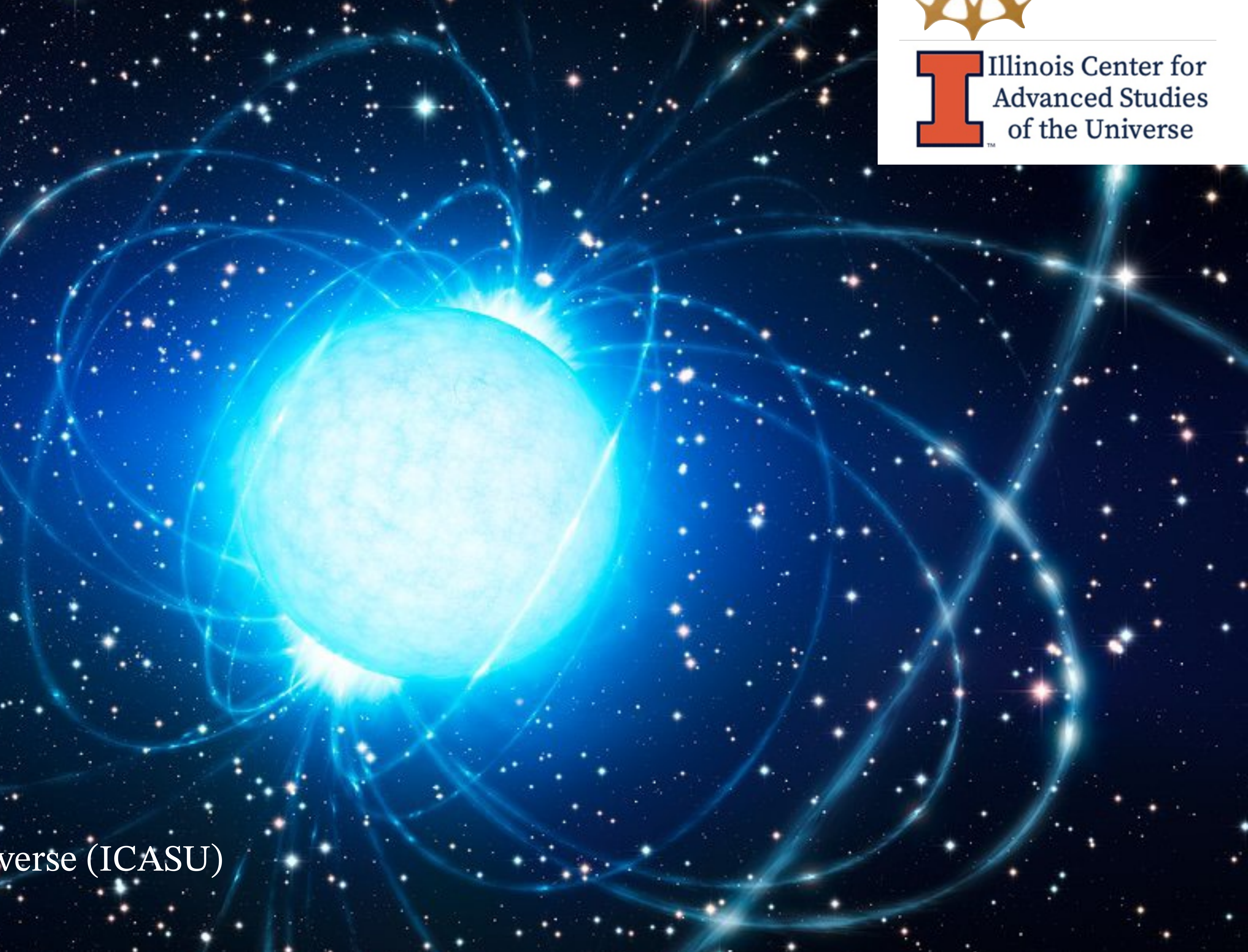
What is hiding in the core of a neutron star?

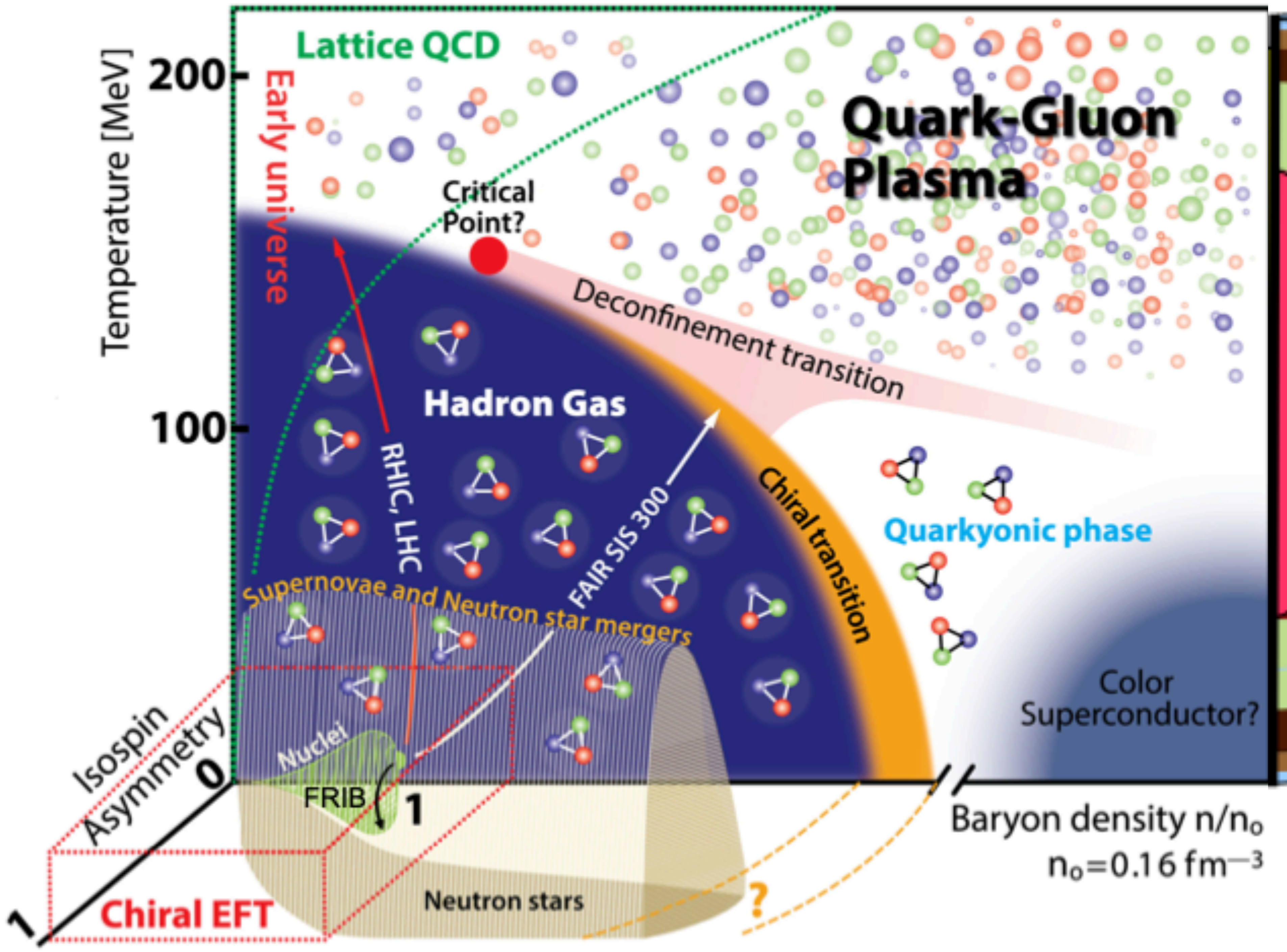


Débora Mroczek
Illinois Center for Advanced Studies of the Universe (ICASU)
University of Illinois at Urbana-Champaign

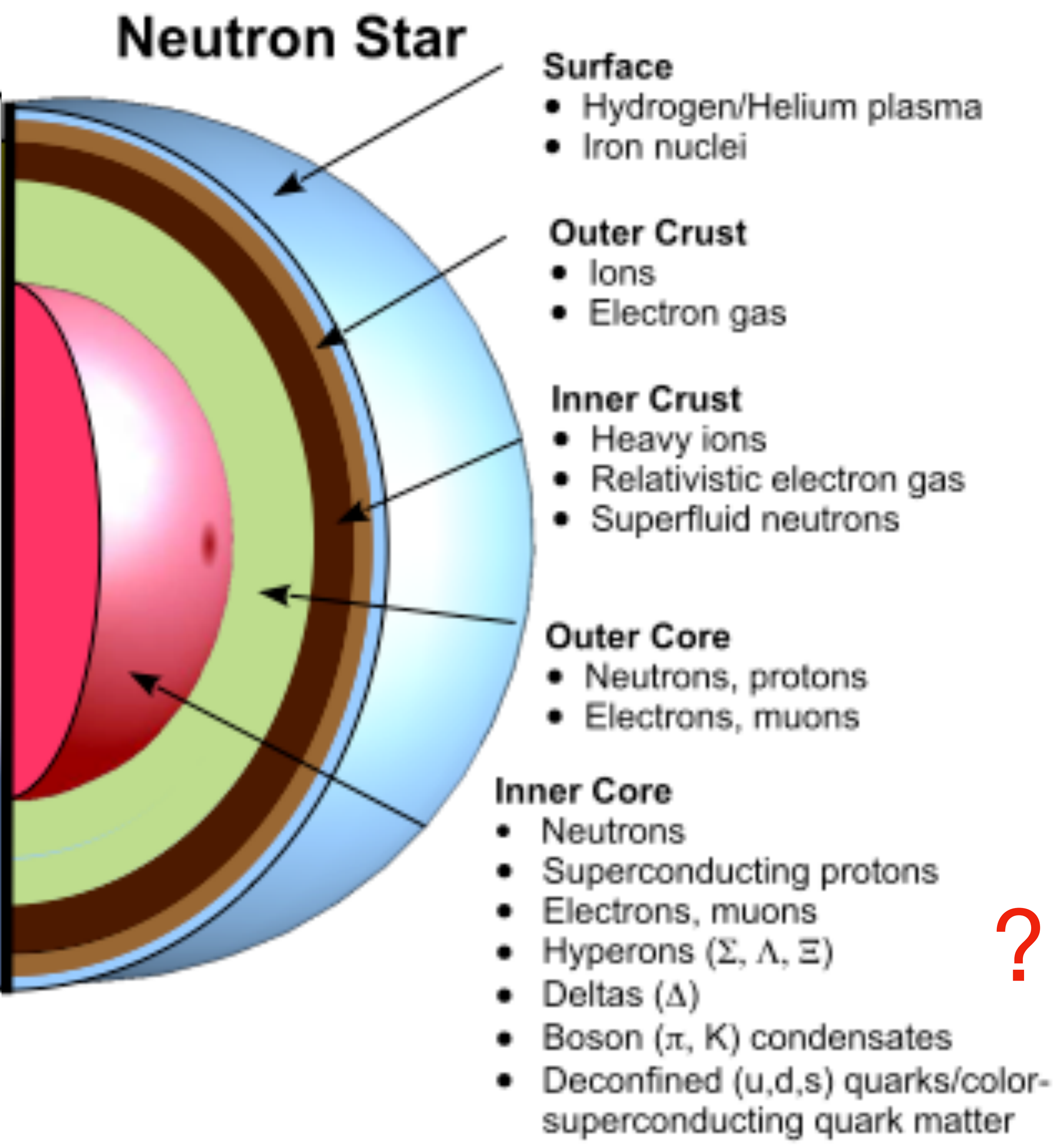
Image credits: ESO/L. Calçada

QCD matter in equilibrium

Drischler, Holt, Wellenhofer, Annu. Rev. Nucl. Part. Sci. (2021)



Weber et al. Mod.Phys.Lett.A (2014)



?

Quantum Chromodynamics (QCD): theory that describes the **strong interaction** governing the behavior of **quarks + gluons and hadrons.**

Phase diagram: phase boundaries + physics of different phases in **thermal and chemical** equilibrium.

Phase transitions are **thermodynamic singularities** in the phase diagram.

Changes in degrees of freedom and interactions leave **thermodynamic imprints**

Phase transition phenomenology

A system in thermal/chemical equilibrium can be described by thermodynamic **state variables**:

T: temperature, p: pressure, s: entropy, ε: energy density, μ_i: chemical potential, n_i: number density

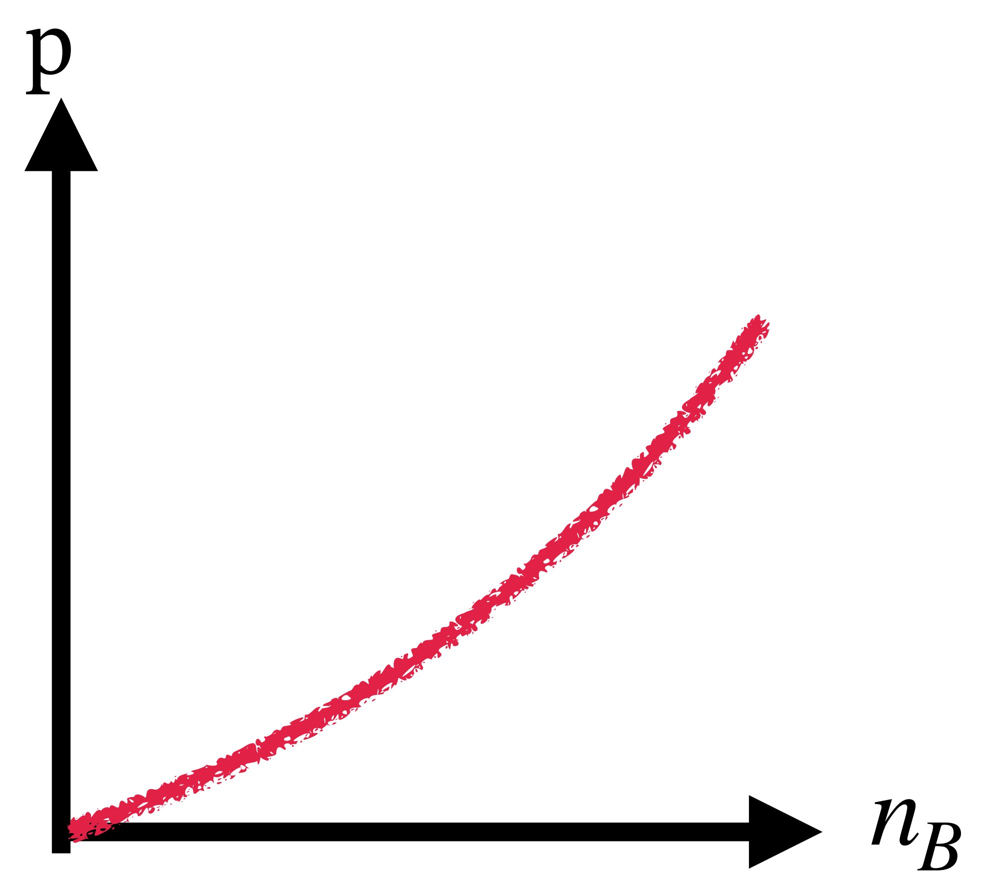
Equation of state (EoS): relationship between thermodynamic variables, e.g. $p(\varepsilon)$

We care about how **state variables change** and **how they're related to each** other inside a neutron star

A phase transition is characterized by the lowest-order derivative of the free energy which is discontinuous at the transition.

Susceptibilities: $\partial^n_{\mu_B} p$

$$\left(\frac{\partial^n p}{\partial \mu_B^n}\right)_{\text{crossover}} \neq \infty \qquad \left(\frac{\partial^n p}{\partial \mu_B^n}\right)_{\text{nth-order}} \to \infty$$



What can we learn about QCD from neutron stars

The set of relevant independent thermodynamic state variables depends on the system.

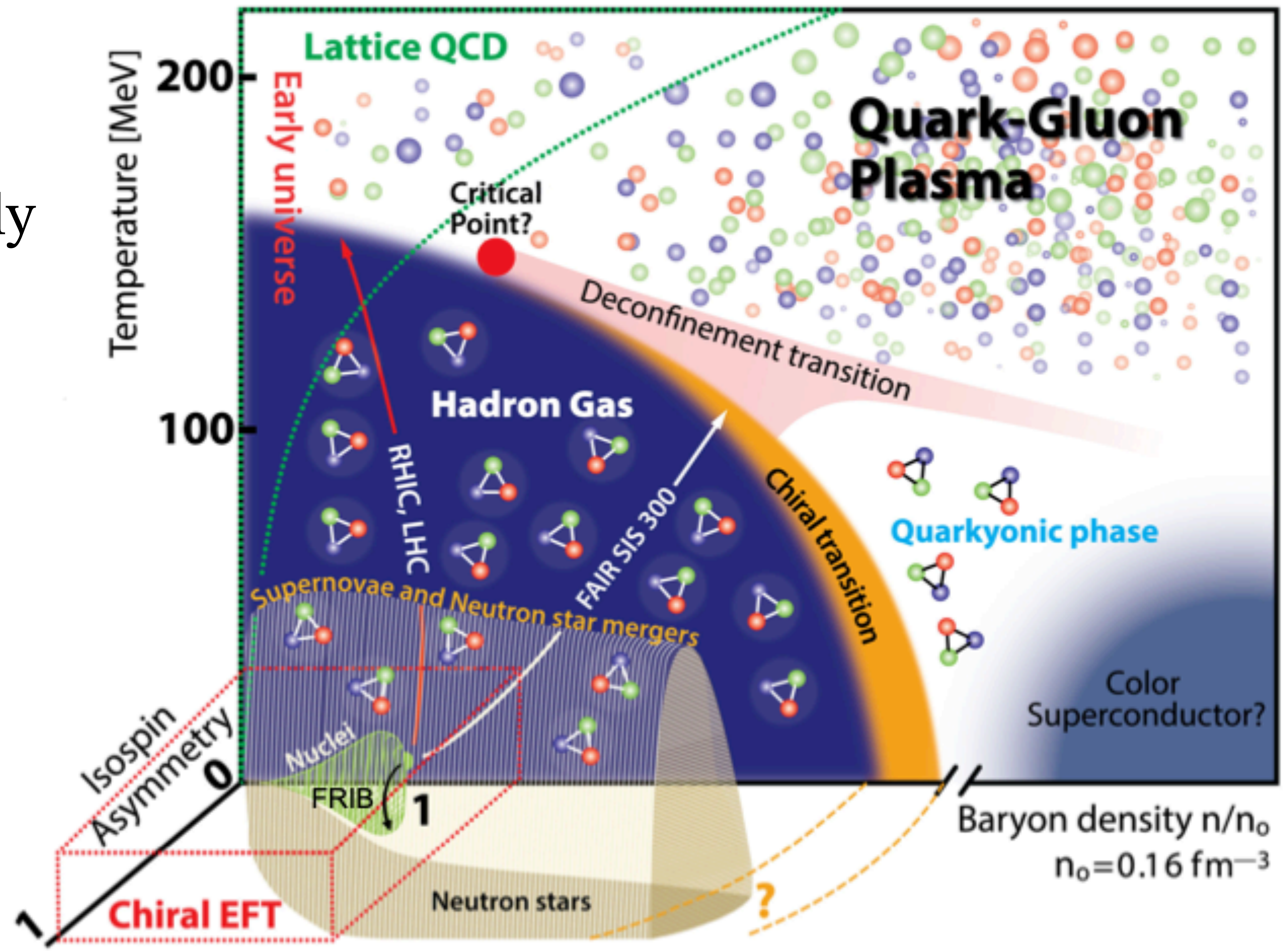
For isolated, slowly-rotating neutron stars:

1) $T = 0$, since $T_F(\sim 10^{12}\ K) \gg T(\sim 10^{8-10}\ K)$
2) β-equilibrium, producing neutrons is energetically favorable at high densities.

Neutron decay : $n \to p + e^- + \bar{\nu}_e$
Electron capture: $p + e^- \to n + \nu_e$

$\to$ fraction of charged baryons, $Y_Q^{\mathrm{QCD}} = n_Q^{\mathrm{QCD}}/n_B$, is a function of density

3) The star is electrically neutral $\to n_{l^-} = n_Q^{\mathrm{QCD}}$

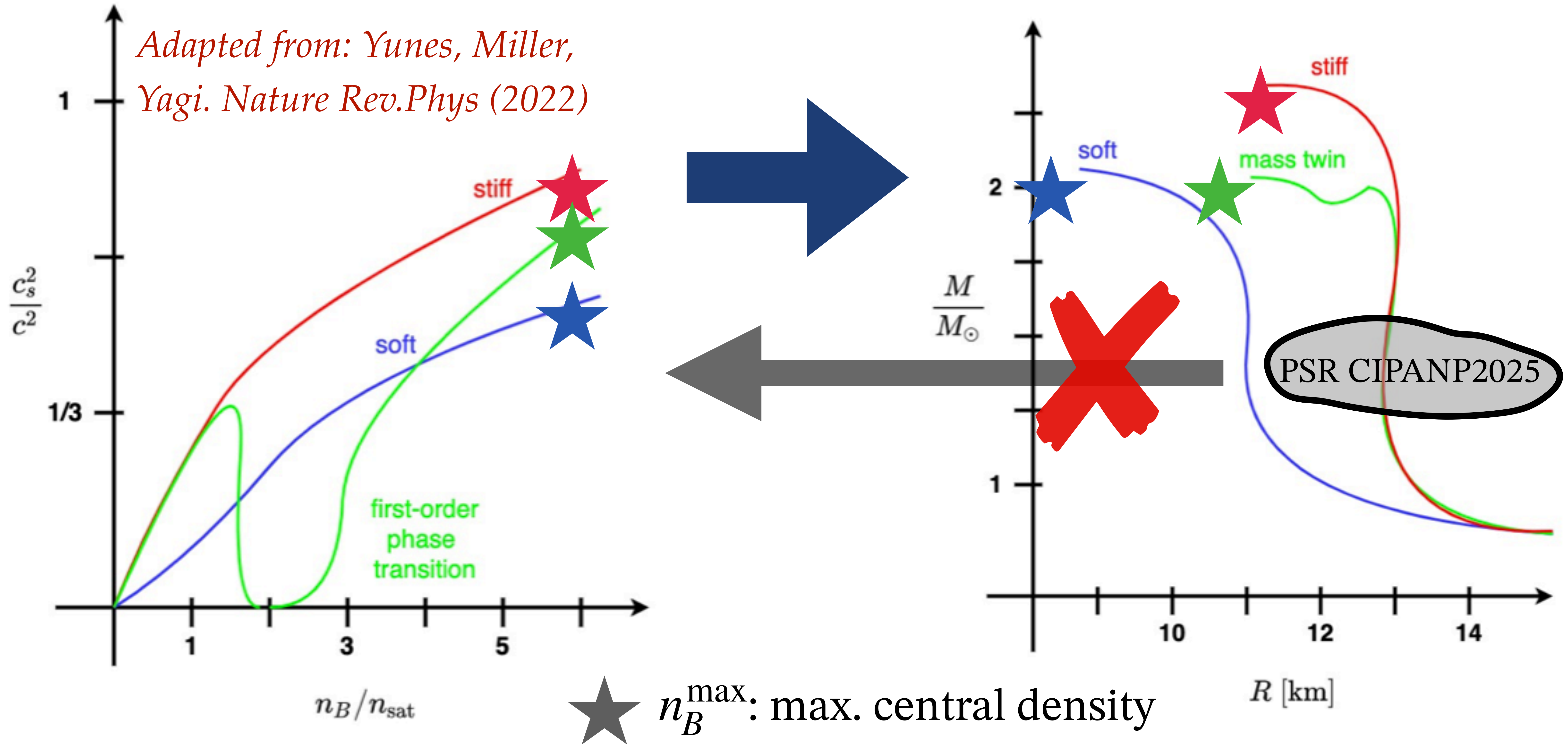


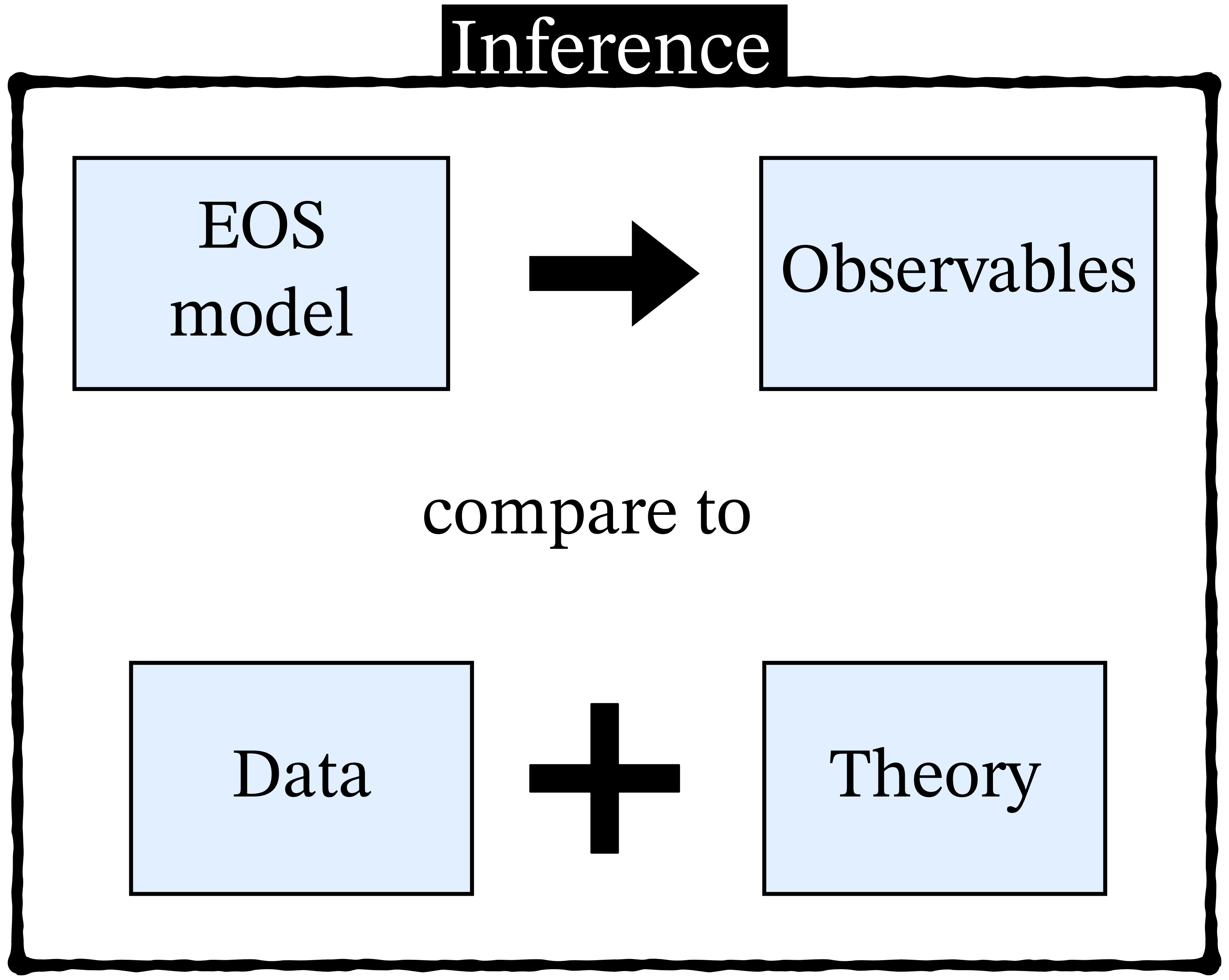
Drischler, Holt, Wellenhofer, Annu. Rev. Nucl. Part. Sci. (2021)

How do we learn about equilibrium QCD from neutron stars?

- Neutron stars have macroscopic properties that we can measure → how **big*** and **squishy**** as a function of the total mass (M) of the star
- For isolated, slowly-rotating stars, these observables depend **only on the EoS**.

From any EoS → M-R, M-Λ sequence





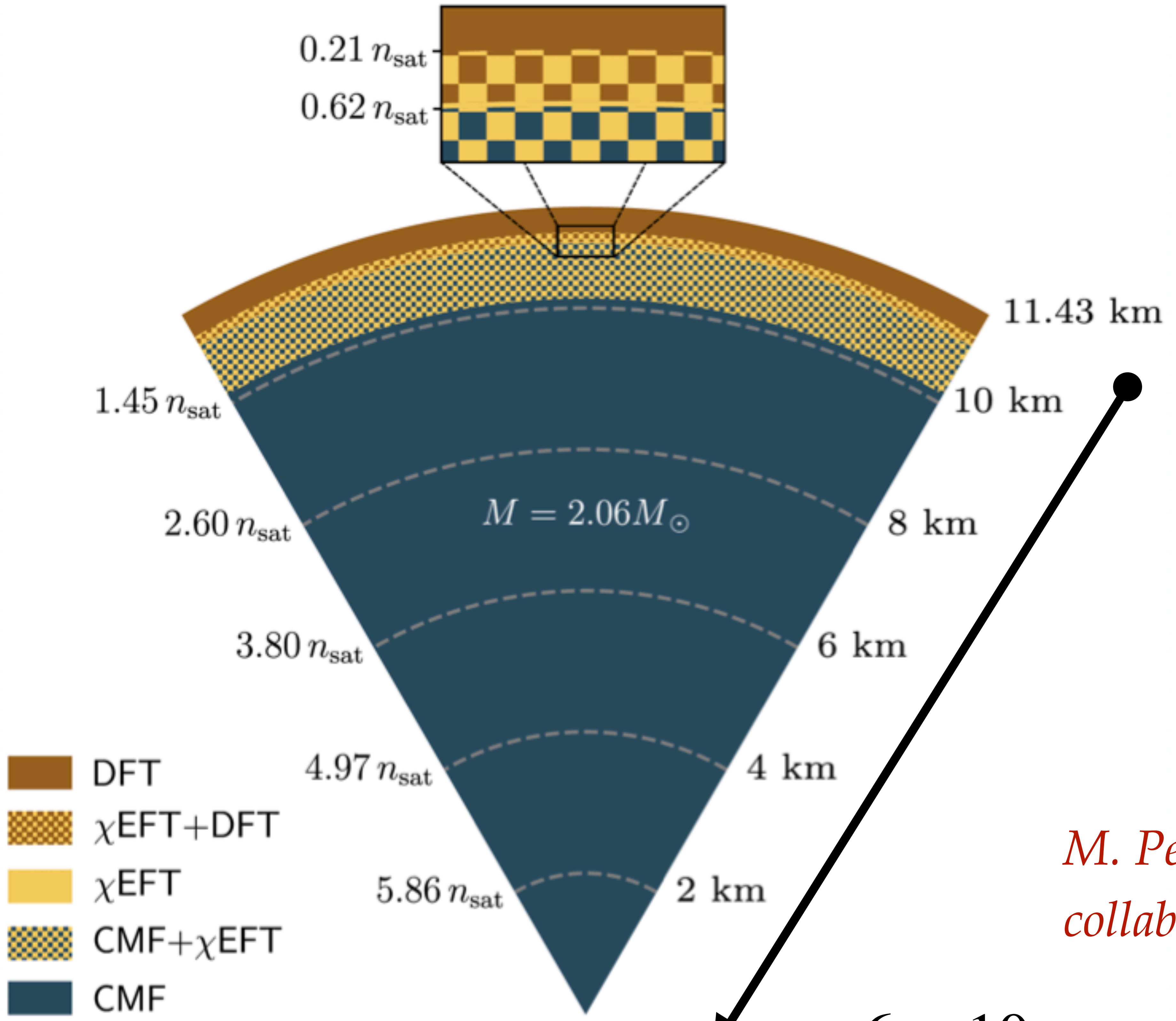
* equatorial radius (R)
** tidal deformability (Λ)

Modeling the EoS

Baryon number density (isolated, stable NS)

$$n_B = \left.\frac{\partial p}{\partial \mu_B}\right|_{\mu_Q}$$

Relevant scale: nuclear saturation density, $n_{sat} \equiv 0.16\ \mathrm{fm}^{-3}$



0

$\sim 6 - 10\ n_{sat}$

M. Pelicer et al (MUSES collaboration), 2502.07902

What are the relevant degrees of freedom and interactions?

crust   outer core   inner core

?

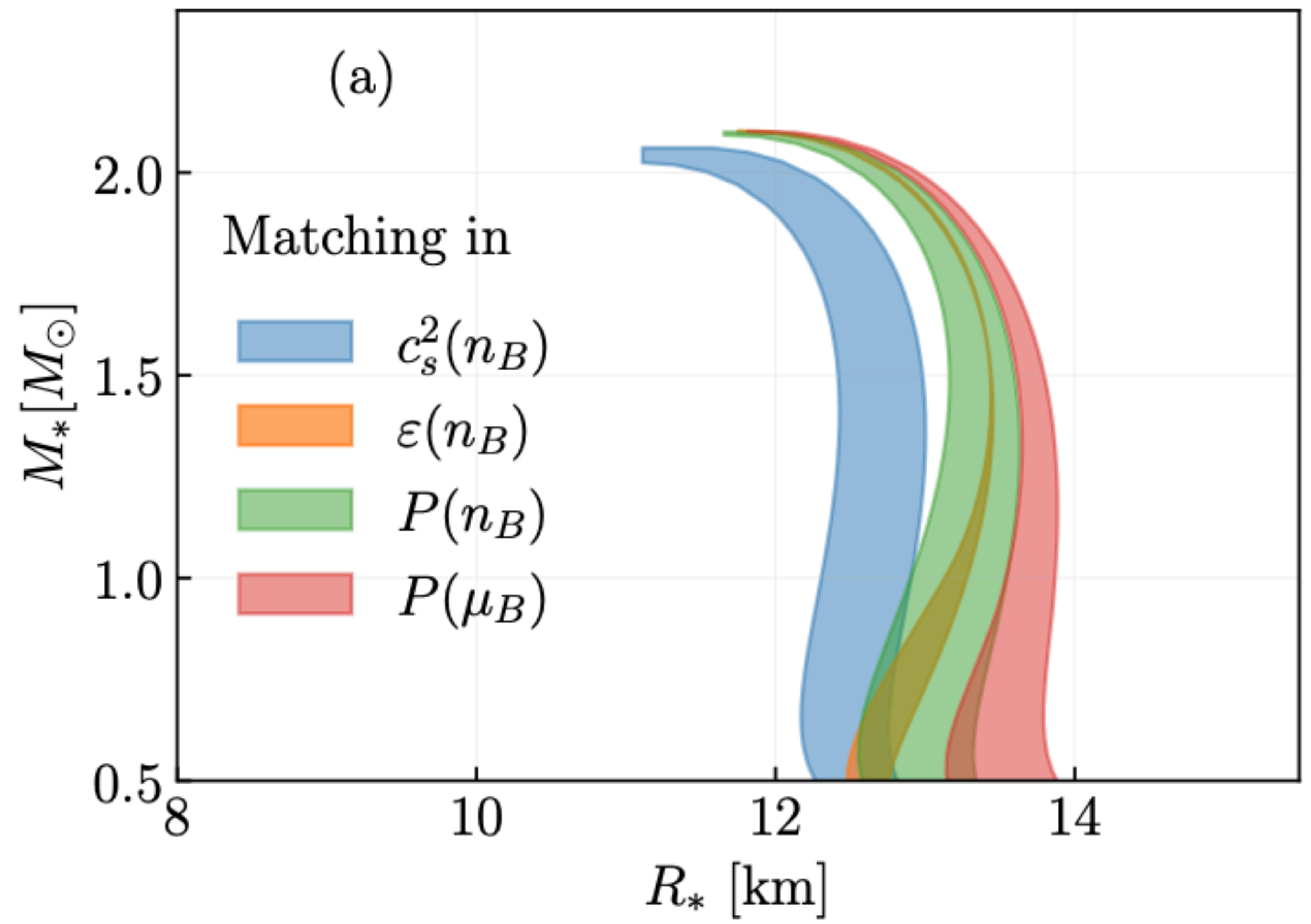
nuclei   n + p   n? + p? + ??????

Effective theories available →

- How to quantify and propagate theory uncertainties?
- What are the relevant parameters/interactions?

Bonus question: how do we piece different regimes of the EoS together?

Systematic biases are introduced by different choices!



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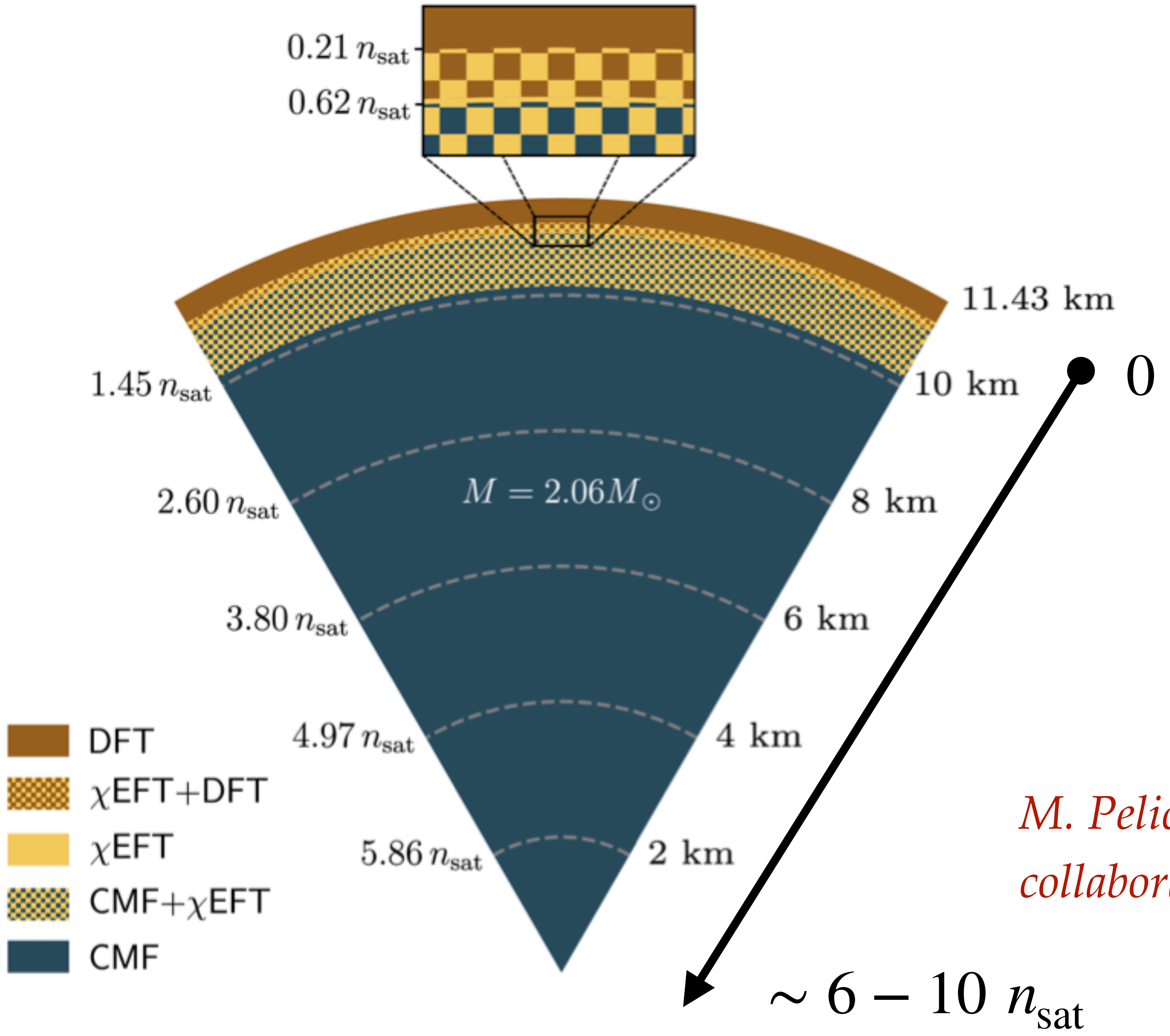
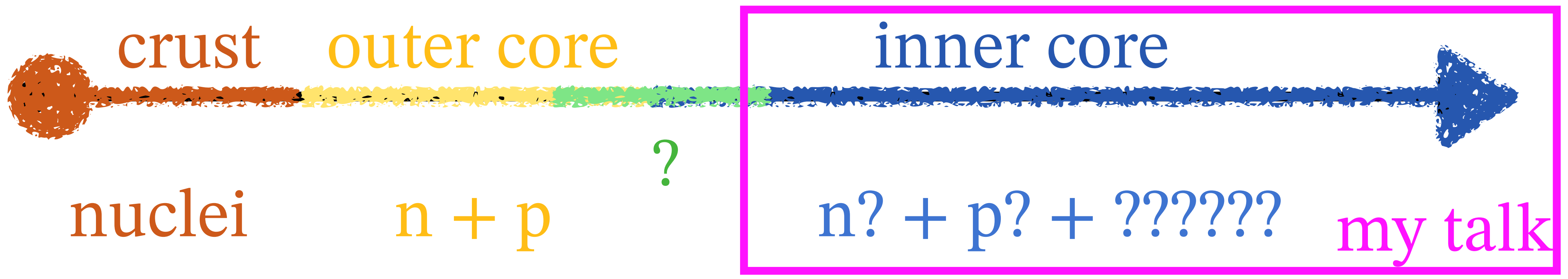


What are the relevant degrees of freedom and interactions?



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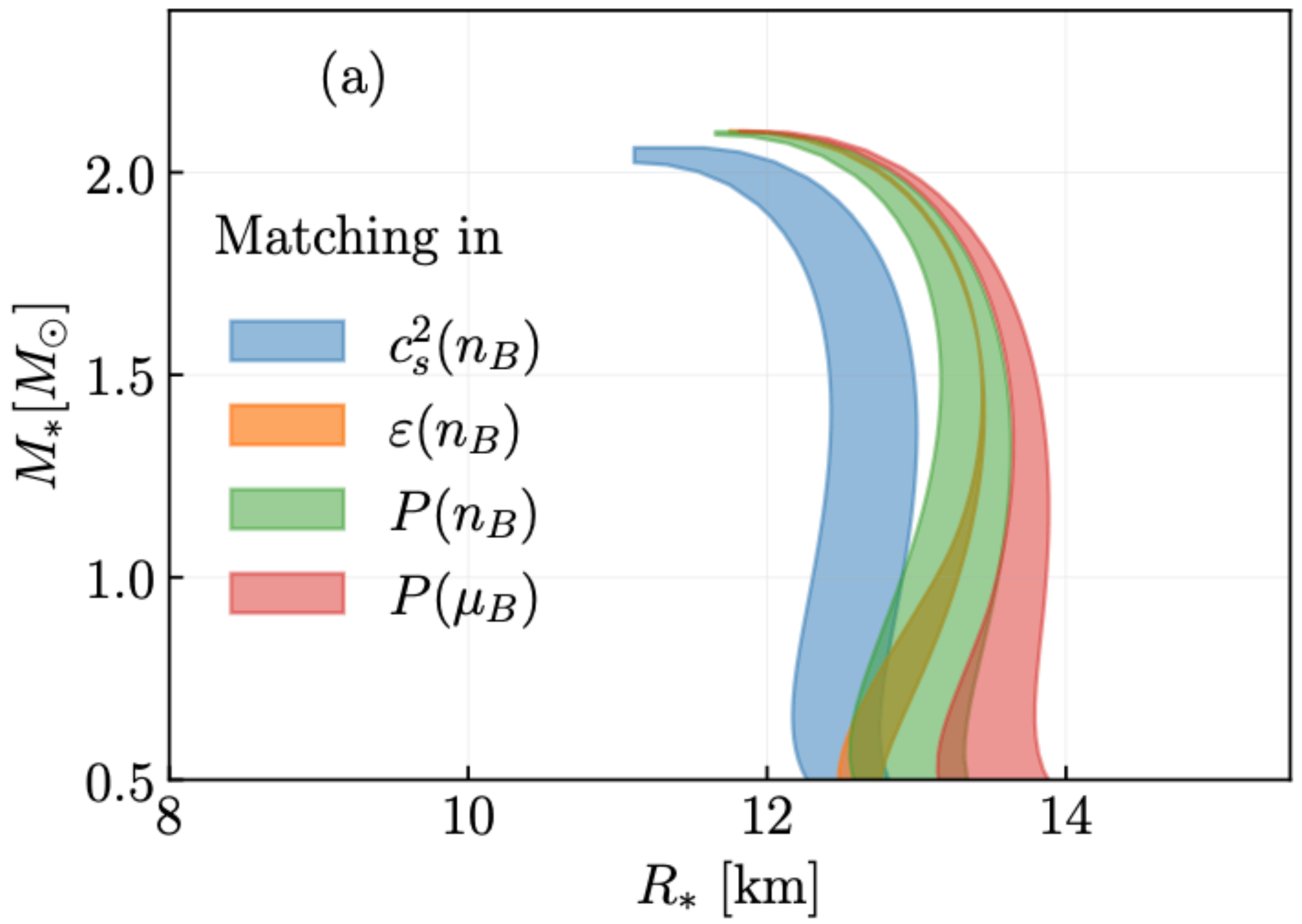


M. Pelicer et al (MUSES collaboration), 2502.07902

Bayesian statistics and choosing a prior

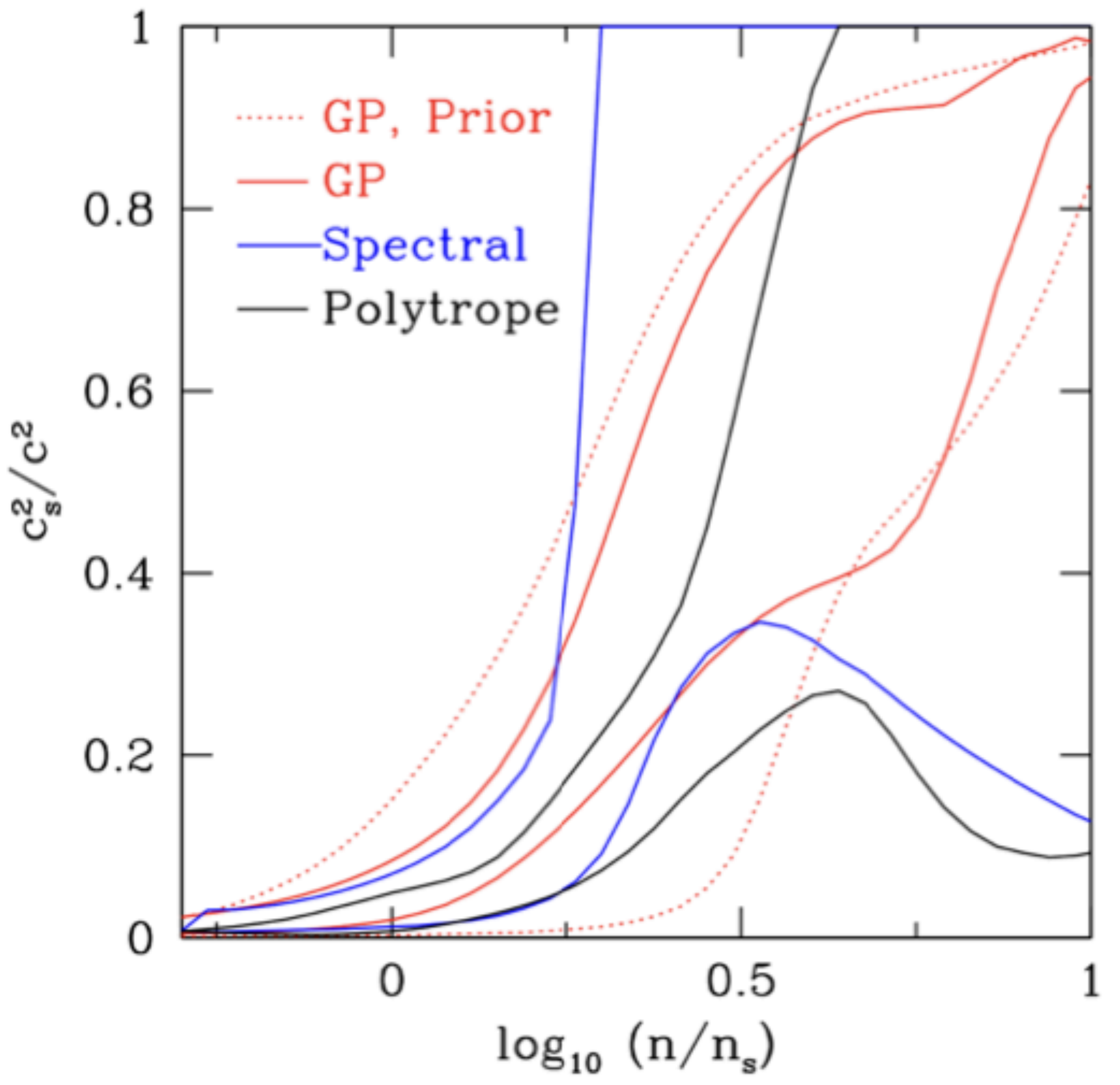
$$p_k = \frac{q_k \mathscr{L}_k}{\int q_l \mathscr{L}_l dl}$$

p_k: posterior, q_k: prior, $\mathscr{L}_k$: likelihood

⟶ model evidence

Infinitely many possible EoS:
How do we account for all possibilities?

Prior dependence: test different priors



Miller et al. AJL (2021)

- Common approach: **sample from a statistical distribution**

→ Gaussian processes (GPs):

EoS modeled via: $\phi(x) = \log(1/c_s^2 - 1)$, **stable and causal**

$$\phi \sim \mathcal{N}(\mu_i, \Sigma_{ij})$$

Collection of functions, behavior specified by a mean and covariance kernel

Squared-exponential is a common choice:

$$K_{se}\left(x_i, x_j\right) = \sigma^2 \exp\left[-\left(x_i - x_j\right)^2/2\ell^2\right]$$

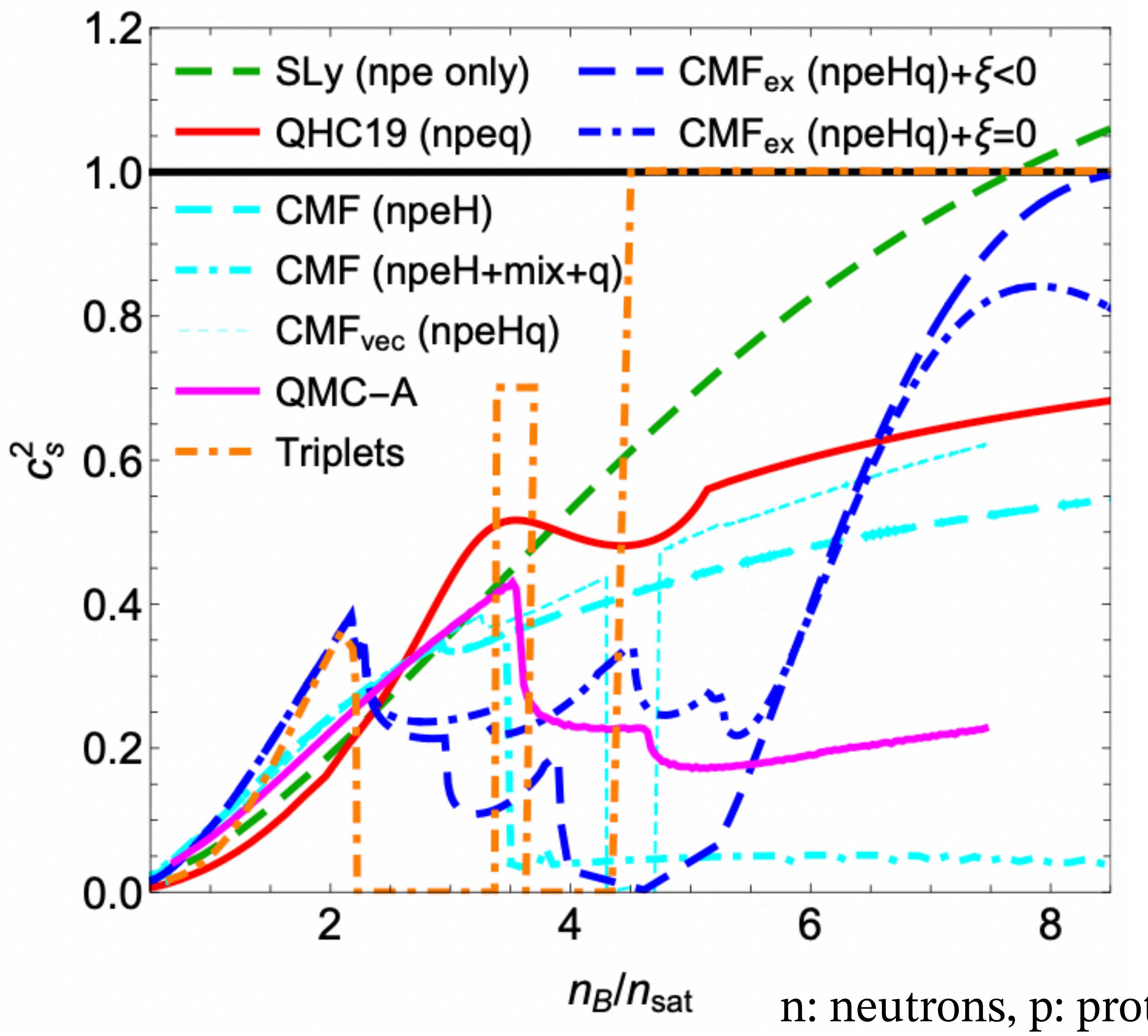
ℓ: correlation length

σ: correlation strength

Influence of exotic degrees of freedom on the EoS from nuclear physics models

*exotic = beyond p + n

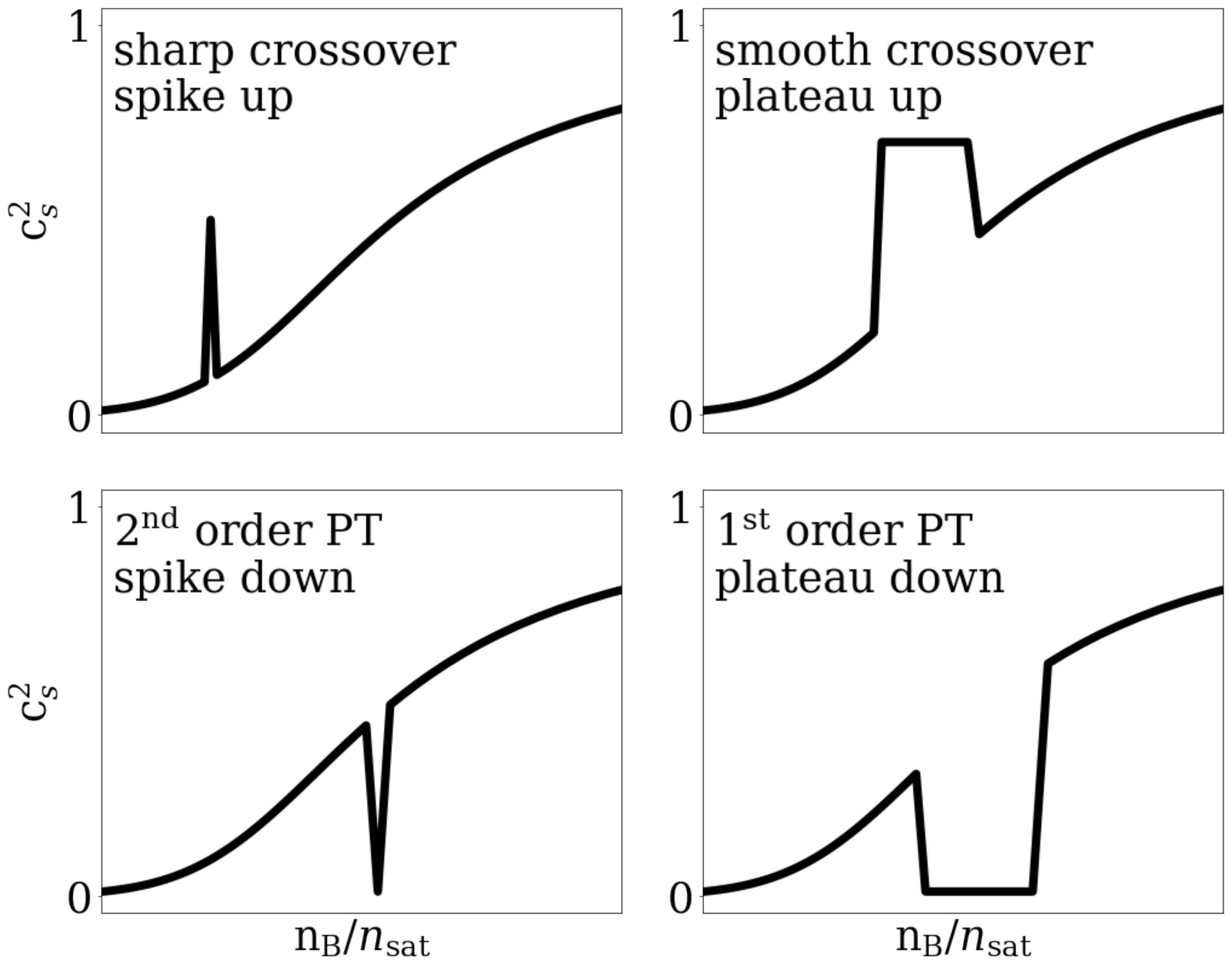
Multi-scale correlations characterize the onset of exotic phases



from: Tan et al. PRD (2022), see for refs.

n: neutrons, p: protons, e: electrons, q: quarks, H: hyperons

Modeling **thermodynamic imprints of new phases of matter** from a phenomenological perspective



Physically-motivated long + short/ medium length correlations in n_B

Bayesian friendly!

→ systematic study + model comparison

Mroczek et al., PRD (2024)

Are there nontrivial features in the c_s^2 inside neutron stars?

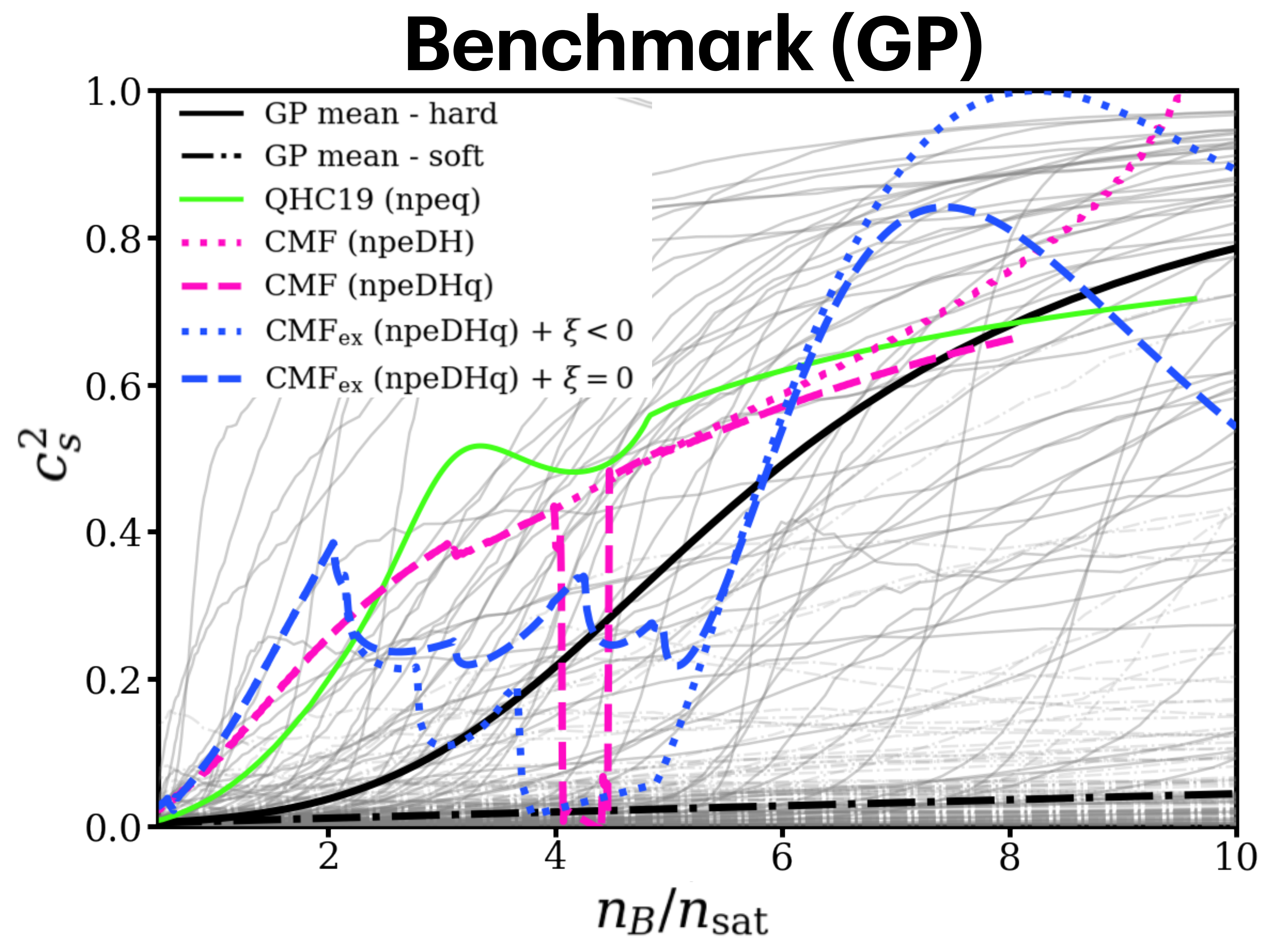
$$p_k = \frac{q_k \mathscr{L}_k}{\int q_l \mathscr{L}_l dl}$$

model evidence ($\mathscr{E}$): quantifies level of support of the data for a given model

Bayesian model comparison:

$$\text{Bayes factor } K = \frac{\mathscr{E}_{\text{benchmark}}}{\mathscr{E}_{\text{structure}}}$$

- Benchmark model in gray: GP with long-range correlations fixed across all densities
- Modified GP (mGP): multi-scale correlations → emergence of exotic degrees of freedom



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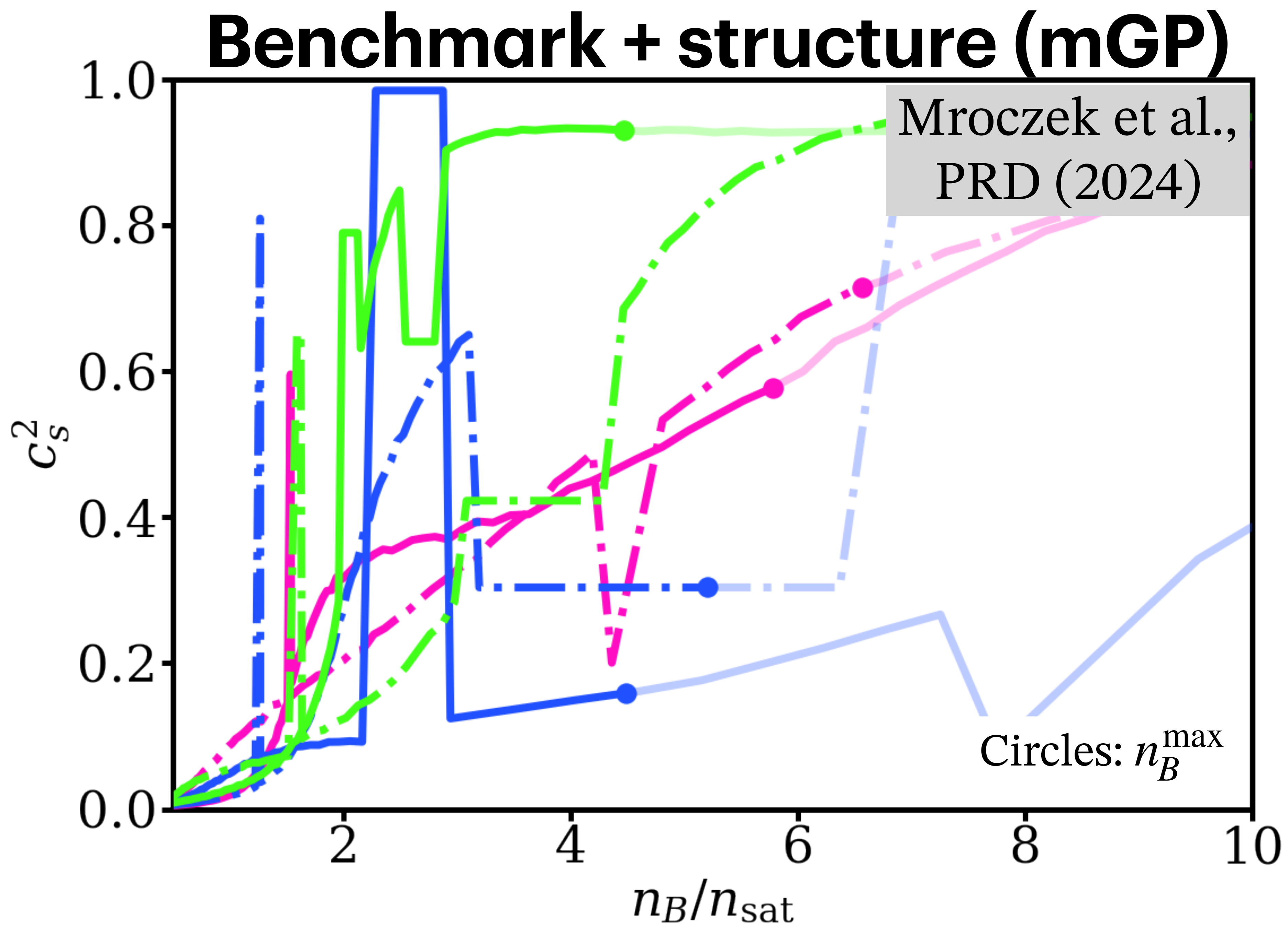


Are $c_s^2(n_B)$ posteriors sensitive to structure in $c_s^2(n_B)$?

Mroczek et al., PRD (2024)

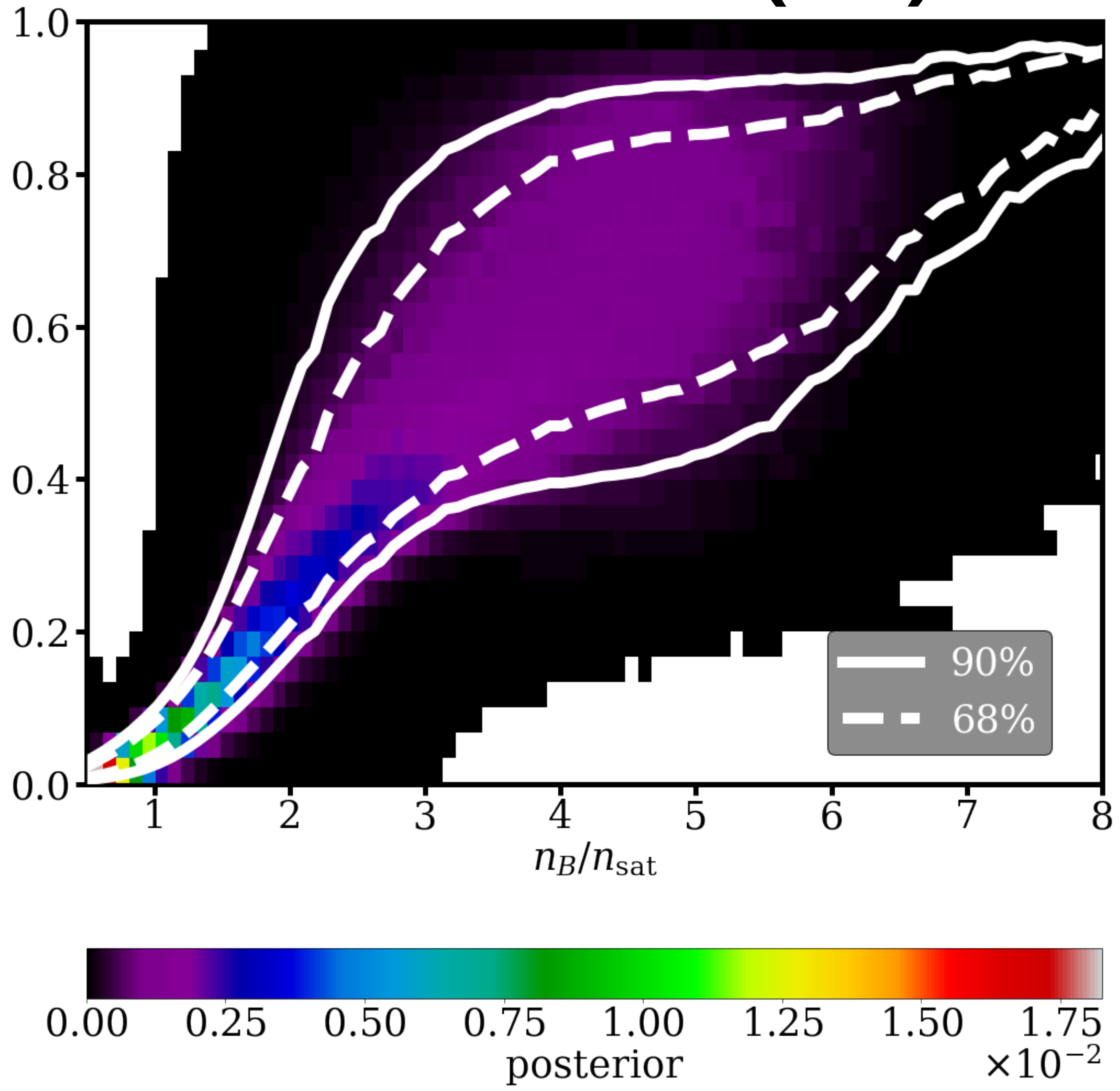
Constraints affect priors differently:

Long-range correlations → **tighter** c_s^2 posterior
New phases (structure) → **broader** c_s^2 posterior

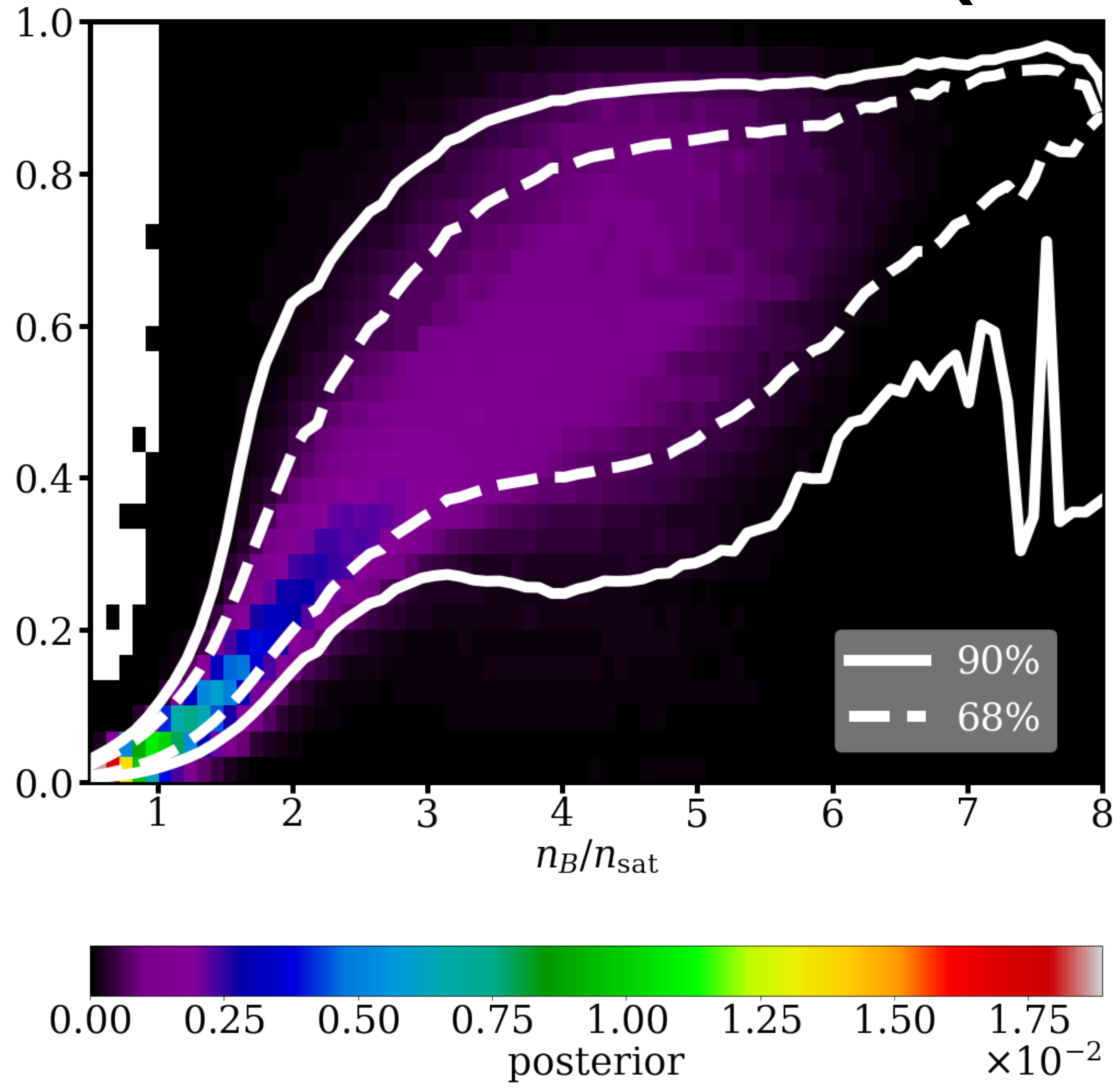
- EoS are shown up to $n_B^{\max}$

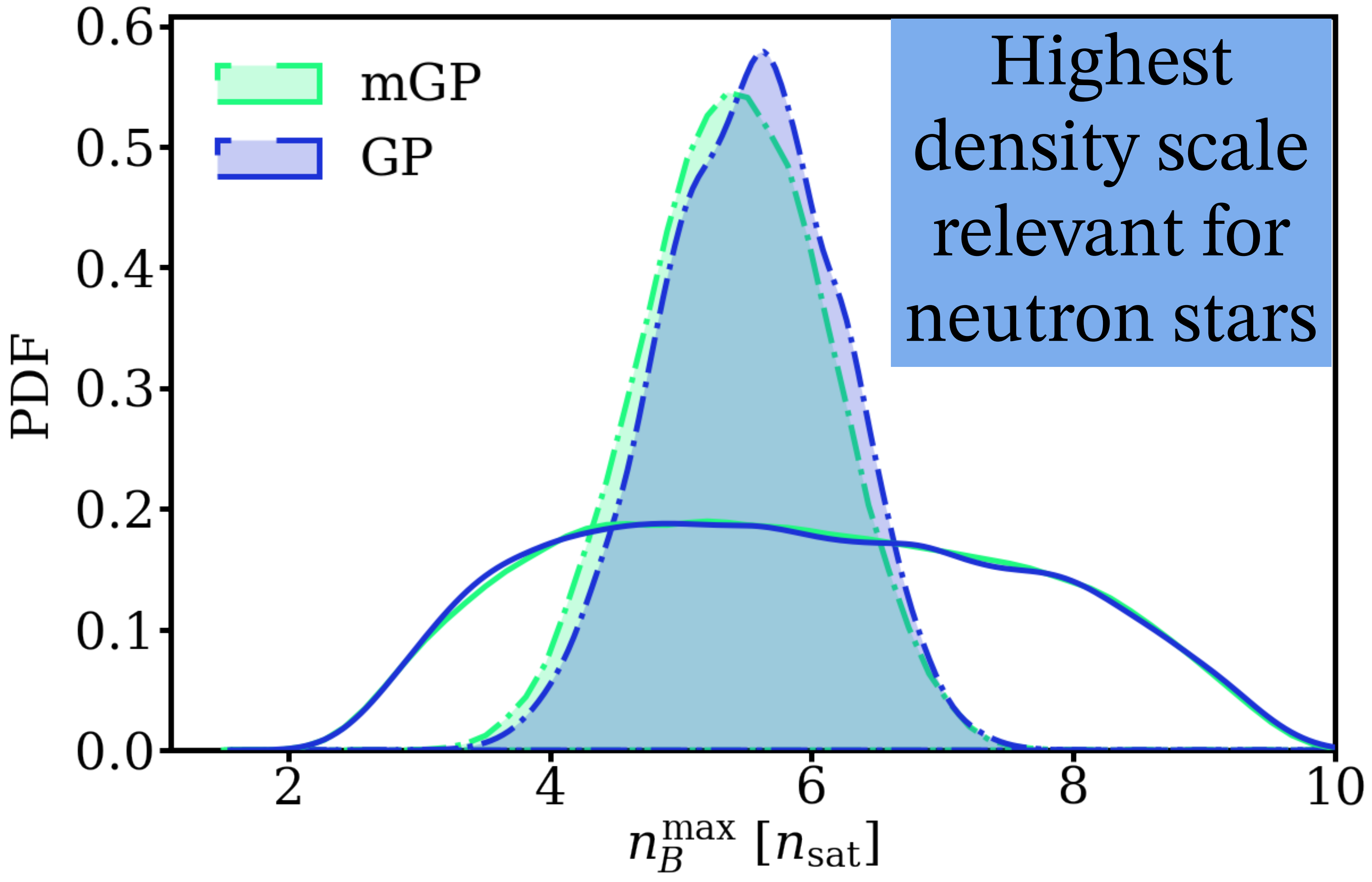
→ credibility bands are correlated with posterior for $n_B^{\max}$

Benchmark (GP)



Benchmark + structure (mGP)





- Constraints favor $n_B^{\max} \sim 5-7\ n_{\rm sat}$

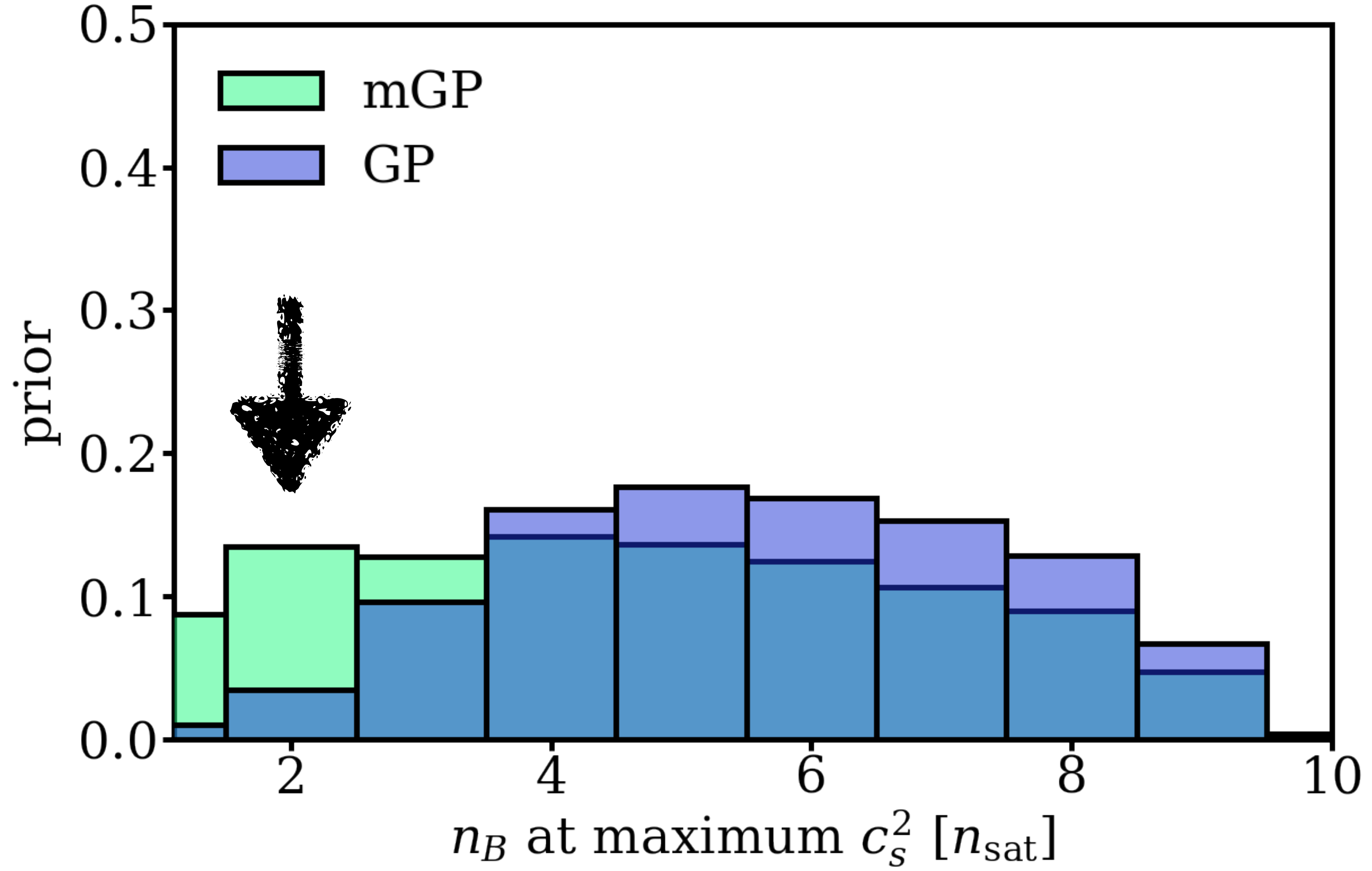
Does $c_s^2(n_B)$ display a peak within neutron star densities?

Bump in c_s^2: softening of the EoS signaling crossover to new degrees of freedom.

→ **global maximum in c_s^2 that occurs within neutron star densities**

Prior

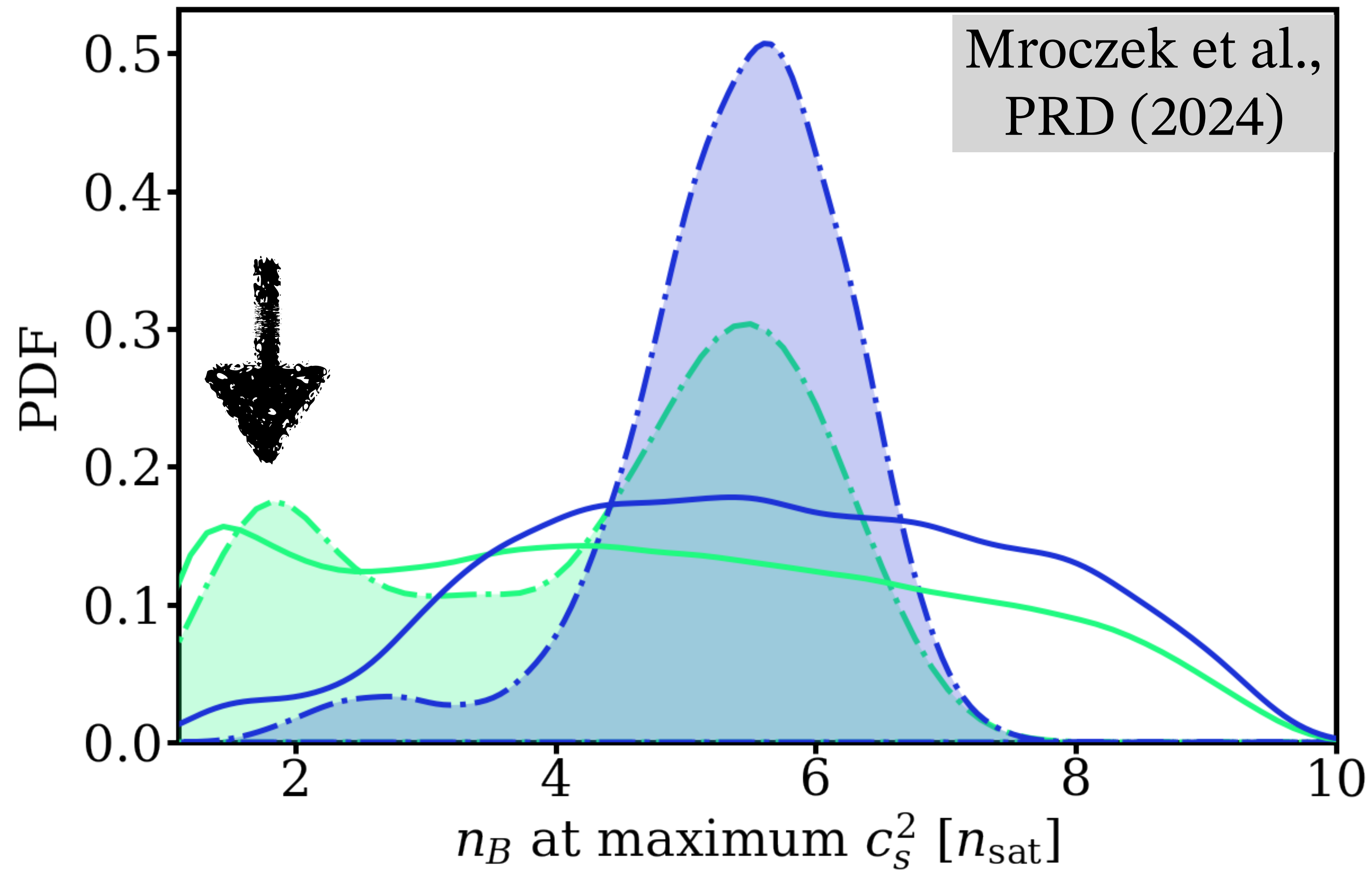
Multi-scale correlations allow for a bump **before** 3 n_{sat}



Posterior

Benchmark (GP): c_s^2 peak near $n_B^{\max}$ → **monotonic** $c_s^2(n_B)$

Benchmark + structure (mGP): **bump allowed** $\sim 2-3\ n_{\text{sat}}$



Takeaway and summary

- We find a **Bayes factor of K = 1.5** between GP and mGP → current constraints do not favor either model.

Physical interpretation: multi-scale correlations and nontrivial features in $c_s^2(n_B)$ which signal the onset of new phases of matter inside neutron stars are **not ruled out** by current constraints, but **neither are they required**.

- Nuclear physics models predict **nontrivial features** in c_s^2 and **multi-scale correlations** across densities when **exotic degrees of freedom are present.**
- Introduced **modified Gaussian processes** as novel approach for **modeling nontrivial features in** c_s^2.
- Performed a fully Bayesian analysis including astrophysical, low-energy, and pQCD constraints.
- **Multi-scale correlations** important for **searches for a crossover** within NS densities.

Neutron stars probe a regime of QCD that we cannot recreate in labs. The only way to extract information about QCD from neutron stars is through inference. **Quantifying theory uncertainty on the EoS is a requirement.**

Other approaches

CONSTRAINING THE SPEED OF SOUND INSIDE NEUTRON STARS WITH CHIRAL EFFECTIVE FIELD THEORY INTERACTIONS AND OBSERVATIONS

I. Tews,[1,2] J. Carlson,[3] S. Gandolfi,[3] and S. Reddy[1,2]

[1] *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195-1550, USA*
[2] *JINA-CEE, Michigan State University, East Lansing, MI, 48823, USA*
[3] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, NM 87545, USA*

Investigating Signatures of Phase Transitions in Neutron-Star Cores

R. Somasundaram,[1,*] I. Tews,[2] and J. Margueron[1]

[1] *Univ Lyon, Univ Claude Bernard Lyon 1, CNRS/IN2P3, IP2I Lyon, UMR 5822, F-69622, Villeurbanne, France*
[2] *Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA*

(Dated: March 22, 2023)

Phase Transition Phenomenology with Nonparametric Representations of the Neutron Star Equation of State

Reed Essick,[1,2,3,4,*] Isaac Legred,[5,6,†] Katerina Chatziioannou,[5,6,‡] Sophia Han (韩君),[7,8,9,§] and Philippe Landry[1,¶]

[1] *Canadian Institute for Theoretical Astrophysics, University of Toronto, Toronto, ON M5S 3H8*
[2] *Department of Physics, University of Toronto, Toronto, ON M5S 1A7*
[3] *David A. Dunlap Department of Astronomy, University of Toronto, Toronto, ON M5S 3H4*
[4] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada, N2L 2Y5*
[5] *TAPIR, California Institute of Technology, Pasadena, California 91125, USA*
[6] *LIGO Laboratory, California Institute of Technology, Pasadena, CA 91125, USA*
[7] *Tsung-Dao Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China*
[8] *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195, USA*
[9] *Department of Physics, University of California, Berkeley, CA 94720, USA*

(Dated: June 9, 2023)

Nonparametric extensions of nuclear equations of state: probing the breakdown scale of relativistic mean-field theory

Isaac Legred,[1,2,*] Liam Brodie,[3,†] Alexander Haber,[4,3,‡] Reed Essick,[5,6,7,§] and Katerina Chatziioannou[1,2,¶]

[1] *TAPIR, California Institute of Technology, Pasadena, CA 91125, USA*
[2] *LIGO Laboratory, California Institute of Technology, Pasadena, California 91125, USA*
[3] *Department of Physics, Washington University in St. Louis, St. Louis, MO 63130, USA*
[4] *Mathematical Sciences and STAG Research Centre, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[5] *Canadian Institute for Theoretical Astrophysics, 60 St. George St, Toronto, Ontario M5S 3H8*
[6] *Department of Physics, University of Toronto, 60 St. George Street, Toronto, ON M5S 1A7*
[7] *David A. Dunlap Department of Astronomy, University of Toronto, 50 St. George Street, Toronto, ON M5S 3H4*

+ many others

Microscopic constraints for the equation of state and structure of neutron stars: a Bayesian model mixing framework

A. C. Semposki,[1,*] C. Drischler,[1,2,†] R. J. Furnstahl,[3,‡] and D. R. Phillips[1,4,§]

[1] *Department of Physics and Astronomy and Institute of Nuclear and Particle Physics, Ohio University, Athens, OH 45701, USA*
[2] *Facility for Rare Isotope Beams, Michigan State University, East Lansing, MI 48824, USA*
[3] *Department of Physics, The Ohio State University, Columbus, OH 43210, USA*
[4] *Department of Physics, Chalmers University of Technology, SE-41296 Göteborg, Sweden*

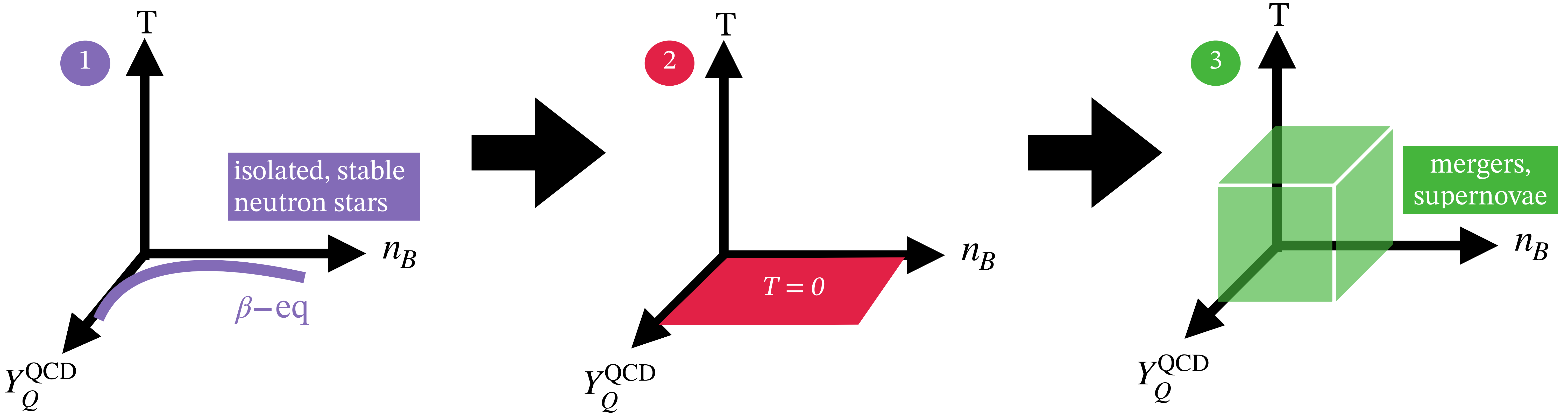
(Dated: June 10, 2025)

Consensus: posteriors are **sensitive to** changes in modeling assumptions (**priors**) → **data is not yet informative** w.r.t. to details in the EoS representation.

Finite temperature expansion of the dense matter EoS

<u>Dense matter</u> (in this work) → **hadron/quark** state of matter with **no strange degrees of freedom** in the regime relevant for **neutron stars.**

Starting from an **arbitrary NS EOS, reconstruct a 3D EOS** for numerical relativity simulations.

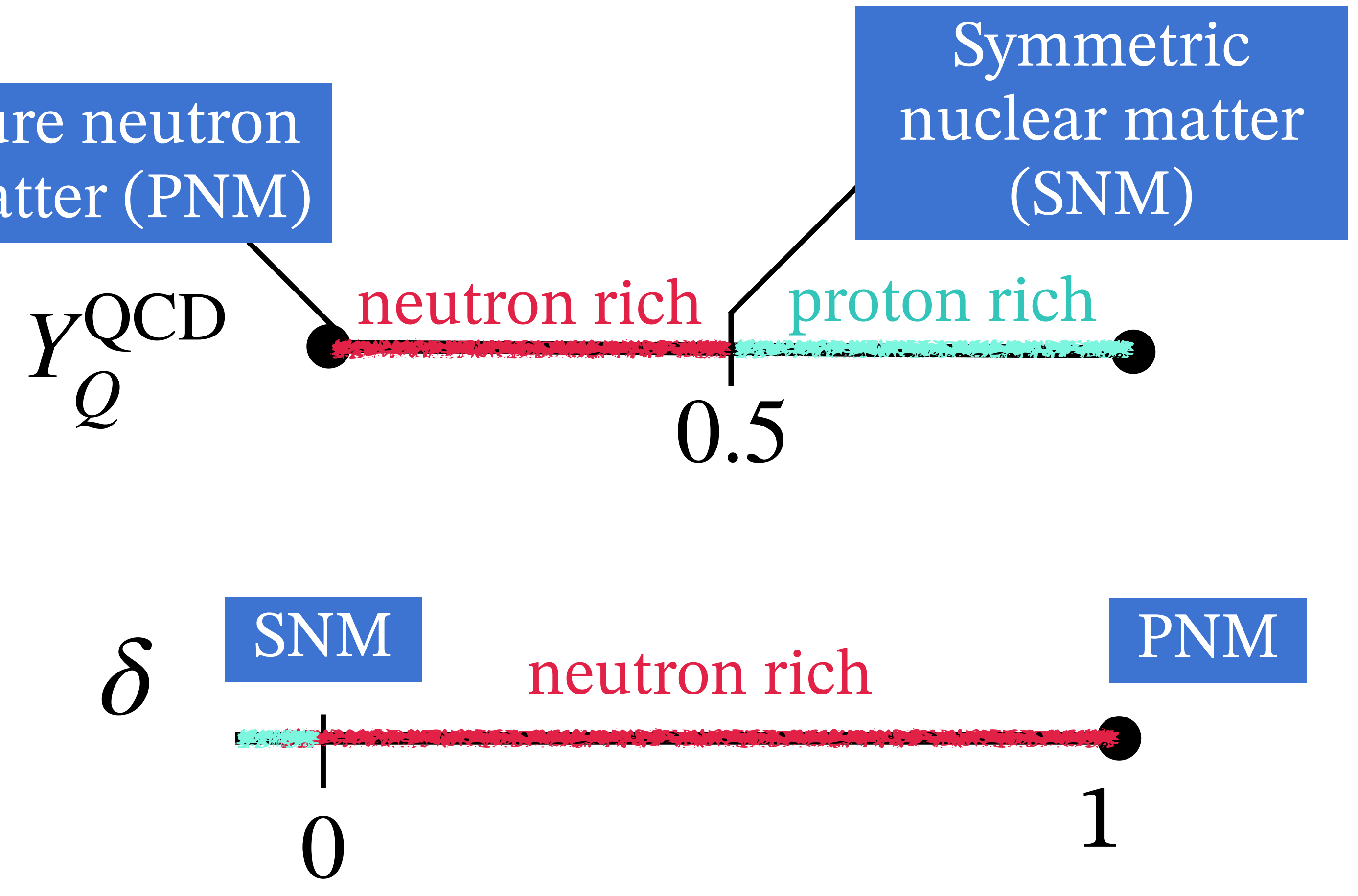


1) **Baryon number density**
(isolated, stable NS)

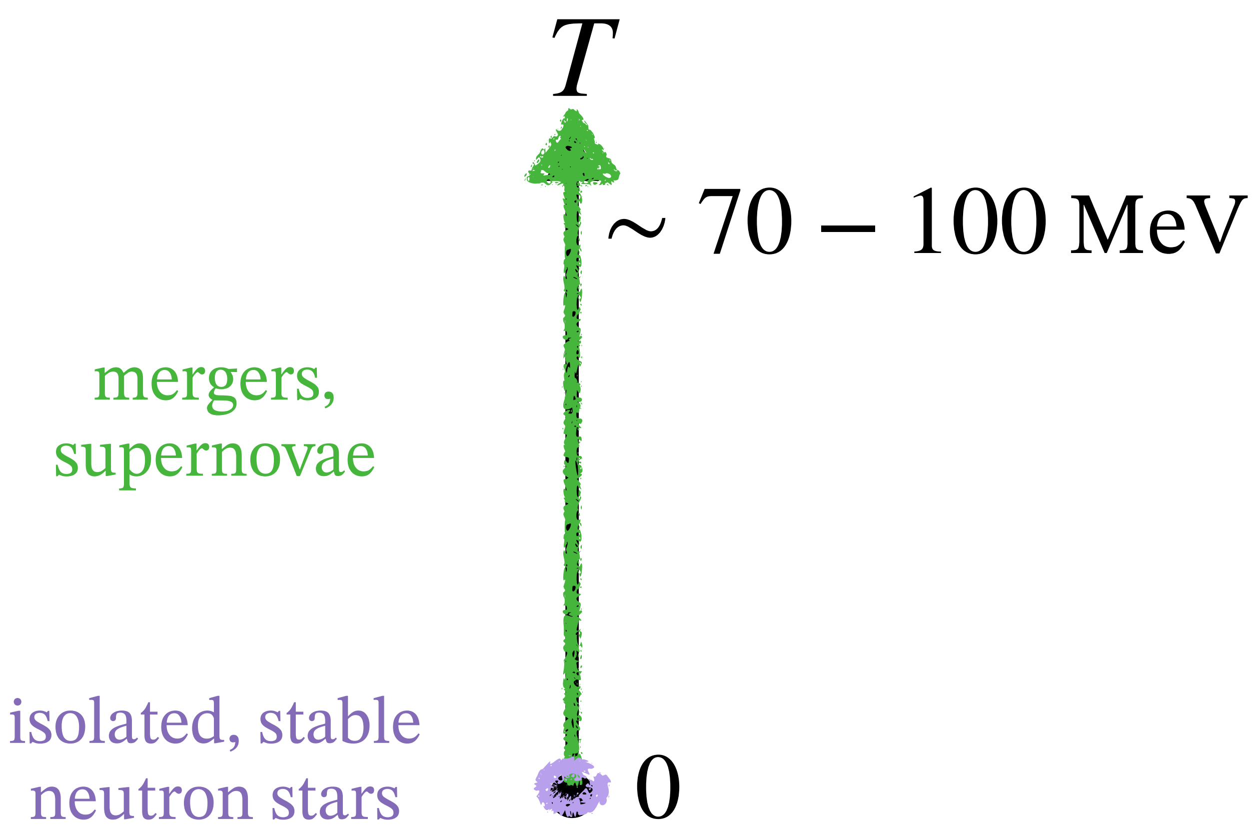
2) **Charge fraction / isospin asymmetry**

$$Y_Q^{QCD} = \frac{n_Q^{QCD}}{n_B} \rightarrow \delta = 1 - 2Y_Q^{QCD}$$

No leptons, hadrons/quarks only.

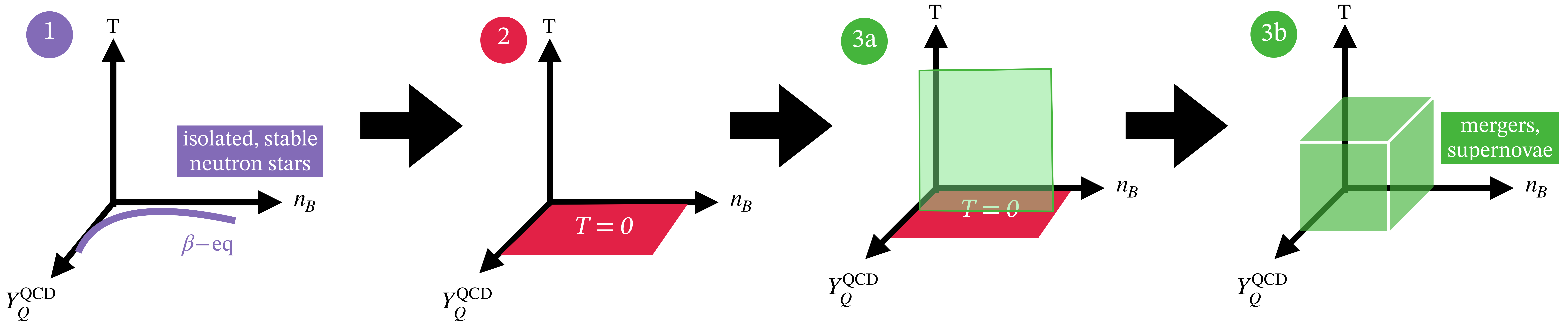


3) **Temperature**

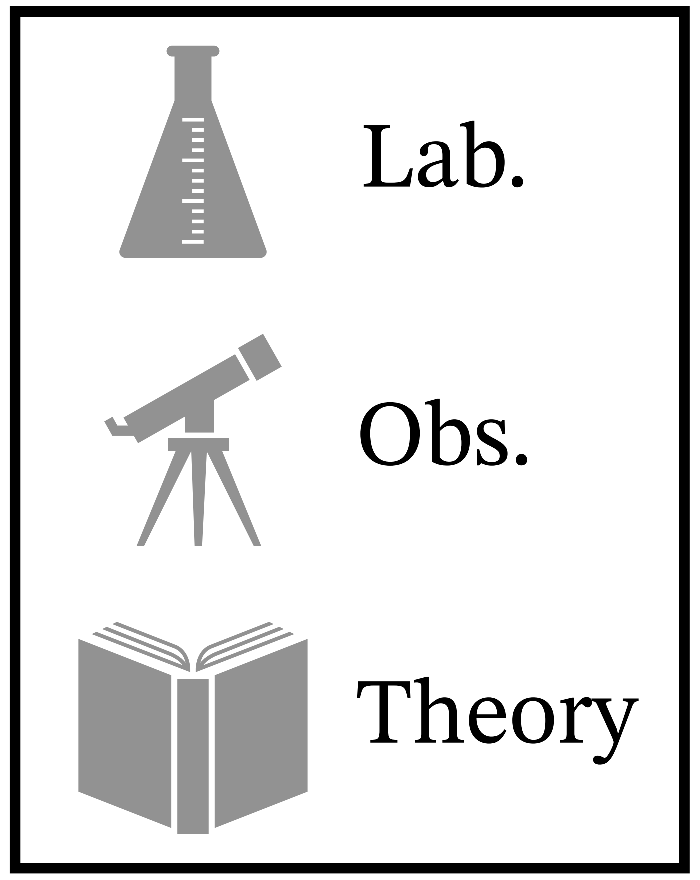


What is needed (pt. 2) and our approach

Mroczek et al., 2404.01658



- Thermodynamically consistent
- Beyond n+p degrees of freedom
- Connection to available experiments, observations, and theory predictions



1 → 2: Expansion of the symmetry energy about NS EOS

Yao et al, PRC 109 (2024)

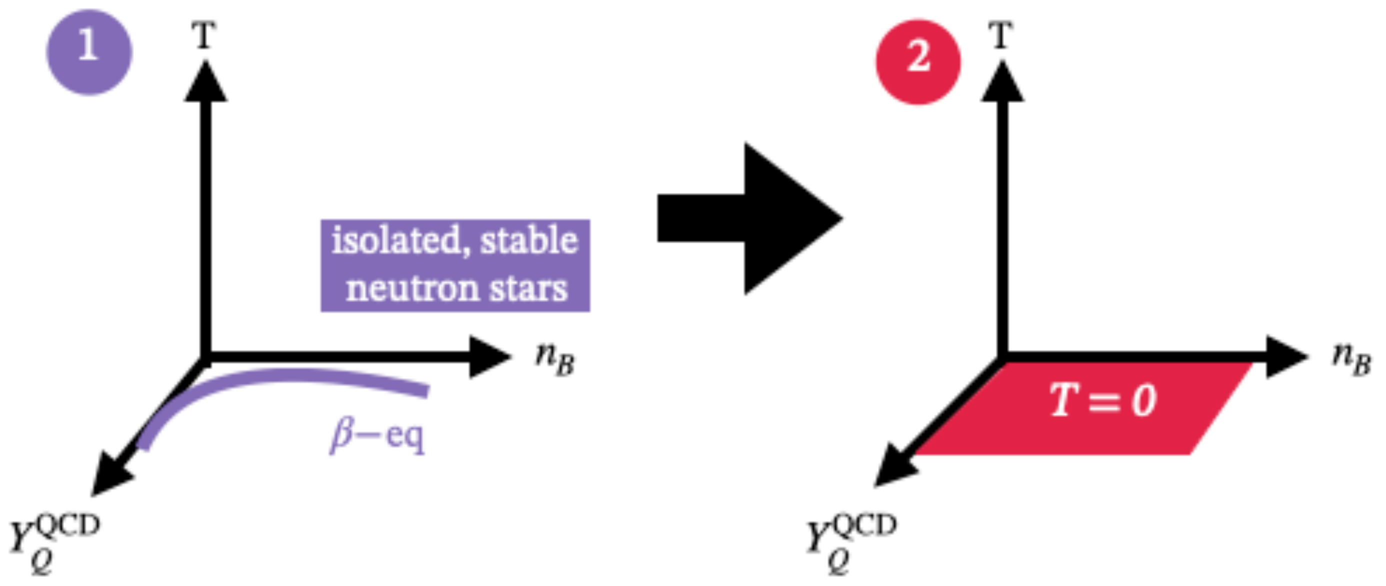
2 → 3a: Finite temperature expansion at fixed μ_Q

New!

3a → 3b: Expansion of charge fraction dependence of finite temperature effects

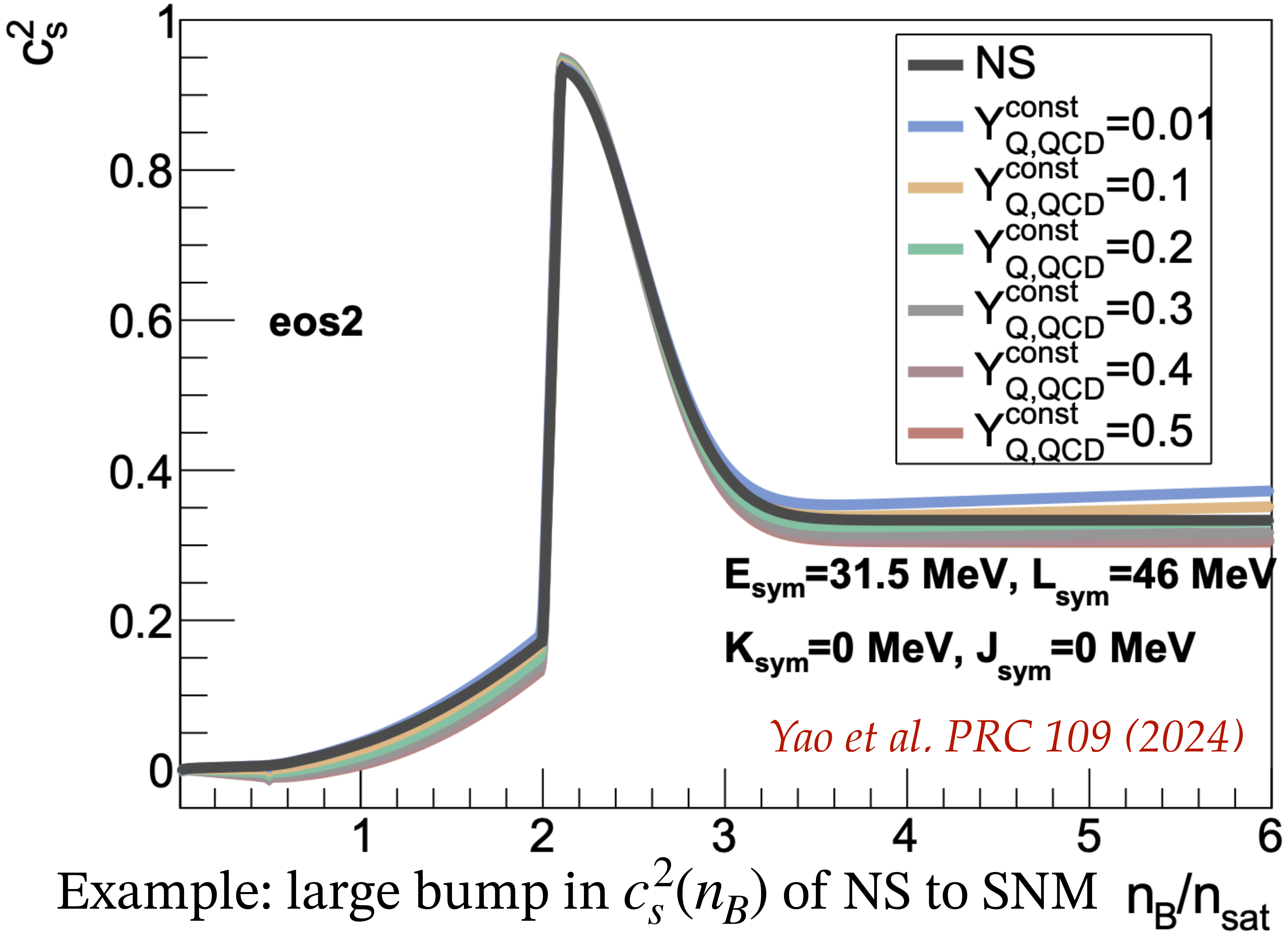
New!

From β-equilibrium to arbitrary charge fraction



- Symmetry energy expansion derived in Bombaci and Lombardo (1991), modified in Yao et al. (2024):

$$\frac{E_{\rm HIC,sym}}{N_B} = \underline{\frac{E_{\rm NS,QCD}}{N_B}} - \left[\underline{E_{\rm sym,sat}} + \underline{\frac{L_{\rm sym,sat}}{3}}\left(\frac{n_B}{n_0}-1\right) + \underline{\frac{K_{\rm sym,sat}}{18}}\left(\frac{n_B}{n_0}-1\right)^2 + \underline{\frac{J_{\rm sym,sat}}{162}}\left(\frac{n_B}{n_0}-1\right)^3\right](1-2Y_{Q,QCD})^2$$

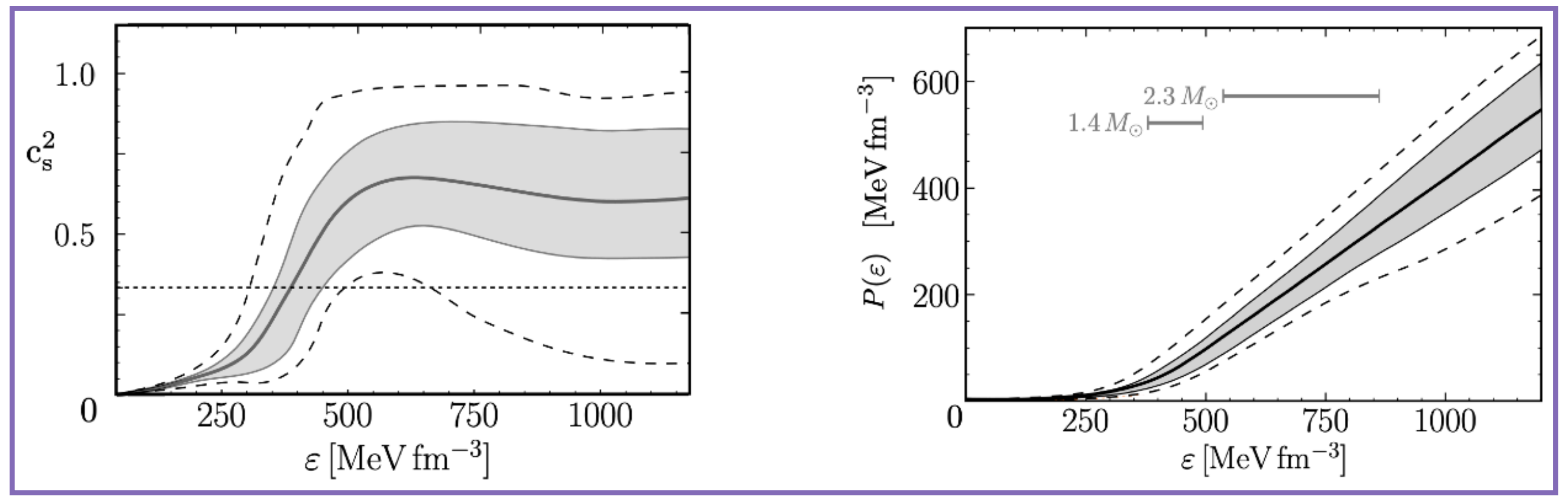


Example: large bump in $c_s^2(n_B)$ of NS to SNM

Input:

- NS EOS
- Symmetry energy coefficients

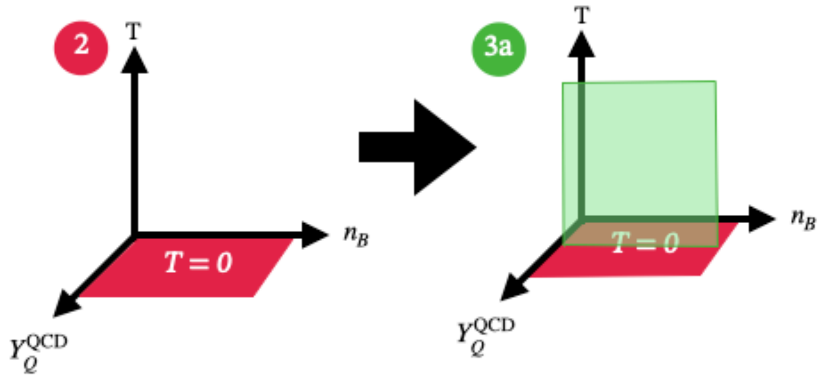
Brandes & Weise, PRD 111 (2025)



Coefficient	Definition	Range	References
$E_{\rm sym,sat}$	$\left(\frac{E_{\rm PNM}-E_{\rm SNM}}{N_B}\right)_{n_{\rm sat}}$	31.7 ± 3.2 [MeV]	Multiple data analyses from nuclear physics and astrophysics [121]
$L_{\rm sym,sat}$	$3n_{\rm sat}\left(\frac{dE_{\rm sym,2}}{dn_B}\right)_{n_{\rm sat}}$	58.7 ± 28.1 [MeV]	Multiple data analyses from nuclear physics and astrophysics [121]
$K_{\rm sym,sat}$	$9n_{\rm sat}^2\left(\frac{d^2E_{\rm sym,2}}{dn_B^2}\right)_{n_{\rm sat}}$	106 ± 37 [MeV]	PREXII [122, 123]
		-120^{+80}_{-100} [MeV]	Bayesian analyses inferred from GW170817 and PSR J0030+0451[124]
$J_{\rm sym,sat}$	$27n_{\rm sat}^3\left(\frac{d^3E_{\rm sym,2}}{dn_B^3}\right)_{n_{\rm sat}}$	300 ± 500 [MeV]	Many-body nuclear theory [125]

Yao et al. PRC 109 (2024)

From T = 0 to finite T



- Taylor expansion about $p(T=0,\ \mu_B,\ \mu_Q)$ New!

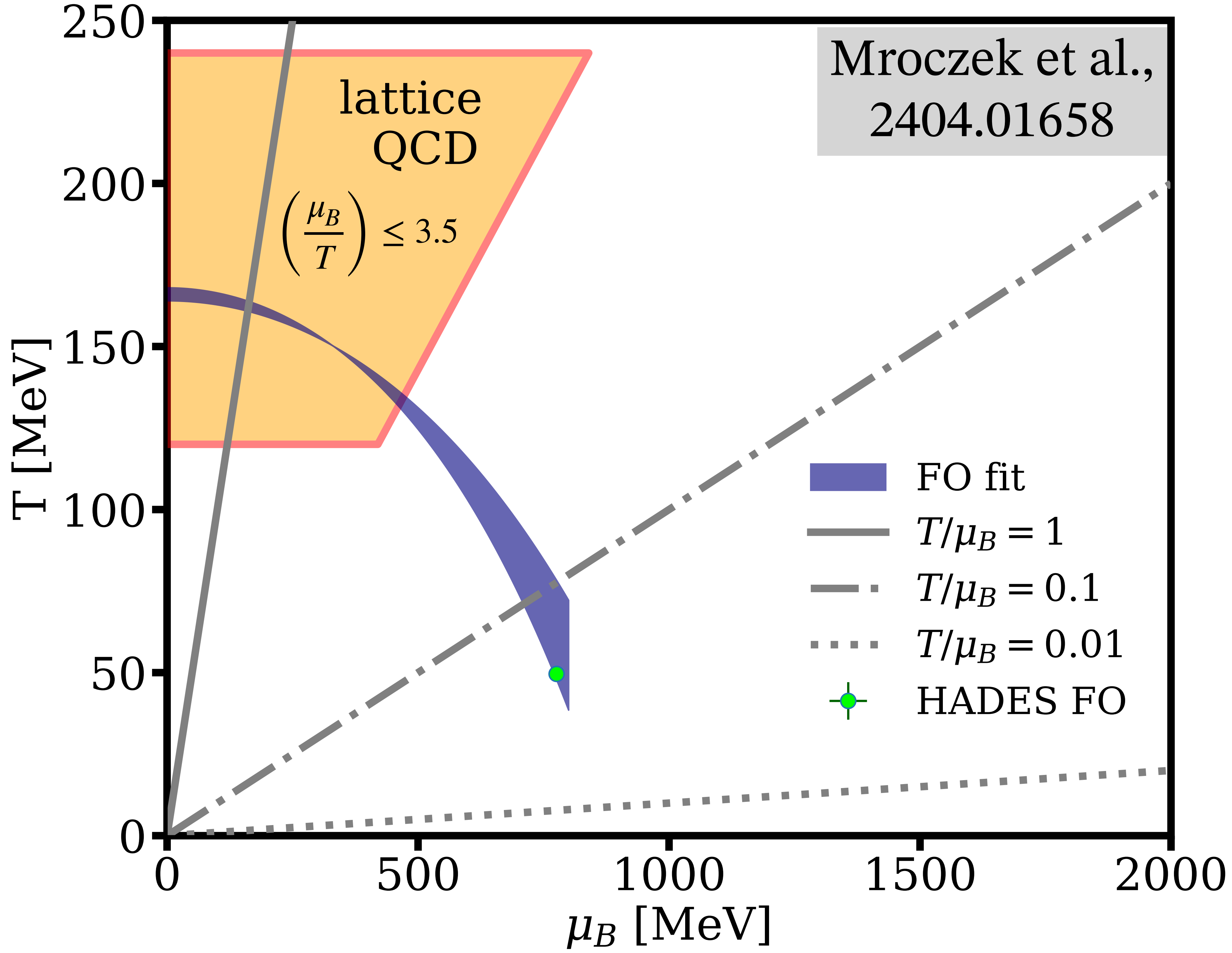
$$p(T,\vec{\mu}) = p_{T=0} + \cancel{\left.\frac{\partial p}{\partial T}\right|_{T=0,\vec{\mu}}}^{\,0} T + \frac{1}{2}\boxed{\left.\frac{\partial^2 p}{\partial T^2}\right|_{T=0,\vec{\mu}}} T^2 + \dots$$

Entropy!
$s(T=0)=0$

Heat capacity $\left.\dfrac{\partial s}{\partial T}\right|_{T=0} > 0$

$$\boxed{p(T,\vec{\mu}) \approx p_{T=0} + \frac{1}{2}\left.\frac{\partial s}{\partial T}\right|_{T=0,\vec{\mu}} T^2}$$

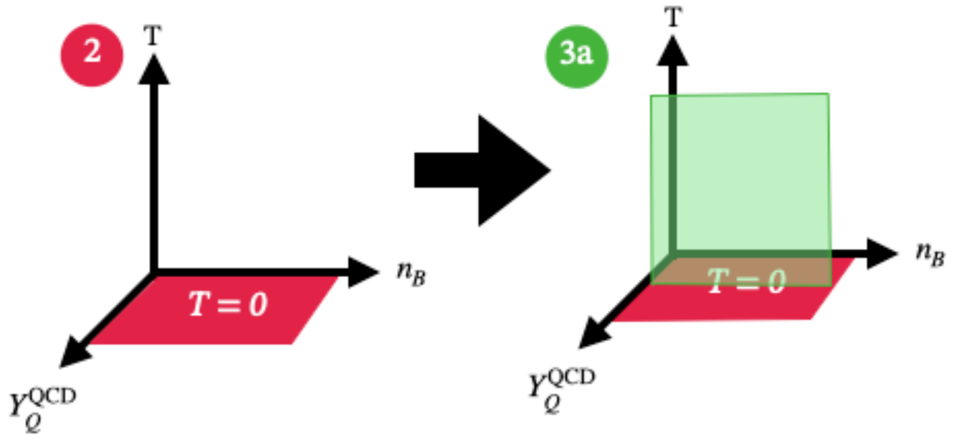
- Special case: Sommerfeld (1928) expansion
- Ideal Fermi systems at $T \ll T_F$,
  $p \approx p_{T=0} + aT^2 + bT^4 + \dots$
- Fermionic **quasi-particles**



- Physical motivation
- Expansion parameter $(T/\mu_B) < 0.1$ in relevant regime
- Overlap with few-GeV $\sqrt{s_{NN}}$ freeze-out (FO)

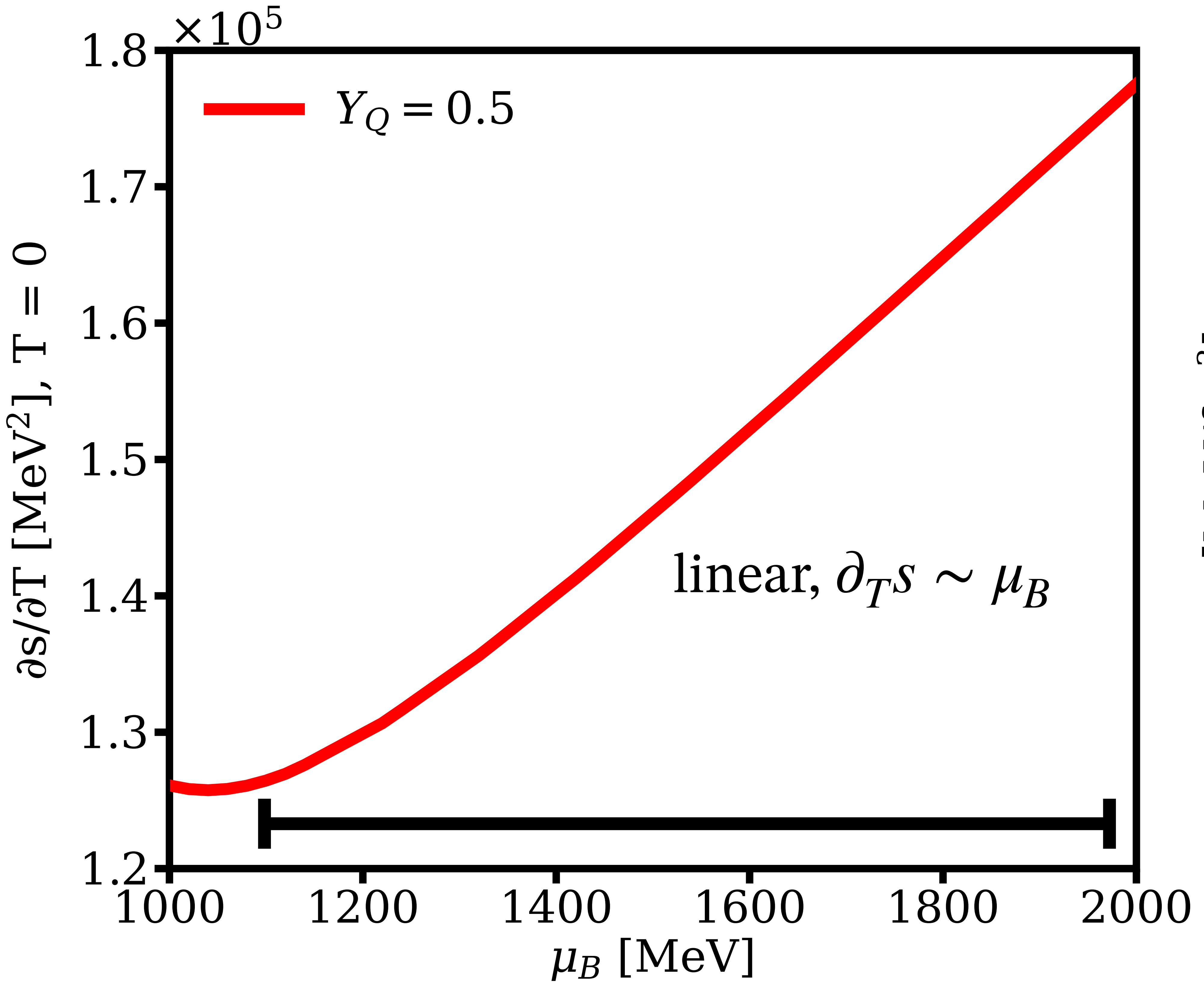
FO fit from Cleymans et al, PRC 73 (2006), HADES FO from Harabasz et al, PRC 102 (2020)

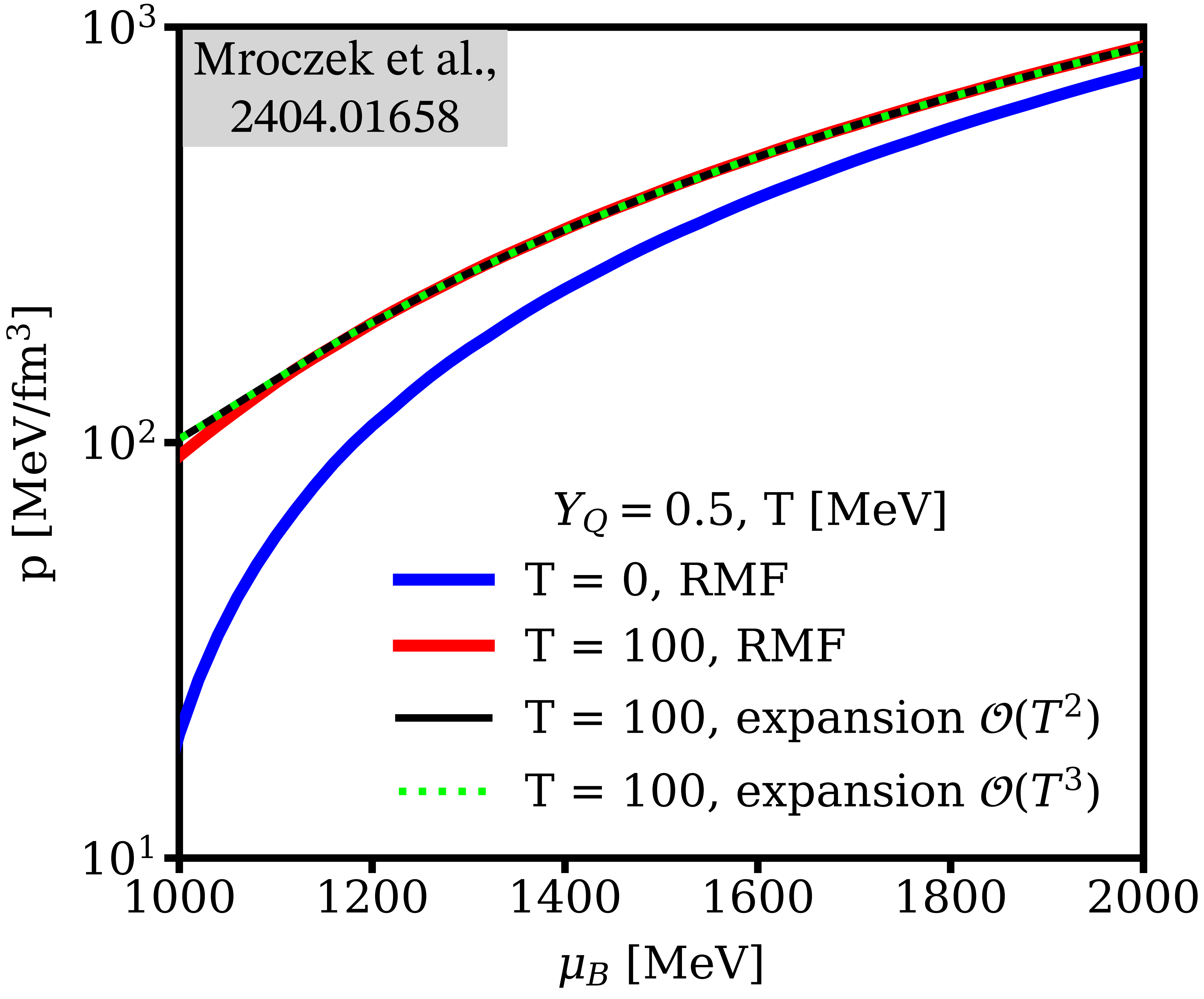
From T = 0 to finite T, test with microscopic model



- Numerical tests with relativistic mean-field (RMF) theory (n+p) well suited for the expansion

T^2 term captures the finite temperature behavior of the pressure to high accuracy when $\partial s/\partial T$ is known





- Breakdown near liquid-gas PT
- Linear coefficient

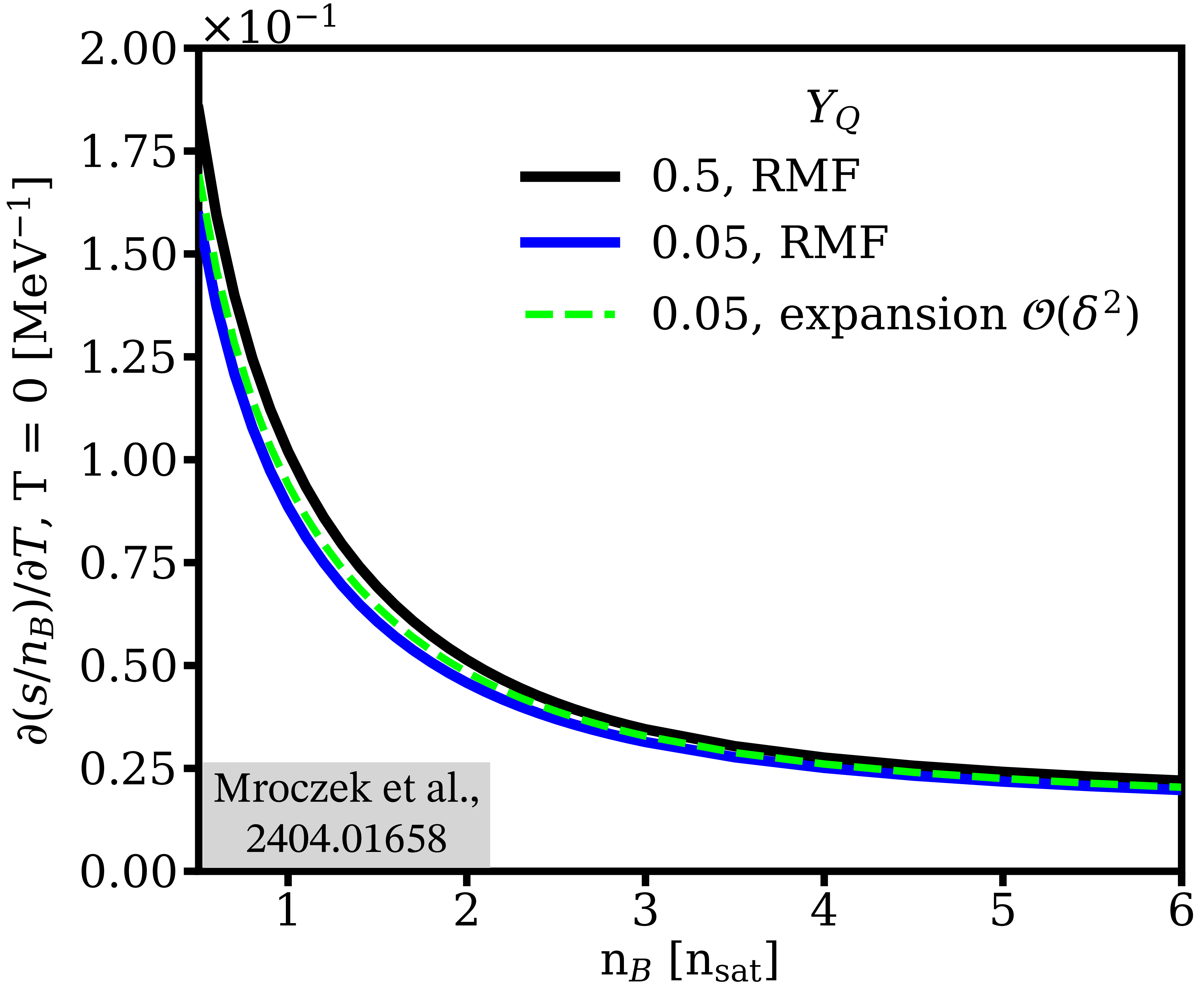
→ easy to parametrize

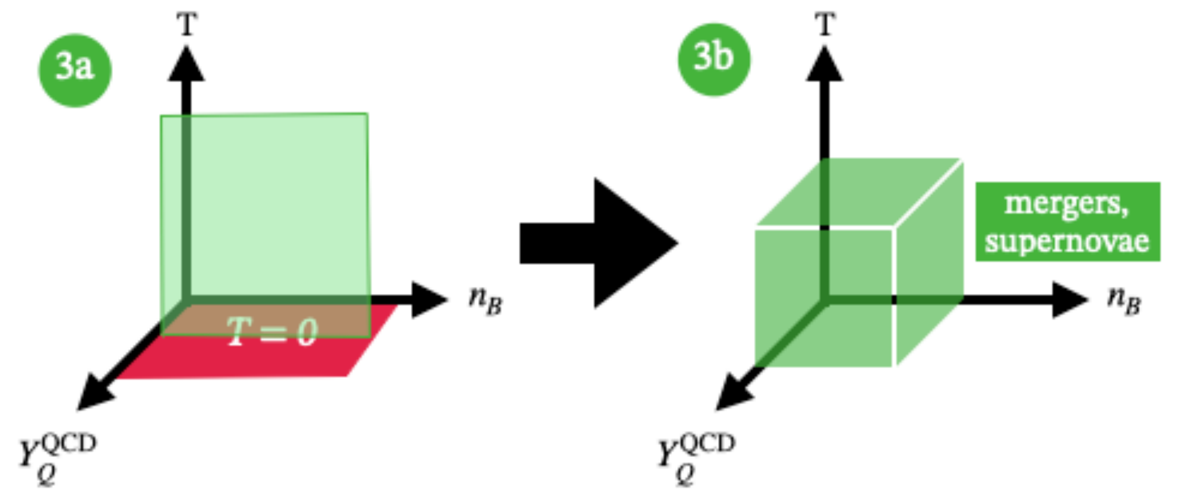
- T^2 term dominates

But: must know $\partial_T s$ for all μ_B, μ_Q

Microscopic model: RMF theory from Alford et. al PRC 106, (2022)

Charge fraction dependence of finite temperature effects





- Motivation: s/n_B for a given (Z/A, $\sqrt{s_{NN}}$) can be extracted from thermal fits of particle yields

→ Expand $\partial_T(s/n_B)$ about SNM assuming isospin symmetry

- **New expansion**:

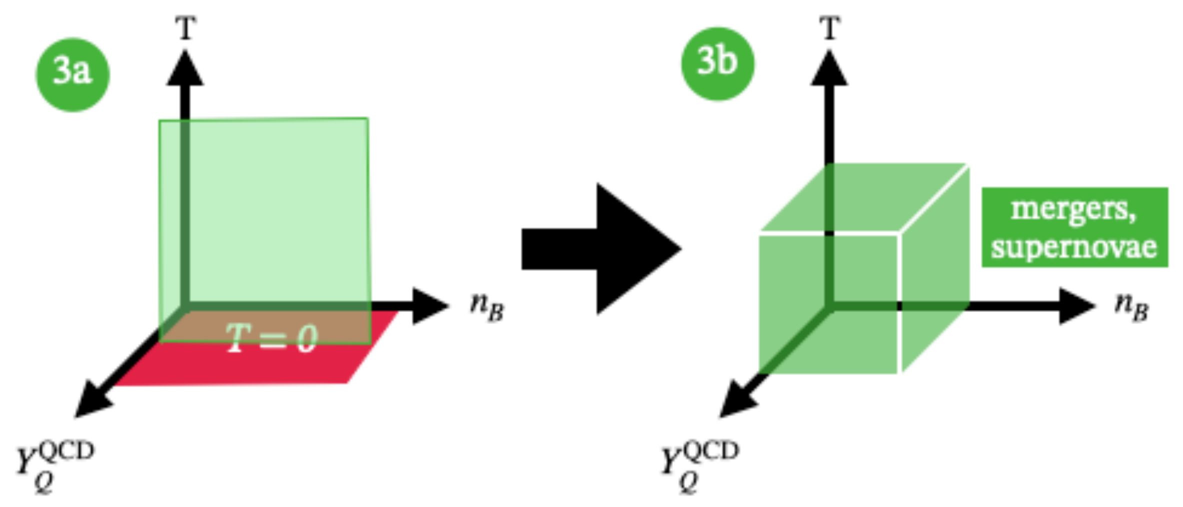
Heat capacity at $Y_Q^{QCD} = 0.5$

$$\left.\frac{\partial \tilde{S}(T, n_B, Y_Q)}{\partial T}\right|_{T=0} = \frac{1}{n_B}\boxed{\left.\frac{\partial s_{SNM}(T, n_B, Y_Q)}{\partial T}\right|_{T=\delta=0}} + \frac{1}{2}\left(1 - 2Y_Q\right)^2 \boxed{\left.\frac{\partial^3 \tilde{S}_{SNM,2}(T, n_B, \delta = 0)}{\partial T \partial \delta^2}\right|_{T=\delta=0}}$$

Heat capacity dependence on Y_Q

Heat capacity across all $\vec{\mu}$ can be extracted from microscopic models

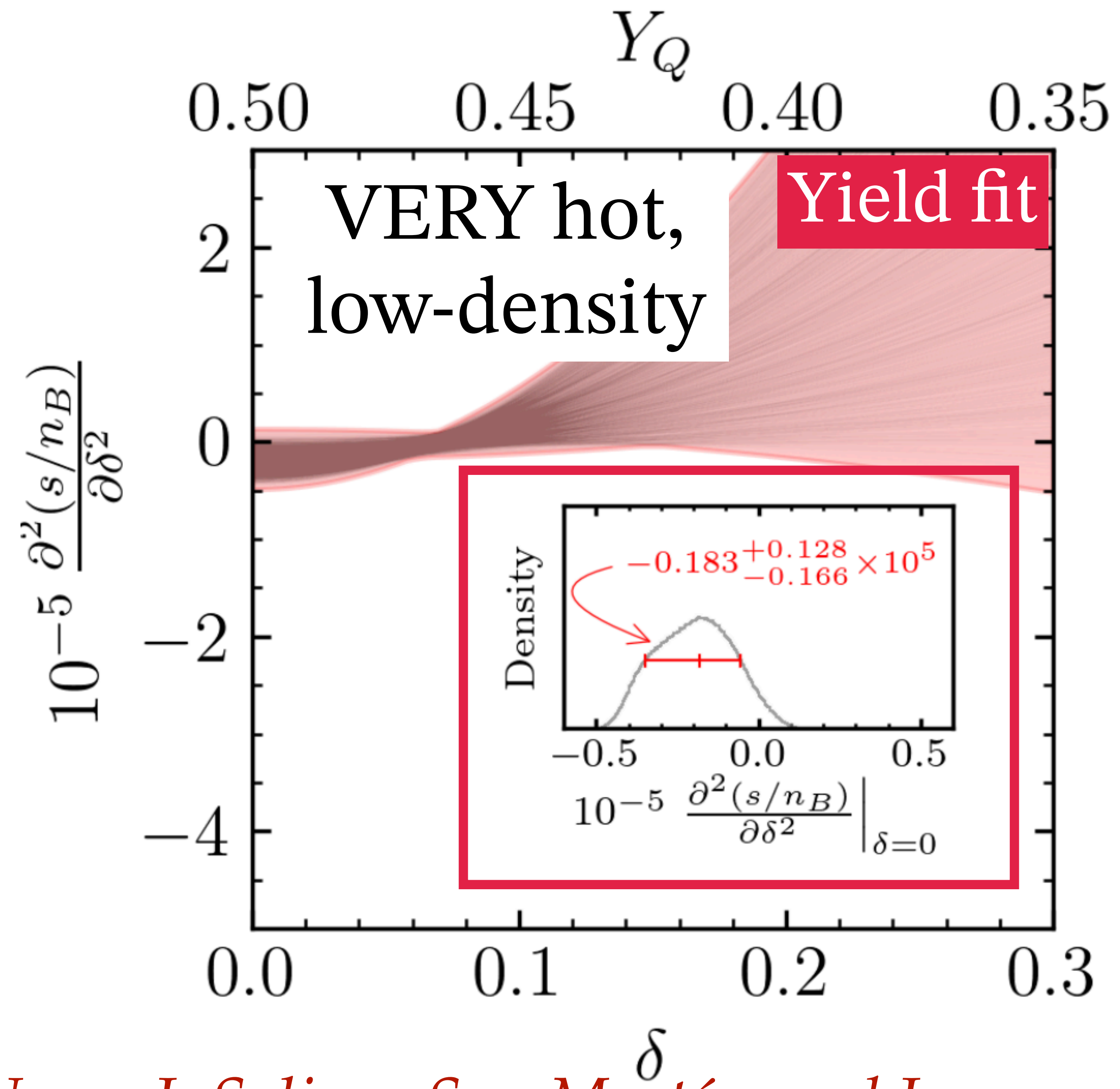
Connection to heavy-ion collisions: system scan



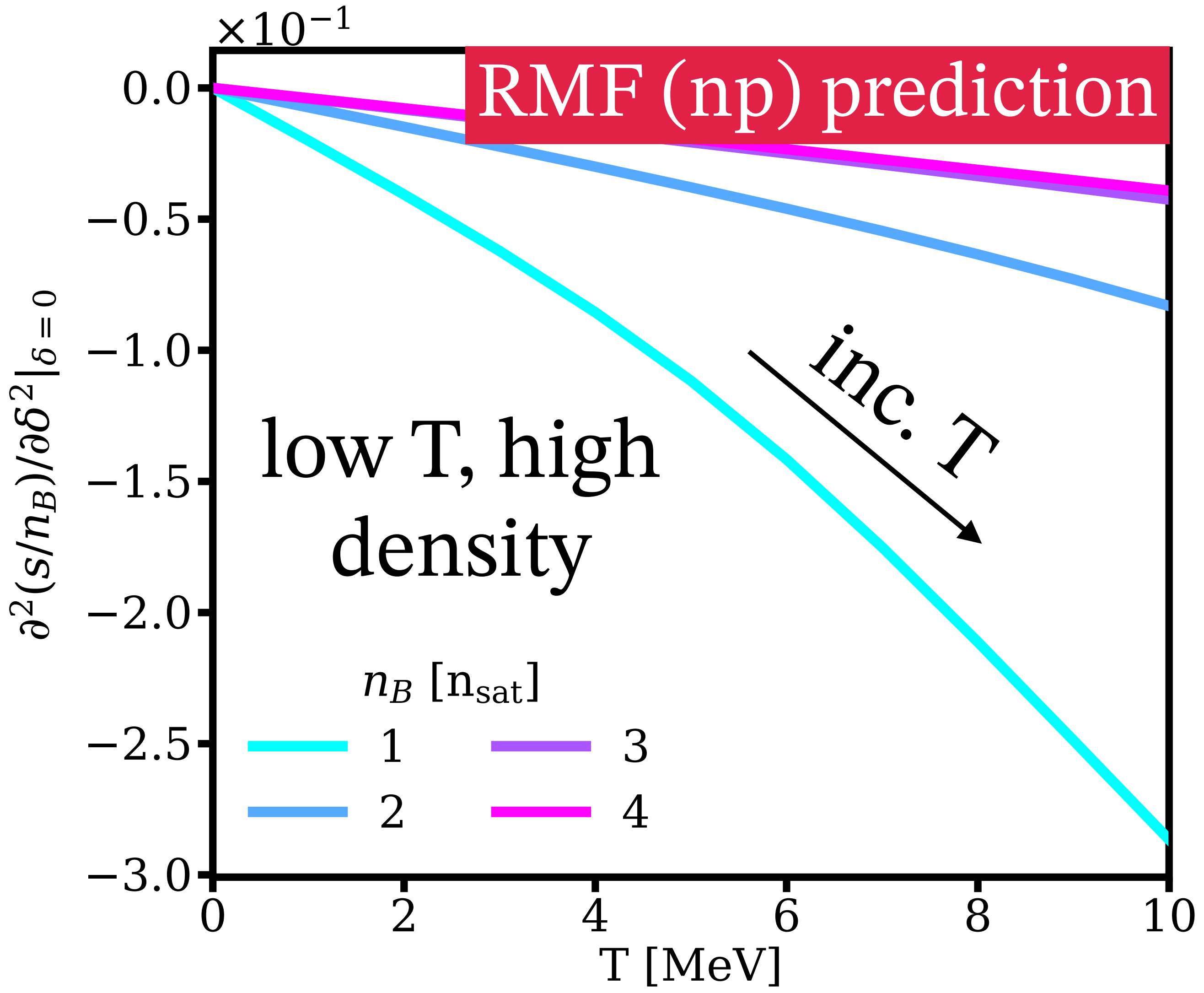
- Nana et al extracted $\partial^2(s/n_B)/\partial\delta^2$ from particle yields across different colliding species, central collisions at $\sqrt{s_{NN}} = 200$ GeV

System	Z	A	Y_Q	Published yield data?
O+O	8	16	0.500	no
Cu+Cu	29	63	0.460	yes
Ru+Ru	44	96	0.458	no*
Zr+Zr	40	96	0.417	no*
Au+Au	79	198	0.399	yes
U+U	92	238	0.387	yes

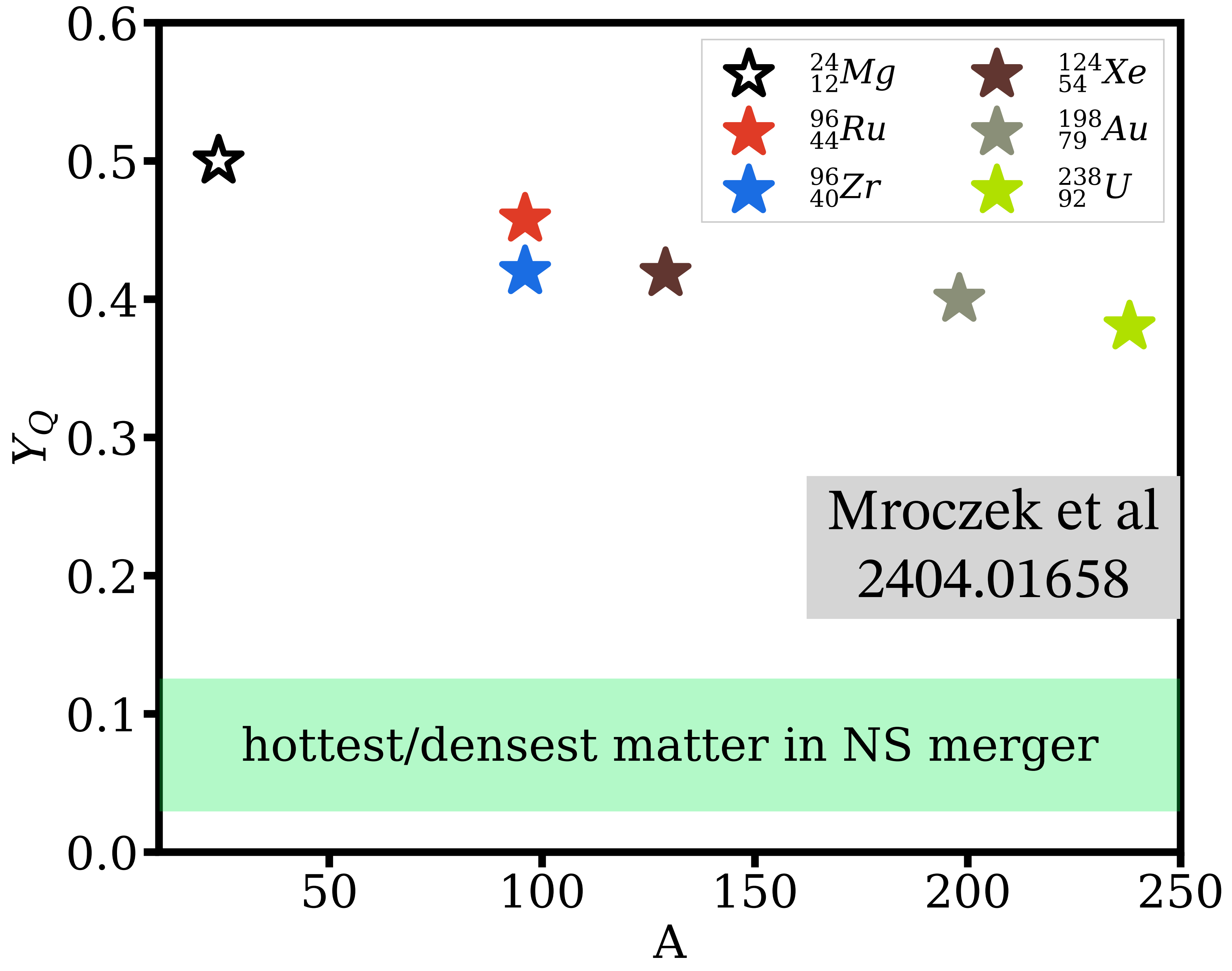
Fits predict a **large and negative** value for $\partial^2(s/n_B)/\partial\delta^2$ at $T_{FO} \sim 145$ MeV, $n_B \sim 0.025\ n_{sat}$, in **qualitative agreement** with RMF (n+p) results



F. Nana, J. Salinas San Martín, and J. Noronha-Hostler, 2411.03705



Mroczek et al 2404.01658



- Needed: **system + energy scan**
- **Symmetric nuclei**, e.g., O+O, crucial for extracting the expansion coefficient at $\delta = 0$

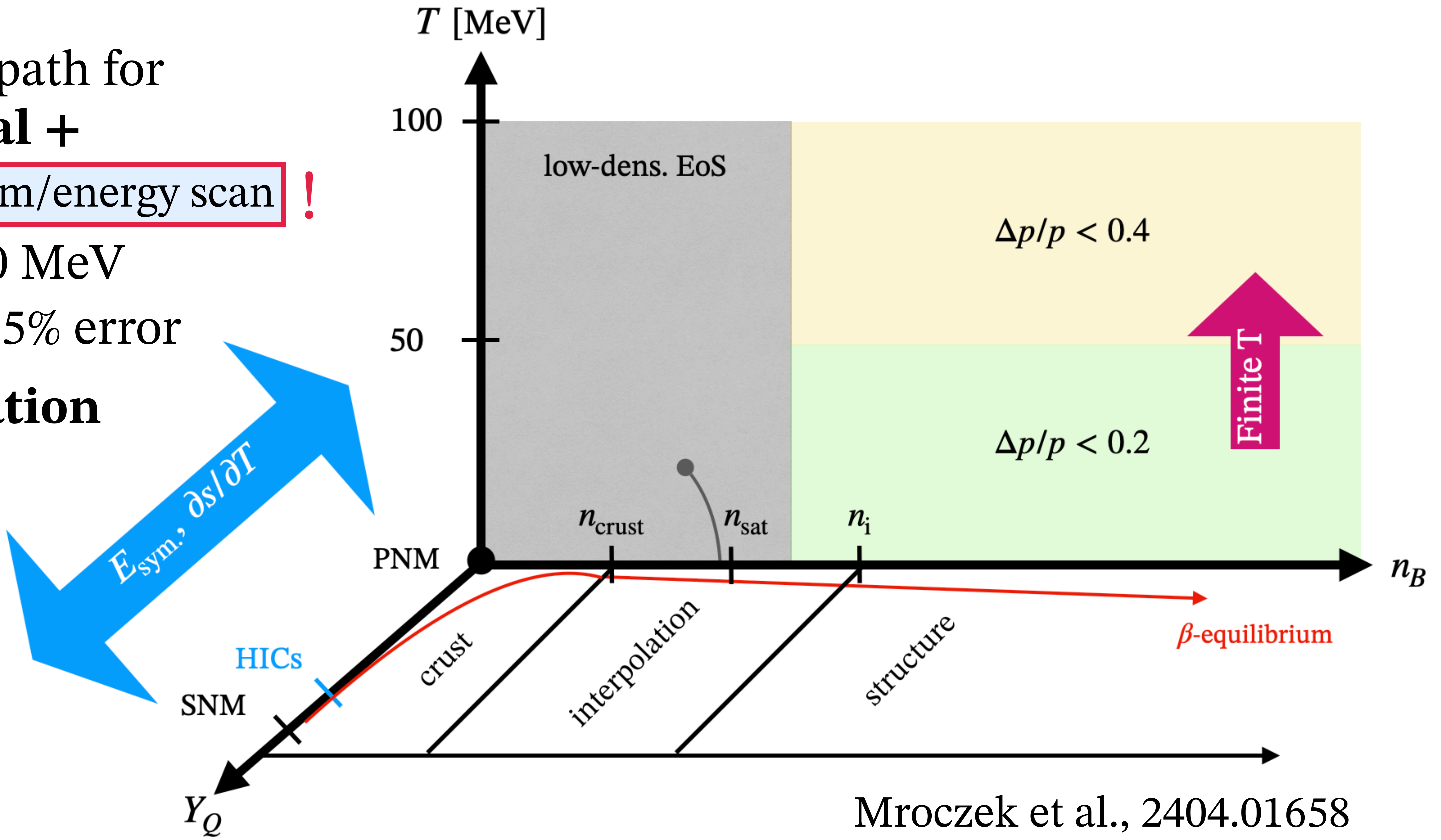
LHC, CBM @ FAIR?

Summary

- Proposed: **two new expansions** for obtaining finite T, Y_Q equation of state
- Allows for beyond np degrees of freedom, path for incorporating **theoretical + experimental + observational information →** HIC system/energy scan !
- Reproduce a microscopic EOS up to T=100 MeV for $\mu_B \gtrsim 1100$ MeV ($\sim 1-2\ n_{\rm sat}$) within 5% error
- Clear method for **uncertainty quantification**

problem:

β-equilibrium $\{p(n_B), Y_Q(n_{n_B})\} \rightarrow$ 3D EOS (T, n_B, Y_Q)



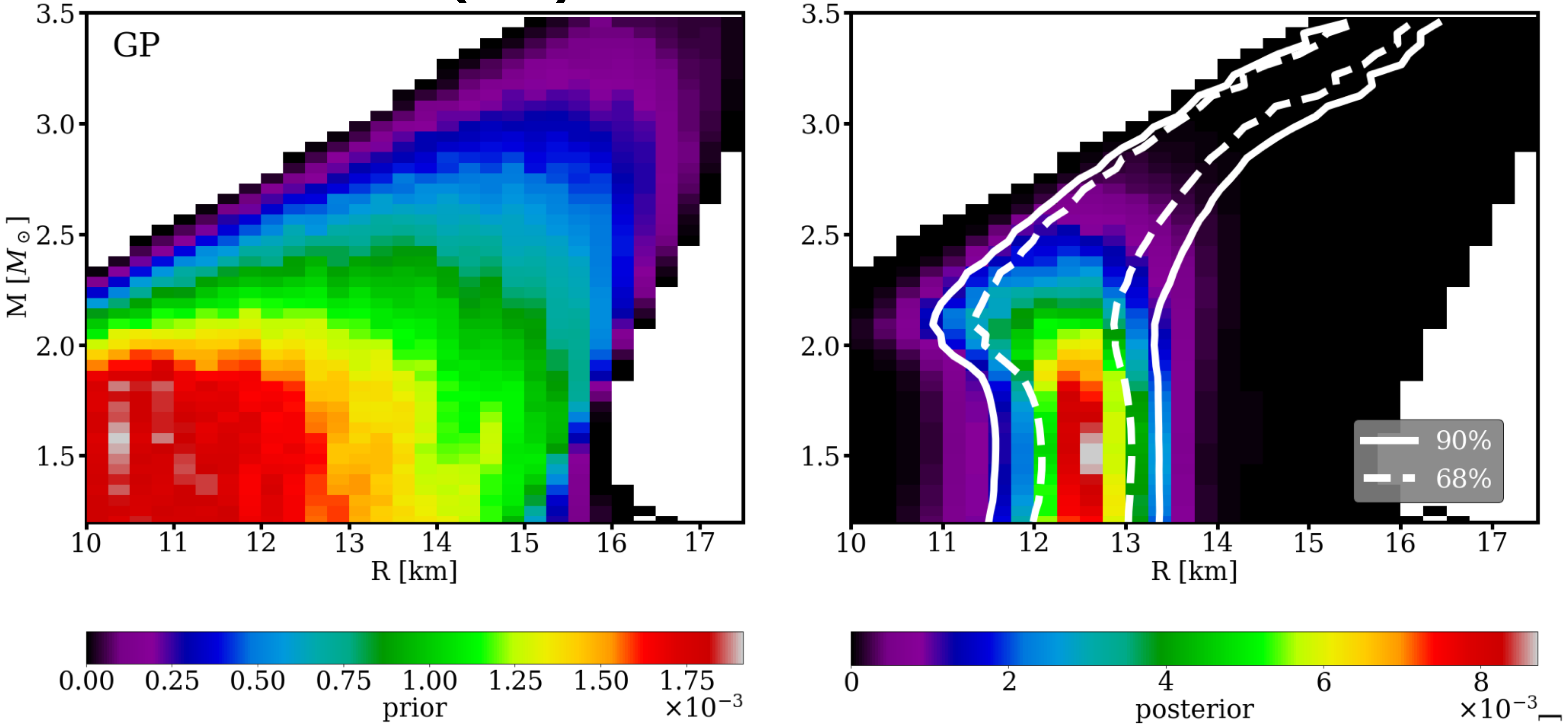
Mroczek et al., 2404.01658

Outlook

- Caveats: no strangeness, no phase transitions → both solvable
- Future study: reducing numerical error, **low-density EOS** at finite T, Y_Q (e.g. hadron resonance gas)

Are M-R posteriors sensitive to structure in $c_s^2(n_B)$?

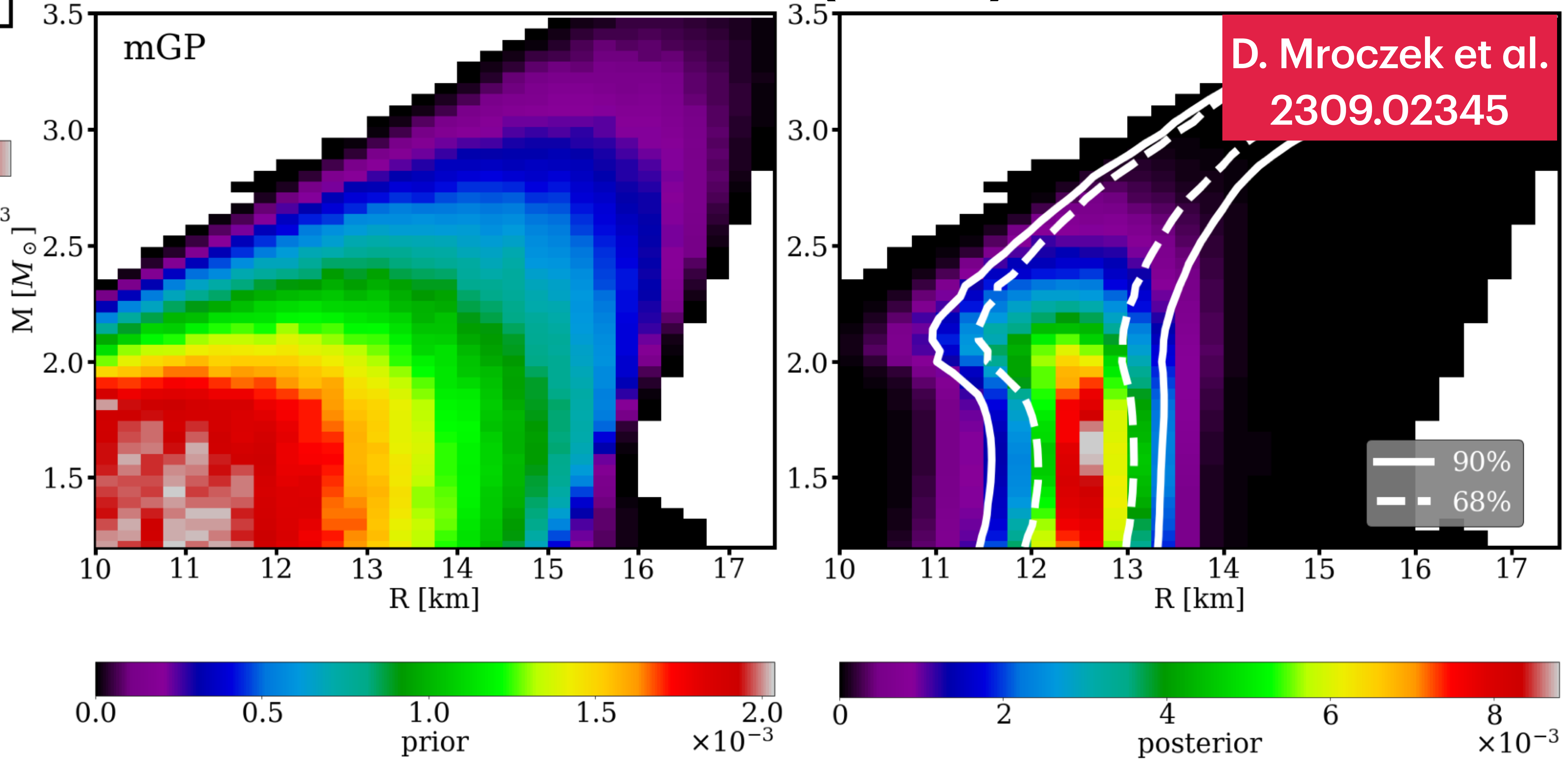
Benchmark (GP)



Why check M-R priors and posteriors?

- Diverse neutron star EoS prior = broad prior in M-R
- Sanity check: can we reproduce measurements when we assume nontrivial features in $c_s^2(n_B)$?

Benchmark + structure (mGP)



We find **no differences** in the prior or posterior probability distributions between benchmark (GP) and benchmark + structure (mGP)

Astrophysical and theoretical constraints

*more on pQCD later

Astrophysics

- $\mathbf{M_{max}}$: 3 highest measured NS masses from Shapiro-delay measurements (~2.0 $M_{\odot}$)
- $\mathbf{\Lambda}$: GW170817 and GW190425 ($\tilde{\Lambda}, M_{ch}, M_1, M_2$)
- **M-R:** NICER (IL/MD) PSR J0740 + 6620, PSR J0030 + 0451

See Mroczek et al. 2309.02345 for refs.

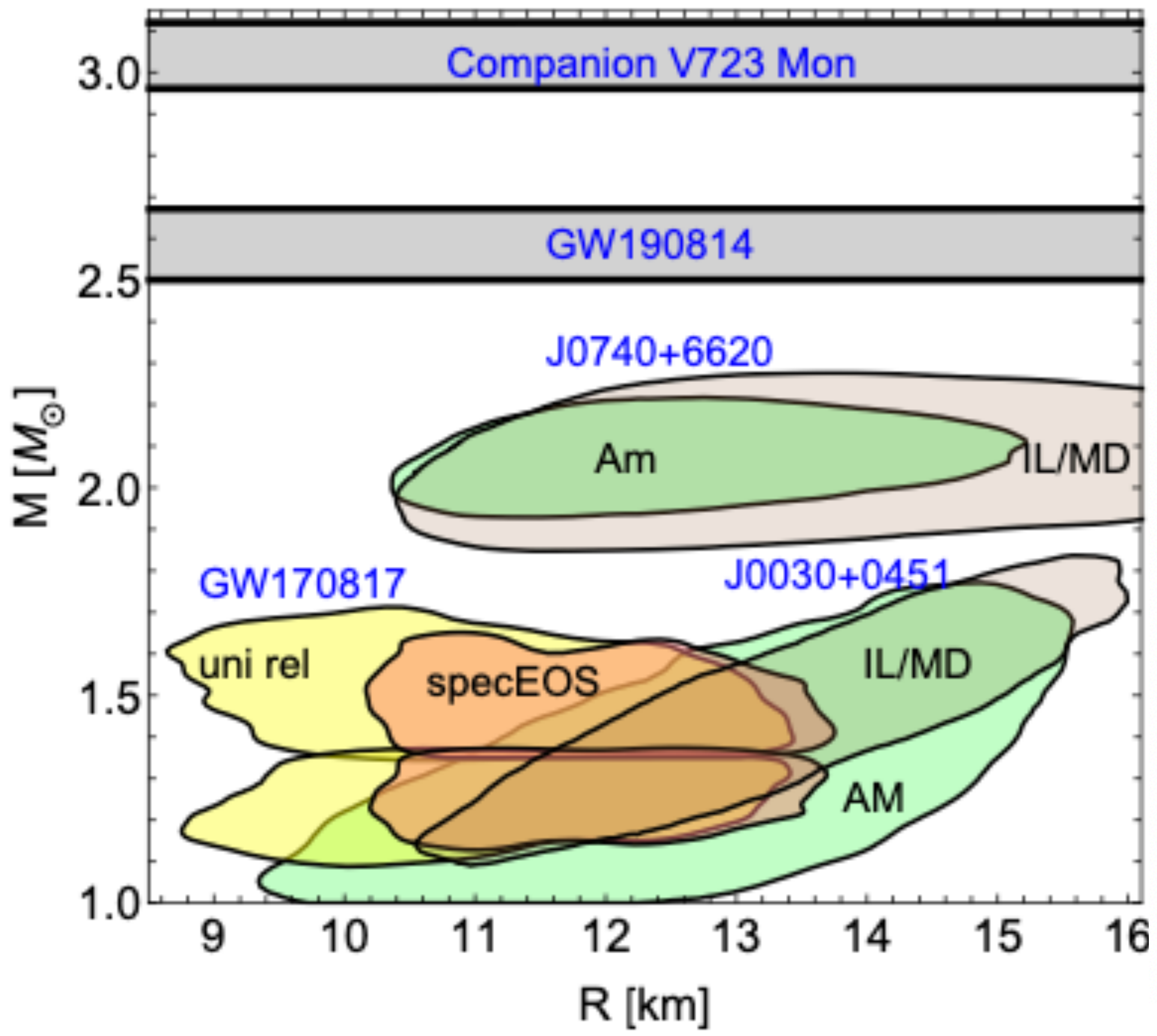
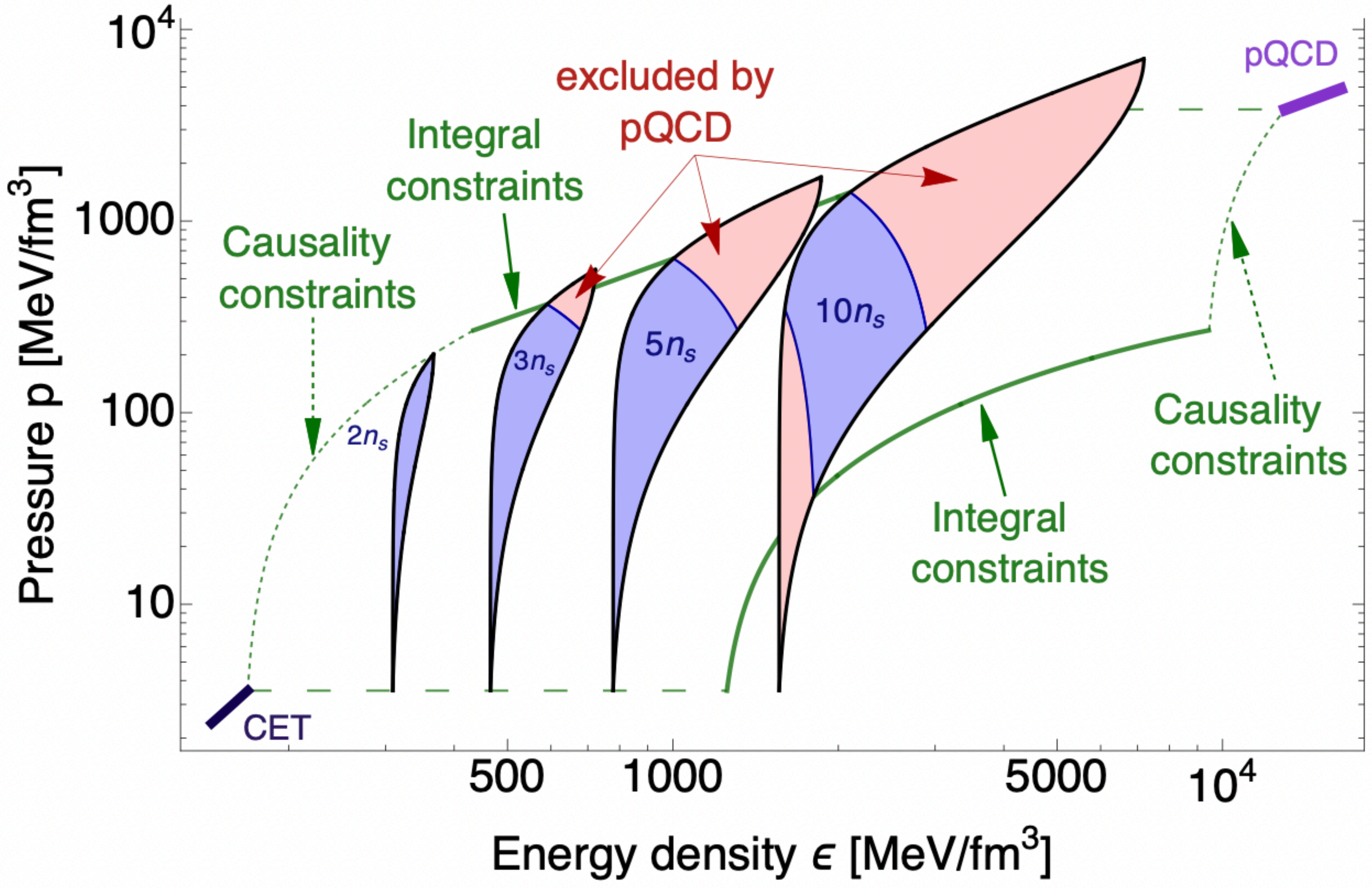


Fig. from: Tan et al. PRD (2022), see for refs.



pQCD constraints: Komoltsev, Kurkela PRL (2022), Gorda et al. Astrophys. J. (2023)

Low-energy

Symmetry energy:
$E_{sym} = 32 \pm 2$ MeV

Tsang et al. PRC (2012)

pQCD*

- partial N3LO results, propagated using causality, stability, and integral constraints down to $\mathbf{n_B^{max}}$ for each EoS.
- Truncated expansion uncertainty accounted for with scale-averaging.

pQCD results: Gorda et al. PRL 127 (2021) and PRD 104 (2021)