High Quality Axion Models and Their Phenomenology

K.S. Babu

Oklahoma State University

Phenomenology at the Frontiers

University of Wisconsin

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Based Upon:

- Primary:

  Fermion Mass Hierarchy and a High Quality Axion From Gauged $U(1)$ Flavor Symmetry, K.S. Babu, Sai Chandrasekar, Zurab Tavartkiladze, to appear

- Secondary:

  Hybrid SO(10) Axion Model Without Quality Problem, K.S. Babu, Bhaskar Dutta, Rabindra N. Mohapatra, e-Print: 2410.07323 [hep-ph], Phys. Rev. Lett. **134** 111803 (2025)

- Tertiary:

  Accidental Peccei-Quinn Symmetry From Gauged U(1) and a High Quality Axion, K.S. Babu, Bhaskar Dutta, Rabindra N. Mohapatra, e-Print: 2412.21157 [hep-ph].

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Supported by the US Department of Energy

Open Questions in the Standard Model

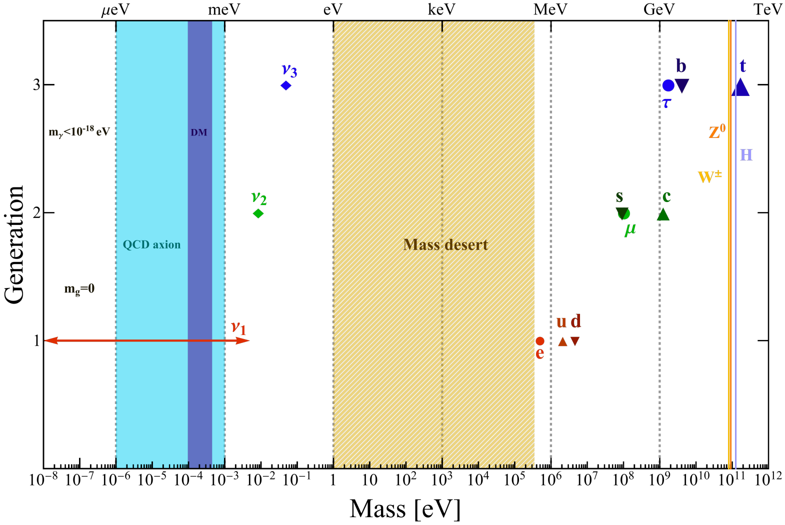
- Fermion mass and mixing puzzle: Why 3 families? How do mass-mixing hierarchies arise?
- How do neutrino masses arise?
- What particle constitutes dark matter in the universe?
- How is the strong CP problem solved?
- What is the origin of matter-antimatter asymmetry in the universe?
- How is inflation realized?
- How is the Higgs boson mass stabilized?
- What is the theory of quantum gravity?

Addressing Open Questions with Flavor Gauge Symmetry

A framework with a family-dependent $U(1)_F$ gauge symmetry can address many of the open questions

- Fermion mass-mixing hierarchies explained by Froggatt-Nielsen mechanism with the help of $U(1)_F$ gauge symmetry
- Gauge anomaly cancellation requires presence of right-handed neutrinos which lead to neutrino masses via seesaw mechanism
- An accidental global symmetry is realized within the framework with a QCD anomaly leading to axion, solving the strong CP problem
- The axion is of high quality owing to the $U(1)_F$ gauge symmetry, and constitutes the dark matter of the universe
- Predictive framework for baryogenesis via leptogenesis
- Scalar fields used for gauge symmetry breaking can serve as inflaton

Understanding Particle Masses



Fermion Mass Hierarchy from Gauged $U(1)_F$

- Hierarchical structure of fermion masses and mixings can be explained with a family-dependent $U(1)_F$ symmetry. Usually $U(1)_F$ is assumed to be global, here we demand it to be a local symmetry.
- Order one differences in flavor charges can lead to hierarchical mass matrices via the Froggatt-Nielsen mechanism.

$$\mathcal{L}_{\rm Yuk} \supset y_{ij}^f F_i F_j^c H \left(\frac{X^{(*)}}{\Lambda_{\rm FN}}\right)^{n_{ij}^f}$$

- X is the flavon field, n_{ij}^f are positive integers, $\Lambda_{\rm FN}$ is the flavor cut-off scale with a small parameter ϵ defined as

$$\epsilon \equiv \frac{\langle X \rangle}{\Lambda_{\rm FN}} \sim 0.22$$

- With all the Yukawa couplings y_{ij}^f being order one observed mass hierarchies can be explained. Lighter fermion masses have higher powers of ϵ.
- Eg: $(m_t, m_c, m_u) \sim (1, \epsilon^4, \epsilon^8)\, v$ with flavor charges $(0, 2, 4)$.

CP Invariance of Strong Interactions

- Strong interactions appear to conserve Parity (P) and Time Reversal (T) symmetries, and therefore also CP symmetry. However, QCD Lagrangian admits a source of P and T violation:

$$\mathcal{L}_{\rm QCD} \supset \theta_{QCD} \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

- Chiral rotations on the quark fields can be done, but the parameter

$$\overline{\theta} = \theta_{QCD} + {\rm ArgDet}(M_Q)$$

is invariant. It contributes to neutrton electric dipole momnent $d_n \simeq 10^{-16}\,\overline{\theta}$ e-cm.Current limits require $\overline{\theta} \leq 10^{-10}$, which is the strong CP problem

- Axion is proposed to solve this problem dynamically

Axion Solution to Strong CP Problem

- The Peccei-Quinn (PQ) mechanism, which leads to a light pseudoscalar particle, the axion (a), is an elegant solution to the strong CP problem Peccei, Quinn (1977)

- The PQ mechanism assumes a global $U(1)_{PQ}$ symmetry that has a QCD anomaly. This $U(1)$ is spontaneously broken by a Higgs scalar, and also explicitly by the QCD anomaly term

- Axion is the pseudo-Goldstone boson associated with the $U(1)_{PQ}$ symmetry breaking. The QCD anomaly induces a coupling of axion to the gluon field so that

$$\mathcal{L} = \frac{g_s^2}{32\pi^2}\left(\overline{\theta} + \frac{a}{f_a}\right) G_{\mu\nu}\tilde{G}^{\mu\nu}$$

- This coupling in turn induces a potential for the axion field,

$$V \approx \Lambda^4 \cos\left(\overline{\theta} + \frac{a}{f_a}\right)$$

which upon minimization dynamically sets $\overline{\theta} + \frac{a}{f_a} = 0$

Axion Quality Problem

- Quantum gravity is expected to break all global symmetries, including $U(1)_{PQ}$. This gives rise to the axion quality problem

- For example, a gravity-induced term in the Higgs potential of canonical axion models (DFSZ and KSVZ),

$$V_{gravity} = \frac{\kappa}{M_{\rm Pl}} |\Phi|^4 (e^{i\delta}\Phi + h.c.)$$

  would shift the vacuum value $\bar{\theta} + \frac{a}{f_a} = 0$

- Minimizing the potential in presence of the quantum gravity correction one has

$$\Delta\bar{\theta} \simeq \frac{\kappa \sin\delta}{2\sqrt{2}} \frac{f_a^5}{\Lambda^4 M_{\rm Pl} N^2}$$

- For currently favored values of $f_a = (10^9 - 10^{12})$ GeV, with $\Lambda \simeq 200$ MeV and $M_{\rm Pl} = 1.22 \times 10^{19}$ GeV, one finds the limits

$$\kappa \sin\delta \leq \{10^{-38} - 10^{-53}\}$$

- This is rather severe, much worse than the strong CP problem itself!

  Holman et. al. (1992); Kamionkowski, March-Russell (1992); Barr, Seckel (1992); Ghigna, Lusignoli, Roncadelli (1992)

Attempts to Solve Axion Quality Problem

- Various ideas have been suggested:
  - Gauge symmetries leading to accidental $U(1)_{\mathrm{PQ}}$ Barr, Seckel (1992); Babu, Barr (1993); Qiu, Yang, Yanagida (2023); Di Luzio (2020); Ardu et. al. (2020); Babu, Dutta, Mohapatra (2024)
  - Composite axion Randall (1992); Lillard, Tait (2018); Gaillard, Gavela, Houtz, Quilez, Del Rey (2018); Lee, Yin (2019); Vechi (2021); Contino, Podo Rivello (2022); Cox, Gherghetta, Paul (2023)
  - Discrete gauge symmetries Babu, Gogoladze, Wang (2003); Hook (2018)
  - Mirror world Berezhiani, Gianfagna, Giannotti (2001); Hook, Kumar, Liu, Sundrum (2022)
  - Extra dimensional setups Choi (2004); Nakai, Suzuki (2021); Cox, Gherghetta, Ngyuen (2021); Reece (2024); Craig, Konsgrov (2024)
  - String theory axion Svrcek, Witten (2006)
- Realizing accidental PQ symmetry from a gauge symmetry is nontrivial, since PQ symmetry should have a QCD anomaly, but the original gauge symmetry has no anomaly
- In the rest of the talk I shall present several models with gauged $U(1)$ and discuss briefly phenomenology of successful models

General Framework for Gauged Flavor $U(1)_F$

- Flavor charges of fermions:

$$Q_i(3, 2, \frac{1}{6}) = (q_1, q_2, q_3), \quad u_i^c(3^*, 1, -\frac{2}{3}) = (u_1, u_2, u_3), \quad d_i^c(3^*, 1, \frac{1}{3}) = (d_1, d_2, d_3),$$
$$L_i(1, 2, -\frac{1}{2}) = (l_1, l_2, l_3), \quad e_i^c(1, 1, 1) = (e_1, e_2, e_3), \quad N_i(1, 1, 0) = (n_1, n_2, n_3)$$

- Two Higgs doublets (H_u, H_d) and two singlets (X, S) are used, as in DFSZ axion model. The additional singlet is needed to break the $U(1)_F$ gauge symmetry. X acts as the flavon.

$$H_u(1, 2, \frac{1}{2}) = h_u, \quad H_d(1, 2, -\frac{1}{2}) = h_d, \quad X(1, 1, 0) = q_X, \quad S(1, 1, 0) = q_S$$

- Yukawa couplings:

$$\begin{aligned} \mathcal{L}_{\rm Yukawa} &= y_{ij}^u Q_i u_j^c H_u \left(\frac{X^{(*)}}{\Lambda_{\rm FN}}\right)^{n_{ij}^u} + y_{ij}^d Q_i d_j^c H_d \left(\frac{X^{(*)}}{\Lambda_{\rm FN}}\right)^{n_{ij}^d} + y_{ij}^\ell L_i e_j^c H_d \left(\frac{X^{(*)}}{\Lambda_{\rm FN}}\right)^{n_{ij}^\ell} \\ &+ y_{ij}^\nu L_i N_j \tilde{H}_d \left(\frac{X^{(*)}}{\Lambda_{\rm FN}}\right)^{n_{ij}^\nu} + y_{ij}^N M_N N_i N_j \left(\frac{X^{(*)}}{\Lambda_{\rm FN}}\right)^{n_{ij}^N} + h.c. \end{aligned}$$

Anomay Cancellation

- For $U(1)_F$ to be gauged, all six anomaly coefficeints should vanish:

$$\begin{aligned}
A[SU(3)_c^2 \times U(1)_F] &= \sum_i (2q_i + u_i + d_i) = 0, \\
A[SU(2)_L^2 \times U(1)_F] &= \sum_i (3q_i + l_i) = 0, \\
A[U(1)_Y^2 \times U(1)_F] &= \frac{1}{6}\sum_i (q_i + 4u_i + 2d_i + 3l_i + 6e_i) = 0, \\
A[(\mathrm{gravity})^2 \times U(1)_F] &= \sum_i (6q_i + 3u_i + 3d_i + 2l_i + e_i + n_i) = 0, \\
A[U(1)_Y \times U(1)_F^2] &= \sum_i (q_i^2 - 2u_i^2 + d_i^2 - l_i^2 + e_i^2) = 0, \\
A[U(1)_F^2] &= \sum_i (6q_i^3 + 3u_i^3 + 3d_i^3 + 2l_i^3 + e_i^3 + n_i^3) = 0
\end{aligned}$$

- Charges should also satisfy:

$$\begin{aligned}
q_i + u_j + h_u \pm q_X\, n_{ij}^u &= 0, \\
q_i + d_j + h_d \pm q_X\, n_{ij}^d &= 0, \\
l_i + e_j + h_d \pm q_X\, n_{ij}^\ell &= 0, \\
l_i + n_j - h_d \pm q_X\, n_{ij}^\nu &= 0, \\
n_i + n_j \pm q_X\, n_{ij}^N &= 0
\end{aligned}$$

- Nontrivial to find solutions to anomaly conditions and desired fermion mass matrix structures

Emergence of Axion

- Among 4 complex scalar fields, a single cross coupling is allowed:

$$V \supset -\lambda_{HS}\frac{H_u H_d S^n (X^{(*)})^k}{M_{\rm Pl}^{n+k-2}} + h.c.$$

  (n, k) are positive integers with values $(3,1)$ preferred.

- Two phases are eaten up by Z and Z'_F, one phase gets mass from potential – the pseudoscalar of 2HDM – and the 4th phase remains massless – axion

- Axion is orthogonal to two Goldstone bosons eaten up by Z and Z'_F and the pseudoscalar from the Higgs doublet:

$$a = \sum_{\alpha=u,d,X,S} K_\alpha \eta_\alpha$$

- η_i are phases of the fields and

$$\begin{aligned}
K_u &= N_a v_u v_d^2 (n\, q_X v_X^2 \mp k\, q_S v_S^2), \\
K_d &= N_a v_d v_u^2 (n\, q_X v_X^2 \mp k\, q_S v_S^2), \\
K_X &= N_a v_X \left(q_S\, v_S^2 (v_u^2 + v_d^2) - n\,(h_u + h_d) v_u^2 v_d^2\right), \\
K_S &= N_a v_S \left(\pm k (h_u + h_d) v_u^2 v_d^2 - q_X v_X^2 (v_u^2 + v_d^2)\right)
\end{aligned}$$

Explicit Model 1

- Flavor charges of fermions and scalars:

$$(q_1, q_2, q_3) = (1, 0, -2), \quad (u_1^c, u_2^c, u_3^c) = (3, 0, -2), \ (d_1^c, d_2^c, d_3^c) = \left(-\frac{13}{3}, \frac{8}{3}, \frac{8}{3}\right),$$

$$(l_1, l_2, l_3) = \left(\frac{2}{3}, \frac{16}{3}, -\frac{5}{3}\right), \quad (e_1^c, e_2^c, e_3^c) = \left(-\frac{8}{3}, -\frac{8}{3}, \frac{7}{3}\right), \quad (n_1, n_2, n_3) = (4, -6, -1)$$

$$h_u = 4, \quad h_d = -\frac{2}{3}, \quad q_X = 1, q_S = -\frac{13}{9}$$

- All anomalies cancel. Fermion mass matrices:

$$\mathcal{L} = q^T \cdot \begin{pmatrix} \bar{\varepsilon}^8 & \bar{\varepsilon}^5 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^7 & \bar{\varepsilon}^4 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^5 & \bar{\varepsilon}^2 & 1 \end{pmatrix} \cdot u^c H_u + q^T \cdot \begin{pmatrix} \varepsilon^4 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \varepsilon^5 & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \varepsilon^7 & 1 & 1 \end{pmatrix} \cdot d^c H_d + l^T \cdot \begin{pmatrix} \varepsilon^4 & \varepsilon^4 & \bar{\varepsilon} \\ \bar{\varepsilon}^2 & \bar{\varepsilon}^2 & \bar{\varepsilon}^7 \\ \varepsilon^5 & \varepsilon^5 & 1 \end{pmatrix} \cdot e^c H_d$$

$$+ l^T \cdot \begin{pmatrix} \bar{\varepsilon}^4 & \varepsilon^6 & \varepsilon \\ \bar{\varepsilon}^{10} & 1 & \bar{\varepsilon}^5 \\ \bar{\varepsilon}^3 & \varepsilon^7 & \varepsilon^2 \end{pmatrix} \cdot N \tilde{H}_d + N^T \cdot \begin{pmatrix} \bar{\varepsilon}^8 & \varepsilon^2 & \bar{\varepsilon}^3 \\ \varepsilon^2 & \varepsilon^{12} & \varepsilon^7 \\ \bar{\varepsilon}^3 & \varepsilon^7 & \varepsilon^2 \end{pmatrix} \cdot N M_N + h.c.$$

- Mass and mixing hierarhcies explained:

$$\lambda_t \sim 1, \quad \frac{\lambda_u}{\lambda_c} \sim \frac{\lambda_c}{\lambda_t} \sim \epsilon^4, \quad \frac{\lambda_e}{\lambda_\mu} \sim \epsilon^2, \quad \frac{\lambda_\mu}{\lambda_\tau} \sim \epsilon^2, \quad |V_{us}| \sim \epsilon, \quad |V_{cb}| \sim \epsilon^2, \quad |V_{ub}| \sim \epsilon^3$$

Model 2

- Flavor charges of fermions and scalars:

$$(q_1, q_2, q_3) = (-\frac{2}{3}, \frac{1}{3}, -\frac{7}{3}), \quad (u_1^c, u_2^c, u_3^c) = (-\frac{22}{3}, -\frac{13}{3}, -\frac{7}{3}), \ (d_1^c, d_2^c, d_3^c) = (2, 4, 4),$$

$$(l_1, l_2, l_3) = \left(-\frac{4}{3}, -\frac{7}{3}, -\frac{7}{3}\right), \quad (e_1^c, e_2^c, e_3^c) = \left(\frac{8}{3}, \frac{20}{3}, \frac{26}{3}\right), \quad (n_1, n_2, n_3, n_4) = (-2, -6, -6, 8)$$

$$h_u = 0, \quad h_d = -\frac{19}{3}, \quad q_X = 1, q_S = -\frac{16}{9}$$

- All anomalies cancel. Fermion mass matrices:

$$\mathcal{L}_{\text{Yuk}} \supset Q^T H_u \begin{pmatrix} \varepsilon^8 & \varepsilon^5 & \varepsilon^3 \\ \varepsilon^7 & \varepsilon^4 & \varepsilon^2 \\ \varepsilon^5 & \varepsilon^2 & 1 \end{pmatrix} u^c + Q^T H_d \begin{pmatrix} \varepsilon^5 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^4 & \varepsilon^2 & \varepsilon^2 \\ \varepsilon^2 & 1 & 1 \end{pmatrix} d^c + L^T H_d \begin{pmatrix} \varepsilon^5 & \varepsilon & \overline{\varepsilon} \\ \varepsilon^6 & \varepsilon^2 & 1 \\ \varepsilon^6 & \varepsilon^2 & 1 \end{pmatrix} e^c$$

$$+ L^T \tilde{H}_d \begin{pmatrix} \overline{\varepsilon}^3 & \varepsilon & \varepsilon & \overline{\varepsilon}^{13} \\ \overline{\varepsilon}^2 & \varepsilon^2 & \varepsilon^2 & \overline{\varepsilon}^{12} \\ \overline{\varepsilon}^2 & \varepsilon^2 & \varepsilon^2 & \overline{\varepsilon}^{12} \end{pmatrix} N + M_R N \begin{pmatrix} \varepsilon^4 & \varepsilon^8 & \varepsilon^8 & \overline{\varepsilon}^6 \\ \varepsilon^8 & \varepsilon^{12} & \varepsilon^{12} & \overline{\varepsilon}^2 \\ \varepsilon^8 & \varepsilon^{12} & \varepsilon^{12} & \overline{\varepsilon}^2 \\ \overline{\varepsilon}^6 & \overline{\varepsilon}^2 & \overline{\varepsilon}^2 & \overline{\varepsilon}^{16} \end{pmatrix} N.$$

- Mass and mixing hierarhcies explained:

$$\lambda_t \sim 1, \ \frac{\lambda_u}{\lambda_c} \sim \frac{\lambda_c}{\lambda_t} \sim \epsilon^4, \quad \frac{\lambda_e}{\lambda_\mu} \sim \frac{\lambda_d}{\lambda_s} \sim \epsilon^3, \ \frac{\lambda_\mu}{\lambda_\tau} \sim \frac{\lambda_s}{\lambda_b} \sim \epsilon^2, \ |V_{us}| \sim \epsilon, \quad |V_{cb}| \sim \epsilon^2, \quad |V_{ub}| \sim \epsilon^3$$

Model 3

- Flavor charges of fermions and scalars:

$$(q_1, q_2, q_3) = (\frac{5}{3}, -\frac{1}{3}, -\frac{7}{3}), \quad (u_1^c, u_2^c, u_3^c) = (\frac{5}{3}, -\frac{1}{3}, -\frac{7}{3}), \ (d_1^c, d_2^c, d_3^c) = \left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right),$$

$$(l_1, l_2, l_3) = \left(\frac{5}{3}, \frac{2}{3}, \frac{2}{3}\right), \quad (e_1^c, e_2^c, e_3^c) = (\frac{5}{3}, -\frac{1}{3}, -\frac{7}{3}), \quad (n_1, n_2, n_3) = (-6, -9, -10)$$

$$h_u = \frac{14}{3}, \quad h_d = \frac{14}{3}, \quad q_X = 1, q_S = -\frac{25}{9}$$

- All anomalies cancel. Fermion mass matrices:

$$\mathcal{L}_{\text{Yuk}} \supset Q^T H_u \begin{pmatrix} \bar\varepsilon^8 & \bar\varepsilon^6 & \bar\varepsilon^4 \\ \bar\varepsilon^6 & \bar\varepsilon^4 & \bar\varepsilon^2 \\ \bar\varepsilon^4 & \bar\varepsilon^2 & 1 \end{pmatrix} u^c + Q^T H_d \begin{pmatrix} \bar\varepsilon^8 & \bar\varepsilon^7 & \bar\varepsilon^7 \\ \bar\varepsilon^6 & \bar\varepsilon^5 & \bar\varepsilon^5 \\ \bar\varepsilon^4 & \bar\varepsilon^3 & \bar\varepsilon^3 \end{pmatrix} d^c + L^T \begin{pmatrix} \bar\varepsilon^8 & \bar\varepsilon^6 & \bar\varepsilon^4 \\ \bar\varepsilon^7 & \bar\varepsilon^5 & \bar\varepsilon^3 \\ \bar\varepsilon^7 & \bar\varepsilon^5 & \bar\varepsilon^3 \end{pmatrix} e^c$$

$$+ L^T \tilde H_d \begin{pmatrix} \varepsilon^9 & \varepsilon^{12} & \bar\varepsilon^7 \\ \varepsilon^{10} & \varepsilon^{13} & \bar\varepsilon^6 \\ \varepsilon^{10} & \varepsilon^{13} & \bar\varepsilon^6 \end{pmatrix} N + M_R N^T \begin{pmatrix} \varepsilon^{12} & \varepsilon^{15} & \bar\varepsilon^4 \\ \varepsilon^{15} & \varepsilon^{18} & \bar\varepsilon \\ \bar\varepsilon^4 & \bar\varepsilon & \bar\varepsilon^{20} \end{pmatrix} N,$$

$$+ l^T \cdot \begin{pmatrix} \bar\varepsilon^4 & \varepsilon^6 & \varepsilon \\ \bar\varepsilon^{10} & 1 & \bar\varepsilon^5 \\ \bar\varepsilon^3 & \varepsilon^7 & \varepsilon^2 \end{pmatrix} \cdot N \tilde H_d + N^T \cdot \begin{pmatrix} \bar\varepsilon^8 & \varepsilon^2 & \bar\varepsilon^3 \\ \varepsilon^2 & \varepsilon^{12} & \varepsilon^7 \\ \bar\varepsilon^3 & \varepsilon^7 & \varepsilon^2 \end{pmatrix} \cdot N M_N + h.c.$$

- Mass and mixing hierarhcies explained:

$$\lambda_t \sim 1, \ \frac{\lambda_u}{\lambda_c} \sim \frac{\lambda_c}{\lambda_t} \sim \epsilon^4, \quad \frac{\lambda_e}{\lambda_\mu} \sim \frac{\lambda_d}{\lambda_s} \sim \epsilon^3, \ \frac{\lambda_\mu}{\lambda_\tau} \sim \frac{\lambda_s}{\lambda_b} \sim \epsilon^2, \ |V_{us}| \sim \epsilon, \quad |V_{cb}| \sim \epsilon^2, \quad |V_{ub}| \sim \epsilon^3$$

Neutrino Sector: Model 1

- One quantitative prediction for neutrino oscillation parameters:

$$m_D \simeq \begin{pmatrix} d & 0 & b \\ 0 & 1 & 0 \\ c & 0 & a \end{pmatrix} \lambda v_d , \quad M_R \simeq \begin{pmatrix} 0 & M_1 & M_2 \\ M_1 & 0 & 0 \\ M_2 & 0 & M_3 \end{pmatrix},$$

$$M_\nu = \begin{pmatrix} \beta^2 & \gamma' & \beta \\ \gamma' & \gamma & \alpha \\ \beta & \alpha & 1 \end{pmatrix} \bar{m}$$

- Cofactor of $(M_\nu)_{22}$ is zero: Liao, Marfatia, Whisnant (2014)

$$M_\nu^{(1,1)} M_\nu^{(3,3)} - (M_\nu^{(1,3)})^2 = 0.$$

- This leads to a consistent prediction. If we choose input as:

$$\bar{m} = 0.0307 \text{ eV}, \quad \{\alpha, \beta, \gamma, \gamma'\} = \{-0.717891, -0.2923, 0.7812, 0.0184\}$$

we obtain:

$$\{m_1, m_2, m_3\} = \{0.00234,\ 0.00893,\ 0.05071\}\mathrm{eV},$$

$$\{\sin^2\theta_{12}, \sin^2\theta_{23}, \sin^2\theta_{13}\} = \{0.343, 0.407, 0.02235\}.$$

$$\Delta m^2_{\mathrm{sol}} = m_2^2 - m_1^2 = 7.42 \times 10^{-5}\mathrm{eV}^2, \quad \Delta m^2_{\mathrm{atm}} = m_2^2 = 2.492 \times 10^{-3}\mathrm{eV}^2 \ .$$

Axion Quality in Gauged $U(1)_F$ Model

- Axion of the gauged flavor model is of high quality. The lowest Planck-induced operator that violates PQ symmetry is

$$V \supset \frac{S^9 X^{13}}{M_{\rm Pl}^{18}}$$

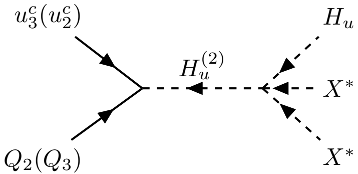
- This leads to extremely tiny shift in $\overline{\theta}$.

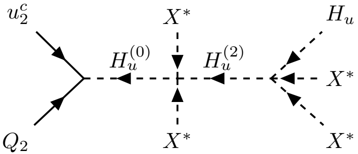
- Since the Froggatt-Nielsen flavor scale is of order $f_a \sim 10^{11}$ GeV, one should UV-complee the flavor model and see if axion quality is preserved with new fields introduced.

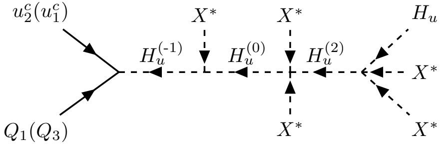
- We have UV-completed the model with new scalar doublets which acquire induced VEVs, suppressed by powers of ϵ.

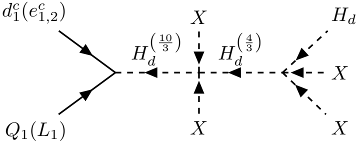
- Axion quality remains robust under the UV-completion, as shown next.

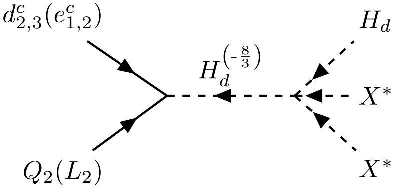
UV-Completion: Charged Fermions



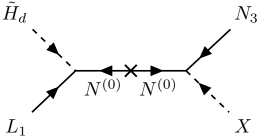


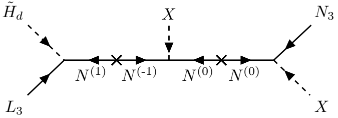


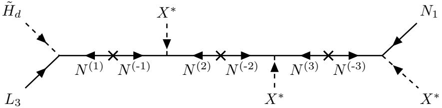


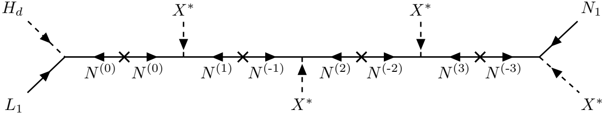


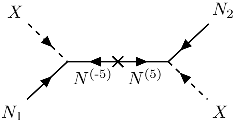
UV-Completion: Neutrino Sector

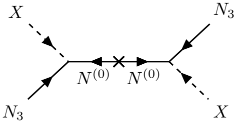


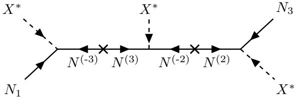




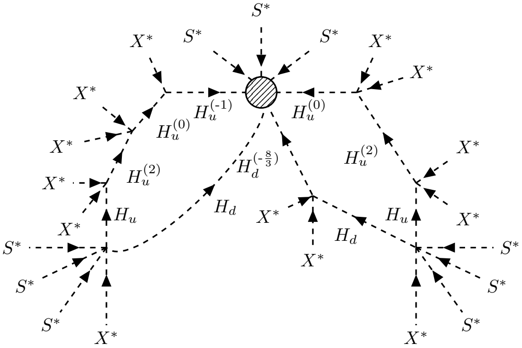








Loop Corrections to $\overline{\theta}$

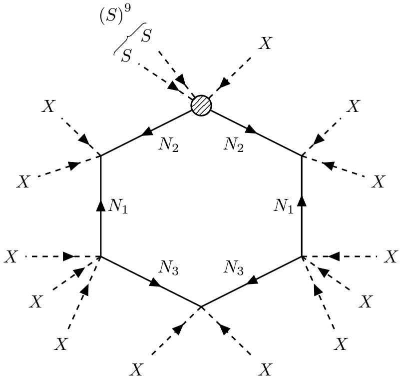


- Loop diagrams may induce additional shifts in $\overline{\theta}$ with lower powers of $M_{\rm Pl}$ suppression

$$V_{\cancel{PQ}}^{\rm ind} \supset \left(\frac{\ln \Lambda_{\rm FN}/M_{A_H}}{16\pi^2}\right)^2 \frac{S^9 X^2}{M_{\rm Pl}^7} \left(\frac{X}{\Lambda_{\rm FN}}\right)^{11} + h.c.$$

- These diagrams do not upset the strong CP solution

Loop Corrections to $\overline{\theta}$



- Tilt in axion potential:

$$V_{\cancel{PQ}}^{\rm ind} \supset \frac{1}{16\pi^2}\frac{S^9}{M_{\rm Pl}^9}\frac{X^{13}}{\Lambda_{\rm FN}^7 M_N^2} + \text{h.c.}$$

- Consistent with axion quality

Axion Quality and Dark Matter

- All QCD axion models lead to degenerate vacuua owing to a Z_N symmetry that is respected by the QCD anomaly. This results in topological structures called domain walls, which are potentially dangerous cosmologically
- When domain wall number $N_{DW} = 1$, the wall is not cosmologically stable and thus harmless Sikivie (1982)
- When axion is a mixed combination of fields in presence of a gauge symmetry, N_{DW} needs careful consideration:

$$N_{\mathrm{DW}} = \text{minimum integer} \left\{ \frac{1}{f_a} \sum_i n_i \, c_i \, v_i \; , \quad n_i \in \mathcal{Z} \right\}, \quad a = \sum_i c_i a_i$$

Ernst, Ringwald, Tamarit (2018)

- In the present model, there are 3 flavors, but $N_{DW} = 1$ nonetheless. The domain wall is another source of axion production in early universe. Range of f_a for the right axion dark matter abundance is Kawaskai, Saikawa, Sekiguchi (2015)

$$f_a = (4.6 - 7.2) \times 10^{10} \ \mathrm{GeV}$$

Successful Models with High Quality Axion

n	k	q_S	High Quality	Axion DM
3	1^+	$-31/9$	✔	✔
3	1^-	$-25/9$	✔	✔

n	k	q_S	High Quality	Axion DM
3	1^+	$16/9$	✔	✔
1	1^-, 2^-, 3^-	$22/3$, $25/3$, $28/3$	✔	✘, ✘, ✘
2	2^-	$25/6$	✔	✔
3	1^-	$22/9$	✔	✔

n	k	q_S	High Quality	Axion DM
3	1^+	$-31/9$	✔	✔
3	1^-	$-25/9$	✔	✔

Axion as a Flavon

- Since axion arises from a family symmetry it has tree-level flavor violating couplings to fermions

- An important parameter is

$$r = \frac{\langle S \rangle}{\langle x \rangle} = \frac{v_S}{v_X}$$

  For $r \geq 1$ is primarily a flavon leading to FCNC

- Most stringent constraint from $K^+ \to \pi^+ + a$ decay:

$$f_a \gtrsim (4.1, 5.0) \times 10^7 \mathrm{GeV} \, \frac{q_S r^2 \sqrt{|\kappa_{(d,s)b}(\epsilon)|}}{n\,(q_X^2 + q_S^2 r^2)}$$

Flavaxion models (global): Calibbi, Goertz, Redigolo, Ziegler, Zupan (2016); Ema, Hamaguchi, Moroi, Nakayama (2016)

Baryogenesis via Leptogenesis

- CP asymmetry in decays of N fields can be estimated. In Model 1 resonant leptogeneis arises:

$$M_N = \begin{pmatrix} 0 & M_1\epsilon^2 & M_2\epsilon^3 \\ M_1\epsilon^2 & 0 & 0 \\ M_2\epsilon^3 & 0 & M_3\epsilon^2 \end{pmatrix}$$

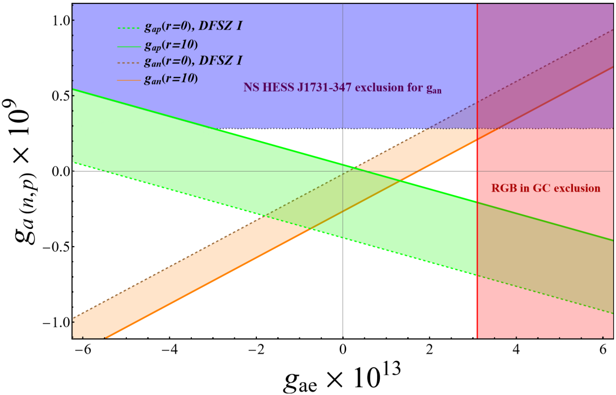
- CP asymmetry in $N_i \to L + H_d$ decays:

$$\epsilon_1 = \frac{\mathrm{Im}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)^2_{21}}{(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{11}(\hat{Y}_\nu^\dagger \hat{Y}_\nu)_{22}} \frac{(\hat{M}_2^2 - \hat{M}_1^2)\hat{M}_1\Gamma_2}{(\hat{M}_2^2 - \hat{M}_1^2)^2 + \hat{M}_1^2\Gamma_2^2}, \qquad \epsilon_2 = \epsilon_1(1 \leftrightarrow 2)$$

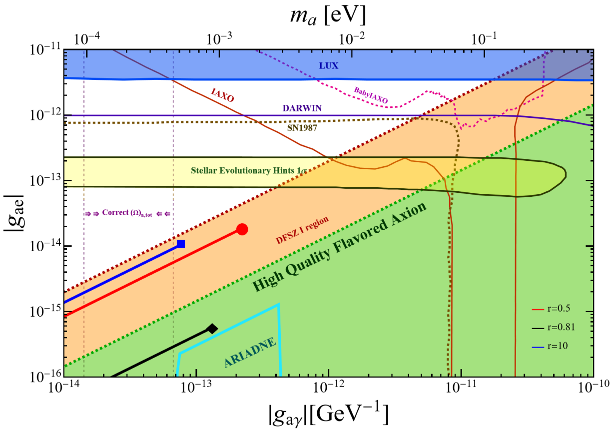
$$\epsilon_1 \simeq -\frac{128\,\pi}{\lambda^2}\mathrm{Im}(a^* c)\,\epsilon^8 \frac{M_2^3 M_3}{(M_1^2 - M_3^2)^2}$$

- ϵ^8 suppression leads to right order of baryon asymmetry

Axion Couplings to Fermions



Axion coupling to electron and Photon



High Quality Hybrid Axion in $SO(10) \times U(1)$

- Unified $SO(10) \times U(1)$ where $U(1)$ is a gauge symmetry, can lead to high quality axion
- The attractive features of $SO(10)$ GUT are preserved, including coupling unification and predictive fermion spectrum
- The fermion content and transformation under $SO(10) \times U(1)$:

$$\{3 \times 16_1 + 1 \times 10_{-6} + 1 \times 1_{12}\} + \{2 \times 1_{-4} + 1 \times 1_8\}$$

- All gauge anomalies cancel.

$$\begin{aligned}
A[SO(10)^2 \times U(1)_a] &= 3 \times 2 \times 1 + 1 \times 1 \times (-6) = 0 \\
A[(\text{gravity})^2 \times U(1)_a] &= 3 \times 16 \times 1 + 1 \times 10 \times (-6) + 1 \times 1 \times 12 + 2 \times 1 \times (-4) \\
&+ 1 \times 1 \times 8 = 0 \\
A[(U(1)_a)^3] &= 3 \times 16 \times (1)^3 + 1 \times 10 \times (-6)^3 + 1 \times 1 \times (12)^3 + 2 \times 1 \times (-4)^3 \\
&+ 1 \times 1 \times (8)^3 = 0
\end{aligned}$$

- This is the simplest $U(1)$ model that can be gauged with $SO(10)$. (Some resemblenece with E_6.)
- Such a model automatically has an axion which is of high quality.

Hybrid $SO(10) \times U(1)$ Axion

- Higgs sector contains the usual $SO(10)$ fields and two singlets:

$$\{10_H(-2) + \overline{126}_H(-2) + 45_H(0) + 10'_H(0) + T(1_{+1}) + S(1_{12})\}$$

- New features are the two singlet scalars T, S and a real $10'_H$. The $10'_H$ is needed to avoid weak scale axion: $V \supset HH'T^2$.

Fermion	$SO(10)$ irrep	gauge $U(1)_a$ charge	global $U(1)$ charge
ψ_a	$\mathbf{16}_a$	$+1$	$+1$
F	$\mathbf{10}$	-6	0
χ	$\mathbf{1}$	$+12$	0
$N_{1,2,3}$	$\mathbf{1}$	$(-4, -4, +8)$	$(0, 0, +2)$
Scalar	**$SO(10)$ rep**	**$U(1)_a$ charge**	**global $U(1)$**
H	$\mathbf{10}$	-2	-2
H'	$\mathbf{10}$	0	0
$\overline{\Delta}$	$\overline{\mathbf{126}}$	-2	-2
T	$\mathbf{1}$	$+1$	$+1$
S	$\mathbf{1}$	$+12$	0
A	$\mathbf{45/210}$	0	0

High Quality SO(10) Axion

- Yukawa couplings:

$$\mathcal{L}_{\rm Yuk} = Y_{10} 16\, 16\, 10_H + Y_{126} 16\, 16\, \overline{126}_H + y_{10} 10_{-6} 10_{-6} S_{12} + y'_{10} 10_{-6} 1_8\, 10_H + 1_{12} 1_{12} \frac{(S^*)^2}{M_{\rm Pl}} + ...h.c.$$

- Realistic fermion masses are induced, including exotics

- Model has two decoupled sectors, one with 16-fermions, and one with 10-fermion. This results in accidental PQ symmetry

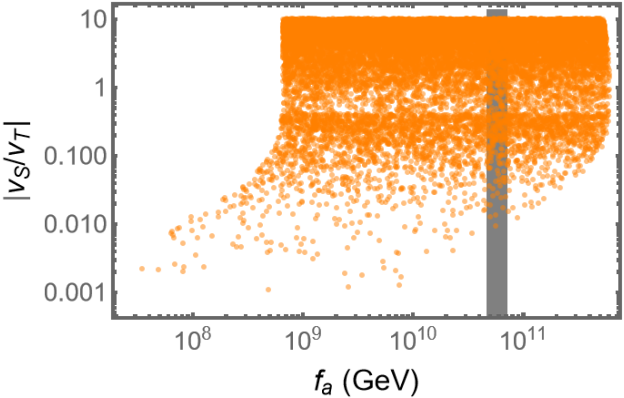
- Leading correction to PQ symmetry from gravity is

$$V \supset \frac{T^{12} S^*}{M_{\rm Pl}^9}$$

- Resulting shift in $\overline{\theta}$ is

$$\Delta\overline{\theta} \simeq \frac{\kappa \sin\delta}{(12)!\, 2^{11/2}} \frac{f_a\, v_T^{12}}{M_{\rm Pl}^9\, m_\pi^2\, f_\pi^2} \frac{(m_u + m_d)^2}{m_u\, m_d} \frac{\left(1 + \frac{144\, v_S^2}{v_T^2}\right)}{\sqrt{1 + \frac{144\, v_S^2\, v^2}{X}}}$$

- This is highly suppressed. $f_a < 7 \times 10^{11}$ GeV (1.5×10^{11} GeV) is required for quality, consistent with dark matter density of axion.



Orange points satisfy axion quality. Shaded band corresponds to the correct relic abundance of axion dark matter. Uses domain wall number $N_{\rm DW} = 1$ in the model.

SO(10) Axion Phenomenology

- Axion field is orthogonal to pseudoscalars and Goldstones

$$
\begin{aligned}
a &\simeq 1/\sqrt{1+\frac{144 v_S^2 v^2}{X}}\left(\eta_S - 12 v_T v_S v^2 \eta_T / X\right) + \ldots \\
f_a &= v_S/\sqrt{1+\frac{144 v_S^2 v^2}{X}} \\
X &= v_T^2\, v^2 + 4\, \tilde{v}^2 (V_u^2 + V_d^2) + 16\, V_u^2 V_d^2
\end{aligned}
$$

- Axion couplings to fermions and gauge bosons are defined as

$$
\mathcal{L}_a^{\rm int} \supset \frac{\alpha_s}{8\pi}\frac{a}{f_a} G^a_{\mu\nu}\tilde{G}^{a,\mu\nu} + \frac{\alpha}{8\pi}\frac{C_{a\gamma}}{f_a} F_{\mu\nu}\tilde{F}^{\mu\nu} + C_{af}\frac{\partial_\mu a}{2 f_a}(\bar{f}\gamma^\mu\gamma_5 f)\ .
$$

where $f = e, p, n$.

- The coupling coefficients are:

$$
\begin{aligned}
C_{a\gamma} &= \frac{E}{N} - 1.92 \\
C_{ap} &= -0.47 + 0.88\, c_u^0 - 0.39\, c_d^0 - C_{a,\rm sea} \\
C_{an} &= -0.02 + 0.88\, c_d^0 - 0.39\, c_u^0 - C_{a,\rm sea} \\
C_{a,\rm sea} &= 0.038\, c_s^0 + 0.012\, c_c^0 + 0.009\, c_b^0 + 0.0035\, c_t^0 \\
C_{ae} &= c_e^0 + \frac{3\alpha^2}{4\pi^2}\left[\frac{E}{N}\log\left(\frac{f_a}{m_e}\right) - 1.92\log\left(\frac{\rm GeV}{m_e}\right)\right]
\end{aligned}
$$

SO(10) Axion Phenomenology – cont.

- In the $SO(10)$ model the coefficients are

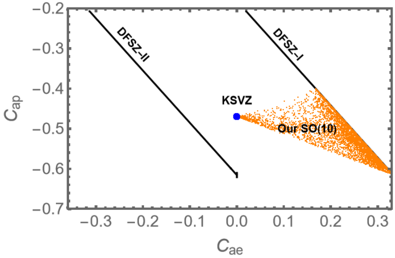
$$
\begin{aligned}
C_{ae} &= \frac{24r}{1+144r}K_e + \frac{3\alpha^2}{4\pi^2}\left[\frac{E}{N}\log\left(\frac{f_a}{m_e}\right) - 1.92\log\left(\frac{\mathrm{GeV}}{m_e}\right)\right] \\
C_{ap} &= -0.47 + \frac{r}{1+144r}(20.75K_u - 10.49K_e) \\
C_{an} &= -0.02 + \frac{r}{1+144r}(19.99K_e - 9.73K_u)
\end{aligned}
$$

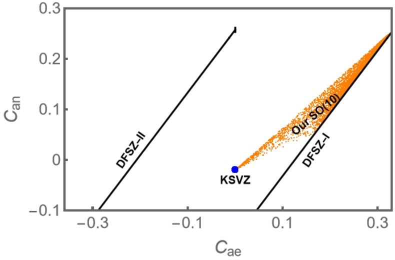
Here we have defined

$$
K_u = \frac{2{V_d}^2 + \tilde{v}^2}{v^2}, \quad K_e = \frac{2{V_u}^2 + \tilde{v}^2}{v^2}
$$

- K_e has a range $(1.5 - 2)$ corresponding to $\tilde{v} = V_u$ and $\tilde{v} \ll V_u$. The value of K_u can be much smaller, with an upper limit of 0.5 (corresponding to $\tilde{v} = v_u$). This gives a range $C_{ae} = (0.25 - 0.33)$ for $r \equiv v_S^2/v_T^2 = (0.1 - 1)$.
- As $r \to 0$ model approaches KSVZ axion. For $r \gg 1$, it is similar to DFSZ axion. In fact, the model is a hybrid version that interpolates between KSVZ and DFSZ models.

Hybrid Nature of Axion

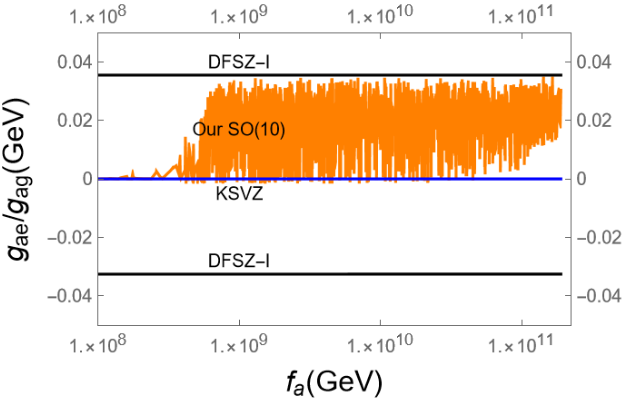


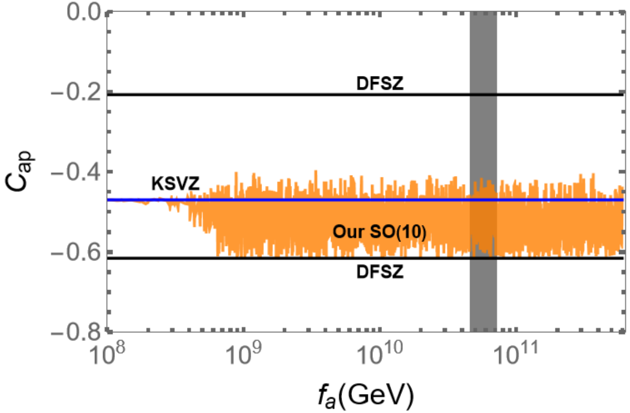


Model interpolates between KSVZ and DFSZ-I models.

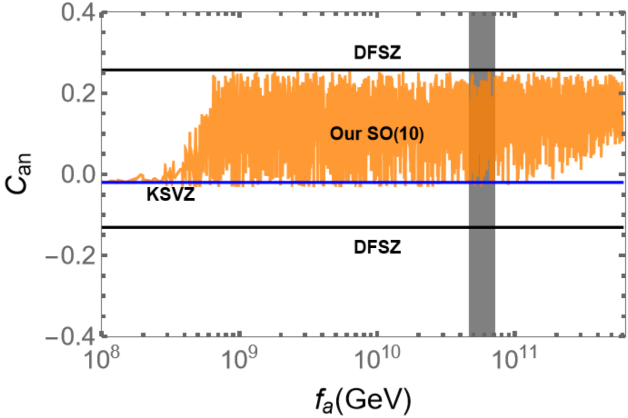
Axion couplings in SO(10) model

$$g_{af} = \frac{m_f}{f_a} C_{af}, \quad g_{ag} = \frac{\alpha_s}{2\pi f_a}, \quad g_{a\gamma} = \frac{\alpha C_{a\gamma}}{2\pi f_a}$$

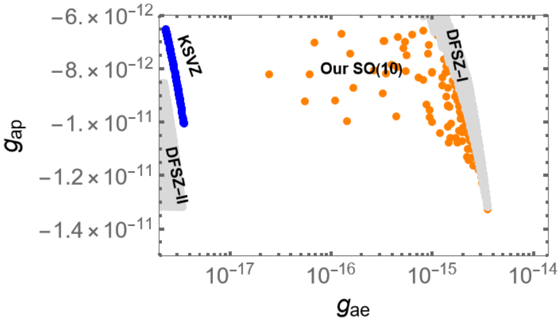




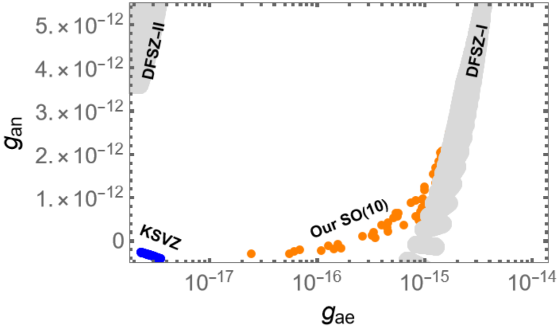
Proton-axion coupling



Neutron-axion couplings



$-6. \times 10^{-12}$
$-8. \times 10^{-12}$
$-1. \times 10^{-11}$
-1.2×10^{-11}
-1.4×10^{-11}
g_{ap}
KSVZ
DFSZ–II
Our SO(10)
DFSZ–I
10^{-17}
10^{-16}
10^{-15}
10^{-14}
g_{ae}



$5. \times 10^{-12}$
$4. \times 10^{-12}$
$3. \times 10^{-12}$
$2. \times 10^{-12}$
$1. \times 10^{-12}$
0
g_{an}
DFSZ–II
DFSZ–I
Our SO(10)
KSVZ
10^{-17}
10^{-16}
10^{-15}
10^{-14}
g_{ae}

Conclusions

- Several classes of models presented which have an accidental PQ symmetry
- $N_{DW} = 1$ in these models for domain wall number, causing no cosmological issues
- Flavor gauge symmetry framework can address many of open questions
- Axion couplings to fermions can potentially distinguish these models from standard benchmarks