VECTOR DYNAMICS IN LOCALLY INVARIANT BRANE WORLD MODELS

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Outline

- **Appearance of vector field(s) generic feature of locally invariant brane world models**

- **Couple vector to the Standard Model. Construct invariant operators.**

- **Examine various constraints on the parameters in effective operators as well as some vector decay properties.**

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D-dimensional invariant interval

$$ds^2 \;=\; \bar{g}_{\mu\nu}(x)A(y)dx^\mu dx^\nu + g'_{ab}(y)dy^a dy^b$$

with

$$\mu, \nu = 0, 1, 2, 3 \quad ; \quad \text{coordinates } x^\mu$$
$$a, b = 1, 2, ..., D-4 \quad ; \quad \text{coordinates } y^a$$

Warp factor $A(y)$ **normalized as** $A(0) = 1$

Insert 4-dimensional probe brane
at position $(x^\mu, y^a(x))$

Ground state: $y^a(x) = 0$

Focus on case:

No warping: $A(y) = 1$

Co-dimension 1: $a = 1$

Presence of probe brane breaks 5-dimensional space-time symmetries:

translation in 4^{th} space dimension parametrized by a

Lorentz transformations involving 4^{th} space dimension parametrized by b^μ

Associated with broken translation is Nambu-Goldstone boson field $\phi(x)$

Transforms nonlinearly under broken global space-tiome symmetries

$$\delta\phi(x) \;=\; aF + b^\mu\Big(F^2 x_\mu - \frac{1}{F^2}\phi(x)\partial_\mu\phi(x)\Big)$$

Nambu-Goldstone field dynamics gives motion of probe brane into the 5^{th} dimension

No independent Nambu-Goldstone modes associated with broken Lorentz generators

Brane oscillations gives rise to induced metric on the brane

Invariant 4-dimensional spce-time interval:

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{F^4} \partial_\mu \phi(x) \partial_\nu \phi(x) dx^\mu dx^\nu$$

Action for Nambu-Goldstone dynamics:

$$\begin{aligned} S_{NG} &= -F^4 \int d^4x [\sqrt{\det \ g_{\mu\nu}} - 1] \\ &= -F^4 \int d^4x [\sqrt{1 + \frac{1}{F^4} \partial_\mu \phi(x) \partial^\mu(x)} - 1] \end{aligned}$$

Nambu-Goto action: brane tension $\sigma = F^4$

Have subtracted a constant thereby setting the vacuum energy to zero. It can be adjusted accordingly

$4\pi F$ **acts as scale above which effective theory breaks down**

Action invariant under nonlinear Nambu-Goldstone transformations associated with broken global space-time symmetries

Make 5^{th} dimension translations local: $a = a(x)$

Construction of invariant action follows usual procedure

Introduce gauge field $X^{\mu}(x)$ and covariant derivative:

$$\partial_\mu \phi \rightarrow \partial_\mu \phi - gFX_\mu$$

S_{NG} **in unitary gauge, $\phi(x) = 0$, gives vector mass term**

$$S_{mass} = -\frac{1}{2} M_X^2 \int d^4x X^\mu X_\mu + \ldots$$

$M_X \equiv gF$ **independent mass scale: Higgs mechanism**

Presence of such a vector generic consequence of brane world models

Include kinetic term for X^μ via field strength: $X_{\mu\nu}$

Massive $U(1)$ vector field Proca action:

$$S_{Proca} = -\frac{1}{4} \int d^4x X^{\mu\nu} X_{\mu\nu} - \frac{1}{2} M_X^2 \int d^4x X^\mu X_\mu + \ldots$$

Couplings to Standard Model:

X **transforms as** $SU(3) \times SU(2) \times U(1)$ **singlet**

• **Induced metric couples to Standard Model symmetric energy momentum tensor** $T^{\mu\nu}_{SM}$

$$S_{gT} \ = \ \frac{1}{F^2} \int d^4x X_\mu X_\nu T^{\mu\nu}_{SM} + ...$$

Coupling is bilinear in X^μ

• **Linear couplings in** X^μ **arise from extrinsic curvature.**

Measures curvature of embedded 4-d brane relative to enveloping 5-d geometry

Extrinsic curvature tensor in unitary gauge:

$$K^{\mu\nu} \ = \ -\frac{1}{F}\partial^\mu X^\nu + \frac{1}{2F^3}X_\lambda X^\lambda \partial^\mu X^\nu + \frac{1}{2F^3}X^\mu X_\lambda \partial^\nu X^\lambda + ...$$

Linear couplings to X^μ **always contain derivatives**

Invariant couplings constructed by contracting $K^{\mu\nu}$ with other tensors

Effective coupling of X^μ to SM fermion pairs

$$\begin{aligned}
\mathcal{O}_{f1} &= \frac{1}{F}\int d^4x(\partial^\nu X_\nu)\bar{f}_i(c_{1V_{ij}} + c_{1A_{ij}}\gamma_5)f_j \\
\mathcal{O}_{f2} &= \frac{1}{F}\int d^4x(\partial^\nu X_\mu)\bar{f}_i\sigma^{\mu\nu}(c_{2V_{ij}} + c_{2A_{ij}}\gamma_5)f_j \\
\mathcal{O}_{f3} &= \frac{1}{F^2}\int d^4x(\partial^2 X_\nu)\bar{f}_i\gamma^\nu(c_{3V_{ij}} + c_{3A_{ij}}\gamma_5)f_j \\
\mathcal{O}_{f4} &= \frac{1}{F^2}\int d^4x(\partial_\mu X_\nu)\bar{f}_i\gamma^\nu(c_{4V_{ij}} + c_{4A_{ij}}\gamma_5)\partial^\mu f_j \\
\mathcal{O}_{f5} &= \frac{1}{F^2}\int d^4x(\partial_\mu X_\nu)\bar{f}_i\gamma^\mu(c_{5V_{ij}} + c_{5A_{ij}}\gamma_5)\partial^\nu f_j \\
\mathcal{O}_{f6} &= \frac{1}{F^2}\int d^4x(\partial_\mu\partial^\nu X_\nu)\bar{f}_i\gamma^\mu(c_{6V_{ij}} + c_{6A_{ij}}\gamma_5)f_j
\end{aligned}$$

f_i is i^{th} generation fermion field
In general X can mix generations

Different from generic Z' which can couple directly to fermionic current.

Effective coupling of X^μ to Standard Model gauge bosons:

$$\mathcal{O}_{VV'} = \frac{1}{F^2}\int d^4x(\partial_\mu X_\nu)[c_{V_{VV'}}V_{\mu\lambda}V'^{\ \lambda}_{\ \nu} + c_{A_{VV'}}V_{\mu\lambda}\tilde{V}'^{\ \lambda}_{\ \nu}]$$

where $V, V' = W_\pm, Z^0, \gamma$

Want to constrain/determine values of M_X, F and dimensionless constants c_i

- **LEP2 limits**

Absence of X discovery for $\sqrt{s} < 205\ GeV$ dictates $M_X > 205\ GeV$

Further requires that any signal produced via virtual propagation of mass $M_X > 205\ GeV$ X vector be less than a discovery which is taken as 5σ above the Standard Model physics

$$\sigma(e^+e^- \to X \to \text{ hadrons})|_{\sqrt{s}=200\ GeV} \ < \ 5\sqrt{\frac{\sigma_{had}}{\mathcal{L}}} \simeq 0.1\ pb$$

where $\sigma_{had} \sim .1\ nb$ is total observed hadronic cross section and $\mathcal{L} \sim 700\ pb^{-1}$ is integrated luminosity

Using the effective operator couplings $\mathcal{O}_{f1} - \mathcal{O}_{f6}$ of X to fermion pairs $(\bar{f}f)$ yields the leading in $1/F$ off resonance cross section $(m_f = 0)$

$$\sigma(e^+e^- \to X \to \text{ hadrons}) = \sum_q \frac{1}{16\pi} c_{1q}^2 c_{1e}^2 \frac{s^3}{M_X^4 F^4}$$

where $c_{1_f}^2 \equiv c_{1V_{ff}}^2 + c_{1A_{ff}}^2$**.**

Plugging in the numbers taking $c_{1q}^2 = c_{1e}^2 \equiv c_1^2$ **yields the constraint on the brane tension as a function of vector mass:**

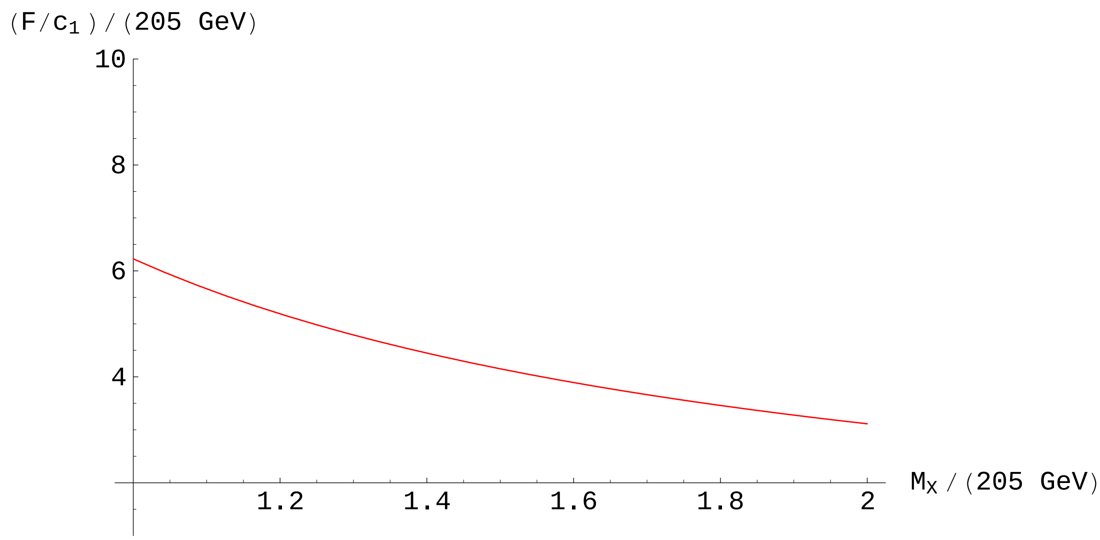

Figure 1: Allowed F/c_1 values lie above the curve

Restricts $\frac{F}{c_1} > \mathcal{O}\ (TeV)$ **for** $M_X > 205 - 400\ GeV$

- **X decays:** $X \to \bar{f}_i f_j$

Leading $1/F^2$ contribution obtained from effective coupling $\mathcal{O}_{f_2}$ as $(m_f = 0)$

$$\Gamma(X \to f_i \bar{f}_j) \;=\; \frac{c_{2_{ij}}^2}{24\pi}\frac{M_X^3}{F^2}$$

where $c_{2_{ij}}^2 = |c_{2V_{ij}} + c_{2A_{ij}}|^2$

Assuming only flavor diagonal decays and defing $c_2^2 = \sum_q c_{2_{qq}}^2$ gives

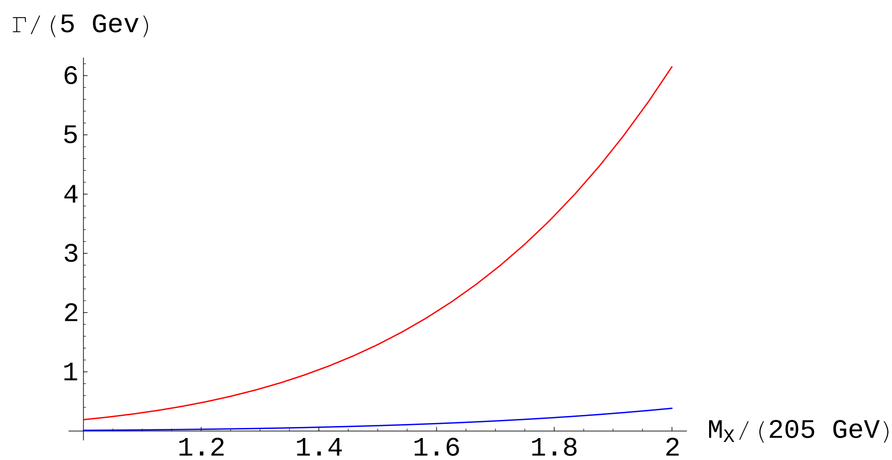

Figure 2: Decay rate $X \to$ hadrons ; red (blue) curve corresponds to $\frac{F}{c_2} = 500\ GeV (1000\ GeV)$

- $X^\mu \to \gamma Z^0, W^+W^-, Z^0Z^0$

Leading linear in X^μ couplings to Standard Model vector bosons go as $1/F^2$.
Thus decay rates $\sim 1/F^4$.

Surpressed relative to fermion decay modes ($\sim 1/F^2$)

Taking various c coefficients to be equal,

$$\frac{\Gamma(Z^0\gamma)}{\Gamma(\bar{f}f)} = \frac{1}{16}\frac{M_X^2}{F^2}\Big(1-\frac{M_Z^2}{M_X^2}\Big)(1+6\frac{M_Z^2}{M_X^2}+\frac{M_Z^4}{M_X^4})$$

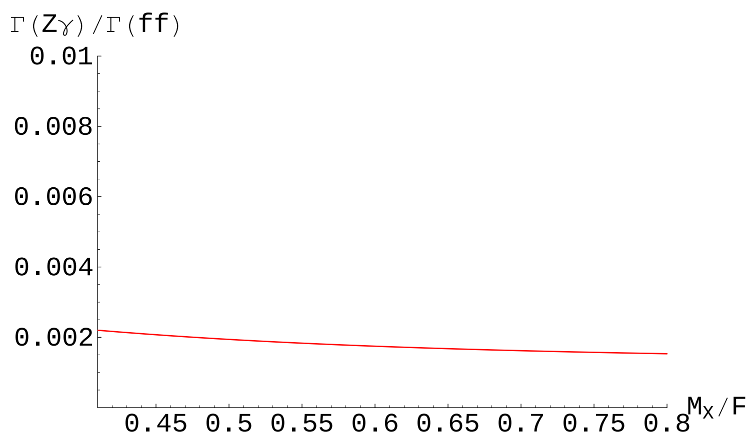

Figure 3: $\frac{\Gamma(X\to\gamma Z)}{\Gamma(X\to\bar{f}f)}$ for $F = 500\ GeV$ and $M_X > 205\ GeV$ taking all couplings c equal

Alternate probe of $M_X > 205\ GeV$

- **X^μ in loops:**

- **Anomalous magnetic moment of muon:**

Brookhaven $g_\mu - 2$ experiment:

$$a_\mu(exp) - a_\mu(SM) \ = \ (23.4 \pm 9.1) \times 10^{-10}$$

Assume deviation from SM result entirely due to virtual X exchange

Using representative coupling: $\frac{c_{l_i l_j}}{F^2} \int d^4x (\partial^2 X_\mu) \bar{l}_i \gamma^\mu l_j$ **gives**

$$\Delta a_\mu \ = \ \frac{\Delta(g_\mu - 2)}{2} = \frac{1}{24\pi^2} (\frac{\Lambda}{F})^2 \frac{m_\mu}{F} \sum_l c_{\mu l}^2 \frac{m_l}{F}$$

where $\Lambda < 4\pi F$ is an ultraviolet cutoff on the internal loop momentum: treat as independent scale

To get idea of scales: assume no lepton flavor mixing and take $\Lambda = \pi F$

$$\frac{F}{c_{\mu\mu}} \ \sim \ \mathcal{O}(TeV)$$

- $\mu^- \to e^- \gamma$

Contribution from virtual X exchange:

Use same operator as in $g_\mu - 2$ calculation

Non-zero result requires X to mix μ with e

$$\Gamma(\mu^- \to e^- \gamma) = \frac{m_\mu}{128\pi}\Big[\frac{c_{\mu\mu} c_{\mu e}}{32\pi^2}(\frac{\Lambda}{F})^4(\frac{m_\mu}{F})^2\Big]^2$$

Using same numbers as in $g_\mu - 2$ result

$$\begin{aligned}\Gamma(\mu^- \to e^- \gamma) &= \frac{m_\mu}{128\pi}(\frac{c_{\mu e}}{c_{\mu\mu}})^2(\Delta a_\mu)^2 \\ &= 1.5 \times 10^{-19}(\frac{c_{\mu e}}{c_{\mu\mu}})^2 \; GeV\end{aligned}$$

Experimentally

$$\begin{aligned}\Gamma(\mu^- \to e^- \gamma) &< 1.2 \times 10^{-11}\Gamma(\mu^- \to e^- \nu_\mu \bar{\nu}_e) \\ &< 3.7 \times 10^{-30} \; GeV\end{aligned}$$

so that

$$\frac{c_{\mu e}}{c_{\mu\mu}} < 5 \times 10^{-5}$$

Lepton flavor mixing surpressed

Summary

- **Embedded 4-dimensional probe brane into 5 dimensional space-time which breaks 5^{th} dimension translation invariance. Dynamics of associated Nambu-Goldstone mode describes oscillations of brane into extra dimension.**

- **Gauging broken 5^{th} dimensional translations leads to massive Proca vector field X which is a Standard Model singlet**

- **Coupled X to the Standard Model**
Focused on terms linear in X which arise due presence of extrinsic curvature tensor
Also manifest in models with additional anisotropic co-dimensions

Constructed effective operators coupling X to fermions and Standard model gauge bosons
All such terms involve derivative couplings

- **Examined various constraints on the brane tension arising from $LEP2, g_\mu - 2$, $\mu^- \to e^-\gamma$ as well as decay properties of the vector.**
If $M_X \sim 200 - 400\ GeV$, require $\frac{F}{c} > \mathcal{O}(TeV)$