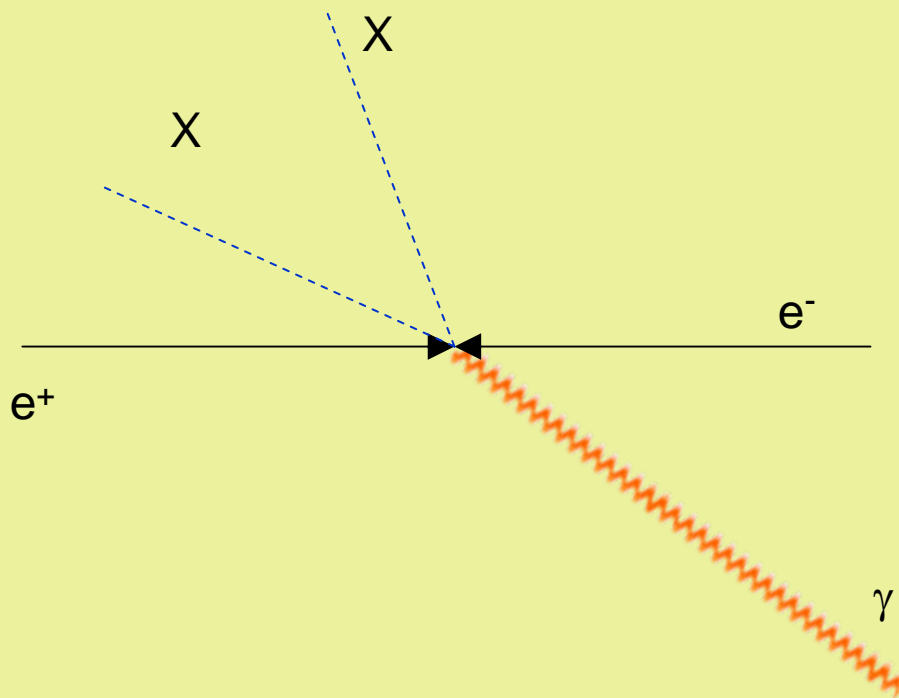


# Brane Oscillations In Collider Physics

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## Outline:

1. Brane-Standard Model  
Effective Action
2. Collider Physics:  $e^+ e^- \rightarrow \gamma + \cancel{E}$
3. Brane Oscillations =World  
Volume Massive Vector  
(Proca) Fields—Re-visit  
 $e^+ e^- \rightarrow \gamma + XX \rightarrow \gamma + \cancel{E}$
4. LEP-II Bounds on Parameter  
Space
5. ILC Reach In Parameter  
Space

# 1. Brane Dynamics Effective Lagrangian

Brane World Nambu-Goto Effective Action:

$$\Gamma_{Brane} = -f^4 \int d^4x \det e$$

Where the 3-brane embedded in  $D=5,6,\dots$  bulk space-time induces a vierbein  $e_\mu^m$  (and metric  $g_{\mu\nu} = e_\mu^m \eta_{mn} e_\nu^n$ ) on the brane

$$e_\mu^m = \delta_\mu^m + \frac{1}{f^4} \partial_\mu \phi^i G_{ij} \partial^m \phi^j = \delta_\mu^m - \frac{1}{2f^4} \partial_\mu \phi^i \partial^m \phi^i + \dots$$

with  $G_{ij} = H_{ik}^{-1/2} \left[ \sqrt{(1-H)} - 1 \right]_{kl} H_{lj}^{-1/2}$ , where  $H_{ij} = \frac{1}{f^4} \partial_\mu \phi^i \partial^\mu \phi^j$ . The brane oscillation coordinates into the

$N$ -dimensional covolume are given by the Nambu-Goldstone boson world volume fields  $\phi^i(x)$ , with  $i = 1, 2, \dots, N$ .

$D$ -dimensional Poincare' invariant coupling to the Standard Model given by induced gravitational interaction

$$\Gamma = \int d^4x \det e \left[ -f^4 + L_{SM}(e) \right]$$

For small oscillations  $\partial\phi \ll f$ , expand action in powers of the branon field  $\phi^i$ . Standard Model fields couple to the branon fields via the SM energy-momentum tensor  $T_{\mu\nu}^{SM}(x)$

$$\Gamma = \int d^4x \det e \left[ -f^4 + L_{SM}(\eta) + \frac{1}{2} \partial_\mu \phi^i \partial^\mu \phi^i + \frac{1}{2} \frac{1}{f^4} \partial^\mu \phi^i T_{\mu\nu}^{SM} \partial^\nu \phi^i + \dots \right]$$

A **curved bulk** requires energy to deform the brane which appears as **mass for the branons**

$$\Gamma = \int d^4x \det e \left[ -f^4 + L_{SM}(\eta) + \frac{1}{2} \partial_\mu \phi^i \partial^\mu \phi^i - \frac{1}{2} m^2 \phi^i \phi^i + \frac{1}{2f^4} \partial^\mu \phi^i T_{\mu\nu}^{SM} \partial^\nu \phi^i - \frac{m^2}{8f^4} \phi^i \phi^i \eta^{\mu\nu} T_{\mu\nu}^{SM} + \dots \right]$$

- Since the bulk is curved, the Nambu-Goldstone branon fields will be eaten by the bulk gravi-photon fields which now appear as massive vector (Proca) brane oscillation fields in the world volume.

The brane effective action is obtained by gauging the previous Nambu-Goto-SM action, replacing

$\partial_\mu \phi^i \rightarrow \partial_\mu \phi^i + g X_\mu^i$ , and adding the gauge field  $X_\mu^i$  kinetic energy term (as well as the world volume gravitational fields—which we ignore here) to the action.

Using the broken extra dimensional general coordinate transformations to transform to the **unitary gauge** in which  $\phi^i$  is **eliminated** the resultant effective action containing the  $X_\mu^i$  is obtained

## Brane Gauge Field Effective Action

$$\Gamma_{\text{eff}} = \int d^4x \left[ L_{SM}(\eta) - \frac{1}{4} F_{\mu\nu}^i F_i^{\mu\nu} + \frac{1}{2} M_X^2 X_\mu^i X_i^\mu + \frac{1}{2} \frac{1}{F_X^2} X_i^\mu T_{\mu\nu}^{SM} X_i^\nu \right]$$

with phenomenologically determined mass  $M_X$  and effective brane tension  $F_X$  and where  $F_{\mu\nu}^i = \partial_\mu X_\nu^i - \partial_\nu X_\mu^i$  are the  $N$  extra dimension-Abelian field strength tensors for the brane (Proca) gauge fields.

## 2. Collider Physics:

$e^+ e^- \rightarrow \gamma + XX$  where the 2  $X$  particles appear as missing energy. Similar missing energy processes occur for  $Z + XX$  and in **hadron colliders** like the TeVatron:  $p\bar{p} \rightarrow \gamma + XX$  as well as  $p\bar{p} \rightarrow jet + XX$  and the LHC:  $pp \rightarrow \gamma + XX$  and  $pp \rightarrow jet + XX$

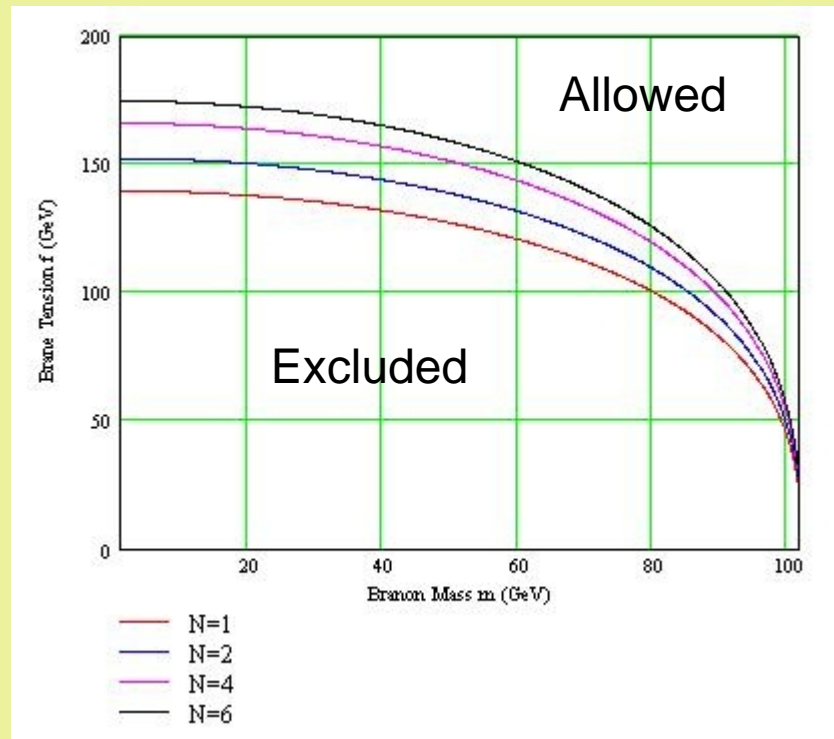
**LEP-II** has searched for  $e^+ e^- \rightarrow \gamma + XX \rightarrow \gamma + \cancel{E}$  and determined an excluded/allowed region of  $f$  vs.  $m$  parameter space for the **massive scalar branon** cross-section based on the **lack of branon production**.

# Branon Scattering Summary: LEP-II average center of mass energy= 206 GeV

$$e^+ e^- \rightarrow \gamma + \phi\phi \rightarrow \gamma + \cancel{E}$$

## Massive Scalar Branon

### LEP II Excluded/Allowed Parameter Regions



Creminelli and Strumia: Nucl. Phys. **B596**, 125 (2001);

Alcaraz, Cembranos, Dobado and Maroto: Phys. Rev. D **67**, 075010 (2003);

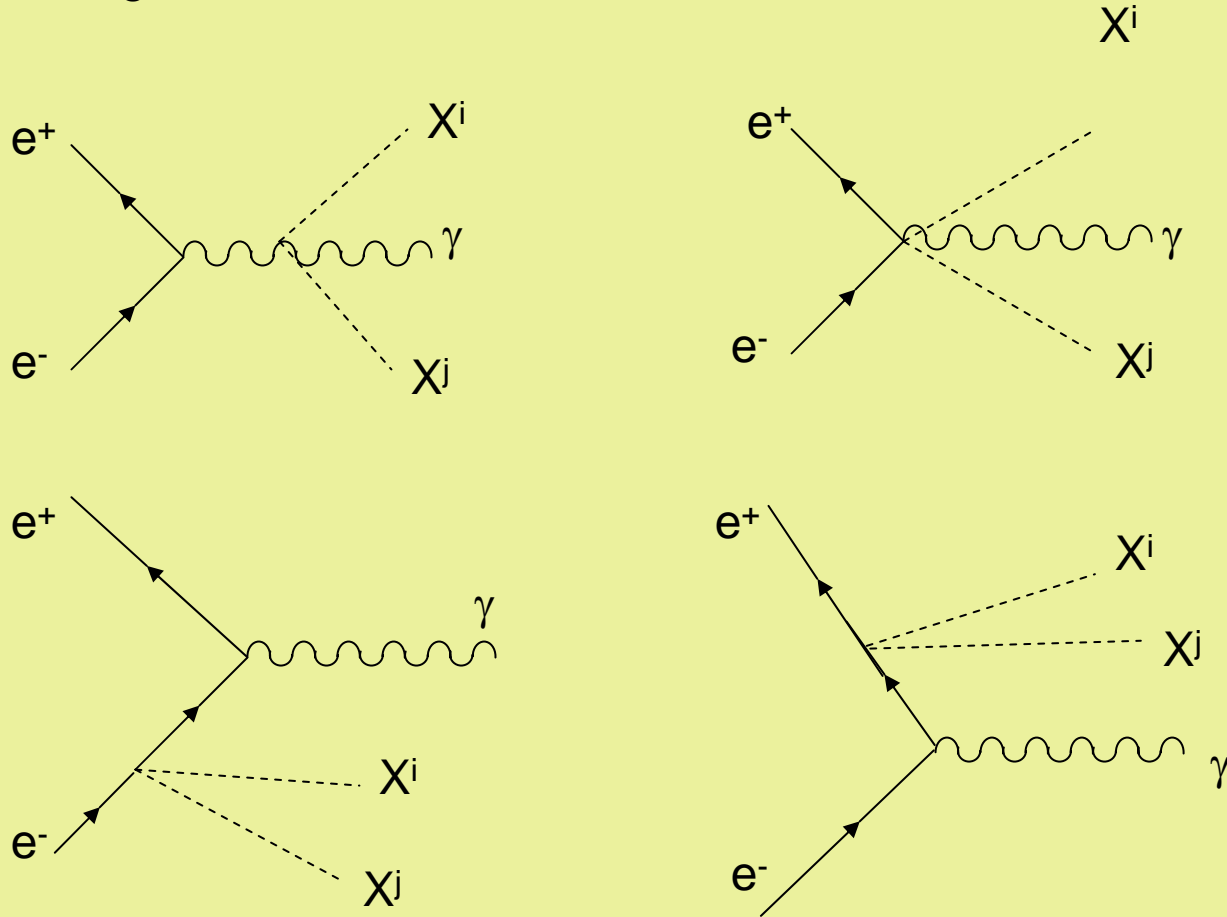
L3 Collaboration, P. Achard et al.: Phys. Lett. B 597 (2004) 145;

S. Mele, Search for Branons at LEP, Int. Europhys. Conf. on High Energy Phys., PoS(HEP2005)153.

### 3. Re-visit $e^+ e^- \rightarrow \gamma + XX \rightarrow \gamma + \cancel{E}$

## Massive Vector Brane Oscillation Fields

The Feynman Diagrams for Brane Particle Production:



The differential cross-section for spin averaged  $e^+e^-$  collisions producing a photon and 2  $X$  particles with summed over polarizations and the  $X$  species,  $i=1,2,\dots,N = \#$  of extra dimensions

$$\frac{d^2\sigma_\gamma}{dx d(\cos\theta)} = \frac{\alpha}{4\pi} \frac{1}{15,360\pi} \left[ \frac{N}{F_X^4 M_X^4} \right] \frac{\sqrt{s} \sqrt{s(1-x) - 4M_X^2}}{\sqrt{(1-x)}} \times$$

$$\times \left[ \left[ s(1-x) - 4M_X^2 \right]^2 + 20M_X^2 \left( s(1-x) + 2M_X^2 \right) \right] \times$$

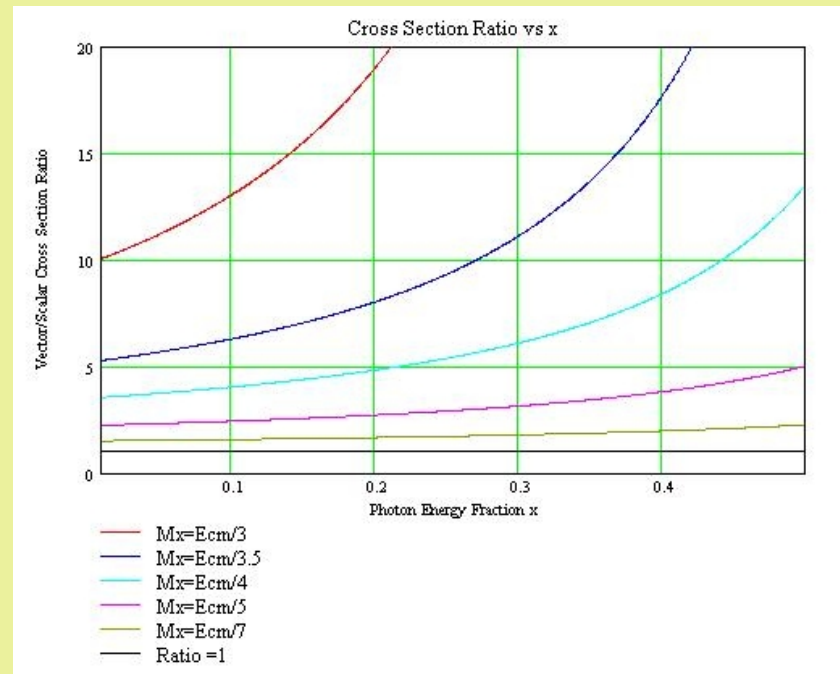
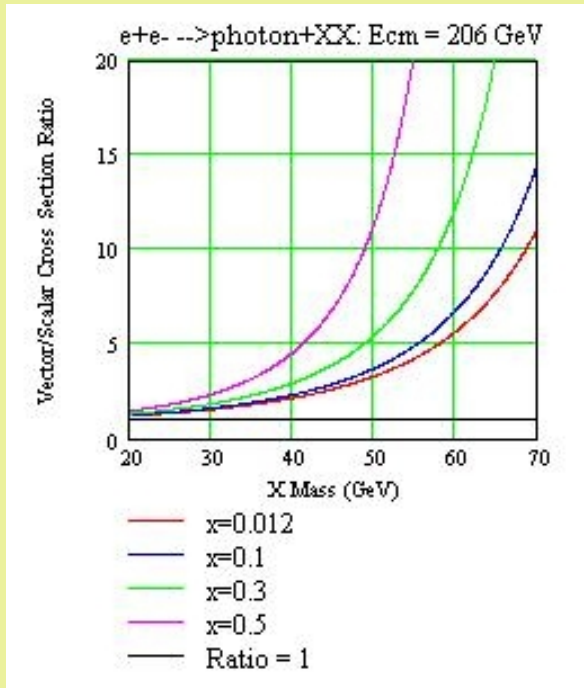
$$\times \left[ x(3 - 3x + 2x^2) - x^3 \sin^2 \theta + \frac{2(1-x) \left[ 1 + (1-x^2) \right]}{x \sin^2 \theta} \right]$$

$E_{CM} = \sqrt{s}$  and the outgoing  $\gamma$  carries the fraction  $x$  of the beam energy  $E_\gamma = x\sqrt{s}/2$  and  $\theta$  is the photon's polar angle from the beam axis.  $\alpha$  is the electromagnetic fine structure constant  $\alpha = 1/128$ .

## Importance of vector field degrees of freedom:

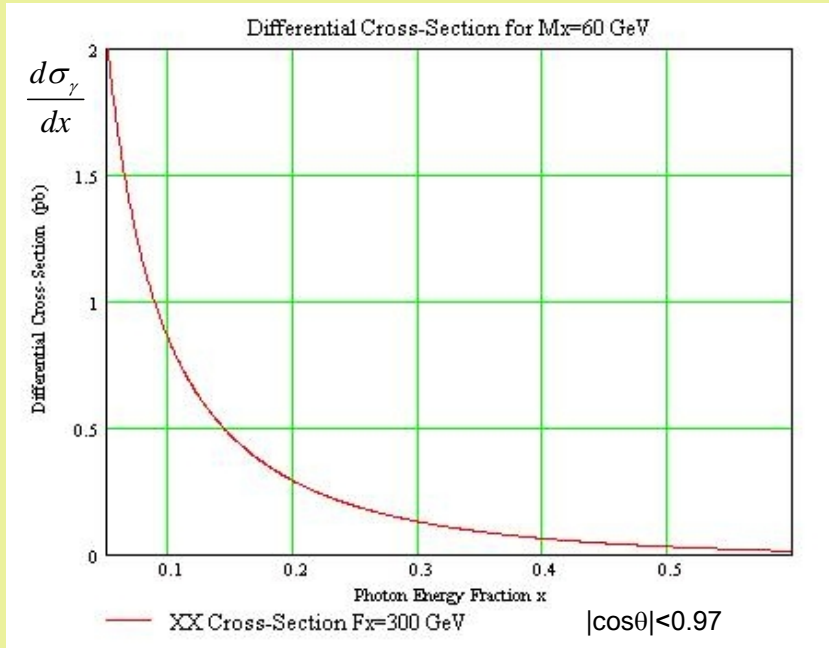
The ratio,  $R$ , of the brane vector  $X$  cross-section, scaled by  $F_X^4 M_X^4$ , to the scalar branon  $\phi$  cross-section, scaled by  $f^8$ , can become appreciable for  $M_X \approx (\frac{1}{3}, \frac{1}{4})\sqrt{s}$ , (with  $m=M_X$ ),

$$R = 1 + \frac{20M_X^2 [s(1-x) + 2M_X^2]}{[s(1-x) - 4M_X^2]^2}$$

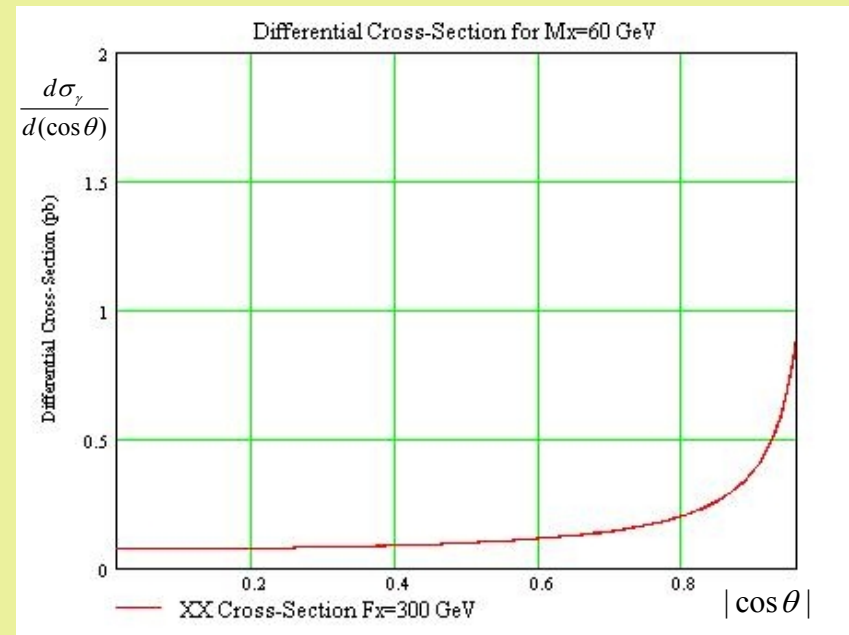




- The differential cross-section vs. photon energy fraction and polar angle



$$\sqrt{s} = 206 \text{ GeV}$$



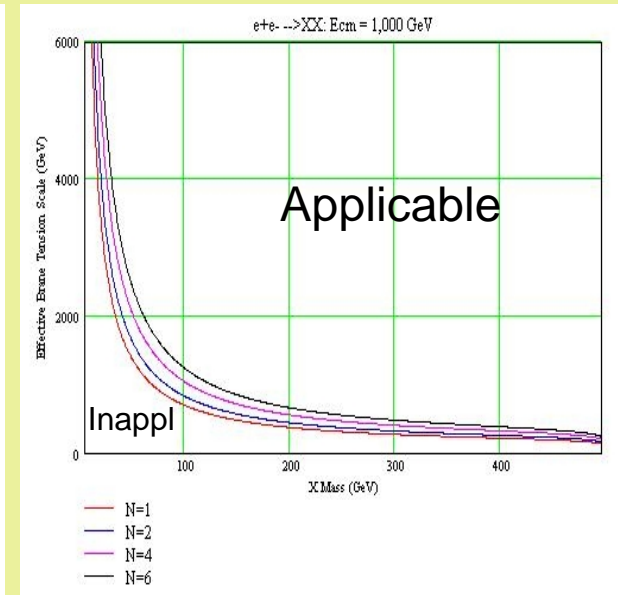
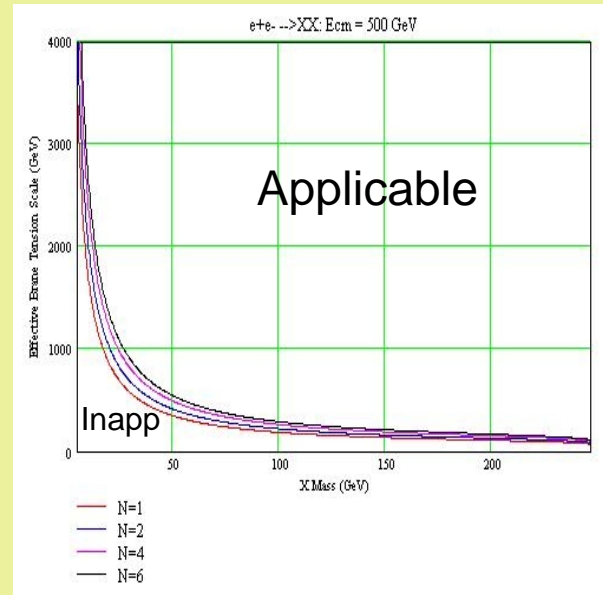
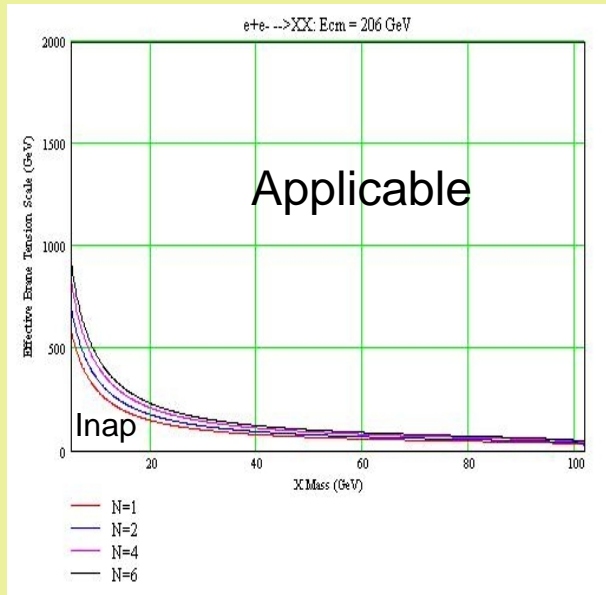
$$x \in \left[0.012, 1 - \frac{4M_X^2}{s} = 0.661\right]$$

## Effectiveness of the Effective Action—Model Applicability

- A theoretical estimate on the total cross-section for  $e^+ e^- \rightarrow XX$  from the unitarity bound requires  $\sigma_{XX} \sim 1/s$ .
- This yields a region of applicability in parameter space.
- The total XX production cross-section is

$$\sigma_{XX} = \frac{N}{15,360 \pi F_X^4 M_X^4} \sqrt{s} \sqrt{s - 4M_X^2} \left[ s^2 + 12M_X^2 s + 56M_X^4 \right]$$

- The unitarity limit implies a region of applicability:

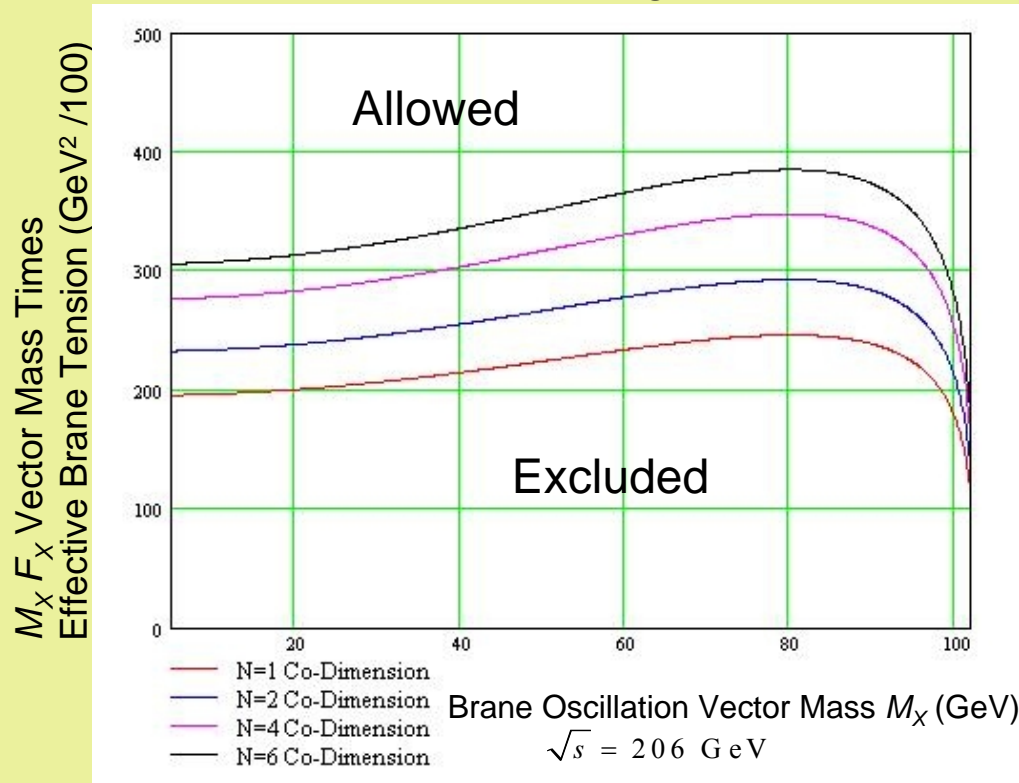


## 4. LEP-II Determined Allowed/Excluded Regions of Parameter Space

- LEP-II agreement with Standard Model has put a limit on the new physics contribution to the single  $\gamma$  plus missing energy cross-section of  $\sigma_{new} \lesssim 0.1\text{pb}$ . This limit on  $\sigma_\gamma \leq \sigma_{new}$  results in
- LEP-II excluded and allowed regions in the  $F_X$ ,  $M_X$  and  $N$  parameter space.

### Brane Oscillation Massive Vector

#### LEP-II Excluded/Allowed Parameter Regions

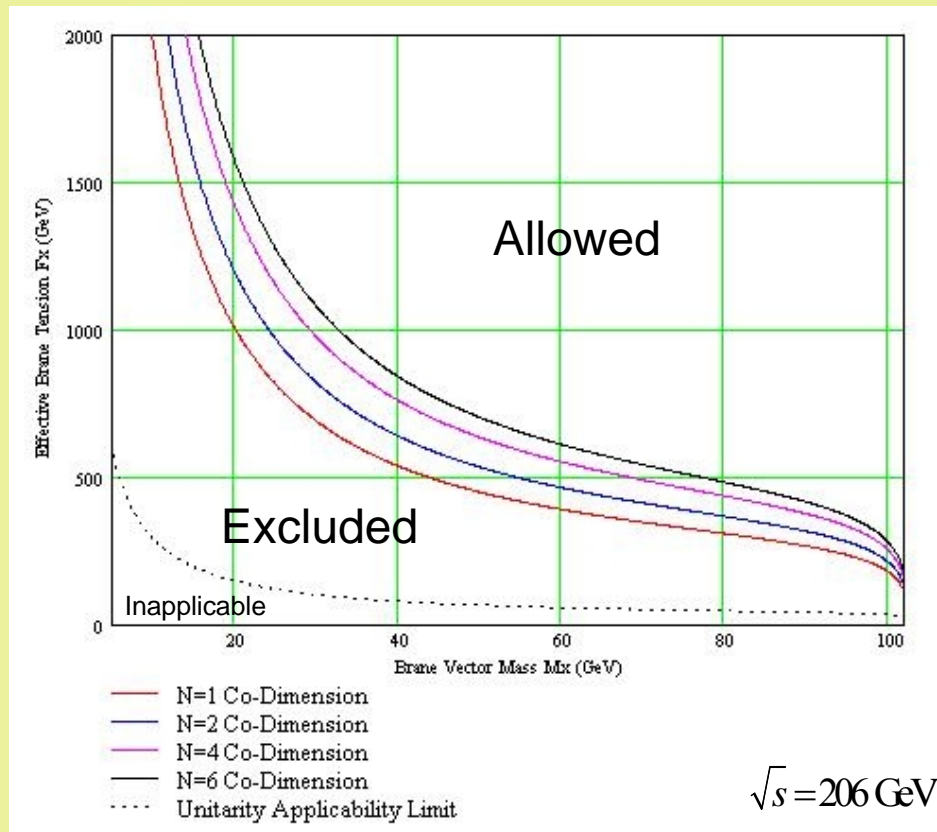


$$\sigma_{\text{Discovery}} \mathcal{I}_{\text{LEP-II}} = 5 \sqrt{\sigma_{\text{SMBkgnd}} \mathcal{I}_{\text{LEP-II}}} \rightarrow \sigma_{\text{Discovery}} \approx 0.1 \text{ pb}$$

$$\left[ 0.012 \leq x \leq \left( 1 - \frac{4M_X^2}{s} \right), |\cos\theta| \leq 0.96 \right]$$

- In terms of the parameters directly

## Brane Oscillation Massive Vector LEP II Excluded/Allowed Parameter Regions



$$\left[ 0.012 \leq x \leq \left( 1 - \frac{4M_x^2}{s} \right), |\cos\theta| \leq 0.96 \right]$$

## 5. ILC: Single Photon Missing Energy Reach For The Brane Oscillation Vector Parameters

Assume SM uncertainties at femtobarn level and the cross-section for new physics is estimated by the gain in statistics from the ratio of integrated luminosities

$$\sigma_{new}^{ILC} = \sqrt{\frac{IL_{LEP-II}}{IL_{ILC}}} \times \sigma_{new}^{LEP-II} \simeq 3 - 6 \text{ fb}$$

Alcaraz, et al., Phys. Rev. D 67, 075010 (2003)

Creminelli and Strumia: Nucl. Phys. **B596**, 125 (2001)

The reach of the ILC can be expressed in terms of the accessible ( $\sigma_\gamma \geq \sigma_{new}^{ILC}$ ) and inaccessible ( $\sigma_\gamma \leq \sigma_{new}^{ILC}$ ) regions of the  $F_X$ - $M_X$  parameter space

Conservative Discovery Criteria:

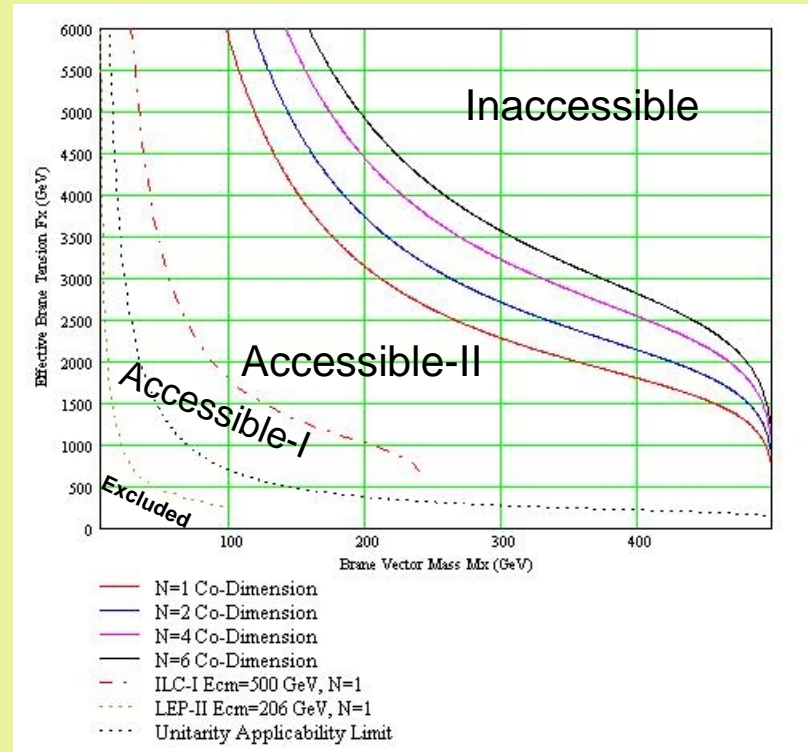
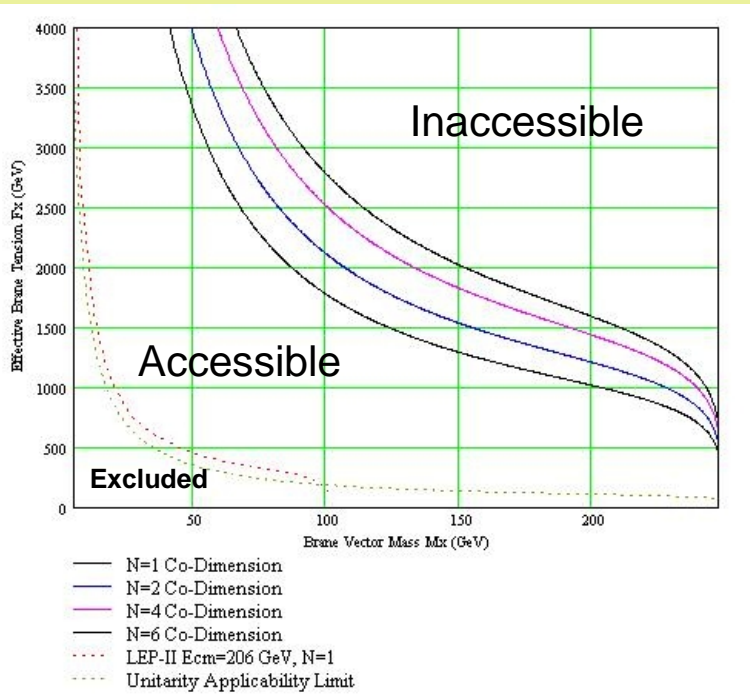
$$\sigma_{Discovery}^{ILC} IL_{ILC} = 5\sqrt{\sigma_{SMBkgnd} IL_{ILC}} = 5\sqrt{\sigma_{SMBkgnd} / IL_{LEP-II}} \sqrt{IL_{LEP-II} IL_{ILC}} \rightarrow \sigma_{Discovery}^{ILC} = \sqrt{\frac{IL_{LEP-II}}{IL_{ILC}}} \sigma_{Discovery}^{LEP-II} \simeq 3 - 6 \text{ fb}$$

# Brane Oscillation Massive Vector

## ILC Accessible/Inaccessible Parameter Regions

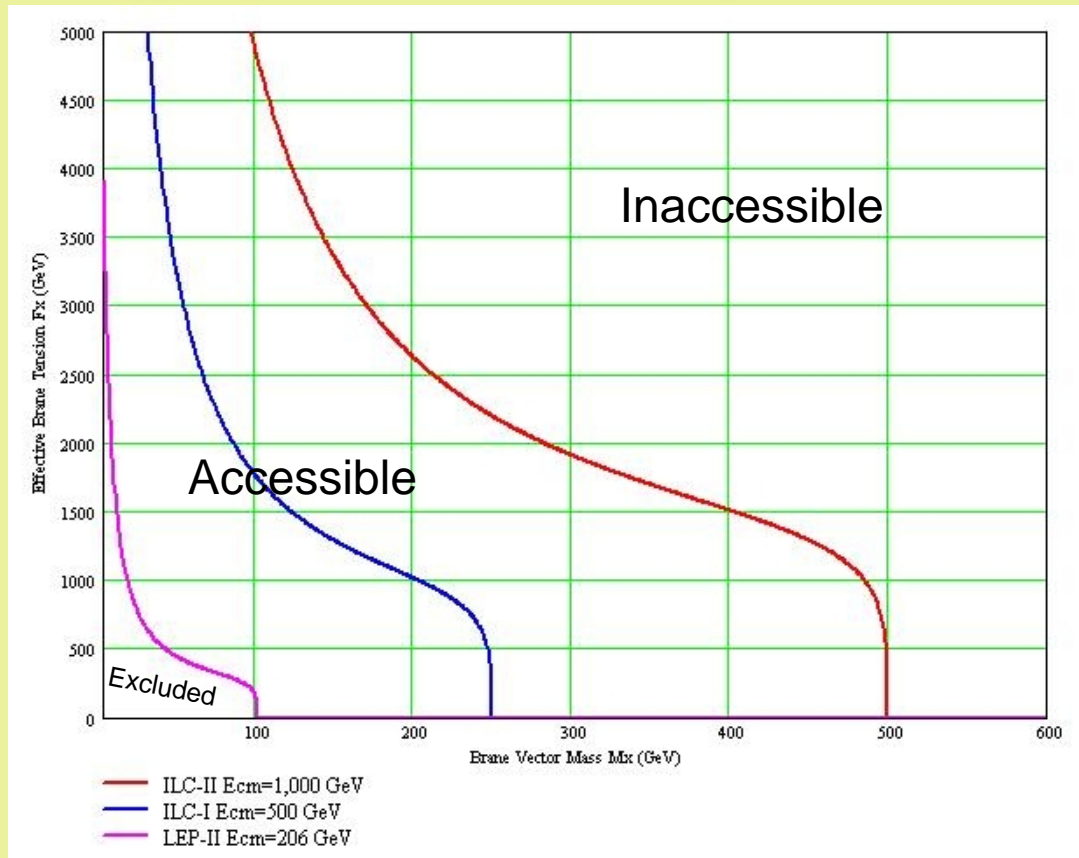
ILC-I:  $E_{\text{cm}} = 500 \text{ GeV}$ ,  $IL = 200 \text{ fb}^{-1}$   
 $\sigma_{\text{newILC}} = 6 \text{ fb}$

ILC-II:  $E_{\text{cm}} = 1,000 \text{ GeV}$ ,  $IL = 1,000 \text{ fb}^{-1}$   
 $\sigma_{\text{newILC}} = 3 \text{ fb}$



$$\left[ 0.012 \leq x \leq \left( 1 - \frac{4M_x^2}{s} \right), \quad |\cos \theta| \leq 0.96 \right]$$

## Summary of Effective Brane Tension and Brane Vector Mass Parameter Space For N=1



$$\left[ 0.012 \leq x \leq \left( 1 - \frac{4M_x^2}{s} \right), |\cos\theta| \leq 0.96 \right]$$