INSTITUTE for NUCLEAR THEORY

A NEW CLASS OF THREE NUCLEON FORCES

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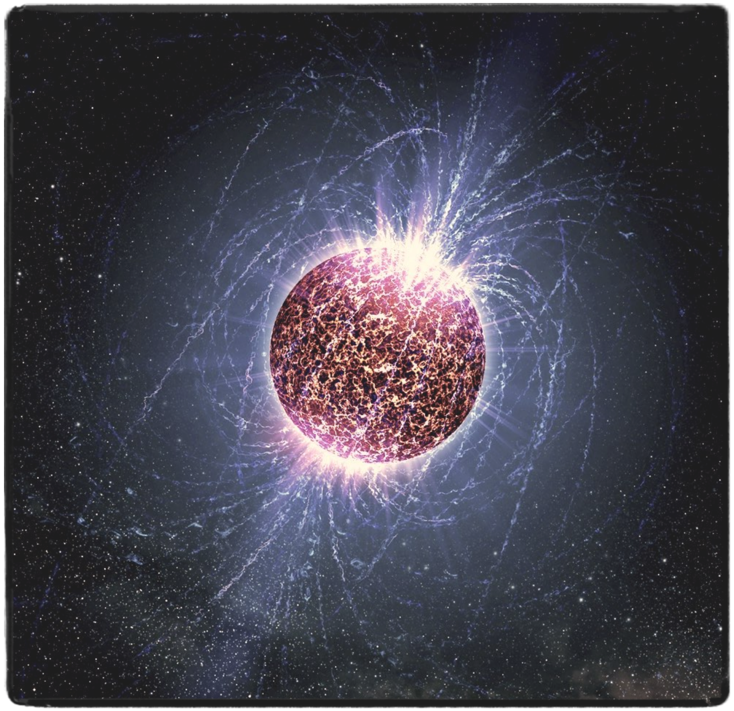
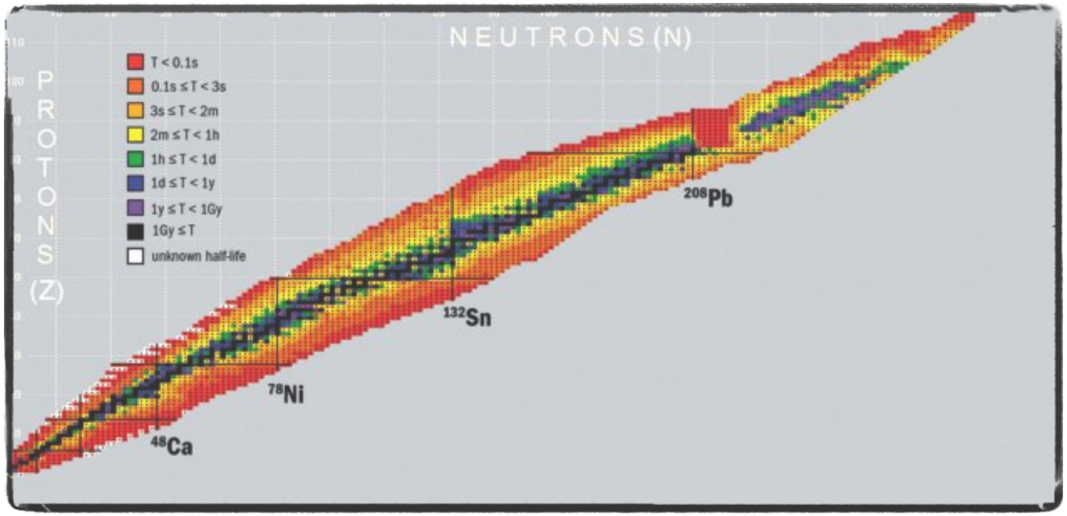
Wouter Dekens
Sanjay Reddy

NUCLEAR FORCES AND THEIR ROLE

Describes nuclear systems

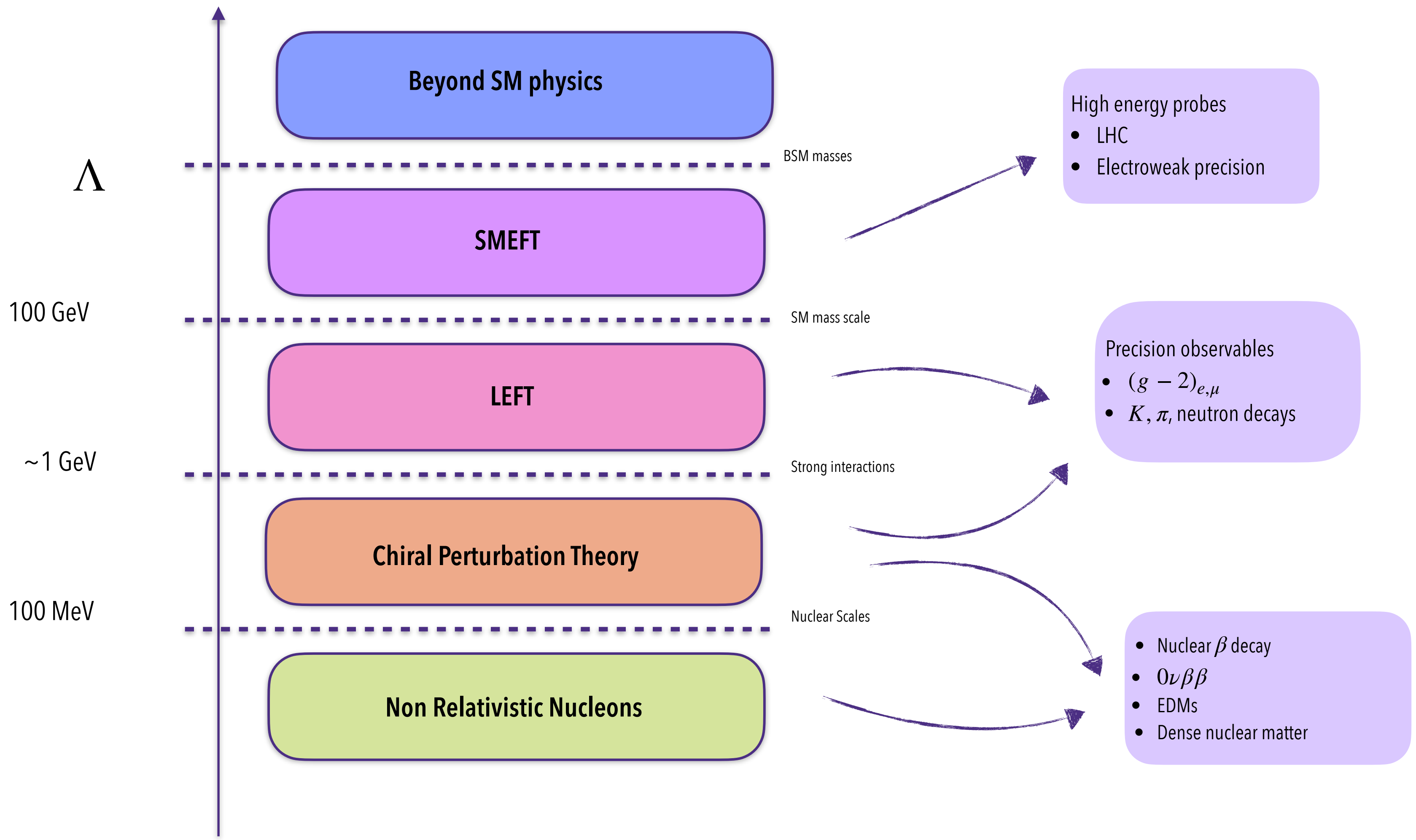
- every property of nuclei
- the equation of state of dense neutron-rich matter in neutron stars,
- probe new physics



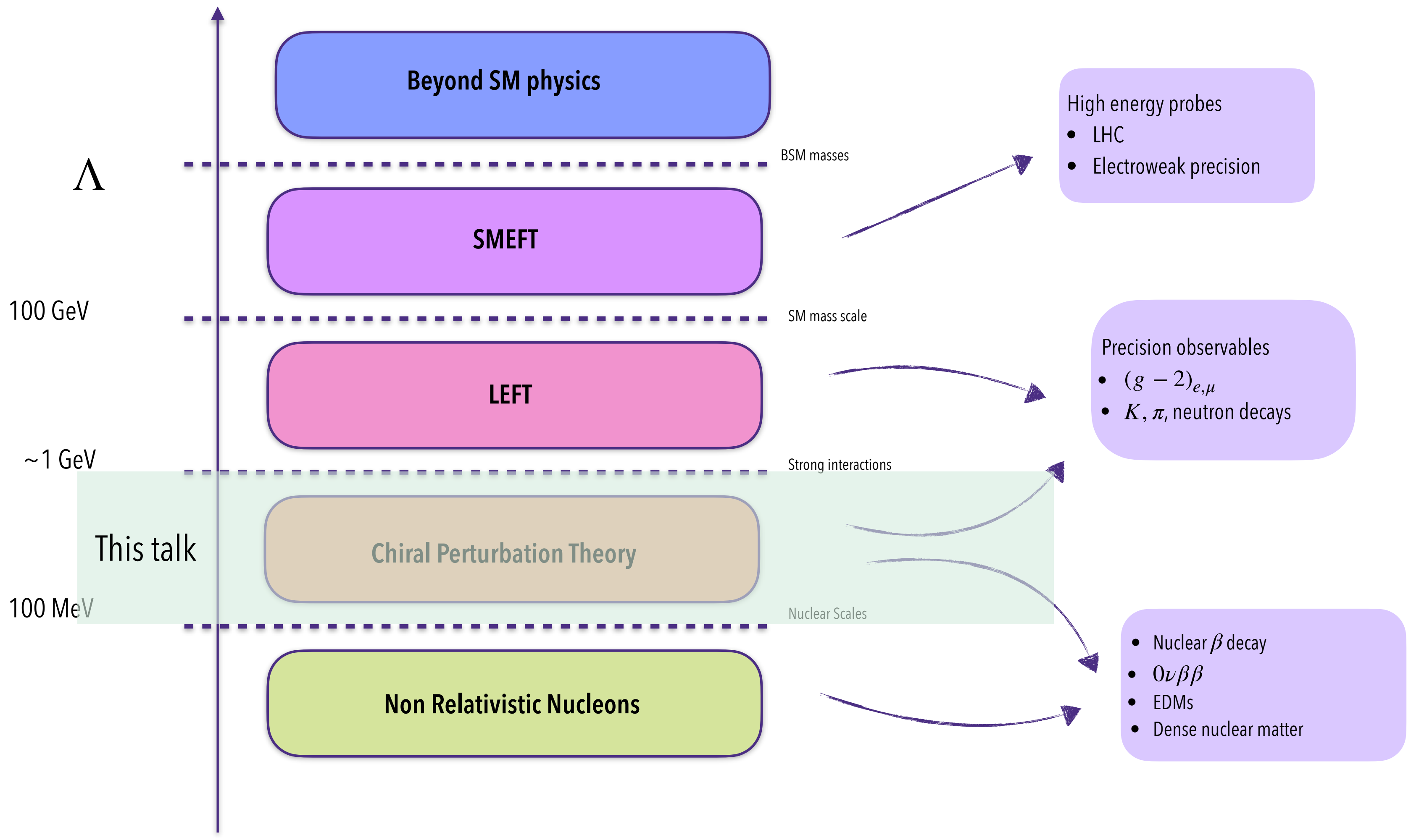


This might be described by Effective Field Theories

EFFECTIVE FIELD THEORY



EFFECTIVE FIELD THEORY



CHIRAL PERTURBATION THEORY

- **Symmetries**: Lorentz invariance, spontaneously broken **chiral symmetry** of QCD Lagrangian

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

- **Degrees of freedom: Goldstone bosons** (π, K, η), and **matter fields** e.g., nucleons (n, p), and other light particles (e, μ, ν, γ)
- **Expansion parameter**
  - $\frac{p}{\Lambda}, \frac{m_\pi}{\Lambda}$,
  - Counting rules $\partial \sim p,\ m_q \sim p^2$

$$\mathscr{L}_{\pi\pi} = \frac{1}{2}\partial_\mu \pi \cdot \partial^\mu \pi - \frac{1}{2}M_\pi^2 \pi^2 + \frac{1}{2F_\pi^2}(\pi \cdot \partial_\mu \pi)(\pi \cdot \partial^\mu \pi) + \ldots$$

$$\mathscr{L}_{\pi N} = N^\dagger(i\partial_0 + \frac{\vec{\nabla}^2}{2m})N - \frac{1}{2F_\pi^2}N^\dagger \tau \cdot (\pi \times \dot{\pi})N + \frac{g_A}{2F_\pi}N^\dagger \vec{\sigma}\tau \cdot \vec{\nabla}\pi N + \ldots$$

$$\mathscr{L}_{NN} = -\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger \vec{\sigma} N)(N^\dagger \vec{\sigma} N) + \ldots$$

Coefficients fixed from pion decay, $F_\pi \simeq 92$ MeV

Infinite sum

POWER COUNTING

Weinberg 1979, 1991

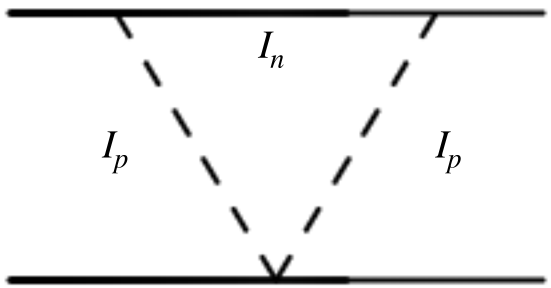
Amplitude

$$\mathscr{A} \sim \left(\frac{p}{\Lambda}\right)^{\nu}$$

$$\nu = \sum_i V_i d_i - 2I_p - I_n + 4L$$

I_n - internal nucleon lines,
I_p - internal pion lines,
V_i - vertices of type I,
d_i - number of derivatives
L - loop

Example



MANY BODY POTENTIAL

Weinberg's idea: Use chiral EFT to calculate $V_{eff} = \sum (all\ irreducible\ diagrams)$

Irreducible diagram = diagram that is not generated through iterations in the dynamical equation

$$H|\psi> = E|\psi> \qquad H = \sum_i T_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \dots$$

V is symmetric under permutation $V_{ij} = V_{ji}$, and invariant under all QCD symmetries

Generate observables by solving the Lippmann-Shwinger equation:

$$T = V_{eff} + V_{eff}\, G_0\, T$$

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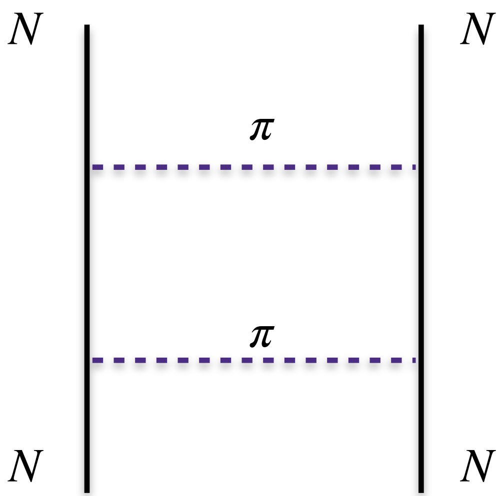
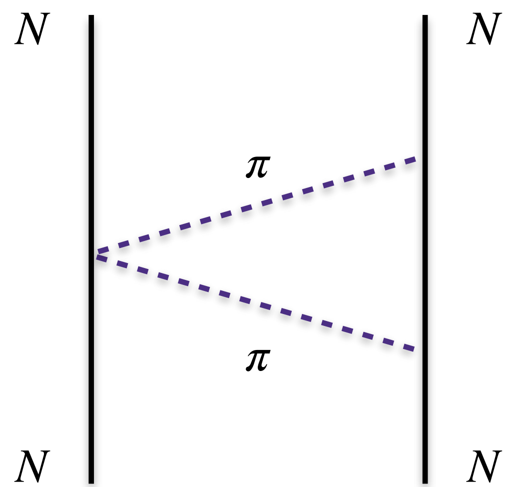
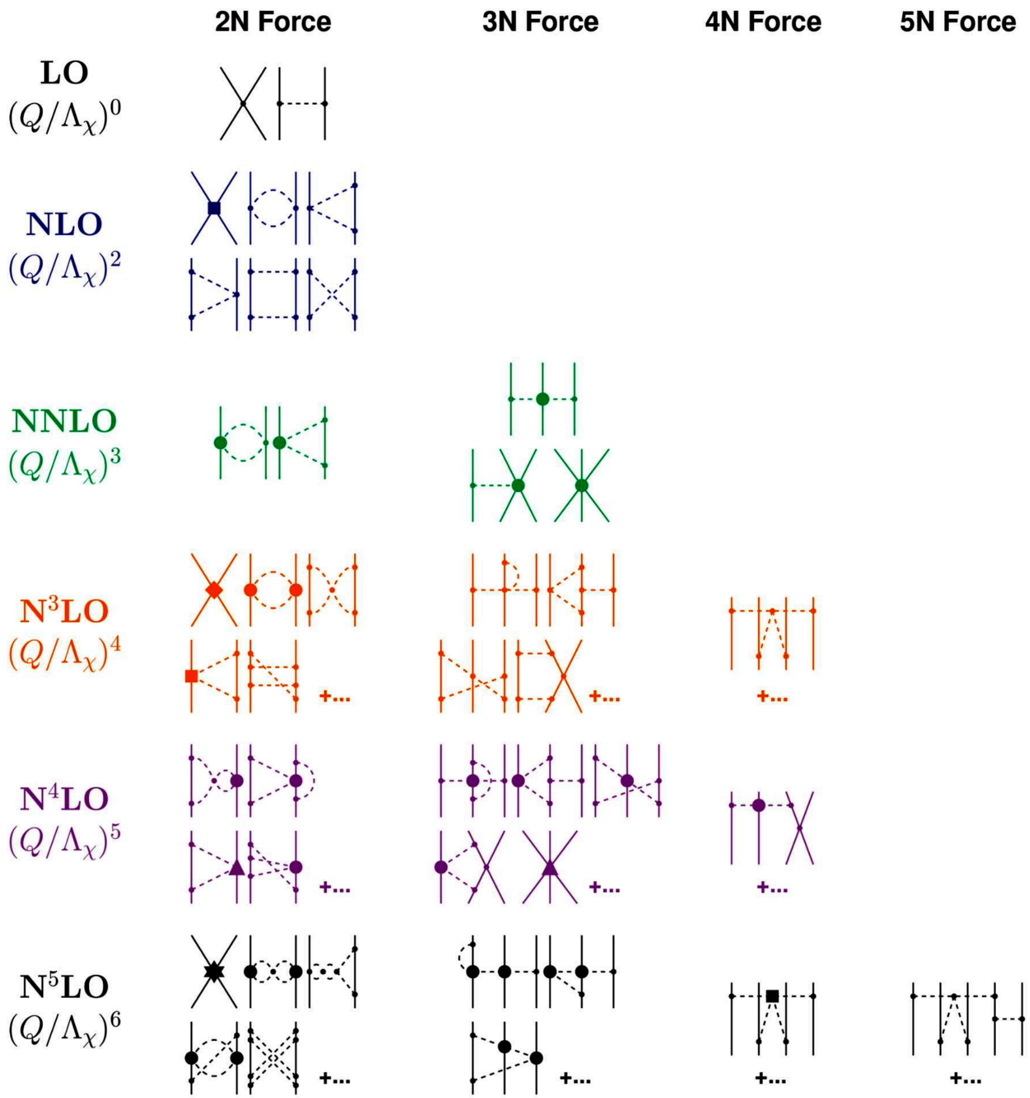
Reducible



Irreducible

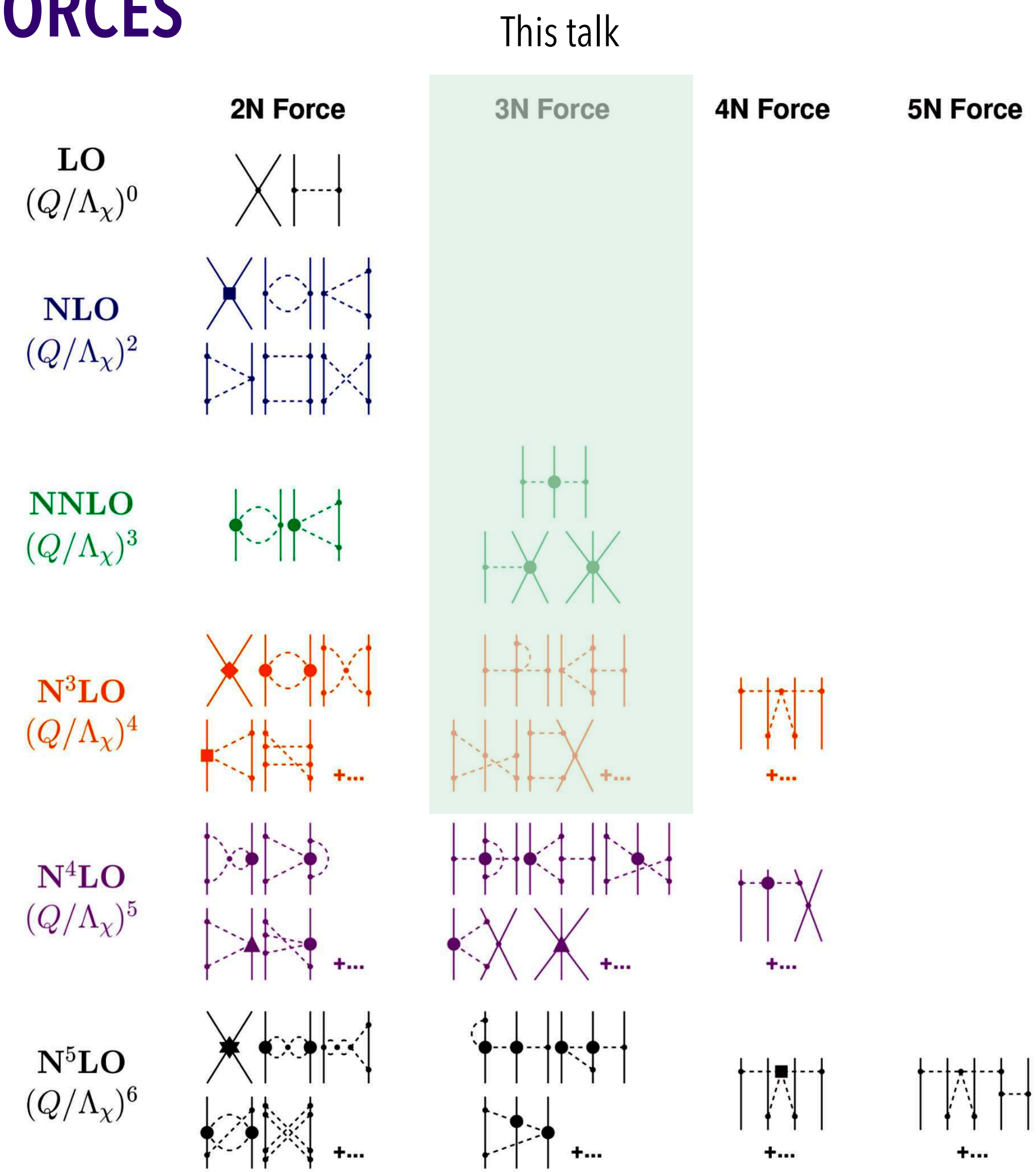


NUCLEAR FORCES



Hierarchy of nuclear forces up to N5 LO in ChiPT. Solid lines represent nucleons and dashed lines pions. Entem, Machleidt, Y. Nosyk, (arXiv:1703.05454)

NUCLEAR FORCES



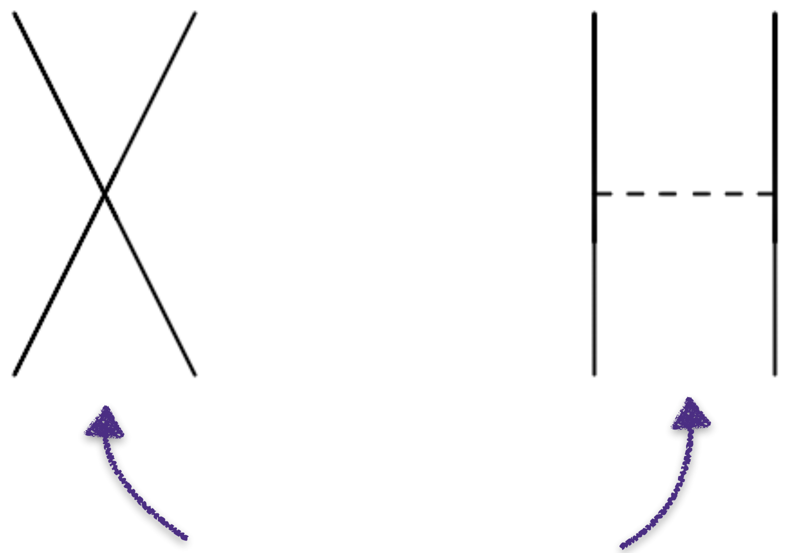
Hierarchy of nuclear forces up to N5 LO in ChiPT. Solid lines represent nucleons and dashed lines pions. Entem, Machleidt, Y. Nosyk, (arXiv:1703.05454)

TWO BODY POTENTIAL

$$\nu = \sum_i V_i d_i - 2I_p - I_n + 4L$$

$$M_N \sim Q \sim m_\pi$$

Leading order:



$$V_0 = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{g_A^2}{4F_\pi^2} \frac{(\vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)}{\vec{q}^2 + m_\pi^2}$$

LO TWO BODY POTENTIAL

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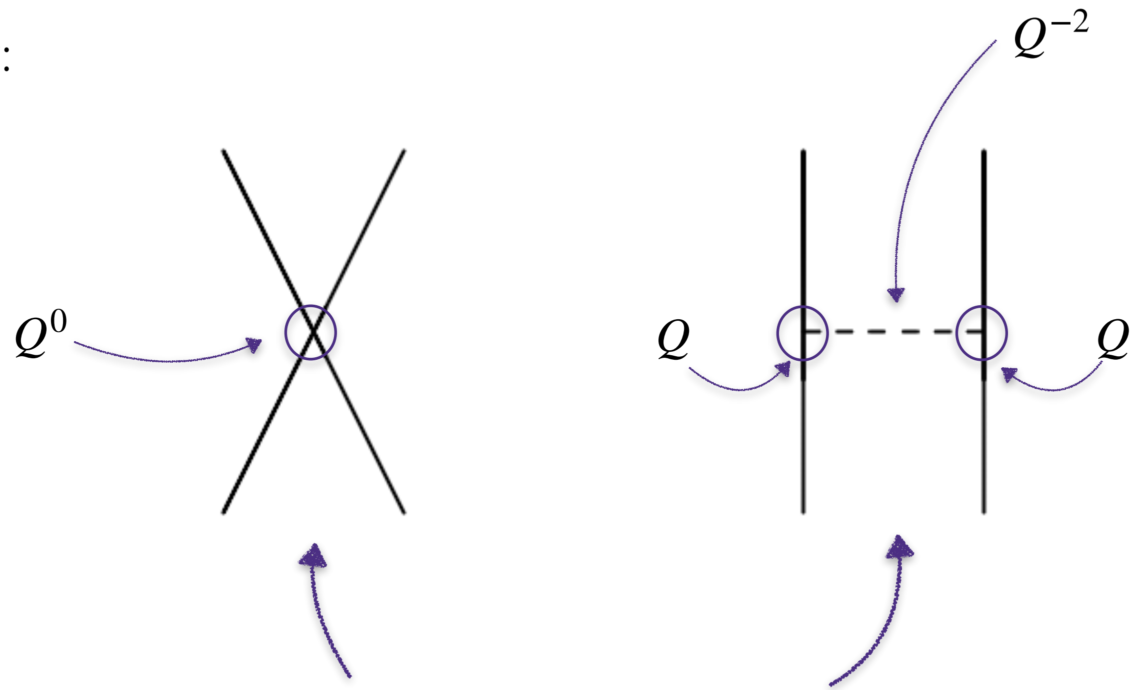
Leading order:



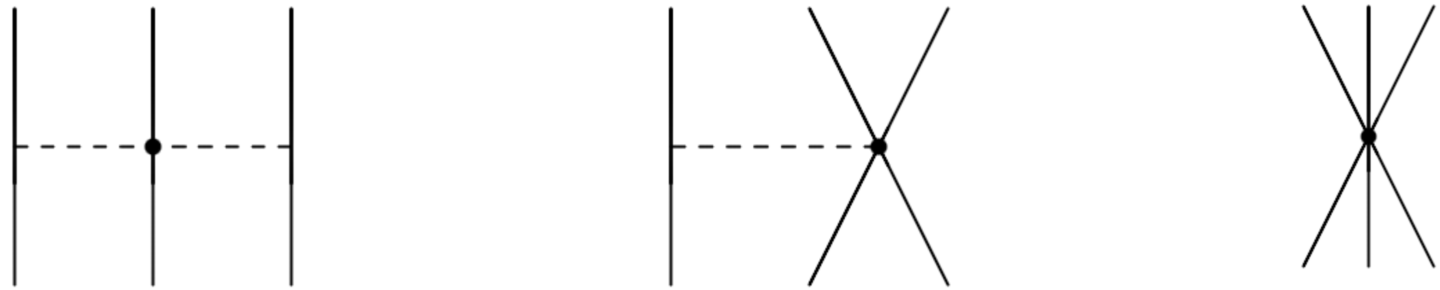
$\nu = 0$

$$V_0 = C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2 - \frac{g_A^2}{4F_\pi^2} \frac{(\vec{q} \cdot \vec{\sigma}_1 \vec{q} \cdot \vec{\sigma}_2)(\vec{\tau}_1 \cdot \vec{\tau}_2)}{\vec{q}^2 + m_\pi^2}$$

$$C_0 = C_S - 3C_T \qquad \sim \frac{4\pi}{m_N} \frac{1}{Q} \quad \sim \quad \frac{1}{F_\pi^2}$$

THREE BODY POTENTIAL N2LO

$$\nu = \sum_i V_i d_i - 2I_p - I_n + 4L$$

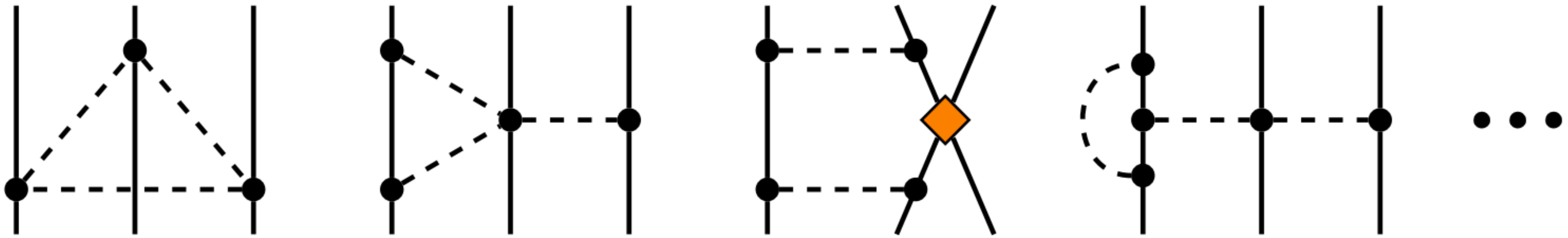


$c_1,\ c_3,\ c_4$ c_D c_E

Neutron matter   **Symmetric matter**

- Interaction range increases with the number of pions
- LEC's determined by
  - c_i's known well and determined by **pion-nucleon** scattering data. Independent of multi nucleon information. **Errors are small** because there are no 3NF short-distance contributions.
  - $c_D,\ c_E$ from Nd scattering, light nuclei and tritium β decay

THREE BODY POTENTIAL N3LO

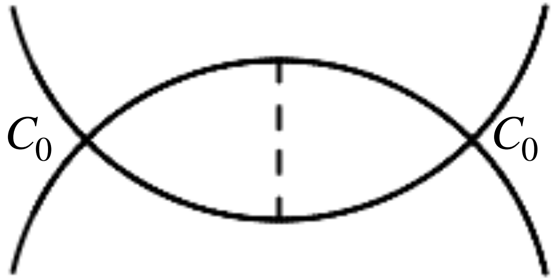


- Consists of
    - Loop diagrams with LO vertices
    - Tree graphs involving relativistic corrections

- ***No new LECs***

Bernard, Epelbaum, Krebs, Meissner '08,'11; Ishikawa, Robilotta '07,

ISSUES WITH POWER COUNTING



For ${}^1S_0$ channel:

$$\sim m_\pi^2 C_0^2 \left(\frac{1}{\epsilon} + \log \mu^2 \right)$$

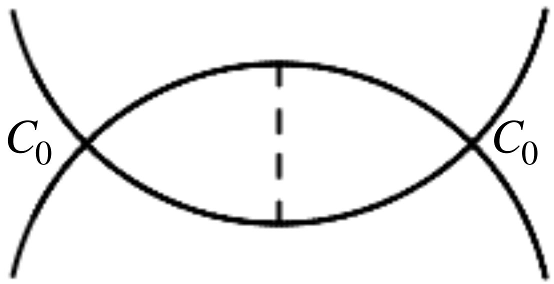
$$\mathscr{L} = D_2 \bar{N} N \bar{N} N < \chi_+ > = D_2 \bar{N} N \bar{N} N m_\pi^2 \left(1 - \frac{1}{2F_\pi} \pi^a \pi^b \delta^{ab} + \mathcal{O}\left(\frac{\pi^4}{F_\pi^4} \right) \right)$$

Weinberg implies: $D_2 \sim C_0^2 \frac{1}{\Lambda^2}$

To absorb the divergencies: $D_2 \sim C_0^2$

D. Kaplan, M. Savage, M. Wise
(ArXiv: 9605002)

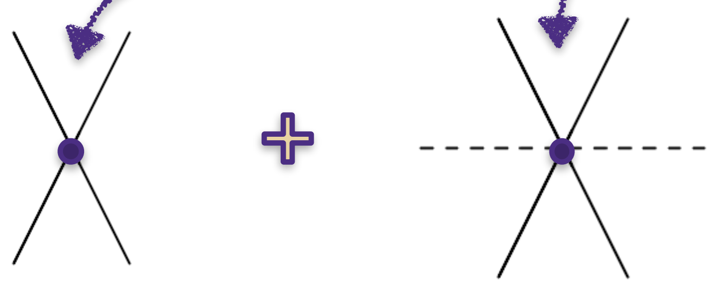
ISSUES POWER COUNTING



For $^1S_0$ renormalization requires:

$$\frac{d}{d\ln\mu}\left[\frac{m_\pi^2 D_2}{\tilde{C}_0^2}\right] = \frac{g_A^2 m_\pi^2 m_N^2}{64\pi^2 f_\pi^2},$$

$$\mathscr{L} = D_2 \bar{N} N \bar{N} N < \chi_+ > = D_2 \bar{N} N \bar{N} N m_\pi^2 \left(1 - \frac{1}{2F_\pi}\pi^a \pi^b \delta^{ab} + \mathscr{O}\left(\frac{\pi^4}{F_\pi^4}\right)\right)$$

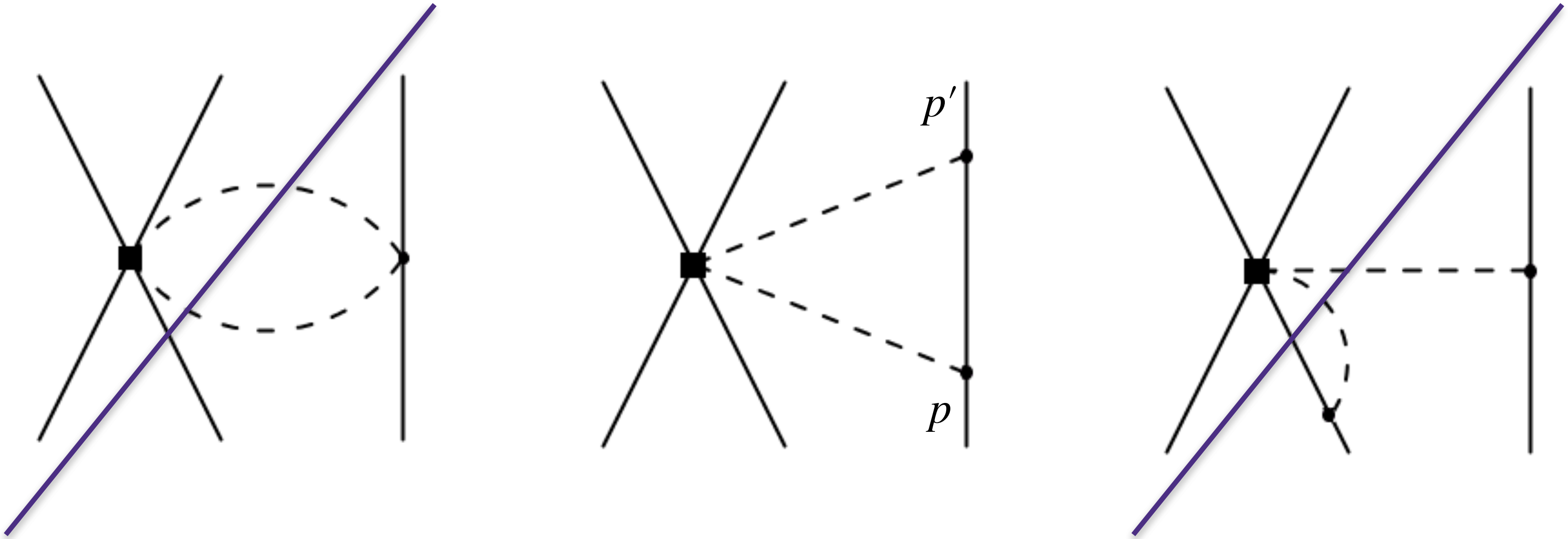


$m_\pi^2 D_2$ $\qquad$ $\frac{m_\pi^2}{2F_\pi^2} D_2$

D. Kaplan, M. Savage, M. Wise
(ArXiv: 9605002)

V. Cirigliano, M. D, W. Dekens, S. Reddy
(ArXiv:2411.00097)

3NF DUE TO D_2 operator

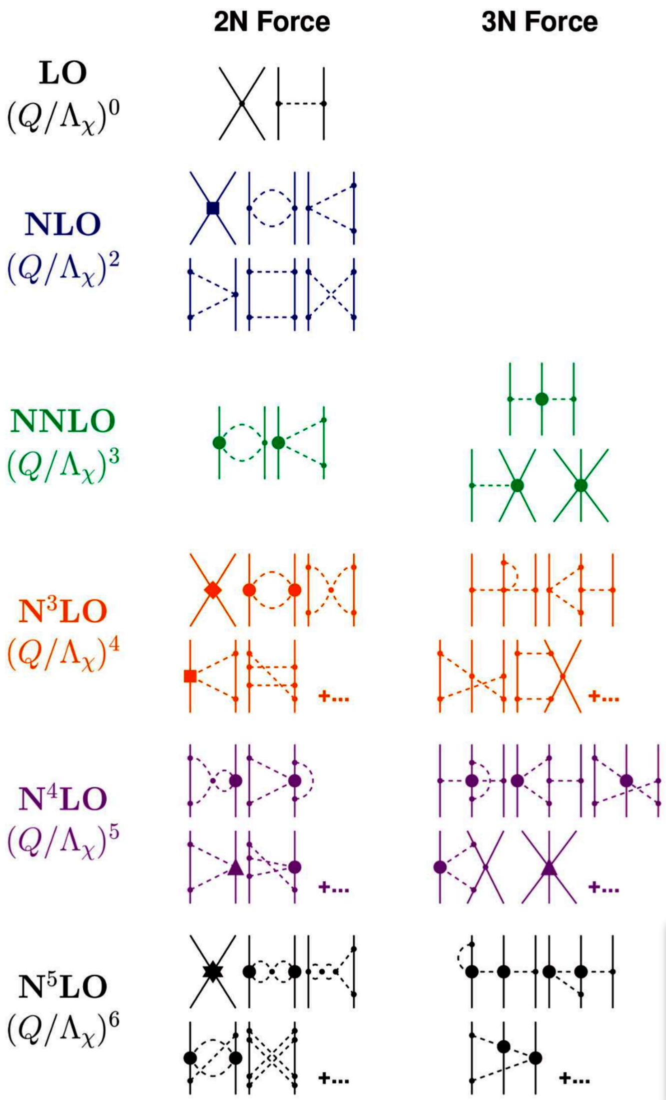


$\vec{q} = \vec{p}' - \vec{p}$

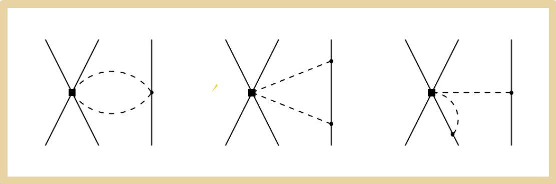
$$V(\vec{q}) = \frac{9 g_A^2 D_2 m_\pi^3}{128 \pi^2 f_\pi^4} \, \pi \, \mathcal{J}\left(\frac{\vec{q}^{\,2}}{4 m_\pi^2}\right) ,$$

$$\mathcal{J}(b) = \frac{2}{3}\left[1 + \left(\frac{1}{2\sqrt{b}} + \sqrt{b}\right)\tan^{-1}(\sqrt{b})\right] .$$

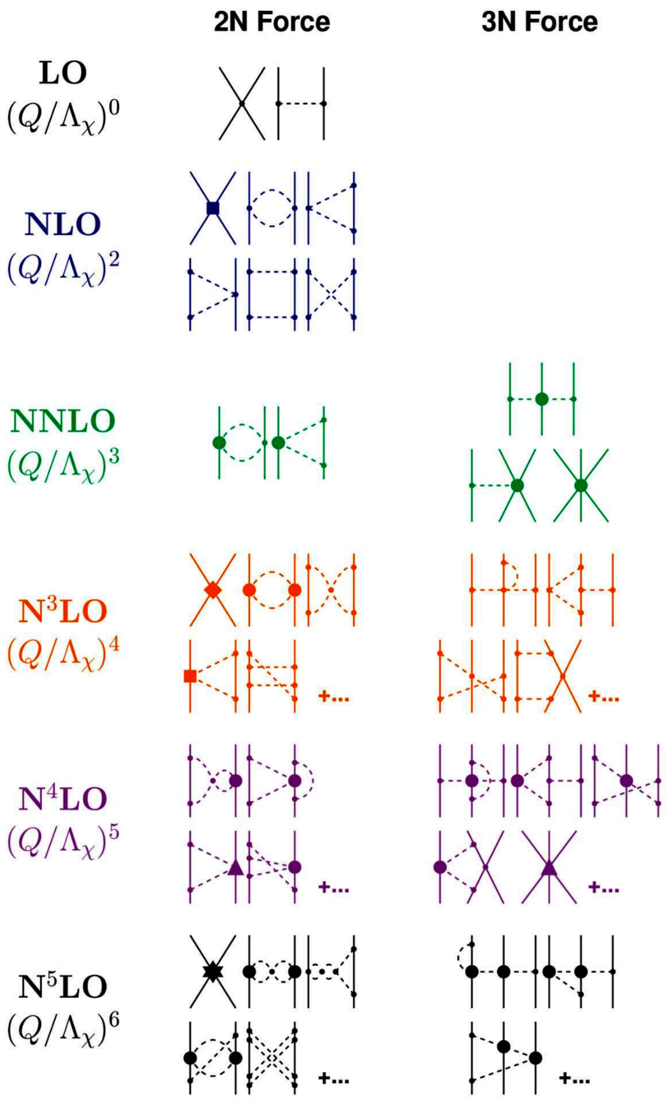
NUCLEAR FORCES



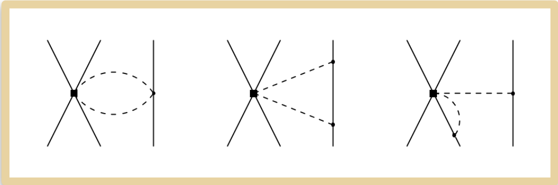
In Weinberg power counting scheme, diagrams with D_2 operator appear in N^5LO



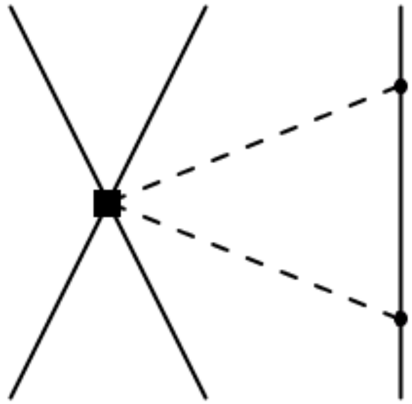
NUCLEAR FORCES



Now they appear in N3LO!



3NF DUE TO D_2 operator



$$V(\vec{q}) = \frac{9 g_A^2 D_2 m_\pi^3}{128 \pi^2 f_\pi^4} \, \pi \, \mathscr{I}\left(\frac{\vec{q}^{\,2}}{4 m_\pi^2}\right),$$

$$\mathscr{I}(b) = \frac{2}{3}\left[1 + \left(\frac{1}{2\sqrt{b}} + \sqrt{b}\right)\tan^{-1}(\sqrt{b})\right].$$

Enhacement by π

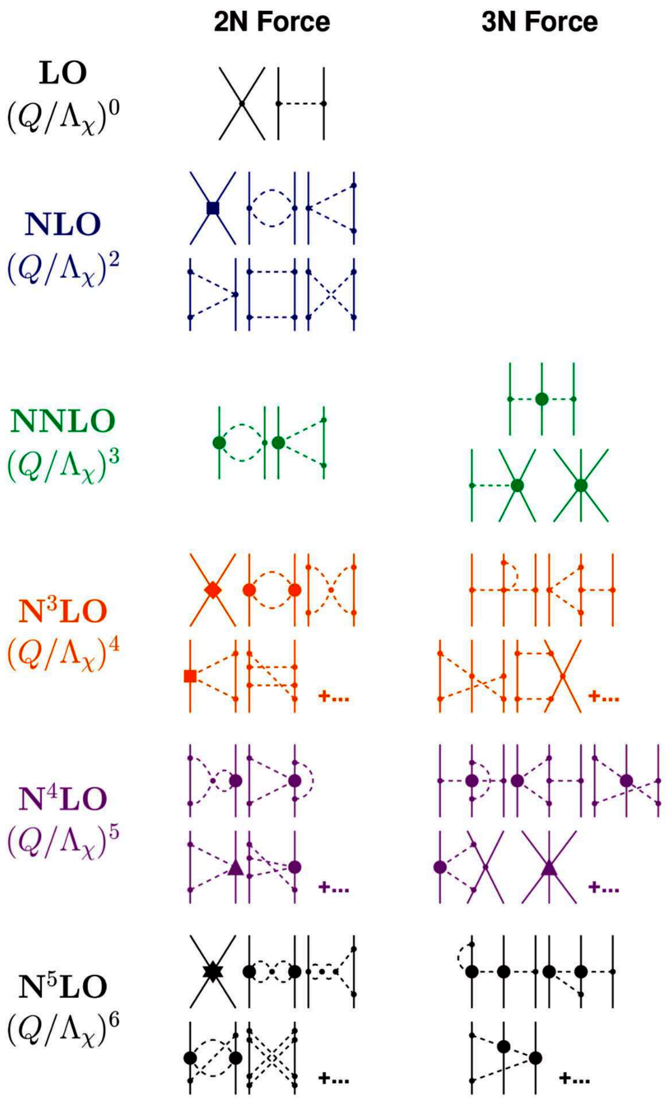
NUCLEAR FORCES

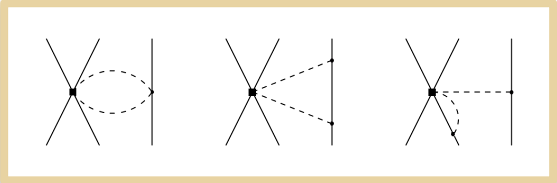
Expansion parameter

$$\frac{m_\pi}{\Lambda}$$

m_π ~ 100 MeV

Λ ~ 1000 MeV





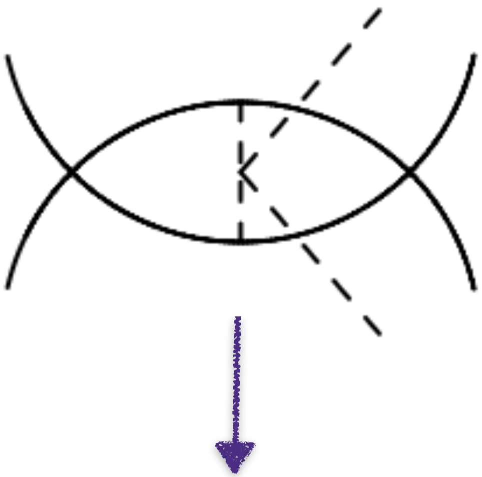
N3LO $\times \pi$

Effectively this contributions are similar in size to usual **NNLO** three body force

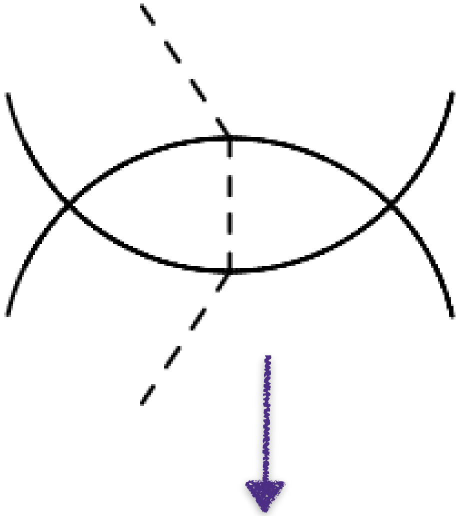
F_2 and E_2 operators

V. Cirigliano, M. D, W. Dekens, S. Reddy
(ArXiv:2411.00097)

E_2, & F_2 are enhanced for the same reason as D_2



$F_2\ \vec{q}^2$



$E_2\ \omega$

$$\mathscr{L} = \frac{1}{4}\left[E_2\langle(v\cdot u)^2\rangle + F_2\langle u\cdot u - (v\cdot u)^2\rangle\right]\ (N^T P_i N)^\dagger\,(N^T P_i N)$$

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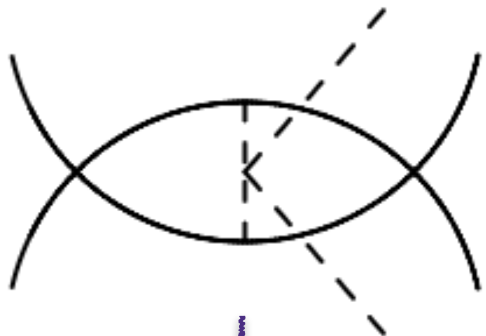
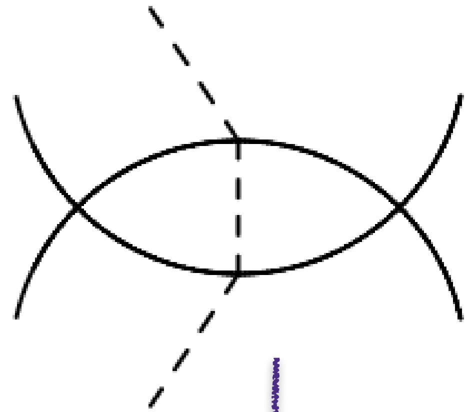
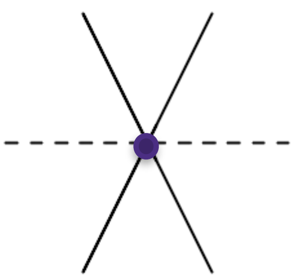


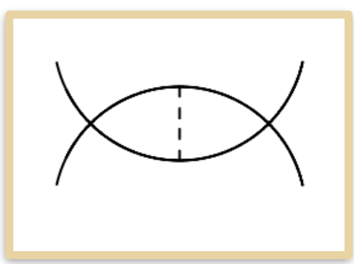
$F_2\ \vec{q}^2$



$E_2\ \omega$

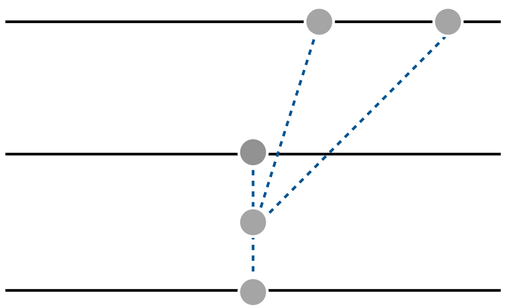
Required counter-term





$D_2 m_\pi^2$

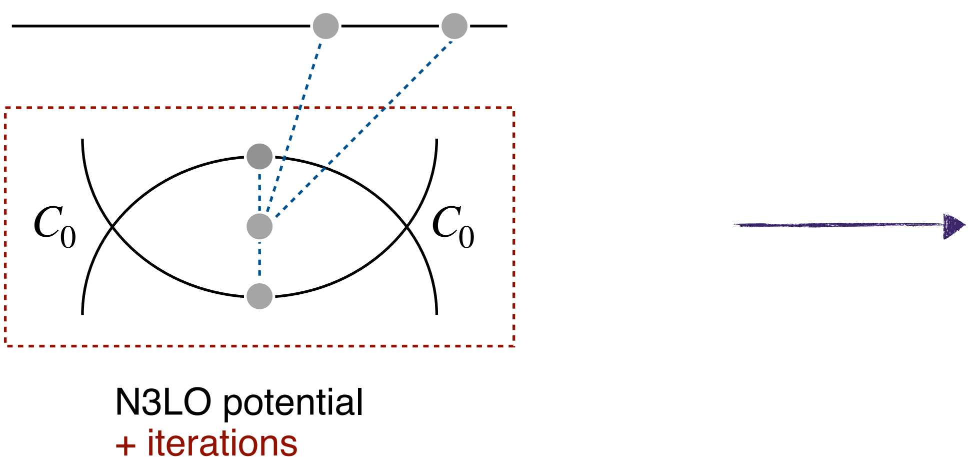
CONNECTION TO USUAL N3LO GRAPHS

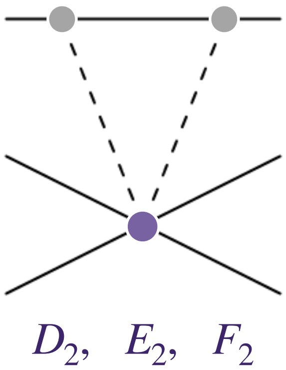


N3LO potential

Bernard, Epelbaum, Krebs, Meissner '08,'11; Ishikawa, Robilotta '07,

CONNECTION TO USUAL N3LO GRAPHS





- Part of the 'conventional' N3LO potential is connected to D_2, E_2, F_2
  - Generates the divergent diagrams that induce D_2, E_2, F_2
  - Need to be considered simultaneously for a consistent calculation

Bernard, Epelbaum, Krebs, Meissner '08,'11; Ishikawa, Robilotta '07,

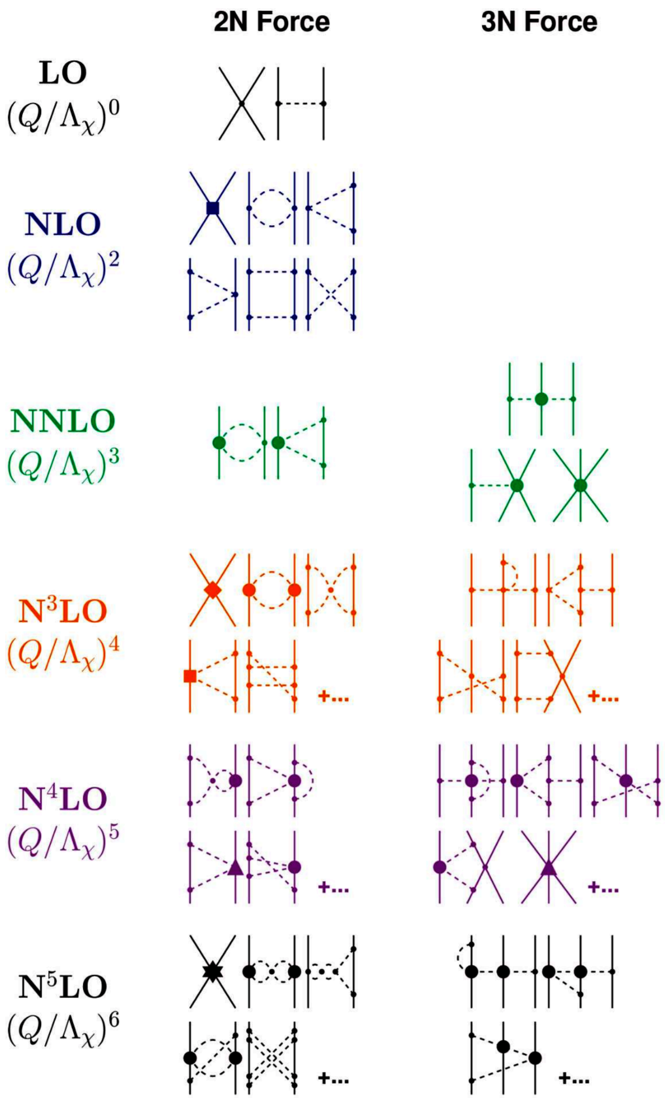
NUCLEAR FORCES

$$\frac{d}{d\ln\mu}\left[\frac{X}{\tilde{C}_0^2}\right] = \gamma_X \left(\frac{m_N}{4\pi f_\pi}\right)^2,$$

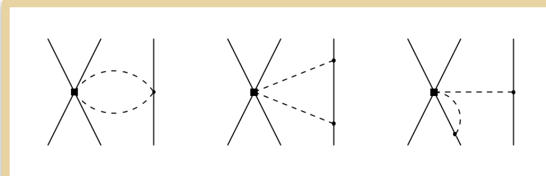
with $X \in \{D_2, E_2, F_2\}$, $\gamma_{E_2} = -(1+g_A^2)/3$,

$\gamma_{F_2} = -g_A^2/3.$

$\gamma_{D_2} = g_A^2/4$



F_2, D_2 and E_2



What are their sizes?

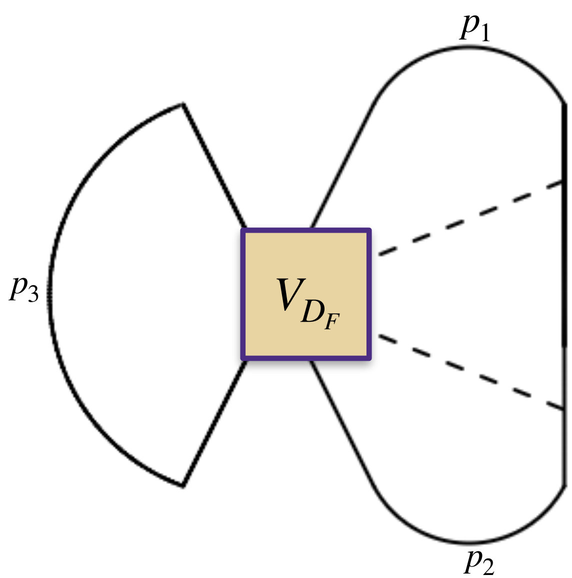
$$D_2, F_2(\mu) \approx \frac{g_a^2 m_\pi^2 m_N^2}{64\pi^2 f_\pi^2} C_0(\mu)^2$$

$$|D_2, F_2| \sim 1/(5 f_\pi^4)$$

ENERGY PER PARTICLE

The interaction energy density is obtained by calculating the matrix element of the potential (Hartree-Fock)

$$\langle \mathscr{H}(0) \rangle = \int_{\vec{p}_1,\vec{p}_2,\vec{p}_3} \theta(k_f - |\vec{p}_1|)\theta(k_f - |\vec{p}_2|)\theta(k_f - |\vec{p}_3|) \times \Big[V^{ijk}_{ijk}(0,0,0) - V^{ikj}_{ijk}(0,\vec{p}_{32},\vec{p}_{23}) + V^{jki}_{ijk}(\vec{p}_{21},\vec{p}_{32},\vec{p}_{13}) + V^{kij}_{ijk}(\vec{p}_{31},\vec{p}_{12},\vec{p}_{21}) - V^{kji}_{ijk}(\vec{p}_{31},0,\vec{p}_{13}) - V^{jik}_{ijk}(\vec{p}_{21},\vec{p}_{12},0) \Big]$$



ENERGY PER PARTICLE

V. Cirigliano, M. D, W. Dekens, S. Reddy
(ArXiv:2411.00097)

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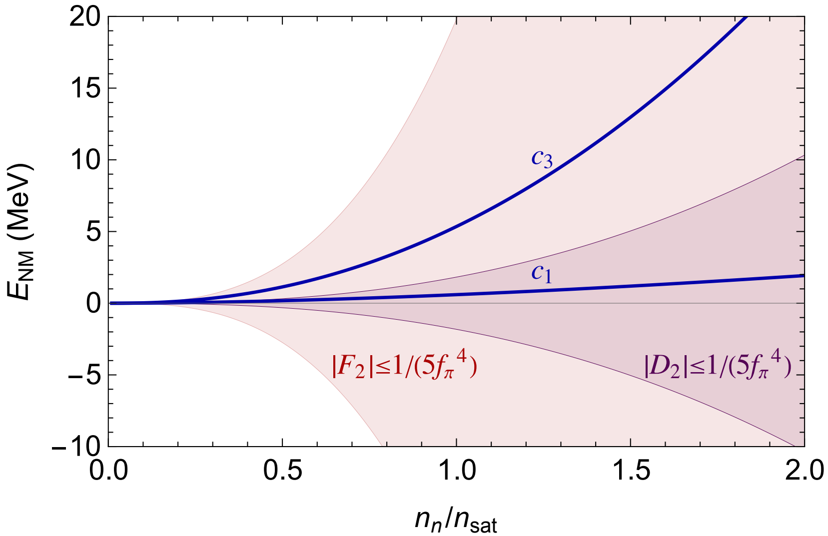
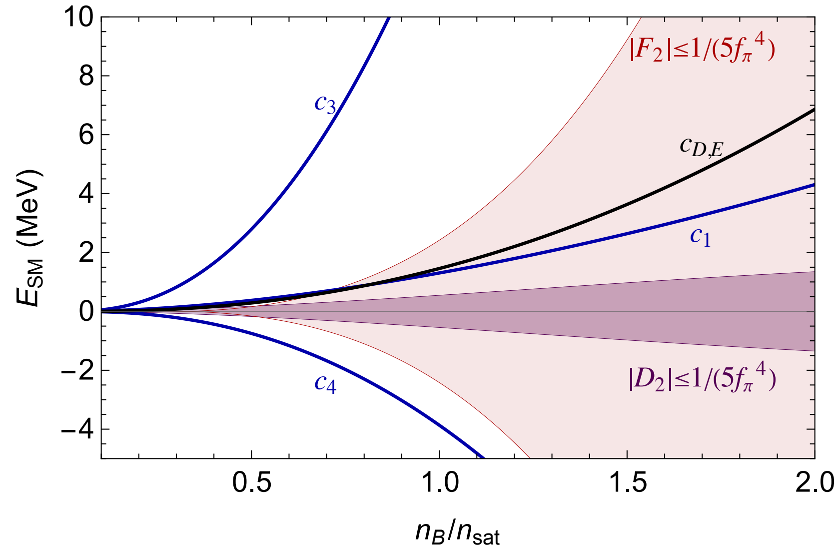




D_2 and F_2 contributions to the energy per particle in **neutron** matter as a function of the density.

D_2 and F_2 contributions to the energy per particle in **symmetric** matter as a function of the density.

$$|D_2|, |F_2| < 1/(5F_\pi^4)$$

PRESSURE

V. Cirigliano, M. D, W. Dekens, S. Reddy
(ArXiv:2411.00097)

$$P = n^2 \frac{\partial(E/N)}{\partial n}$$

Chiral EFT NNLO predicts

- $P(n_{sat}) = 3.1 \pm 0.5$ MeV/fm^3

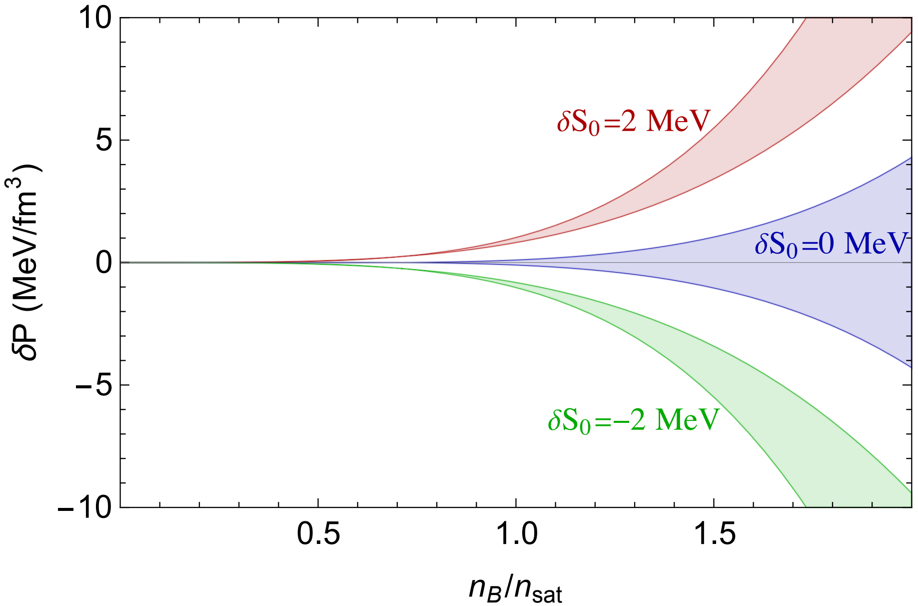
C. Drischler, R.J. Furnstahl, J.A, Melendez, D.R. Phillips (2021)

- $P(n_{sat}) = 2.2 \pm 0.4$ MeV/fm^3

I. Tews, R. Somasundaram, D. Lonardoni, H.Göttling, R. Seutin, J. Carlson, S. Gandolfi, K. Habeler, A. Schwenk (2024)

Estimations

$$\delta P_{3NF} = \left[0.7\left(\frac{D_2}{D_2^{ref}}\right) + 8.8\left(\frac{F_2}{F_2^{ref}}\right)\right] \frac{\text{MeV}}{fm^3}$$



$D_2 + F_2$ contribution to the pressure of neutron matter

How to determine D_2, F_2

From theory:

- First principles determination using Lattice QCD
  - Currently only calculations at unphysical m_π

e.g. Beane, Bedaque, Orginos, Savage, '06; Beane et al '15;

From experiment:

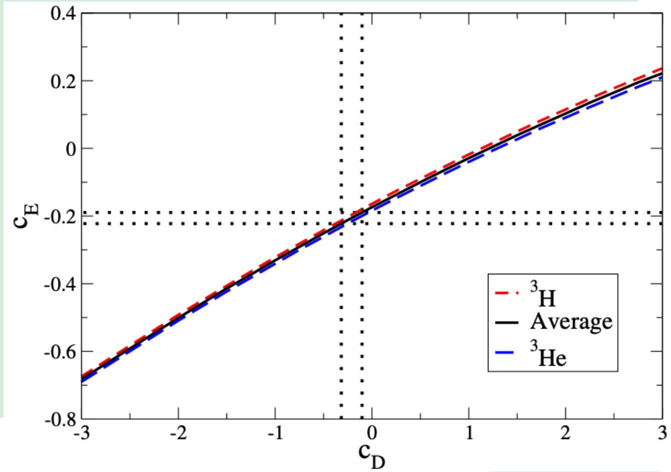
- Determine D_2, F_2 together with $c_{D,E}$ from
  - Light systems:
    - Nd scattering
    - Binding energies
    - tritium β decay

with W. Dekens, C. Drischler, M. Kumamoto, S. Reddy

- Properties of dense matter
- Properties of neutron stars

with C. Armstrong, W. Dekens, I. Tews, S. Reddy

- π-nucleus scattering

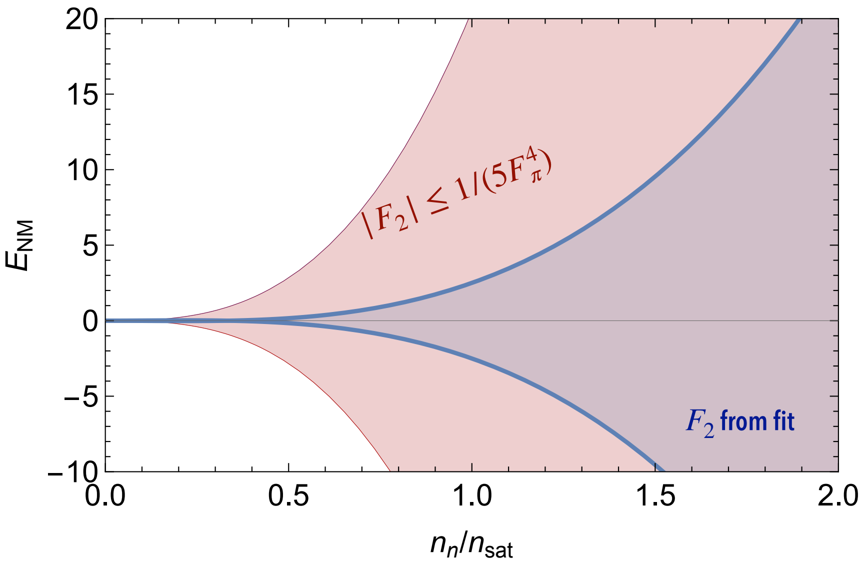


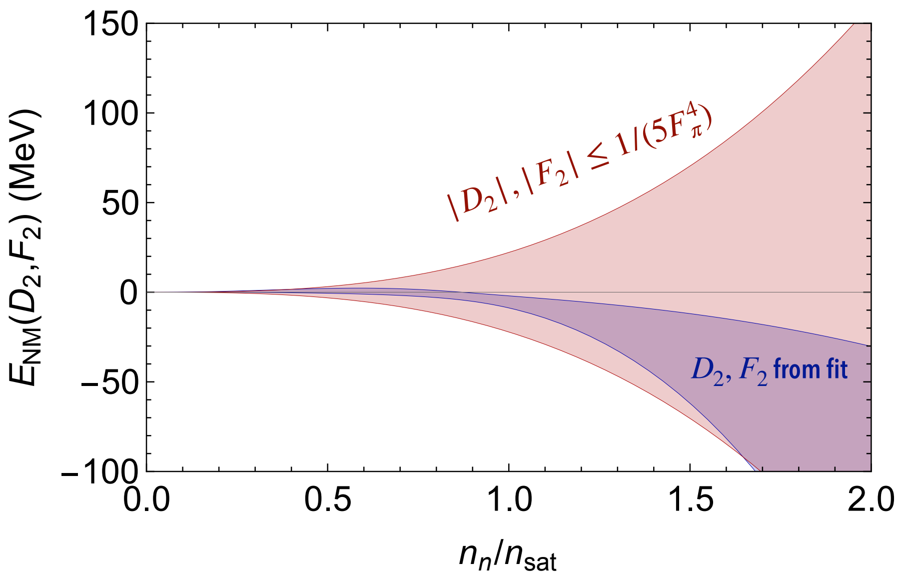
Gazit et al '09

FUTURE WORK: CONSTRAINING D_2, AND F_2

Constraint the operators

- Combines HF estimates of 3-nucleon force with 2nucleon contributions
- Fits to properties of dense matter near saturation





Total contributions to the energy per particle in **neutron** matter as a function of the density.

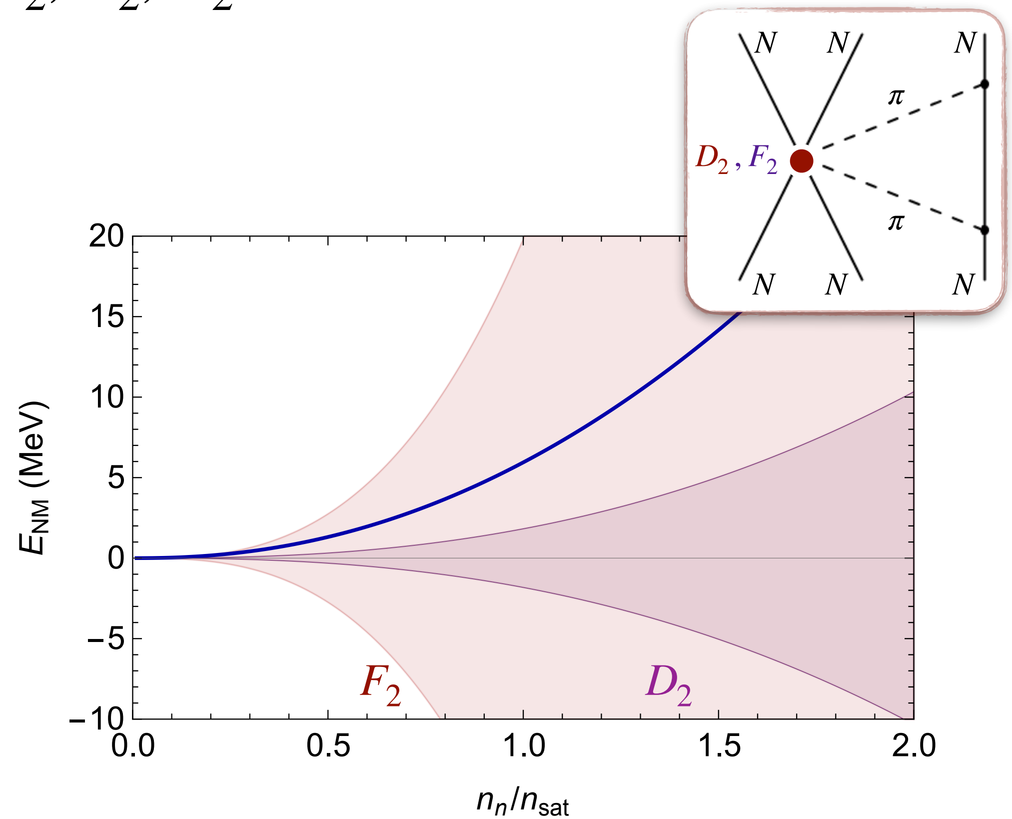
M. D, C. Drischler, W. Dekens, M. Kumamoto, S. Reddy (Preliminary)

CONCLUSIONS

1. We identified a new class of Three Nucleon-Forces

2. We estimate their contribution to the energy of neutron and nuclear matter

3. Future directions: constraining $D_2,\ F_2,\ E_2$



INSTITUTE for
NUCLEAR THEORY

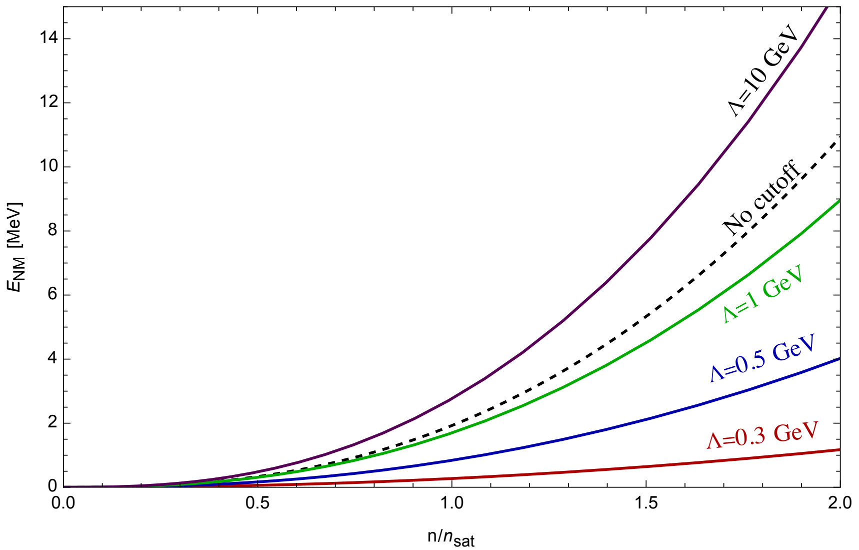
THANK YOU!

New class of three-nucleon forces

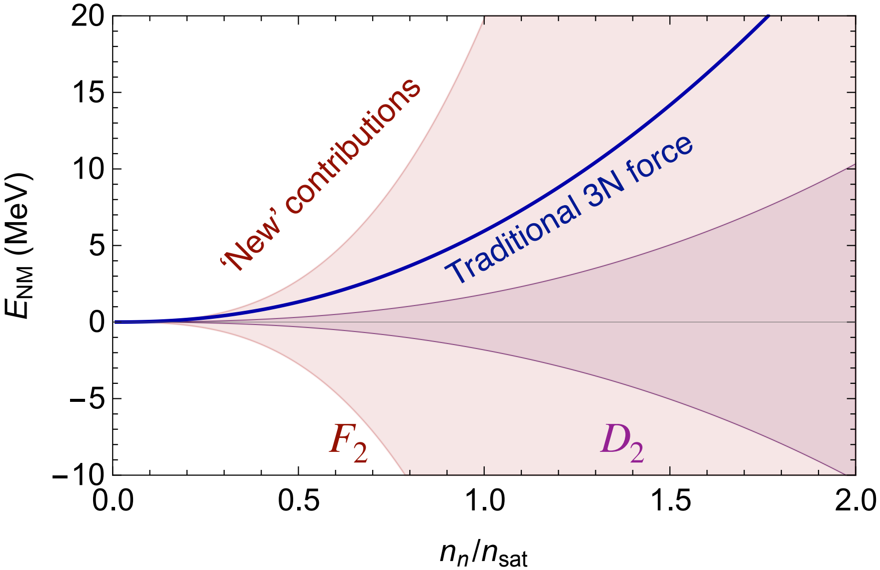
Different regulators

$|D_2| \leq 1/(5\,F_\pi^4)$,
$|F_2| \leq 1/(5\,F_\pi^4)$

Long-range



Dim reg



Long-range regulator

- Picks out the long-range part of the potential
- Reduces the contributions by factor of a few

$$X_{\rm long-range}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty}\frac{d\mu}{\mu^3}\,\rho_X(\mu)\left(\frac{q^4}{\mu^2+q^2}+C_1(\mu)+C_2(\mu)q^2\right)e^{-\frac{\mu^2+q^2}{2\Lambda^2}}\,,$$

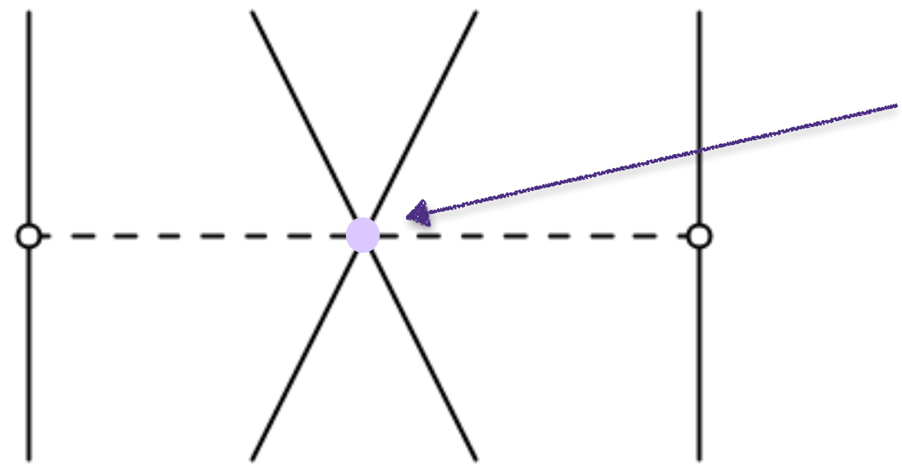
$$C_1(\mu) = \frac{2\Lambda\mu^2\left(2\Lambda^4-4\Lambda^2\mu^2-\mu^4\right)+\sqrt{2\pi}\mu^5 e^{\frac{\mu^2}{2\Lambda^2}}\left(5\Lambda^2+\mu^2\right)\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}\Lambda}\right)}{4\Lambda^5}\,,$$

$$C_2(\mu) = -\frac{2\Lambda\left(6\Lambda^6-2\Lambda^2\mu^4-\mu^6\right)+\sqrt{2\pi}\mu^5 e^{\frac{\mu^2}{2\Lambda^2}}\left(3\Lambda^2+\mu^2\right)\mathrm{erfc}\left(\frac{\mu}{\sqrt{2}\Lambda}\right)}{12\Lambda^7}\,,$$

$$\rho_{\mathcal{I}}(\mu) = \frac{1}{M_\pi}(2M_\pi^2-\mu^2)\frac{\pi}{4\mu},$$

Epelbaum, Krebs, Reinert' 18; Epelbaum & Krebs, '23,'23

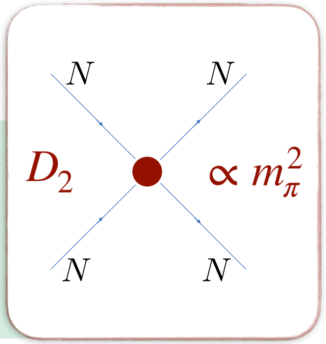
FOUR NUCLEON FORCE



$$i\mathscr{T} = -\, i4(\bar{N}N)^4 \, \frac{d_2 g_A^2}{F^4} \, \frac{S \cdot q_2 \tau^d \ S \cdot q_1 \tau^a}{((p_4 - p_4')^2 - m_\pi^2)((p_1 - p_1')^2) - m_p i^2)} \,,$$

Effects on BSM scenarios

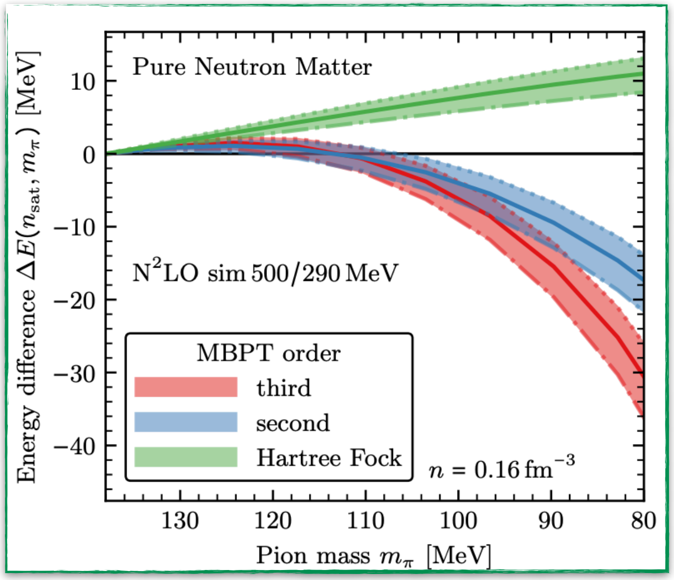
- D_2 induces m_π dependence of NN interactions



BSM scenarios can affect the quark masses

- Variations of fundamental constants
  - Lead to time dependent $m_q(t)$
- Axion scenarios
  - Axion could condense in dense matter like neutron stars
  - Would change $m_\pi(\theta = 0) \rightarrow m_\pi(\theta = \pi) \simeq 80\,\mathrm{MeV}$

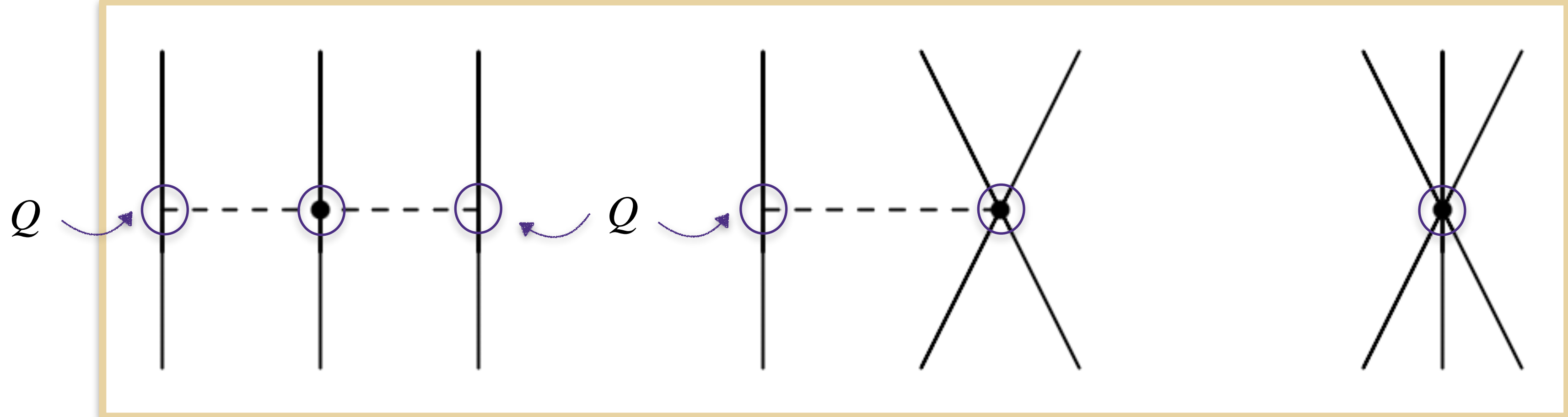
- Can be probed through their effect on the nuclear force
  - Requires m_π dependence of the nuclear force and D_2



Kumamoto, Huang, Drischler, Baryakhtar, Reddy, '24

THREE BODY POTENTIAL

$$\nu = \sum_i V_i d_i - 2I_p - I_n + 4L$$



$c_1,\ c_2,\ c_3$ c_D c_E

$$V_c = \frac{1}{2}\left(\frac{g_A}{2F_\pi}\right)^2 \sum_{i\neq j\neq k} \frac{(\vec{\sigma}_i\cdot\vec{q}_i)(\vec{\sigma}_j\cdot\vec{q}_j)}{(q_i^2+m_\pi)(q_j^2+m_\pi)} F_{ijk}^{\alpha\beta}\tau_i^\alpha\tau_i^\beta$$

$$F_{ijk}^{\alpha\beta} = \delta^{\alpha\beta}[-\frac{4c_1 m_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2}\vec{q}_i\cdot\vec{q}_j] + \sum_\gamma \frac{c_4}{F_\pi^2}\epsilon^{\alpha\beta\gamma}\tau_k^\gamma\vec{\sigma}_k\cdot(\vec{q}_i\times\vec{q}_j)$$

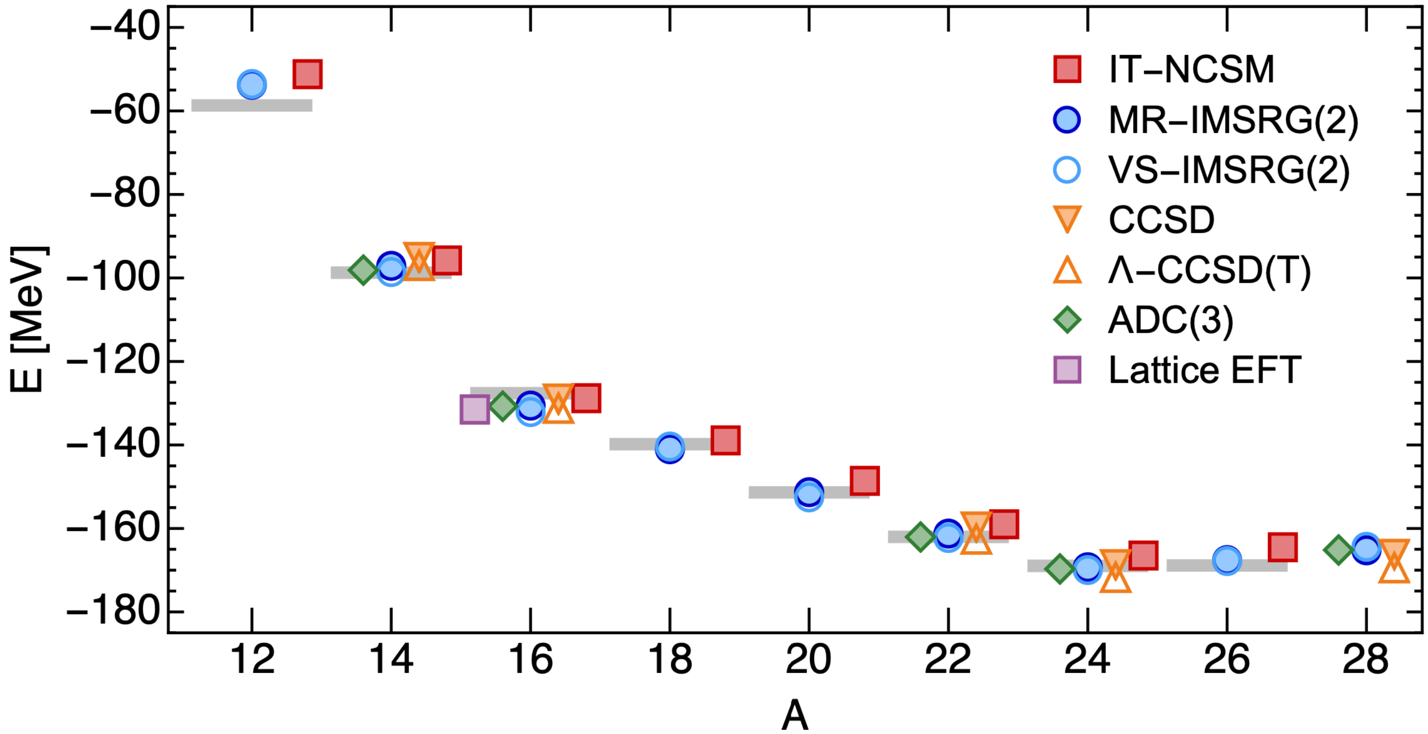
$$V_D = -\frac{g_A}{8F_\pi^2}\frac{c_D}{F_\pi^2\Lambda_\chi}\sum_{i\neq j\neq k}\frac{\sigma_j\cdot\vec{q}_j}{q_j^2+m_\pi^2}(\vec{\tau}_i\cdot\vec{\tau}_j)(\vec{\sigma}_i\cdot\vec{q}_j)$$

$$V_E = \frac{c_E}{2F_\pi^4\Lambda_\chi}\sum_{j\neq k}(\tau_j\cdot\tau_k)$$

SUCCESSFUL PREDICTIONS OF CHIPT

Highly successful in

- Meson sector
- Single baryon sector
- Multi nucleon (plus many body methods)



Ground-state energies of the oxygen isotopes for various many-body approaches, using the chiral NN+3N(400) interaction at $\lambda = 1.88\mathrm{fm}^{-1}$. H. Hergert (ArXiv:2008.0506)