A Designer's Guide to Lunar Far-Side Interferometer Array: Power Spectrum Measurement and Cosmological Constraints from the Dark Ages

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Overview

Scientific Motivation

- Dark Ages ($30 \lesssim z \lesssim 200$) is a pristine era for primordial physics
- 21-cm signal from neutral hydrogen is the sole probe
- Access to scales inaccessible to CMB & LSS surveys due to linearity on small scales
- Potentially a good and independent probe of initial conditions and particle physics

What is the measuring power of a lunar far-side interferometer array for the Dark Ages 21-cm signal, after realistically accounting for thermal noise?

How do fundamental design trade-offs—such as array compactness or station distribution—determine how well we can measure the 21-cm power spectrum and, in turn, use it to constrain cosmological and inflationary parameters?

Outline

1. **The 21-cm Power Spectrum during the Dark Ages**
   - Theoretical framework (CAMB)
   - Impact of Minihalos?
2. **Lunar Far-Side Interferometer Design**
   - Noise power spectrum modeling
   - Baseline density distributions
   - Survey specifications
3. **Power Spectrum Measurement Forecasts**
   - Single-array detection thresholds
   - Multi-array configurations for small-scale sensitivity
4. **Constraining Inflation via the Running of the Spectral Index α_s**
   - Fisher analysis
   - Comparison with previous estimates
5. **Summary**

The 21-cm Power Spectrum during the Dark Ages

CAMB computes a 21-cm monopole transfer function

$$T_{\rm mono}(k) = \Delta_s + (r_\tau - 1)(\Delta_{\rm HI} - \Delta_{T_s})$$

$\Delta_s \equiv \Delta_{\rm HI} +$ fluctuations in gas temperature and photon temperature.
The second term accounts for perturbations in the 21-cm optical depth.

Redshift-Space Power Spectrum

$$P_{21}(k,\mu) = (\delta \bar{T}_b)^2 \left[P_{\mu^0} + \mu^2 P_{\mu^2} + \mu^4 P_{\mu^4} \right]$$

Multipole Components

$$P_{\mu^0} = \mathcal{P}_\chi T_{\rm mono}^2, \quad P_{\mu^2} = 2\mathcal{P}_\chi T_{\rm mono} T_b, \quad P_{\mu^4} = \mathcal{P}_\chi T_b^2$$

where $\mathcal{P}_\chi$ is the primordial power spectrum and T_b is the baryon transfer function.

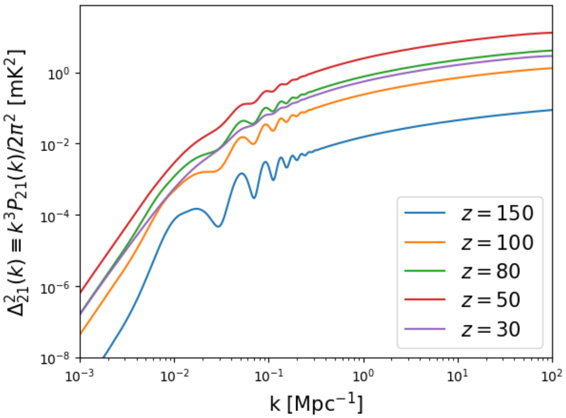


Figure: The dimensionless, spherically-averaged 21-cm power spectrum at various redshifts during the Dark Ages calculated with CAMB.

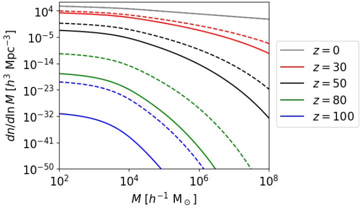
Impact of Minihalos

Halo Model Approach

$$P^{1h}_{\mathrm{mh}}(k) = \delta \bar{T}_b^2 \int dm \left(\frac{m}{\bar{\rho}}\right)^2 n_h(m) \mathcal{T}^2_{\mathrm{mh}} |u(k|m)|^2$$

$$P^{2h}_{\mathrm{mh}}(k) = \delta \bar{T}_b^2 P_{\mathrm{lin}}(k) I^2_{\mathrm{mh}}(k)$$

$$I_{\mathrm{mh}}(k) = \int dm \left(\frac{m}{\bar{\rho}}\right) n_h(m) b(m) \mathcal{T}_{\mathrm{mh}}(m) u(k|m)$$

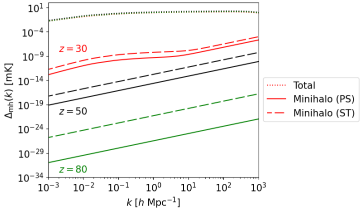


Halo mass functions assuming PS (solid) or ST (dashed)

Halo Mass Function

- Press-Schechter vs. Sheth-Tormen
- Truncated-isothermal-spherical ('TIS') model for gas temperature inside halos

Deep into the Dark Ages, minihalo contribution is **negligible** up to Jeans scale and linearity assumption is valid to $k \sim 1000\, h\,\mathrm{Mpc}^{-1}$.



Minihalo vs. total 21-cm signal

Lunar Far-Side Interferometer: Noise & Survey

Noise Power Spectrum

$$P_N(k_\perp) = T_{\rm sys}^2 r^2 y \left(\frac{\lambda^2}{A_e}\right)^2 \frac{1}{n_b(u_\perp) t_{\rm tot}} \frac{S_{\rm area}}{\Omega_{\rm FoV}}$$

Fiducial Survey Parameters

Parameter	Value
$D_{\rm min}$	4.5 m
$T_{\rm sys}$	$5000\,\mathrm{K}\,(\nu/50\,\mathrm{MHz})^{-2.5}$
$N_{\rm pol}$	2
A_e	$\lambda^2 G/(4\pi)$
$\Omega_{\rm FOV}$	10,300 deg^2
$S_{\rm area}$	20,000 deg^2
$\Delta\nu/\nu$	30%
$\delta\nu$	10 kHz

Foreground

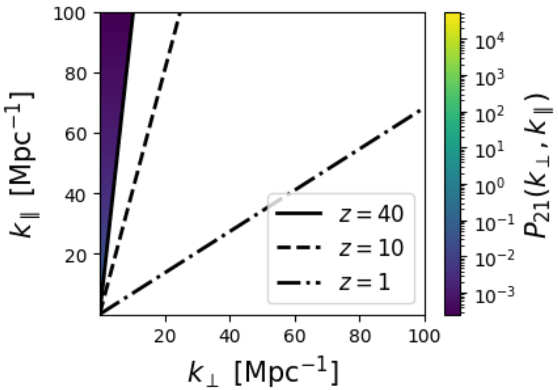


Figure: Foreground wedge in $k_\perp$–$k_\parallel$ space at $z = 40$. In our analysis, we assume a future optimistic foreground removal and do not discard modes in the wedge.

Baseline Density Distributions: Single Array

Random Circular Array

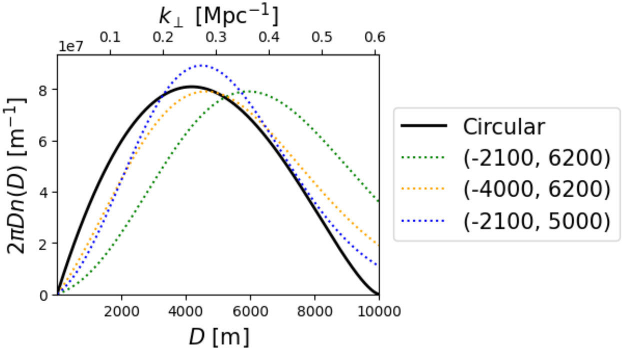
$$n_b(d) = \frac{N_{\rm ant}(N_{\rm ant}-1)}{2}\,\frac{8d}{\pi^2 D_{\rm max}^2}\left[\arccos\left(\frac{d}{D_{\rm max}}\right) - \frac{d}{D_{\rm max}}\sqrt{1-\left(\frac{d}{D_{\rm max}}\right)^2}\right]$$

Approximate Model (FarView)

$$n_b^{\rm phys}(d) = \mathcal{A}\,(d-D_0)^2 \exp\left[-\left(\frac{d-D_0}{w}\right)^2\right]$$

where $\mathcal{A}$ is set by normalization and (D_0, w) control the shape.

The FarView model tends to predict more long baselines than the random circular distribution.



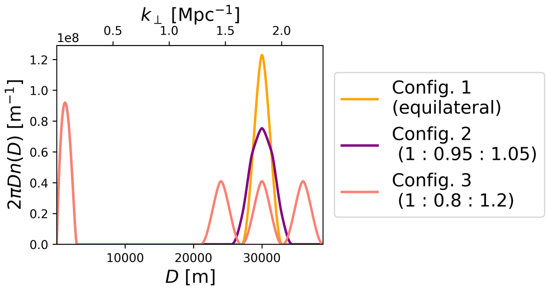
Baseline Density Distributions: Multi-Array Configurations

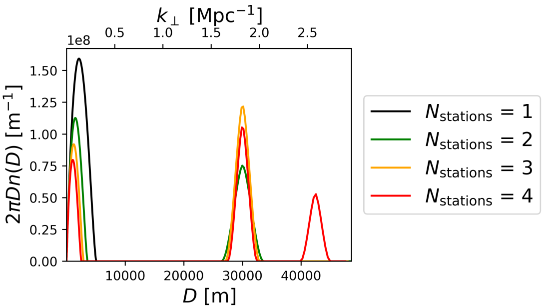
Density of inter-array baselines (two stations separated by L)

$$n_{\mathrm{cross}}(d) = N^2 d \int_0^{2\pi} \frac{\mathbb{P}_{\mathrm{intra}}(\sqrt{d^2 + L^2 - 2dL\cos\phi})}{2\pi\sqrt{d^2 + L^2 - 2dL\cos\phi}}\, d\phi$$

The total baseline density for multiple stations placed at the vertices of a polygon is

$$n_b(d) = \frac{1}{2\pi d}\left[s\binom{N_{\mathrm{ant}}}{2}\mathbb{P}_{\mathrm{intra}}(d) + N_{\mathrm{ant}}^2 \sum_{i<j} \mathbb{P}_{\mathrm{cross}}(d; L_{ij}) \right]$$





Power Spectrum Measurement: Single-Array Detection

Detection Threshold: $\mathrm{S/N} \gtrsim 1$ in key bins, $\sim 4\sigma$ total

Compact Design

- $D = 3.6$ km, $N_{\mathrm{ant}} = 10^5$
- $t_{\mathrm{int}} = 60,000$ hours
- $k_\perp^{\mathrm{max}} \sim 0.15$ Mpc^{-1}

Sparse Design (similar to FarView Stage III)

- $D = 10$ km, $N_{\mathrm{ant}} = 2 \times 10^5$
- $t_{\mathrm{int}} = 100,000$ hours
- $k_\perp^{\mathrm{max}} \sim 0.7$ Mpc^{-1}

Key Trade-off: Accessing smaller scales requires $\sim 2\times$ more antennas and $> 1.6\times$ integration time

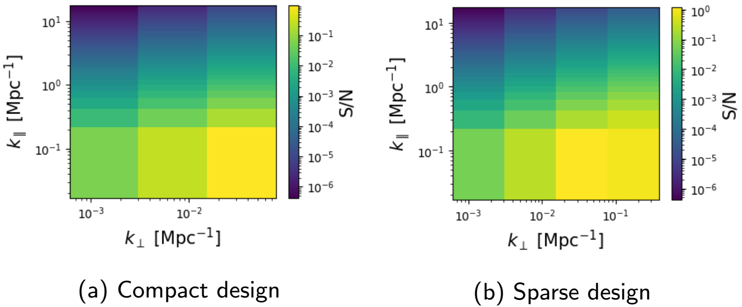


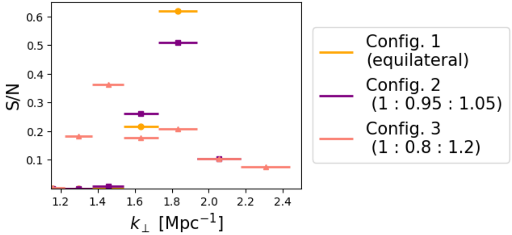
Figure: Signal-to-noise ratio maps for two single-array configurations reaching the detection threshold.

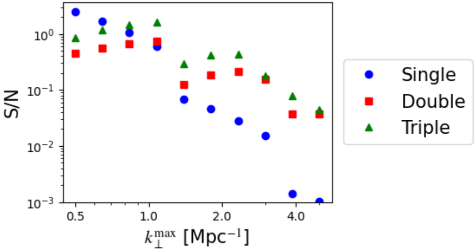
Multi-Array Configurations: Small-Scale Sensitivity

Advantages

- Flexibility and Tunability
- Sacrifice mid-length baselines to have far more long baselines under the same total $N_{\rm ant}$
- Tune station separation L to match target scale
- Up to **two orders of magnitude** improvement at $k_\perp \gtrsim 1\ \mathrm{Mpc}^{-1}$

Key Insight: Equilateral triangle concentrates baselines at one scale; non-regular triangles provide broader coverage





Constraining Inflation via the Running of the Spectral Index α_s

Running of the Spectral Index

$$\alpha_s \equiv dn_s/d\log k$$

Fisher Matrix

$$F_{ij} = \sum_{k_\perp, k_\parallel} \frac{\partial P_{21}}{\partial p_i} \frac{\partial P_{21}}{\partial p_j} \frac{1}{[\delta P_{21}]^2}$$

with $p_i = \{\Omega_b h^2, \Omega_c h^2, H_0, A_s, n_s, \alpha_s\}$.

Target: $\sigma(\alpha_s) \sim 0.012$ from Planck 2018 (for comparison)

Model Predictions Summary

Model	α_s
Scale-invariant R^2 inflation	$\sim 10^{-2}$
Multi-natural (sinusoidal)	-0.06 to 0.08
Multi-natural (η-function)	-0.013 ± 0.009
Linear potential + oscillations	~ 0.05
Quadratic potential + oscillations	~ 0.10
Multifield natural	$\sim -5 \times 10^{-3}$

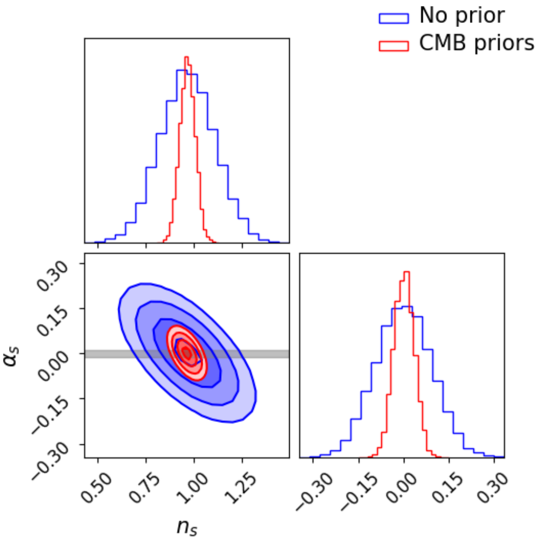
Single-Array Constraints on α_s

Minimal Configuration

- $D = 5.1$ km, $N_{\mathrm{ant}} = 10^6$
- $t_{\mathrm{int}} = 100,000$ hours
- $30 \leq z \leq 100$
- $k_\perp^{\max} \sim 0.4\ \mathrm{Mpc}^{-1}$

Results

- Without CMB priors: $\sigma(\alpha_s) = 0.091$
- **With Planck priors:** $\sigma(\alpha_s) = 0.034$
- After foreground wedge removal: $\sigma(\alpha_s) = 0.22$

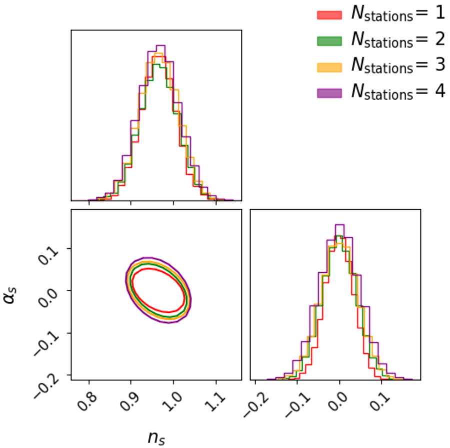


Multi-Array Constraints: 1-4 Stations

Setup

- $N_{\mathrm{ant,total}} = 10^6$ (fixed)
- $t_{\mathrm{int}} = 100,000$ hours
- Regular polygon, $L = 10$ km

L [km]	1	2	3	4
5	0.034	0.039	0.041	0.044
10		0.041	0.045	0.049
20		0.041	0.047	0.051
100		0.041	0.047	0.052



Single compact array is optimal for α_s constraints; separating the stations worsens the constraints slightly

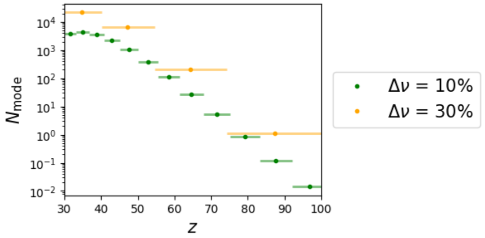
What about the estimated 10^{12} modes?

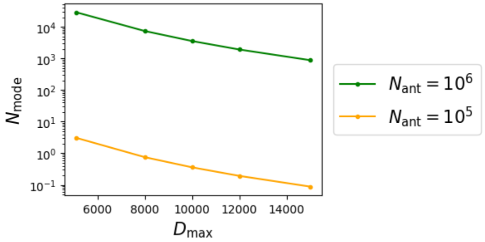
Number of Accessible Modes

$$N_{\text{modes}} = \sum_{z} \sum_{k_\perp, k_\parallel} \left(\frac{P_{21}}{\delta P_{21}} \right)^2$$

Insights:

- $N_{\text{modes}} \sim \times 10^5$ for our configurations
- Far below the $\sim 10^{12}$ cosmic variance limit
- Below Planck's $\sim 10^6$ modes
- Thermal noise is the dominant bottleneck
- Increasing the redshift space volume does not significantly increase #modes
- Increasing D to reach smaller $k_\perp$ reduces N_{modes} by $\sim 10^2$





Summary & Conclusions

- Nonlinear contributions from minihalos at $z > 30$ is negligible
- Provided a minimum configuration for power spectrum detection when noise is included
- Multi-array configurations are a more economic alternative—sensitivity at tunable small scales can be improved by up to 2 orders of magnitude
- To constrain α_s at a comparable level with Planck, one needs a single circular array with 10^6 antennas, a diameter of 5,100 m reaching a $k_\perp^{\max} \sim 0.4$ and 100,000 hours of integration
- Accessible Fourier modes from the Dark Ages is limited, in practice, by thermal noise
- Despite the limitations, the Dark Ages 21-cm signal can still be a crucial complementary cosmological probe by providing an independent measurement