Space-Time Propagation of Neutrino Wave-Packets in the Early Universe

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Introduction

★ Neutrino wave-packets:
(I) involved during production and detection processes in realistic experiments
(II) produced before BBN through nuclear beta decays

★ Current studies:
(I) focus on the vacuum case
(II) reveal the existence of a *coherence time limit* beyond which neutrino oscillation vanishes

★ Propagation of neutrino wave-packets with *dispersions* at *high temperature (for example, the early universe)* has never been investigated

Neutrino Wave-Packet

- ★ At $t = 0$, a gaussian wave-packet of ν_e and no ν_μ: $\exp\left[-\frac{(\vec{k}-\vec{k}_0)^2}{4\sigma^2}\right]$
- ★ How does this neutrino wave-packet evolve and propagate (in *space-time*) through the high temperature thermal medium?
- ★ Transition probability $\mathcal{P}_{\nu_e \to \nu_\mu}(\vec{r}, t) = ?$

Space-Time Evolution

★ Implement *finite-temperature QFT*
$\Rightarrow$ the effective Dirac equation in the medium
$\Rightarrow$ include self-energy corrections from the medium

★ Laplace transform solves the effective Dirac equation
$\Rightarrow$ time evolution of ν_μ for any momentum k

★ Fourier transform to obtain space-time evolution of ν_μ
$\Rightarrow$ transition probability $\mathcal{P}_{\nu_e \to \nu_\mu}(\vec{r}, t)$

Intuitive Picture # 1

★ Physically propagating modes are two mass eigenstates (almost degenerate):
$M_a = \overline{M}\left[1 + (-1)^{a-1}\frac{\delta M^2}{4\overline{M}^2}\right]$; $|\delta M^2|/\overline{M}^2 \ll 1$

★ WMAP3 data:
$\overline{M} = \frac{1}{2}(M_1 + M_2) < \frac{1}{2}(M_1 + M_2 + M_3) \approx 0.34\,\text{eV}$

★ Solar + KamLAND data:
(I) $|\delta M^2| = |M_1^2 - M_2^2| \approx 7.9 \times 10^{-5}\,(\text{eV})^2$
(II) $\tan^2\theta \approx 0.40$

Intuitive Picture # 2

★ Small difference in group velocities:
$\Rightarrow$ two mass eigenstates separate progressively

★ Neutrino oscillations will be exponentially suppressed when two mass eigenstates cease to overlap:
$\Rightarrow$ medium coherence time limit $T^{\text{coherence}}_{\text{medium}}$

★ Oscillatory term in $\mathcal{P}_{\nu_e \to \nu_\mu}(\vec{r}, t)$ is proportional to

$$\exp\left[-\left(\frac{t}{T^{\text{coherence}}_{\text{medium}}}\right)^2\right] \cos\left(\frac{t}{T^{\text{oscillate}}_{\text{medium}}}\right)$$

What about the Dispersions?

★ Two new characteristic time scales emerge!

★ For $t > T_\perp$:
$\Rightarrow$ transverse dispersion becomes important

★ For $t > T_{||} \approx \left(k_0/\overline{M}\right)^2 T_\perp$:
$\Rightarrow$ longitudinal dispersion becomes important
$\Rightarrow$ *competes* with progressive separation between the two mass eigenstates

Effects of Dispersions # 1

★ Oscillatory term in $\mathcal{P}_{\nu_e \to \nu_\mu}(\vec{r},\, t \gtrsim T_\parallel)$ is proportional to

$$\exp\left[-\left(\frac{T_\parallel}{T^{\text{coherence}}_{\text{medium}}}\right)^2\right]\cos\left(\frac{t}{T^{\text{oscillate}}_{\text{medium}}}\right)$$

★ If $T_\parallel < T^{\text{coherence}}_{\text{medium}}$:
$\Rightarrow$ longitudinal dispersion will be able to catch up with progressive separation between the two mass eigenstates *before the coherence time limit is exceeded*

★ Neutrino oscillations will *never* be exponentially suppressed due to progressive separation between the two mass eigenstates
$\Rightarrow$ *frozen-coherence*

Effects of Dispersions # 2

★ Recall that $T_{\|} \gg T_{\perp}$

★ *Large suppression* of transition probability due to enormous transverse dispersion:

$$\mathcal{P}_{\nu_e \to \nu_\mu}(\vec{r}, t \gg T_{\perp}) \propto \left(\frac{T_{\perp}}{t}\right)^2 \left(\text{oscillatory term} + \ldots\right)$$

Resonance Regime in the Early Universe

★ Assume $B - L = 0$ is a good symmetry:
$\Rightarrow$ charged-lepton and neutrino asymmetries are of the same order as $\mathcal{L}_B \sim 10^{-9}$

★ Medium mixing angle:
$$\sin 2\theta_m = \frac{\sin 2\theta}{\left[\left(\cos 2\theta - \frac{\mathrm{PHENO}}{\delta M^2}\right)^2 + \sin^2 2\theta\right]^{\frac{1}{2}}}$$

★ Resonance ($\sin 2\theta_m \approx 1$) occurs at $T \sim$ a few MeV
$\Rightarrow$ right before BBN

Comparison of Time Scales

★ Take $T \sim \text{a few MeV}$ as the typical energy scale

★ Comparison:
$T_\perp <<< T^{\text{oscillate}}_{\text{medium}} \ll T_{\text{collision}} \ll T_{\text{Hubble}} \ll T^{\text{coherence}}_{\text{medium}} \sim T_\parallel$

★ $\mathcal{P}_{\nu_e \to \nu_\mu}(\vec{r},\, t \gg T_\perp) \propto \left(\frac{T_\perp}{t}\right)^2 \left(\text{oscillatory term} + \ldots\right)$

is highly suppressed on the time scale of $T^{\text{oscillate}}_{\text{medium}}$!!!

Conclusions

★ characteristic time scales $T_\perp$ and $T_\parallel$ arise due to dispersions

★ *frozen-coherence* can occur if $T_\parallel < T^{\rm coherence}_{\rm medium}$

★ large suppression of transition probability due to enormous transverse dispersion
$\Rightarrow$ distortion of ν_e abundance due to neutrino oscillations right before BBN is negligible