Gravitational Brane Gauge Fields from Extra Dimensions

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References

- **Kaluza-Klein theory**

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  Quantum gravity and extra dimensions at high-energy colliders, G. Giudice, R. Rattazzi and J. Wells, Nucl. Phys. B 544(1999) 3

- **Phenomenology of “branon”**

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  Alcaraz, Cembranos, Dobado and Maroto, Phys. Rev. D 67, 075010(2003)

  Search for Branons at LEP, S. Mele, Int. Eruophys. Conf. on high energy phys., PoS(HEP2005)153

  Probing extra dimensions using Nambu-Goldstone bosons, T. Kugo and K. Yoshioka, Nucl. Phys. B 594(2001)301

Outline

- Kaluza-Klein gravity in higher dimensional spacetime;
- Bulk-brane world scenario: Kaluza-Klein gravity with branes;
- Brane oscillations: Gauge field vs. Goldstone boson (“branon”);
- The mass of brane gauge field;
- The coupling of brane gauge field to standard model and the effective action;

Kaluza-Klein Gravity

- Instead of using higher dimensional Kaluza-Klein gravity theories for gauge/gravity unification, we are studying the geometry and the gravity themselves in higher dimensions, i.e. we will obtain extra gauge fields, not the photon in 4D;

- In Kaluza-Klein gravity, the higher dimensional metric will lead to lower dimensional graviton (spin 2) and gauge bosons (spin 1) and dilaton/radion (spin 0);

- Example: 5D ($M_4 \times S^1$) $\longrightarrow$ 4D (M_4) (d.o.f 5 = 2 + 2 + 1)

$$G_{MN} = \phi^{-\frac{1}{3}} \begin{pmatrix} g_{\mu\nu} + \phi A_\mu A_\nu & \phi A_\mu \\ \phi A_\nu & \phi \end{pmatrix}$$

Kaluza-Klein Gravity With Branes

- The 4D effective action is

$$S_G = -\frac{1}{16\pi G_5}\int \mathrm{d}^4x\,\mathrm{d}y\;\, e^{(5)}\; R^{(5)} \qquad\qquad \left(G_5 = 2\pi r G_4 \right)$$

$$= -\frac{1}{16\pi G_4}\int \mathrm{d}^4x\;\, e^{(4)}\left[R^{(4)} + \frac{1}{4}\phi\, F_{\mu\nu}\, F^{\mu\nu} + \frac{1}{6\phi^2}\partial^{\mu}\phi\,\partial_{\mu}\phi \right]$$

- The 5D Einstein-Hilbert action gives the 4D one plus the Maxwell term if we set the scalar $\phi = 1$;

- Now we add a 3-brane in 5D spacetime. It spontaneously breaks the translational symmetry along the fifth direction. The corresponding Nambu-Goldstone field is $Y^5(x)$ is dynamical (in static gauge). Based on the induced metric $h_{\mu\nu}$, the effective action for the brane is

Kaluza-Klein Gravity With Branes

$$S_{brane} = -\,\mathrm{f}^{\,4}\int \mathrm{d}^4 x\,\sqrt{\det h_{\mu\nu}} \quad , \quad h_{\mu\nu} = G_{MN}\,\partial_\mu Y^M\,\partial_\nu Y^N$$

$$S_{4D} = S_G + S_{brane}$$

which is in the form of an Nambu-Goto action and the induced metric is found to be (in static gauge $Y^{M=\mu}(x) = x^\mu$)

$$h_{\mu\nu} = g_{\mu\nu} + (A_\mu + \partial_\mu Y^5)(A_\nu + \partial_\nu Y^5) = g_{\mu\nu} + X_\mu X_\nu$$

We see the vector becomes massive (Proca field), by “eating” the Goldstone boson (Higgs effect).

What is the mass of this brane gauge field ?

Kaluza-Klein Gravity With Branes

If we work in the Einstein frame, set $\phi = 1$ and rescale the vector X_μ to get the normalized Maxwell term, the mass of X_μ is found to be

$$m_{X_\mu}^2 = \frac{f^4}{M_{(4)}^2}$$

For $f \sim$ TeV, this mass is very small. In previous work people do not take into account this Higgs effect and simply take the Goldstone boson term $\partial_\mu Y^5 \ \partial_\nu Y^5$ in the induced metric for the phenomenology.

However, this is not the general case. The dilaton(s) from the higher dimension(s) could change the mass by a “dilaton factor”.

Dilaton factor in K-K Gravity With Branes

This is because in general even we start with the higher dimensional Einstein gravity, the dimensionally-reduced action is that of a scalar-tensor theory, with the scalar (dilaton or radion) characterizing the size of the extra-dimensional spacetime.

Consider N co-dimensions case (Y.M. Cho and P.G.O. Freund, Phys. Rev. D, V12, No.6, 1975)

$$G_{MN} = \begin{pmatrix} g_{\mu\nu} + g_{ij} A_\mu^i A_\nu^j & g_{li} A_\mu^i \\ g_{ki} A_\nu^i & g_{kl} \end{pmatrix}$$

$$S_G = -\int \mathrm{d}^D x \; e^{(D)} \left[\frac{1}{16\pi G_D} R^{(D)} + \frac{1}{4} g_{ij} F_{\mu\nu}^i F^{j\mu\nu} + L_{scalar} \right]$$

$$e^{(D)} = \sqrt{\det g_{\mu\nu}} \sqrt{\det g_{ij}}$$

→ Dilaton factor

Mass of the brane gauge field

In order to write the action in the Einstein frame, one has to rescale the metric by field redefinition, which will lead to the rescaling of the induced metric on the brane;

To simplify the calculation and see how this dilaton factor could change the mass of the brane gauge field, we use a toy model based on a two-step dimensional reduction:

$$M_4 \times S^1 \times E \Rightarrow M_4 \times S^1 \Rightarrow M_4$$

$$D = 4 + 1 + \delta \rightarrow 4 + 1 \rightarrow 4$$

The D-dimensional metric is taken to be

$$\hat{G}_{\hat{M}\hat{N}} = \begin{pmatrix} G_{MN}(x^\mu, y) & 0 \\ 0 & e^{-2\sigma(x^\mu, y)}\gamma_{ij}(z^k) \end{pmatrix}$$

Mass of the brane gauge field

The 5D action is $S_{5D} = S_{d-G} + S_{brane}$

$$S_{d-G} = -\frac{1}{16\pi G_5}\int \mathrm{d}^4x\,\mathrm{d}y\ \sqrt{G}\,(e^{-\sigma})^{\delta} R^{(5)}$$

Dilaton factor

$$S_{brane} = \mathrm{f}^{\,4}\int \mathrm{d}^4x\sqrt{\det h_{\mu\nu}} \quad , \quad h_{\mu\nu} = G_{MN}\,\partial_{\mu}Y^{M}\,\partial_{\nu}Y^{N}$$

Note that the R-term is not in Einstein frame and one has to rescale the metric as

$$G_{MN} = \tilde{G}_{MN}\, e^{\frac{2}{3}\sigma\delta}$$

Which leads to the rescaling of the induced metric on the brane

$$h_{\mu\nu} = \tilde{h}_{\mu\nu}\, e^{\frac{2}{3}\sigma\delta} \longrightarrow \mathrm{f}_{new} = \mathrm{f}\ e^{\frac{1}{3}\sigma\delta}$$

Mass of the brane gauge field

Now we can take the 5D metric as in the original Kaluza-Klein gravity and we obtain the same action (we did not include the dynamics of radion) as before, but the mass of brane gauge field X_μ is changed to

$$m_{X_\mu}^2 = \frac{f_{new}^{\ 4}}{M_{(4)}^2} = \frac{f^{\,4}}{M_{(4)}^2} e^{\frac{4}{3}\sigma\delta}$$

The coupling of X_μ to standard model can be read from

$$S_{brane} = \int d^4x \sqrt{\det h_{\mu\nu}}\, [-\mathrm{f}^{\,4} + L_{SM}(h_{\mu\nu}, e_\mu^a)\,]$$

which is $$S_{X_\mu - SM} = \int d^4x \frac{1}{F_X^2} T_{SM}^{\mu\nu} X_\mu X_\nu, \quad T_{SM}^{\mu\nu} \equiv \left.\frac{\delta S_{brane}}{\delta h_{\mu\nu}}\right|_{h_{\mu\nu}=\eta_{\mu\nu}}$$

The 4D effective action

To summarize, the 4D effective action of the brane gauge field is

$$S_{\text{eff}} = \int \mathrm{d}^4 x \left[L_{SM} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} M_X^2 X_\mu X^\mu + \frac{1}{F_X^2} T_{SM}^{\mu\nu} X_\mu X_\nu \right]$$

The constraints on the mass of X_μ and the coupling constant F_X will be discussed in the presentations of T. Clark and T. ter Veldhuis, from the collider physics and dark matter phenomenology, respectively.

Discussions

- It can be generalized to multiple Proca vector field case;
- Their masses and effective coupling constants will be determined by the experiments;
- These parameters provide the profile of extra dimensions and the information of brane dynamics;
- Supergravity and graviphoton;
- PVLAS experiments;