Constraints on leptogenesis from SO(10) GUT models

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Motivations

- Theories of fermion masses, for both quarks and leptons, have acquired a good level of predictivity and agreement with physics at ElectroWeak Scale.

- Simple *models where type I see-saw mechanism is used,* give a good fit to /or/ predict the neutrino oscillation observables:

$$\Delta m^2_{\rm sol}, \quad \Delta m^2_{\rm atm}, \quad \theta_{\rm atm}, \quad \theta_{\rm sol}, \quad \theta_{\rm react},$$

which requires masses for right-handed neutrinos in the range

$$M_R \sim (10^6, 10^{16})\ {\rm GeV}$$

- On the other hand *thermal leptogenesis* is a simple mechanism to explain the observed amount of baryon asymmetry in the universe

- Lightest of N_R produced by thermal scattering after inflation $\longrightarrow$ decays out of equilibrium to a lepton and a Higgs doublet producing a CP and lepton number violation asymmetry, bound on their masses

$$M_R \gtrsim 10^9\ {\rm GeV}.$$

$=:$ **Natural to try to implement in GUT based models of fermion masses the leptogenesis mechanism**

Outline

- Introduction
- Fermion masses and flavour symmetries in $\mathrm{SO(10)}$.
- Structure of m_ν^D and M_R compatible with low energy observables for:

(i) $(m_\nu^D)_{11} = 0$

(ii) $(m_\nu^D)_{11} \neq 0$

- Incompatibility of "standard" leptogenesis and the structures for $m_{11}^D = 0$
- Soft leptogenesis can work with an underlying supergravity theory
- Conclusions

Introduction

- In models where the parameter expansion describing the Yukawa couplings of neutrinos is of the order of the parameter expansion describing the Yukawa couplings of u-type quarks,

$$\epsilon_\nu = O(\epsilon_u),$$

sometimes happens that the value needed for the lightest right handed Majorana neutrino

$$M_1 \leq 10^8 \quad \mathrm{GeV}$$

is quite below the bounds that this neturino mass should satisfy in order to get agreement with thermal leptogenesis conditions:

$$M_1 \gtrsim 10^9 \quad \mathrm{GeV}$$

[Which flavour symmetries are incompatible with leptogenesis?!]

- How general is this statement and which alternatives for a kind of leptogenesis are compatible with these scenarios?

sometimes $\longrightarrow$ **type I see-saw** and $\mathbf{m^\nu_{11} = 0}$

Fermion masses and flavour symmetries ($\mathbf{SO(10)}$)

Definition. Flavour symmetry: A symmetry that distinguishes among the species of fermions and or its generations.

Definition. Horizontal symmetry: A symmetry that distinguishes the generations of fermions.

Question. How to describe fermion masses and mixings with flavour symmetries?

Use experimental information $+$ your favourite form of mass matrices

Experimental information

- Fermion Masses

$$\begin{aligned}\{\mathbf{m_u},\ \mathbf{m_c},\ \mathbf{m_t}\} &= \{(\mathbf{0.0015},\mathbf{0.004}),(\mathbf{1.15},\mathbf{1.35}),\mathbf{174.3}\pm\mathbf{5.1}\}\mathrm{GeV}\\ \{\mathbf{m_d},\ \mathbf{m_s},\ \mathbf{m_b}\} &= \{(\mathbf{0.004},\mathbf{0.008}),(\mathbf{0.080},\mathbf{0.130}),(\mathbf{4.1},\mathbf{4.4})\}\ \mathrm{GeV}\end{aligned}$$

- CKM matrix

$$\begin{aligned}V_{\mathrm{CKM}} &= \begin{pmatrix} \mathbf{1}-\frac{\lambda^{\mathbf{2}}}{\mathbf{2}} & \lambda & A\lambda^{\mathbf{3}}(\rho-\mathbf{i}\eta) \\ -\lambda & \mathbf{1}-\frac{\lambda^{\mathbf{2}}}{\mathbf{2}} & \mathbf{A}\lambda^{\mathbf{2}} \\ \mathbf{A}\lambda^{\mathbf{3}}(\mathbf{1}-\rho-\mathbf{i}\eta) & -\mathbf{A}\lambda^{\mathbf{2}} & \mathbf{1}\end{pmatrix}\\ &= \begin{pmatrix} (\mathbf{0.9739},\mathbf{0.9751}) & (\mathbf{0.221},\mathbf{0.227}) & (\mathbf{0.0029},\mathbf{0.0045}) \\ (\mathbf{0.221},\mathbf{0.227}) & (\mathbf{0.9730},\mathbf{0.9744}) & (\mathbf{0.039},\mathbf{0.044}) \\ (\mathbf{0.0048},\mathbf{0.014}) & (\mathbf{0.037},\mathbf{0.043}) & (\mathbf{0.9990},\mathbf{0.9992})\end{pmatrix}\end{aligned}$$

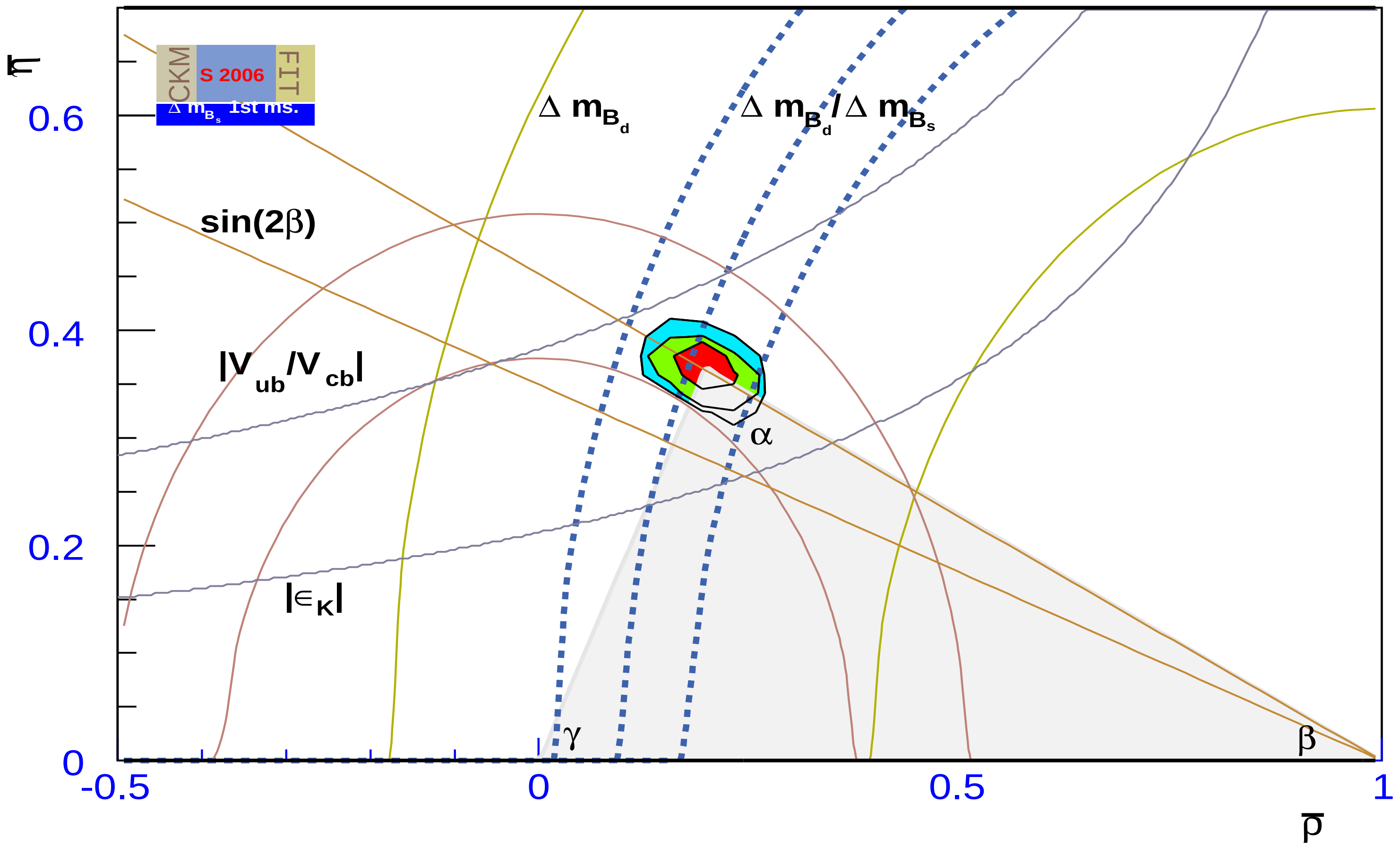

Figure 1: $(\bar{\rho}, \bar{\eta})$ including the measurement on Δm_{B_s}.

Favourite form of mass matrices

Determine your form of mass matrices

$$M^u_{\rm diag} = L^{u\dagger} M^u R^u, \qquad M^d_{\rm diag} = L^{d\dagger} M^d R^d$$

$$V_{\rm CKM} = L^{u\dagger} L^d \downarrow \qquad U_{\rm MNS} = L^{l\dagger} L^\nu$$

$\rightarrow$ **we can determine the structure above the diagonal and the eigenvalues and constrain elements below the diagonal.**

We have many possibilities for the structure of mass matrix but a **natural description of masses in terms of** $\varepsilon = O(\lambda),\ \lambda = 0.224$ **it is a hierarchical description**

$$M^d = m_b \begin{pmatrix} \varepsilon^{\geq 6} & \varepsilon^3 & \varepsilon^{\geq 3} \\ & \varepsilon^2 & \varepsilon^2 \\ & & 1 \end{pmatrix},\ M^u = m_t \begin{pmatrix} \varepsilon^{\geq 6} & \varepsilon^6 & \varepsilon^{\geq 6} \\ & \varepsilon^{\geq 4} & \varepsilon^4 \\ & & 1 \end{pmatrix},M^e = m_\tau \begin{pmatrix} \varepsilon^{\geq 6} & \varepsilon^3 & \varepsilon^{\geq 3} \\ & \varepsilon^2 & \varepsilon^{\geq 2} \\ & & 1 \end{pmatrix}$$

Need to make extra assumptions

- Elements below diagonal: Symmetric matrix, anti-symmetric
- Which powers to keep in certain places?

$\rightarrow$ Gatto-Sartori-Tonin Relation $V_{us} = |s^d_{12} - e^{i\phi_1} s^u_{12}| \approx \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi_1} \sqrt{\frac{m_u}{m_c}} \right|$

Choose your flavour symmetries

Which GUT?, which horizontal symmetry?

Some possibilities

Just GUT's [Senjanovic et. al.]

$SU(5) + \nu_R$ + horizontal symmetries [Masina & Savoy, Z. Tavartkiladze, Z. Berezhiani, K. Babu et. al.]

$SO(10)$ + non-Abelian horizontal symmetries [Ross, V-S, Raby & Dermisek, M-C. Chen & K.T. Mahanthapa, Bando & et al.]

Just horizontal symmetries, e.g. $U(1)$ [Dreiner & Thormeier et. al.]

Emerging scenarios

Symmetric	Non-symmetric
Non-Abelian	Abelian or Non-Abelian
$m^f_{11} = 0$	$m^f_{11} \neq 0$
$\downarrow$	$\downarrow$
$SU(4)_C \times SU(2)_R \times SU(2)_L$	$SU(5)$
Non compatible with thermal leptogenesis	Compatible with thermal leptogenesis

Structure of m_ν^D and M_R compatible with low energy observables for: $(m_\nu^D)_{11} = 0$

In order to identify elements of M_R and m_ν^D, in a particular basis, with low energy observables (mixings and mass differences) we can use the following relation

$$m_\nu = U^T \begin{bmatrix} m_{\nu_1} & & \\ & m_{\nu_2} & \\ & & m_{\nu_3} \end{bmatrix} U^* = -m_\nu^D M_R^{-1} (m_\nu^D)^T,$$

where U is the neutrino oscillation mixing matrix and m_{ν_i} are the neutrino mass eigenvalues. This expression is valid in the basis where charged leptons are diagonal, if their matrix is not diagonal then $U = U^\nu U^{e*}$.

$$\begin{aligned} \Delta m^2_{\rm sol} &= (8.2 \pm 0.3) \times 10^{-5}\,{\rm eV}^2, \quad \Delta m^2_{\rm atm} = (2.2^{+0.6}_{-0.4}) \times 10^{-3} \times 10^{-3}\,{\rm eV}^2 \\ t^2_{\rm atm} &= 1^{+0.35}_{-0.26}, \qquad t^2_{\rm sol} = 0.39^{+0.05}_{-0.04}, \qquad {\rm s}^2_{\rm rct} \leq 0.041 \end{aligned}$$

Limit of $s_{13} \to 0, t \to 0$ and contributions proportional to $1/M_3$ neglegible

We can then use the standard parameterization of the neutrino oscillation mixing matrix, along with the Majorana phases:

$$m_\nu = m_{\nu_3} \begin{bmatrix} r s_{12}^2 & c_{12}c_{23}s_{12}r & -c_{12}s_{12}s_{23}r \\ c_{12}c_{23}s_{12}r & c_{12}^2c_{23}^2r + s_{23}^2e^{-2i\sigma} & -c_{12}^2c_{23}s_{23}r + c_{23}s_{23}e^{-2i\sigma} \\ -c_{12}s_{12}s_{23}r & -c_{12}^2c_{23}s_{23}r + c_{23}s_{23}e^{-2i\sigma} & c_{12}^2s_{23}^2r + e^{-2i\sigma}c_{23}^2 \end{bmatrix}$$

The complete form of m_ν in terms of a diagonal matrix M_R and a general matrix m_D is given by

$$m_\nu = \frac{1}{M_1}\begin{bmatrix} m_{11}^2 & m_{11}m_{21} & m_{11}m_{31} \\ m_{11}m_{21} & m_{21}^2 & m_{21}m_{31} \\ m_{11}m_{31} & m_{21}m_{31} & m_{31}^2 \end{bmatrix} + \frac{1}{M_2}\begin{bmatrix} m_{12}^2 & m_{12}m_{22} & m_{12}m_{32} \\ m_{12}m_{22} & m_{22}^2 & m_{22}m_{32} \\ m_{12}m_{32} & m_{22}m_{32} & m_{32}^2 \end{bmatrix}$$

$$+ \frac{1}{M_3}\begin{bmatrix} m_{13}^2 & m_{13}m_{23} & m_{13}m_{33} \\ m_{13}m_{23} & m_{23}^2 & m_{23}m_{33} \\ m_{13}m_{33} & m_{33}m_{23} & m_{33}^2 \end{bmatrix}$$

Now when $m^D_{\nu 11} \to 0$, this matrix acquires a simple form.

$$m^D = \begin{bmatrix} 0 & \frac{t_{12}}{c_{23}} m_{22} & x_1 \\ p m_{31} & m_{22} & x_2 \\ m_{31} & -t_{23} m_{22} & x_3 \end{bmatrix}, \quad t_{23} = \ 1 \to \ p = 1$$

second column **is not hierarchical**, all the entries are of comparable order, however we could have all possible relations between m_{22} and m_{31}:

$$m^2_{22} r \gtrsim m^2_{31} \to \frac{M_1}{M_2} \gtrsim 1 \qquad m^2_{22} r \lesssim m^2_{31} \to \frac{M_1}{M_2} \lesssim 1$$

$$\mathbf{m_{22} = O(m_{31})} \to \frac{M_1}{M_2} = O(r m^2_{22}/m^2_{31})$$

$$\Downarrow$$

Compatible option with and underlying GUT theory

Fermion masses in $\mathrm{SO}(10)$

At the **renormalizable** level in the Higgs fields the allowed Yukawa couplings are described by the matter Lagrangian [Mohapatra:1980nn]

$$\mathcal{L}_M = \kappa_{ij}^{10}(16)_i(16)_j(10) + \kappa_{ij}^{120}(16)_i(16)_j(120) + \kappa_{ij}^{126}(16)_i(16)_j(\overline{126})$$

since

$$16 \otimes 16 = 10 \oplus 120 \oplus \overline{126}.$$

Then the fermion masses are given in general by

$$(M^u)_{ij} = \kappa_{\mathbf{ij}}^{\mathbf{10}}\langle\mathbf{10}\rangle^+ + \kappa_{ij}^{120}(\langle 120\rangle^+ + \frac{1}{3}\langle 120'\rangle^+) + \frac{1}{3}\kappa_{ij}^{\overline{126}}\langle\overline{126}\rangle^+ \quad \to Y^u\frac{v_u}{\sqrt{2}}$$

$$(M^d)_{ij} = \kappa_{\mathbf{ij}}^{\mathbf{10}}\langle\mathbf{10}\rangle^- + \kappa_{ij}^{120}(-\langle 120\rangle^- + \frac{1}{3}\langle 120\rangle^-) - \frac{1}{3}\kappa_{ij}^{\overline{126}}\langle\overline{126}\rangle^- \to Y^d\frac{v_d}{\sqrt{2}}$$

$$(M^\nu_{LR})_{ij} = \kappa_{\mathbf{ij}}^{\mathbf{10}}\langle\mathbf{10}\rangle^+ + \kappa_{ij}^{120}(\langle 120\rangle^+ - \langle 120\rangle)^+ + \kappa_{ij}^{\overline{126}}\langle\overline{126}\rangle^+ \quad \to Y^\nu\frac{v_u}{\sqrt{2}}$$

$$(M^l)_{ij} = \kappa_{\mathbf{ij}}^{\mathbf{10}}\langle\mathbf{10}\rangle^- + \kappa_{ij}^{120}(-\langle 120\rangle^- - \langle 120\rangle)^- + \kappa_{ij}^{\overline{126}}\langle\overline{126}\rangle^- \quad \to Y^l\frac{v_d}{\sqrt{2}}$$

$$M^\nu_{RR} = \kappa_{ij}^{\overline{126}}\langle\overline{126}\rangle^0, \qquad M^\nu_{LL} = \kappa_{ij}^{\overline{126}}\langle\overline{126}\rangle^+$$

Before $\mathbf{SO(10)}$ is broken

$\longrightarrow$

$$M^u_{ij} = Y^u_{ij}\langle 10\rangle^+ \quad \rightarrow \quad (M^\nu_{LR})_{ij} = Y^\nu_{ij}\langle 10\rangle^+$$
$$M^d_{ij} = Y^d_{ij}\langle 10\rangle^- \quad \rightarrow \quad M^l_{ij} = Y^l_{ij}\langle 10\rangle^-$$

$\longrightarrow$

After breaking of $\mathbf{SO(10)}$

- We know we need a different structure in

$$Y^d_{ij} \quad \text{and} \quad Y^l_{ij}$$

because we know well masses and mixing angles in these sectors $\longrightarrow$ Use $\langle 45\rangle$

45 vev lies in the two dimensional subspace of $U(1)'$ generators of $SO(10)$ that commute with $SU(3)\times SU(2)\times U(1)$ (to leave it unbroken). There are four special directions in this subspace [Anderson:1994fe]

$$X, \quad Y, \quad B-L \quad \text{and} \quad T_{R,3}.$$

For a breaking to $SU(4)_c \times SU(2)_R \times SU(2)_L$, we can choose

$$\langle 45\rangle = \langle 45_{B-L}\rangle + k_{T_{R,3}}\,\langle 45_{T_{R,3}}\rangle$$

$$m_\nu^D = \begin{bmatrix} 0 & \frac{t_{12}}{c_{23}} m_{22} & x_1 \\ m_{31} & m_{22} & x_2 \\ m_{31} & -t_{23} m_{22} & x_3 \end{bmatrix}, \quad m^u = \begin{bmatrix} 0 & O(\lambda^6) & O(\lambda^6) \\ O(\lambda^6) & O(\lambda^4) & O(\lambda^4) \\ O(\lambda^6) & O(\lambda^4) & 1 \end{bmatrix}$$

$$\begin{aligned} (m_\nu^D)_{22} &= a_\nu \lambda^4 + x_\nu \lambda^6 \\ m_{22}^u &= a_u \lambda^4 + x_u \lambda^6 \end{aligned}$$

$$\mathbf{m_{31}^D = O(\lambda^6)} \;\rightarrow\; \mathbf{m_{22}^D = O(\lambda^6) \neq m_{22}^u}$$

We can achieve this for $\mathbf{k_{T_{R,3}} = 2}$

$$a_\nu \propto \left[(B-L) + k_{T_{R,3}} T_{R,3}\right]_\nu = 0$$

	Q	u^c	d^c	L^c	l^c	ν^c
X	1	1	-3	-3	1	5
Y	$-1/3$	$4/3$	$-2/3$	$+1$	-2	0
$B-L$	$-1/3$	$+1/3$	$+1/3$	$+1$	-1	-1
$T_{R,3}$	0	$+1/2$	$-1/2$	0	$-1/2$	$+1/2$

In the present analysis then the mixing angles are simply and exactly given by

$$\mathrm{t}_{23} = -\frac{m_{32}}{m_{22}} \qquad \mathrm{t}_{12} = \frac{m_{12}}{m_{22}} \frac{m_{32}}{\sqrt{m_{22}^2 + m_{32}^2}},$$

in order for M_R to be consistent with these results then we need

$$M_2 = \frac{\mathrm{t}_{12}^2}{\mathrm{s}_{12}^2 \mathrm{c}_{23}^2 \mathrm{m}_{\nu_3} \mathrm{r}} m_{22}^2 \qquad \frac{M_1}{M_2} = \left[\frac{\mathrm{c}_{12}^2 r}{\mathrm{c}_{23}^2} \frac{m_{22}^2}{m_{31}^2}\right] \frac{\mathrm{e}^{-2i\sigma} + r\mathrm{c}_{12}^2(1 - \mathrm{c}_{23}^2(1 + \mathrm{t}_{23}^2))}{1 + p^2}$$

$$\mathbf{m}_{\nu_3} = \sqrt{\mathbf{m}^2_{\mathrm{atm}}}, \quad \mathbf{m}_{\nu_2} = \sqrt{\mathbf{m}^2_{\mathrm{sol}}}, \quad \mathbf{y}^{\nu}_{22} = \sqrt{\frac{\mathbf{m_u}}{\mathbf{m_c}}} \approx \mathbf{0.054},$$

$$\rightarrow \mathbf{M_2} = \sin^2_\beta(\mathbf{0.5}, \mathbf{2.4}) \times \mathbf{10^8}\ \mathrm{GeV}, \quad \frac{\mathbf{M_1}}{\mathbf{M_2}} = \mathbf{0.1}$$

What we have learnt? The hierarchical structure, with underlying GUT symmetry

$$M^d = m_b \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^{\geq 3} \\ & \varepsilon^2 & \varepsilon^2 \\ & & 1 \end{pmatrix}, \; M^u = m_t \begin{pmatrix} 0 & \varepsilon^6 & \varepsilon^{\geq 6} \\ & \varepsilon^{\geq 4} & \varepsilon^4 \\ & & 1 \end{pmatrix}, M^e = m_\tau \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^{\geq 3} \\ & \varepsilon^2 & \varepsilon^{\geq 2} \\ & & 1 \end{pmatrix}$$

$\longrightarrow$ Gatto-Sartori-Tonin Relation $V_{us} = |s^d_{12} - e^{i\phi_1} s^u_{12}| \approx \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi_1} \sqrt{\frac{m_u}{m_c}} \right|$

$$m^D_\nu \propto \begin{bmatrix} 0 & \frac{t_{12}}{c_{23}}\varepsilon^6 & \varepsilon^6 \\ \varepsilon^6 & \varepsilon^6 & \varepsilon^6 \\ \varepsilon^6 & t_{23}\varepsilon^6 & 1 \end{bmatrix}, \qquad M_R \;\; \text{Diagonal}$$

$\longrightarrow \quad M_{R_1} \sim 10^6$ GeV

[M. C. Chen and K. T. Mahanthappa hep-ph/0409096]

[M. Bando and S. Kaneko, M. Obara hep-ph/0405071]

[R. Dermisek, M. Harada and S. Raby hep-ph/0606055]

[G. Ross, L. V-S, hep-ph/0208218, hep-ph/0401064]

Incompatibility of "standard" leptogenesis and the structures for $(m_\nu^D)_{11} = 0$

"Standard" Leptogenesis in MSSM $+\nu_R$

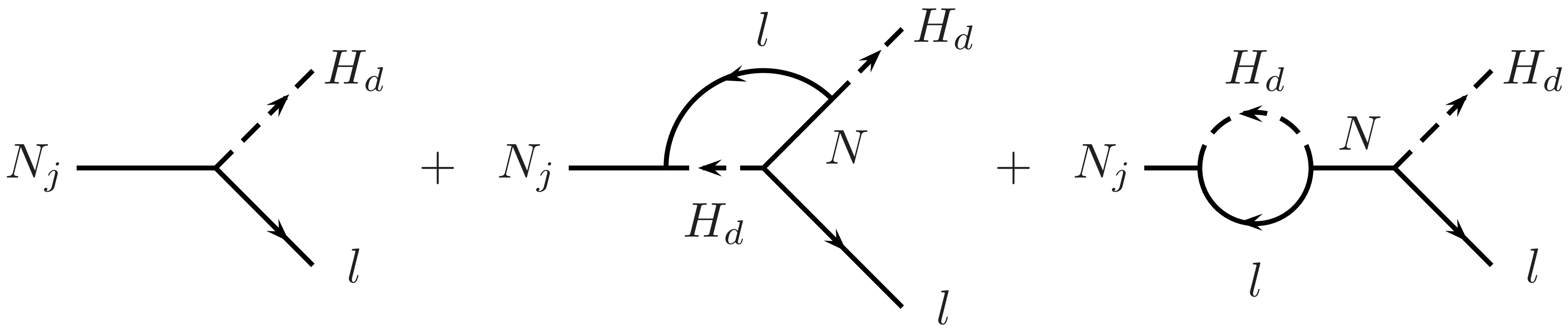

Figure 2: *Tree level and one-loop diagrams contributing to heavy neutrino decays.*

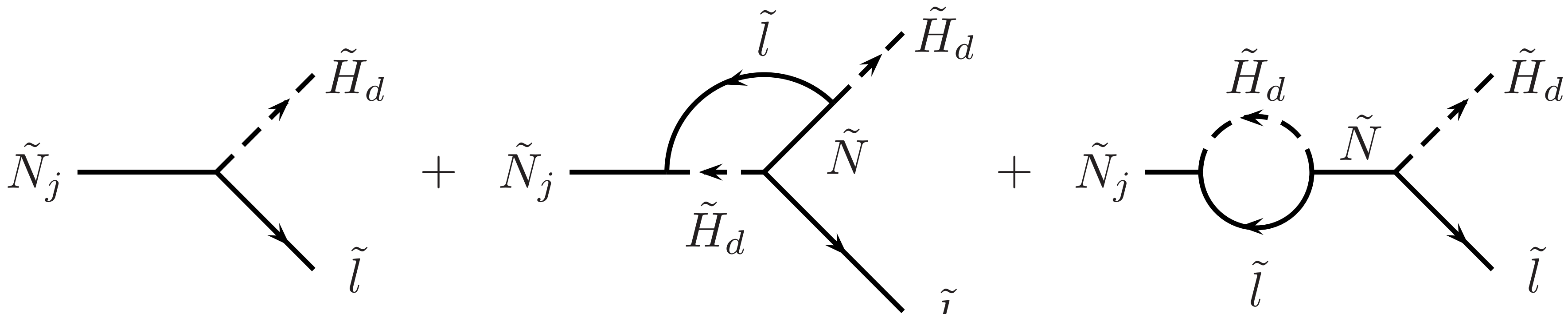

Figure 3: *Tree level and one-loop diagrams contributing to heavy s-neutrino decays.*

CP asymmetry produced in the decay of right handed neutrinos and their superpartners

In a basis, where M_R is diagonal,

$$\varepsilon_1 = \frac{\Gamma(N_1 \to l\, H_d) - \Gamma(N_1 \to l^c\, H_d^c)}{\Gamma(N_1 \to l\, H_d) + \Gamma(N_1 \to l^c\, H_d^c)} \simeq \frac{1}{8\pi} \frac{1}{(Y^\nu Y^{\nu\dagger})_{11}} \sum_{i=2,3} \mathrm{Im}\Big[\left(Y^\nu Y^{\nu\dagger}\right)^2_{1i} \Big] \Big[f\Big(\frac{M_i^2}{M_1^2}\Big) + g\Big(\frac{M_i^2}{M_1^2}\Big) \Big]$$

For $M_1 \ll M_2, M_3$

$$\varepsilon_1 \simeq -\frac{3}{8\pi}\frac{1}{(Y^\nu Y^{\nu\dagger})_{11}}\sum_{i=2,3}\mathrm{Im}\left[\left(Y^\nu Y^{\nu\dagger}\right)^2_{1i}\right]\frac{M_1}{M_i}$$

In the case of mass differences of order the decay widths one expects an enhancement from the self-energy contribution.

The CP asymmetry then leads to a lepton asymmetry

$$Y_L = \frac{n_L - n_{\bar{L}}}{s}$$

Baryon Asymmetry and Lepton asymmetry are related by

$$Y_B = \frac{n_B - n_{\bar{B}}}{s} = \frac{C}{C+1}Y_L = 1.3\times 10^{-3}\eta_i\epsilon_i \sim 10^{-10}$$

$$\eta_i \simeq \frac{2}{z_i K_i}\left(1 - e^{-z_i K_i/2}\right), \qquad K_i = \frac{\frac{4\pi\Gamma_i v^2}{M_i^2}}{1.6\times 10^{-3}\ \mathrm{eV}}$$

$$K_i \lesssim 1 \quad \longrightarrow \quad \eta_i \approx 1$$

$$K_i \geq 1 \quad \longrightarrow \quad \eta_i \ll 1$$

For the present case $\mathbf{Y_B \sim 10^{-14}}$

$\longrightarrow$ **not compatible!**

Soft leptogenesis

We need an underlying effective supergravity Lagrangian compatible with the flavour symmetry. Once we have the soft terms

$$\begin{aligned} -\mathcal{L}_{\rm soft}({\rm lep}) &= (m^2_{\tilde{l}})^j_i \tilde{l}^{\dagger i}_L \tilde{l}_{Lj} + (m^2_{\tilde{e}})^j_i \tilde{e}^{*i}_R \tilde{e}_{Rj} + (m^2_{\tilde{\nu}})^j_i \tilde{\nu}^{*i}_R \tilde{\nu}_{Rj} \\ &+ (m^2_{\tilde{h}_d}) \tilde{h_d}^\dagger \tilde{h_d} + (m^2_{\tilde{h}_u}) \tilde{h_u}^\dagger \tilde{h_u} + (a^{ije} \tilde{l}_{Lj} \tilde{e}^*_{Ri} h_d + a^{ij\nu} \tilde{l}_{Lj} \tilde{\nu}^*_{Ri} h_u + {\rm h.c}) \\ &+ (b_h h_u h_d + \frac{1}{2} (b_\nu)^i_j \tilde{\nu}^{*i}_R \tilde{\nu}^*_{Rj} + {\rm h.c.}) \end{aligned}$$

In the MSSM only the Higgs fields can have a b term, such that $b_h = B\mu$, but when right-handed neutrinos are included, in general there is a b_ν term entering in $\mathcal{L}_{\rm soft}$ that we can write as

$$(b_\nu)_{ij} = B_\nu M_{ij}$$

$\tilde{N}$ and $\tilde{N}^\dagger$ behave like the mixing system of Kaons, $\bar{K} - K$.

Due to all the mass terms in $\mathcal{L}_{\rm soft}({\rm lep})$, $\tilde{N}$ and $\tilde{N}^\dagger$ are not mass eigenvalues, instead

$$\tilde{N}_+ = \frac{1}{\sqrt{2}}(e^{i\phi/2}\tilde{N} + e^{-i\phi/2}\tilde{N}^\dagger), \quad \tilde{N}_- = \frac{-i}{\sqrt{2}}(e^{i\phi/2}\tilde{N} - e^{-i\phi/2}\tilde{N}^\dagger),$$

are the mass eigenvalues. Its evolution (in the non-relativistic limit) can be understood in terms of the Hamiltonian $H = \hat{M} - i\hat{\Gamma}/2$

$$\hat{M} = M \begin{pmatrix} 1 & \frac{B}{2M} \\ \frac{B}{2M} & 1 \end{pmatrix}, \quad \hat{\Gamma} = \Gamma \begin{pmatrix} 1 & \frac{A^*}{M} \\ \frac{A}{M} & 1 \end{pmatrix}, \quad \Gamma = \frac{(Y^\nu Y^{\nu\dagger})}{4\pi} M, \quad a_{ij} = A_{ij} Y_{ij}$$

Then the total CP asymmetry is

$$\varepsilon_1 = \frac{4\Gamma B}{4B^2 + \Gamma^2} \frac{\mathrm{Im} A}{M} \Delta_{BF}, \quad \Delta_{BF} = \frac{c_B - c_F}{c_B + c_F}$$

For the present case in order to have a Baryon asymmetry of $O(10^{-10})$, given by

$$\frac{n_B}{s} = -\left(\frac{24 + 4n_H}{66 + 13n_H} \frac{\epsilon_1}{\Delta_{BF}} \right) \eta Y^{eq}_{\tilde{N}}$$

we need

$$B \sim (10\,\mathrm{MeV}, 1\,\mathrm{GeV}), \quad A \sim 10^3\,\mathrm{GeV}$$

$\longrightarrow$ $\mathbf{B}$ extremely small

$\longrightarrow$ Use an appropriate supergravity description to generate it at the right scale!

What we can learn from the Higgs case?

$$\begin{aligned}
\mathbf{W}_{Obs} &= \mathbf{W}_{Yuk} + M_H \mathbf{H}_u \mathbf{H}_d \nearrow^0 \\
\mathbf{W}_{Hid} &= M_{Hid} f(\mathbf{X}) \\
\mathbf{W}_{Obs-Hid} &= \frac{\lambda_H}{2} X \mathbf{H}_u \mathbf{H}_d \nearrow^0 \\
\mathbf{K}_{Obs} &= \mathbf{H}_u^\dagger \mathbf{H}_u + \mathbf{H}_d^\dagger \mathbf{H}_d \\
\mathbf{K}_{Hid} &= \mathbf{X}^\dagger \mathbf{X} \\
\mathbf{K}_{Obs-Hid} &= \frac{\lambda_H}{M_P^2} \mathbf{X}^\dagger \mathbf{X} H_u H_d + h.c. \ (\mathrm{Giudice - Masiero})
\end{aligned}$$

$$\Downarrow$$

Effective mass parameters of Higgs sector at the right scale

$$\begin{aligned}
\mu &= \lambda_H < X >< F_X > \sim M_{Hid} => m_{3/2} \\
b_H &= \frac{\lambda_H}{M_P^2} \langle F_X \rangle^2 \sim M_{Hid}^2 => m_{3/2}^2
\end{aligned}$$

i.e. successful EW symmetry breaking

$$\begin{aligned}
\mu &< \Lambda = M_P \\
b_H &= B_H \mu \lesssim \mu^2
\end{aligned}$$

$N, \tilde{N}$ case

$$\mathbf{W}_{Obs} = \mathbf{W}_{Yuk} + M_H \mathbf{H}_u \mathbf{H}_d \nearrow^0 + \lambda_{ij} \nu_{Ri} \Sigma \nu_{Rj} \nearrow^0 ?$$

$$\mathbf{W}_{Hid} = M_{Hid} f(\mathbf{X})$$

$$\mathbf{W}_{Obs-Hid} = \frac{\lambda_H}{2} X \mathbf{H}_u \mathbf{H}_d \nearrow^0 + \frac{(\lambda_N)_{ij}}{2} X {\nu^{\mathbf{c}}}_i {\nu^{\mathbf{c}}}_j \nearrow^0 ?$$

$$\mathbf{K}_{Obs} = \mathbf{H}_u^\dagger \mathbf{H}_u + \mathbf{H}_d^\dagger \mathbf{H}_d + \nu^{\mathbf{c}\dagger} \nu^{\mathbf{c}}$$

$$\mathbf{K}_{Hid} = k_{XX} \mathbf{X}^\dagger \mathbf{X}$$

$$\mathbf{K}_{Obs-Hid} = \frac{\lambda_H}{M_P^2} \mathbf{X}^\dagger \mathbf{X} \mathbf{H}_u \mathbf{H}_d + h.c. \frac{(\lambda_N)_{ij}}{M_P^2} \mathbf{X}^\dagger \mathbf{X} {\nu^\dagger}_i \nu_j ?$$

$$\Downarrow$$

Effective mass parameters of N and $\tilde{N}$ at the right scale?

$$\langle \Sigma \rangle = M_P => (M_R)_{ij} = \lambda_{ij} M_P = \mathcal{O}(10^7 \mathrm{GeV})(+\mathrm{see} - \mathrm{saw}) \spadesuit$$

$$b_\nu = \frac{\lambda_N}{M_P^2} \langle F_X \rangle^2 \sim M_{Hid}^2 => m_{3/2}^2 ?$$

Do we have a sufficiently small $B = b_\nu M_R$ term?

Yes if we can achieve

$$M_N \lesssim M_G < M_P$$

$$b_N = B_N M_P < m_{3/2}^2 \rightarrow B_N \sim \frac{m_{3/2}^2}{M_P}$$

$$W = \mu(\Phi_i)NN + Af(\Phi_i)XK_N \sim \frac{N_i^\dagger N_j}{M}\left(\lambda^{i\dagger}\lambda^j(a_0 + a_1 XX^\dagger) + ...\right),$$

X a field breaking the supersymmetry

Φ_i observable field multiplet of $SO(10)$

Set a vanishing B through the minimisation of V : [Yamaguchi, M & K. Yoshioka hep-ph/0204293]

$$V = e^K\left[K^{i\bar{j}}F_iF_{\bar{j}} - 3|W|^2\right],$$

For V_X :

$$\begin{aligned} V_X &= K^{i\bar{j}}\left[F_{\bar{j}}(W_{iX} + K_iW_X)\right] - 3W_X\overline{W} \\ &+ K^{X\bar{X}}\left[F_X\overline{W}K_{\overline{X}X} + F_{\overline{X}}(W_{XX} + K_XW_X + K_{XX}W)\right] \end{aligned}$$

On the other hand the b term associated to N

$$b = e^K\left[K^{i\bar{j}}F_{\bar{j}}(\mu_l + K_{i\mu}) - 3\mu\overline{W} + 2\mu\overline{W}\right]$$

Possible with a specific choice of $K \rightarrow$

$$b = 0 \quad @ \quad \text{at} \quad O(M_P) \quad \rightarrow$$

only contribution to b coming from the hidden-obs. $K \rightarrow \quad B \sim \frac{m_{3/2}^2}{M_R} \sim (0.01, 1)$ GeV.

Simplest form to arrange $B = 0 \to$ No-Scale Supergravity

$$\mathbf{K} = -\mathbf{3}\log(\phi + \phi^*)$$

Coupling to matter fields

$$\mathbf{Y_{10\ 120}}(\phi) = -\mathbf{e}^{-\mathbf{c}\phi}, \quad \mathbf{Y}_{126} = \ \mathbf{126} = \mathrm{const.}$$

(This can be a $U(1)$ symmetry)

$$\phi \quad \to \phi \, + \, i \, \alpha$$

$$\mathbf{16.16.10} \qquad \mathbf{16.16.10} \qquad \mathbf{16.16.\overline{126}}$$

$$AY = -m_{3/2}(\phi \, + \, \phi^*)\partial_\phi Y \to$$

$$A_{10,\ 120} \quad \sim \quad m_{3/2}, \qquad B = A_{126} = 0 \quad @ \quad \mathrm{GUT}$$

1 Loop corrections due to gauge interactions of N_R produce small B term

In $SO(10)$ the gauge coupling $N - \tilde{N} - \tilde{X}$ gives

$$b \; = \; BM \; = \; \frac{\alpha}{4\pi} m_{1/2} M \log \frac{M_X}{M}$$

M_X is the mass of the heavy gauge boson X (or $B - L$ scale)

$$\alpha = \frac{1}{30} \quad M = 10^8 \text{ GeV} \quad M_X = 10^{10} \text{ GeV} \; \rightarrow$$

B$\approx 10^{-2} m_{1/2} \quad B \approx 1 \text{ Gev}, \; m_{1/2} = 100 \text{ Gev}$

Remarks

●The hierarchical structure, with underlying GUT symmetry, such that $m^f_{11} = 0$
$(V_{us} = \left| \sqrt{\frac{m_d}{m_s}} - e^{i\phi_1} \sqrt{\frac{m_u}{m_c}} \right|$)
$\rightarrow \quad M_{R_1} = 10^6$ GeV $\rightarrow$

Not compatible with thermal leptogenesis

●Soft Leptogenesis

$$\varepsilon_1 = \frac{4\Gamma B}{4B^2 + \Gamma^2} \frac{\mathrm{Im}A}{M} \Delta_{BF}, \quad \Delta_{BF} = \frac{c_B - c_F}{c_B + c_F}$$

$\rightarrow B = (0.01, 1)$ GeV

● No-scale supergravity $A = B = 0$ @ GeV 1 Loop corrections in the coupling $X - N - \tilde{N}$

$$B = \frac{\alpha}{4\pi} m_{1/2} \log \frac{M_X}{M}$$

$\rightarrow B = (0.01, 1)$ GeV