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Constraints on the weak mixing angle from future facilities & global QCD analysis

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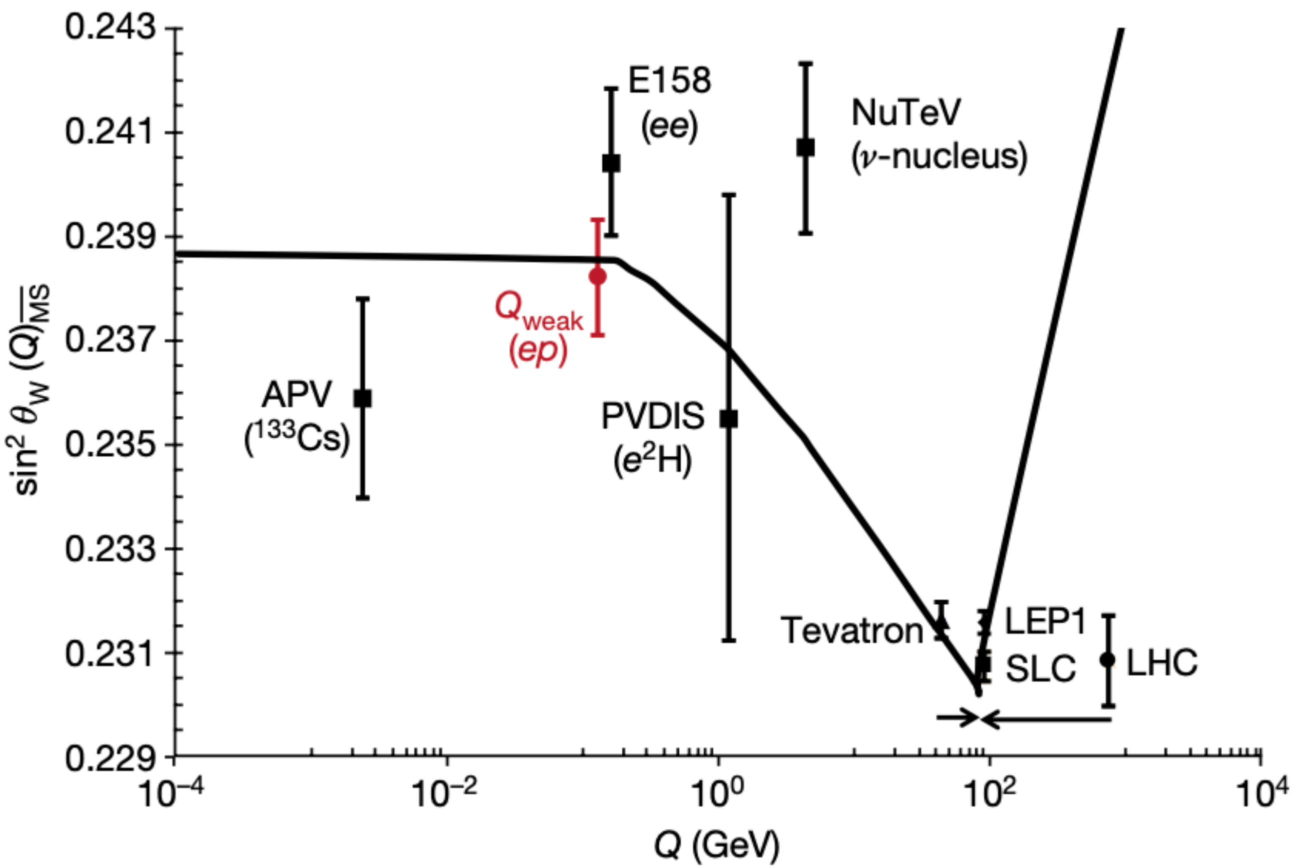
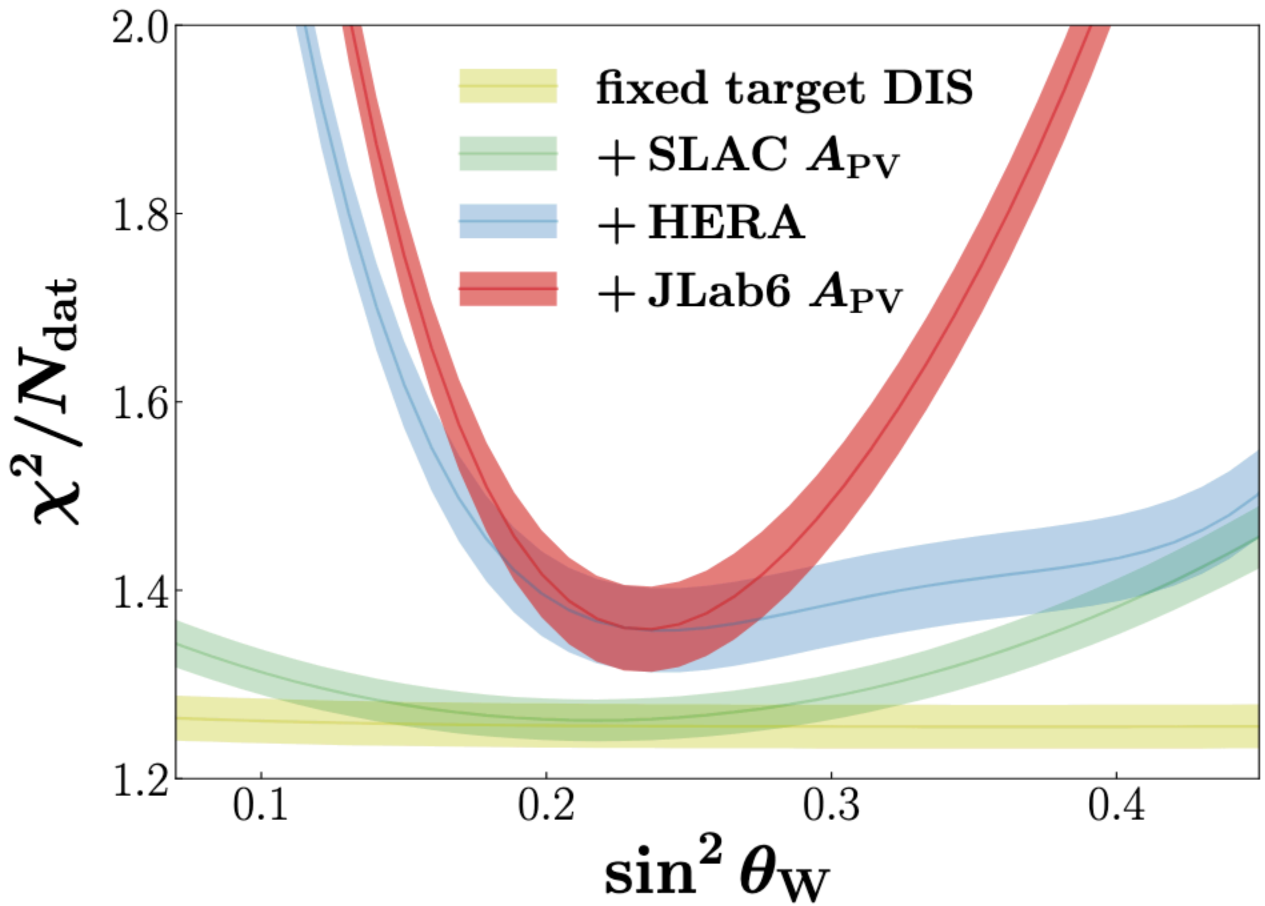
Existing empirical determinations of the running of $\sin^2\theta_W$



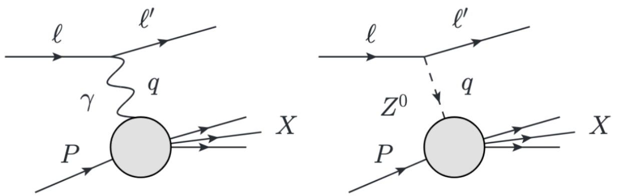
Jefferson Lab Qweak Collaboration
*Nature **557**, 207 (2018)*

■ Sensitivity to $\sin^2\theta_W$ from global QCD analysis



⟶ existing PVDIS data from JLab 6 GeV experiment provides some constraint on weak mixing angle

Parity-violating deep-inelastic scattering



$$\frac{\mathrm{d}\sigma_{\lambda_\ell S}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y} = \frac{2\pi\alpha^2 y}{Q^4}\sum_i \eta_i C_i L^{\gamma}_{\mu\nu} W^{\mu\nu}_i$$

$$\eta_\gamma = 1, \quad \eta_{\gamma Z} = \frac{M_Z^2 G_F}{2\sqrt{2}\pi\alpha}\frac{Q^2}{Q^2+M_Z^2}, \quad \eta_Z = \eta_{\gamma Z}^2$$

$$C_\gamma = 1, \quad C_{\gamma Z} = -(g_V^e + Q_\ell \lambda_l g_A^e), \quad C_Z = C_{\gamma Z}^2$$

$$L^{\gamma}_{\mu\nu} = 2\left(\ell_\mu \ell'_\nu + \ell'_\mu \ell_\nu - g_{\mu\nu}\ell\cdot l' - i\lambda_\ell \epsilon_{\mu\nu\alpha\beta}\ell^\alpha \ell'^\beta\right)$$

$$W^{\mu\nu}_i = -\tilde{g}^{\mu\nu}F_1^i(x_\mathrm{B},Q^2) + \frac{\widetilde{P}^\mu \widetilde{P}^\nu}{P\cdot q}F_2^i(x_\mathrm{B},Q^2) + i\epsilon^{\mu\nu\alpha\beta}\frac{P_\alpha q_\beta}{2P\cdot q}F_3^i(x_\mathrm{B},Q^2)$$

⟶ PVDIS asymmetry

$$A_\mathrm{PV} = \frac{\mathrm{d}\sigma_{LU}}{\mathrm{d}\sigma_{UU}}$$

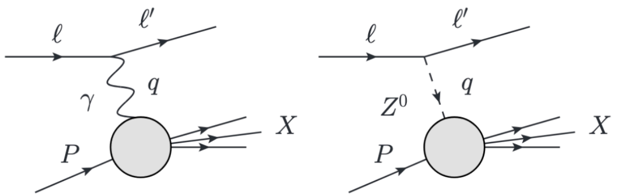
$$\frac{\mathrm{d}\sigma_{UU}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y} = \frac{1}{4}\left[\frac{\mathrm{d}\sigma_{++}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y} + \frac{\mathrm{d}\sigma_{+-}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y} + \frac{\mathrm{d}\sigma_{-+}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y} + \frac{\mathrm{d}\sigma_{--}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y}\right]$$

unpolarized

$$\frac{\mathrm{d}\sigma_{LU}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y} = \frac{1}{4}\left[\left(\frac{\mathrm{d}\sigma_{++}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y} + \frac{\mathrm{d}\sigma_{+-}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y}\right) - \left(\frac{\mathrm{d}\sigma_{-+}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y} + \frac{\mathrm{d}\sigma_{--}}{\mathrm{d}x_\mathrm{B}\,\mathrm{d}y}\right)\right]$$

lepton polarized

Parity-violating deep-inelastic scattering



→ unpolarized

$$\frac{\mathrm{d}\sigma_{UU}}{\mathrm{d}x_{\mathrm{B}}\,\mathrm{d}y} = \frac{4\pi\alpha^2}{x_{\mathrm{B}}\,y\,Q^2}\Big[K_1(x_{\mathrm{B}},y,Q^2)F_1^{UU}(x_{\mathrm{B}},Q^2) + K_2(x_{\mathrm{B}},y,Q^2)F_2^{UU}(x_{\mathrm{B}},Q^2) - Q_\ell K_3(x_{\mathrm{B}},y,Q^2)F_3^{UU}(x_{\mathrm{B}},Q^2)\Big]$$

$$\begin{aligned} F_1^{UU} &= F_1^{\gamma} - g_V^e\eta_{\gamma Z}F_1^{\gamma Z} + \big[(g_V^e)^2 + (g_A^e)^2\big]\eta_Z F_1^Z \\ F_2^{UU} &= F_2^{\gamma} - g_V^e\eta_{\gamma Z}F_2^{\gamma Z} + \big[(g_V^e)^2 + (g_A^e)^2\big]\eta_Z F_2^Z \\ F_3^{UU} &= \qquad - g_A^e\eta_{\gamma Z}F_3^{\gamma Z} + 2g_V^e g_A^e\eta_Z F_3^Z \end{aligned}$$

$$K_1(x_{\mathrm{B}},y,Q^2) = x_{\mathrm{B}}\,y^2, \quad K_2(x_{\mathrm{B}},y,Q^2) = 1 - y - \frac{x_{\mathrm{B}}^2y^2M^2}{Q^2}, \quad K_3(x_{\mathrm{B}},y,Q^2) = x_{\mathrm{B}}\,y\Big(1-\frac{y}{2}\Big)$$

→ lepton polarized

$$\frac{\mathrm{d}\sigma_{LU}}{\mathrm{d}x_{\mathrm{B}}\,\mathrm{d}y} = \frac{4\pi\alpha^2}{x_{\mathrm{B}}\,y\,Q^2}\Big[Q_\ell K_1(x_{\mathrm{B}},y,Q^2)F_1^{LU}(x_{\mathrm{B}},Q^2) + Q_\ell K_2(x_{\mathrm{B}},y,Q^2)F_2^{LU}(x_{\mathrm{B}},Q^2) - K_3(x_{\mathrm{B}},y,Q^2)F_3^{LU}(x_{\mathrm{B}},Q^2)\Big]$$

$$\begin{aligned} F_1^{LU} &= -g_A^e\eta_{\gamma Z}F_1^{\gamma Z} + 2g_V^e g_A^e\eta_Z F_1^Z \\ F_2^{LU} &= -g_A^e\eta_{\gamma Z}F_2^{\gamma Z} + 2g_V^e g_A^e\eta_Z F_2^Z \\ F_3^{LU} &= -g_V^e\eta_{\gamma Z}F_3^{\gamma Z} + \big[(g_V^e)^2 + (g_A^e)^2\big]\eta_Z F_3^Z \end{aligned}$$

PVDIS asymmetry

$$A_{\rm PV} = \frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\Big[a_1(x_{\rm B},Q^2)Y_1(x_{\rm B},y,Q^2) + a_3(x_{\rm B},Q^2)Y_3(x_{\rm B},y,Q^2)\Big]$$

$$a_1 = 2g_A^e \frac{F_1^{\gamma Z}}{F_1^{\gamma}}, \quad Y_1 = \left(\frac{1+R^{\gamma Z}}{1+R^{\gamma}}\right)\frac{1+(1-y)^2 - \frac{y^2}{2}\left[1+r^2-\frac{2r^2}{1+R^{\gamma Z}}\right]}{1+(1-y)^2 - \frac{y^2}{2}\left[1+r^2-\frac{2r^2}{1+R^{\gamma}}\right]}$$

$$a_3 = g_V^e \frac{F_3^{\gamma Z}}{F_1^{\gamma}}, \quad Y_3 = \left(\frac{1+R^{\gamma Z}}{1+R^{\gamma}}\right)\frac{1-(1-y)^2}{1+(1-y)^2 - \frac{y^2}{2}\left[1+r^2-\frac{2r^2}{1+R^{\gamma}}\right]}$$

$$R^i = \frac{F_2^i}{2x_{\rm B}F_1^i}r^2 - 1$$

$$r^2 = 1 + 4M^2x_{\rm B}^2/Q^2$$

⟶ structure functions at leading order

$$F_1^{[\gamma,\gamma Z,Z]}(x_{\rm B},Q^2) = \frac{1}{2}\sum_q \left[e_q^2,\ 2e_q g_V^q,\ (g_V^q)^2+(g_A^q)^2\right] q^+(x_{\rm B},Q^2)$$

$$F_3^{[\gamma,\gamma Z,Z]}(x_{\rm B},Q^2) = \sum_q \left[0,\ 2e_q g_A^q,\ 2g_V^q g_A^q\right] q^-(x_{\rm B},Q^2)$$

$$A_{\rm PV} = \frac{G_F Q^2}{2\sqrt{2}\pi\alpha}\frac{\sum_q e_q\left[2g_A^e g_V^q\, q^+(x_{\rm B},Q^2) + 2g_V^e g_A^q\, Y_3(y)\, q^-(x_{\rm B},Q^2)\right]}{\sum_q e_q^2\, q^+(x_{\rm B},Q^2)}$$

PVDIS asymmetry for deuteron

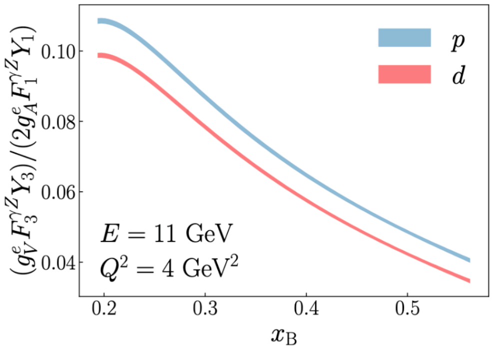
$$A_{\mathrm{PV}}^{D} = \frac{3G_F Q^2}{2\sqrt{2}\pi\alpha}\left[\frac{2g_A^e\left[(2g_V^u - g_V^d) + 2g_V^u R_c - g_V^d R_s\right] + 2g_V^e(2g_A^u - g_A^d)Y_3(y)R_V}{5 + R_s + 4R_c}\right]$$

assuming
$s \approx \bar{s} \quad c \approx \bar{c}$

$$R_s = 2s^+/(u^+ + d^+) \qquad R_c = 2c^+/(u^+ + d^+) \qquad R_V = (u^- + d^-)/(u^+ + d^+)$$

⟶ further assuming $R_c,\, Y_3(y)R_V \ll R_s$ gives

$$A_{\mathrm{PV}}^{D} \approx -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\left[\left(\frac{9}{5} - 4\sin^2\theta_{\mathrm{W}}\right) + \frac{R_s}{25}\right]$$



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sensitivity to weak mixing angle

competing contribution from strangeness

PVDIS asymmetry for deuteron

$$A_{\rm PV}^{D} = \frac{3G_F Q^2}{2\sqrt{2}\pi\alpha}\left[\frac{2g_A^e\left[(2g_V^u - g_V^d) + 2g_V^u R_c - g_V^d R_s\right] + 2g_V^e(2g_A^u - g_A^d)Y_3(y)R_V}{5 + R_s + 4R_c}\right]$$

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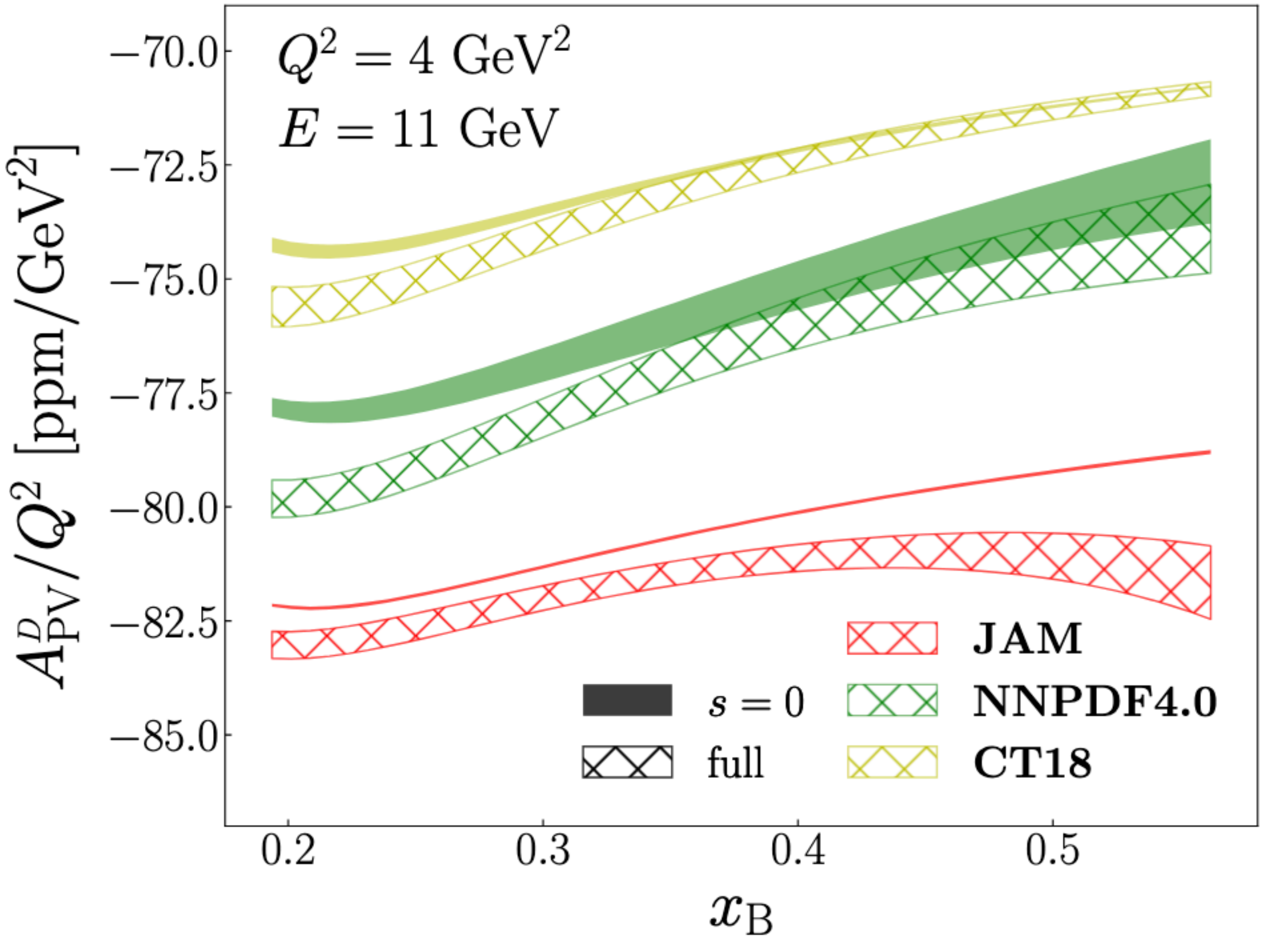
PVDIS asymmetry for proton

$$A_{\rm PV}^{p} \approx -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}(3g_V^u)\frac{1 - (g_V^d/2g_V^u)(d^+/u^+)}{1 + \frac{1}{4}(d^+/u^+)}$$

$\longrightarrow$ unique sensitivity to d/u PDF ratio at high x

PVDIS asymmetry for deuteron



⟶ interplay between weak mixing angle and strange PDF dependence

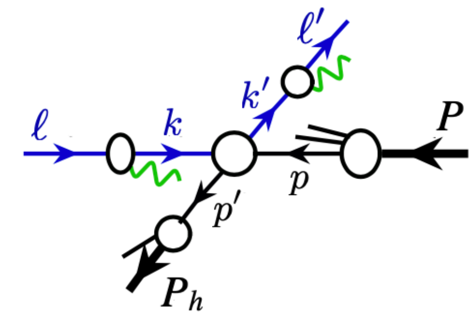
QED radiative corrections

→ hybrid QED+QCD factorization framework

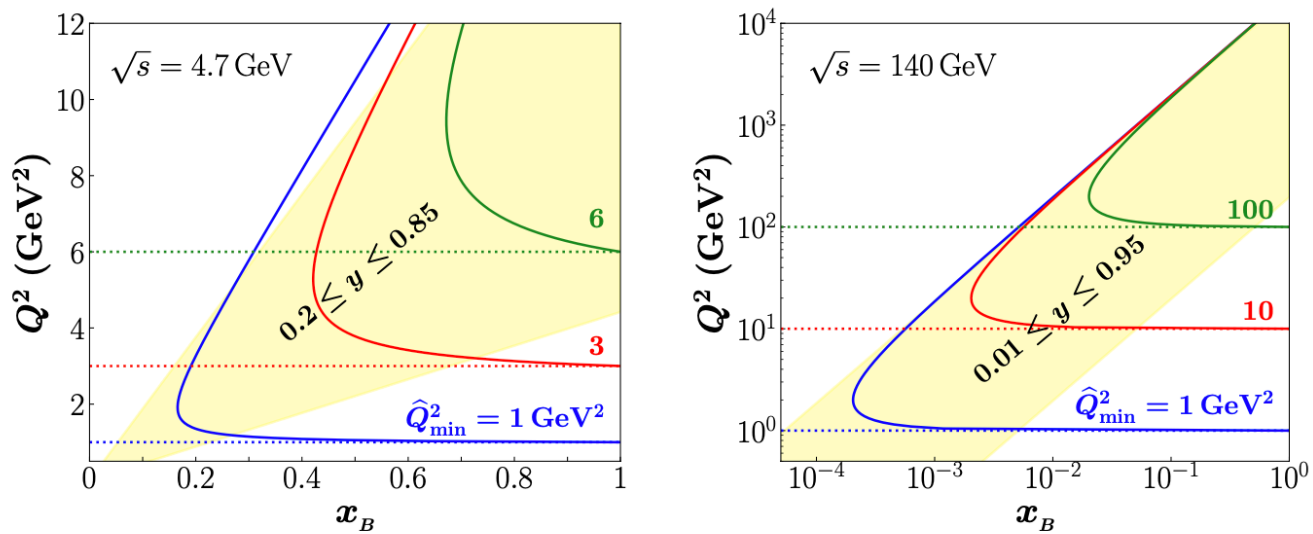
$$\frac{d\sigma_{(U/L)U}}{dx_B\,dy} = \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} D_{e/e}(\zeta,\mu^2) \int_{\xi_{\min}}^{1} d\xi\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\hat{Q}^2}\right] \frac{d\hat{\sigma}_{(U/L)U}}{d\hat{x}_B\,d\hat{y}}$$

lepton fragmentation function (LFF)

lepton distribution function (LDF)



*Liu, WM, Qiu, Sato, JHEP **11**, 157 (2021)*



→ induced QED radiation changes hard scale $Q^2 \to \widehat{Q}^2 = (\xi/\zeta)\,Q^2$, could push "true" scale out of DIS regime even when Q^2 above cut

QED radiative corrections

→ hybrid QED+QCD factorization framework

$$\frac{d\sigma_{(U/L)U}}{dx_B\, dy} = \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} D_{e/e}(\zeta,\mu^2) \int_{\xi_{\min}}^{1} d\xi\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\hat{Q}^2}\right] \frac{d\hat{\sigma}_{(U/L)U}}{d\hat{x}_B\, d\hat{y}}$$

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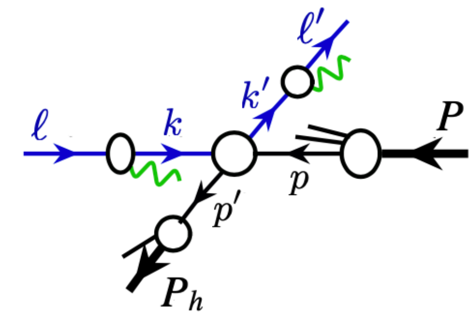

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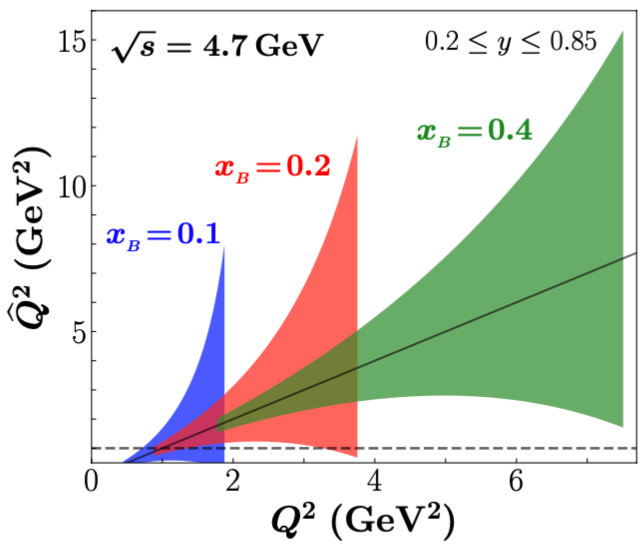
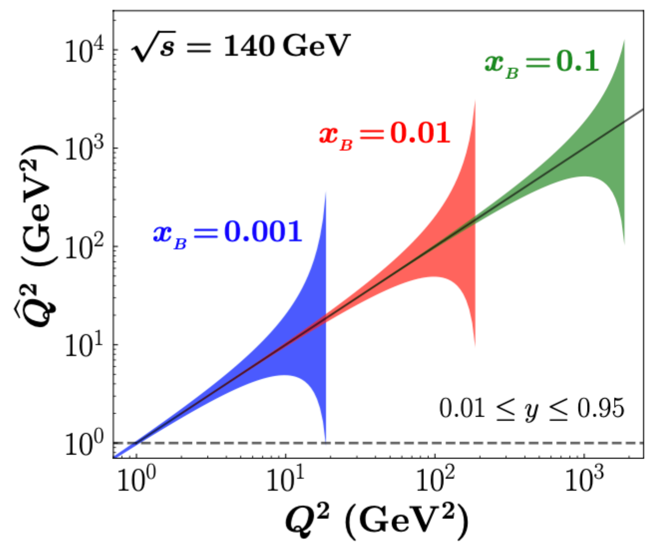


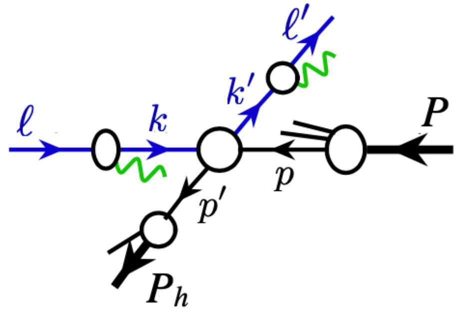


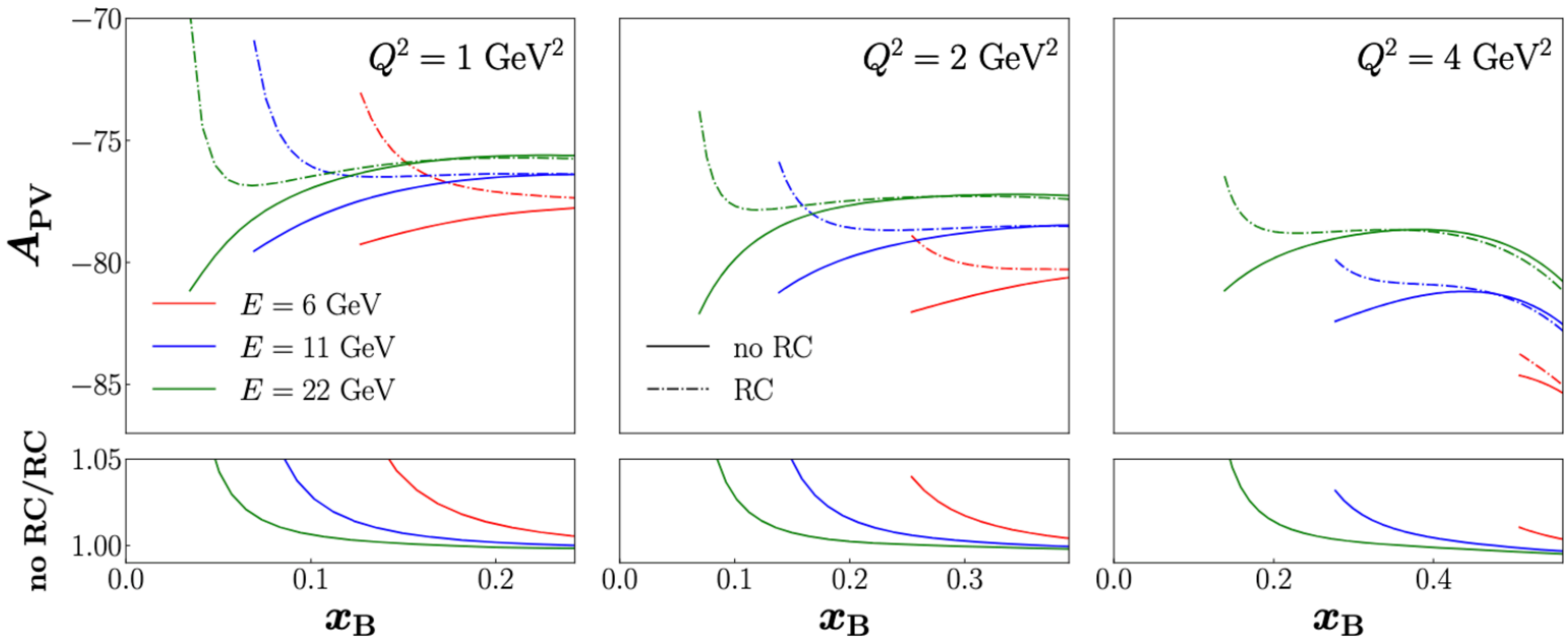
→ for given measured (x_B, Q^2), actually probe colliding nucleon over much wider kinematic region of $(\hat{x}_B, \hat{Q}^2)$

QED radiative corrections

→ hybrid QED+QCD factorization framework

$$\frac{d\sigma_{(U/L)U}}{dx_B\,dy} = \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} D_{e/e}(\zeta,\mu^2) \int_{\xi_{\min}}^{1} d\xi\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\hat{Q}^2}\right] \frac{d\hat{\sigma}_{(U/L)U}}{d\hat{x}_B\,d\hat{y}}$$



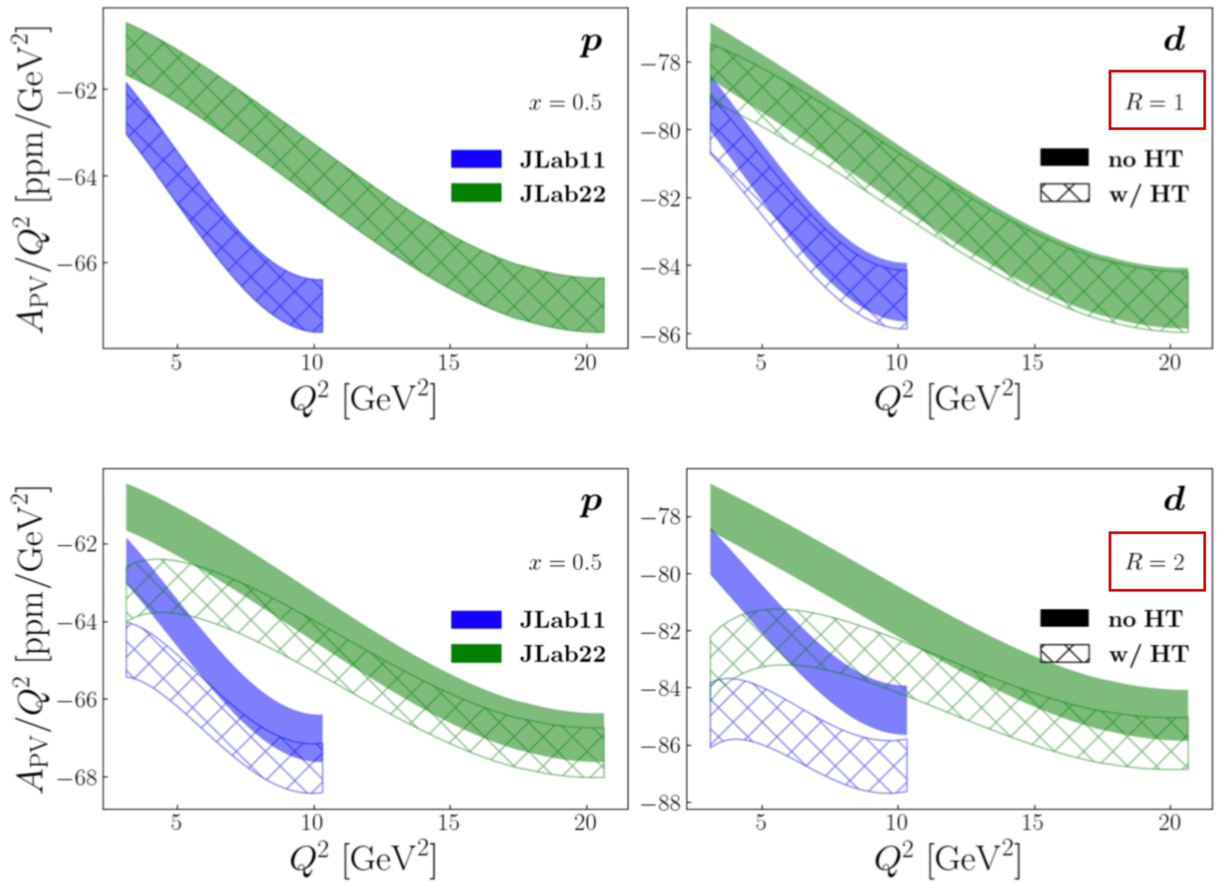


→ effects of RCs ameliorated with increasing energy of the lepton beam

Higher twist corrections

→ assume multiplicative form $F_i^j = F_{i,\mathrm{LT}}^j \left(1 + \frac{H_i^j}{Q^2}\right)$

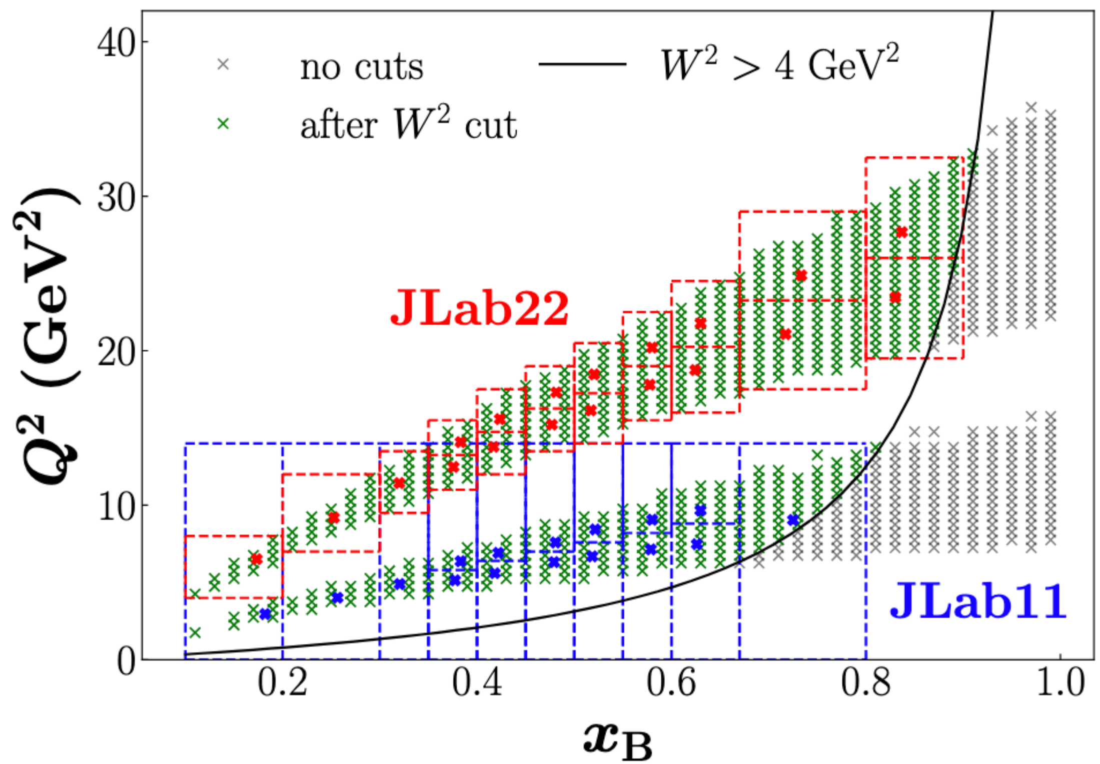
→ H_2^γ extracted from JAM global QCD analyses,
but $H_i^{\gamma Z} = R\, H_2^\gamma$ unknown … study dependence on R



→ need to include possible HT effects global analysis of PVDIS data

Impact study

→ pseudo-data for JLab 11 GeV and 22 GeV kinematics with SoLID



Experimental configuration

→ $P = 85\,\%$

→ $d\mathscr{L}/dt = 4.85 \times 10^{38}\,\text{cm}^{-2}\,\text{s}^{-1}$

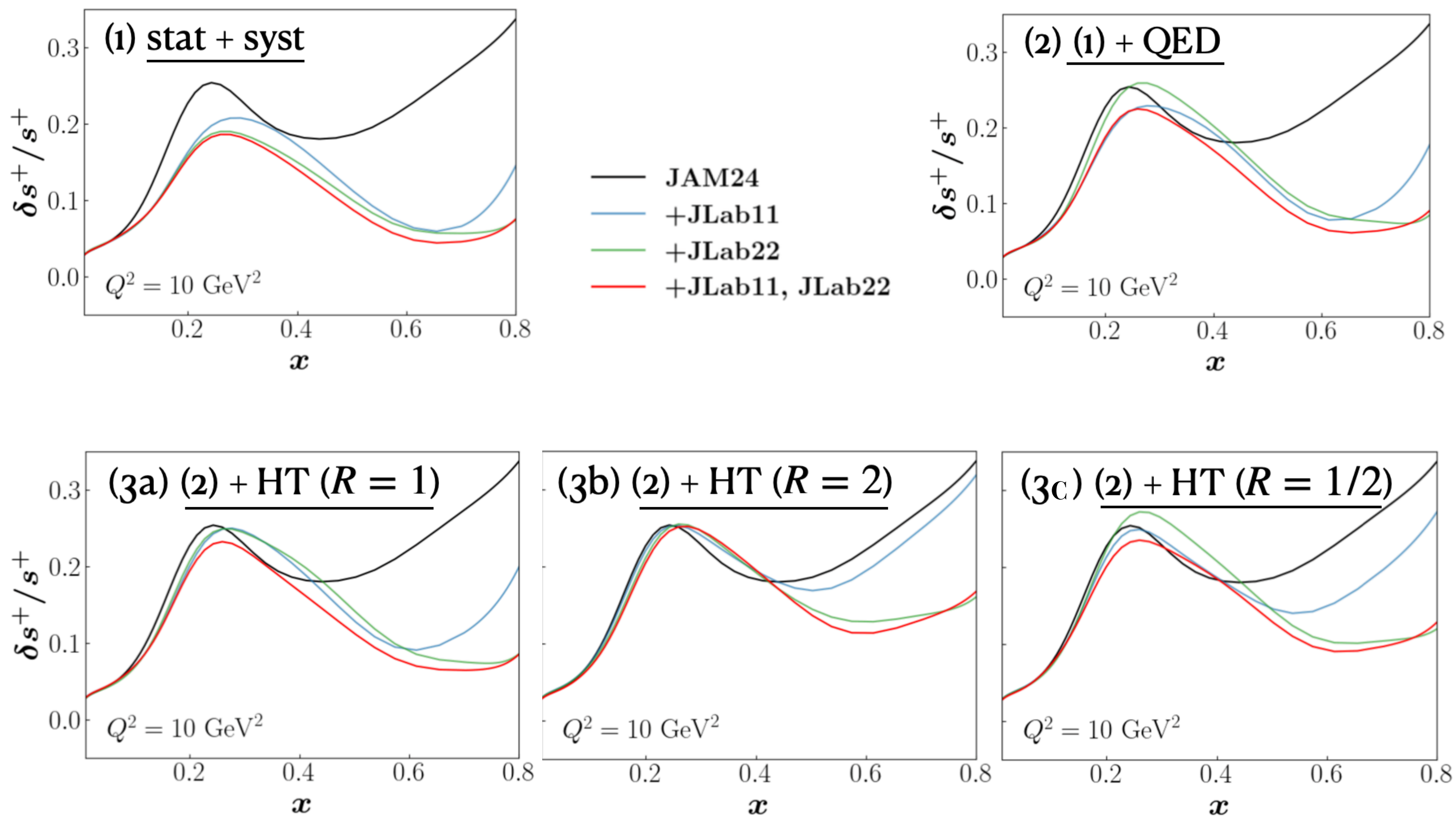
→ run time: 50 days/target

→ $\delta^{\text{syst}} A_{\text{PV}} = 0.5\,\%$

Scenarios

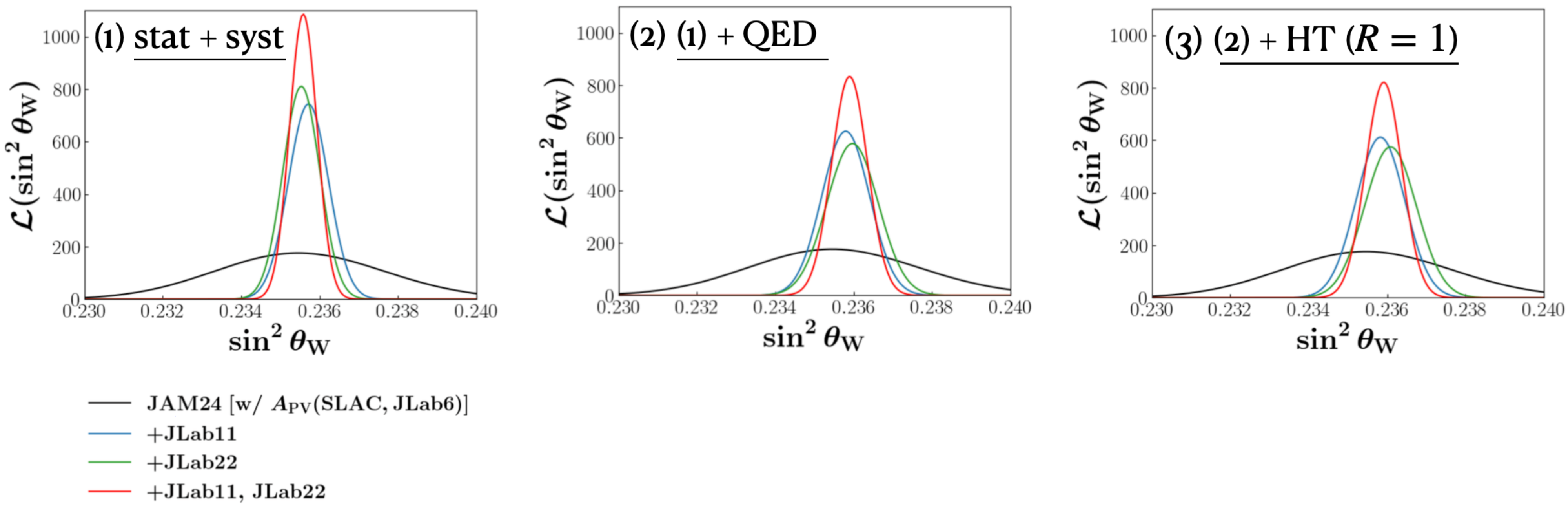
1. stat. + exp. syst. uncertainties
2. (1) + QED effects
3. (2) + HT effects

Impact on strange PDF determination



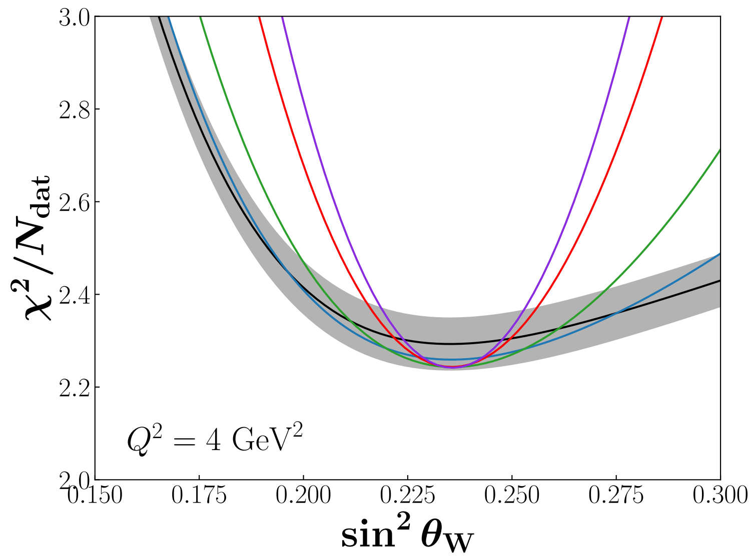
→ degree of uncertainty reduction does depend on specific value of R

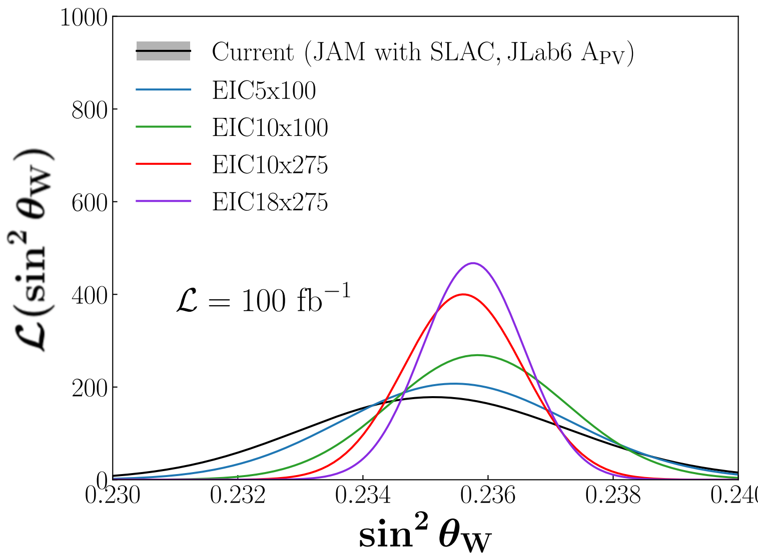
Impact on weak mixing angle



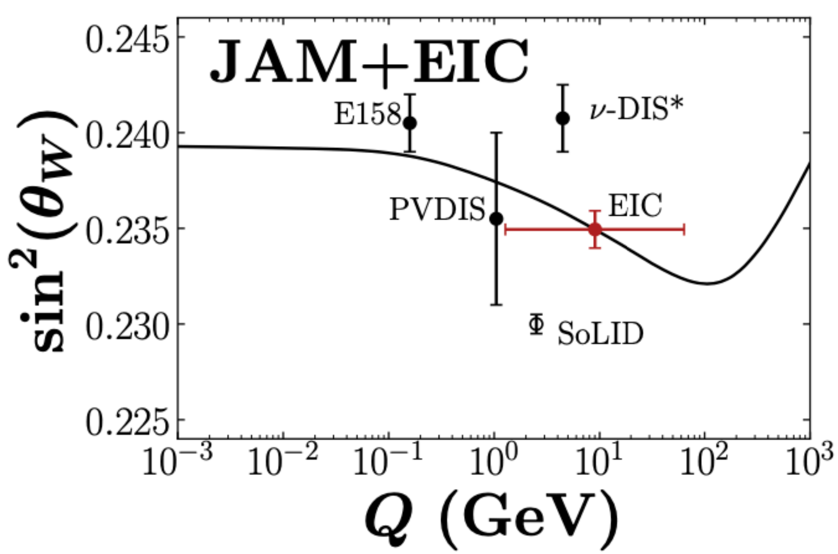
⟶ likelihood function $\mathcal{L} \sim \exp\left[-\frac{1}{2}\chi^2(\sin^2\theta_W)\right]$

Impact on weak mixing angle — EIC kinematics





Whitehill et al. in preparation (2025)

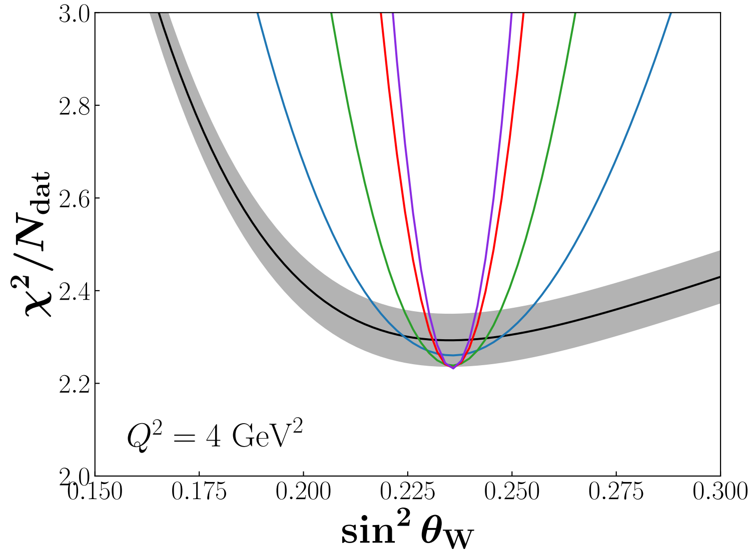


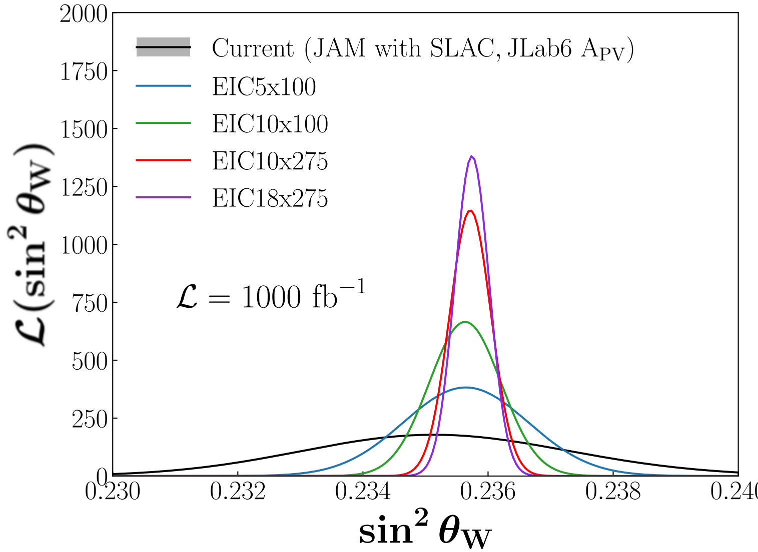
EIC Yellow Report, Nucl. Phys. ***A1026****, 122447 (2022)*

→ smaller uncertainties on extracted $\sin^2\theta_W$ for SoLID due to high CEBAF luminosity

→ larger EIC Q^2 range will sample running of $\sin^2\theta_W$

Impact on weak mixing angle — EIC kinematics





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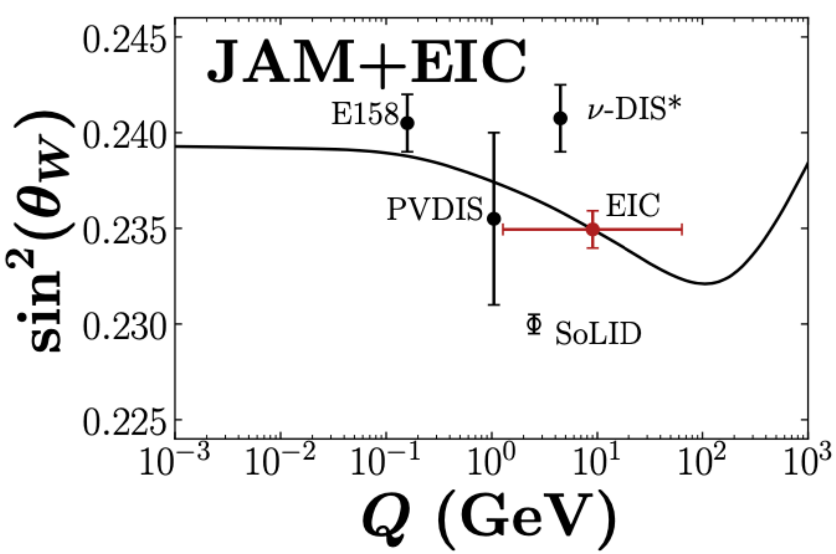

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Outlook

- PVDIS is a unique process which can provide (clean) data for input into global analyses:

  $\longrightarrow$ improve determination of weak mixing angle (test BSM physics)

  $\longrightarrow$ also constrain strange quark PDF in the nucleon

  $\longrightarrow$ important to perform simultaneous analysis of PDF and EW parameters

- In future, consider also:

  $\longrightarrow$ e^+/e^- PVDIS to constrain $s - \bar{s}$ asymmetry

  $\longrightarrow$ polarized PVDIS?

- Thanks to Richard Whitehill (grad student at ODU) & Nobuo Sato (JLab)