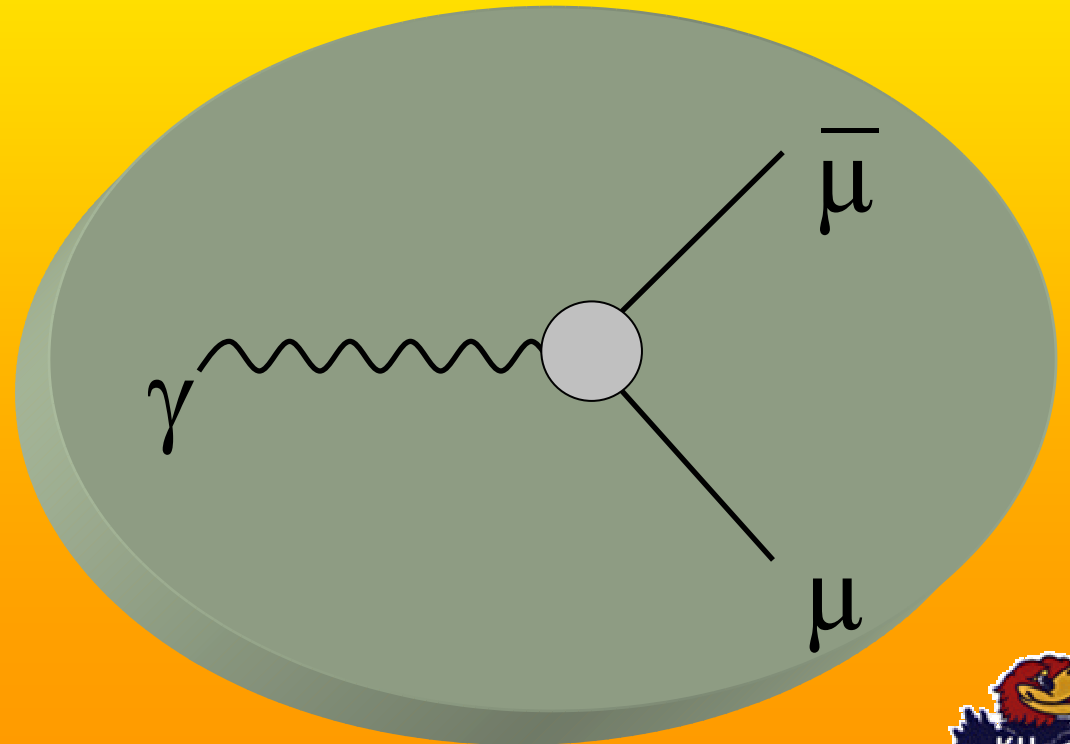


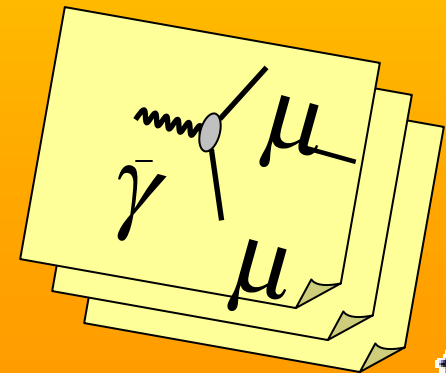
Explaining the Muon Magnetic Moment Anomaly

Rainer Schiel
with
John Ralston



Outline

- a_μ : Experiment vs Theory
- Hadronic Contribution
...Previously Overlooked
- Consequences



The Muon Magnetic Moment

$$\vec{\mu} = g \frac{e}{2m} \vec{S}$$

Magnetic moment

Landé g-factor
(expect "2" for Dirac particle)

Spin

Charge over twice the mass

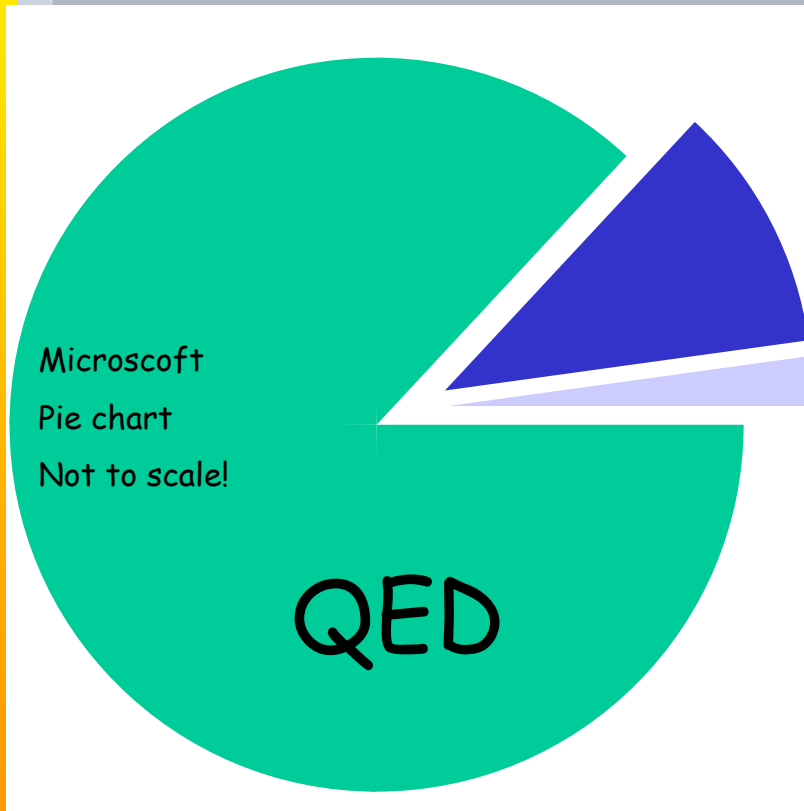
Magnetic moment anomaly:

$$a_{\mu} = \frac{g - 2}{2}$$

- $a_{\mu}^{\text{exp}} = (11659208.0 \pm 6.3)10^{-10}$

- $a_{\mu}^{\text{theory}} = (11659180.0 \pm 5.1)10^{-10}$

Expt -BNL 2006; Theory - Hagiwara et al,
Davier et al, Jagerlehner ...

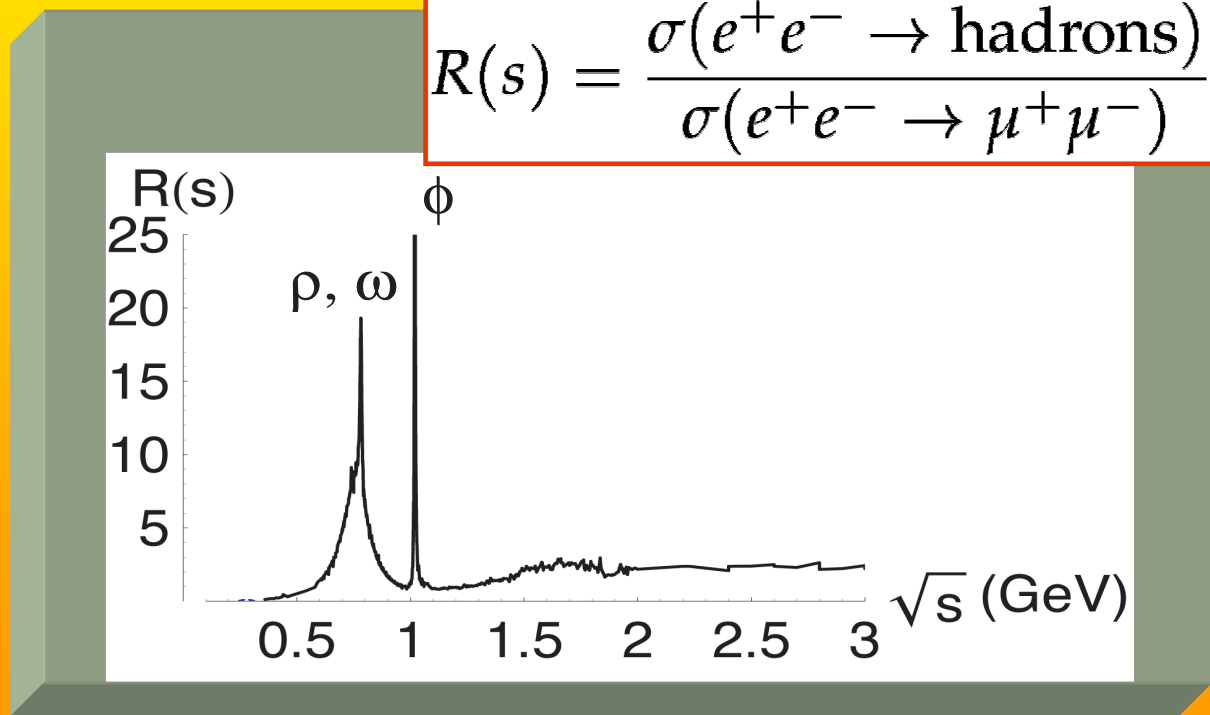
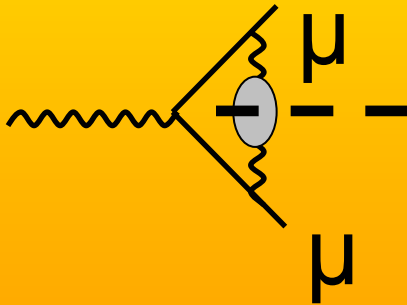


hadronic
vacuum
polarization
 $(689.4 \pm 4.6)10^{-10}$

hadronic contributions

$$a_{\mu}^{had,LO} = \frac{\alpha^2}{3\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \frac{K(x(s))}{s} R(s)$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$



and the famous kernel is...

$$\begin{aligned} K(x) &= x^2 \left(1 - \frac{x^2}{2}\right) \\ &+ (1+x)^2 \left(1 + \frac{1}{x^2}\right) \left(\ln(1+x) - x + \frac{x^2}{2}\right) \\ &+ \frac{1+x}{1-x} x^2 \ln x; \end{aligned}$$

$$x = (1 - \beta_\mu) / (1 + \beta_\mu);$$

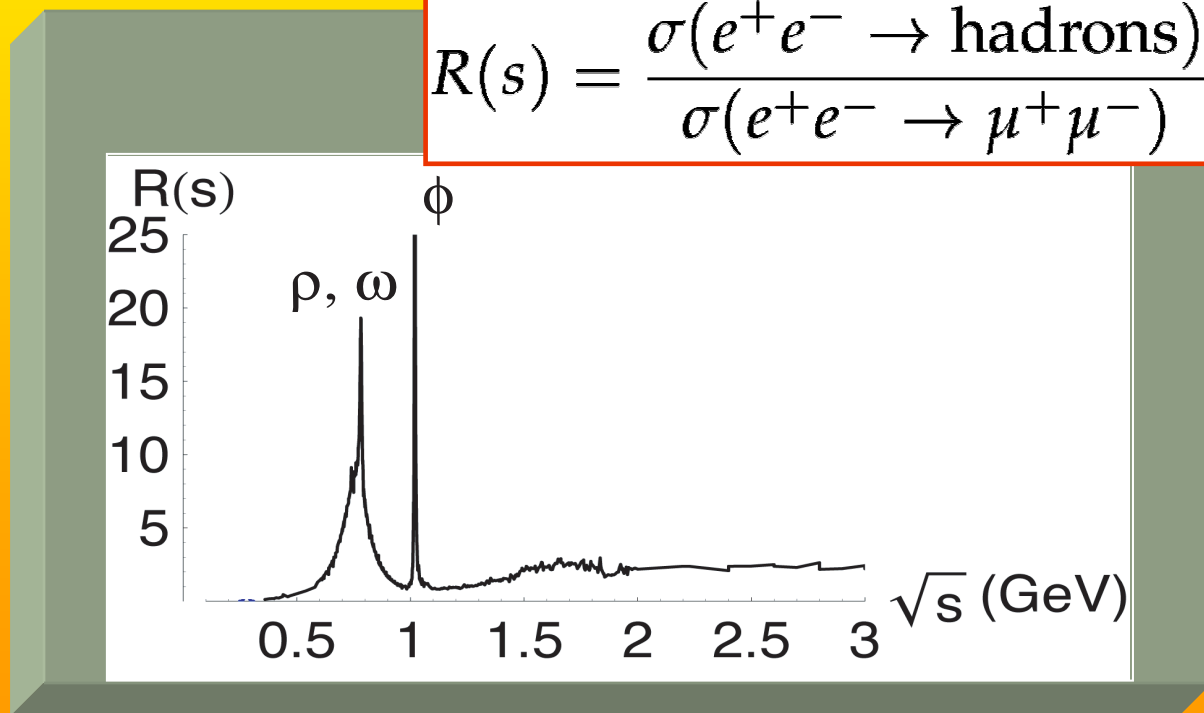
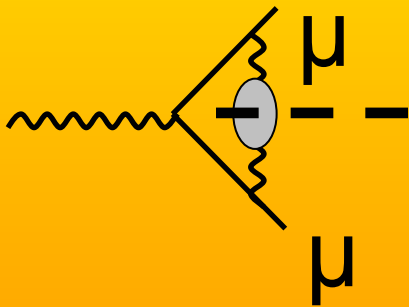
$$\beta_\mu = \sqrt{1 - 4m_\mu^2/s}.$$



hadronic contributions

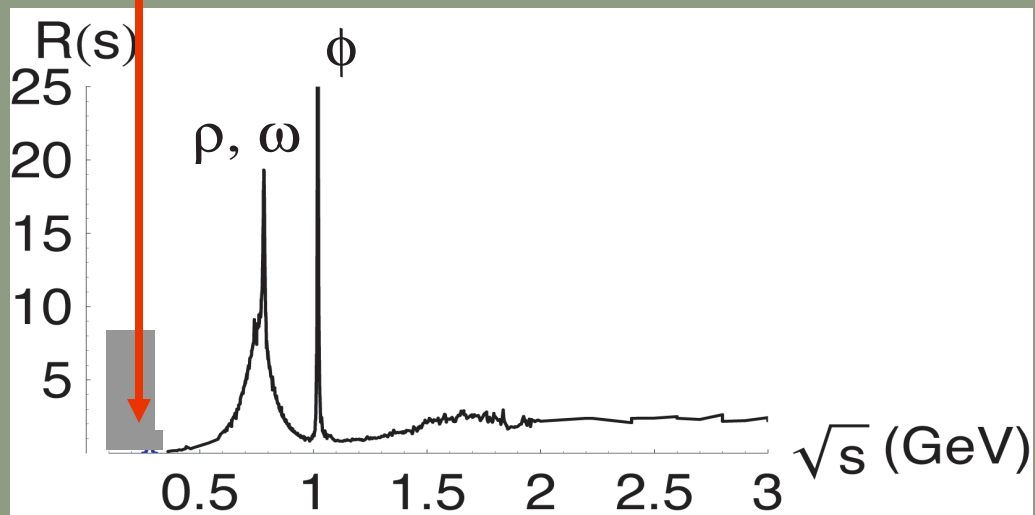
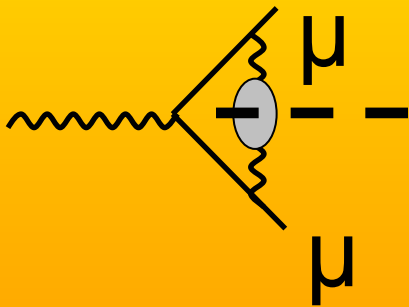
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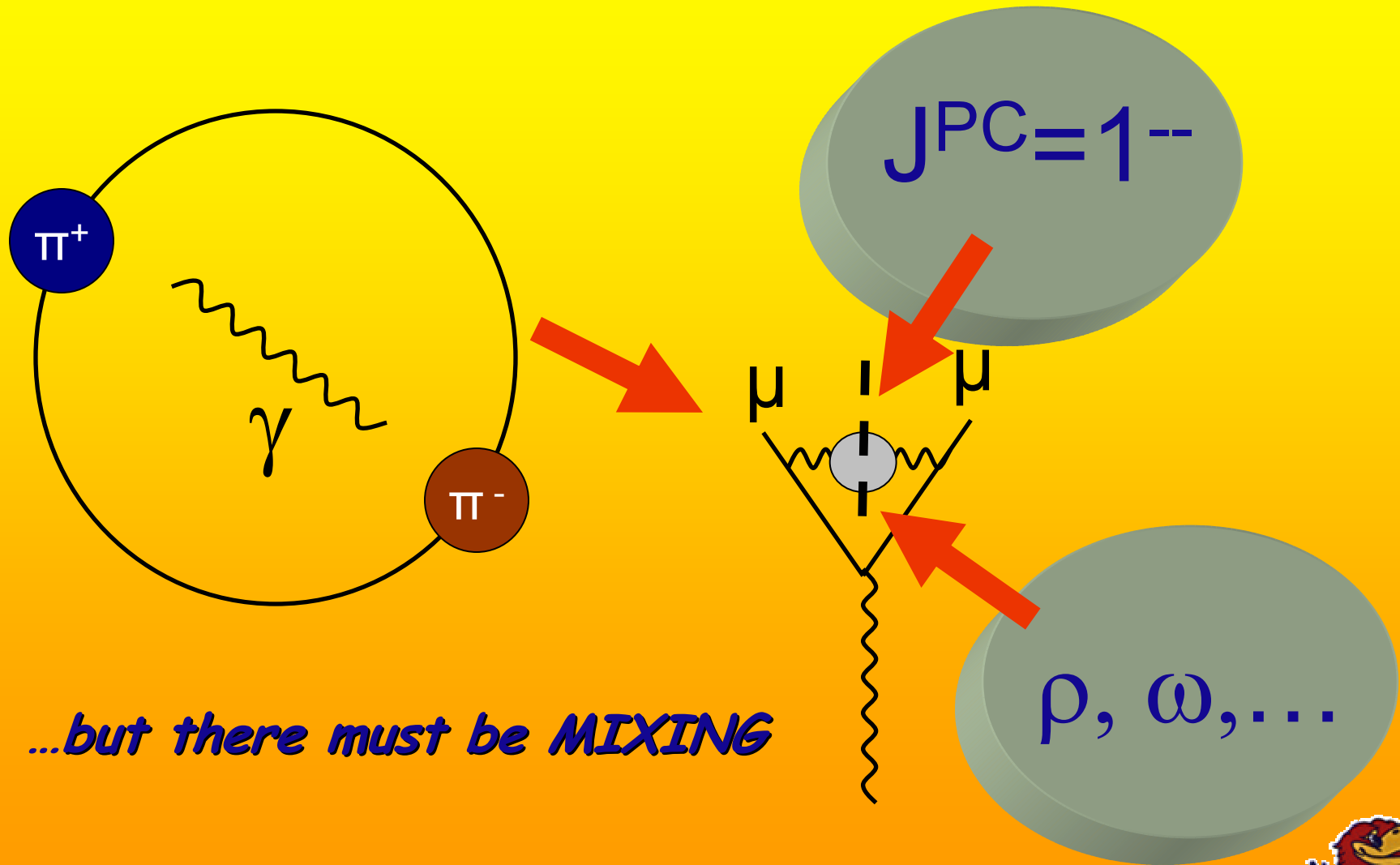


hadronic contributions

$$a_{\mu}^{had,LO} = \frac{\alpha^2}{3\pi^2} \int_{s_{min}}^{\infty} ds \frac{K(x(s))}{s} R(s)$$



$\pi^+ - \pi^-$ "atoms" are *pionium*



...but there must be MIXING



2 state mixing !

Pirhonium

$$\begin{pmatrix} |\pi_2/\rho\rangle \\ |\rho\rangle \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} |\tilde{\pi}_2\rangle \\ |\tilde{\rho}\rangle \end{pmatrix}$$

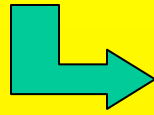
physical

bare

$$H_{int} = \begin{pmatrix} m_{\tilde{\pi}_2} & m_{ix} \\ m_{ix} & m_{\tilde{\rho}} \end{pmatrix}$$

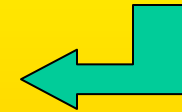


bare atomic state...

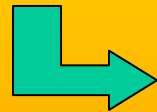


$$R_{\tilde{\pi}_2}(r) = \frac{1}{\sqrt{24}} \frac{r}{a_0^{5/2}} e^{-r/2a_0}$$

bare mass = $2 m_\pi - 13.6 \text{ eV} / 2^2 (m_\pi / 2 m_e)$...
...fixes one mixing parameter

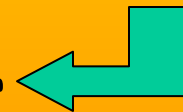


diagonalize...



$$\tan \beta = \frac{1}{2m_{ix}} (m_{\tilde{\rho}} - m_{\tilde{\pi}_2} - \sqrt{(m_{\tilde{\rho}} - m_{\tilde{\pi}_2})^2 + 4m_{ix}^2})$$

$\Gamma(\pi_2/\rho \rightarrow e^+e^-) = 28 \text{ eV}$...fixes second parameter



mass eigenvalues...

bare atomic state...

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diagonalize...

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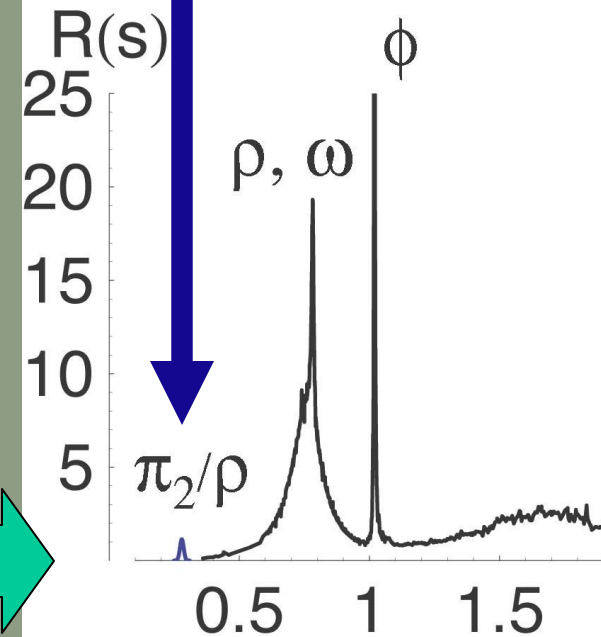
mass eigenvalues...

Rainer W. Schiel and John P. Ralston

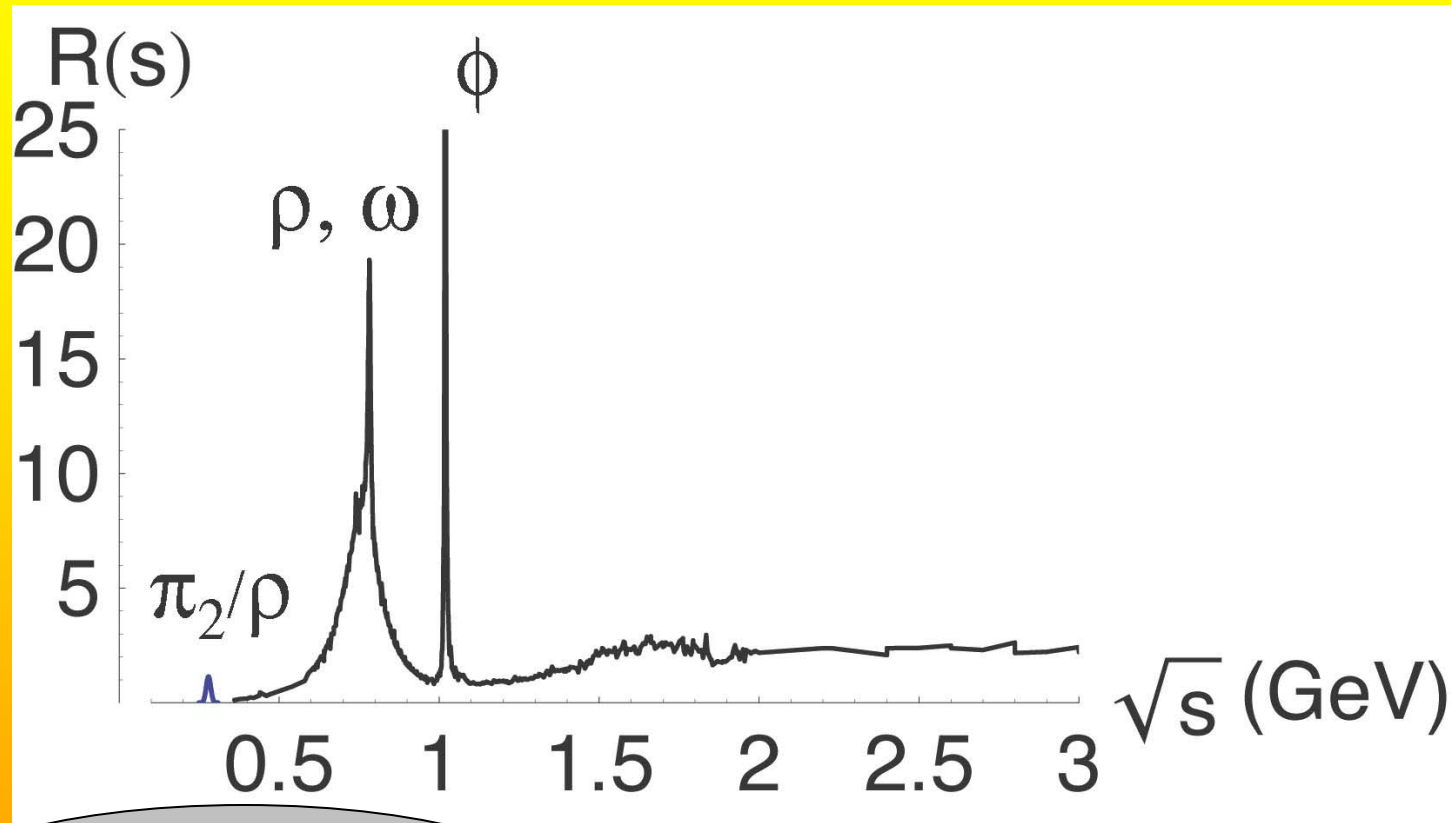
$m_\rho = 775 \text{ MeV}$... fixes last parameter

$$\sin \beta = 0.81\%$$
$$\beta = 0.46^\circ$$

279.11 MeV



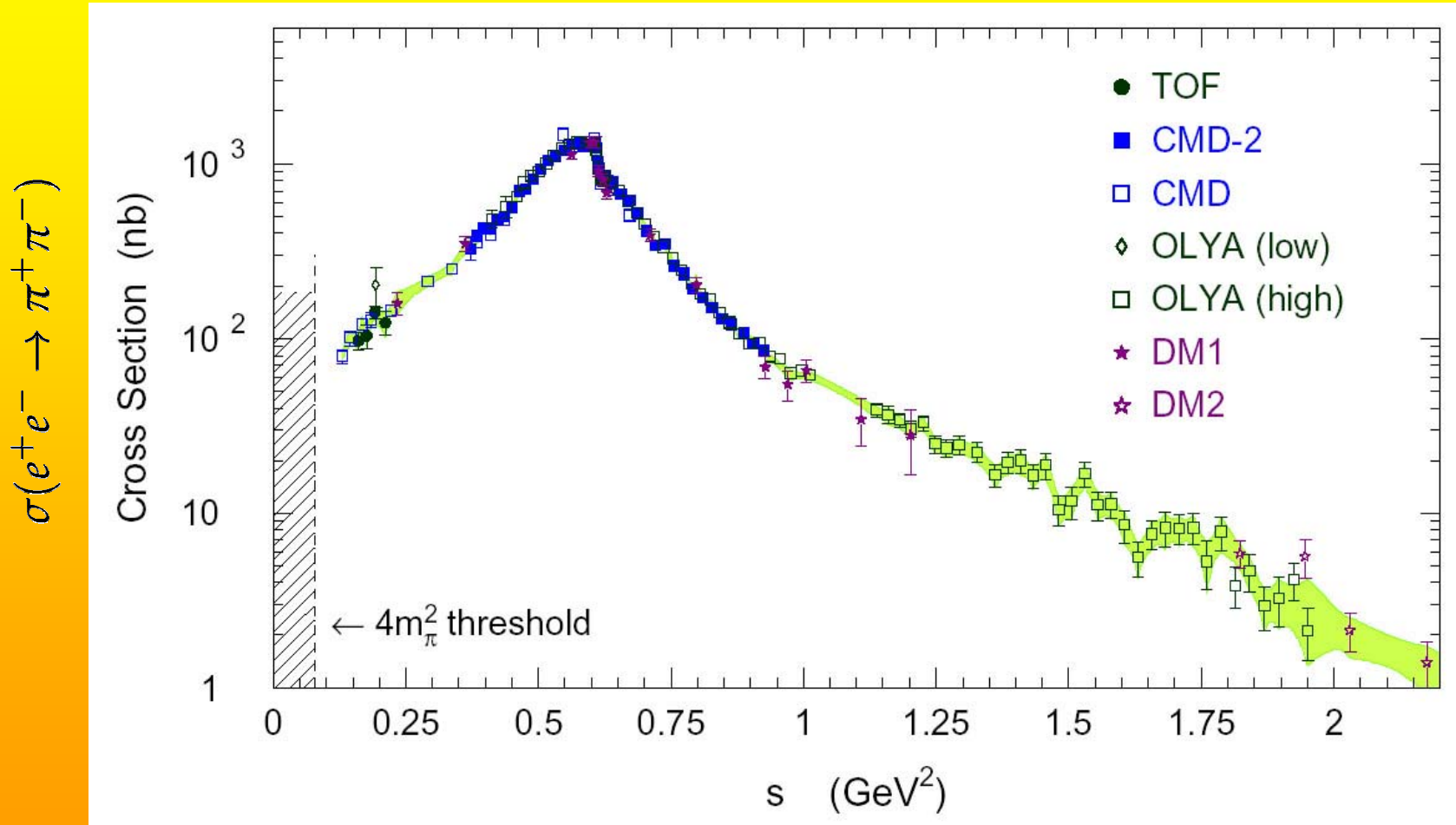
Pirhonium contribution



$\sin \beta \approx 0.81\%$



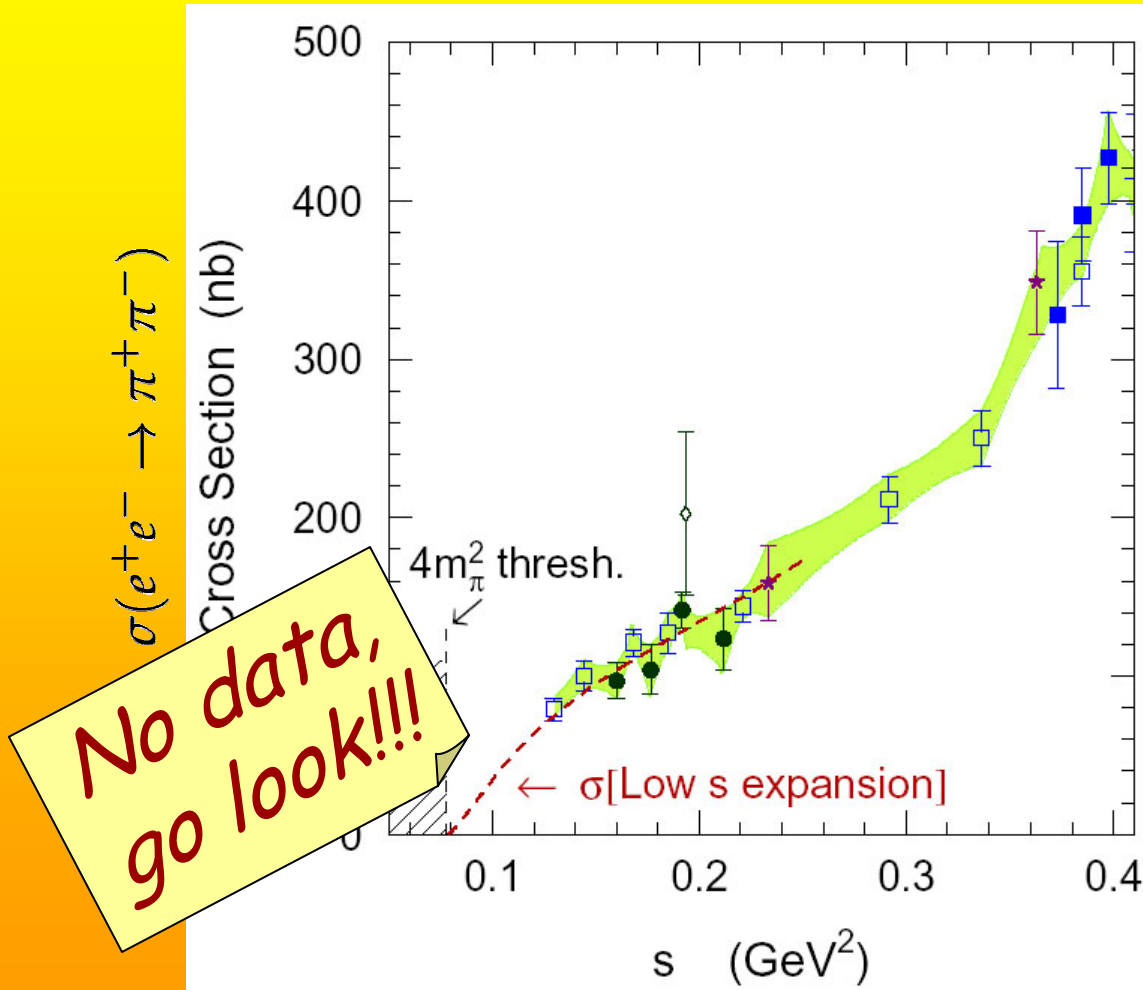
The unexplored...low- s ! data



(Davier et al., 2002)



Very, very low- s data



(Davier et al., 2002)



Predicted Decay Channels

$$\Gamma(\pi_2/\rho \rightarrow e^+e^-) \sim 28 \text{ eV}$$

$$\Gamma(\pi_2/\rho \rightarrow \mu^+\mu^-) \sim 18 \text{ eV}$$

$$\Gamma(\pi_2/\rho \rightarrow \pi^0\pi^0) \sim 150 \text{ keV}$$

(estimates based on phase space only!)



Conclusions

- Pirhonium explains the muon magnetic moment anomaly
- Mixing with the ρ of order 1% is sufficient
- There is no doubt this particle exists: the burden of proof is reversed!

