Precision Magnetic Field Measurements in Muon g-2

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Muon g-2 Collaboration

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The g-2 Magnetic Field

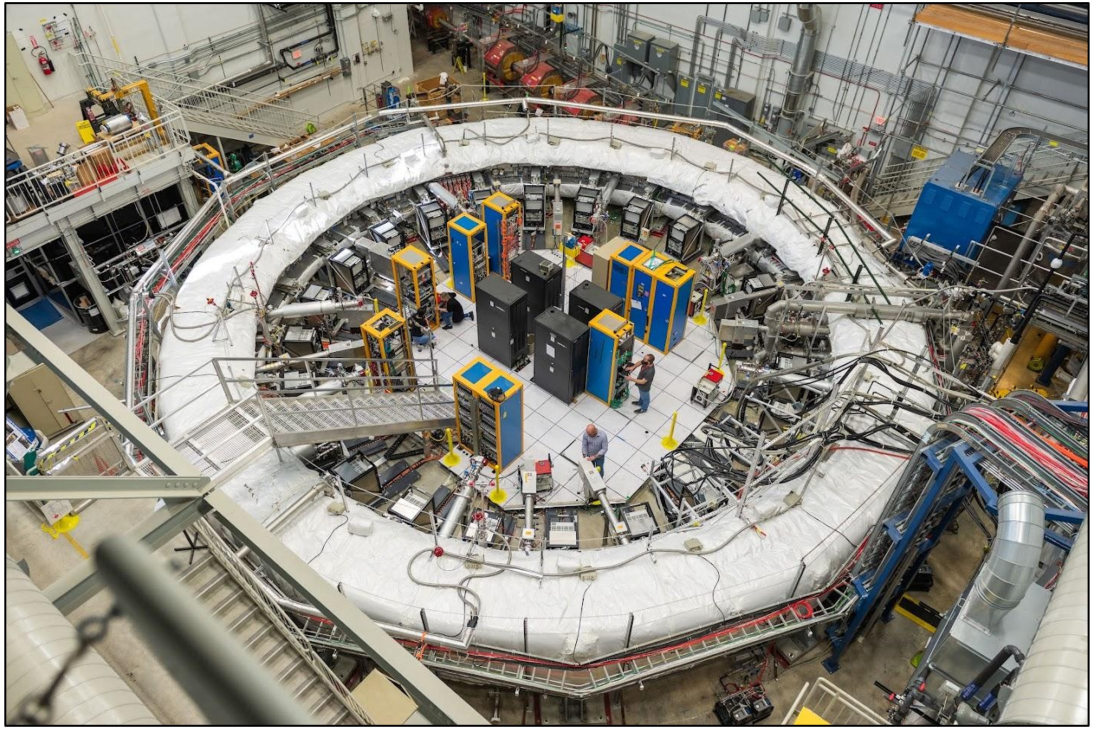
- The Muon g-2 experiment uses a **magnetic storage ring at 1.45 Tesla.**

- Muon anomalous moment, equation for a_μ:

$$\omega_a = -a_\mu * \left(\frac{q}{m_\mu}\right) * B$$

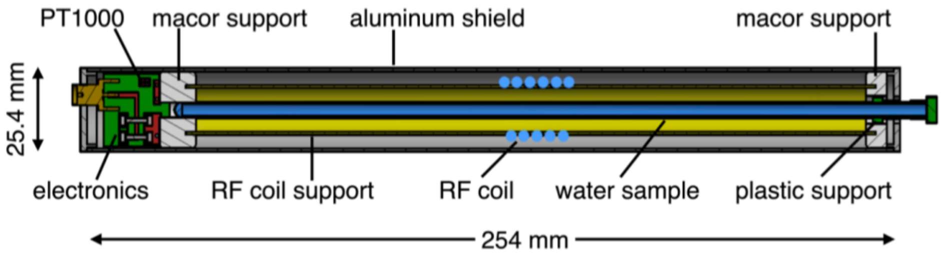
- a_μ precision goal: 140 ppb.

- B precision goal: 70 ppb.

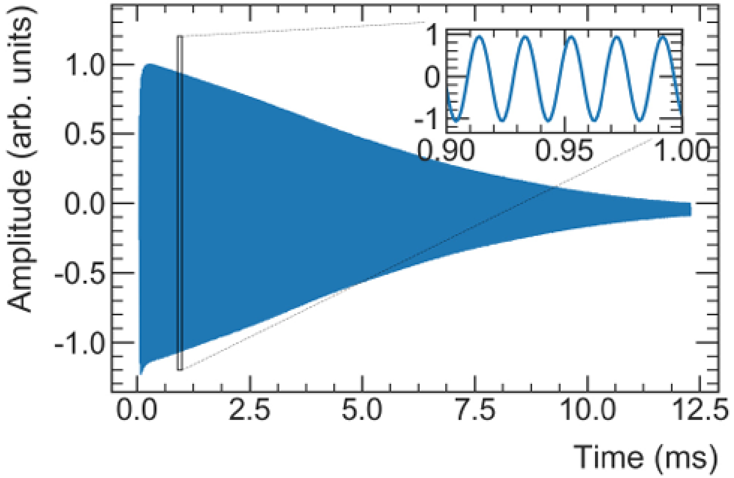


Muon g-2 Magnetic Storage Ring.

Measuring with Nuclear Magnetic Resonance (NMR)



g-2 Calibration Probe Diagram.



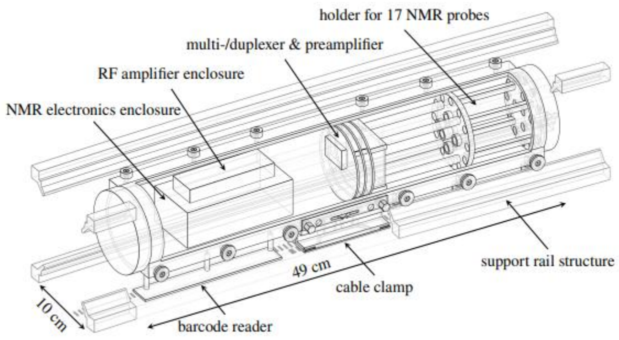
- FID signals from proton precession in sample.
- FID frequency is proportional to B.

$$\omega_p = \frac{g_p q}{2m_p} B$$

Coefficient known to 4.1 ppb.

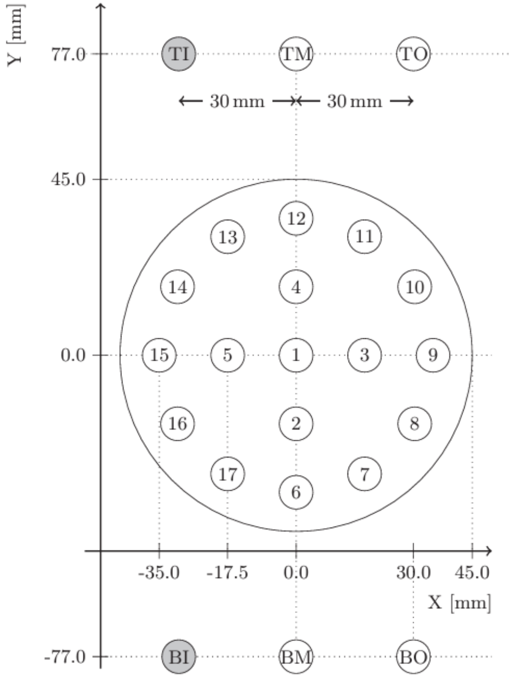
Measurement Overview

- Trolley Probes and Fixed Probes:



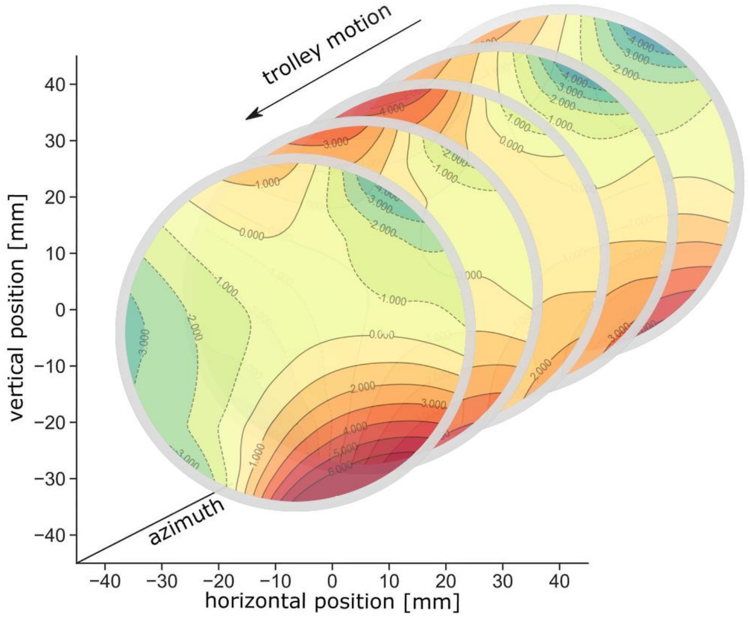
17 **trolley probes** map the field along the muon beam path.

378+ **fixed probes** track changes in the field while muons are running.



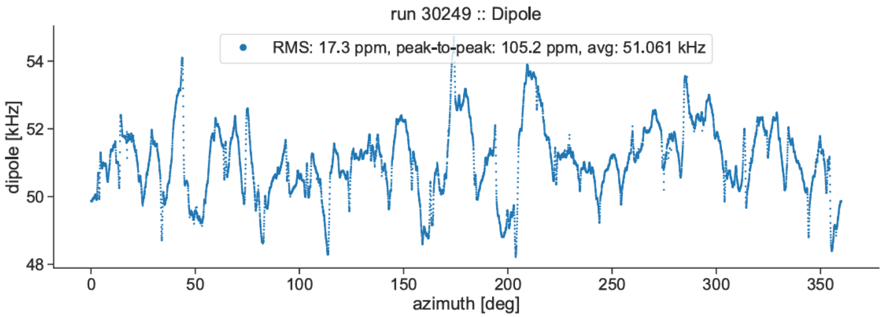
Relative positions of trolley and fixed probes.

Trolley Runs



Trolley readings per slice are fit to 2D multipole expansions.

Trolley runs were performed every ~3 days, with muon beam off.



Each trolley run covers the circumference of the storage ring.

Analysis Overview

- Tasks and Challenges:
  - Interpolation – Combining trolley and FP data.
  - Calibration – Trolley perturbs field while present.
  - Transient Fields – Synched to muon injection, while trolley/FP are asynchronous.

➢ The Objective:

$$\tilde{\omega}_p{}'$$

Shielded proton

Spherical water sample at 25° Celsius

In field averaged over time/space muon distribution

Using $\widetilde{\omega}_p{}'$

- Shielded proton frequency $\widetilde{\omega}_p{}'$ combines with ω_a, alongside known fundamental constants, to produce a_μ.

$$a_\mu = \left\{\frac{\omega_a}{\widetilde{\omega}_p{}'}\right\} * \underbrace{\frac{\mu_p{}'}{\mu_{Bohr}} * \frac{m_\mu}{m_e}}_{\text{Precisely-known ratios from other experiments (CODATA).}}$$

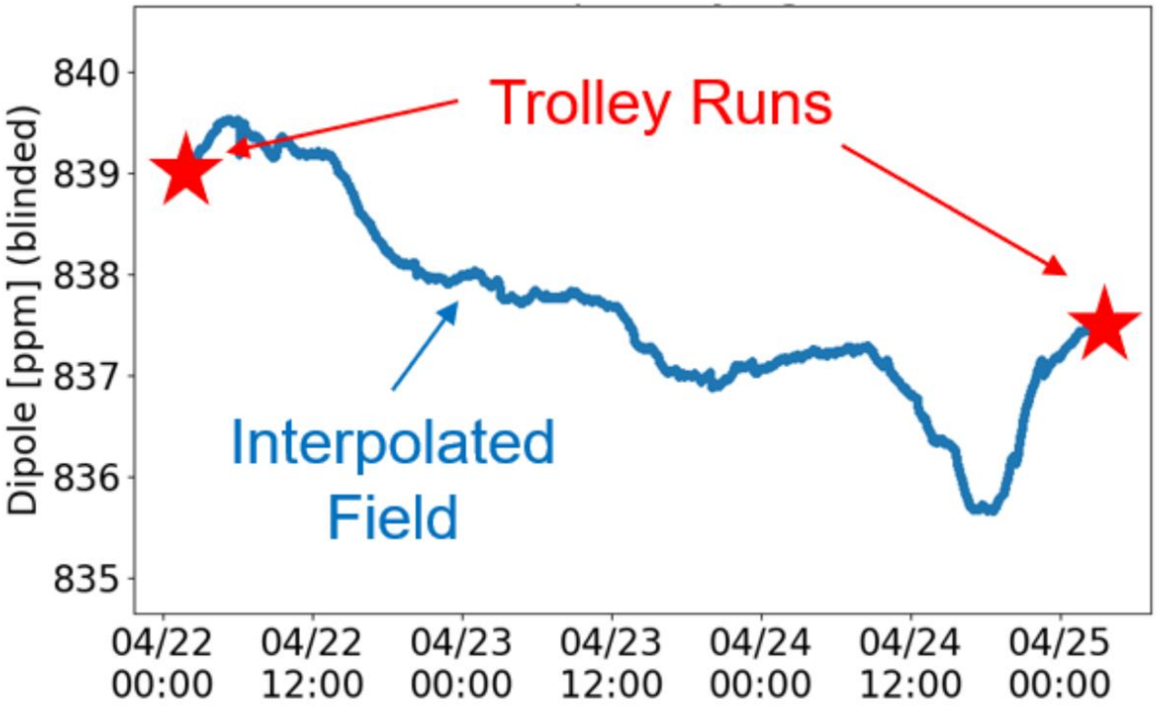
Ratio from our collaboration's analysis.

Precisely-known ratios from other experiments (CODATA).

(Knowing this ratio at 25° C is why we aim for $\widetilde{\omega}_p{}'$.)

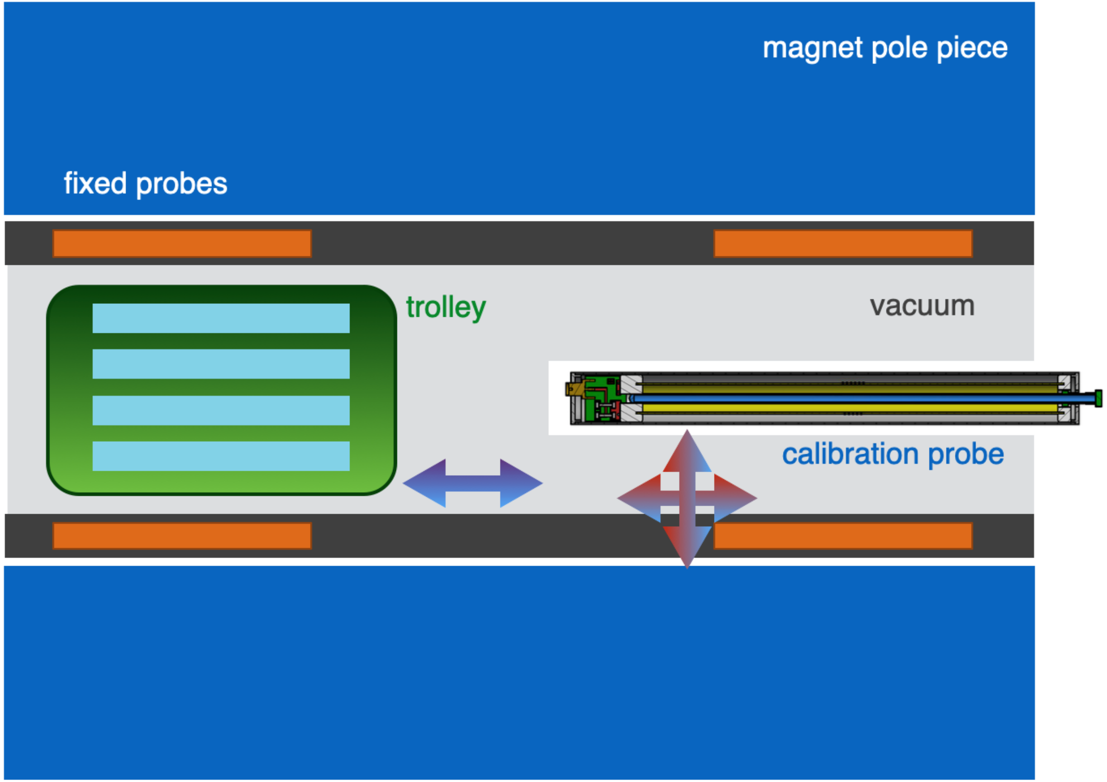
Interpolation

- Fixed probes measure between each trolley run pair, showing evolution from one map to the next.
- Any mismatches between evolved initial map and next map become uncertainties.

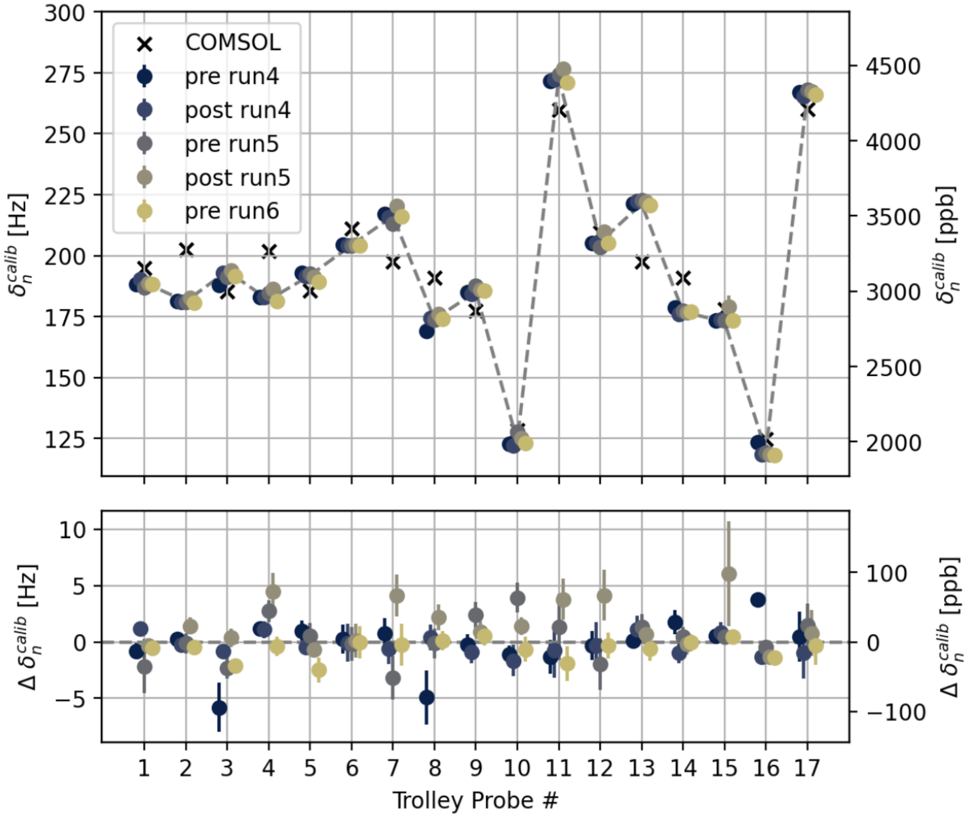


Trolley Calibration

- Determines field perturbation from trolley structure.
- Each trolley probe compared to calibration probe.
    - Cali probe is solo, so less field perturbation.
- Alignment and Rapid Swapping procedure.
- Nine campaigns over the course of g-2 operations.



Trolley Calibration Campaign Results



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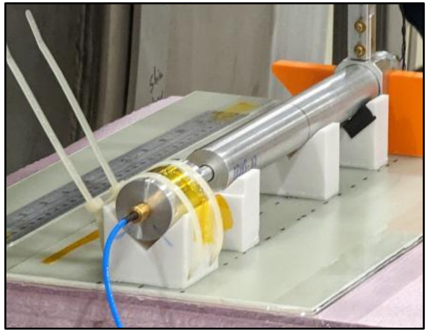
Absolute Calibration

- Getting $\tilde{\omega}'_p$ $(\textit{shielded spherical-water...})$ from $\omega_p(cali\ probe)$.
- Tests performed with Argonne magnet to quantify each effect: Calibration Probe vs Absolute Probe.



$$\tilde{\omega}'_p(T_r) = \left[1 + \delta^T(T_r - T) + \left(\epsilon - \frac{1}{3}\right)\chi_{H_2O}(T) - \delta_{probe}\right]\omega_p(cali\ probe, T)$$

Temperature Correction ~10 ppb

Shape and Susceptibility Effects ~1500 ppb

Material Effects ~10 ppb

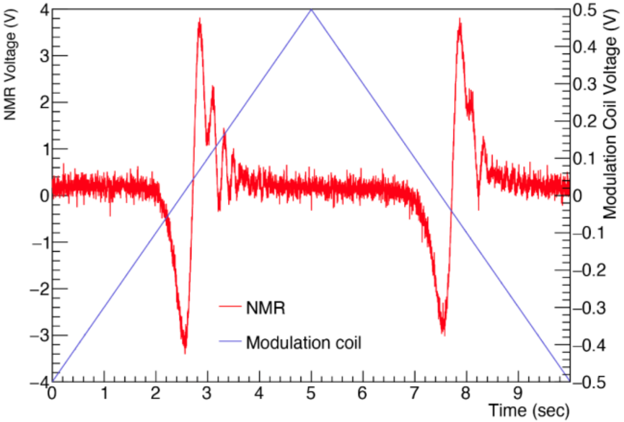
Cross-Calibrations

- Extra tests using probes undergoing independent calibrations.
- Cross-Calibration #1:
  Cali probe vs ^{3}He probe.
- Cross-Calibration #2:
  Cali probe vs J-PARC probe.
- Increased uncertainty by 25ppb, but improved our confidence in the resulting uncertainty.

^{3}He-based spherical alternate probe

Agreement on the 1.7 σ-level, supporting results for material effect and shape factor.

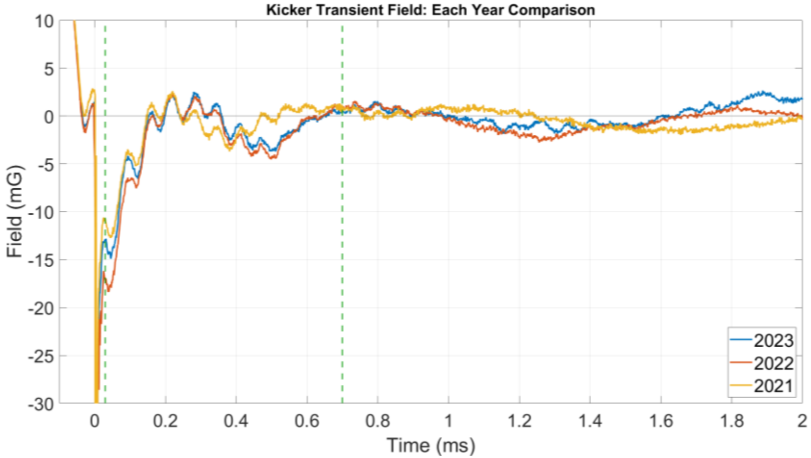
J-PARC
Continuous Wave NMR probe



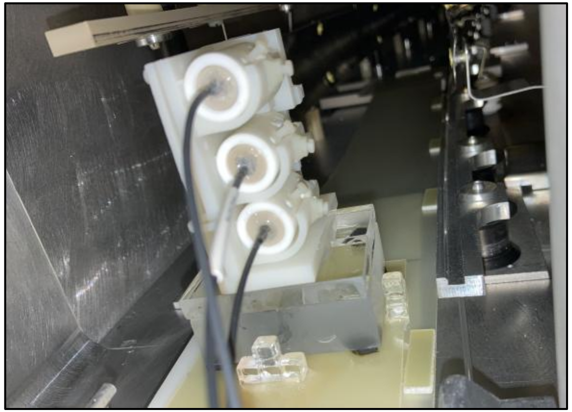
2019 (1.45T), 2019 (1.7T) campaigns: some inconsistencies.

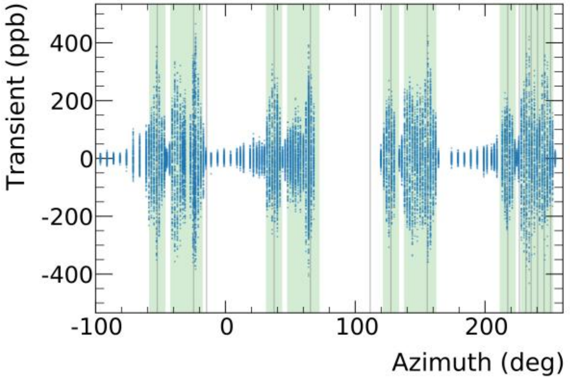
2022 (3T), 2023 (1.45T) campaigns: in good agreement.

Transient Fields

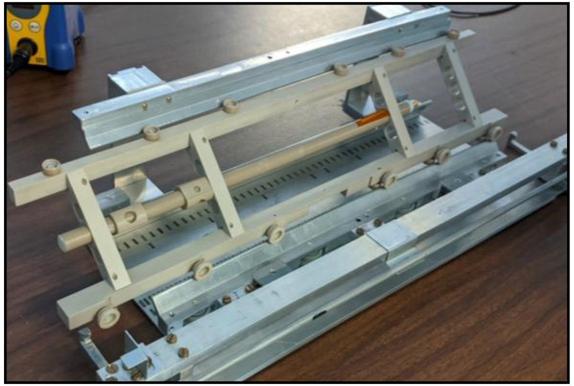


Kicker transient field and Faraday Effect magnetometers.





Electrostatic Quadrupoles transient field and NMR magnetometer.



Uncertainty Improvement

Run-1 Uncertainties Table

Quantity	Correction terms (ppb)	Uncertainty (ppb)
ω_a^m (statistical)	$\cdots$	434
ω_a^m (systematic)	$\cdots$	56
C_e	489	53
C_p	180	13
C_{ml}	−11	5
C_{pa}	−158	75
$f_{\text{calib}} \langle \omega_p(x,y,\phi) \times M(x,y,\phi) \rangle$	$\cdots$	56
B_k	−27	37
B_q	−17	92
$\mu'_p(34.7°)/\mu_e$	$\cdots$	10
m_μ/m_e	$\cdots$	22
$g_e/2$	$\cdots$	0
Total systematic	$\cdots$	157
Total fundamental factors	$\cdots$	25
Totals	544	462

Run-4/5/6 Uncertainties Table

Quantity		Correction (ppb)	Uncertainty (ppb)
ω_a^m (statistical)		$\cdots$	114
ω_a^m (systematic)		$\cdots$	30
C_e	Electric Field	347	27
C_p	Pitch	175	9
C_{pa}	Phase Acceptance	-33	15
C_{dd}	Differential Decay	26	27
C_{ml}	Muon Loss	0	2
$\langle \omega'_p \times M \rangle$ (mapping, tracking)		$\cdots$	34
$\langle \omega'_p \times M \rangle$ (calibration)		$\cdots$	34
B_k	Transient Kicker	-37	22
B_q	Transient ESQ	-21	20
μ'_p/μ_B		$\cdots$	4
m_μ/m_e		$\cdots$	22
Total systematic for $\mathcal{R}'_\mu$		$\cdots$	76
Total for a_μ		572	139

Field uncertainty total: **56 ppb**.

Target Achieved!

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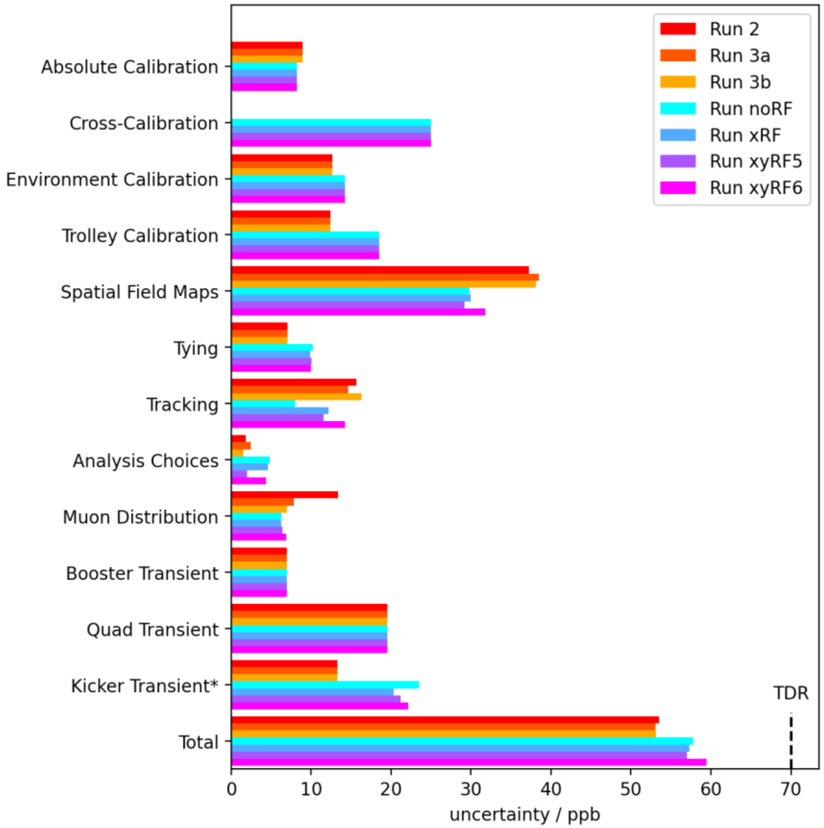
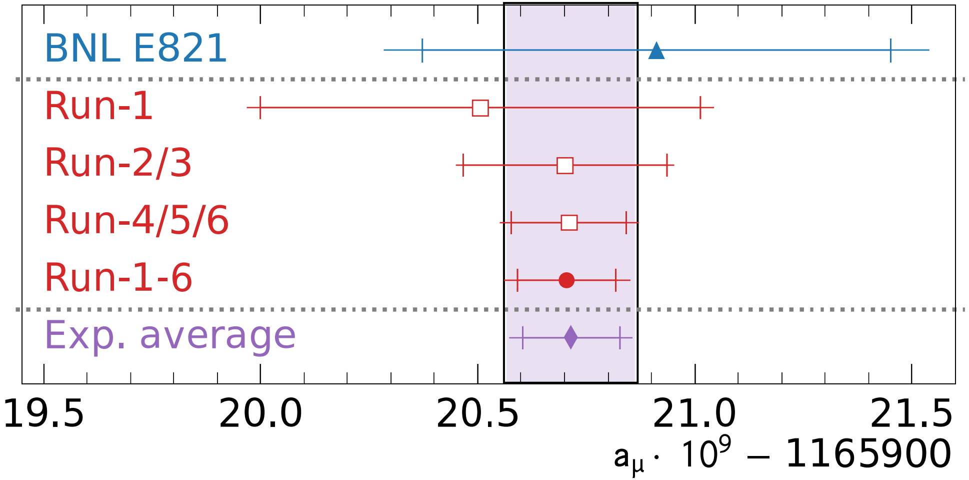
Relative Field Uncertainties



Conclusion

- Muon g-2 result diagram:



- Magnetic field measurements are important to Muon g-2's precision goal.
- NMR systems: Fixed Probes tracking, Trolley Probes mapping, and Calibration Probes calibrating.
  - Plus plenty of analysis, extra measurements, and corrections!
- Goal achieved!