Flavor Swapping and Non-forward Scattering in the Neutrino-driven Wind

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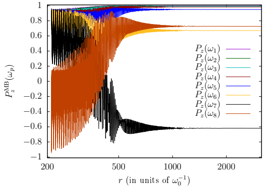
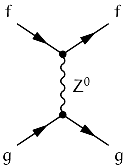
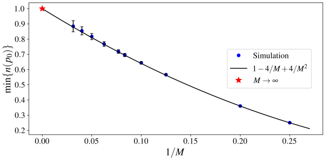
Roadmap

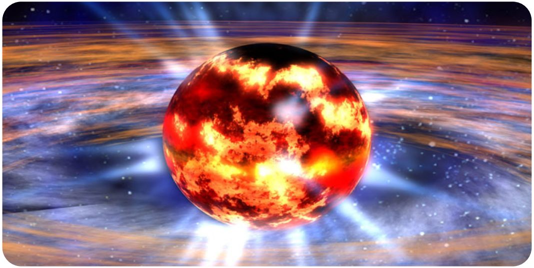
- Introduction to Collective Neutrino Oscillations
- Revisiting how neutrinos exchange momentum & flavor
- Results from full quantum evolution in a simple case







Supernovae: Large ν Sources



- Neutrino luminosity $L_\nu \sim 10^{53}$ ergs/s
- Neutron star temperature $k_B T \sim 10$ MeV

$$\implies \sim 10^{58} \text{ neutrinos}$$

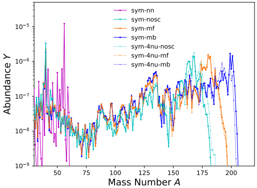
- SN envelope: $\ell_{\mathrm{MFP}} \gg \ell_{\mathrm{osc}}$ neutrinos evolving coherently

Large ν Sources & Nucleosynthesis Sites

- Core-collapse SNe, Binary neutron star mergers: sites for nucleosynthesis beyond Fe-56

- Without collective oscillations, expect:
  $\langle E_{\nu_e} \rangle < \langle E_{\bar{\nu}_e} \rangle < \langle E_{\nu_\mu, \nu_\tau, \bar{\nu}_\mu, \bar{\nu}_\tau} \rangle$

- With collective oscillations:
  Higher energy $\nu_{\mu,\tau} \rightarrow \nu_e \implies$ change n/p
  $\implies$ affect elemental abundances produced



ABB, **MJC**, AVP, RS, XW (2024)

$$\nu_e + n \longleftrightarrow p + e^-$$

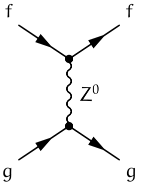
$$\bar{\nu}_e + p \longleftrightarrow n + e^+$$

Neutrino-neutrino interactions

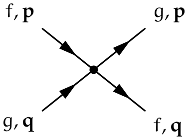
A Standard Model Prediction

- $T \ll m_Z$: Z-boson exchange $\rightarrow$ Fermi 4-point interaction

$$\frac{g^2}{m_Z^2 - p^2} \sim$$



$$\Longrightarrow$$



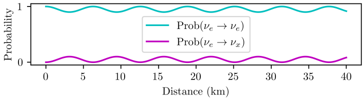
$$\sim \frac{g^2}{m_Z^2} \sim G_F$$

- Interaction potential strength $\sim G_F \rho_\nu$
- Coherent, forward scattering $\iff$ flavor swapping:

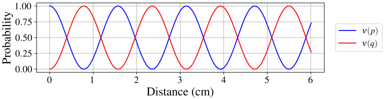
$$H_{\mathrm{swap}} = \sqrt{2}\, G_F \sum_{f,g} \int (1 - \cos\theta_{\mathbf{pq}})\, a_g^\dagger(\mathbf{p}) a_f^\dagger(\mathbf{q}) a_g(\mathbf{q}) a_f(\mathbf{p})\, \mathrm{d}\mathbf{p}\, \mathrm{d}\mathbf{q}$$

- Conserved quantities:
  - total occupancy of a momentum state, $n(\mathbf{p}) = \sum_f n_f(\mathbf{p})$ ✓
  - total flavor occupation numbers, $n_f = \int n_f(\mathbf{p})\, \mathrm{d}\mathbf{p}$ ✓

- Fast oscillations: freq $\sim G_F E^3$
- Bipolar oscillations:



vacuum oscillations

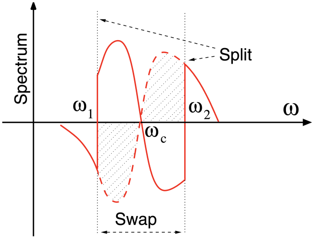


$\nu\nu$ interaction

- Spectral splits:

Linear stability analysis
with flavor mean field:

$$i\partial_t \rho(\omega) = \sqrt{2}\, G_F \left[\rho_{MF}, \rho(\omega)\right]$$

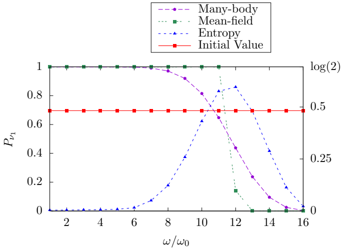


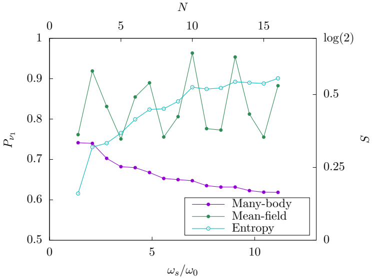
(Dasgupta et al., 2009)

Flavor correlations

Importance of multi-body correlations

- Quantum many-body theory reproduces flavor spectral swap





AVP, **MJC**, ABB (2021)

- Neutrino gas with randomized angles: flavor entanglement across the spectrum [Martin, Neill, Roggero, Duan, Carlson (2024)]
- Quantum kinetics: highlight importance of collision terms [Froustey (2022), Johns (2023)]

See: whole Particle & Nuclear Astrophysics session, an hour ago...

Momentum correlations

Non-forward scattering: A Lattice Problem

More general LEFT interaction Hamiltonian:

$$H_{\nu\nu} = \frac{G_F}{\sqrt{2}} \sum_{f,g} \int_{\{\mathbf{p}+\mathbf{q}=\mathbf{p}'+\mathbf{q}'\}} a_f^\dagger(\mathbf{p}') a_g^\dagger(\mathbf{q}') a_f(\mathbf{p}) a_g(\mathbf{q}) \; F(\mathbf{p},\mathbf{q}) F(\mathbf{p}',\mathbf{q}')^*$$

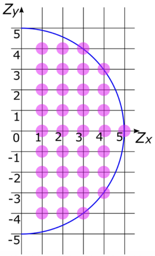
$$\sim H_{\text{flav}} \otimes H_{\text{mom}}$$

where $|F(\mathbf{p},\mathbf{q})|^2 = 1 - \cos\theta_{\mathbf{p},\mathbf{q}} + \mathcal{O}(m/E)$ [from Weyl spinors].

Symmetries?

- Conserve total flavor occupancy, n_f ✓
- Change momentum occupancy, $n(\mathbf{p})$ ×
- *NB:* still elastic scattering

$\Longrightarrow$ Task: take infinite-volume & continuum limits



Cirigliano, Sen, Yamauchi (2024)

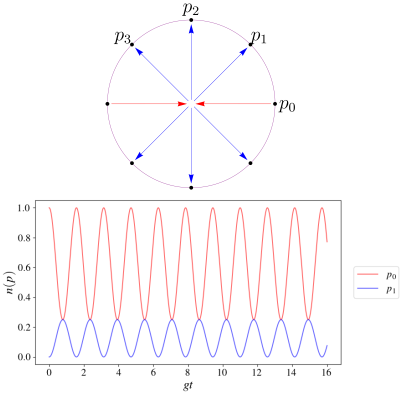
New Analysis in the Center of Momentum

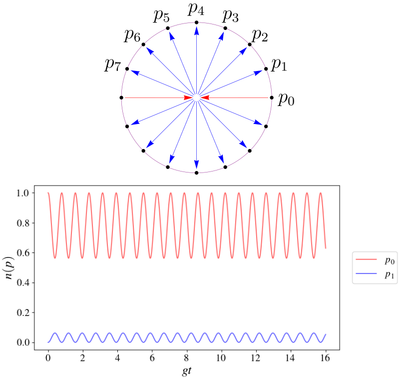
An Easier Lattice Problem

Tricky to take infinite-volume and continuum limits in $\mathbf{p}$ space...

$\implies$ Consider scattering in the Center of Momentum frame

$\implies$ Isotropic coupling strength $1 - \cos\theta_{\mathbf{pq}} = 2$

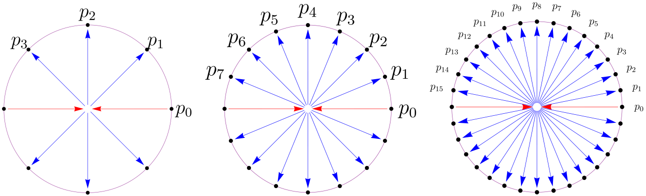


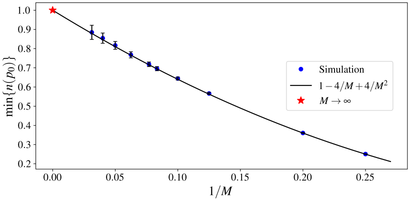


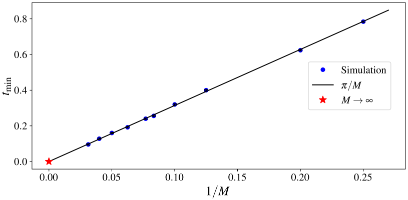
Center of Momentum Results

A Lattice Problem with a Continuum Limit

Trends in CoM scattering of two neutrinos as allowed angles $M \to \infty$







$\implies n(p_0) \to 1$, $\sim$ constant in time... but how?!

Thanks, Yukari Yamauchi (GitHub)!

Center of Momentum $\nu\nu$ Scattering

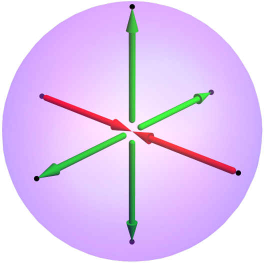
A Solvable Lattice Model

- Consider basis of states [in 3 $\dim$ now]
  $\{|p_1, -p_1\rangle, \ldots, |p_M, -p_M\rangle\}$
- Non-forward scattering Hamiltonian is

$$H \doteq g \begin{pmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{pmatrix}$$

$$\implies H^n = (gM)^{n-1} H$$

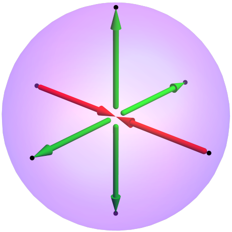
$$\implies e^{-itH} = \mathbb{1} + (e^{-itgM} - 1) H/gM$$

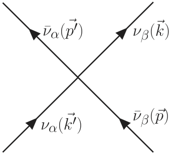


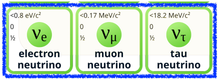
- Continuum limit ($M \to \infty$): non-forward scattering diminishing
- Contrast with flavor swapping: Hilbert space is discrete, finite

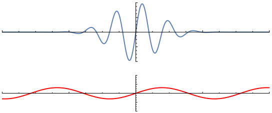
Summary

- Interacting neutrino problem cast in a many-body perspective
- Direct calculation of non-forward scattering in a simple model
- Further analysis, considering more general conditions:
  - More incoming neutrinos
  - Antineutrino interactions
  - Three-flavor simulations
  - Neutrino wave packets









- THANK YOU -

Astrophysical Setting

- "Separation of scales" - Neutrinos evolve differently in varying densities of media
  - High Density: Neutrinos evolving in thermal equilibrium (e.g., SN core, after weak-decoupling in EU)

$$\ell_{\mathrm{MFP}} \ll L_{\mathrm{osc,medium}}$$

  - Low Density: Neutrinos evolving coherently, thermally decoupled (e.g., SN envelope)

$$\ell_{\mathrm{MFP}} \gg L_{\mathrm{osc,medium}}$$

    Forward scattering neutrinos, coherent flavor states oscillating

- For an environment with low matter density, high neutrino flux $\Longrightarrow$ important to consider collective neutrino oscillations

Neutrino Flavor/Mass Isospin

SU(2) Notation

- Fermionic ops for ν flavor and mass states $a_f(\mathbf{p})$ and $a_j(\mathbf{p})$, respectively, for $f = e, x$ and $j = 1, 2$ with mixing angle θ

$$\begin{pmatrix} a_e(\mathbf{p}) \\ a_x(\mathbf{p}) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} a_1(\mathbf{p}) \\ a_2(\mathbf{p}) \end{pmatrix}$$

- Introduce su(2) ops in flavor & mass bases (aka "isospin"):

$$\begin{aligned} J^+_{\mathbf{p}} &= a_1^\dagger(\mathbf{p}) a_2(\mathbf{p}), & \text{mass } 1 : |\nu_1\rangle \longleftrightarrow |\uparrow\rangle \\ J^-_{\mathbf{p}} &= a_2^\dagger(\mathbf{p}) a_1(\mathbf{p}), & \text{mass } 2 : |\nu_2\rangle \longleftrightarrow |\downarrow\rangle \\ J^z_{\mathbf{p}} &= \frac{1}{2}[a_1^\dagger(\mathbf{p}) a_1(\mathbf{p}) - a_2^\dagger(\mathbf{p}) a_2(\mathbf{p})] & \end{aligned}$$

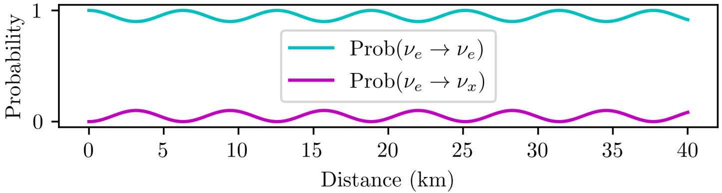
Vacuum Flavor Oscillations

An 1-body Hamiltonian

- Relativistic energy of massive particles:

$$\begin{aligned} H_\nu &= \sum_{\mathbf{p}} (|\mathbf{p}|^2 + m_1^2)^{1/2} a_1^\dagger(\mathbf{p}) a_1(\mathbf{p}) + (|\mathbf{p}|^2 + m_2^2)^{1/2} a_2^\dagger(\mathbf{p}) a_2(\mathbf{p}) \\ &= \sum_{\mathbf{p}} \omega_{\mathbf{p}} \vec{B} \cdot \vec{J}_{\mathbf{p}} + \text{const}, \end{aligned}$$

where $\omega_{\mathbf{p}} = \dfrac{\Delta m_{21}^2}{2|\mathbf{p}|}$ and $\vec{B} = (0, 0, -1)_{\mathcal{M}} = (\sin 2\theta, 0, -\cos 2\theta)_{\mathcal{F}}$



Geometric Interpretation

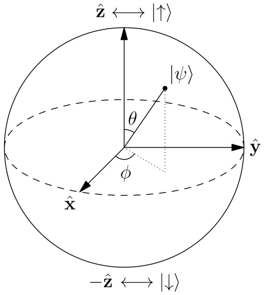
"Polarization" vectors

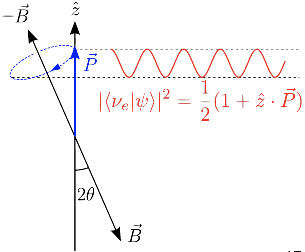
- Define polarization $\vec{P}_{\mathbf{p}} = 2\langle\Psi|\vec{J}_{\mathbf{p}}|\Psi\rangle$
- $\vec{P}_{\mathbf{p}}$: Bloch vector of one neutrino's density $\rho_{\mathbf{p}} = \mathrm{Tr}_{\mathbf{q}(\neq\mathbf{p})}[|\Psi\rangle\langle\Psi|] = \frac{1}{2}(1+\vec{\sigma}\cdot\vec{P}_{\mathbf{p}})$.
- Non-interacting system: for each ω,

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{P}_{\mathbf{p}} = \omega_{\mathbf{p}}\vec{B}\times\vec{P}_{\mathbf{p}}$$

Entanglement entropy: $S = -\mathrm{Tr}[\rho\ln\rho]$ inversely related to P

- $P = 1 \iff S = 0$ (Unentangled)
- $P = 0 \iff S = \ln(2)$ (*Maximally*)

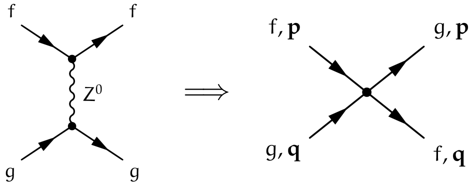




Two-body Hamiltonian

Neutrino-neutrino Interactions

- Low-energy EFT: Z-boson exchange → Fermi 4-point interaction



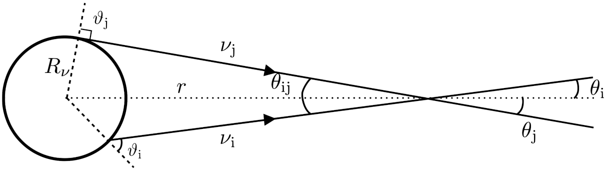
$$H_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\sum_{\mathbf{p},\mathbf{q}}(1-\widehat{\mathbf{p}}\cdot\widehat{\mathbf{q}})\sum_{f,g=e,x} a_f^\dagger(\mathbf{p})a_g(\mathbf{p})a_g^\dagger(\mathbf{q})a_f(\mathbf{q})$$

$$= \frac{\sqrt{2}G_F}{V}\sum_{\mathbf{p},\mathbf{q}}(1-\cos\vartheta_{\mathbf{pq}})\vec{J}_{\mathbf{p}}\cdot\vec{J}_{\mathbf{q}} + \text{const}$$

Reducing the Two-body Hamiltonian

The "Bulb Model"

- Definite-flavor νs emitted isotropically from spherical surface:



- Make the problem more tractable by averaging over $\theta_{\mathbf{pq}}$;

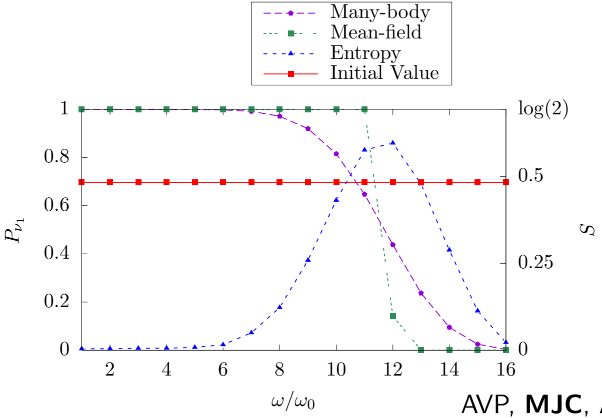
$$H_{\nu\nu} \approx \frac{\sqrt{2}G_F}{V} \langle 1 - \cos\vartheta_{\mathbf{pq}} \rangle \sum_{\mathbf{p}\neq\mathbf{q}} \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

$$= \mu(r) \sum_{\omega,\omega'} \vec{J}_\omega \cdot \vec{J}_{\omega'}, \quad \text{where} \quad \vec{J}_\omega = \sum_{\left\{\mathbf{p}:\omega=\frac{\Delta m^2}{2|\mathbf{p}|}\right\}} \vec{J}_{\mathbf{p}}$$

Final Data of All-Electon Flavor Initial State

$N = 16$ results across the spectrum

- Evolve $|\Psi_0\rangle = |\nu_e\rangle^{\otimes 16}$ to $r \gg R_\nu$ with $\theta = 0.584$
- Compare final P_{ν_1} and S at each ω

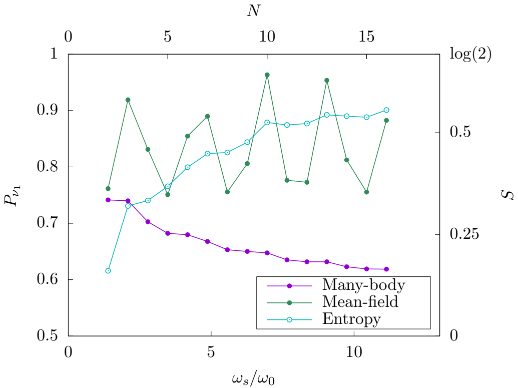


AVP, **MJC**, ABB (2021)

Pinpointing Entanglement

Honing on the spectral split

- Evolve $|\Psi_0\rangle = |\nu_e\rangle^{\otimes N}$ to $r \gg R_\nu$ with $\theta = 0.584$
- Here, spectral split frequency: $\omega_s = \omega_0 N \cos^2(\theta)$
- $P_{\nu_1}(\omega_s)$ & $S(\omega_s)$ vs. N



AVP, **MJC**, ABB (2021)

Neutrino flavor evolution: matter effects

- Matter backgrounds (electrons, nucleons, etc.) modify flavor evolution: neutrinos acquire "effective mass" through forward scattering (like photons in medium, but via weak interactions)

- In typical environments ($T \lesssim 10$ MeV), ν_e experience charged- and neutral-current interactions, unlike ν_μ and ν_τ (only NC)

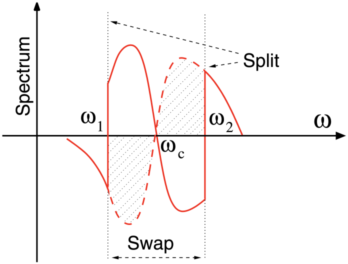
- In such a medium, ν_e acquires additional effective mass compared to ν_μ, ν_τ

$$i \frac{\mathrm{d}}{\mathrm{d}t} \begin{pmatrix} \psi_e(\omega) \\ \psi_x(\omega) \end{pmatrix} = \left[U \frac{1}{2E} \begin{pmatrix} m_1^2 & 0 \\ 0 & m_2^2 \end{pmatrix} U^\dagger + \begin{pmatrix} V_{\mathrm{CC}} & 0 \\ 0 & 0 \end{pmatrix} \right] \begin{pmatrix} \psi_e(\omega) \\ \psi_x(\omega) \end{pmatrix}$$

where $V_{\mathrm{CC}} = \sqrt{2} G_F n_B Y_e$.

Mean-field Effective Hilbert Space

Analyzing its Scaling



(Dasgupta et al., 2009)

- Separated Hilbert spaces for each ω:

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\psi_1(\omega)\\ \psi_2(\omega)\end{pmatrix} = \begin{pmatrix}-\omega+\mu P^z & P^+\\ P^- & +\omega-\mu P^z\end{pmatrix}\begin{pmatrix}\psi_1(\omega)\\ \psi_2(\omega)\end{pmatrix}$$

- MFT Collective Oscillations:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{P}_\omega = (\omega\vec{B}+\mu\vec{P})\times\vec{P}_\omega$$

- "Many-body" wave function simply: $|\Psi\rangle = \bigotimes_\omega |\psi(\omega)\rangle$, $2N$-dim

Mean-field Theory

Random Phase Approximation

- Ansatz that relative phases for different ω are random (RPA) $\implies$ Mean-field approximation of our Hamiltonian:

$$H_{\nu\nu} = \mu\vec{J}\cdot\vec{J} \underset{MFT}{\approx} \mu\vec{P}\cdot\vec{J} - \frac{1}{4}\mu P^2$$

  where $\vec{P} = 2\langle\vec{J}\rangle$ is the "mean field" with state $|\psi\rangle$ satisfying

$$\langle\vec{J}_1\cdot\vec{J}_2\rangle = \langle\vec{J}_1\rangle\cdot\langle\vec{J}_2\rangle$$

- "Many-body" wave function simply: $|\Psi\rangle = \bigotimes_\omega |\psi(\omega)\rangle$, $2N$-dim
- ... But can we neglect the other dimensions?!

How do we study larger N systems?

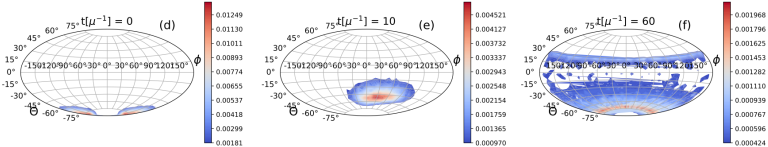
2: Stochastic Mean-field: improving the mean-field approximation

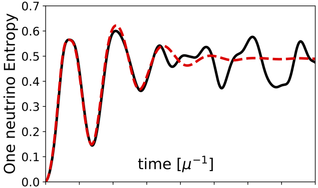
Consider an uniform neutrino beam

SMF: random distribution around initial flavor state

$\rightarrow$ evolve each sample via ordinary mean field (easy!)

$\rightarrow$ average over trajectories (reproduce entanglement!)





DL, ABB, **MJC**, AVP, PS (2022)