Quasielastic Lepton-Nucleus Scattering and the Correlated Fermi Gas Model

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Outline

- Introduction
- Nucleon physics: Form factors
- Nuclear physics: Correlated Fermi Gas (CFG) Model
- Separating nuclear and nucleon effects in neutrino-nucleus interactions
- Conclusions

Introduction

Motivation

- Current and future neutrino experiments aim to
- Precisely measure Standard Model parameters
- Uncover non-standard neutrino interactions (NSI)

- Precision measurements require better control of systematic uncertainties in lepton-nucleus interactions

- Example:
  Quasiealstic $\nu - N$ scattering: $\nu_\ell + \text{nucleus} \to \ell^- + \text{nucleus}'$

Quasiealstic $\nu - N$ scattering

- Quasiealstic $\nu - N$ scattering: $\nu_\ell + \text{nucleus} \to \ell^- + \text{nucleus}'$
- At the quark level: $\nu_\ell + d \to \ell^- + u$. Process "folded" twice

Quark: $\nu_\ell + d \to \ell^- + u$

$\Downarrow$ Form factors

Nucleon $\nu_\ell + n \to \ell^- + p$

$\Downarrow$ Nuclear model

Nucleus: $\nu_\ell + \text{nucleus} \to \ell^- + \text{nucleus}'$

- Form factors: how do nucleons (proton, neutron) react to $\gamma, W^\pm$?
- Nuclear model: how are the nucleons are combined in the nucleus?
- Precision needs separate control of *form factors* and *nuclear effects*

From quark to nucleon: Form factors

- Matrix element of EM or charged weak current J_μ $(q = p' - p)$
  $\langle p'|J_\mu|p\rangle = \bar{u}(p')\Gamma_\mu(q)u(p)$ gives rise to form factors

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2(q^2) + \gamma_\mu\gamma_5 F_A(q^2) + \frac{q_\mu}{m_N}\gamma_5 F_P(q^2)$$

- The free nucleon cross section depends on nucleon tensor $H_{\mu\nu}$

$$H_{\mu\nu} = \mathrm{Tr}[(\not{p}' + m_{p'})\Gamma_\mu(q)(\not{p} + m_p)\bar{\Gamma}_\nu(q)]$$

- The nucleon tensor $H_{\mu\nu}$ can be decomposed as

$$H_{\mu\nu} = -g_{\mu\nu}H_1 + \frac{p_\mu p_\nu}{m_N^2}H_2 - i\frac{\epsilon_{\mu\nu\rho\sigma}}{2m_N^2}p^\rho q^\sigma H_3 + \frac{q_\mu q_\nu}{m_N^2}H_4 + \frac{(p_\mu q_\nu + q_\mu p_\nu)}{2m_N^2}H_5$$

$$H_1 = 8m_N^2 F_A^2 - 2q^2\left[(F_1+F_2)^2 + F_A^2\right],\, H_2 = H_5 = 8m_N^2\left(F_1^2 + F_A^2\right) - 2q^2F_2^2\,,$$

$$H_3 = -16m_N^2\, F_A(F_1+F_2)\,, H_4 = -\frac{q^2}{2}\left(F_2^2 + 4F_P^2\right) - 2m_N^2F_2^2 - 4m_N^2\left(F_1F_2 + 2F_AF_P\right)$$

Notation from [Bhattacharya, Hill, GP PRD **84** 073006 (2011)]

From nucleon to nucleus: Nuclear tensor

- Similar to nucleon tensor need the *nuclear* tensor (m_T is target mass)

$$W_{\mu\nu} = -g_{\mu\nu} W_1 + \frac{p_\mu^T p_\nu^T}{m_T^2} W_2 - \frac{i\epsilon_{\mu\nu\rho\sigma} p_T^\rho q^\sigma}{2m_T^2} W_3 + \frac{q_\mu q_\nu}{m_T^2} W_4 + \frac{p_\mu^T q_\nu + q_\mu p_\nu^T}{2m_T^2} W_5$$

- The (anti-)neutrino-nucleus cross section is

$$\frac{d\sigma^\nu_{\rm nuclear}}{dE_\ell\, d\cos\theta_\ell} = \frac{G_F^2 |\vec{P}_\ell|}{16\pi^2\, m_T} \Bigg\{ 2(E_\ell - |\vec{P}_\ell| \cos\theta_\ell)\, W_1 + (E_\ell + |\vec{P}_\ell| \cos\theta_\ell) W_2$$

$$\pm \frac{1}{m_T} \Big[(E_\ell - |\vec{P}_\ell| \cos\theta_\ell)(E_\nu + E_\ell) - m_\ell^2 \Big] W_3 + \frac{m_\ell^2}{m_T^2} (E_\ell - |\vec{P}_\ell| \cos\theta_\ell) W_4 - \frac{m_\ell^2}{m_T}\, W_5 \Bigg\}$$

where the upper (lower) sign is for neutrino (anti-neutrino) scattering.

- Neglecting the electron mass, the electron-nucleus cross section is

$$\frac{d\sigma^e_{\rm nuclear}}{dE_\ell\, d\cos\theta_\ell} = \frac{\alpha^2 E_\ell^2}{2q^4\, m_T} \Big[2W_1(1-\cos\theta_\ell) + W_2(1+\cos\theta_\ell) \Big]$$

Notation: [Bhattacharya, Carey, Cohen, GP, PRD **111** 096021 (2025)]

From nucleon to nucleus: Nuclear model

- We have two tensors: Nucleon tensor ($H_{\mu\nu}$) and nuclear tensor ($W_{\mu\nu}$)
- Connecting the two is a crucial step not always spelled out in papers
- Here one *assumes* separation between nucleon and nuclear physics
- In the following discuss two nuclear models
- "Classic" Relativistic Fermi Gas (RFG) Model
  [Smith, Moniz, NPB **43**, 605 (1972)]
- Implementation of the Correlated Fermi Gas (CFG) Model of
  [Hen, Li, Guo, Weinstein, Piasetzky, PRC, **91**, 025803 (2015)]
- For both models:
- initial nucleons are distributed as $n_i(\boldsymbol{p})$
- final nucleons are distributed as $n_f(\boldsymbol{p}')$
- single nucleon cross section $\sigma_{\text{nucleon}}(\boldsymbol{p} \to \boldsymbol{p}')$ depends on form factors
- Nuclear cross section is

$$\sigma_{\text{nuclear}} = n_i(\boldsymbol{p}) \otimes \sigma_{\text{nucleon}}(\boldsymbol{p} \to \boldsymbol{p}') \otimes [1 - n_f(\boldsymbol{p}')]$$

From nucleon to nucleus: Nuclear model

- For both RFG and CFG models:
- initial nucleons are distributed as $n_i(\boldsymbol{p})$
- final nucleons are distributed as $n_f(\boldsymbol{p}')$
- single nucleon cross section $\sigma_{\rm nucleon}(\boldsymbol{p} \to \boldsymbol{p}')$ depends on form factors
- Nuclear cross section is

$$\sigma_{\rm nuclear} = n_i(\boldsymbol{p}) \otimes \sigma_{\rm nucleon}(\boldsymbol{p} \to \boldsymbol{p}') \otimes [1 - n_f(\boldsymbol{p}')]$$

- The relation is a model and doesn't follow from factorization theorem.
- More explicitly

$$W_{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{m_T}{E_p} 2Vn_i(\boldsymbol{p}) \int \frac{d^3p'}{(2\pi)^3 2E_{p'}} (2\pi)^4 \delta^4(p - p' + q)\, H_{\mu\nu} \left[1 - n_f(\boldsymbol{p'})\right]$$

$$\text{Number of nucleons} = \int \frac{d^3p}{(2\pi)^3} 2Vn_i(\boldsymbol{p})$$

From nucleon to nucleus: Spectral function

- The closest we get to a statement of separation in the literature is in the "spectral function" notation

- Using the notation of [Joanna E. Sobczyk PRC **96** 045501 (2017)]

$$W^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{3\, m_N^2}{4\pi p_F^3} \int dE\, S^h(E, \boldsymbol{p}) S^p(\omega - E, \boldsymbol{p} + \boldsymbol{q}) H^{\mu\nu},$$

- E is the "removal energy",
- the superscript "h" denotes the initial nucleon ("hole")
- the superscript "p" denotes the final nucleon ("particle")

- Different models have different spectral functions S^h and S^p

- In the next section we discuss the form factors in more detail

Nucleon physics: Form factors

Anthony Tropiano

Form Factors: What is the problem?

- Matrix element of EM or charged weak current J_μ $(q = p' - p)$

$$\langle p'|J_\mu|p\rangle = \bar{u}(p')\Gamma_\mu(q)u(p)$$

gives rise to form factors

$$\Gamma_\mu(q) = \gamma_\mu F_1(q^2) + \frac{i}{2m_N}\sigma_{\mu\nu}q^\nu F_2(q^2) + \gamma_\mu\gamma_5 F_A(q^2) + \frac{q_\mu}{m_N}\gamma_5 F_P(q^2)$$

The form factors are non-perturbative objects.

- **Nobody** knows the *exact* functional form of these form factors

- They don't have to have a dipole/polynomial/inverse polynomial or any other functional form

- What to do?

Form Factors: What we do know

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

- Analytic properties of $G_E^p(t)$ and $G_M^p(t)$ are known $(t = q^2)$
- They are analytic outside a cut $t \in [4m_\pi^2, \infty]$
  [Federbush, Goldberger, Treiman, Phys. Rev. **112**, 642 (1958)]

- $e - p$ scattering data is in $t < 0$ region

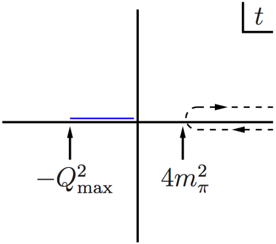


Figure from [Hill, GP PRD **82** 113005 (2010)]

z expansion

- z expansion: map domain of analyticity onto unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$

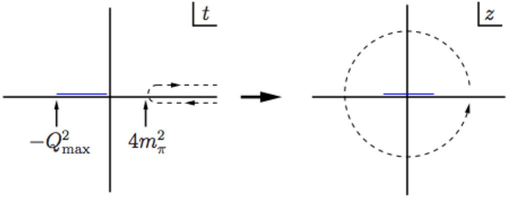
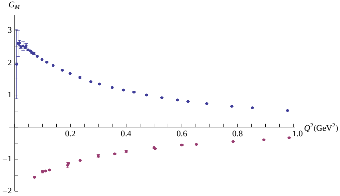


Figure from [Hill, GP PRD **82** 113005 (2010)]

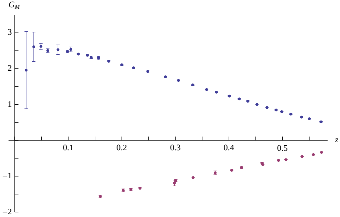
- Expand $G^p_{E,M}$ in a Taylor series in z: $G^p_{E,M}(q^2) = \sum_{k=0}^{\infty} a_k\, z(q^2)^k$

z expansion

- [Zachary Epstein, GP, Joydeep Roy PRD **90**, 074027 (2014)]
  $G_M(Q^2)$ for proton (blue, above axis) and neutron (red, below axis)



  $G_M(z)$ for proton (blue, above axis) and neutron (red, below axis)



- See also R.J. Hill talk at FPCP 2006 [hep-ph/0606023]

z expansion

- **The** method for **meson** form factors
  [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]

- z expansion first applied to **baryon** form factors in
  [Hill, GP PRD **82** 113005 (2010)]

Axial form factor

- The interaction

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{ud}^* \, \bar{\ell}\gamma^\alpha(1-\gamma^5)\nu \, \bar{u}\gamma_\alpha(1-\gamma^5)d$$

  Known current: $\bar{u}\gamma_\alpha(1-\gamma^5)d$

- Parametrize $\langle p(p')|\bar{u}\gamma_\alpha(1-\gamma^5)d|n(p)\rangle$ by form factors:

  F_1, F_2, F_p, F_A, functions of $q^2 = (p'-p)^2$

- F_1, F_2 related via isospin to EM form factors: $F_{1,2} = F_{1,2}^p - F_{1,2}^n$

- F_P contribution suppressed by m_ℓ^2/m_N^2

- What do we know about $F_A(q^2)$?

Axial form factor

- Consider a small q^2 expansion of $F_A(q^2)$

- $F_A(0) = -1.269$ known from neutron decay
- Axial radius: $r_A \equiv \sqrt{\frac{6F_A'(0)}{F_A(0)}}$ or axial mass: $m_A \equiv \sqrt{\frac{2F_A(0)}{F_A'(0)}}$

- Common **historical model** for F_A: the dipole model

$$F_A = F_A(0)\,[1 - q^2/(m_A^{\rm dipole})^2]^{-2}$$

- Warning: dipole model known to be inadequate for EM form factors!

- What can possibly go wrong?

The axial mass problem

- ν experiments before 1990: $m_A^{\text{dipole}} = 1.026 \pm 0.021$ GeV
  [Bernard, Elouadrhiri, Meissner, J. Phys. G 28, R1 (2002)]
- Axial mass $m_A^{\text{dipole}} = 1.35 \pm 0.17$ GeV
  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]
- Discussion in literature focused on nuclear effects:
  use dipole model for F_A and vary nuclear effects
  Assumed to have solved the problem

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- Discussion in literature focused on nuclear effects:
  use dipole model for F_A and vary nuclear effects
  Assumed to have solved the problem
- Does it have to be a nuclear effect?
- What if we fix nuclear effects and vary F_A?
- Expand F_A in a Taylor series in z: $F_A = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$

Example: m_A from MiniBooNE's neutrino data

- Example: extracting m_A using the z expansion

  [Bhattacharya, Hill, GP PRD **84** 073006 (2011)]

  from MiniBooNE's neutrino data

  [MiniBooNE Collaboration, PRD **81** 092005 (2010)]

- Mostly follow MiniBooNE's analysis: use RFG as nuclear model

- Our fit using z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV

  Our fit using dipole model: $m_A^{\text{dipole}} = 1.29 \pm 0.05$ GeV

  MiniBooNE's fit: $m_A^{\text{dipole}} = 1.35 \pm 0.17$ GeV

Example: m_A from MiniBooNE's antineutrino data

- Example: extracting m_A using the z expansion
  [Bhattacharya, GP, Tropiano, PRD **92**, 113011 (2015)]
  from MiniBooNE's antineutrino data
  [MiniBooNE Collaboration, PRD **88**, 032001 (2013)]

- For scattering off mineral oil C_nH_{2n+2} ($n \sim 30$):
- $\bar{\nu}$ can scatter off protons in C *and* protons in H

- MiniBooNE used *different* m_A^{dipole} for C and H
  Problematic! the axial mass is a fundamental property of the nucleon
  Might hide nuclear effects in the axial mass

- We used the same F_A for C and H and extract *one* axial mass

Example: m_A from MiniBooNE's antineutrino data

- Example: extracting m_A using the z expansion
  [Bhattacharya, GP, Tropiano, PRD **92**, 113011 (2015)]
  from MiniBooNE's antineutrino data
  [MiniBooNE Collaboration, PRD **88**, 032001 (2013)]

- Our fit using z expansion: $m_A = 0.84^{+0.12}_{-0.04} \pm 0.11$ GeV
  Our fit using dipole model: $m_A^{\rm dipole} = 1.27^{+0.03}_{-0.04}$ GeV

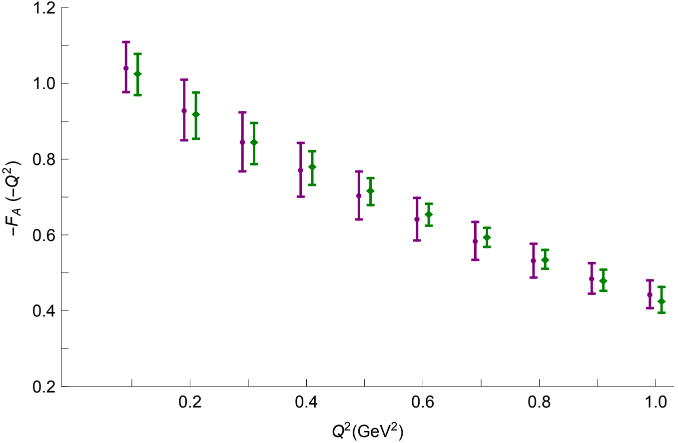
- Neutrino data z expansion: $m_A = 0.85^{+0.22}_{-0.07} \pm 0.09$ GeV
- Neutrino data dipole model: $m_A^{\rm dipole} = 1.29 \pm 0.05$ GeV

- Ultimately $m_A^{\rm dipole}$ is model parameter
  It has **no** physical meaning
  It is a proxy for the form factor

- We should use the full form factor

Comparing F_A from ν and $\bar{\nu}$ data

- Assuming the RFG model
  We can also extract F_A directly from MiniBooNE data
  neutrino data (purple circles) and antineutrino data (green diamonds).



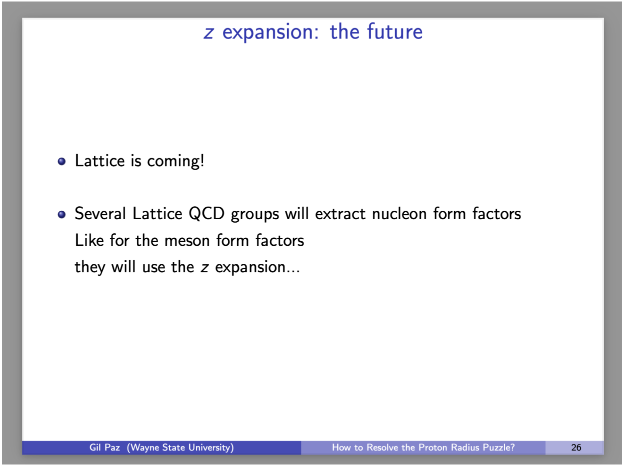
[Bhattacharya, GP, Tropiano, PRD **92**, 113011 (2015)]

Full axial form factor

- We should use the full axial form factor
- For historical reasons a dipole model was used
- A systematic parameterization based on the z-expansion was suggested in [Bhattacharya, Hill, GP PRD **84** 073006 (2011)]

Full axial form factor

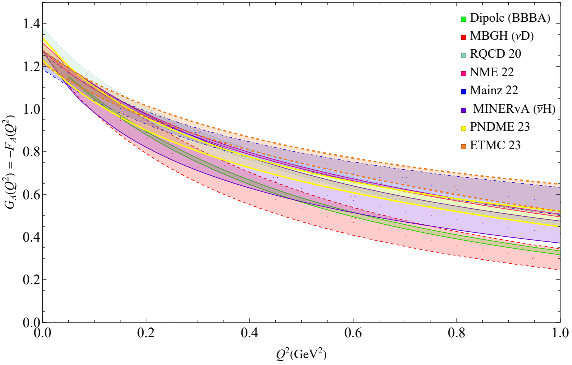
- At a workshop at ECT* in June 2016 I made the following prediction



- In the last few years several LQCD and experimental z-expansion based extractions of the axial form factor became available

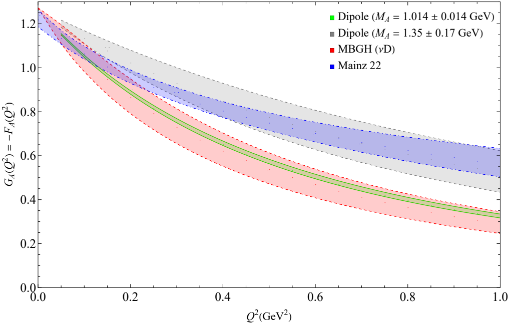
Axial form factor

- Recent lattice extractions:
  - RQCD 20: [Bali, et al., JHEP, 05:126, (2020)]
  - NME 22: [Park, et al., PRD, **105**, 054505, (2022)]
  - Mainz 22: [Djukanovic, et al., PRD, **106**, 074503, (2022)]
  - PNDME 23: [Jang, et al., Phys. PRD, **109**, 014503, (2024)]
  - ETMC 23: [Alexandrou, et al., PRD, **109**, 034503, (2024)]
- Data extractions:
  - MBGH: [Meyer, Betancourt, Gran, Hill, PRD **93**, 113015, (2016)]
  - MINERvA: [Cai, et al., Nature, **614**, 48-53, (2023)].



Axial form factor

- Looking at two of the "extreme" extractions:
- Mainz 22: [Djukanovic, et al., PRD, **106**, 074503, (2022)]
- MBGH: [Meyer, Betancourt, Gran, Hill, PRD **93**, 113015, (2016)]



Maybe this reaction was premature...

Nuclear physics:
Correlated Fermi Gas (CFG) Model

[Bhattacharya, Carey, Cohen, GP, PRD **111** 096021 (2025)]

Sam Carey

Goals

- Quasi-Elastic Lepton Nucleus Scattering and the Correlated Fermi Gas Model [Bhattacharya, Carey, Cohen, GP, PRD **111** 096021 (2025)]

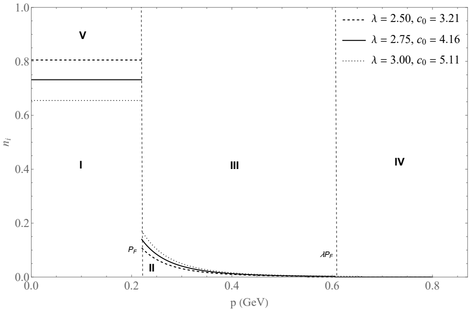
- Goals of the paper
- Implement Correlated Fermi Gas (CFG) Model of [Hen, Li, Guo, Weinstein, Piasetzky, PRC, **91**, 025803 (2015)] for lepton-nucleus scattering
- Use CFG and Relativistic Fermi Gas (RFG) Model to separate form factors and nuclear effects

- For both models:
- initial nucleons are distributed as $n_i(\boldsymbol{p})$
- final nucleons are distributed as $n_f(\boldsymbol{p}')$
- single nucleon cross section $\sigma_{\rm nucleon}(\boldsymbol{p} \to \boldsymbol{p}')$ depends on form factors
- Nuclear cross section is

$$\sigma_{\rm nuclear} = n_i(\boldsymbol{p}) \otimes \sigma_{\rm nucleon}(\boldsymbol{p} \to \boldsymbol{p}') \otimes [1 - n_f(\boldsymbol{p}')]$$

CFG model

- RFG model: nucleons occupy states only up to the Fermi momentum
- Data from last two decades showed $\sim 20\%$ of nucleons have momentum greater than the Fermi momentum
- They appear in short-range correlated (SRC) neutron-proton pairs
- CFG model: add high-momentum tail above the Fermi momentum



Nucleon momentum distribution

- For the RFG model

$$n_i(\boldsymbol{p}) = \theta(p_F - |\boldsymbol{p}|)\,, \quad n_f(\boldsymbol{p}') = \theta(p_F - |\boldsymbol{p}'|)$$

- For the CFG model

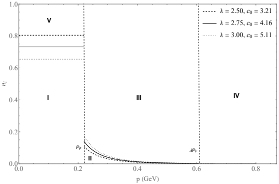
$$n_{\text{CFG}}(\boldsymbol{p}) = \begin{cases} 1 - \left(1 - \dfrac{1}{\lambda}\right)\dfrac{c_0}{\pi^2} \equiv \alpha_0 & |\boldsymbol{p}| \le p_F \\ \dfrac{c_0}{3\pi^2}\left(\dfrac{p_F}{\boldsymbol{p}}\right)^4 \equiv \dfrac{\alpha_1}{|\boldsymbol{p}|^4} & p_F \le |\boldsymbol{p}| \le \lambda p_F \\ 0 & |\boldsymbol{p}| \ge \lambda p_F\,. \end{cases}$$

where $c_0 = 4.16 \pm 0.95$ and $\lambda \approx 2.75 \pm 0.25$

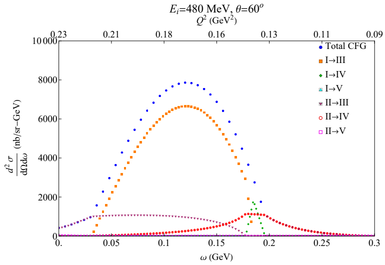
- Since high-momentum tail $\propto |\boldsymbol{p}|^{-4}$
all the phase space integrals are done analytically

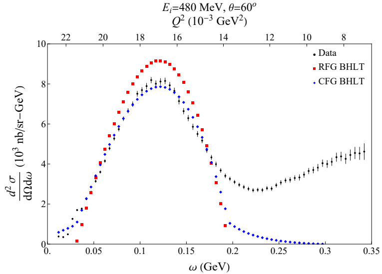
Highlights of CFG implementation

- CFG model: add high-momentum tail above the Fermi momentum



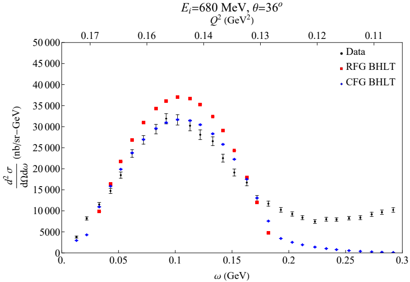
- Initial nucleon can be in regions I or II
- Final nucleon can be in regions III, IV, or V
- Combining all regions we can compare CFG model to data

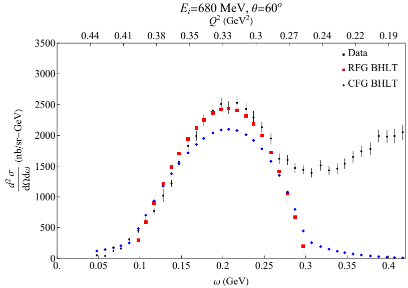




$e - C$ data: fix form factor vary nuclear model

- Fix the vector form factors to the z-expansion based extraction [Borah, Hill, Lee, Tomalak, PRD, **102** 074012, (2020)]
- Compare RFG and CFG models

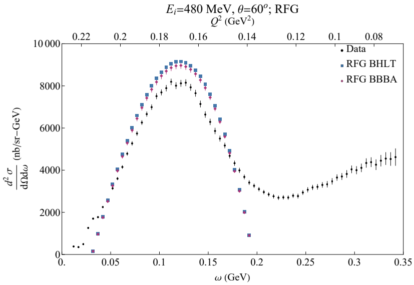


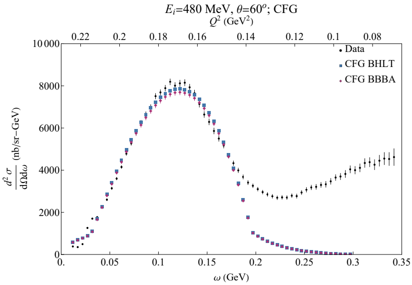


- We observe clear differences between the two nuclear models

$e - C$ data: fix nuclear model vary form factor

- Fix the nuclear model and compare vector form factors from
- [Bradford, Bodek, Budd, Arrington, NPB Proc. Suppl., **159** 127132 (2006)]
- [Borah, Hill, Lee, Tomalak, PRD, **102** 074012, (2020)]
- First fix model to RFG and then to CFG





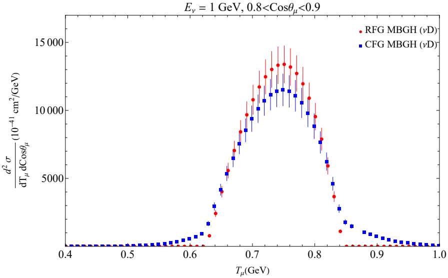
- Differences between form factors are small compared to differences between nuclear models
- What happens for neutrino scattering?

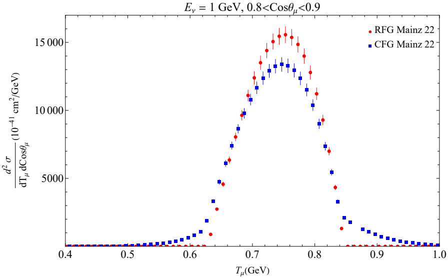
Separating nuclear and nucleon effects in neutrino-nucleus interactions

[Bhattacharya, Carey, Cohen, GP, PRD **111** 096021 (2025)]

Neutrino scattering: hypothetical constant neutrino energy

- Consider the hypothetical scattering of a 1 GeV neutrino on Carbon



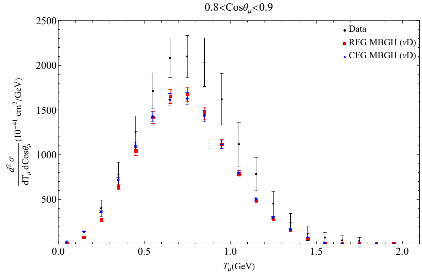


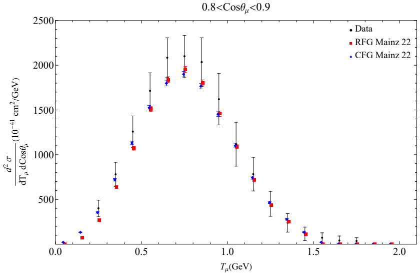
- $T_\mu = E_\nu - m_\mu - \omega$
- Vector form factor from BHLT
- Axial form factor from MBGH (ν D scattering) and Mainz 22 (lattice)
- We observe clear differences between
- the two nuclear models and
- the two axial form factors extractions

MiniBooNE data: fix axial form factor, vary nuclear model

$$\frac{d\sigma_{\text{carbon,per nucleon,avg.}}}{dE_\ell \, d\cos\theta_\ell} = \int dE_\nu \, f(E_\nu) \, \frac{d\sigma_{\text{carbon,per nucleon}}}{dE_\ell \, d\cos\theta_\ell}$$

- Fix axial form factor, vary nuclear models, and compare to MiniBooNE data [Aguilar-Arevalo, et al., PRD **81** 092005, (2010)]



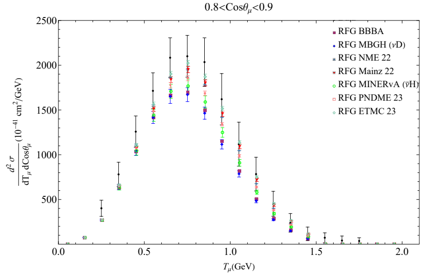


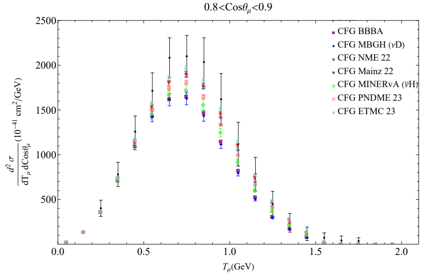
- With flux averaging, indistinguishable RFG and CFG models

MiniBooNE data: fix nuclear model, vary axial form factor

$$\frac{d\sigma_{\text{carbon,per nucleon,avg.}}}{dE_\ell d\cos\theta_\ell} = \int dE_\nu\, f(E_\nu)\, \frac{d\sigma_{\text{carbon,per nucleon}}}{dE_\ell d\cos\theta_\ell}$$

- Fix nuclear model, vary axial form factor, and compare to MiniBooNE data [Aguilar-Arevalo, et al., PRD **81** 092005, (2010)]





- Continuous spread from axial form factors for both nuclear models

Conclusions and outlook

Conclusions

- Current and future neutrino experiments require better control of systematic uncertainties in lepton-nucleus interactions

- Need separate control of form factors and nuclear effects

- Quasi-Elastic Lepton Nucleus Scattering and the Correlated Fermi Gas Model [Bhattacharya, Carey, Cohen, GP, PRD **111** 096021 (2025)]

- Presented analytic implementation of Correlated Fermi Gas Model for lepton nucleus scattering

- Use CFG and RFG models to separate form factors and nuclear effects
  - $e - N$ scattering differences: form factors small, nuclear models large
  - Flux-avg. $\nu - N$ scattering: form factors large, nuclear models small

Future directions

- Correlated Fermi Gas Model:

- Can we distinguish CFG and RFG models for semi-inclusive $\nu - N$ scattering?
- Combining CFG with other effects, e.g., final state interaction (FSI)

- General:

- Separate form factor and nuclear model uncertainties for other nuclear models

  See, e.g., [D. Simons, N. Steinberg, A. Lovato, Y. Meurice, N. Rocco and M. Wagman, arXiv:2210.02455]

- More work to do

Thank you!

Backup slides

Nuclear and hadronic tensor

- The nuclear tensor $W_{\mu\nu}$ can be expressed by the nucleon tensor $H_{\mu\nu}$

$$W_{\mu\nu} = \int \frac{d^3p}{(2\pi)^3}\,\frac{m_T}{E_p}\,2Vn_i(\boldsymbol{p})\int \frac{d^3p'}{(2\pi)^3 2E_{p'}}(2\pi)^4\delta^4(p-p'+q)H_{\mu\nu}\left[1-n_f(\boldsymbol{p'})\right]$$

- Performing some of the integrals

$$W_{\mu\nu} \equiv \int d^3p\, f(\boldsymbol{p}, q^0, \boldsymbol{q})H_{\mu\nu}(\epsilon_{\boldsymbol{p}}, \boldsymbol{p}; q^0, \boldsymbol{q})\,,$$

$$f(\boldsymbol{p}, q^0, \boldsymbol{q}) = \frac{m_T V}{4\pi^2}\,n_i(\boldsymbol{p})[1-n_f(\boldsymbol{p}+\boldsymbol{q})]\frac{\delta(\epsilon_{\boldsymbol{p}} - \epsilon'_{\boldsymbol{p}+\boldsymbol{q}} + q^0)}{\epsilon_{\boldsymbol{p}}\epsilon'_{\boldsymbol{p}+\boldsymbol{q}}}\,.$$

Phase space integrals

$$W_1 = a_1 H_1 + \frac{1}{2}(a_2 - a_3) H_2 \,,$$

$$W_2 = \left[a_4 + \frac{\omega^2}{|\boldsymbol{q}|^2} a_3 - 2\frac{\omega}{|\boldsymbol{q}|} a_5 + \frac{1}{2}\left(1 - \frac{\omega^2}{|\boldsymbol{q}|^2}\right)(a_2 - a_3)\right] H_2 \,,$$

$$W_3 = \frac{m_T}{m_N}\left(a_7 - \frac{\omega}{|\boldsymbol{q}|} a_6\right) H_3 \,,$$

$$W_4 = \frac{m_T^2}{m_N^2}\left[a_1 H_4 + \frac{m_N}{|\boldsymbol{q}|} a_6 H_5 + \frac{m_N^2}{2|\boldsymbol{q}|^2}(3a_3 - a_2) H_2 \right] \,,$$

$$W_5 = \frac{m_T}{m_N}\left(a_7 - \frac{\omega}{|\boldsymbol{q}|} a_6\right) H_5 + \frac{m_T}{|\boldsymbol{q}|}\left[2a_5 + \frac{\omega}{|\boldsymbol{q}|}(a_2 - 3a_3)\right] H_2 \,,$$

$$a_1 = \int d^3p\, f(\boldsymbol{p}, q) \,, \qquad a_2 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{|\boldsymbol{p}|^2}{m_N^2} \,,$$

$$a_3 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{(p^z)^2}{m_N^2} \,, \qquad a_4 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{\epsilon_{\boldsymbol{p}}^2}{m_N^2} \,,$$

$$a_5 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{\epsilon_{\boldsymbol{p}} p^z}{m_N^2} \,, \qquad a_6 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{p^z}{m_N} \,,$$

$$a_7 = \int d^3p\, f(\boldsymbol{p}, q)\, \frac{\epsilon_{\boldsymbol{p}}}{m_N} \,.$$