Cosmological Constant and Condensates

PRD 104,076010 (2021)
[arXiv:2103.15768],

PLB 849, 138418 (2023)
[arXiv:2302.11600]

- Cosmology – Friedmann equations and dark energy. Energy density and pressure.
- Analogy with hadrons: Energy-pressure relation of trace anomaly and confinement
- Analogy with vortices of type II superconductors
- Origin of the cosmological constant

CIPANP June 11, 2025

Cosmology

- A constant cosmological constant is introduced by Einstein in the general relativity equation for a static universe.

$$R_{\mu\nu} + \frac{1}{2} R\, g_{\mu\nu} - \Lambda\, g_{\mu\nu} = 8\pi G\, T_{\mu\nu} \qquad \Lambda = 4\pi G \rho$$

- Friedmann equations of Friedmann-Robertson-Walker scale parameter

$$(\frac{\dot{a}}{a})^2 = \frac{H^2}{H_0^2} = \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda + \frac{1-\Omega_0}{a^2}$$

radiation matter CC curvature

$$H(t) \equiv \dot{a}/a$$

$$\Omega_{r,m,\Lambda} \equiv \epsilon_{r,m,\Lambda}/\epsilon_c$$

$$\epsilon_c \equiv \frac{3c^2}{8\pi G} H^2$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\sum_i \epsilon_i + \epsilon_\Lambda + 3(\sum_i P_i + P_\Lambda))$$

$$\epsilon_\Lambda = \frac{\Lambda}{8\pi G} \qquad P_\Lambda = -\frac{\Lambda}{8\pi G}$$

- $P_\Lambda < 0$ anti-gravitates ⇒ acceleration

Cosmological constant

- Equations of state: $$P = \omega\epsilon$$
  - Non-relativistic matter: $\omega \approx 0$
  - Radiation (γ, υ): $\omega = 1/3$
  - Dark energy: $\omega < -1/3$
  - Cosmological constant: $\boxed{\omega = -1}$

N.B. $\Lambda\, g_{\mu\nu}$ in the GR equation - a salient feature

$$\omega = -1.00 \pm 0.04(?)$$

- Puzzles:
  - Why is $\Lambda > 0$, and ω = -1?
  - Identifying Λ as the vacuum energy, why so small compared to the plack energy density?

$$\epsilon_{\text{Plank}} \sim \frac{E_P}{l_P^3} \sim 3 \times 10^{132}\,\text{eVm}^{-3}$$

$$\frac{\epsilon_c}{\epsilon_{\text{Plank}}} \sim 10^{-123} \text{ !!!}$$

$$\Omega_{\text{baryon}} = 0.048, \Omega_{dm} = 0.262,$$
$$\Omega_r = 9.0 \times 10^{-5}, \Omega_\Lambda \approx 0.69$$

Mass of Hadrons – Energy Momentum Tensor

- Energy momentum tensor

$$T_{\mu\nu} = \frac{1}{4}\bar{\psi}\gamma_{(\mu}\overleftrightarrow{D}_{\nu)}\psi + G_{\mu\alpha}G_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}G^2$$

- Mass from trace of EMT – scalar, frame independent, scale invariant

$$\langle P|T^{\mu\nu}|P\rangle = 2P^\mu P^\nu$$

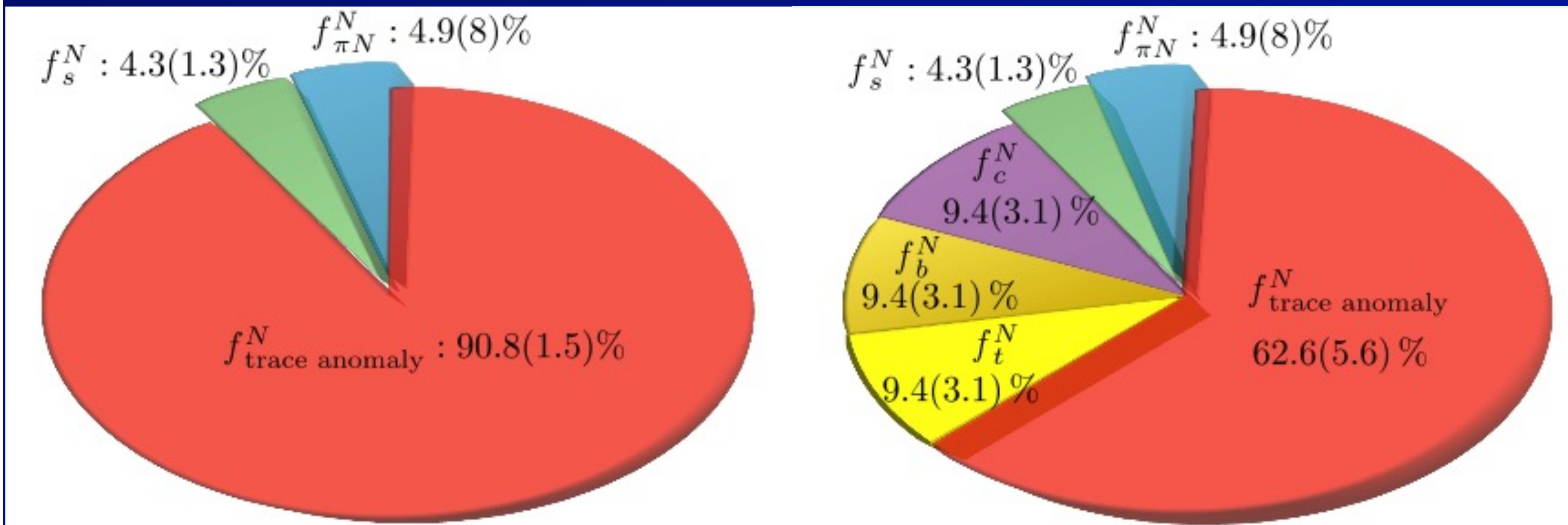
$$\frac{\langle P|\int d^3\vec{x}\,\gamma\, T^\mu_\mu(x)|P\rangle}{\langle P|P\rangle}| = M_N$$

$$T^\mu_\mu = \sum_f m_f\bar{\psi}_f\psi_f + \left[\frac{\beta(g)}{2g}G^{\alpha\beta}G_{\alpha\beta} + \sum_f m_f\gamma_m(g)\bar{\psi}_f\psi_f\right]$$

Chanowitz, Ellis, Crewther, Collin, Duncan, Joglekar

Mass from Trace of EMT

- Lattice calculation of of quark condensate
  - Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]; M. Gong et al, (χ QCD) [arXiv:1304.1194]
  - Overlap fermion (Z_m Z_s =1), 3 lattices (one at physical m_π)



$$m_h \langle N|\bar{\psi}_h \psi_h|N\rangle \sim -\frac{n_f}{3}\frac{\alpha_s}{4\pi}\langle N|G^2|N\rangle + \mathcal{O}(1/m_h)$$

$$f_f^N = \frac{m_f \langle N|\bar{\psi}_f \psi_f|N\rangle}{M_N}$$

$$\beta_0 = 11 - \frac{2}{3}n_f$$

Shifman, et al., Phys.Lett. B 78, 443 (1978)

Decoupling theorem: $f_c^N + f_b^N + f_t^N + f_a^N \sim \sum_H \mathcal{O}_H(1/m_H)$

Rest Energy Decomposition from Hamiltonian

- Separate the EMT into traceless part and trace part (Ji, 1995)

$$T^{\mu\nu} = \overline{T}^{\mu\nu} + \frac{1}{4} g^{\mu\nu} (T^{\rho}_{\rho})$$

- Hamiltonian -- $H = \int d^3\vec{x}\, T^{00}(x)$

$$H_q(\mu) = \int d^3\vec{x} \left(\frac{i}{4} \sum_f \bar{\psi}_f \gamma^{\{0} \overleftrightarrow{D}^{0\}} \psi_f - \frac{1}{4} T^{\mu}_{q\,\mu} \right),$$

Quark energy (scale dependent)

$$H_g(\mu) = \int d^3\vec{x}\, \frac{1}{2}(B^2 + E^2),$$

Glue field energy (scale dependent)

$$H_{tr} = \int d^3\vec{x}\, \frac{1}{4}(T^{\mu}_{\mu}).$$

Caracciolo:1989pt, Makino:2014taa, DallaBrida:2020gux

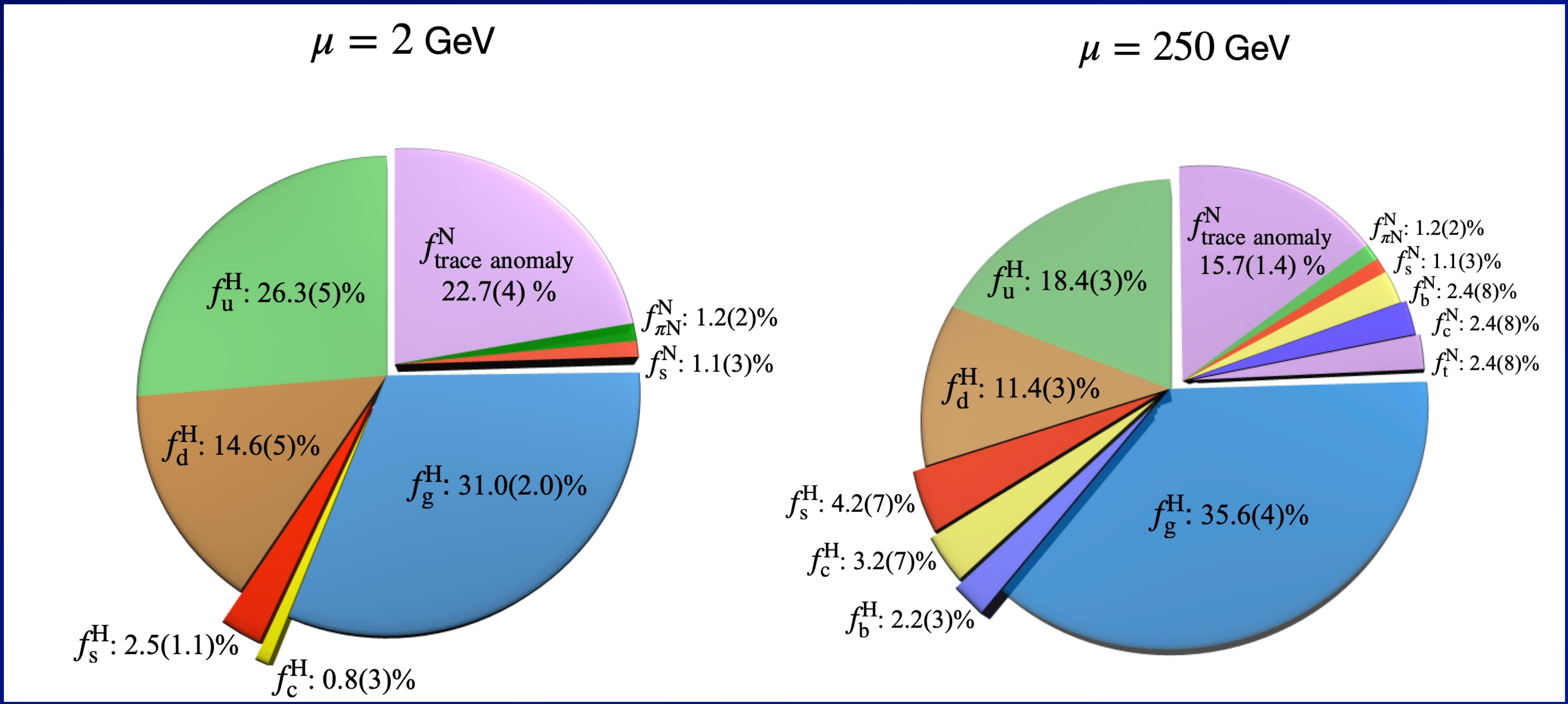
- Rest energy -- $E_0 = \langle H_{q_f}(\mu) \rangle + \langle H_g(\mu) \rangle + \langle H_{tr} \rangle,$

$$\langle H_{q_f}(\mu) \rangle = \frac{3}{4} \sum_f \langle x \rangle_f(\mu) M, \qquad \langle H_g(\mu) \rangle = \frac{3}{4} \langle x \rangle_g(\mu) M,$$

$$\langle H_{tr} \rangle = \frac{1}{4} \langle T^{\mu}_{\mu} \rangle = \frac{1}{4} M.$$

<x> - momentum fraction experimentally measurable

Rest Energy Decomposition from Hamiltonian



$$f_f^H = \langle H_q \rangle / M = \frac{3}{4} \langle x \rangle_f(\mu), \qquad f_g^H = \langle H_g \rangle / M = \frac{3}{4} \langle x \rangle_g(\mu),$$

$$f_{\pi N}^N = \frac{1}{4}\frac{\sigma_{\pi N}}{M}, \qquad f_s^N = \frac{1}{4}\frac{\sigma_s}{M}, \qquad f_{\text{trace anomaly}}^N = \frac{1}{4}\frac{\langle H_{\text{ta}} \rangle}{M}$$

Y.B. Yang et al (χ QCD) [arXiv: 1511.15089]

Momentum fractions from CT18 (T.J. Hou et al, PRD, arXiv:1912.10053) at $\mu = 2$ GeV and 250 GeV.

Rest Energy/Mass from Gravitational FF

- Gravitational Form factors from the EMT matrix elements

$$\langle P'|(T^{\mu\nu}_{q,g})(\mu)|P\rangle/2M_N = \bar{u}(P')[A_{q,g}(q^2,\mu)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(q^2,\mu)\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}q_\alpha}{2M_N}$$

$$+ D_{q,g}(q^2,\mu)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{M_N} + \bar{C}_{q,g}(q^2,\mu)M_N\, g^{\mu\nu}]u(P)$$

  - A(0) and A(0)+ B(0) : momentum and angular momentum - [Ji]
  - D(0): D term (deformation of space = elastic property) - [Polyakov]
  - C-bar term: pressure-volume work - [Lorce, Liu]

$$E_0 = \overline{E}_0 + E_0(\mathrm{tr})$$

$$\overline{E}_0 = \langle \overline{T}^{00}\rangle = \frac{3}{4}\langle x\rangle_{q+g}M$$

$$E_0(\mathrm{tr}) = \frac{1}{4}\mathrm{g}^{00}\langle \mathrm{T}^\mu_\mu\rangle = \frac{1}{4}\mathrm{M}$$

$$PV = \overline{P}V + PV(\mathrm{tr})$$

$$\overline{P}V = \langle \overline{T}^{ii}\rangle = \frac{1}{4}\langle x\rangle_{q+g}M$$

$$PV(\mathrm{tr}) = \frac{1}{4}\mathrm{g}^{\mathrm{ii}}\langle \mathrm{T}^\mu_\mu\rangle = -\frac{1}{4}\mathrm{M}$$

$$\partial_\nu T^{\mu\nu} = 0 \qquad PV = -\frac{dE}{dV}V = 0$$

N.B.

$$PV(\mathrm{tr}) = -\mathrm{E}_0(\mathrm{tr}); \quad \overline{\mathrm{P}}\mathrm{V} = \frac{1}{3}\overline{\mathrm{E}}_0$$

Trace Anomaly and Gluon Condensate

- Equation of state $E_0 = \epsilon V + \epsilon_K V^{-1/3}$, (cf. MIT Bag Model)

  where $\epsilon = \frac{E_S}{V}$, $\epsilon_K = E_T V^{1/3}$ are constants

- Picture: Nucleon is a bubble in the sea of gluon condensate, where

$$\epsilon = -\epsilon_{\rm vac}$$

N.B. $\langle OG_2 \rangle_{\rm correlated} = \langle OG_2 \rangle - \langle O \rangle \langle G_2 \rangle$

$$\epsilon_{vac} = \frac{\beta(g)}{2g} \langle 0 | F^{\alpha\beta} F_{\alpha\beta} | 0 \rangle < 0 \qquad V = \frac{E_S}{|\epsilon_{\rm vac}|}$$

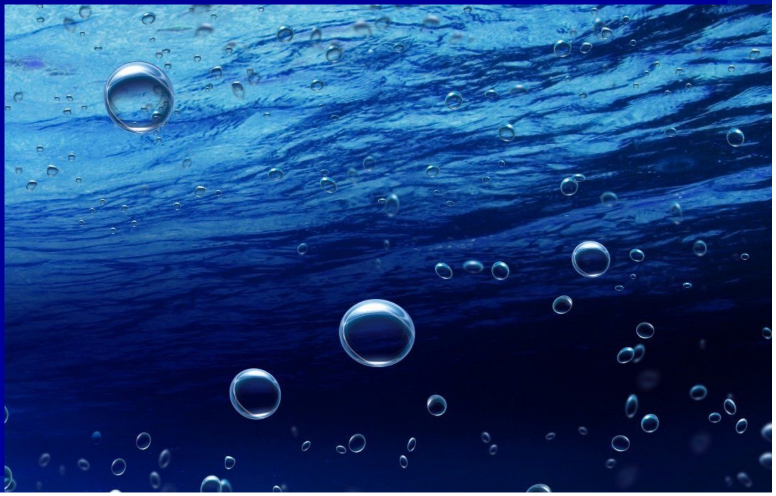
- Trace anomaly gives a negative constant

  pressure ⟹ confinement

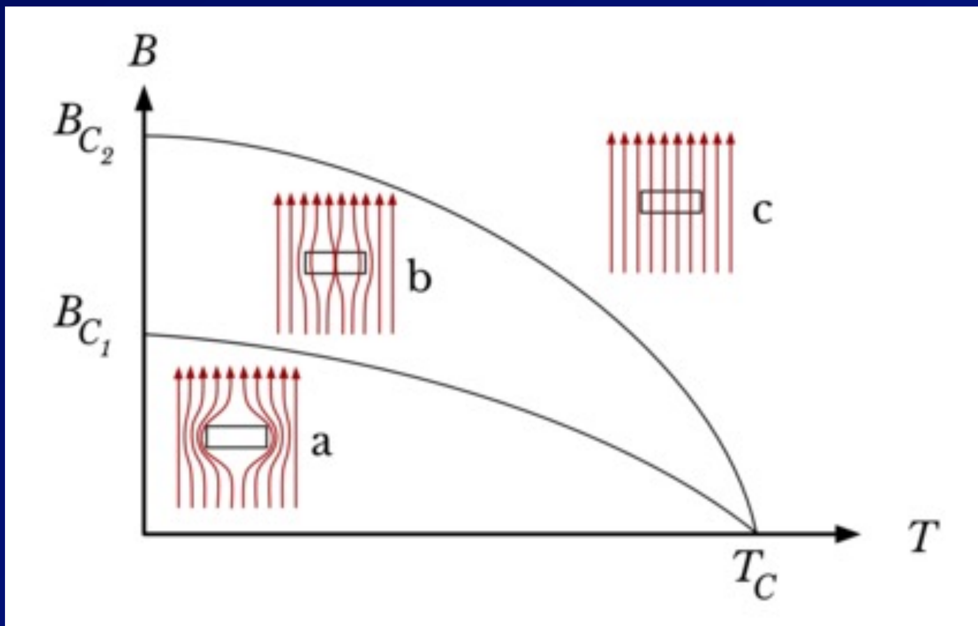
Same as in charmonium $V(r) = |\epsilon_{vac}| \, A \, r = \sigma \, r$

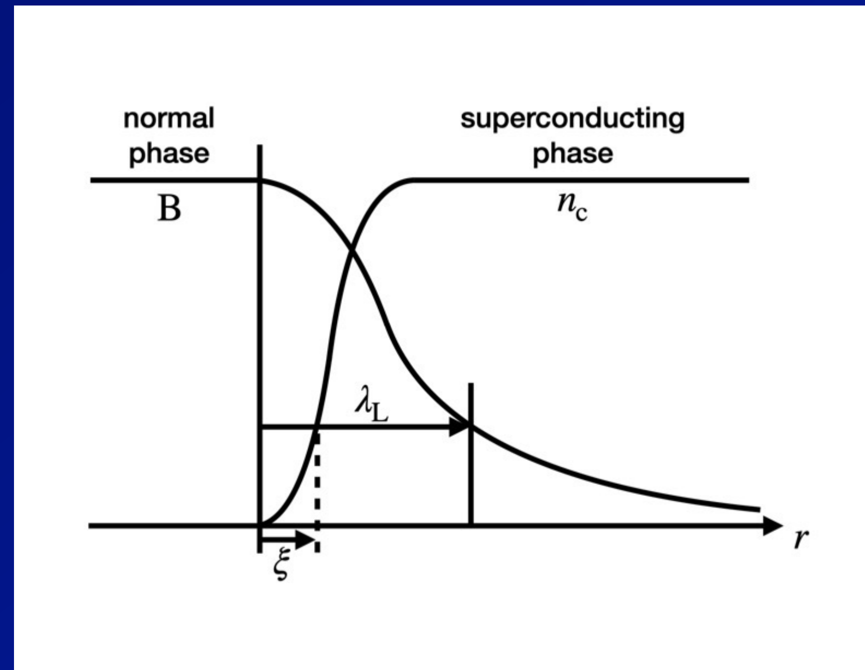
Bali ('97), Baker ('18)

- Many facets of color confinement
  - Dual superconductor
  - Magnetic monopole
  - Center vortices



Type II Superconductor





Ginzburg–Landau equations

$$\alpha\psi + \beta|\psi|^2\psi + \frac{1}{2m}(-i\hbar\nabla - 2e\mathbf{A})^2\psi = 0$$

$$\nabla \times \mathbf{B} = \mu_0\mathbf{j} \;\; ; \;\; \mathbf{j} = \frac{2e}{m}\mathrm{Re}\{\psi^*\left(-i\hbar\nabla - 2e\mathbf{A}\right)\psi\}$$

$$|\psi|^2 = n_s$$

London penetration depth

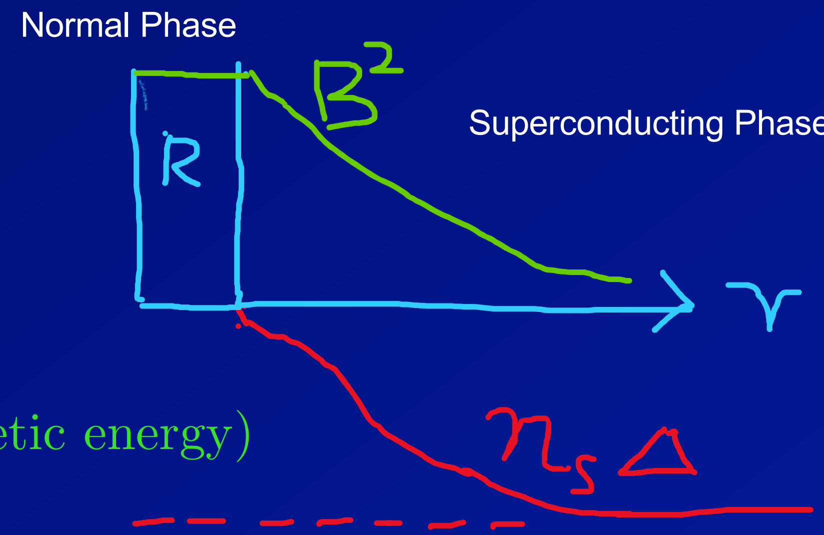
$$\lambda_L = \sqrt{\frac{m}{4\mu_0 e^2\psi_0^2}}$$

Coherent length ξ

Type II: $\kappa = \lambda_L/\xi > 1/\sqrt{2}$

Energetics and Equilibrium

- Type II superconductor



$$F = F_s + F_B + F_{sc}$$

$$F_s = \text{ cost of condensation energy}$$

$$F_B = \int dv\, B^2/2\mu_0 \ \text{(magnetic energy)}$$

$$F_{sc} = 1/2 \int dv\, \lambda_L^2 J_s \cdot J_s \ \text{(supercurrent kinetic energy)}$$

- Variational model (J.R. Clem, Jour. Low Temp. Phys. 18, 5/6 (1975))

$$\frac{|\psi|^2}{n_0} = \frac{n_s}{n_0} = \frac{\rho^2}{\rho^2 + R^2} \underset{\rho \to \infty}{\longrightarrow} 1$$

$$\frac{1}{\sqrt{2}H_c} \frac{E}{l} = \phi_0 H_c'/4\pi \ \text{ where } \phi_0 = hc/2e, \ \sqrt{2}H_c = \kappa\phi_0/2\pi\lambda_L^2$$

Equation of state

$$F'/l = \underbrace{\kappa R'^2/8}_{F_s} + \underbrace{1/8\kappa + K_0(R')/2\kappa R' \, K_1(R')}_{F_B + F_{sc}}, \ \text{ where } R' = R/\lambda_L$$

EEC $-\dfrac{dF'/l}{dA} A = 0$

Spatial Distributions from Graviational FF

- Fourier Transform to obtain spatial distributions

$$G_m(q^2) \text{ from } T^\mu_\mu \longrightarrow m(r) \qquad A(q^2) \longrightarrow A(r)$$

- $$\epsilon(r) = \epsilon_{\rm tr}(r) + \overline{\epsilon}(r)$$

$$\epsilon_{\rm tr}(r) = \frac{M}{4} m(r); \quad \overline{\epsilon}(r) = M(\frac{2}{3} A(r) + \frac{1}{12} m(r))$$

$$p(r) = p_{\rm tr}(r) + \overline{p}(r) \qquad \int d^3 r\, p(r) = 0$$

$$p_{\rm tr}(r) = -\frac{M}{4} m(r); \quad \overline{p}(r) = M(\frac{2}{9} A(r) + \frac{1}{36} m(r))$$

- Equations of state: $p_{\rm tr}(r) = -\epsilon_{\rm tr}(r); \quad \overline{p}(r) = \frac{1}{3}\overline{\epsilon}(r)$

- Vortex: $p_{\rm con} = -\epsilon_{\rm con}; \quad \overline{p} \sim \frac{1}{2}\overline{\epsilon}$ constant

SC Vortices and Hadrons

- The common theme of hadrons and SC vortices is the existence of a condensate
- Hadrons: condensates from breaking of conformal and chiral symmetries. SC: Cooper pair condensate from gauge symmetry breaking. They cause condensates.
- Vortices emerge by replacing the paired electrons with unpaired ones which do not interact through the lattice phonons, thus gaining energy of the gap and a negative static pressure because $p = - dE/dV$ ($k = 0$ mode).
- Similarly, hadrons uncondense the quark-antiquark and gluons in their condensates, gaining mass and ¼ of energy and induce a negative static pressure to balance of the dynamic pressure from the quarks and gluons for confinement.

Conjecture on Origin of Cosmological Constant

- How does $p = -\epsilon \left(\omega = -1\right)$ come about?
  - By fiat (Einstein)
  - Condensate
- Cosmos – by analogy with QCD and superconductivity, there could a diffeomorphism or conformal symmetry breaking in quantum gravity, leading to a trace anomaly and, thus, a condensate with negative energy density

$$\langle T^{\mu}_{\mu} \rangle_{\mathrm{Gvac}} < 0$$

- Universe emerges from the true vacuum.

$$\langle T^{\mu}_{\mu} \rangle_{\mathrm{U}} = -\langle T^{\mu}_{\mu} \rangle_{\mathrm{Gvac}}$$

Cf. -- S. K. Blau. E. I. Guendelman. and A. H. Guth. Phys. Rev. 035. 1747 (1987)

- Cosmological constant

$$\Lambda = \frac{1}{32\pi G}\left[\langle T^{\mu}_{\mu} \rangle_{\mathrm{U}} - |\langle T^{\mu}_{\mu} \rangle_{\mathrm{QCD}}| - |\langle T^{\mu}_{\mu} \rangle|_{\mathrm{EW}} \cdots\right]$$

S.Weinberg, RMP 61 (1989)1

$$200\,\mathrm{MeV}/\mathrm{fm}^3 \sim 10^{44}\epsilon_c$$

Emergent Expanding Universe

Explains why $\Lambda > 0$, but not why it is so small compared to the glue condensate

Multiverse, Bubble Universe

Cappadocia, Turkey

Some ascending and some descending

- To see a World in a Grain of Sand and a Heaven in a Wild Flower, Hold Infinity in the palm of your hand and Eternity in an hour.

  -- William Blake

- In a drop of water, all the oceans are reflected. In a speck of dust, infinite worlds.

  -- Zen teaching

Superconductor Vortex	Hadron
F_S – Cost of compensation energy	H_{ta} -- Trace anomaly
F_B -- Magnetic field energy	H_g -- Glue field energy (E^2+B^2)
F_{SC} -- Supercurrent energy	H_q -- Quark energy
Total Electron mass $m_e\langle\bar{\psi}\psi\rangle \sim m_e\langle\psi^\dagger\psi\rangle$	H_σ -- Sigma terms
Negative constant pressure from F_S	Negative constant pressure from trace anomaly
Confinement due to the superconducting condensate	Confinement due to the glue condensate