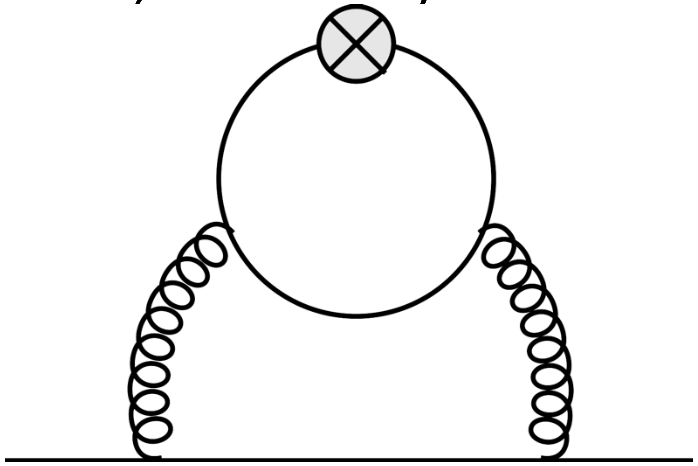
- We explicitly evaluated the 3×3 flavor mixing matrices in RI-sMOM scheme and convert into $\overline{\mathrm{MS}}$ scheme value 2 GeV.
- Flavor singlet axial current is not conserved due to $U_A(1)$ anomaly

$$C_A^{\text{RI-sMOM}\to\overline{\text{MS}}}(\mu) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 - 26.5\, a^2\, n_f \end{pmatrix}$$

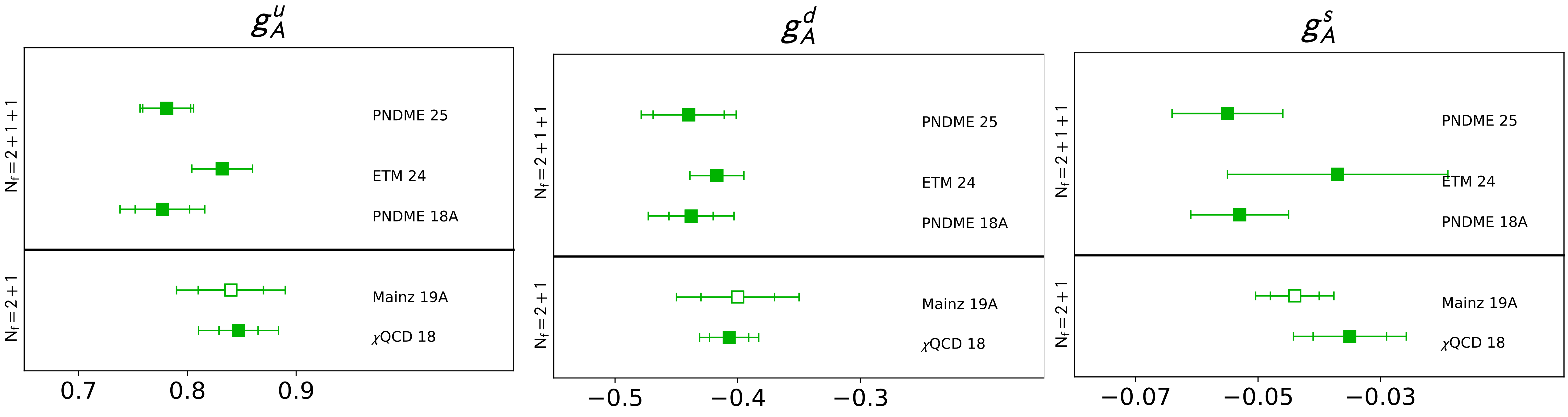
in {3,8,0} basis

- Off-diagonal mixing matrix element $Z_\Gamma^{u+d,s}$ and $Z_\Gamma^{s,u+d}$ are small (largest at $a = 0.15$fm, below 6%)





Axial charges: Comparison with FLAG 2024



q	g_A^q
u	$0.781(22)(11)_{\rm CC}$
d	$-0.440(29)(9)_{\rm CC}(24)_{N\pi}$
s	$-0.055(9)(1)_{\rm CC}$

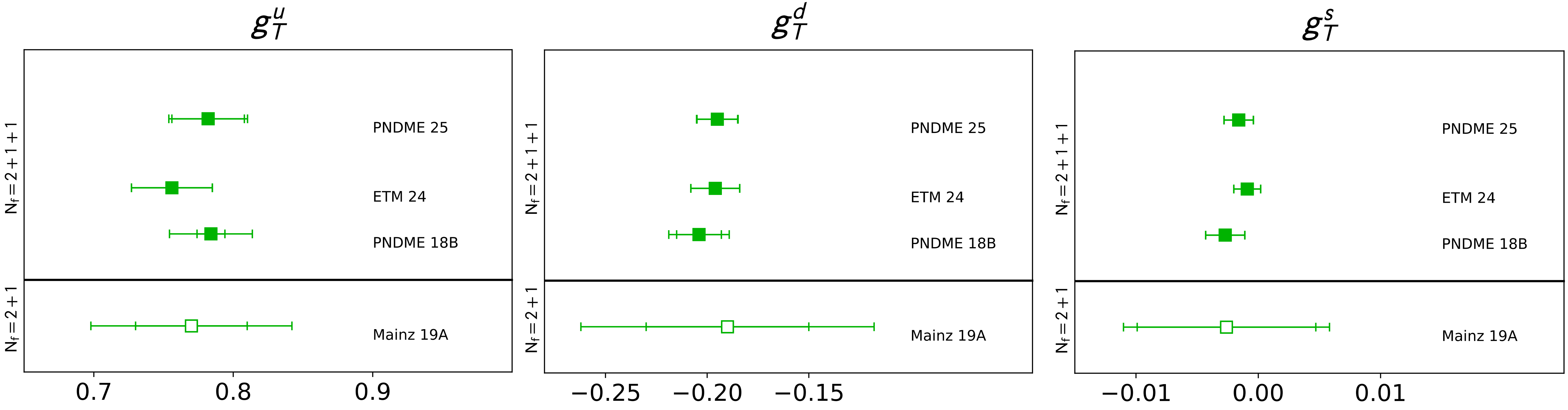
$\sum_q 0.5\Delta q = 0.143(24)$ **PNDME 25**

$0.13 \le \sum_q 0.5\Delta q \le 0.18$

(evaluated at 3GeV^2)

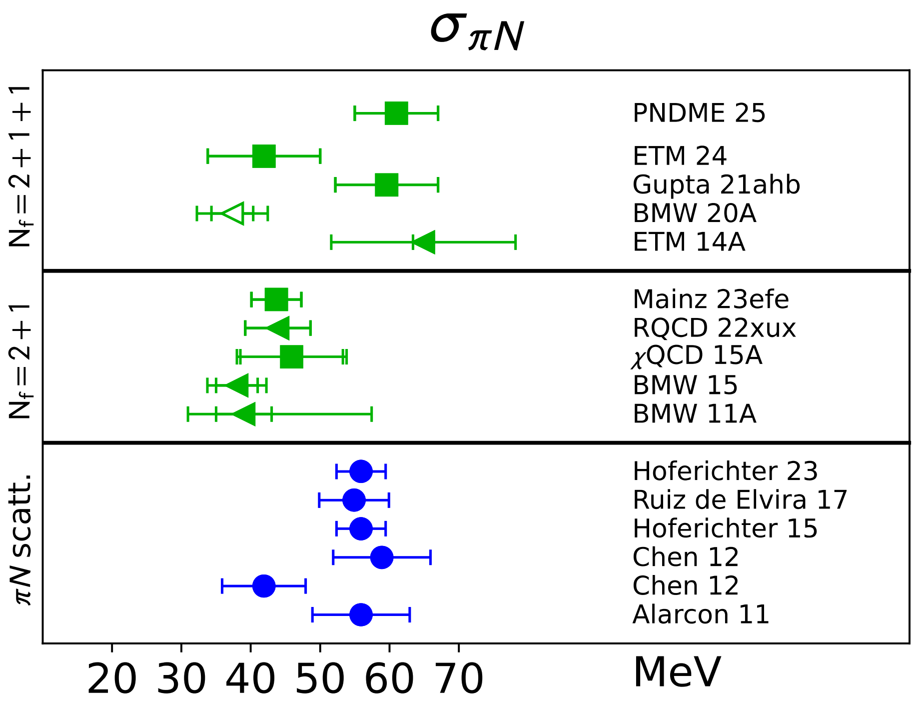
COMPASS, Phys. Lett. B753, 18 (2016)

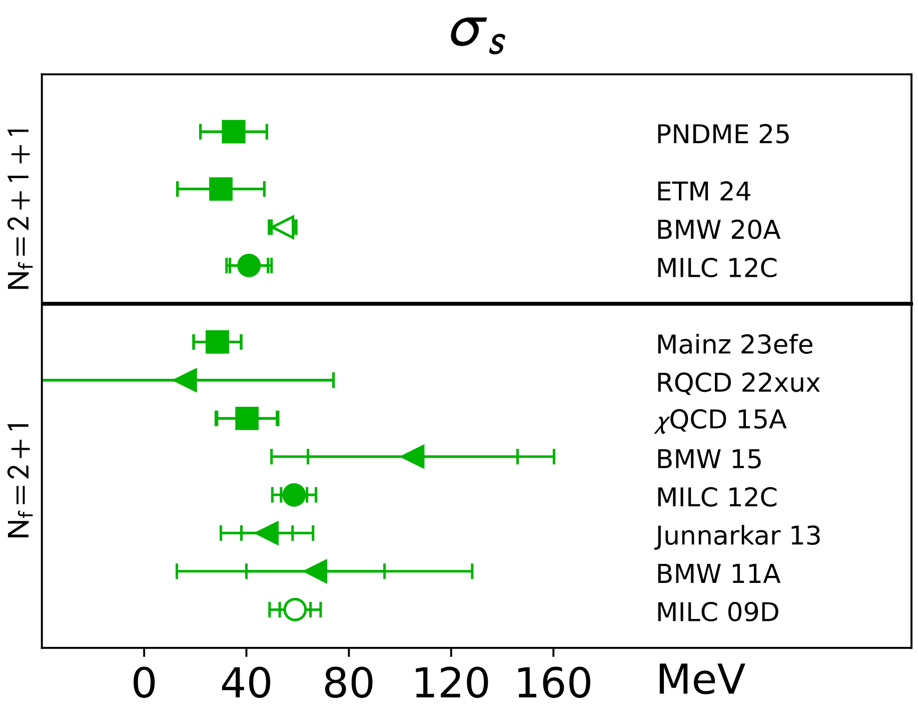
Tensor charges: Comparison with FLAG 2024



q	g_T^q
u	$0.782(26)(11)_{\rm CC}$
d	$-0.195(10)(2)_{\rm CC}$
s	$-0.0016(12)(1)_{\rm CC}$

Sigma terms: Comparison with FLAG 2024





- $\sigma_{\pi N}$: Tension between Lattice QCD and Phenomenology (πN scatt.)

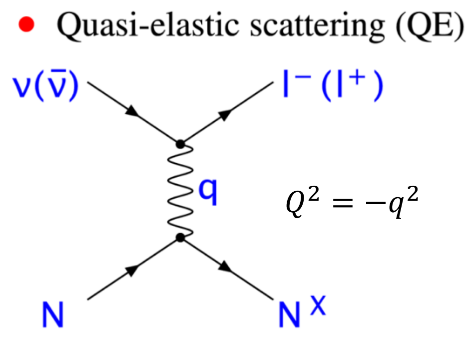
q	$g_S^q\|_{N\pi}$	$g_S^q\|_{\mathrm{St.}}$
u	$9.36(88)(4)_{\mathrm{CC}}$	$6.34(57)(1)_{\mathrm{CC}}$
d	$8.84(93)(1)_{\mathrm{CC}}$	$6.04(63)(1)_{\mathrm{CC}}$
s	$0.66(17)(5)_{\mathrm{CC}}$	$0.37(13)(6)_{\mathrm{CC}}$

Two different chiral-continuum extrapolation result matches

1. fit to scale invariant $\sigma_{\pi N} = m_l^{\mathrm{bare}} g_S^{u+d,\mathrm{bare}}$
2. $\sigma_{\pi N} = m_l^{\mathrm{ren}} g_S^{u+d,\mathrm{ren}}$ with m_l^{ren} from Nf=2+1+1 FLAG average

Nucleon isovector Axial form factors

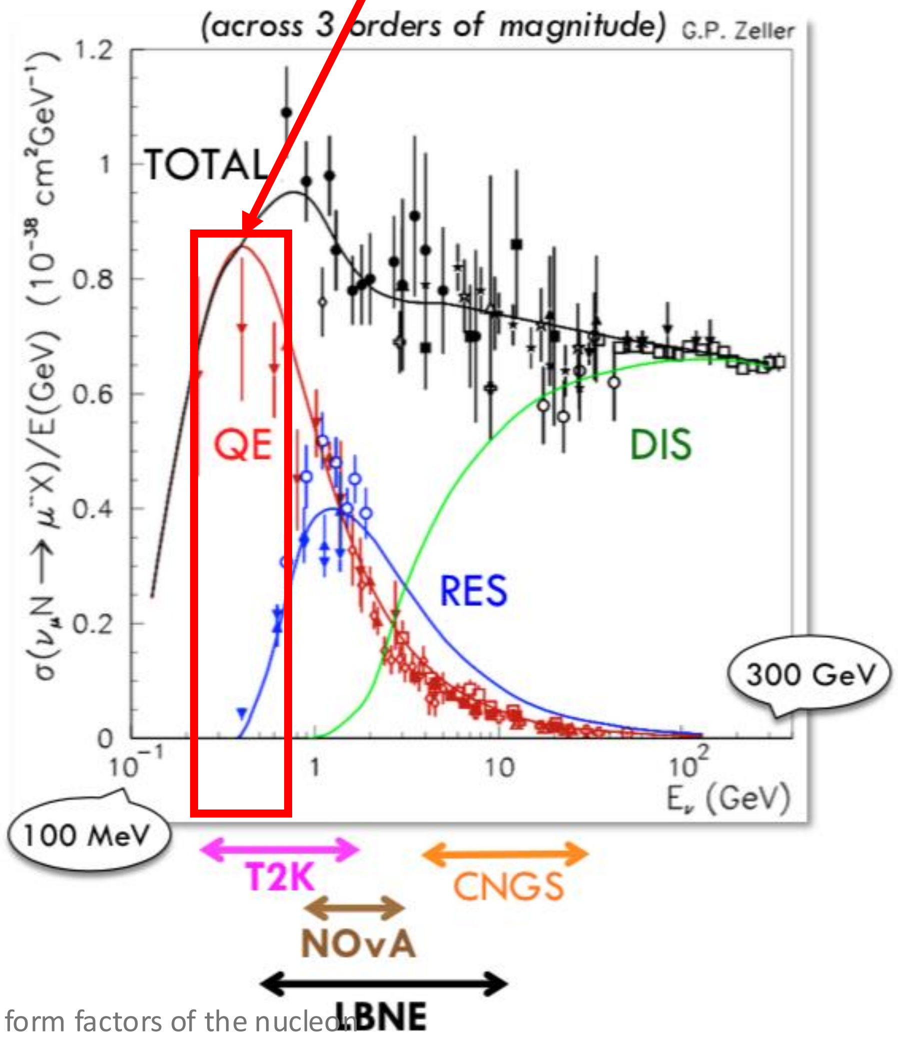
Neutrino Oscillation Experiment



- Upcoming flagship neutrino oscillation experiment such as DUNE (US) and HyperK (Japan), Quasi-elastic (QE) neutrino-nucleon scattering is the dominant interaction process

$$\frac{d\sigma}{dQ^2} \propto f(Q^2, [\mathrm{VFF}], [\mathrm{AFF}], \cdots)$$

- Weak interaction (V-A): Low statistics
  - Vector form factor (VFF)
    → high-statistics electron scattering experiments
  - Axial vector form factor (AFF)
    → Lattice QCD calculation is simple at QE processes and can help constraining experimental cross-section
    → Must provide a complete parametrization function (e.g. z-expansion) of $G_A(Q^2)$ including a covariance matrix of parameters.



This work: two new MILC M_π^{Phys} ensembles

- $N_f = 2+1+1$ dynamical fermion flavors (isospin symmetric u,d quark masses)
- Gauge ensemble generated by MILC collaboration using HISQ (Highly Improved Staggered Quark) seq quark action

ID	β	a (fm)	M_π (MeV)	M_N (MeV)	L	T	$M_\pi L$	Lattices	N_{HP}	N_{LP}	τ
$a09m135$	6.3	0.087(1)	134(1)	947.1(7.6)	64	96	3.80	5,497	16,491	527,712	{10,12,14,16,18}
$a06m135$	6.7	0.057(0)	136(0)	932.2(8.1)	96	192	3.78	3,990	15,960	430,920	{16,18,20,22,24}

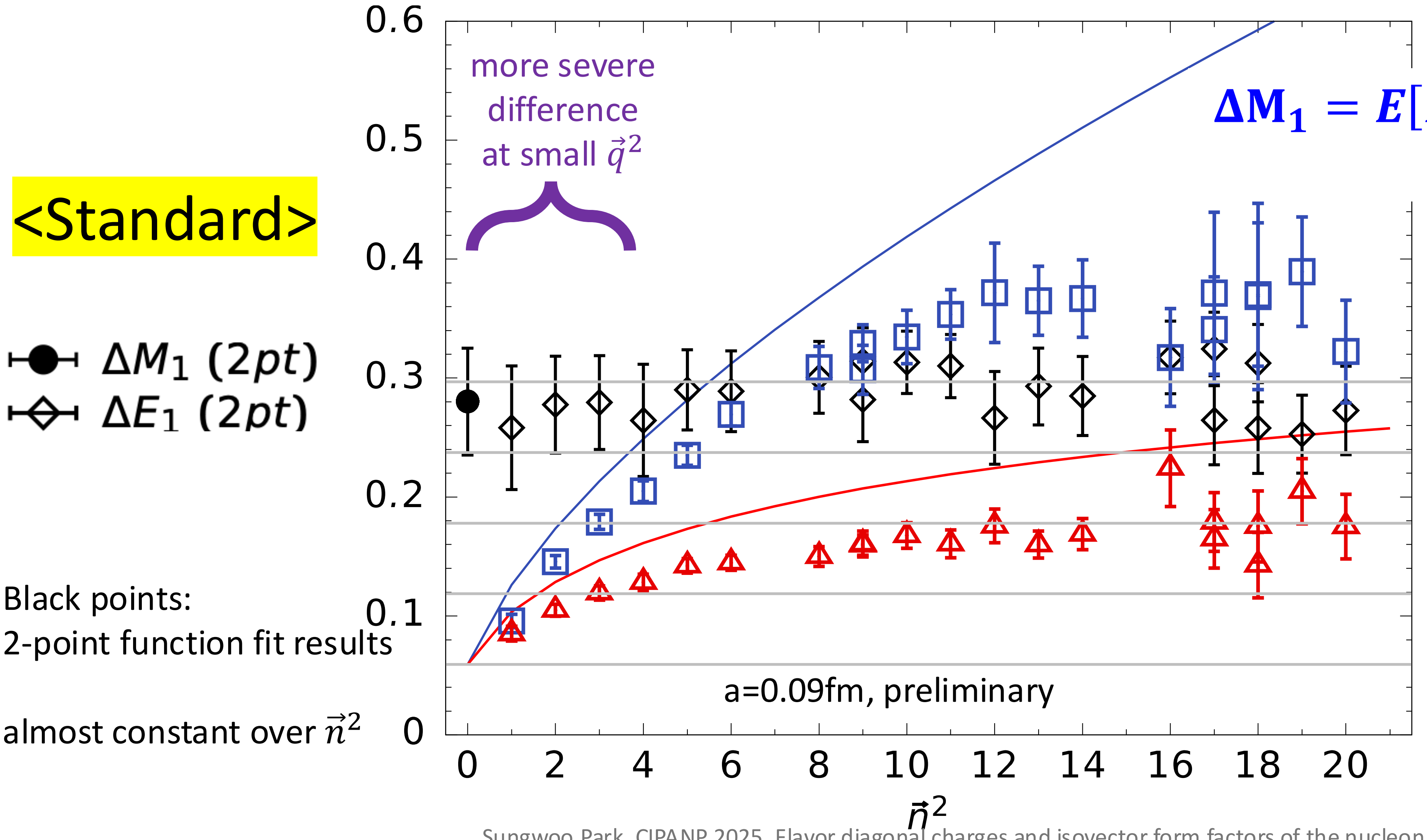
~4x improved statistics

- Improved gauge link smearing, mass parameter tuning
- Larger source-sink time separation
- The largest momentum transfer $\vec{q} = 2\pi\vec{n}/L$ is doubled
    - spacelike 4-momentum transfer $Q^2 = -q^2$ max has increased from $0.45\ \mathrm{GeV}^2$ to $0.82\ \mathrm{GeV}^2$

Jang et al. PRL 124 (2020) 072002
Park et al. PRD 105 (2022) 054505
Jang et al. PRD 109 (2024) 014503

1^{st} excited state masses from two different analysis

<Standard>



Black points:
2-point function fit results

almost constant over $\vec{n}^2$

$$\Delta \mathrm{M}_1 = E[N(\vec{p}) + \pi(-\vec{p})] - M$$

<$N\pi$>

$$\Delta \mathrm{E}_1 = E[N(0) + \pi(\vec{p})] - E_N(\vec{p})$$

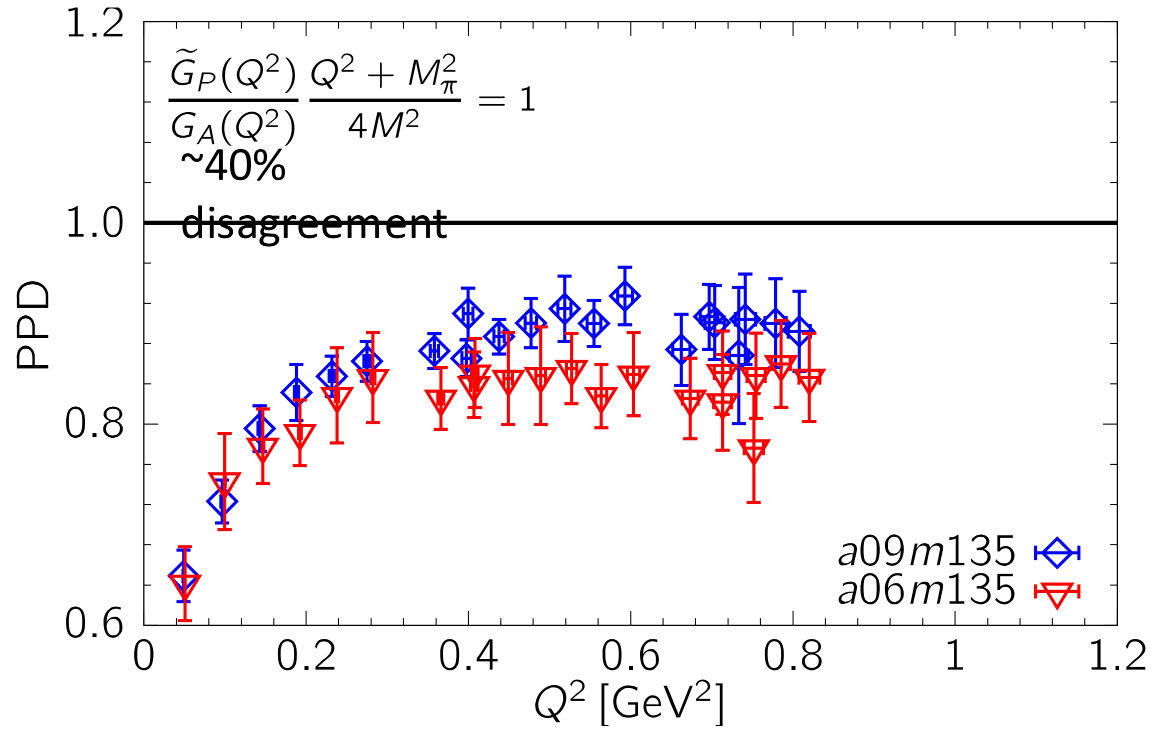
blue / red points
3-point function fit results

similar $\vec{n}^2$ dependence shape as the noninteracting $N\pi$ states where π carries $\vec{p}$

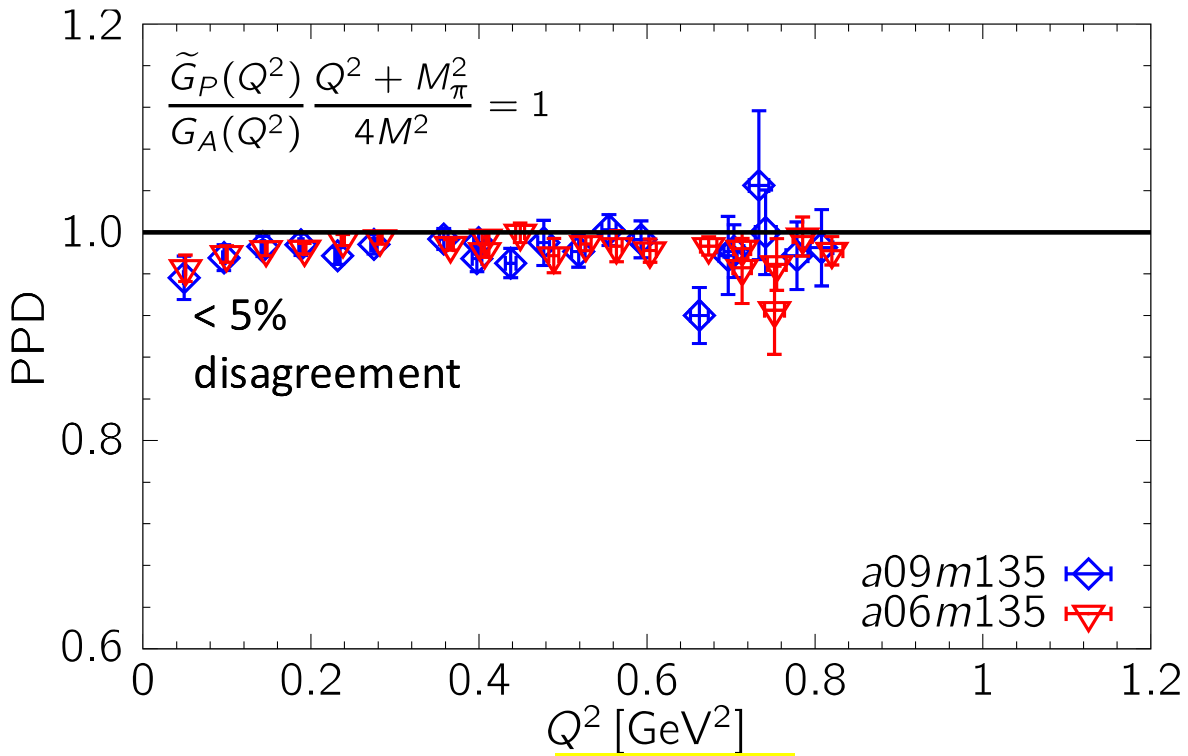
Checking Pion Pole Dominance (PPD) hypothesis

- Relates Induced pseudoscalar and the axial form factors

$$(\widetilde{G_P}(Q^2) = \frac{4M^2}{Q^2+M_\pi^2} G_A(Q^2))$$



<Standard>



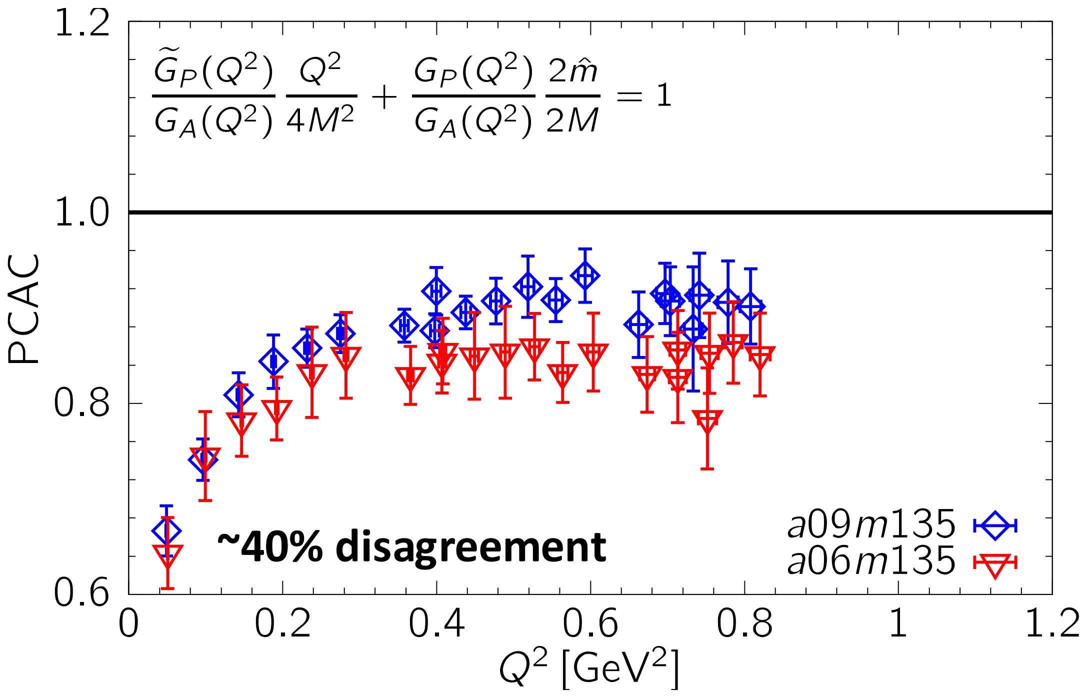
<$N\pi$>

PCAC (Partially Conserved Axial Current) relation

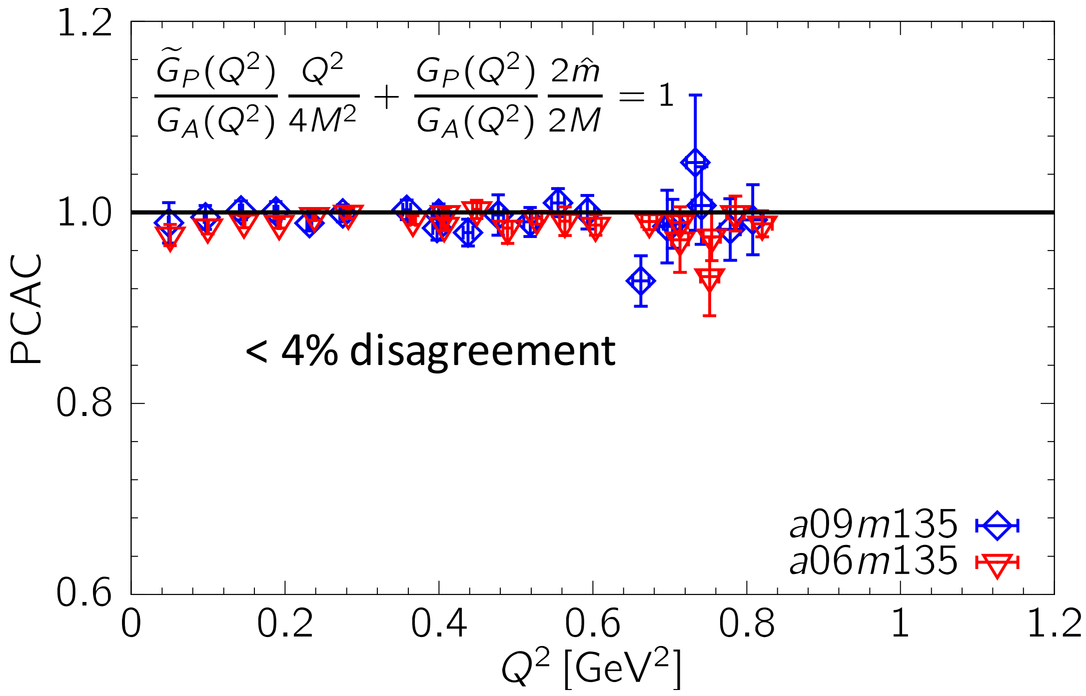
- $\partial_\mu A_\mu(x) = 2\hat{m}P(x)$ where $\hat{m} = Z_m m_{ud} Z_P Z_A^{-1}$
- Applied to nucleon ground state, it relates the 3 nucleon form factors

$$2M_N G_A(Q^2) - \frac{Q^2}{2M_N}\tilde{G}_P(Q^2) = 2\hat{m}G_P(Q^2)$$

Generalized Goldberger-Treiman Relation



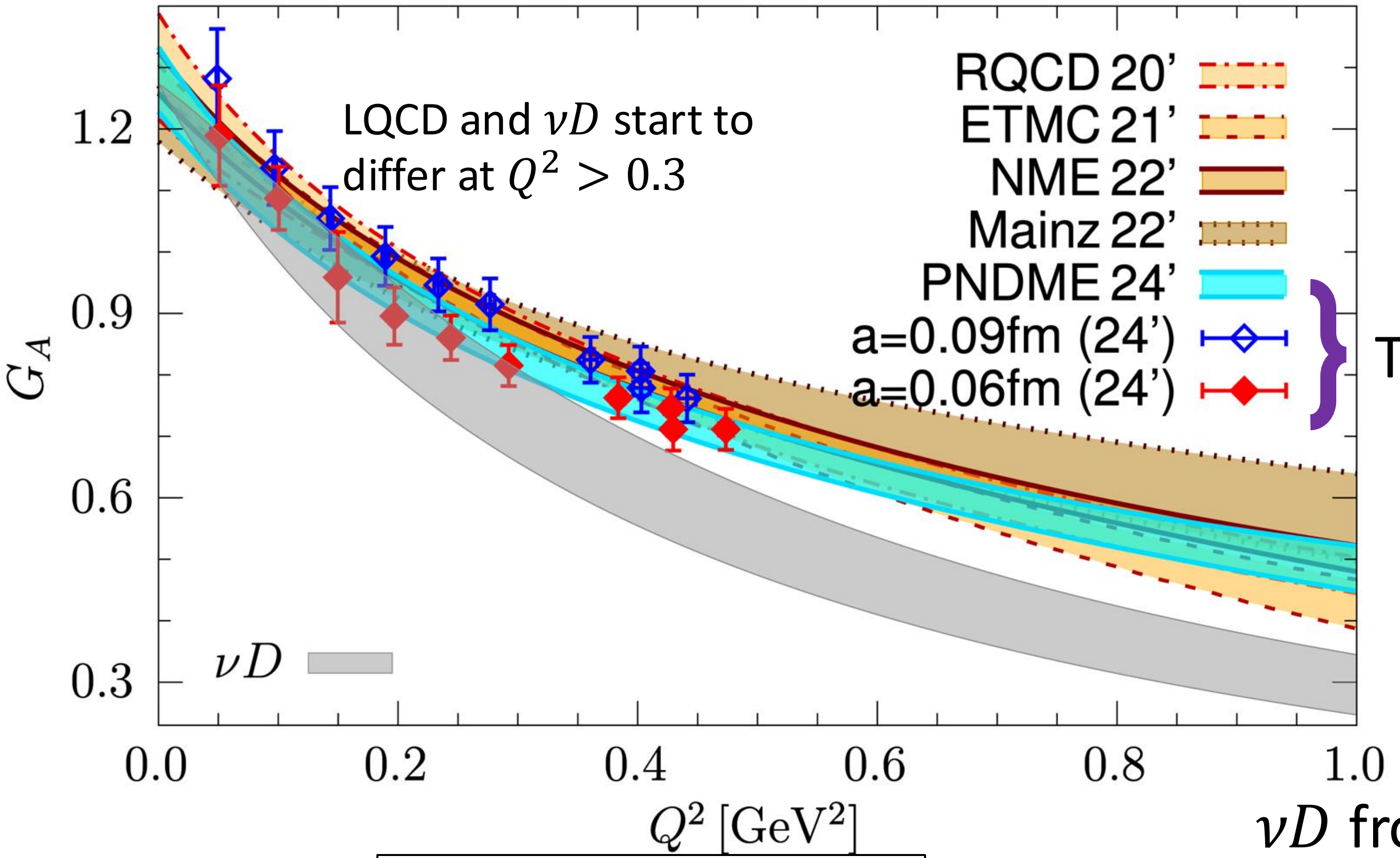
<Standard>



<$N\pi$>

2024 Comparison of lattice results of G_A

RQCD 20' JHEP 05 126
ETMC 21' PRD 103, 034509
NME 22' PRD 105, 054505
Mainz 22' PRD 106, 074503
PNDME 24' PRD 109, 014503



Two physical pion mass data in PNDME 24'

Turquoise Band PNDME 24'

$$G_A(Q^2) = a_0 + a_1 z + a_2 z^2$$
$$= 0.876(28) - 1.669(99)z + 0.483(498)z^2,$$

with the correlation matrix:

$$\begin{array}{cc} & \begin{array}{ccc} a_0 & a_1 & a_2 \end{array} \\ \begin{array}{c} a_0 \\ a_1 \\ a_2 \end{array} & \begin{pmatrix} 1.0 & -0.45170 & -0.02966 \\ -0.45170 & 1.0 & -0.24394 \\ -0.02966 & -0.24394 & 1.0 \end{pmatrix} \end{array}$$

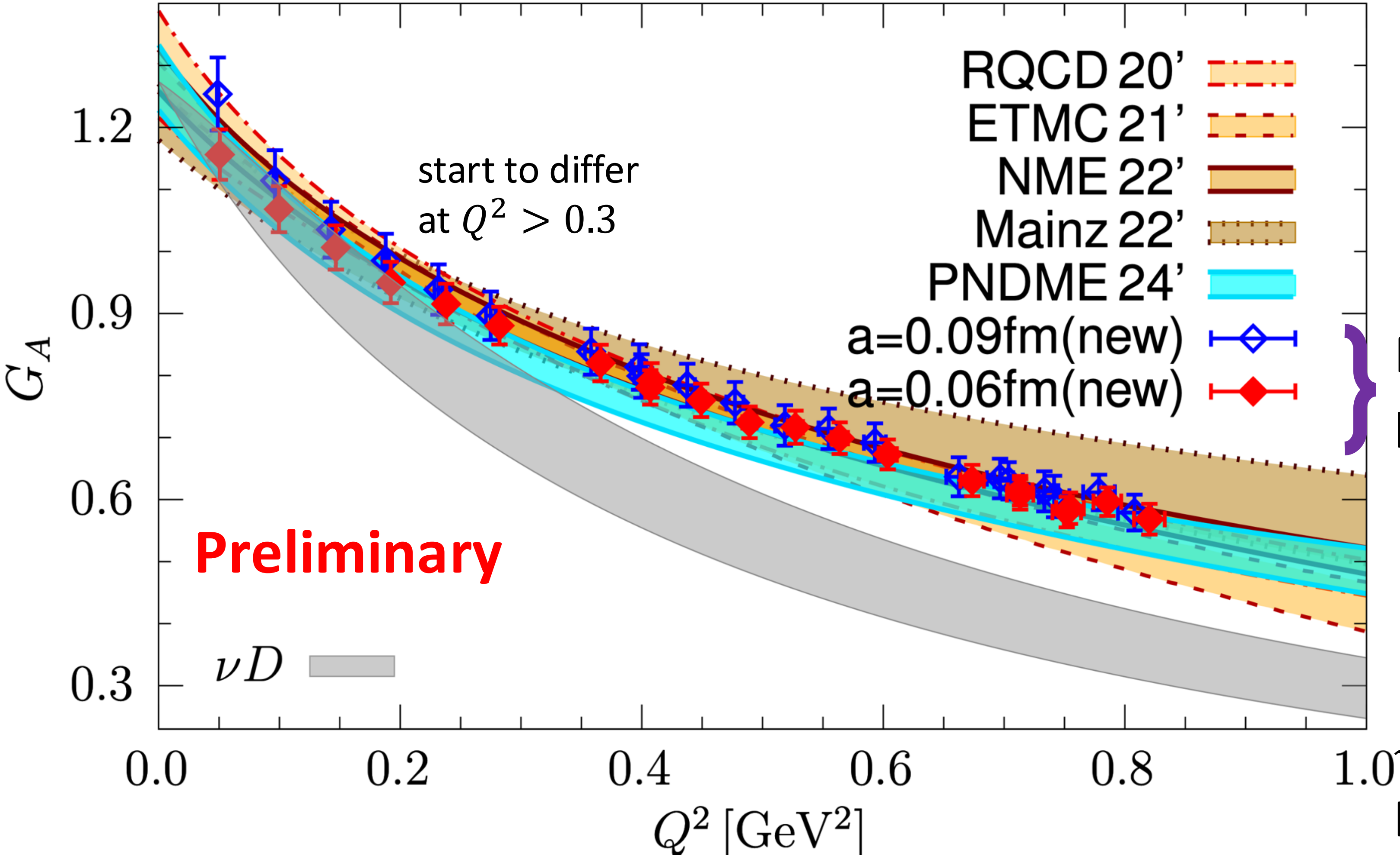
νD from z-expansion fit

[A.Meyer et al., PRD 93,113015(2016)]

- The deuterium bubble chamber experiment data (70-80s) have small statistics (10^3),
- Unknown correction added. Cannot access original data.
- Unresolved nuclear (deuterium) correction
- Dipole fit underestimated G_A uncertainty by x10

Our new data for G_A

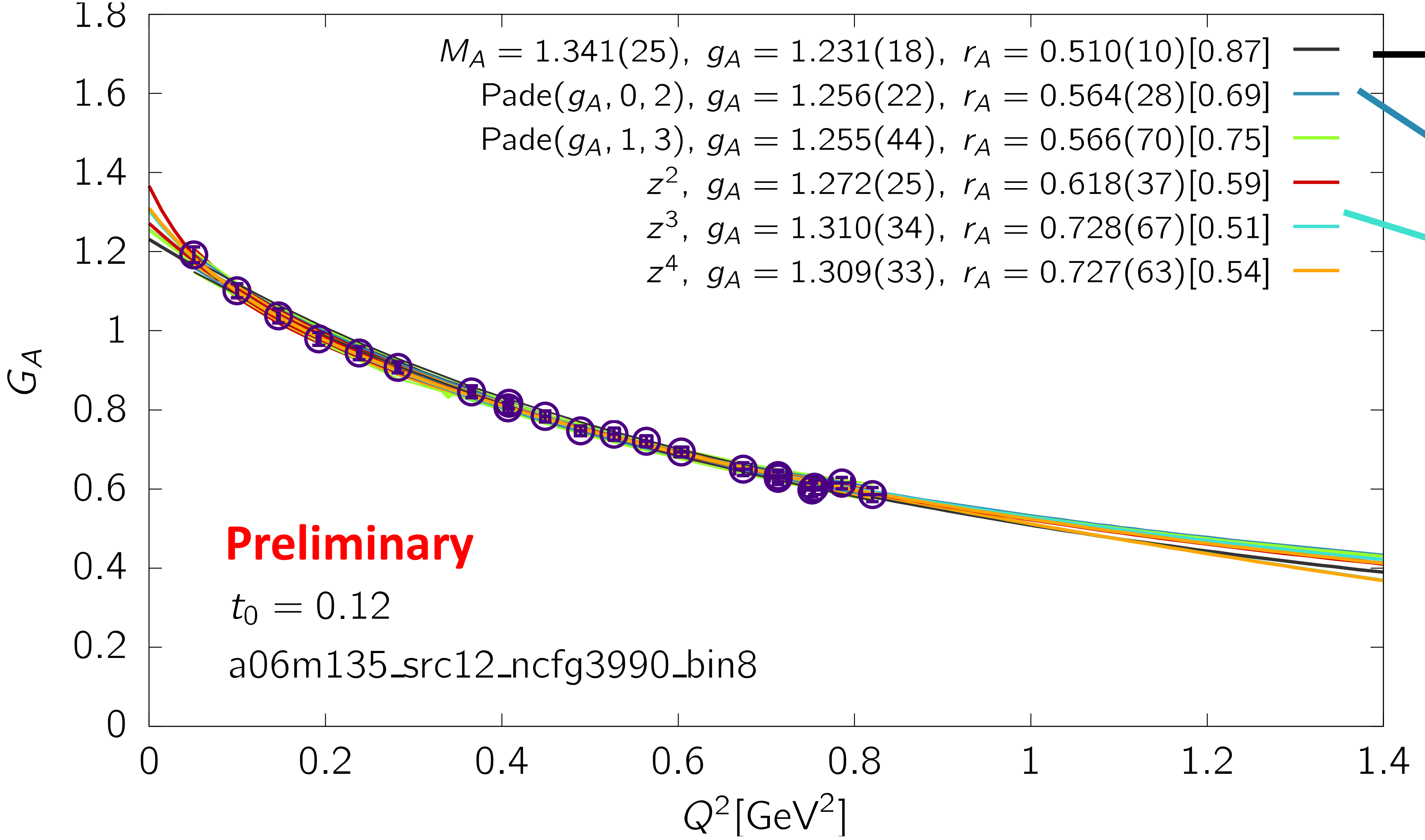
RQCD 20' JHEP 05 126
ETMC 21' PRD 103, 034509
NME 22' PRD 105, 054505
Mainz 22' PRD 106, 074503
PNDME 24' PRD 109, 014503



NEW data on the two physical pion mass ensembles

νD from z-expansion fit
[A.Meyer et al., PRD 93,113015(2016)]

G_A^{u-d} : Examined Dipole, Pade and z-expansion fits



- Dipole: $g_A / \left(1 + \frac{Q^2}{M_A^2}\right)^2$
  - uncertainty underestimated
- Pade: $\frac{g_A}{1+b_1Q^2+b_2Q^4}$
- z^n-expansion: $\sum_{k=0}^{n} a_k z(Q^2)^k$

z^3 result (renormalized)
gA=1.271(51)
GA= 1.038(34) - 1.54(12) z
+0.89(83) z^2 – 3.2(1.9) z^3

Summary

- Calculating the flavor diagonal charges $g_{A,S,T}^{u,d,s}$ and isovector axial form factor $G_A^{u-d}(Q^2)$ as part of a comprehensive analysis of nucleon structure.
- Current data driven excited state analysis of $g_{A,S,T}^{u,d,s}$ do not resolve between "standard" and "$N\pi$" analyses. To improve this, we are increasing statistics at two M_π^{Phys} ensembles by 4x-6x with larger source-sink separation.
- Presented new improved data for G_A^{u-d} on two M_π^{Phys} ensembles over $0.04 < Q^2 \lesssim 1\ \text{GeV}^2$ with significantly improved statistics and systematics control. This enables a more reliable chiral-continuum extrapolation.
- Result: $G_A^{u-d}(Q^2)$ and a z-expansion parametrization with the correlation matrix of the fit parameters is important input in the calculation of the quasi-elastic neutrino-nucleon scattering cross-section. (Needed for DUNE to meet precision goal)

PNDME and NME members

- Tanmoy Bhattacharya (LANL)
- Vincenzo Cirigliano (INT)
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References

PNDME (clover-on-HISQ formulation)

- Charges: Gupta et al, PRD.98 (2018) 034503
- AFF: Gupta et al, PRD 96 (2017) 114503
  Jang et al, PRL 124 (2020) 072002
  Jang et al, PRD 109 (2024) 014503
- VFF: Jang et al, PRD 100 (2020) 014507
- $\sigma_{\pi N}$ Gupta et al, PRL 127 (2021) 242002
- d_n from Θ-term Bhattacharya et al, PRD 103 (2021) 114507
- d_n from qEDM Gupta et al, PRD 98 (2018) 091501
- Moments of PDFs Mondal et al, PRD 102 (2020) 054512
- Proton spin: Lin et al, PRD 98 (2018) 094512
- Flavor diag. charges: Park et al, arXiv:2503.07100
- AFF and VFF Park, Gupta, Yoo, et al., in preparation

NME (clover-on-clover formulation)

- Charges, VFF, AFF: Park et al, PRD 105 (2022) 054505
- Moments of PDFs Mondal et al, JHEP 04 (2021) 044

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BACKUP

Lepton-nucleon scattering

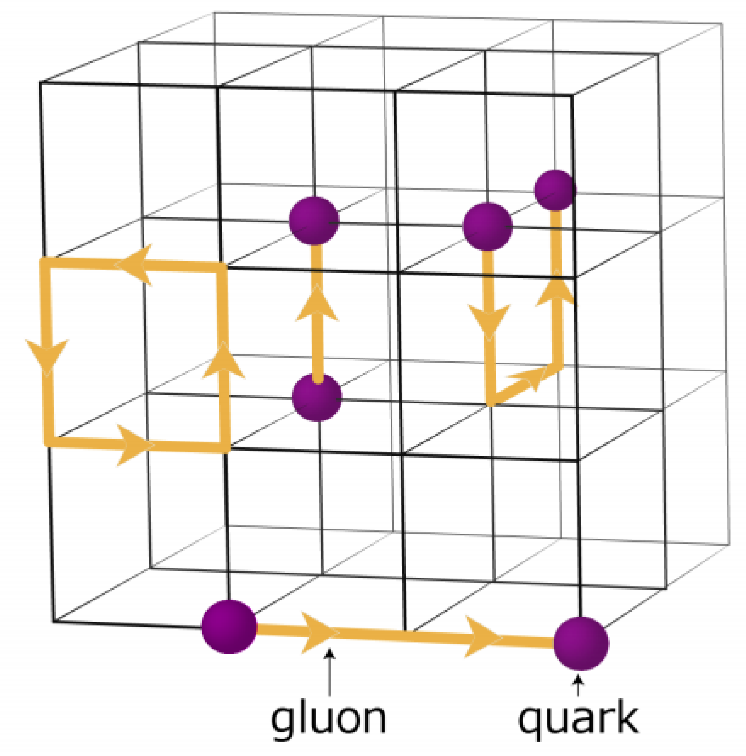
$$\frac{d\sigma}{dQ^2}\begin{pmatrix} \nu_l + n \to l^- + p \\ \bar{\nu}_l + p \to l^+ + n \end{pmatrix} = \frac{M^2 {G_F}^2 cos^2\theta_c}{8\pi {E_\nu}^2}\left\{A(Q^2) \pm B(Q^2)\frac{(s-u)}{M^2} + C(Q^2)\frac{(s-u)^2}{M^4}\right\},$$

$$\begin{aligned} A(Q^2) &= \frac{(m^2+Q^2)}{M^2}\Big[(1+\tau)F_A^2 - (1-\tau)F_1^2 + \tau(1-\tau)F_2^2 + 4\tau F_1 F_2 \\ &\quad -\frac{m^2}{4M^2}\left((F_1+F_2)^2 + (F_A+2F_P)^2 - 4\left(1+\tfrac{Q^2}{4M^2}\right)F_P^2\right)\Big], \\ B(Q^2) &= \tfrac{Q^2}{M^2}F_A(F_1+F_2), \\ C(Q^2) &= \frac{1}{4}(F_A^2 + F_1^2 + \tau F_2^2). \end{aligned}$$

F_A = axial form factor
$\tilde{F}_P$ = induced pseudoscalar
$G_E = F_1 - \tau F_2$ Electric
$G_M = F_1 + F_2$ Magnetic
$\tau = Q^2/4M^2$
M=M_n= $M_p \approx$ 939 MeV
$m = M_\pi$

Lattice QCD

[Formulated by K. Wilson (1974). Numerical computation field opened by M. Creutz (1979)]



Lattice QCD is QCD defined on a 4-dimensional Euclidean space-time lattice

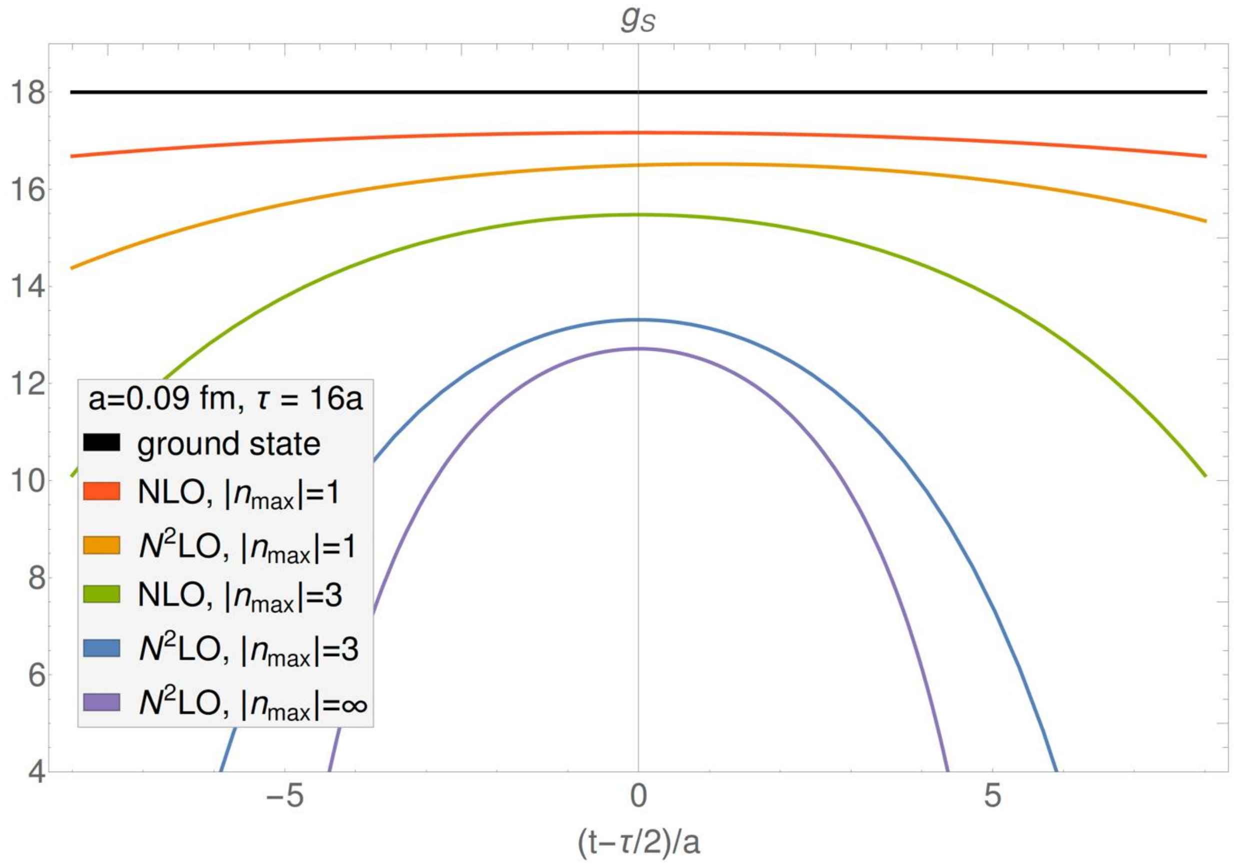
- Finite lattice spacing: (a)
- Quark fields $(q, \bar{q})$, Gauge fields (gluons): (U_μ)
- Perturbative & **Numerical (nonperturbative) calculations**

The simulation allows **ab initio** calculations of nonperturbative QCD interactions of quarks and gluons using the **Feynman path integral** formulation of QFT.

Major systematic errors coming from:

- Finite lattice spacing a (UV cut-off effect)
- Chiral fit to get value at physical pion mass
- Finite Volume
- Statistical errors
- **Excited state contaminations**
- Renormalization

ESC from $\boldsymbol{N\pi}$ and $\boldsymbol{N\pi\pi}$: $\boldsymbol{g_S^{u+d}}$



ChPT estimate of ESC
from $N(\boldsymbol{k})\pi(-\boldsymbol{k})$ and $N(\boldsymbol{0})\pi(\boldsymbol{k})\pi(-\boldsymbol{k})$
intermediate states
Gupta et al., PRL 127, 242002 (2021), Supplemental Material

For an explicit GEVP analysis with the basis $\{N(\boldsymbol{0}), N(\boldsymbol{1})\pi(-\boldsymbol{1})\}$,
see C. Alexandrou et al., PRD 110, 094514 (2024)

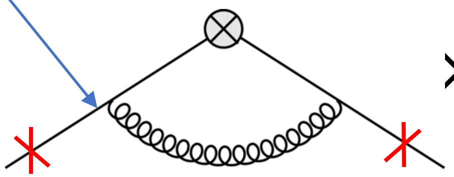
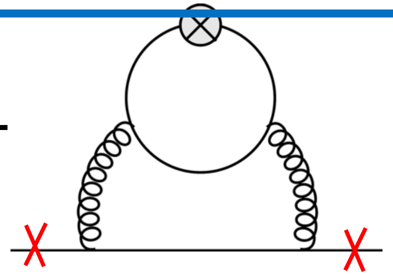
Operator mixing calculation in RI-sMOM

- We explicitly evaluated the 3×3 flavor (u, d, s) mixing matrices in **RI-sMOM**

$$g_\Gamma^f = \sum_{f'} Z_\Gamma^{ff'} g_\Gamma^{f'}|_{\mathrm{bare}}$$

Landau gauge fixed quark propagators using momentum source with $p \propto (1,1,1,1)$

Projected amputated Green's function $\mathrm{Tr}[(..)\mathbb{P}] \equiv \Lambda_\Gamma^{\mathrm{PA}}$

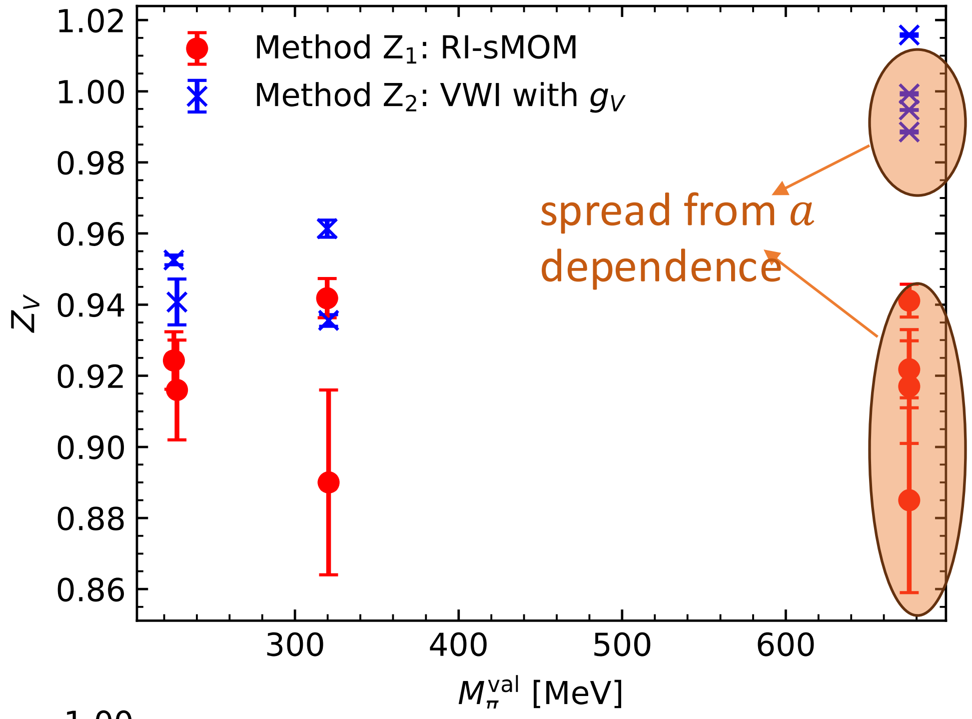
$$\left(Z_\Gamma^{-1}\right)^{ff'} = \sum_{f'} \frac{1}{Z_\psi^f} \mathrm{Tr}\Big[\Big(\quad$$  $$\times \delta^{ff'} - \quad$$  $$\quad\Big)\mathbb{P}(p', p)\Big]$$

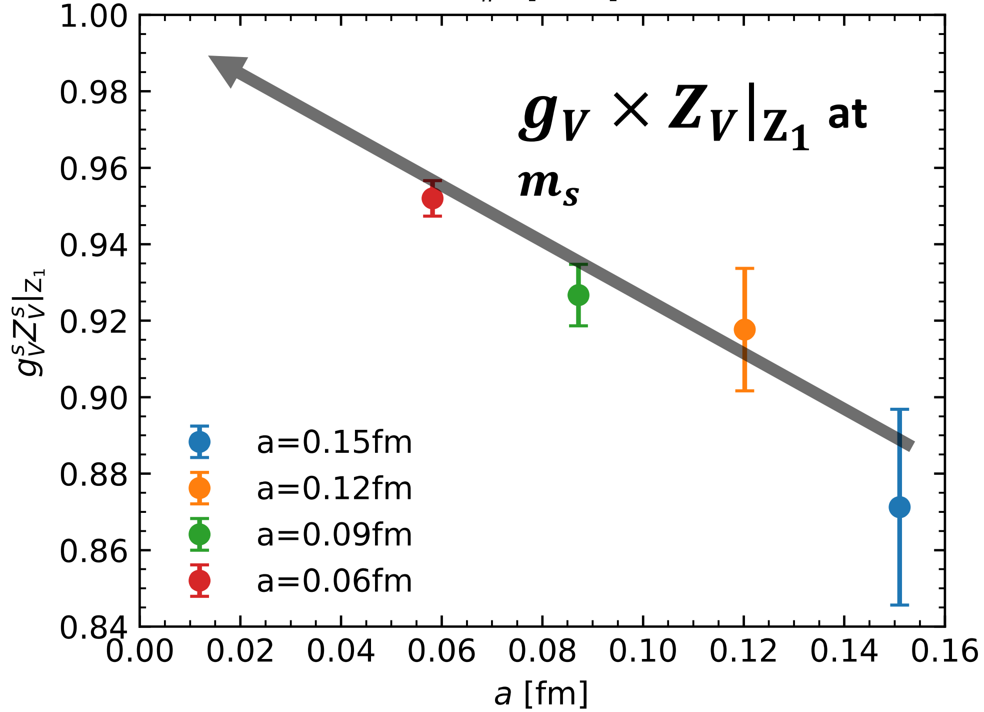
- **Z_1 method**: $Z_\psi(p) \equiv \frac{i}{12p^2} \mathrm{Tr}[S^{-1}(p) p \cdot \gamma]$
- **Z_2 method**: $Z_\psi^{\mathrm{VWI}}(p) \equiv \Lambda_V^{\mathrm{PA}}(p)/g_V$

Using Vector Ward Identity (VWI),

$$g_V Z_V = 1$$

And g_V from separate nucleon matrix element calculation





Z_V from methods $\mathbf{Z_1}$, $\mathbf{Z_2}$

- $Z_V|_{Z_1}$ and $Z_V|_{Z_2}(= 1/g_V)$ have different M_π^{val} and a dependence
- $g_V \times Z_V|_{Z_1}$ deviates from VWI (=1) at large quark mass, but VWI restored in the continuum limit

- To study the systematic effect in two different methods, $\{\mathbf{Z_1}, \mathbf{Z_2}\}$ we do chiral-continuum extrapolate $g_\Gamma|_{Z_1}$ and $g_\Gamma|_{Z_2}$, separately, and compare the results.