

Predicting the outcome of neutrino flavor instabilities

Julien Froustey

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Based on:

 S. Richers, **JF**, S. Ghosh, F. Foucart, J. Gomez, [[2409.04405](#)]

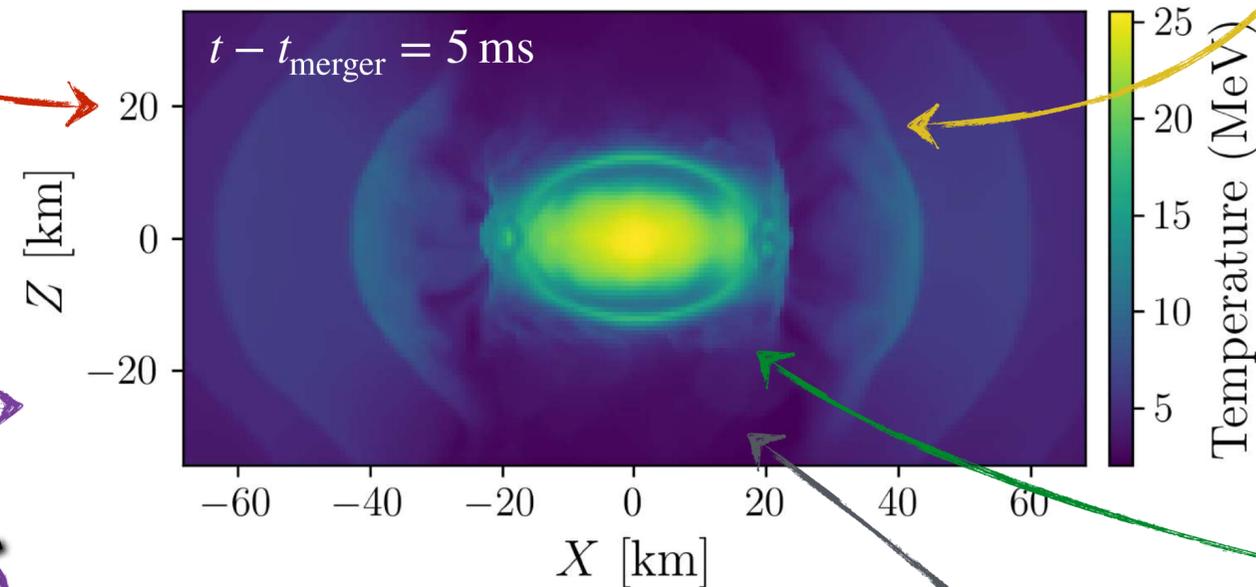
 **JF** [[2505.16961](#)]

Large-scale simulations: what's included and what's not

General relativity

Hydrodynamics

Merger of two $1.2 M_{\odot}$ neutron stars



Neutrino transport*

Equation of state

...

* without neutrino flavor oscillations

Classical and quantum transport

- Classical neutrino transport:

Boltzmann's equation $\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) f_{\nu_\alpha} = C_{\nu_\alpha}$

- Quantum neutrino transport:

$$\varrho = \begin{pmatrix} \varrho_{ee} & \varrho_{ex} \\ \varrho_{ex}^* & \varrho_{xx} \end{pmatrix} = \begin{pmatrix} \text{Distribution function of } \nu_e & \text{Flavor coherence} \\ \text{Flavor coherence} & \text{Distribution function of } \nu_x \end{pmatrix}$$

Quantum Kinetic Equation $\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \varrho = -i [\mathcal{H}, \varrho] + \mathcal{C}$

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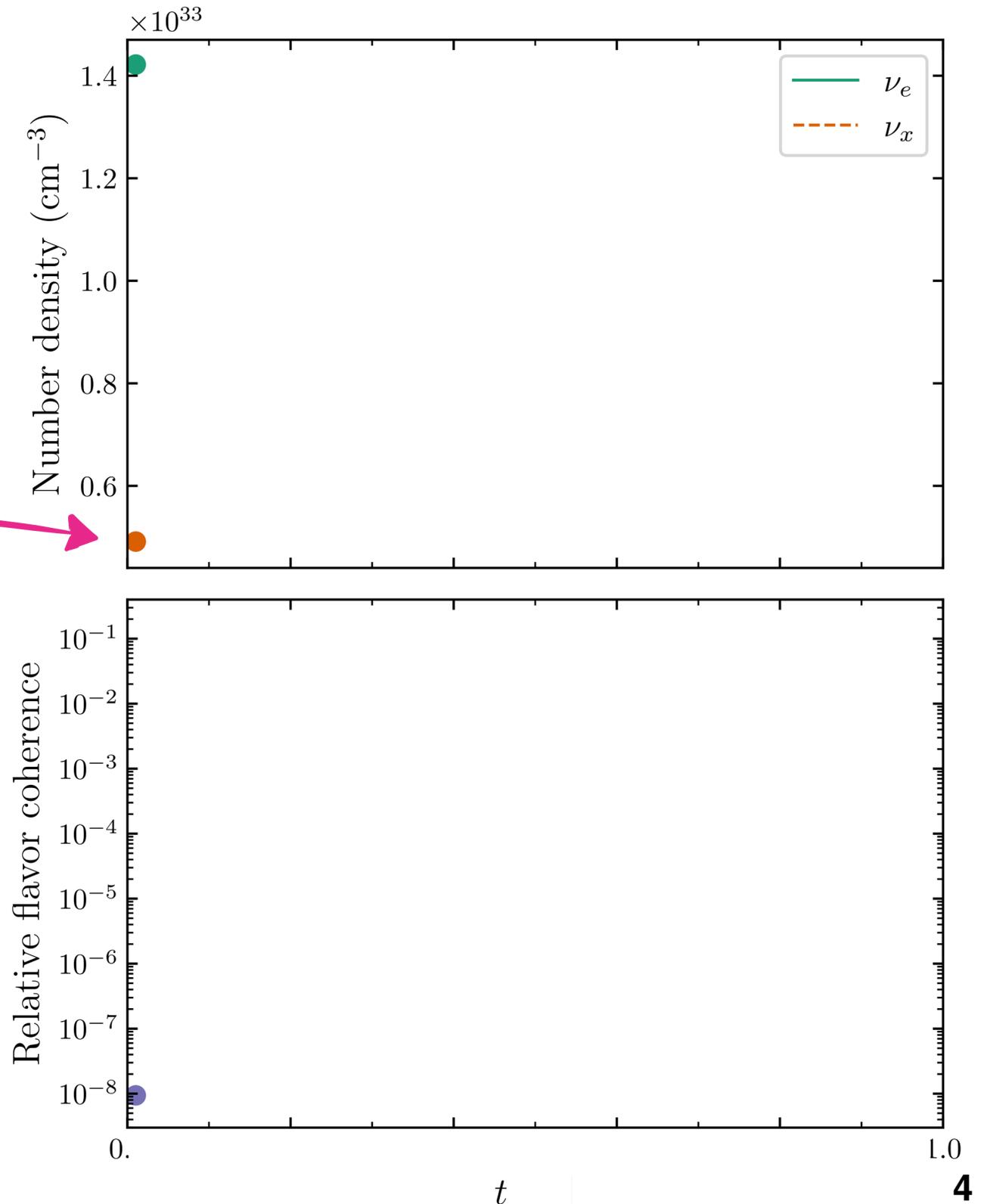
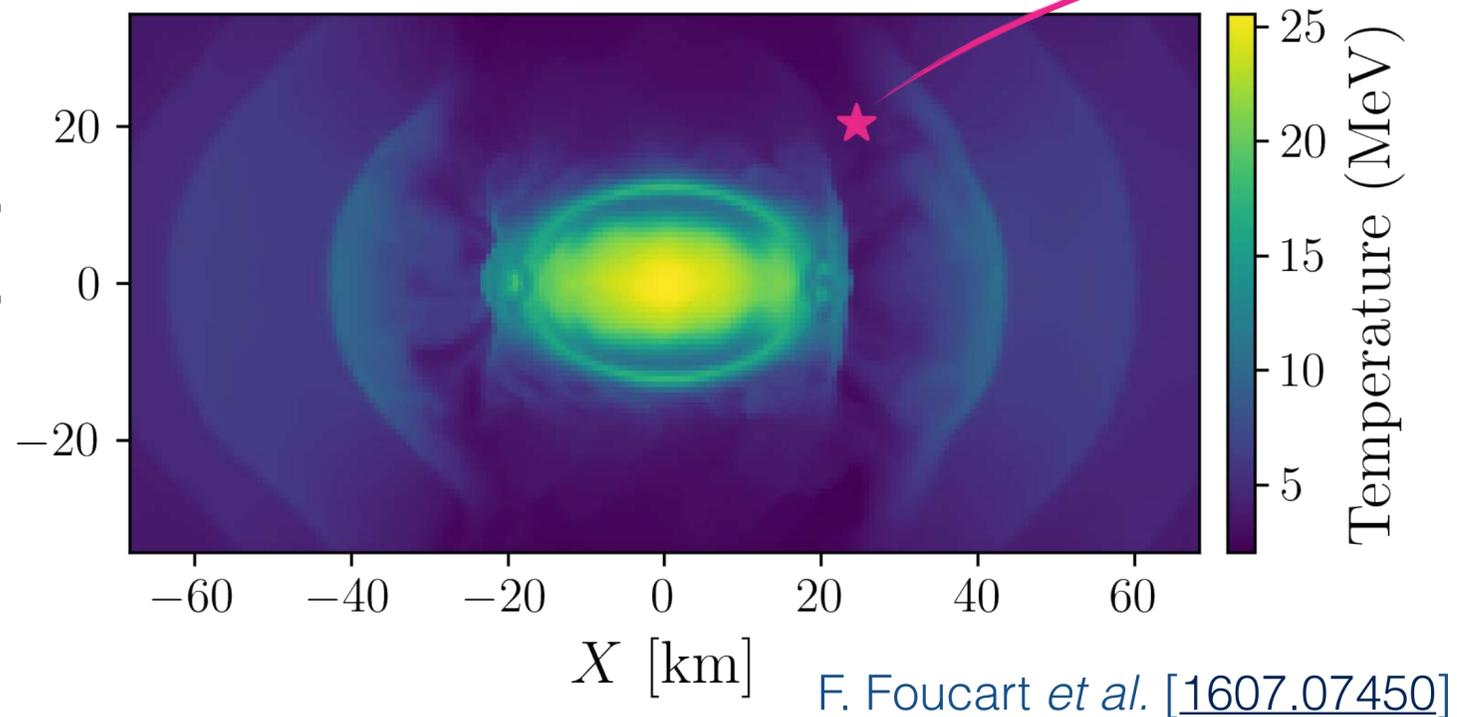
Quantum Kinetic Equation $\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \varrho = -i [\mathcal{H}, \varrho] + \mathcal{C}$ Collisions

Vacuum term
+
mean-field potentials

Flavor instabilities in dense astrophysical environments

- We can check if classical simulations are self-consistent.
- Take the classical results, and solve the QKE.

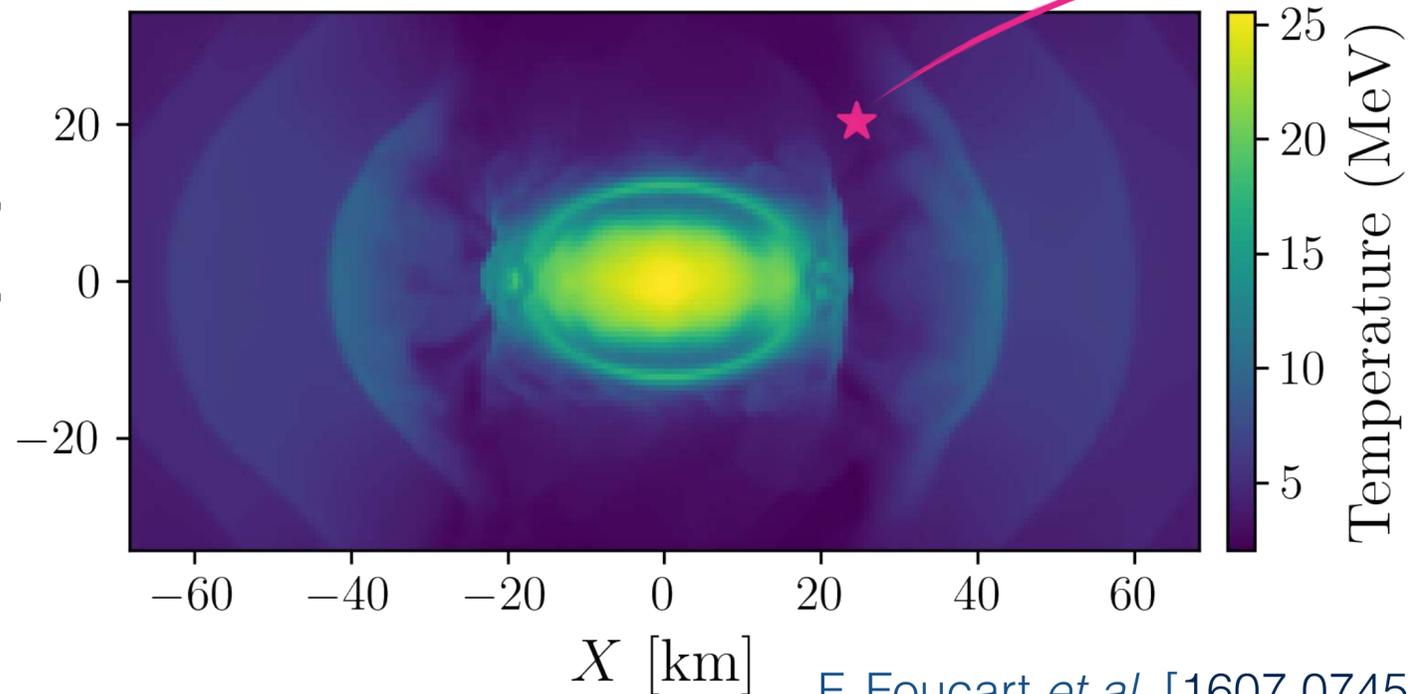
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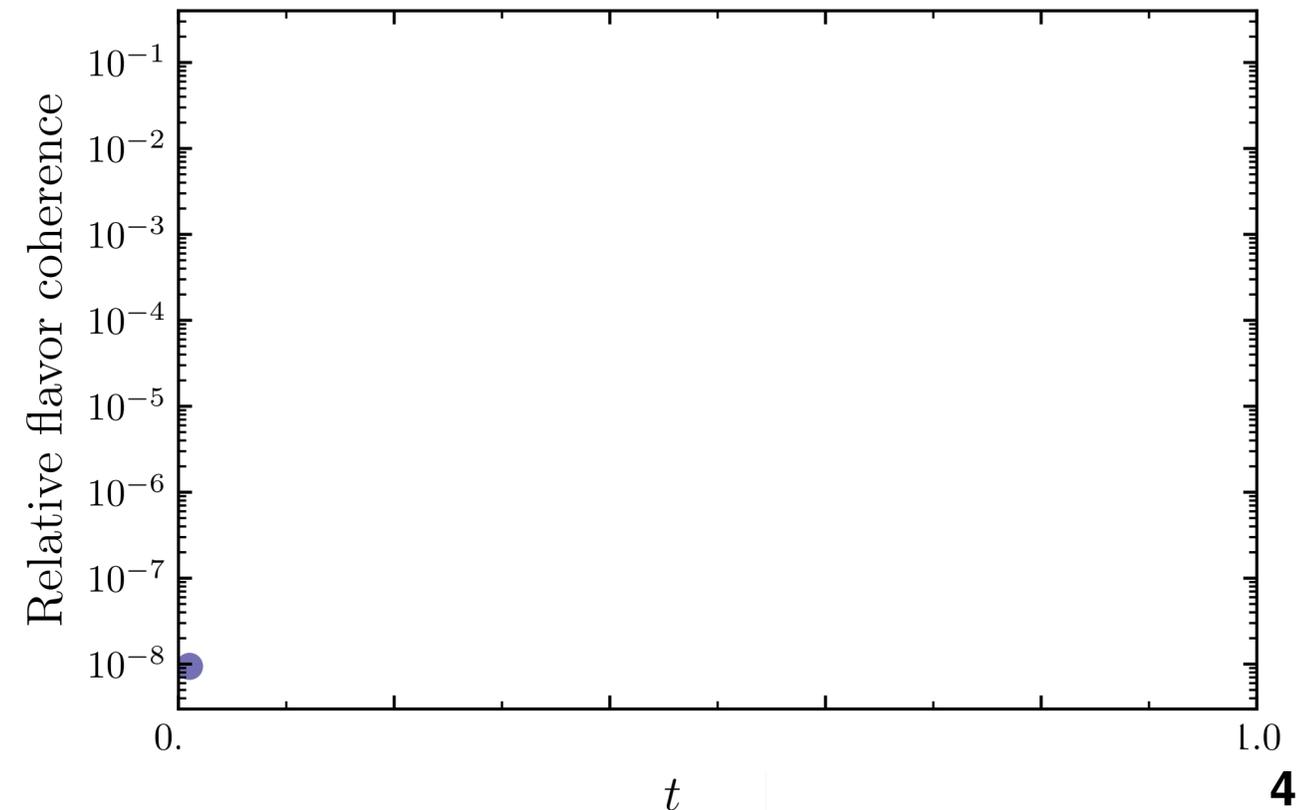
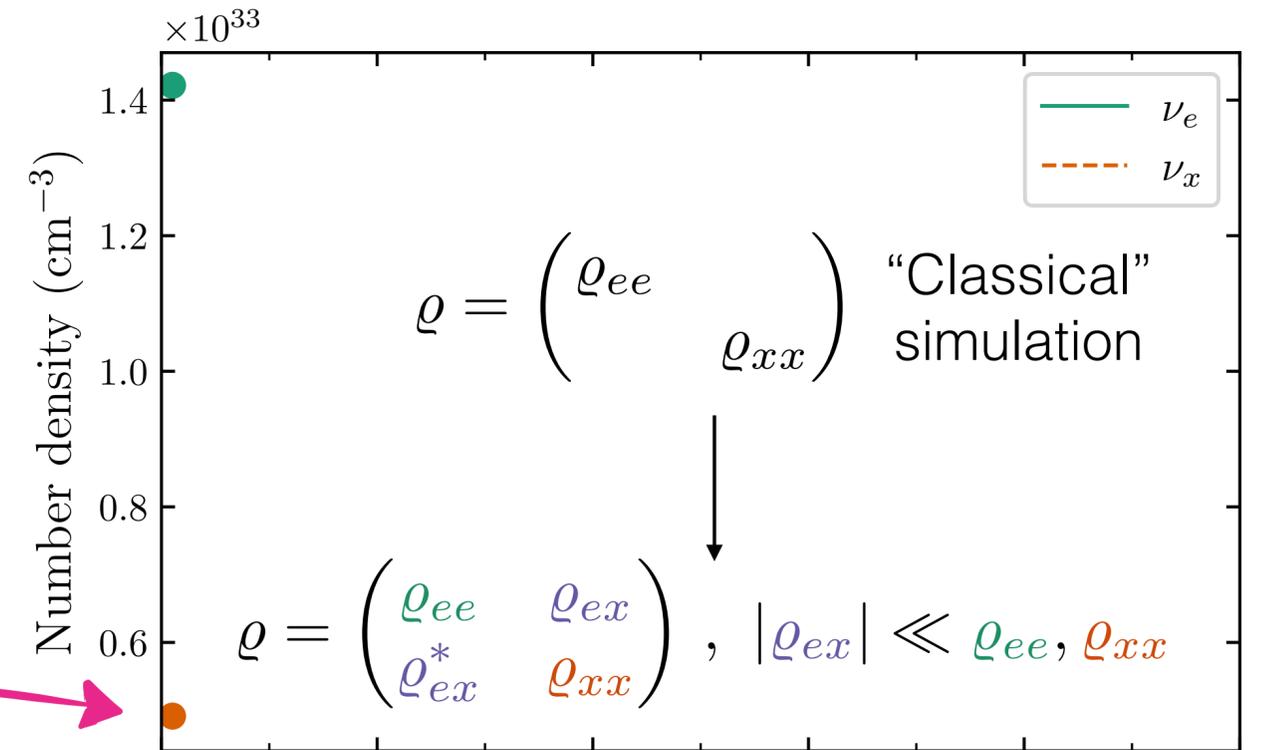
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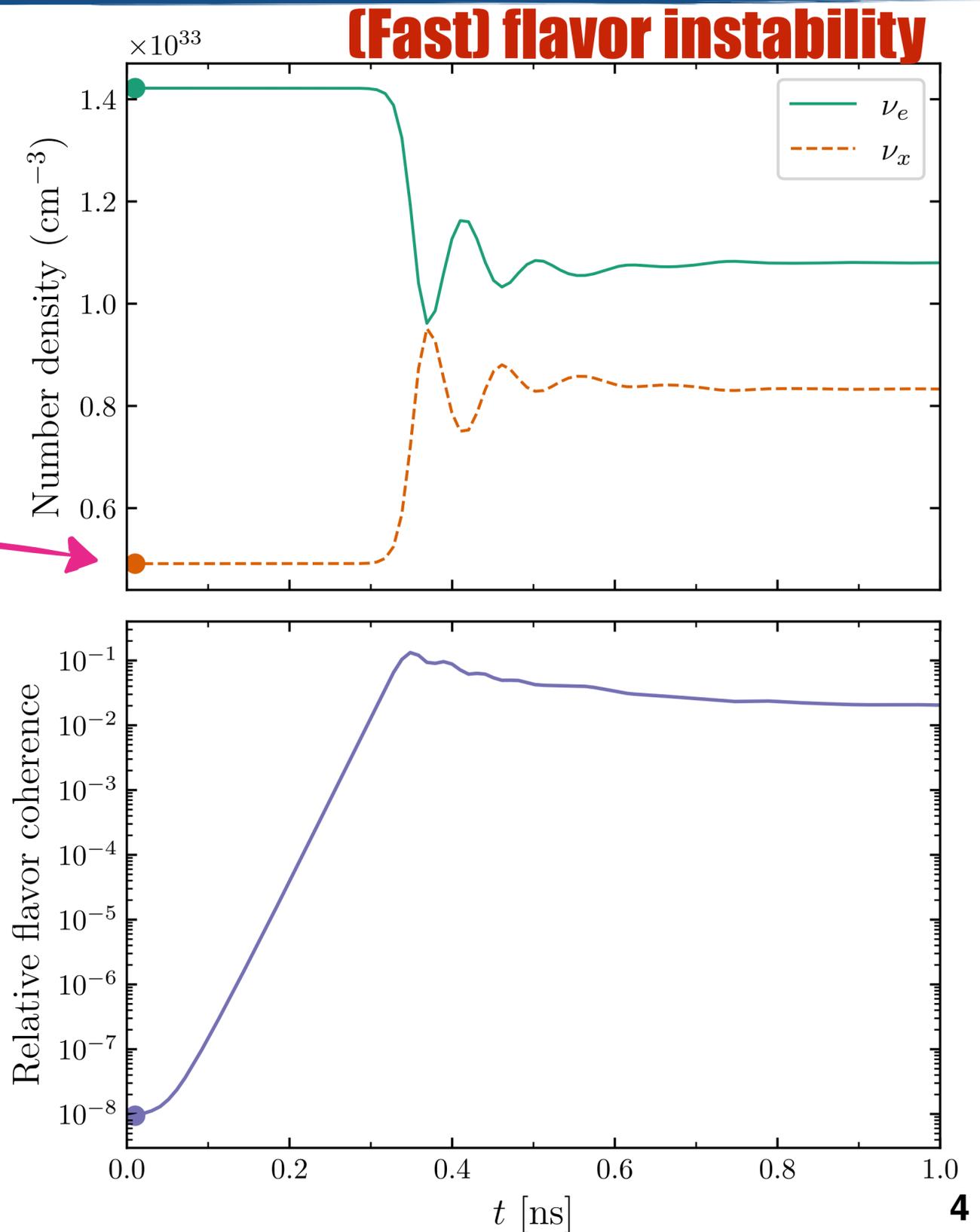
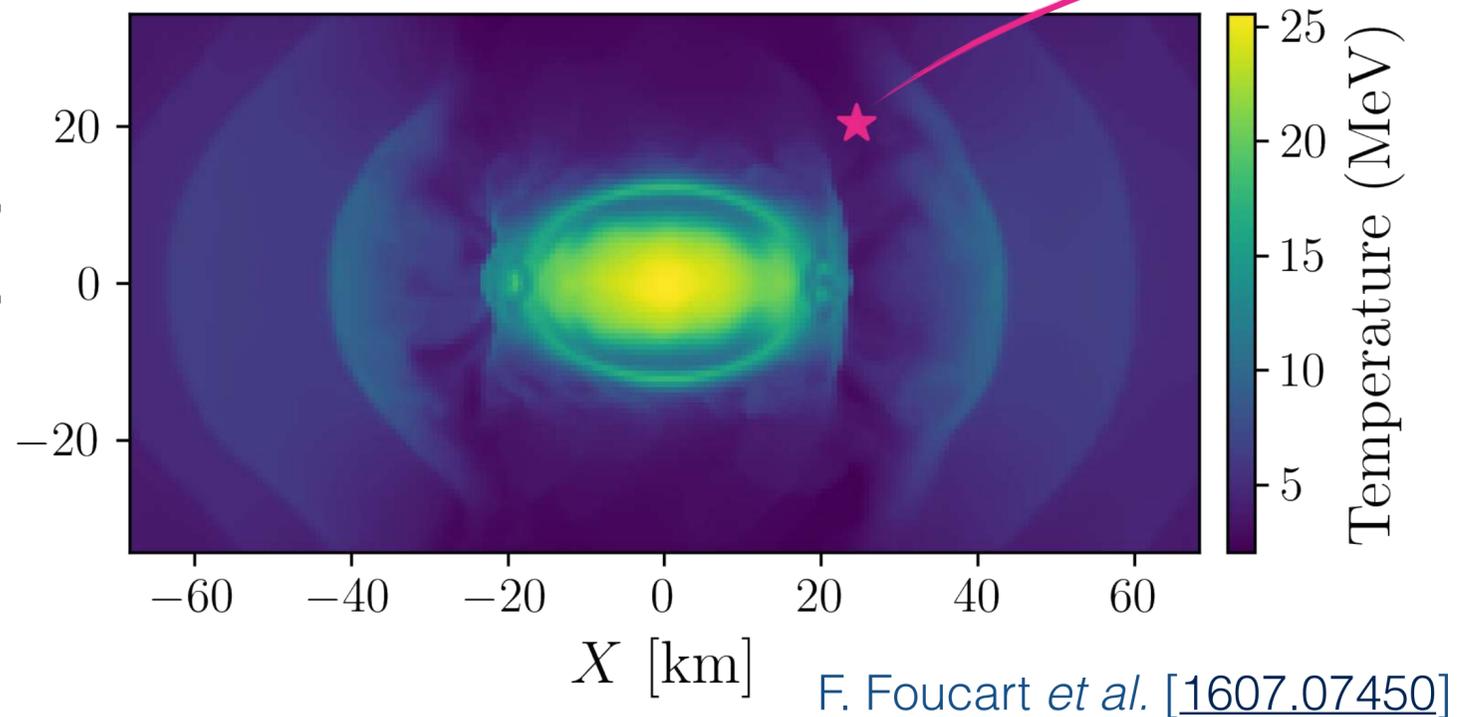
F. Foucart *et al.* [[1607.07450](#)]



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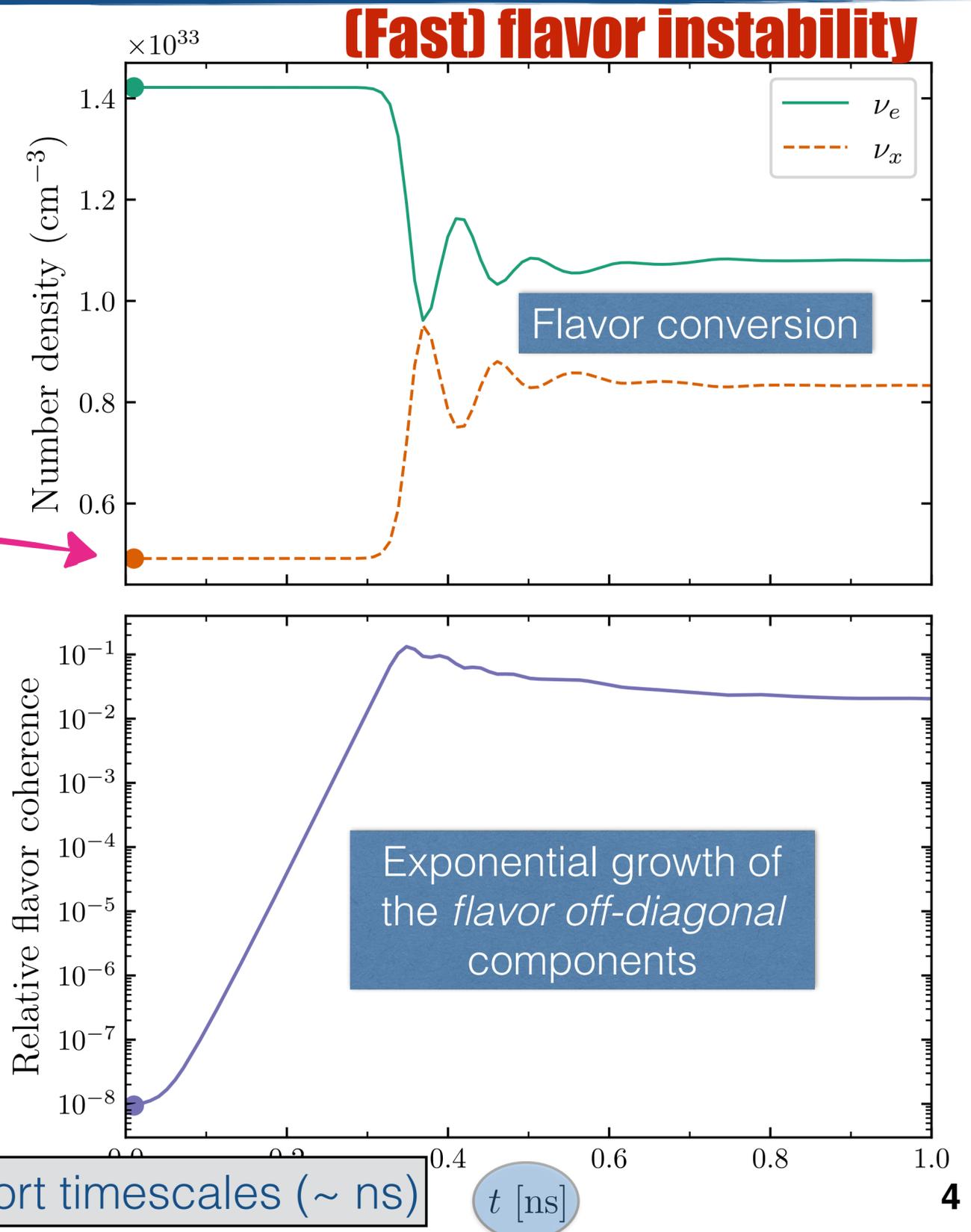
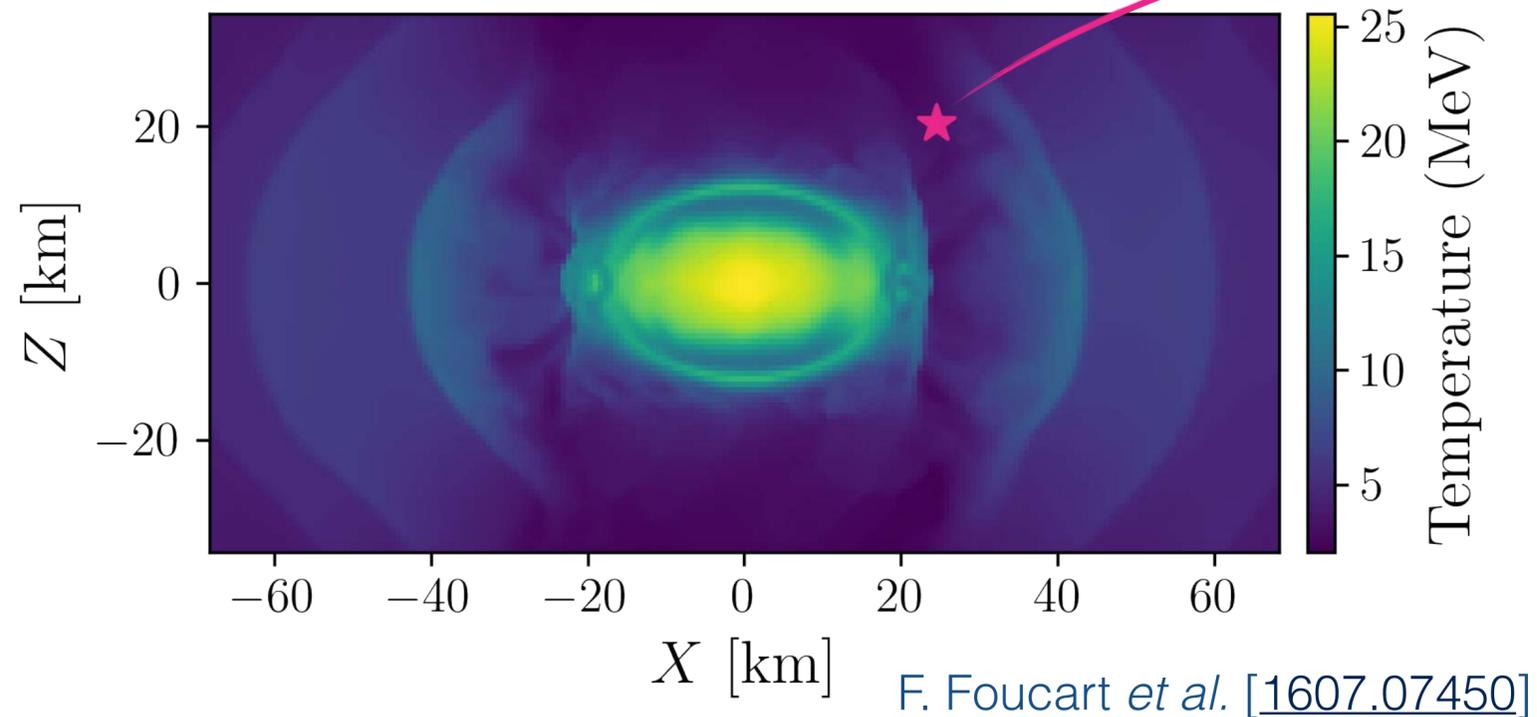
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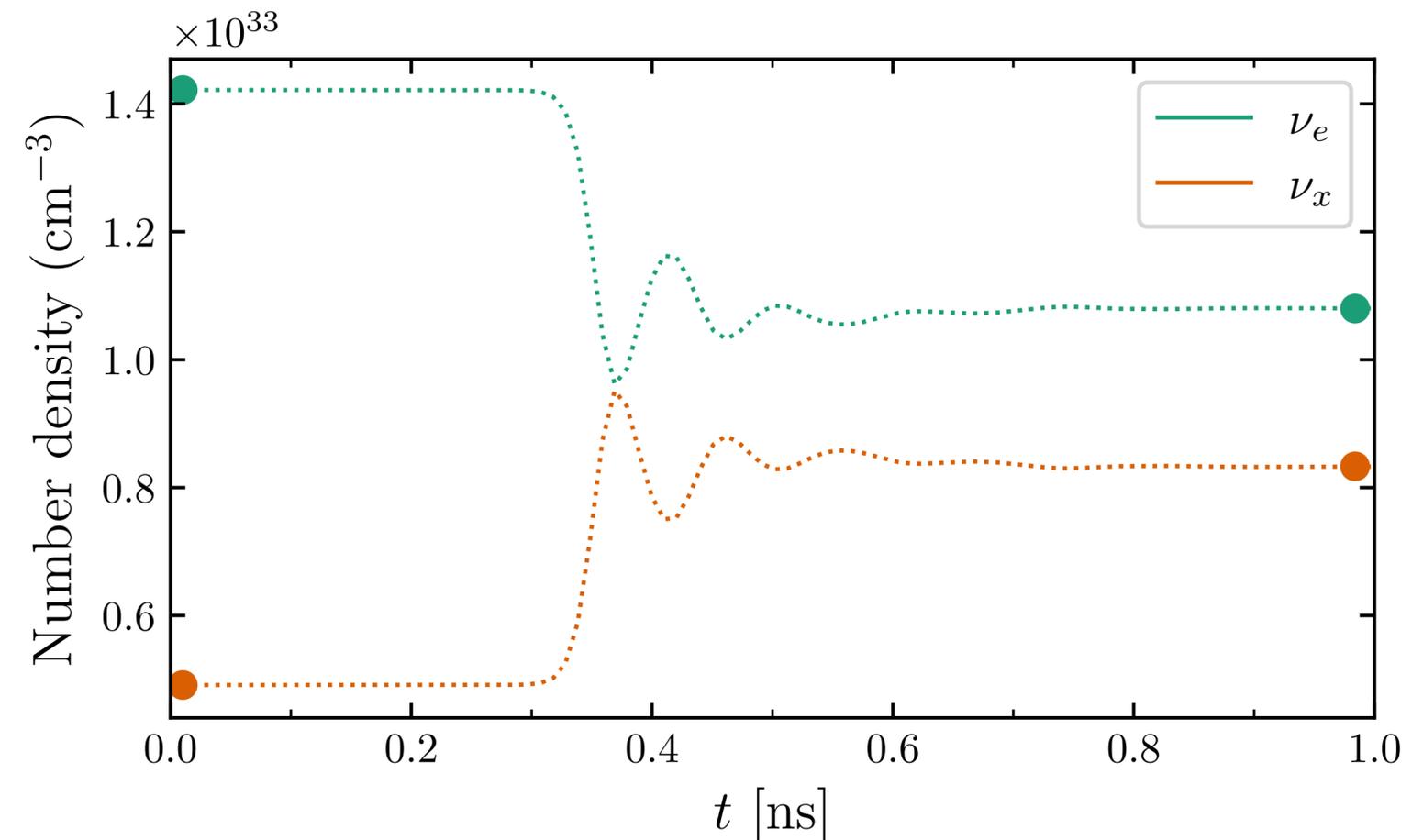


Fast flavor instabilities (FFIs)

- FFIs are driven by the **neutrino/antineutrino self-interaction mean-field**

$$\left(\frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla\right) \varrho(t, \mathbf{x}, \mathbf{p}) = -i [\mathcal{H}_{\text{self}}, \varrho(t, \mathbf{x}, \mathbf{p})] \quad \mathcal{H}_{\text{self}} = \frac{\sqrt{2}G_F}{(2\pi)^3} \int d^3\mathbf{q} (1 - \cos \theta_{\mathbf{p},\mathbf{q}}) [\varrho(t, \mathbf{x}, \mathbf{q}) - \bar{\varrho}(t, \mathbf{x}, \mathbf{q})]$$

- Various approaches to include FFIs in simulations:
 - ▶ Angular moment methods
 - ▶ Attenuation of the Hamiltonian (to increase length- and time- scales)
 - ▶ **Subgrid model with predicted asymptotic state**



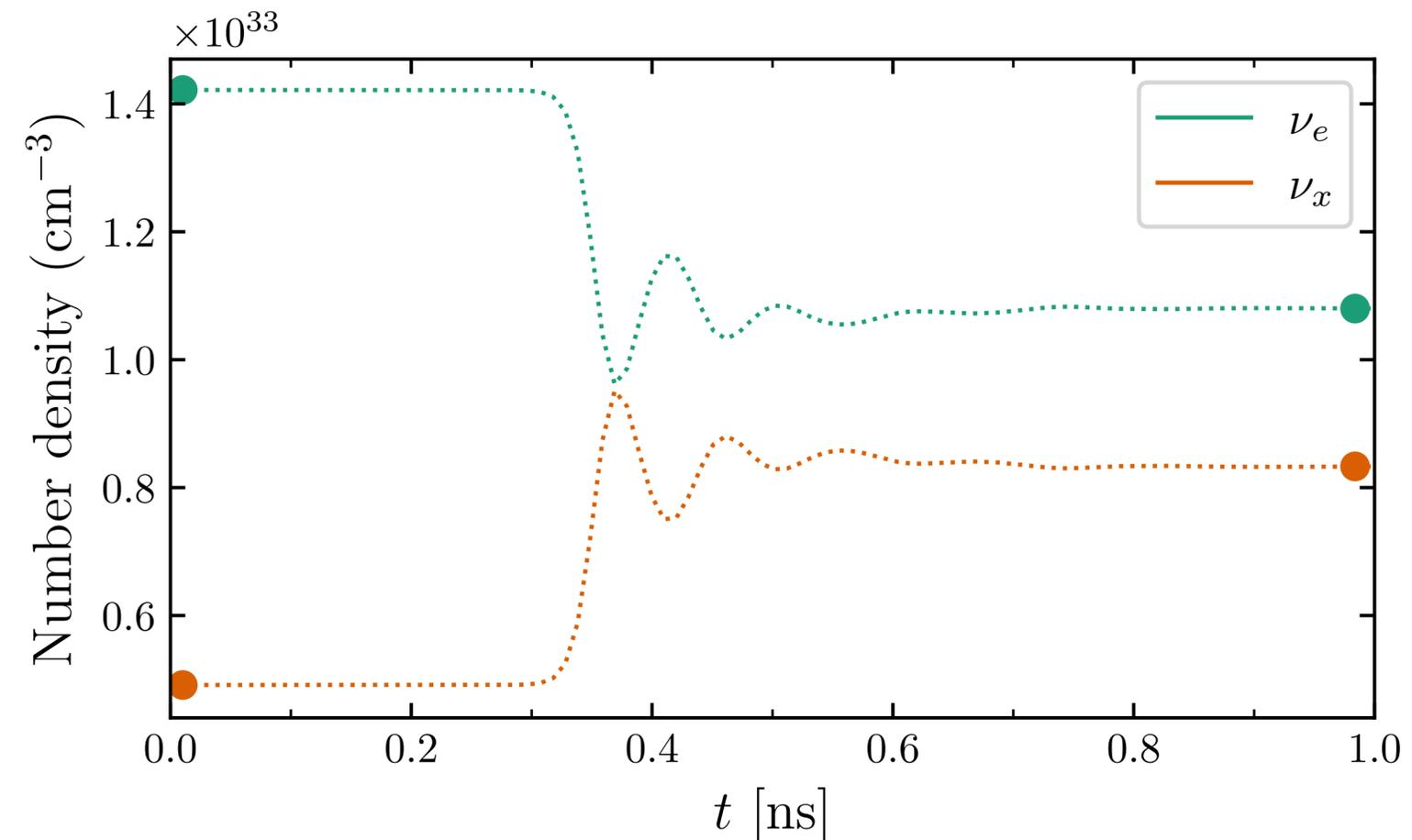
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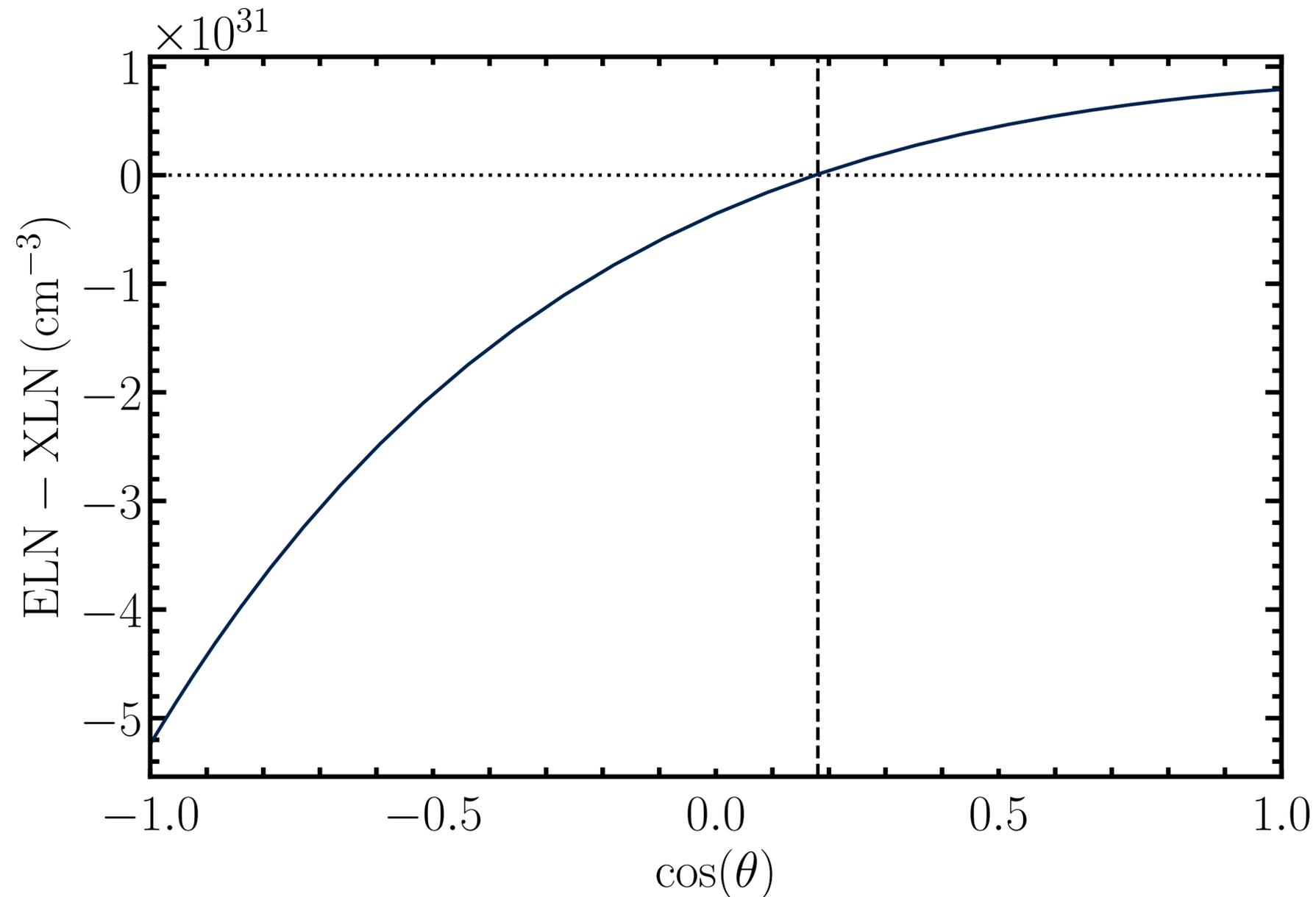
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Method: local (cm-scale) high-resolution QKE calculations with periodic boundary conditions



Asymptotic state of FFI — Axisymmetric case



Data: *E. Urquilla* with the
particle-in-cell code **Emu**

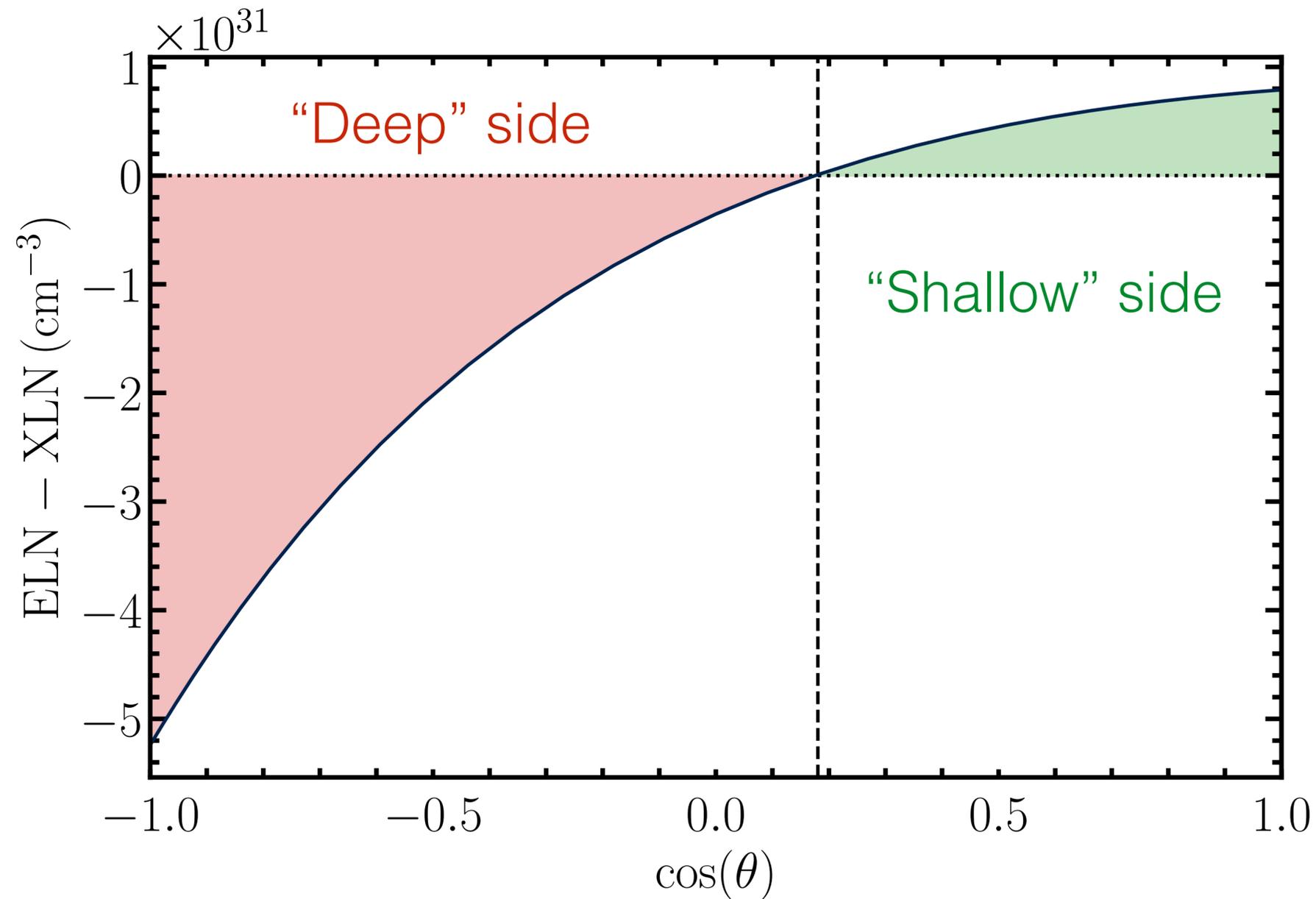
S. Richers et al., [[2101.02745](#)]

$$ELN = \rho_{ee}(\theta) - \bar{\rho}_{ee}(\theta)$$

$$XLN = \rho_{xx}(\theta) - \bar{\rho}_{xx}(\theta)$$

Instability \iff Crossing in $ELN - XLN$

Asymptotic state of FFI — Axisymmetric case



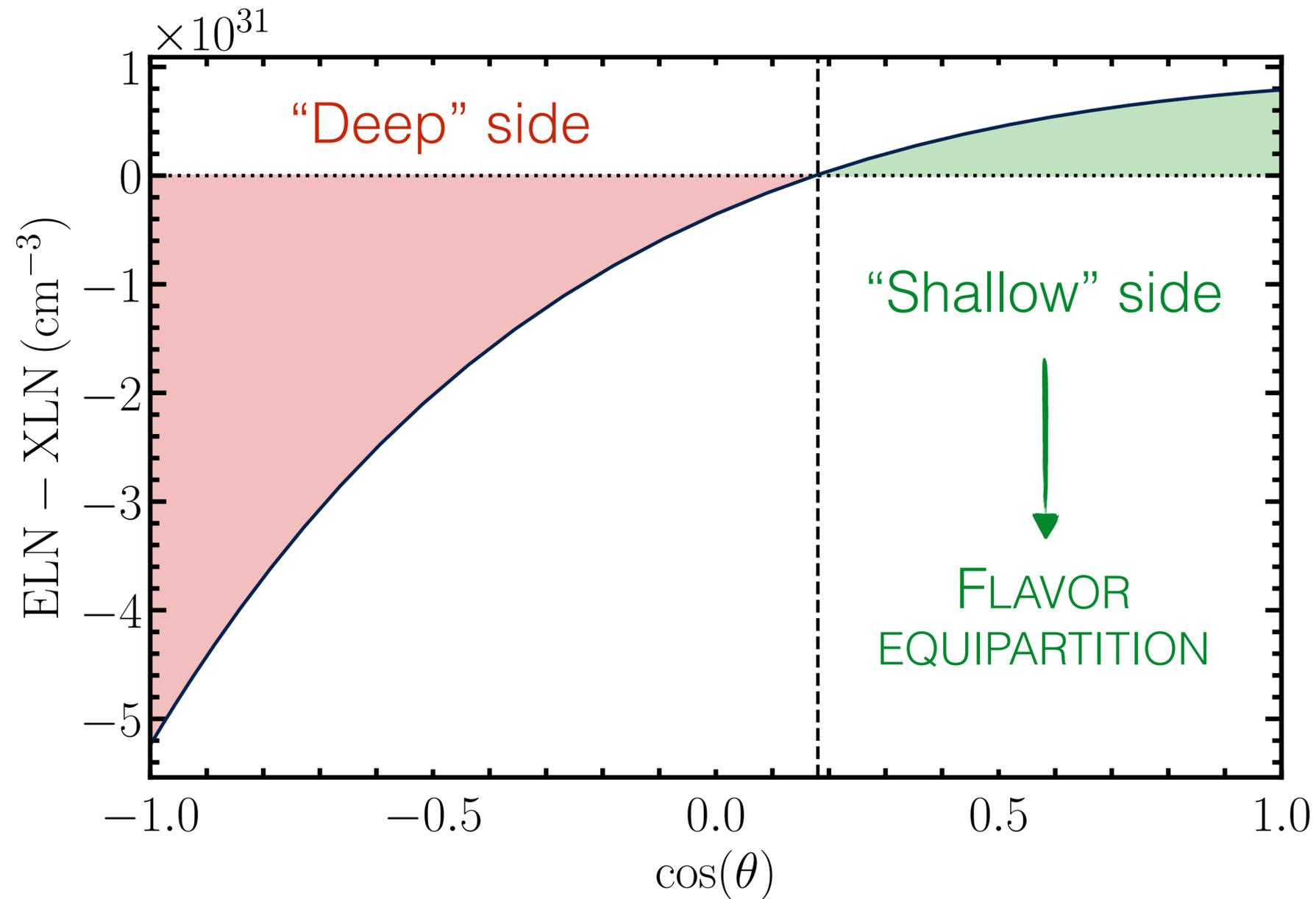
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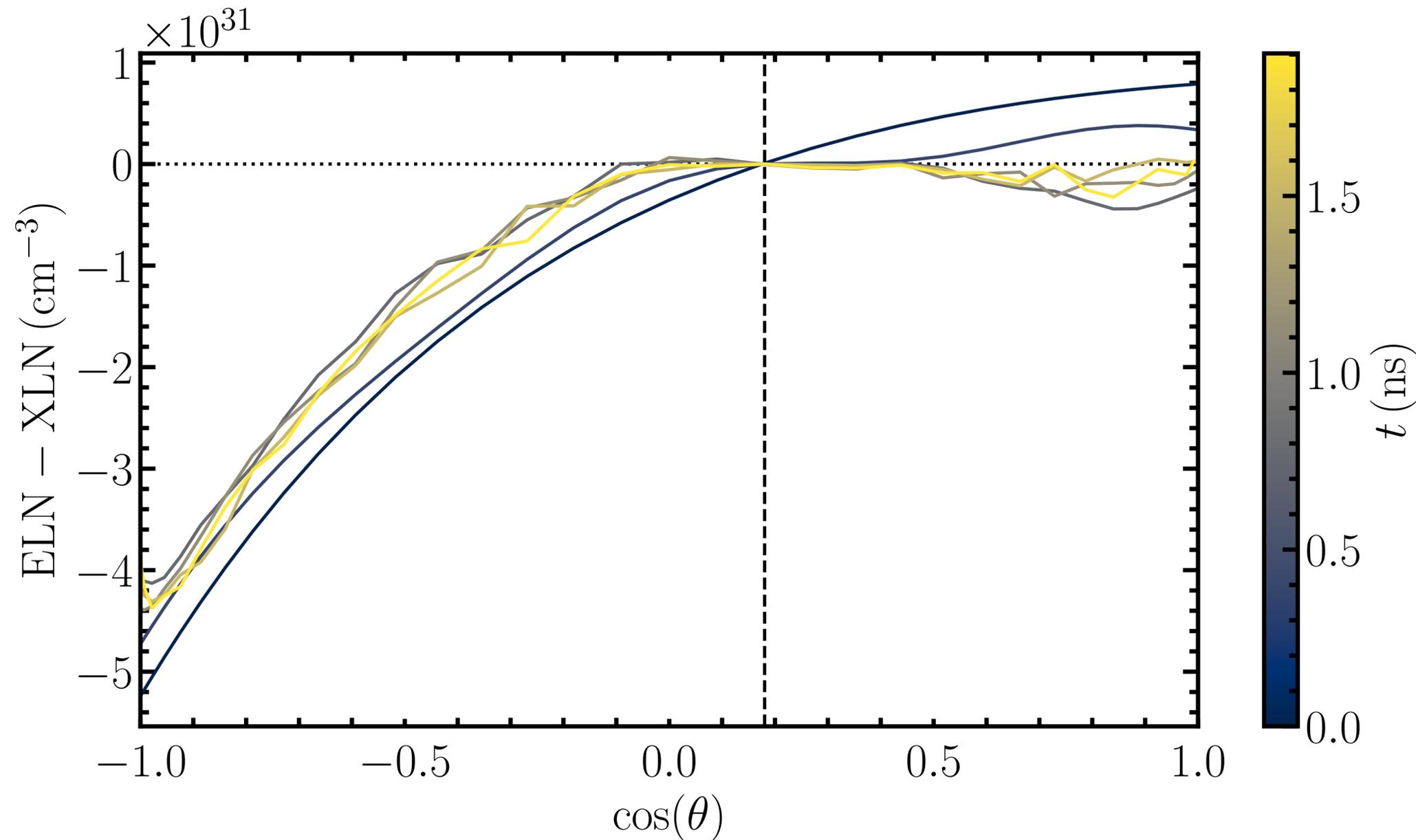
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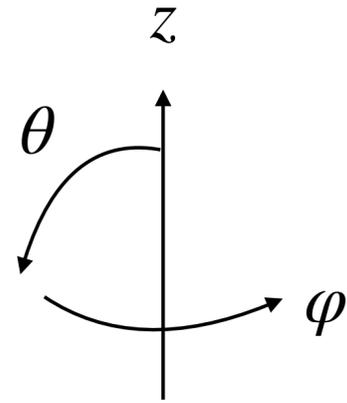
Data: *E. Urquilla* with the particle-in-cell code **Emu**
S. Richers et al., [[2101.02745](#)]

This “localized crossing erasure” can be expressed as a survival probability.
M. Zaizen and H. Nagakura, [[2211.09343](#)]

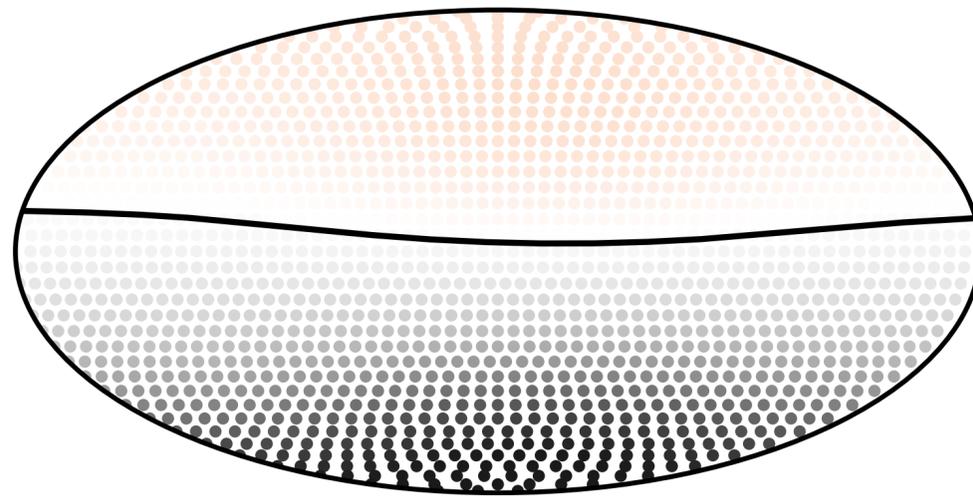
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Instability \iff Crossing in ELN - XLN

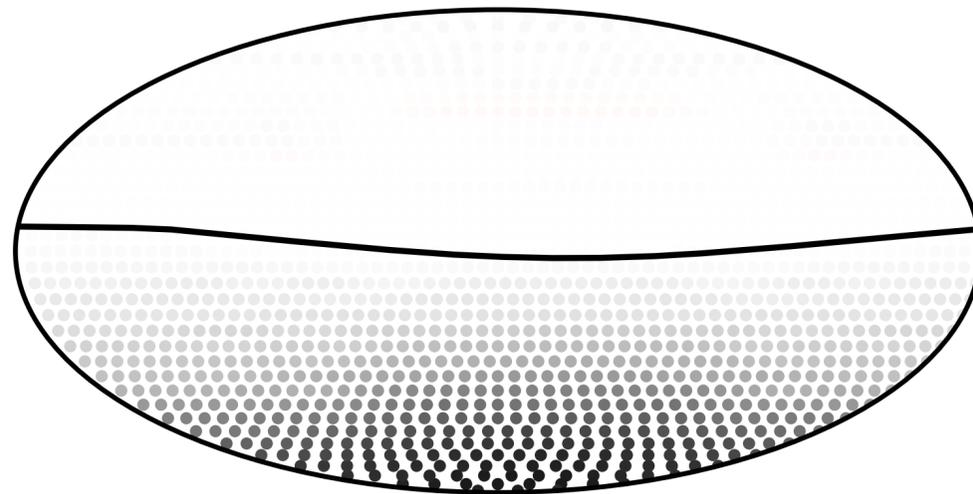
Three-dimensional generalization



Initial distribution



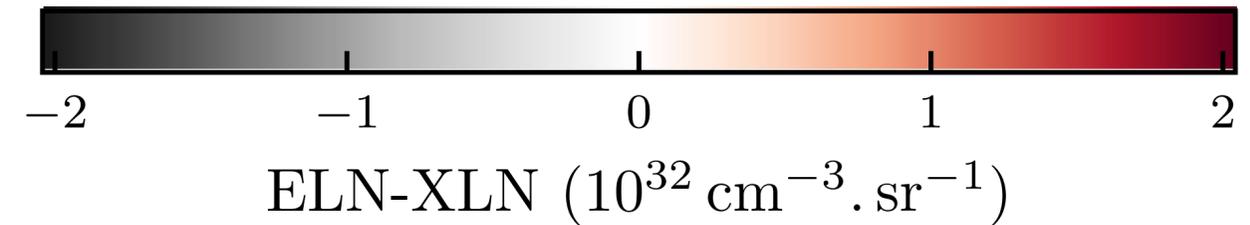
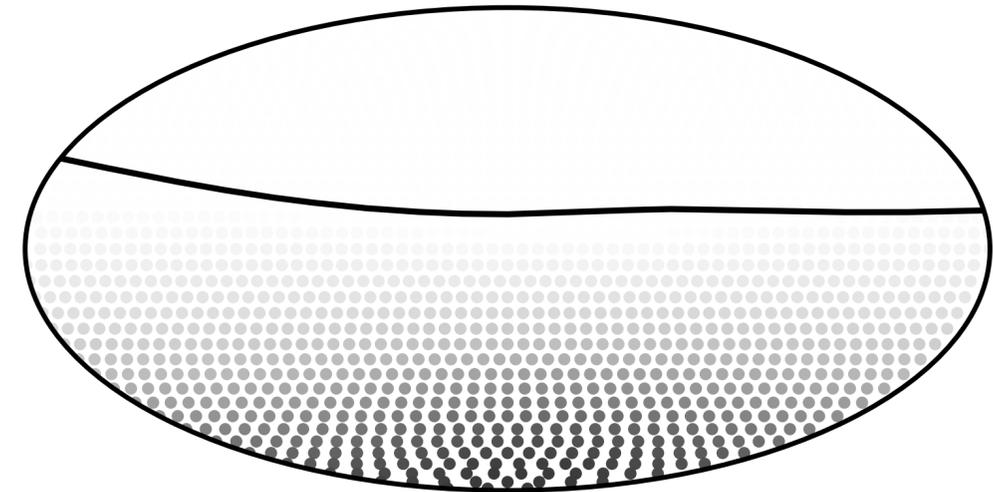
Emu



Final state
obtained with
Emu

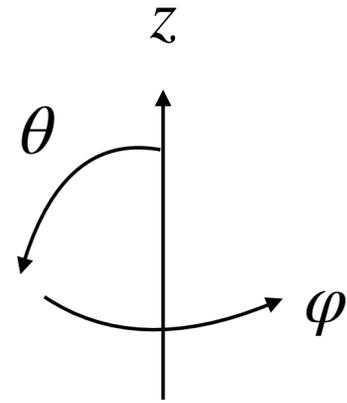
Three-dimensional generalization
of ELN-XLN disappearance

Box3D

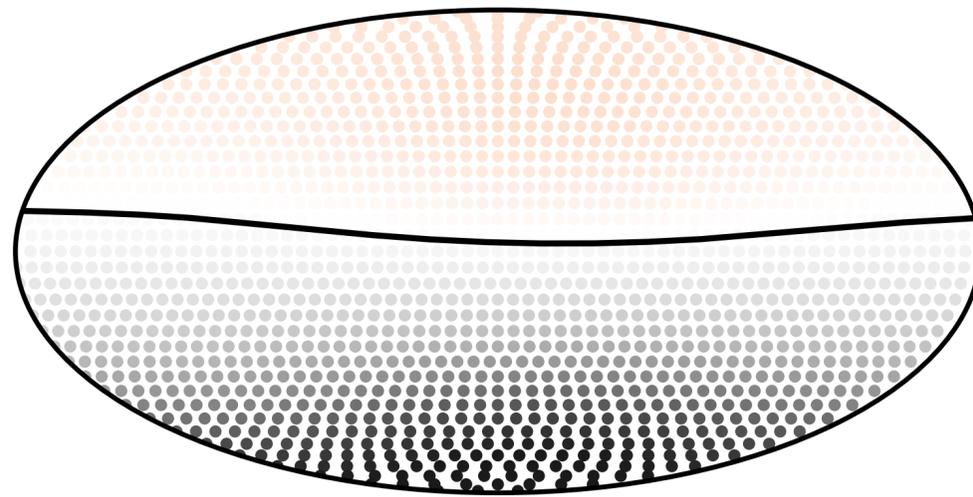


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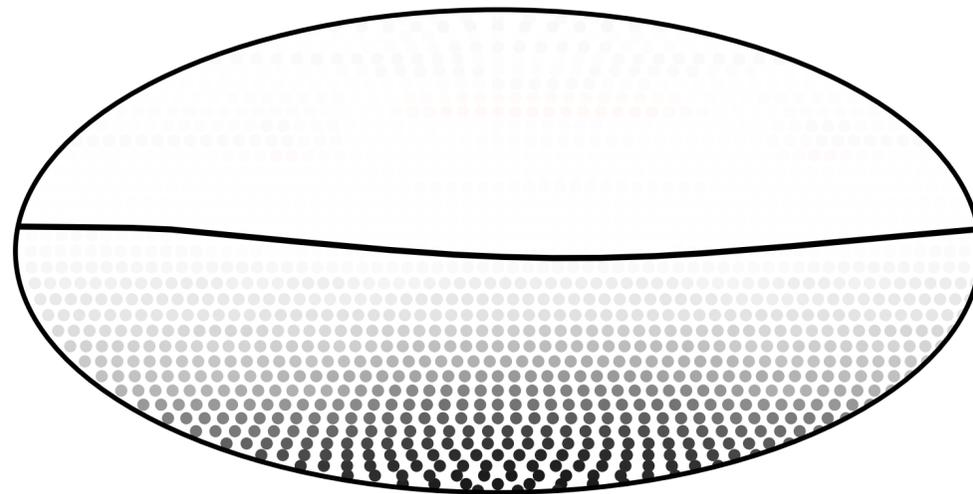
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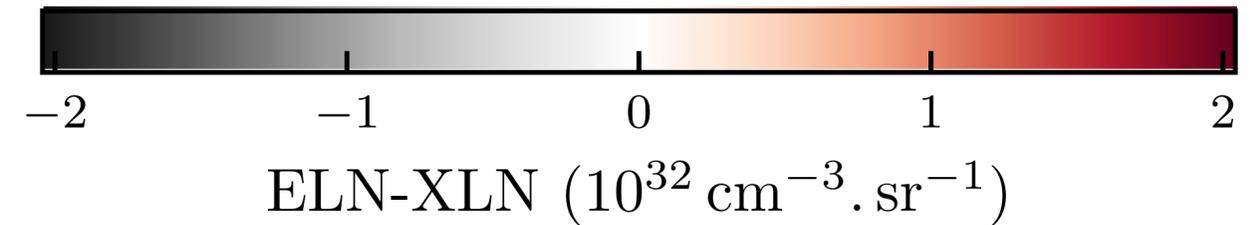
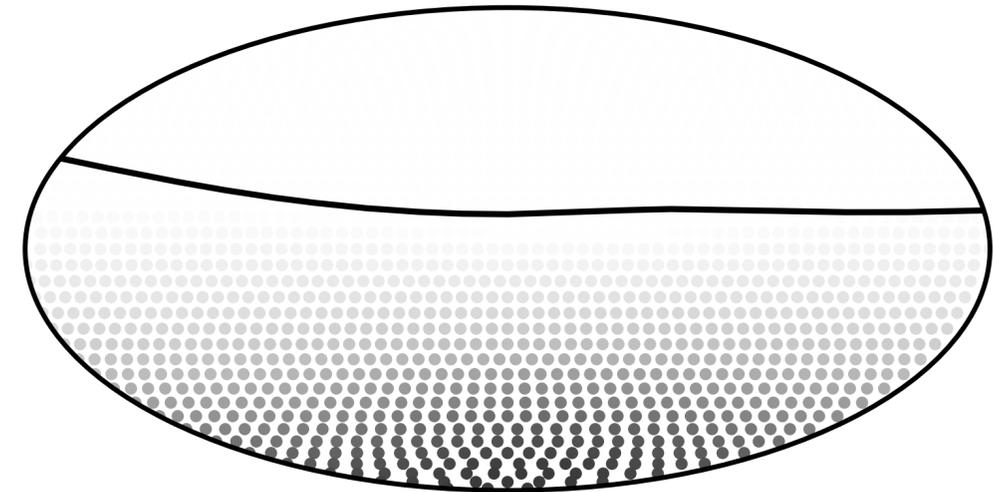
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Three-dimensional generalization
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S. Richers, **JF**, S. Ghosh, F. Foucart, J. Gomez, [[2409.04405](#)]

Scheme tested and validated on more
than 15,000 **Emu** calculations

Collisional flavor instabilities

A quantum/classical competition

Context

- Collisions are generally expected to **destroy** flavor coherence.
- In some regimes, a discrepancy between the neutrino and antineutrino reaction rates can actually **amplify** coherence *through the non-linear self-interaction term*.

PHYSICAL REVIEW LETTERS **130**, 191001 (2023)

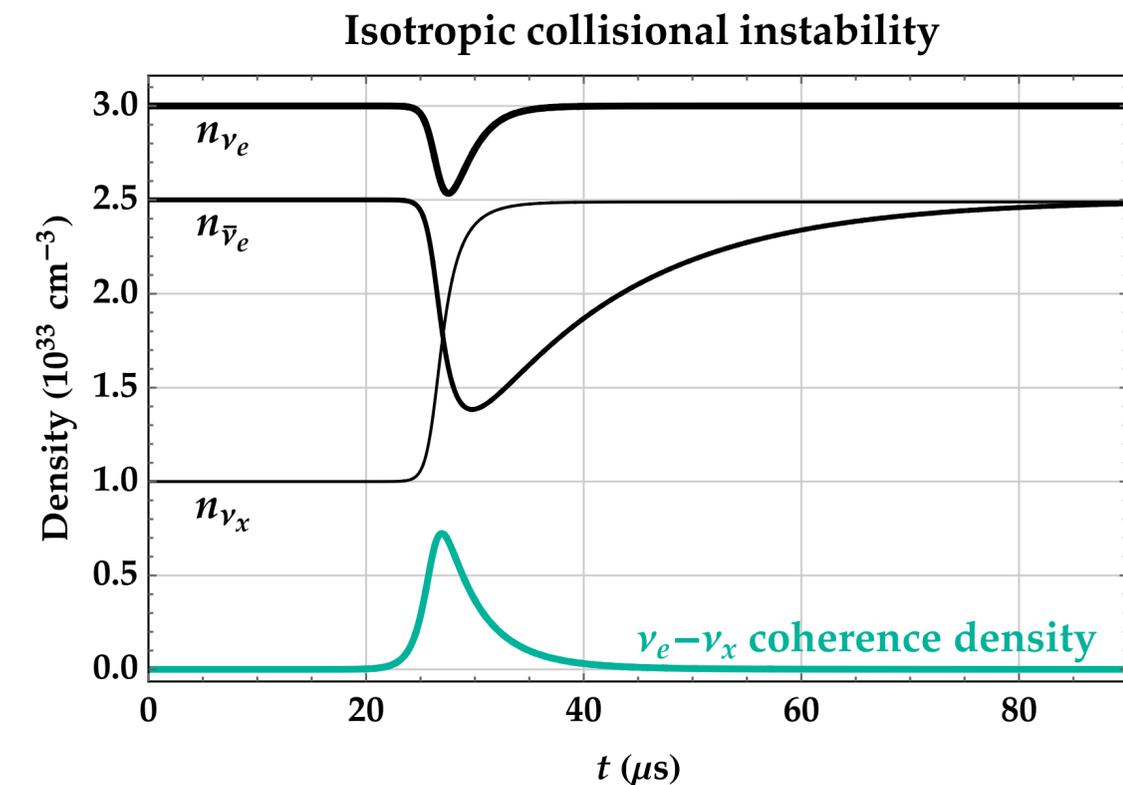
Collisional Flavor Instabilities of Supernova Neutrinos

Lucas Johns *

Departments of Astronomy and Physics, University of California, Berkeley, California 94720, USA

 (Received 27 April 2021; revised 15 December 2022; accepted 24 April 2023; published 8 May 2023)

A lingering mystery in core-collapse supernova theory is how collective neutrino oscillations affect the dynamics. All previously identified flavor instabilities, some of which might make the effects considerable, are essentially collisionless phenomena. Here, it is shown that collisional instabilities exist as well. They are associated with asymmetries between the neutrino and antineutrino interaction rates, are possibly prevalent deep inside supernovae, and pose an unusual instance of **decoherent interactions** with a thermal environment **causing the sustained growth of quantum coherence**.



(Homogeneous and isotropic) Quantum Kinetic Equations

$$\frac{dN}{dt} = -i \left[+\frac{\Delta m^2}{4E_\nu} \begin{pmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} + \sqrt{2}G_F(N - \bar{N}), N \right] + \frac{1}{2} \left\{ \begin{pmatrix} \kappa_e & 0 \\ 0 & \kappa_x \end{pmatrix}, \begin{pmatrix} N_{ee}^{(\text{cl})} & 0 \\ 0 & N_{xx}^{(\text{cl})} \end{pmatrix} - N \right\}$$

$$\frac{d\bar{N}}{dt} = -i \left[-\frac{\Delta m^2}{4E_\nu} \begin{pmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} + \sqrt{2}G_F(N - \bar{N}), \bar{N} \right] + \frac{1}{2} \left\{ \begin{pmatrix} \bar{\kappa}_e & 0 \\ 0 & \bar{\kappa}_x \end{pmatrix}, \begin{pmatrix} \bar{N}_{ee}^{(\text{cl})} & 0 \\ 0 & \bar{N}_{xx}^{(\text{cl})} \end{pmatrix} - \bar{N} \right\}$$

(Homogeneous and isotropic) Quantum Kinetic Equations

$$\begin{aligned} \frac{dN}{dt} &= -i \left[+ \frac{\Delta m^2}{4E_\nu} \begin{pmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} + \sqrt{2}G_F(N - \bar{N}), N \right] + \frac{1}{2} \left\{ \begin{pmatrix} \kappa_e & 0 \\ 0 & \kappa_x \end{pmatrix}, \begin{pmatrix} N_{ee}^{(cl)} & 0 \\ 0 & N_{xx}^{(cl)} \end{pmatrix} - N \right\} \\ \frac{d\bar{N}}{dt} &= -i \left[- \frac{\Delta m^2}{4E_\nu} \begin{pmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{pmatrix} + \begin{pmatrix} \sqrt{2}G_F n_e & 0 \\ 0 & 0 \end{pmatrix} + \sqrt{2}G_F(N - \bar{N}), \bar{N} \right] + \frac{1}{2} \left\{ \begin{pmatrix} \bar{\kappa}_e & 0 \\ 0 & \bar{\kappa}_x \end{pmatrix}, \begin{pmatrix} \bar{N}_{ee}^{(cl)} & 0 \\ 0 & \bar{N}_{xx}^{(cl)} \end{pmatrix} - \bar{N} \right\} \end{aligned}$$

Vacuum
Matter mean-field
Self-interaction mean-field
Collisions (classical relaxation)

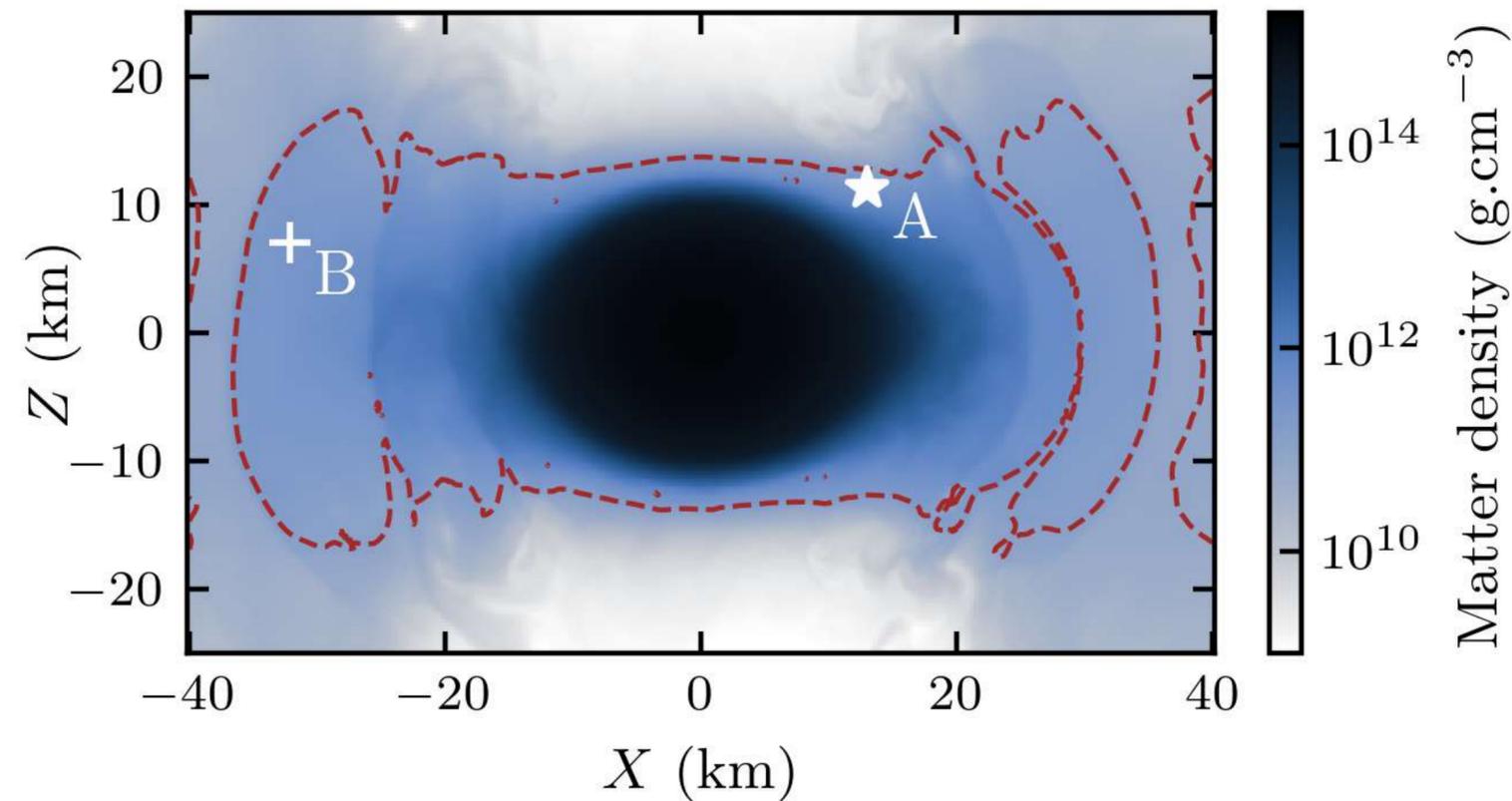
In reality for ν_x , steady-state \neq thermal equilibrium because of advection (escape).

Still, we consider a *relaxation* toward this classical steady-state.

$$\begin{aligned} \frac{dN}{dt} &= \frac{1}{2} \left\{ \begin{pmatrix} \kappa_e & 0 \\ 0 & \kappa_x \end{pmatrix}, \begin{pmatrix} N_{ee}^{(cl)} & 0 \\ 0 & N_{xx}^{(cl)} \end{pmatrix} - N \right\} = 0 \iff N = N^{(cl)} \\ \frac{d\bar{N}}{dt} &= \frac{1}{2} \left\{ \begin{pmatrix} \bar{\kappa}_e & 0 \\ 0 & \bar{\kappa}_x \end{pmatrix}, \begin{pmatrix} \bar{N}_{ee}^{(cl)} & 0 \\ 0 & \bar{N}_{xx}^{(cl)} \end{pmatrix} - \bar{N} \right\} = 0 \iff \bar{N} = \bar{N}^{(cl)} \end{aligned}$$

$$\sim -\kappa(N^{(cl)} - N)$$

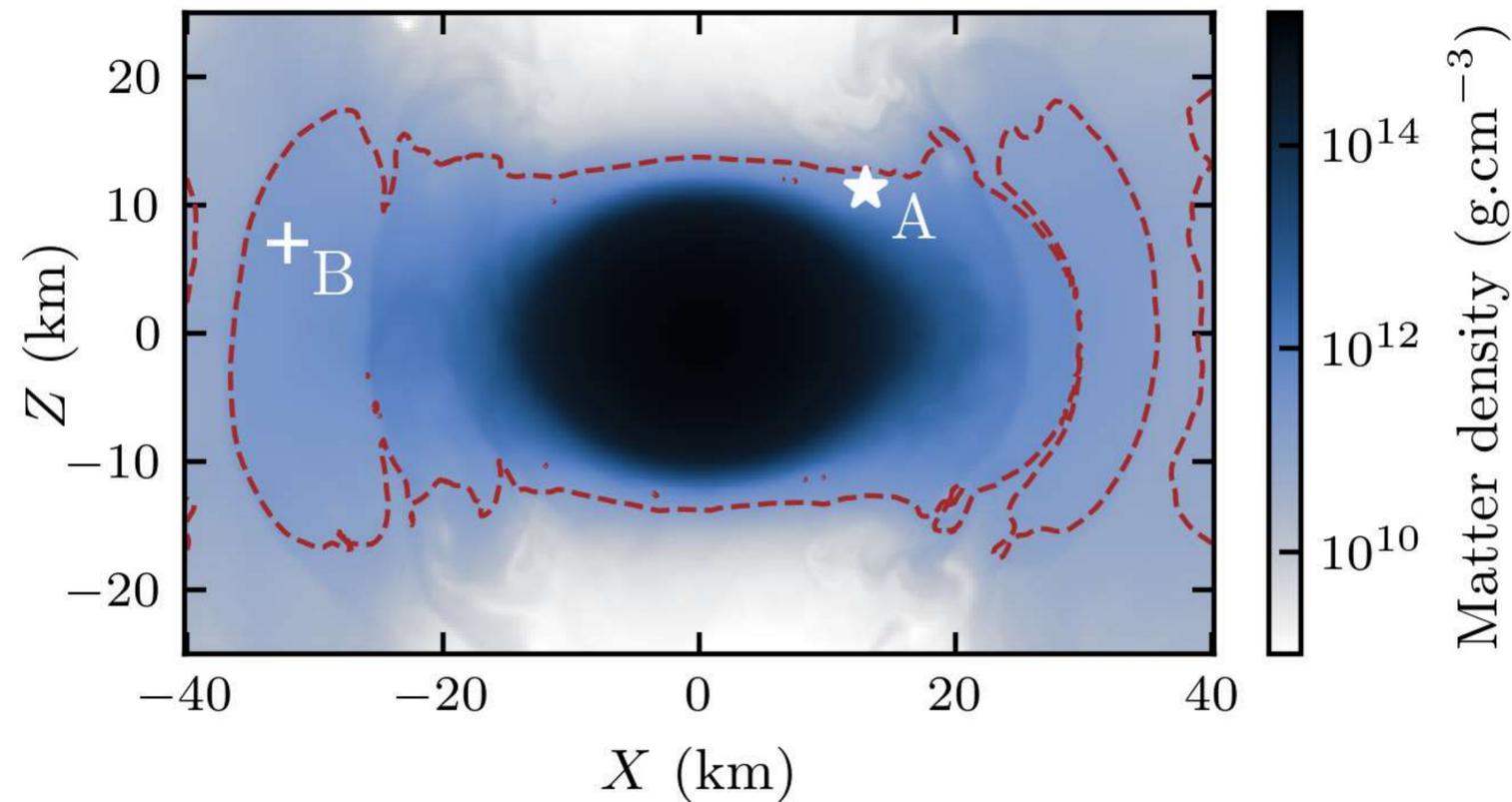
- 7 ms post-merger snapshot of NSM simulation by Foucart *et al.* [2407.15989]



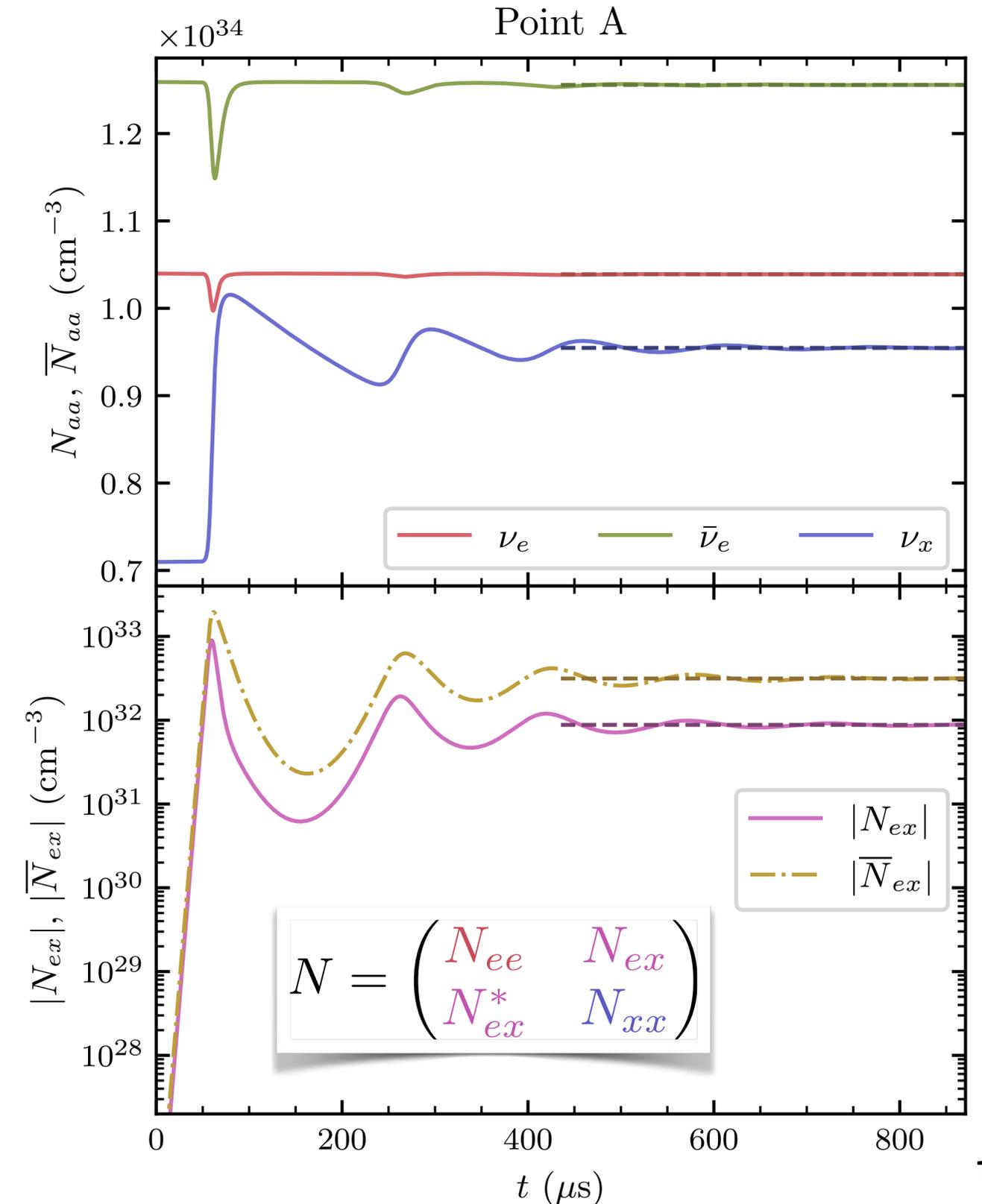
- Assume isotropy, number densities from simulation = classical steady-state
- Solve the homogeneous and isotropic QKEs

NSM-like configuration

- 7 ms post-merger snapshot of NSM simulation by Foucart *et al.* [2407.15989]

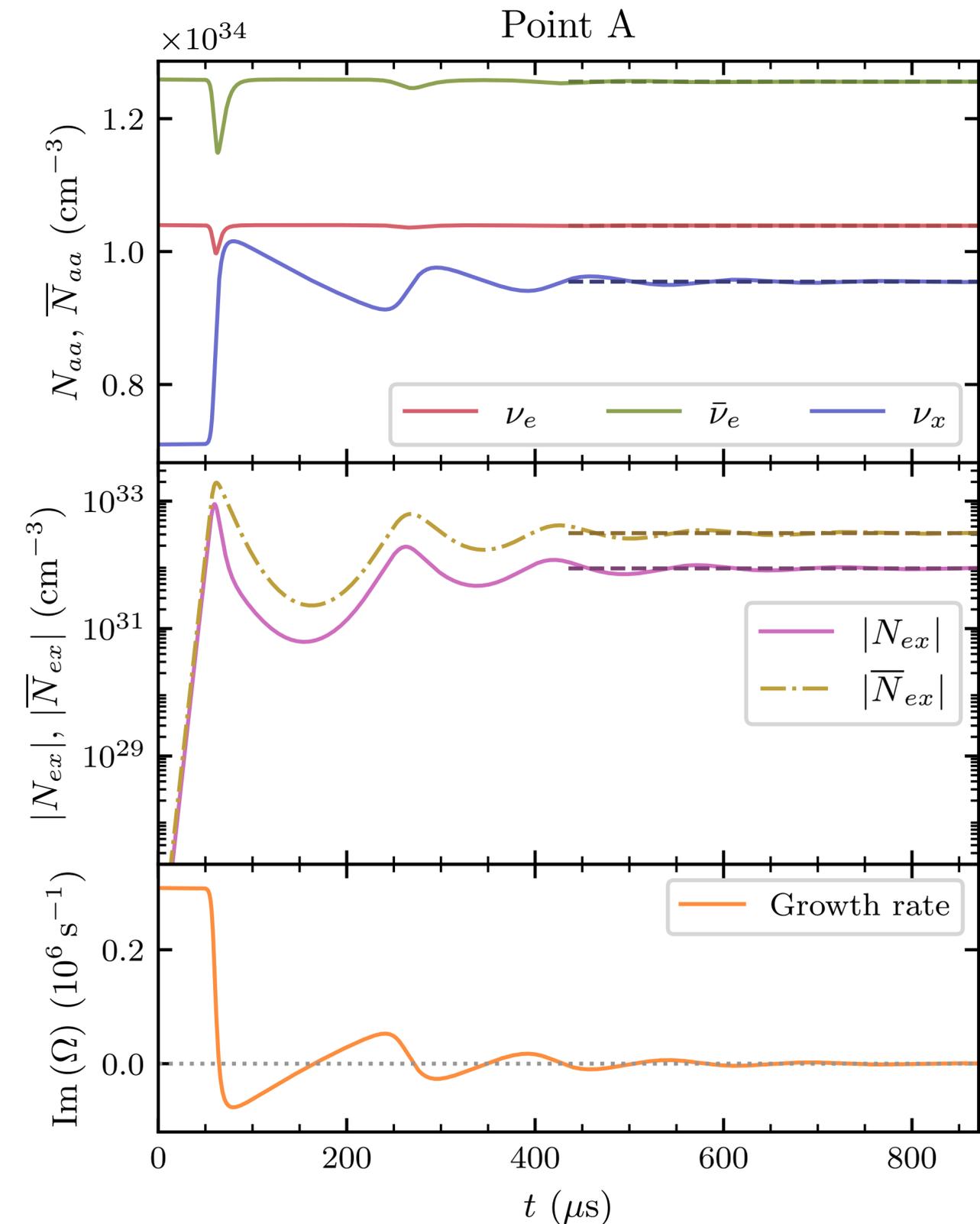


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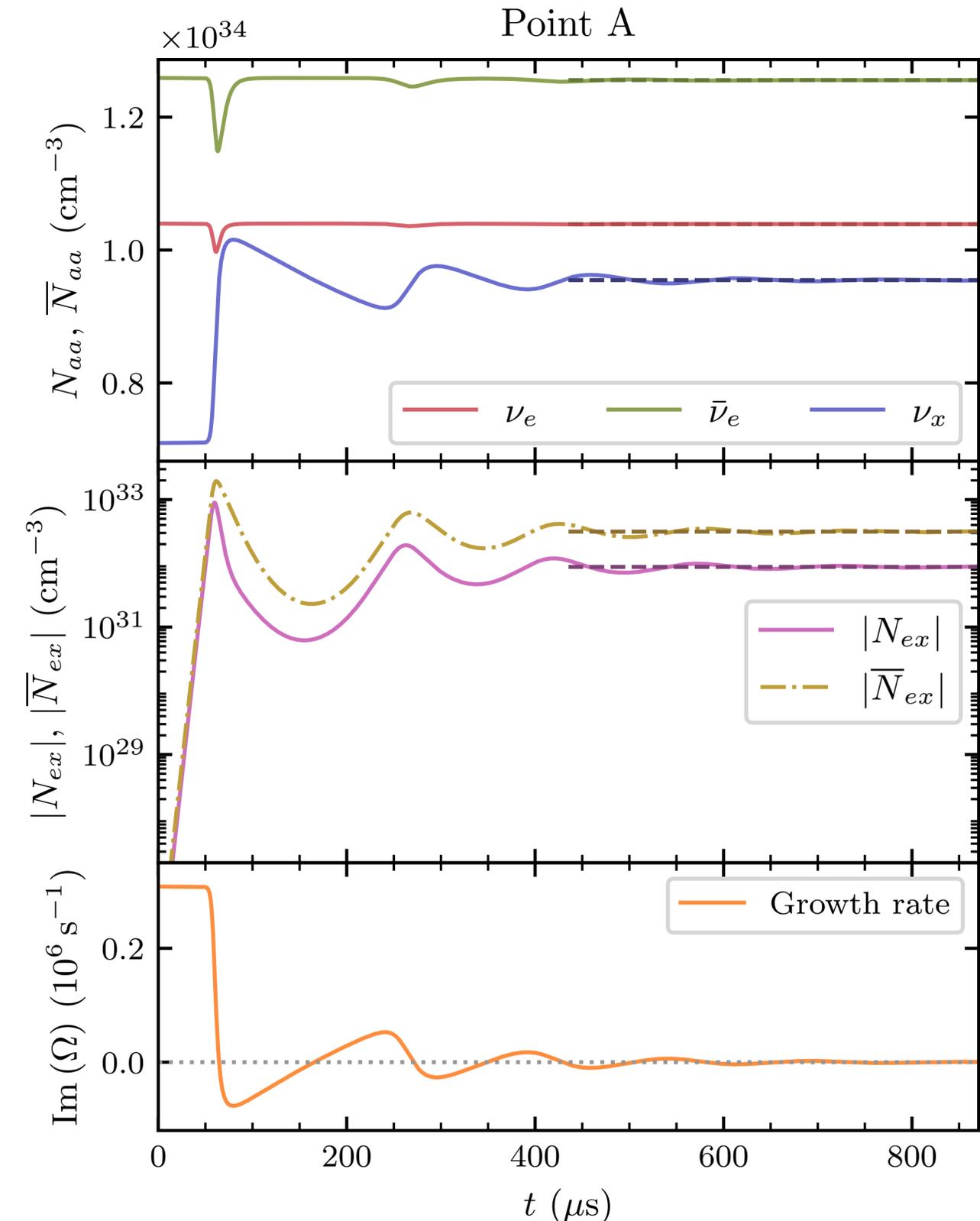
Classical/quantum trade-off

- The classical steady-state is unstable
⇒ instability and flavor conversion



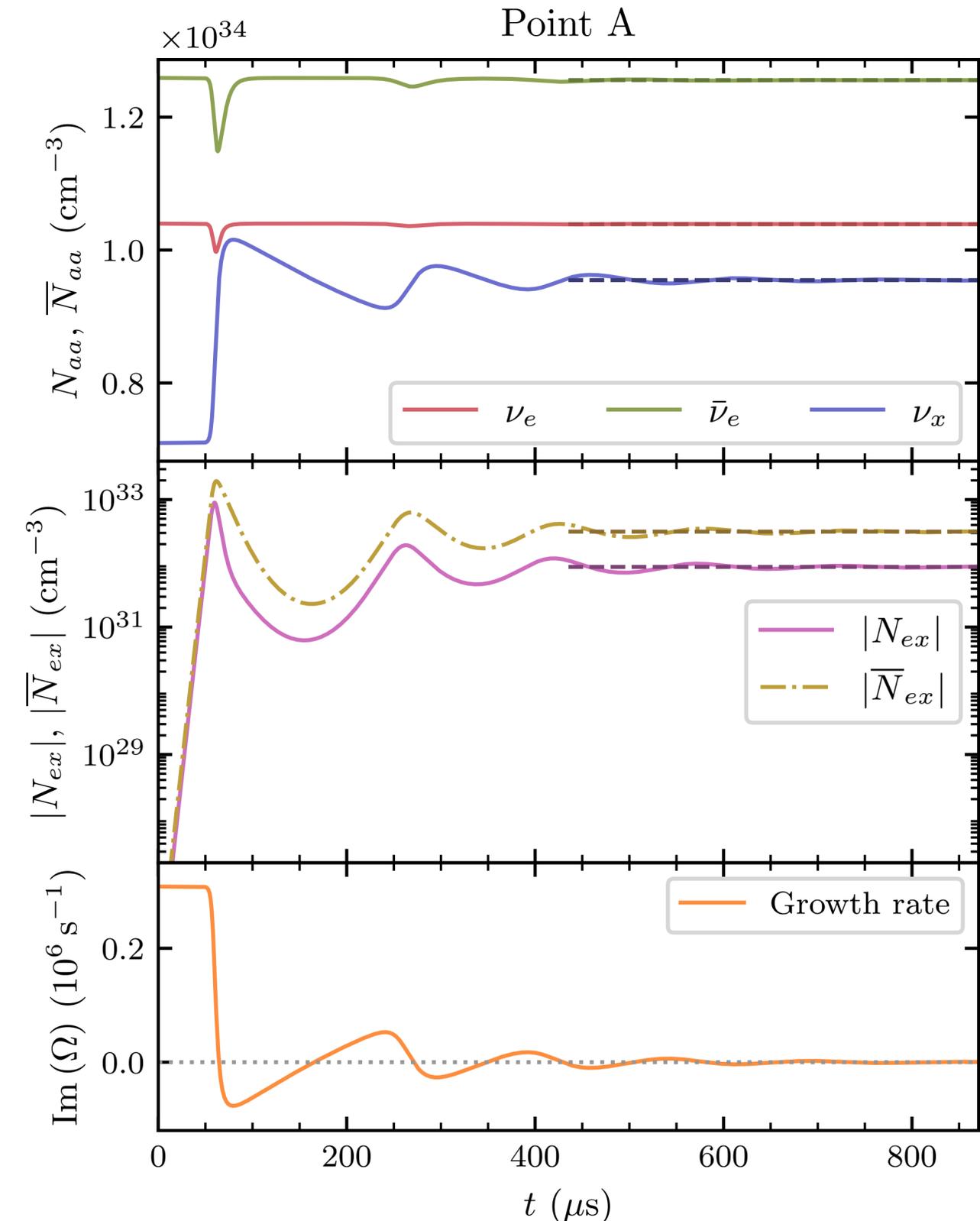
Classical/quantum trade-off

- The classical steady-state is unstable
 \implies instability and flavor conversion
- ν_e and $\bar{\nu}_e$ are quickly brought back to classical equilibrium
- Long-term relaxation of ν_x toward $N_{xx}^{(cl)}$
 (difference of timescales $\kappa_x \ll \kappa_e, \bar{\kappa}_e$)



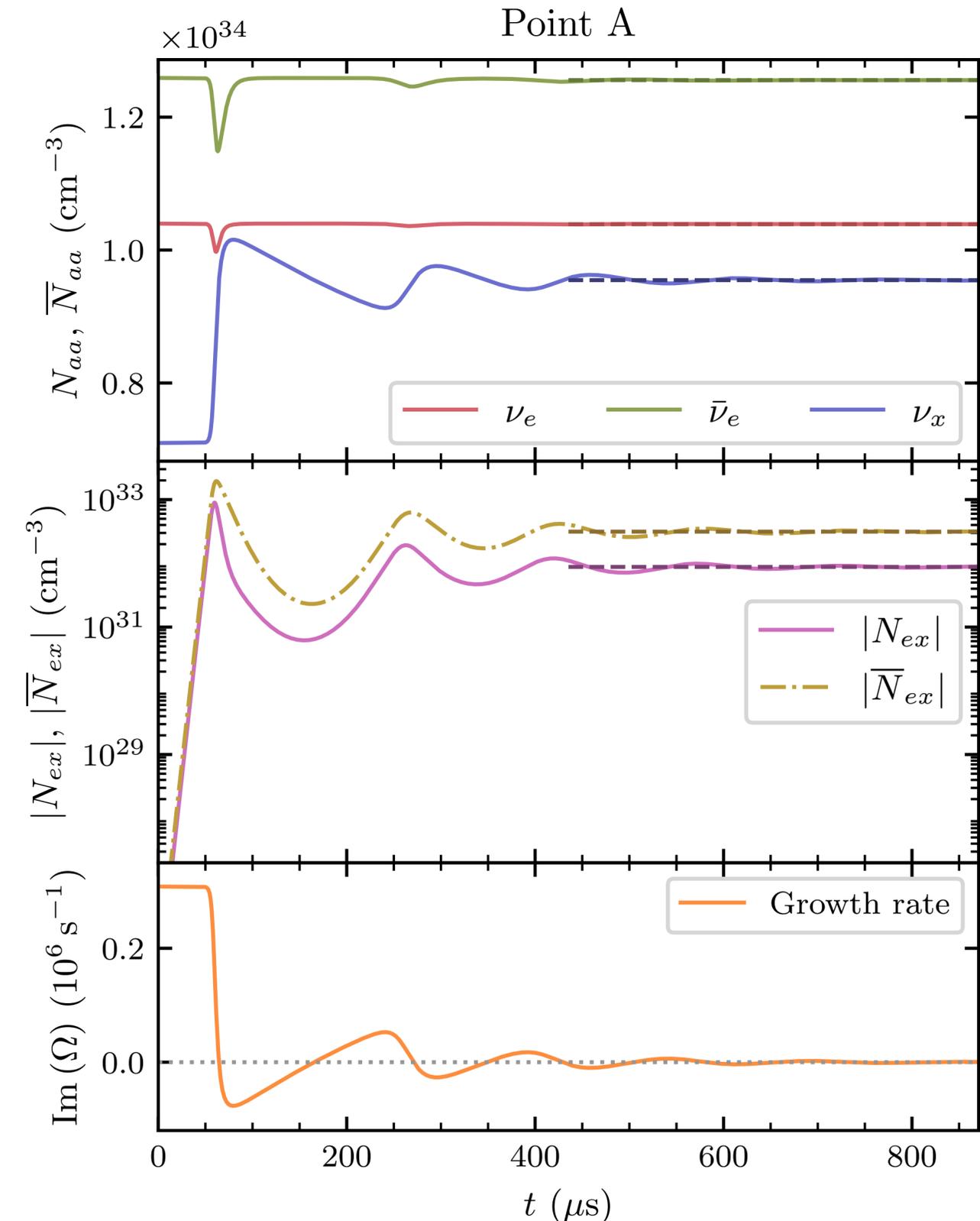
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- System unstable again
⇒ instability and (smaller) flavor conversion



Classical/quantum trade-off

- The classical steady-state is unstable
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- ...



Asymptotic state

- Vacuum term contribution negligible, and same opacities $\kappa_x = \bar{\kappa}_x$

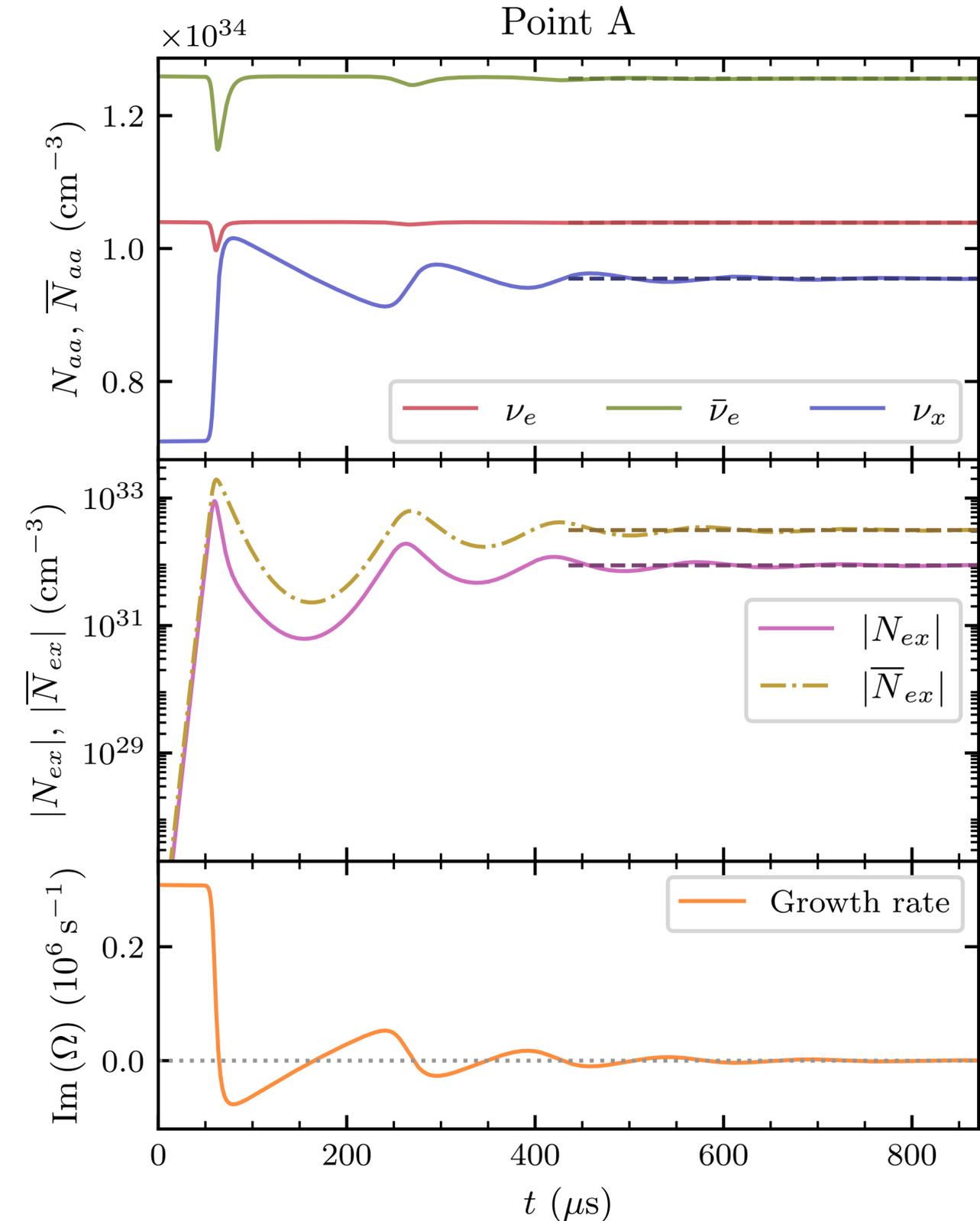
$$\implies N_{xx} = \bar{N}_{xx}$$

- Asymptotic state characterized by

$$\dot{N}_{ee} + \dot{N}_{xx} = 0$$

$$\dot{N}_{ee} - \dot{\bar{N}}_{ee} = 0$$

Instability growth rate = 0



Asymptotic state

- Vacuum term contribution negligible, and same opacities $\kappa_x = \bar{\kappa}_x$

$$\implies N_{xx} = \bar{N}_{xx}$$

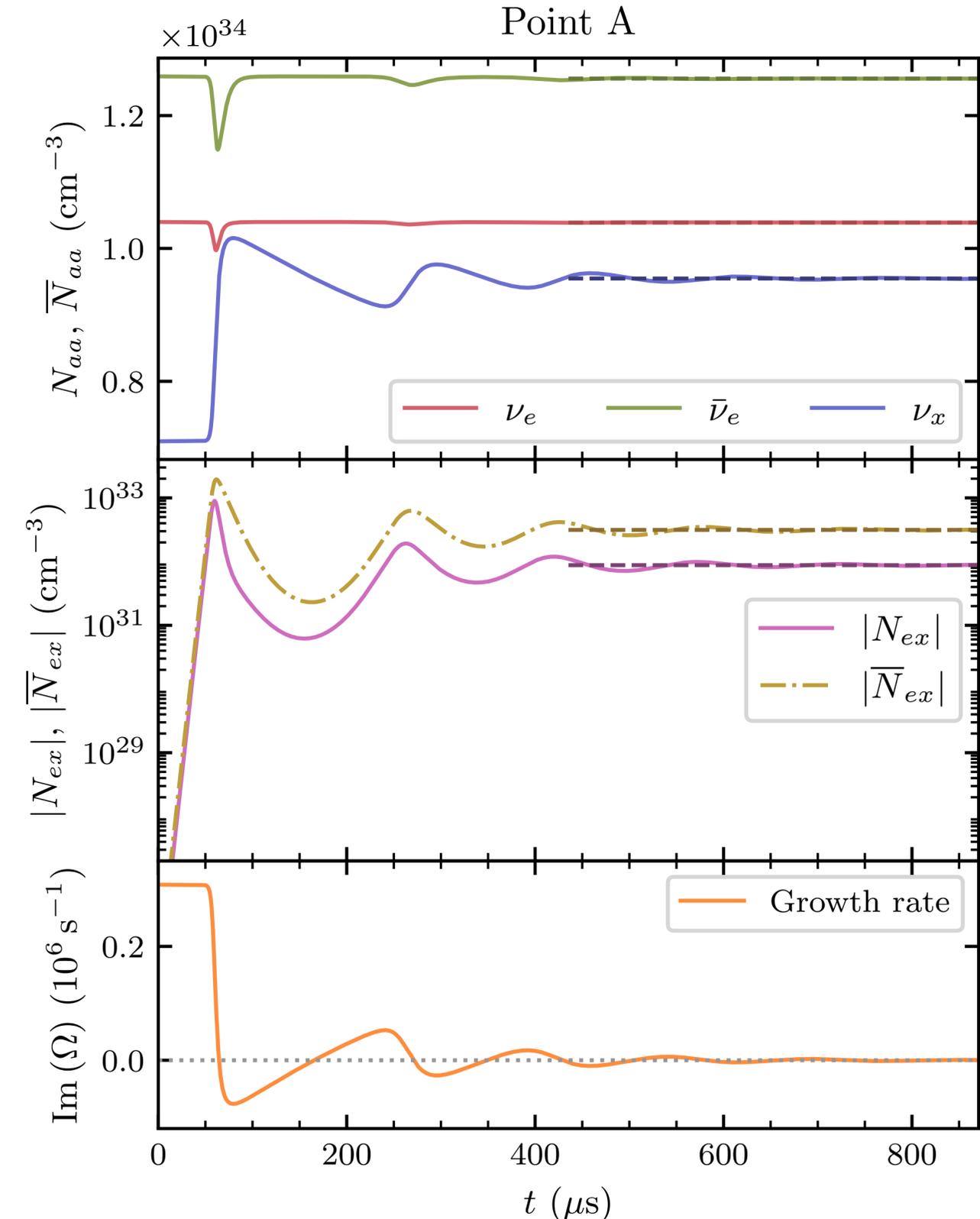
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$$\kappa_e [N_{ee}^{(\infty)} - N_{ee}^{(cl)}] = -\kappa_x [N_{xx}^{(\infty)} - N_{xx}^{(cl)}]$$

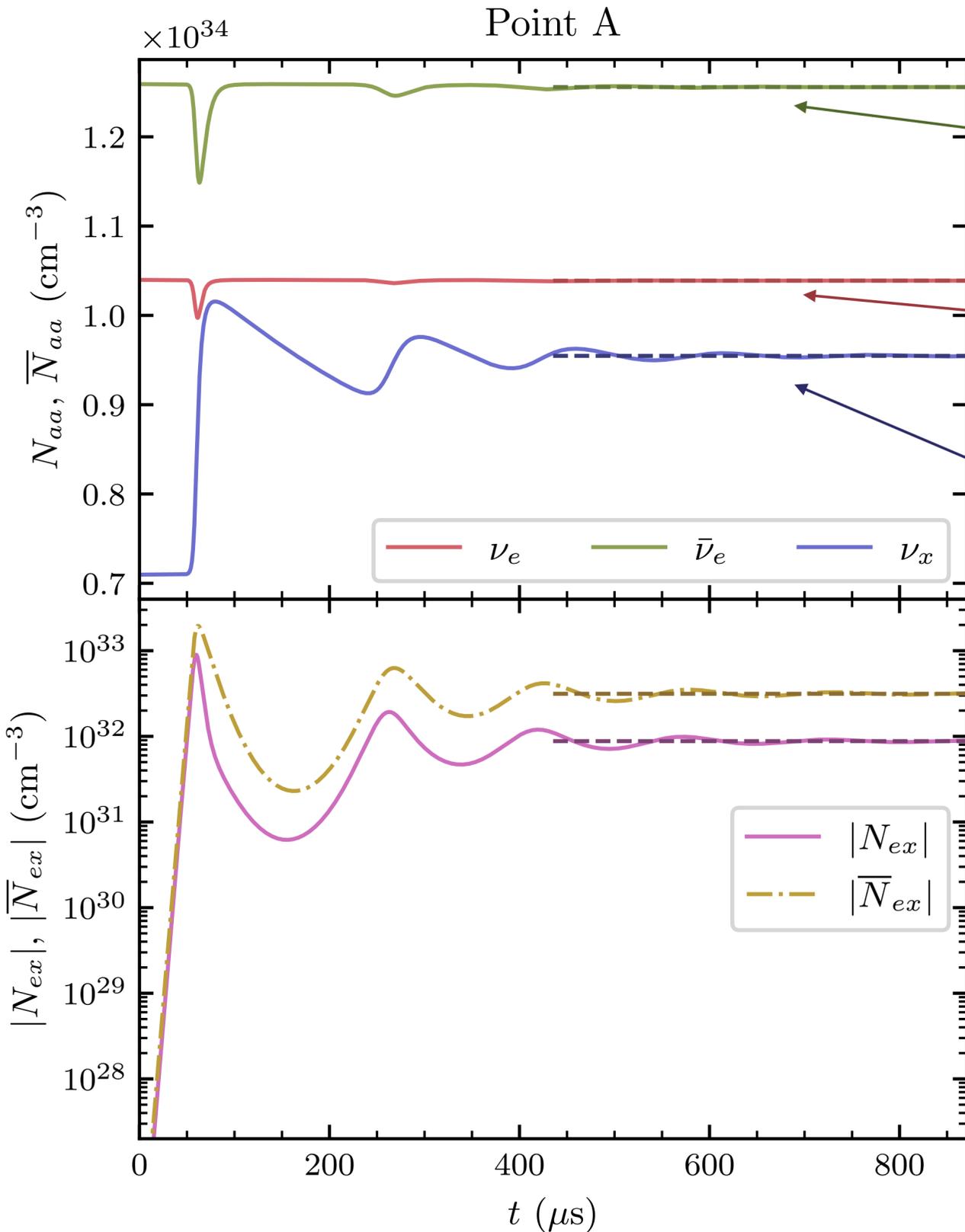
$$\kappa_e [N_{ee}^{(\infty)} - N_{ee}^{(cl)}] = \bar{\kappa}_e [\bar{N}_{ee}^{(\infty)} - \bar{N}_{ee}^{(cl)}]$$

$$\Gamma [N_{ee}^{(\infty)} - N_{xx}^{(\infty)}] = \bar{\Gamma} [\bar{N}_{ee}^{(\infty)} - N_{xx}^{(\infty)}]$$

$$\text{where } \Gamma \equiv \frac{\kappa_e + \kappa_x}{2}, \quad \bar{\Gamma} \equiv \frac{\bar{\kappa}_e + \kappa_x}{2}$$



Asymptotic state

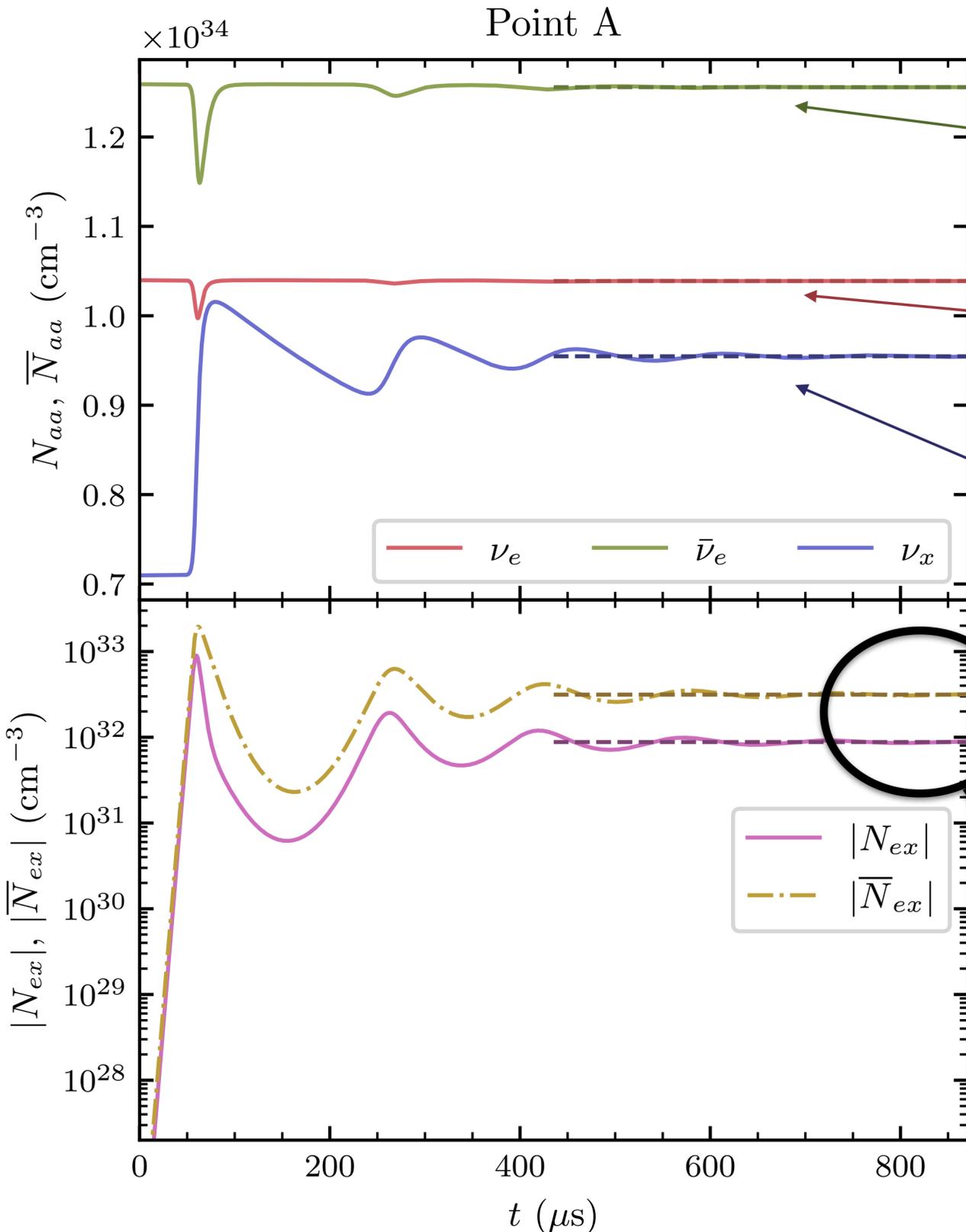


$$\bar{\kappa}_e \left[\bar{N}_{ee}^{(\infty)} - \bar{N}_{ee}^{(\text{cl})} \right] = \kappa_e \left[N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} \right]$$

$$N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} = \frac{1}{\kappa_e} \frac{\bar{\Gamma}[\bar{N}_{ee}^{(\text{cl})} - N_{xx}^{(\text{cl})}] - \Gamma[N_{ee}^{(\text{cl})} - N_{xx}^{(\text{cl})}]}{\Gamma\left(\frac{1}{\kappa_e} + \frac{1}{\kappa_x}\right) - \bar{\Gamma}\left(\frac{1}{\kappa_e} + \frac{1}{\kappa_x}\right)}$$

$$\kappa_x \left[N_{xx}^{(\infty)} - N_{xx}^{(\text{cl})} \right] = -\kappa_e \left[N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} \right]$$

Asymptotic state



$$\bar{\kappa}_e \left[\bar{N}_{ee}^{(\infty)} - \bar{N}_{ee}^{(\text{cl})} \right] = \kappa_e \left[N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} \right]$$

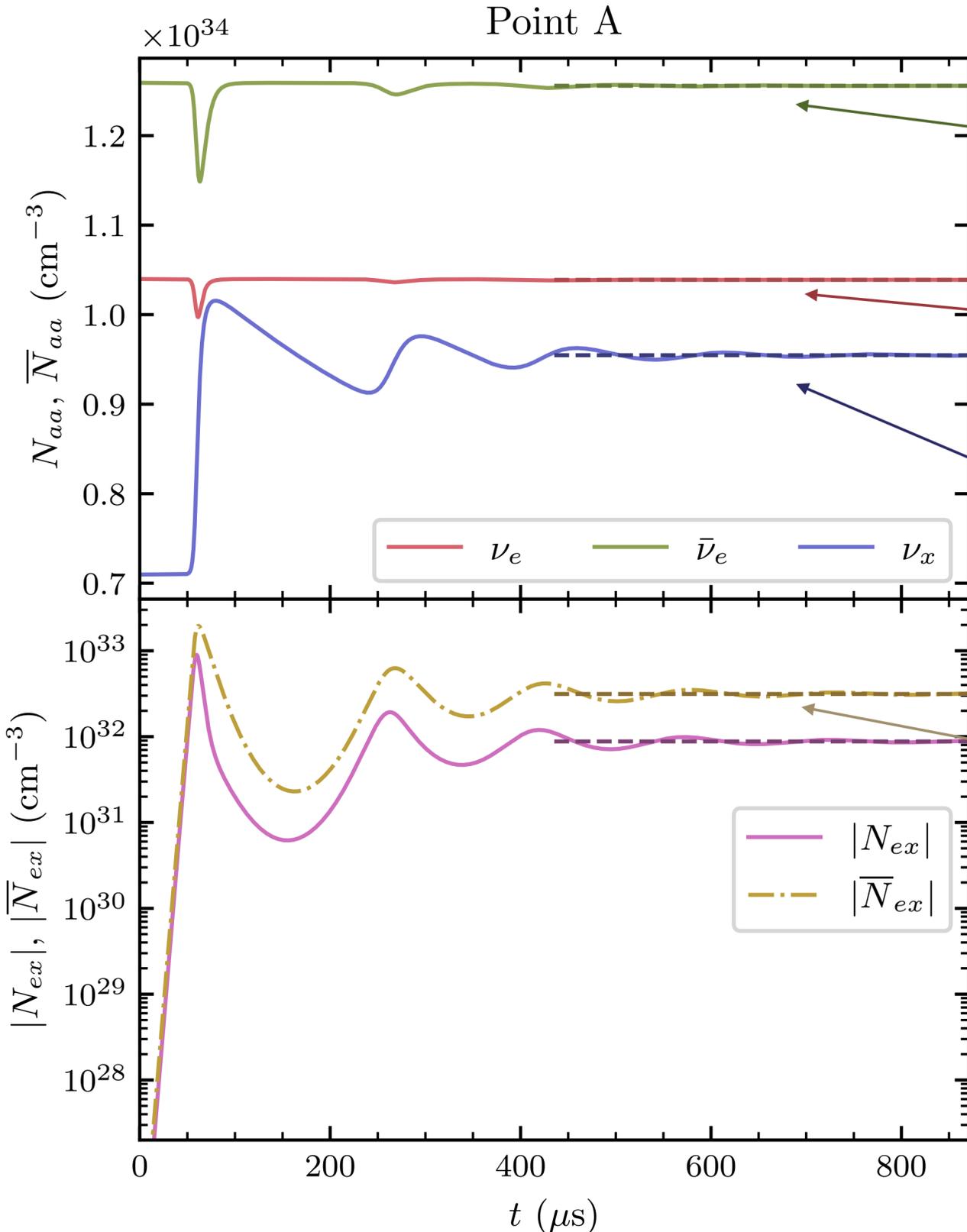
$$N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} = \frac{1}{\kappa_e} \frac{\bar{\Gamma}[\bar{N}_{ee}^{(\text{cl})} - N_{xx}^{(\text{cl})}] - \Gamma[N_{ee}^{(\text{cl})} - N_{xx}^{(\text{cl})}]}{\Gamma\left(\frac{1}{\kappa_e} + \frac{1}{\kappa_x}\right) - \bar{\Gamma}\left(\frac{1}{\kappa_e} + \frac{1}{\kappa_x}\right)}$$

$$\kappa_x \left[N_{xx}^{(\infty)} - N_{xx}^{(\text{cl})} \right] = -\kappa_e \left[N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} \right]$$

In the “quantum” equilibrium, the collision term does not vanish

Nonzero flavor coherence

Asymptotic state and flavor coherence



$$\bar{\kappa}_e \left[\bar{N}_{ee}^{(\infty)} - \bar{N}_{ee}^{(\text{cl})} \right] = \kappa_e \left[N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} \right]$$

$$N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} = \frac{1}{\kappa_e} \frac{\bar{\Gamma}[\bar{N}_{ee}^{(\text{cl})} - N_{xx}^{(\text{cl})}] - \Gamma[N_{ee}^{(\text{cl})} - N_{xx}^{(\text{cl})}]}{\Gamma\left(\frac{1}{\kappa_e} + \frac{1}{\kappa_x}\right) - \bar{\Gamma}\left(\frac{1}{\kappa_e} + \frac{1}{\kappa_x}\right)}$$

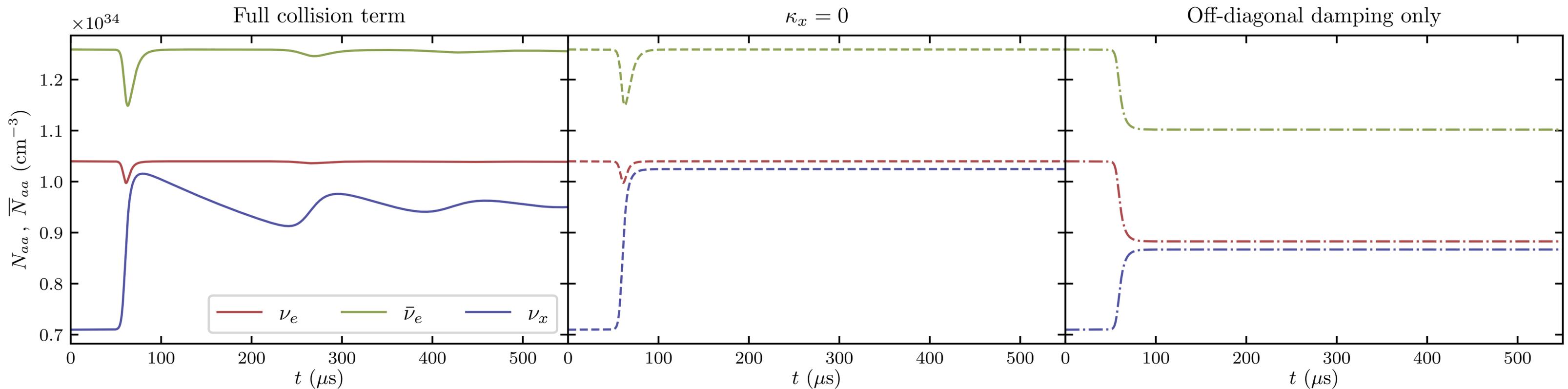
$$\kappa_x \left[N_{xx}^{(\infty)} - N_{xx}^{(\text{cl})} \right] = -\kappa_e \left[N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} \right]$$

$$|N_{ex}|^{(\infty)} = \sqrt{-\frac{\kappa_e}{\kappa_e + \kappa_x} \left[N_{ee}^{(\infty)} - N_{ee}^{(\text{cl})} \right] \left[N_{ee}^{(\infty)} - N_{xx}^{(\infty)} \right]}$$

Treatment of collisions

Collision term:

$$\begin{pmatrix} -\kappa_e [N_{ee} - N_{ee}^{(cl)}] & -\frac{\kappa_e + \kappa_x}{2} N_{ex} \\ -\frac{\kappa_e + \kappa_x}{2} N_{xe} & -\kappa_x [N_{xx} - N_{xx}^{(cl)}] \end{pmatrix} \quad \begin{pmatrix} -\kappa_e [N_{ee} - N_{ee}^{(cl)}] & -\frac{\kappa_e}{2} N_{ex} \\ -\frac{\kappa_e}{2} N_{xe} & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & -\frac{\kappa_e + \kappa_x}{2} N_{ex} \\ -\frac{\kappa_e + \kappa_x}{2} N_{xe} & 0 \end{pmatrix}$$



→ Equipartition is an artifact of neglecting the repopulation of $\nu_e, \bar{\nu}_e$



- Asymptotic state of **FFI**: *erasure of ELN-XLN crossing in the “shallow” angular domain*
 - ▶ Used in large-scale simulations! [[Wang & Burrows, 2503.04896](#)], [[Lund et al., 2503.23727](#)]
 - ▶ Caveat: dependence on the boundary conditions [[Zaizen & Nagakura, 2304.05044](#)]
- Possibility to predict the outcome of **collisional flavor instabilities**
 - ▶ Crucial to include the classical relaxation term
 - ▶ “Compromise” between **relaxation to** / **instability of** classical steady-state
- Limitations: single-energy, homogeneous calculation (large-scale advection effects?)
 - ▶ **Important phenomenology with multi-energies** [[Zaizen, 2502.09260](#)]
- Flavor coherence cannot necessarily be neglected in the asymptotic state!

