



Lepton EDM's from Heavy Right-Handed Majorana Neutrinos

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with
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Electric Dipole Moment (EDM)

Interactions between spin and the electromagnetic field:

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Under CPT:

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Non-zero d violates **P**, **CP**, and **T**.

EDM's as measures of CP violation

⑥ CP violation in the quark sector

⑥ CP violation in the lepton sector

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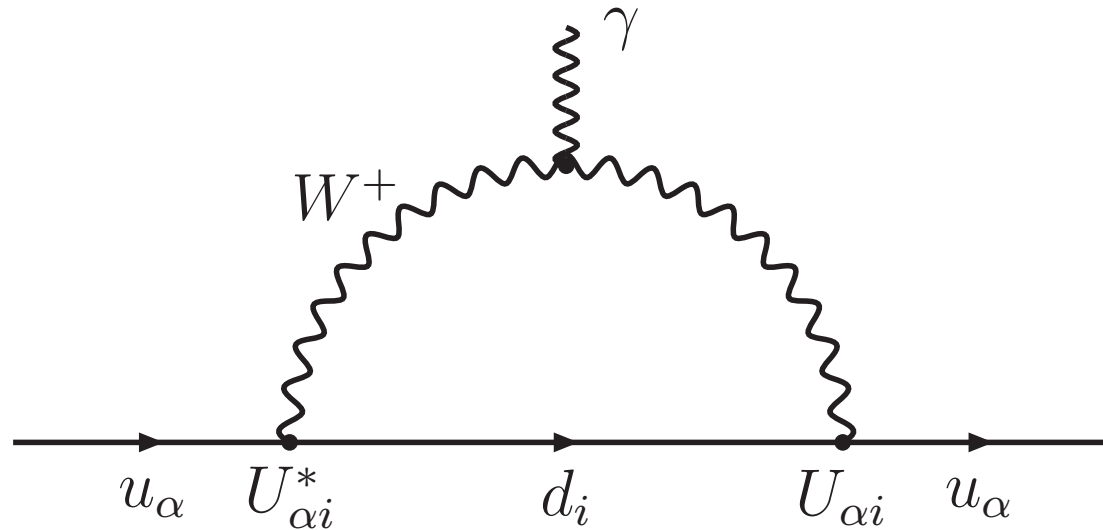
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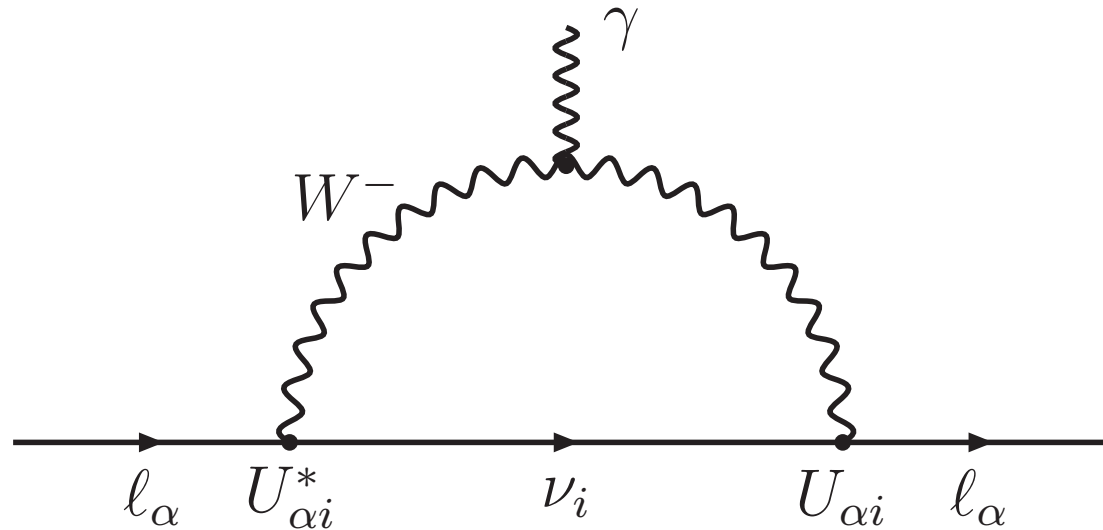
- ⑥ CP violation in the lepton sector
 - △ Unconstrained. Could be big.
 - △ Could be a source of leptogenesis
 - △ Lepton EDM's generated at the **2-loop** level

1-loop diagram



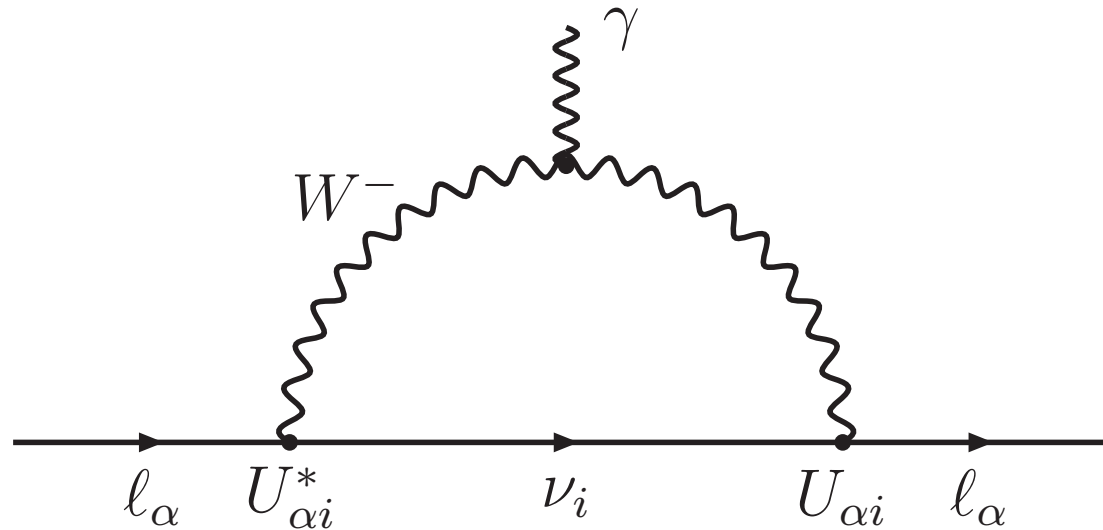
$$\propto |U_{\alpha i}|^2$$

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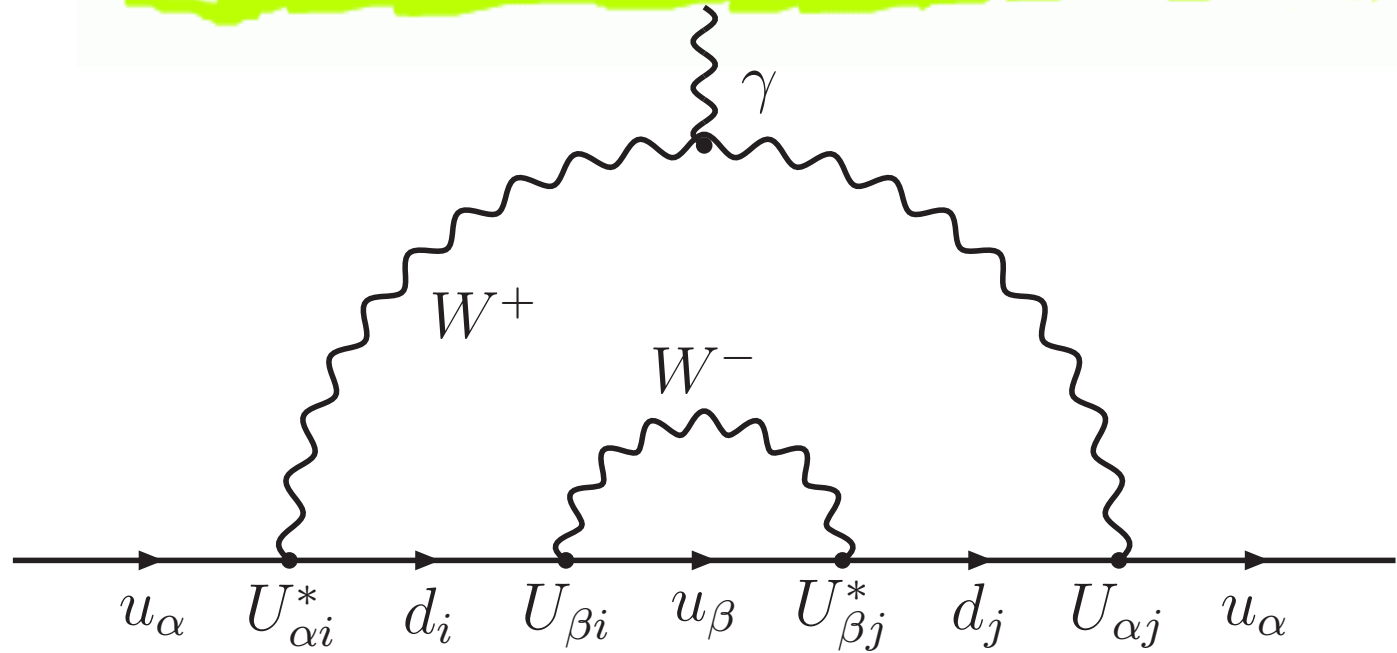
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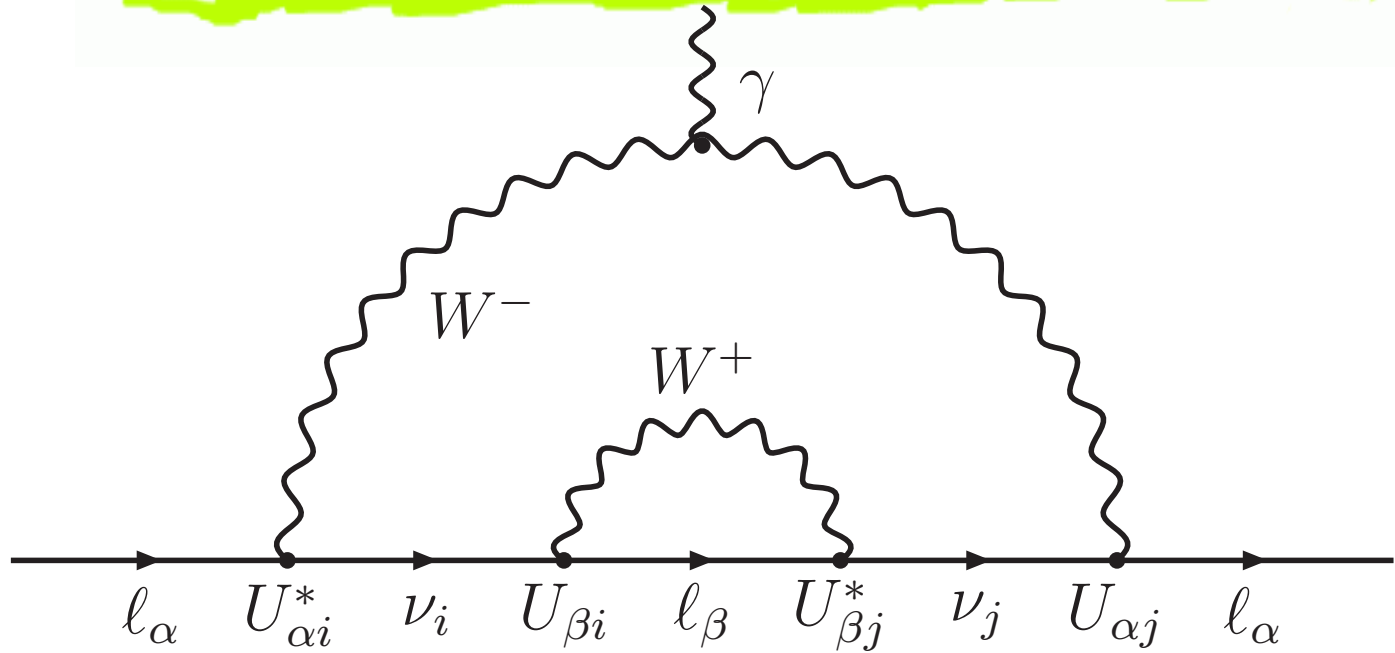
Insensitive to complex phases

2-loop diagram



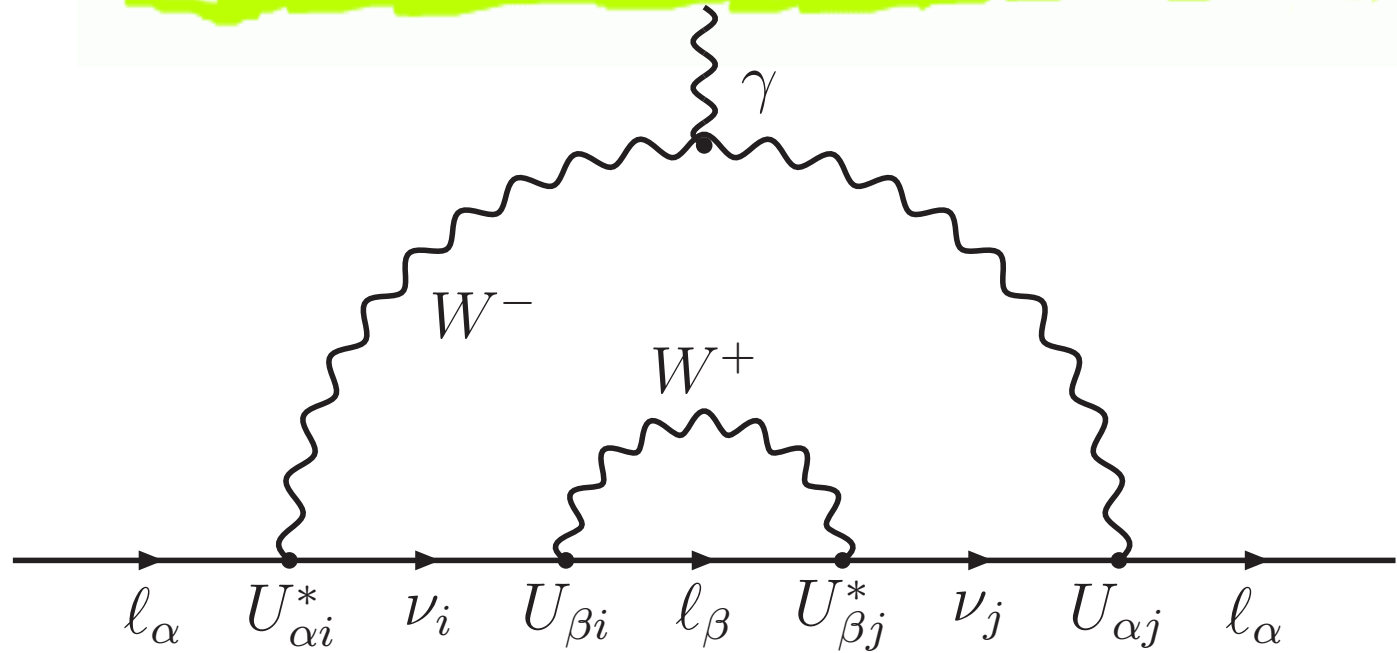
$$\propto (U_{\alpha i}^* U_{\beta i})(U_{\alpha j} U_{\beta j}^*)$$

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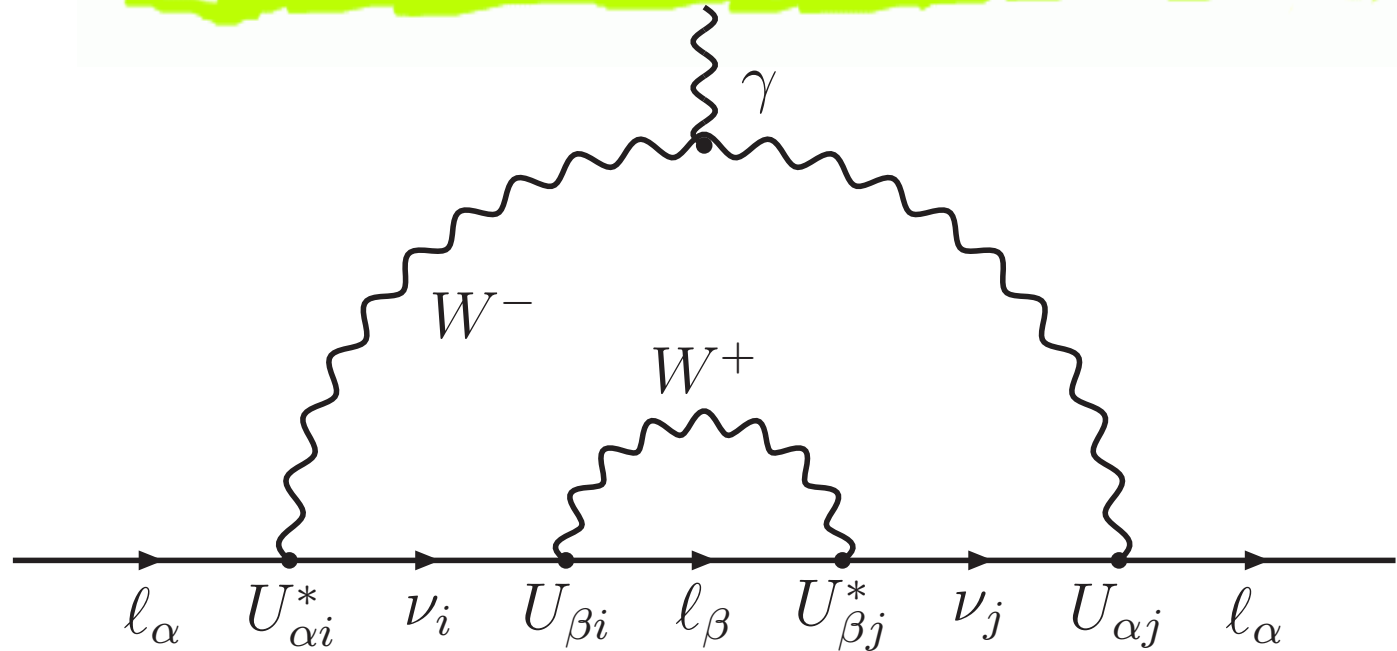
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Diagram is **symmetric** under the interchange $i \leftrightarrow j$

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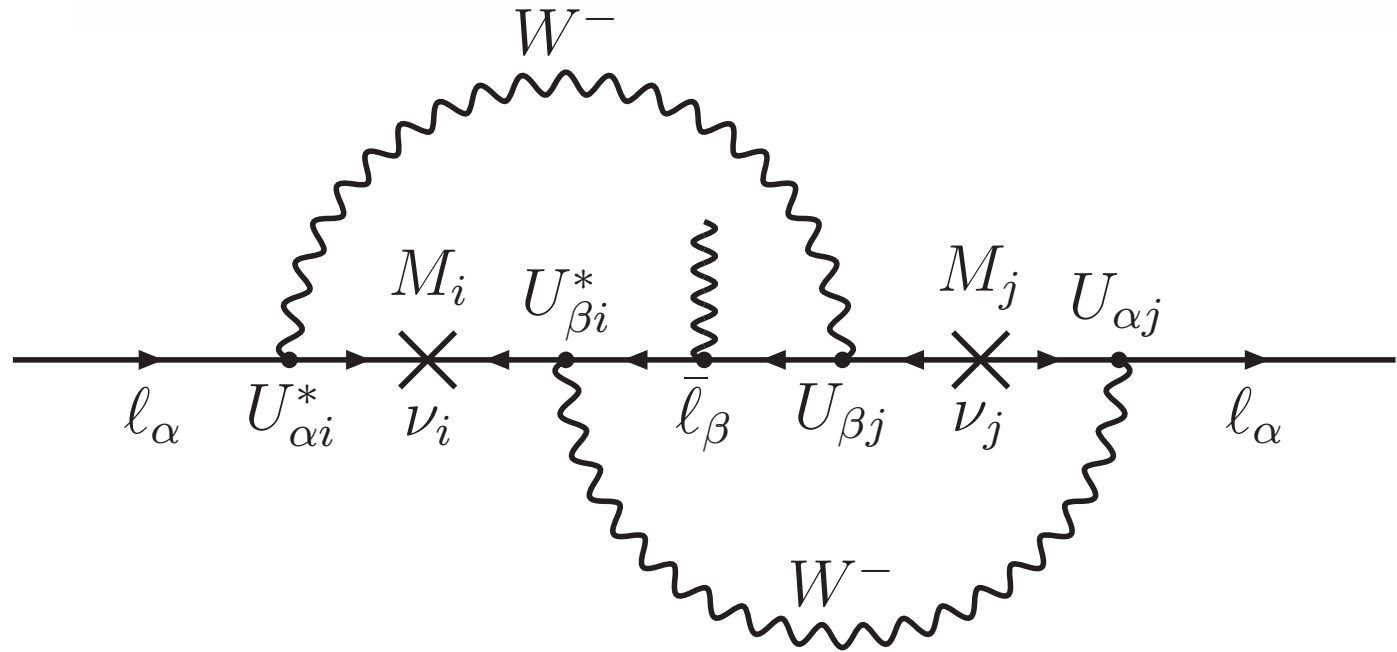
$$\propto (U_{\alpha i}^* U_{\beta i})(U_{\alpha j} U_{\beta j}^*)$$

Diagram is **symmetric** under the interchange $i \leftrightarrow j$

\implies Imaginary parts of $(U_{\alpha i}^* U_{\beta i})(U_{\alpha j} U_{\beta j}^*)$ **cancel**

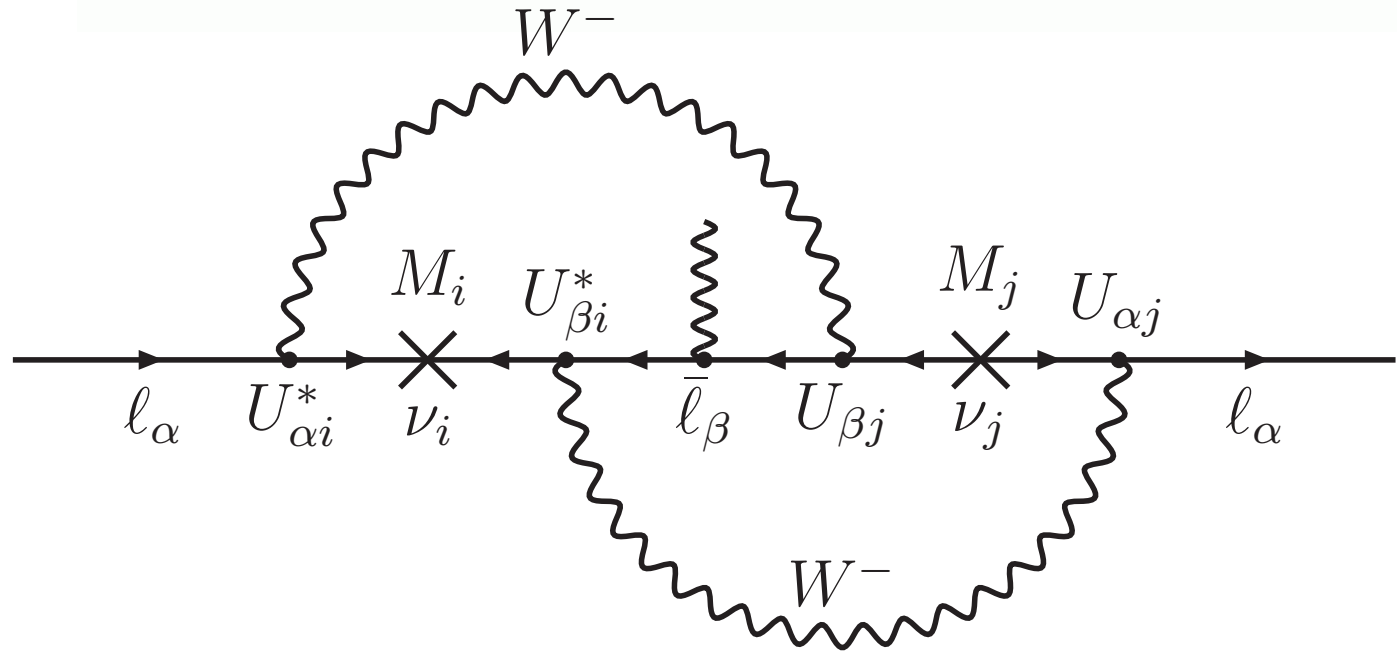
Shabalin, Sov. J. Nucl. Phys. 28 (1978) 75

2-loop diagram unique to leptons



$$\propto (U_{\alpha i}^* U_{\beta i}^*) (U_{\alpha j} U_{\beta j})$$

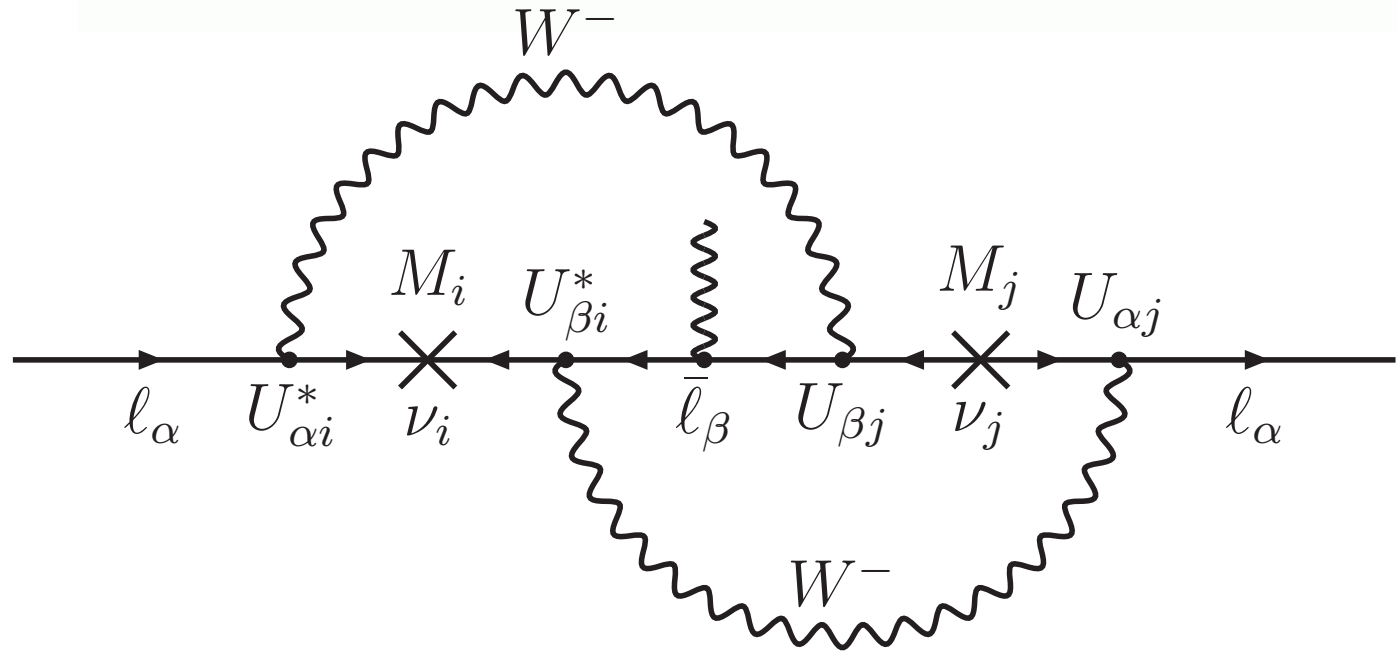
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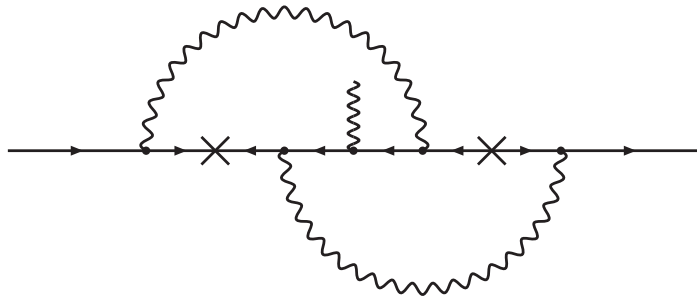


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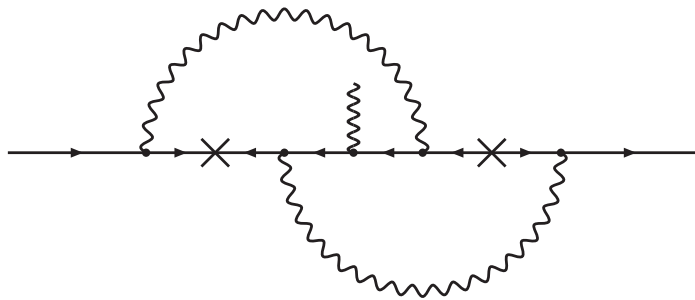
\implies Imaginary parts of $(U_{\alpha i}^* U_{\beta i}^*) (U_{\alpha j} U_{\beta j})$ **survive**

Is it significant?



$$\propto M_i M_j (U_{\alpha i}^* U_{\beta i}^*) (U_{\alpha j} U_{\beta j})$$

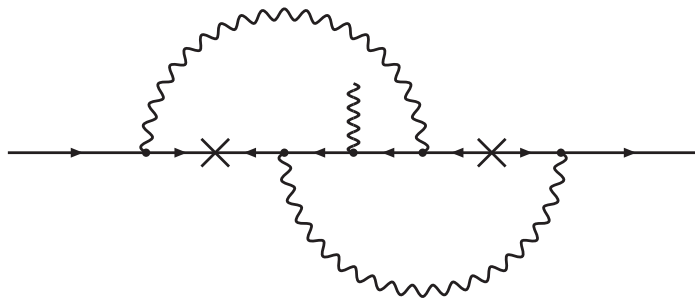
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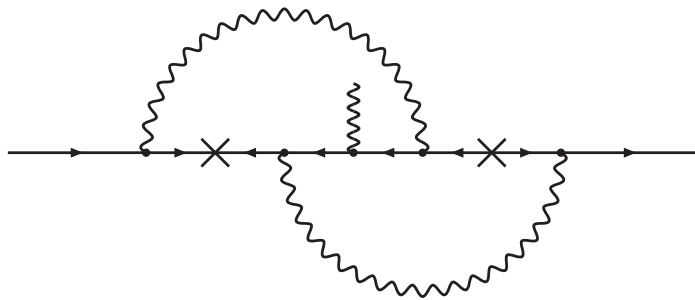
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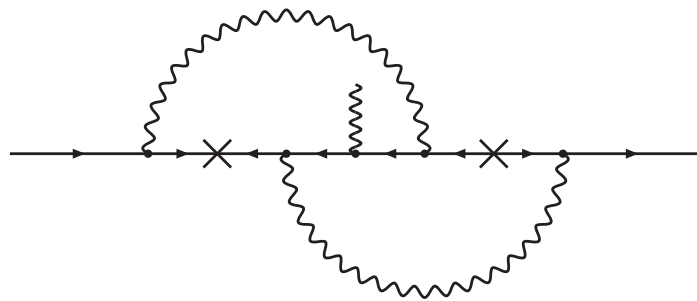
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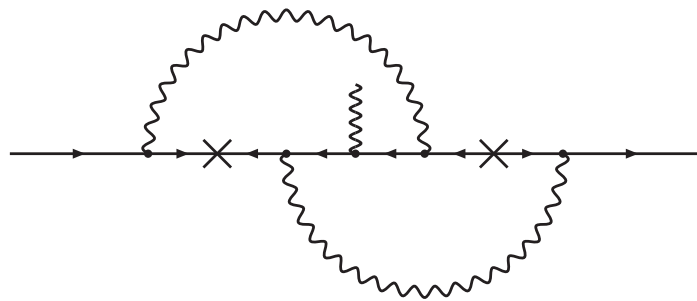
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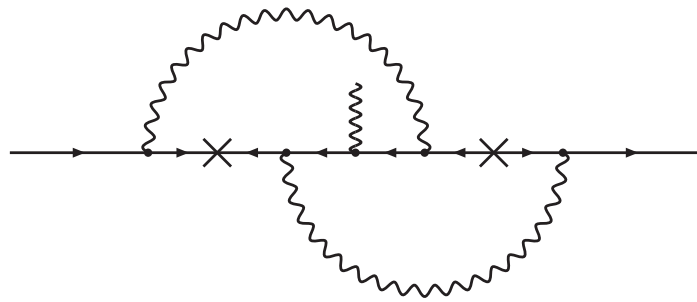
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→ same order as the light neutrino contribution.

TeV-Scale Seesaw Models

Example: Okamura Texture [PRD 68, 073001 \(2003\)](#)

$$\begin{bmatrix} 0 & 0 & 0 & \alpha D & \beta D & \gamma D \\ 0 & 0 & 0 & \alpha D & \beta D & \gamma D \\ 0 & 0 & 0 & \alpha D & \beta D & \gamma D \\ \alpha D & \alpha D & \alpha D & \alpha M & 0 & 0 \\ \beta D & \beta D & \beta D & 0 & \beta M & 0 \\ \gamma D & \gamma D & \gamma D & 0 & 0 & \gamma M \end{bmatrix}$$

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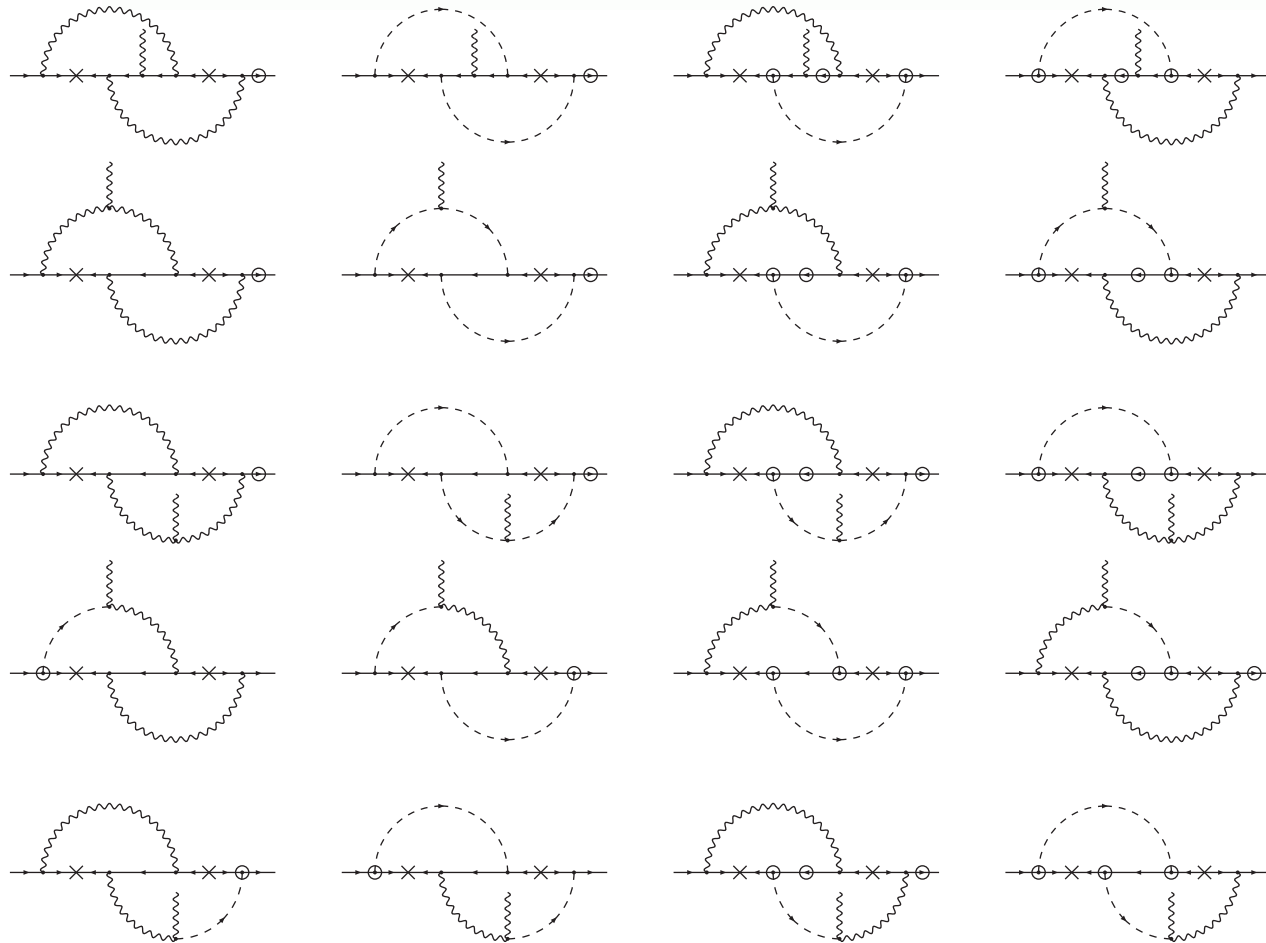
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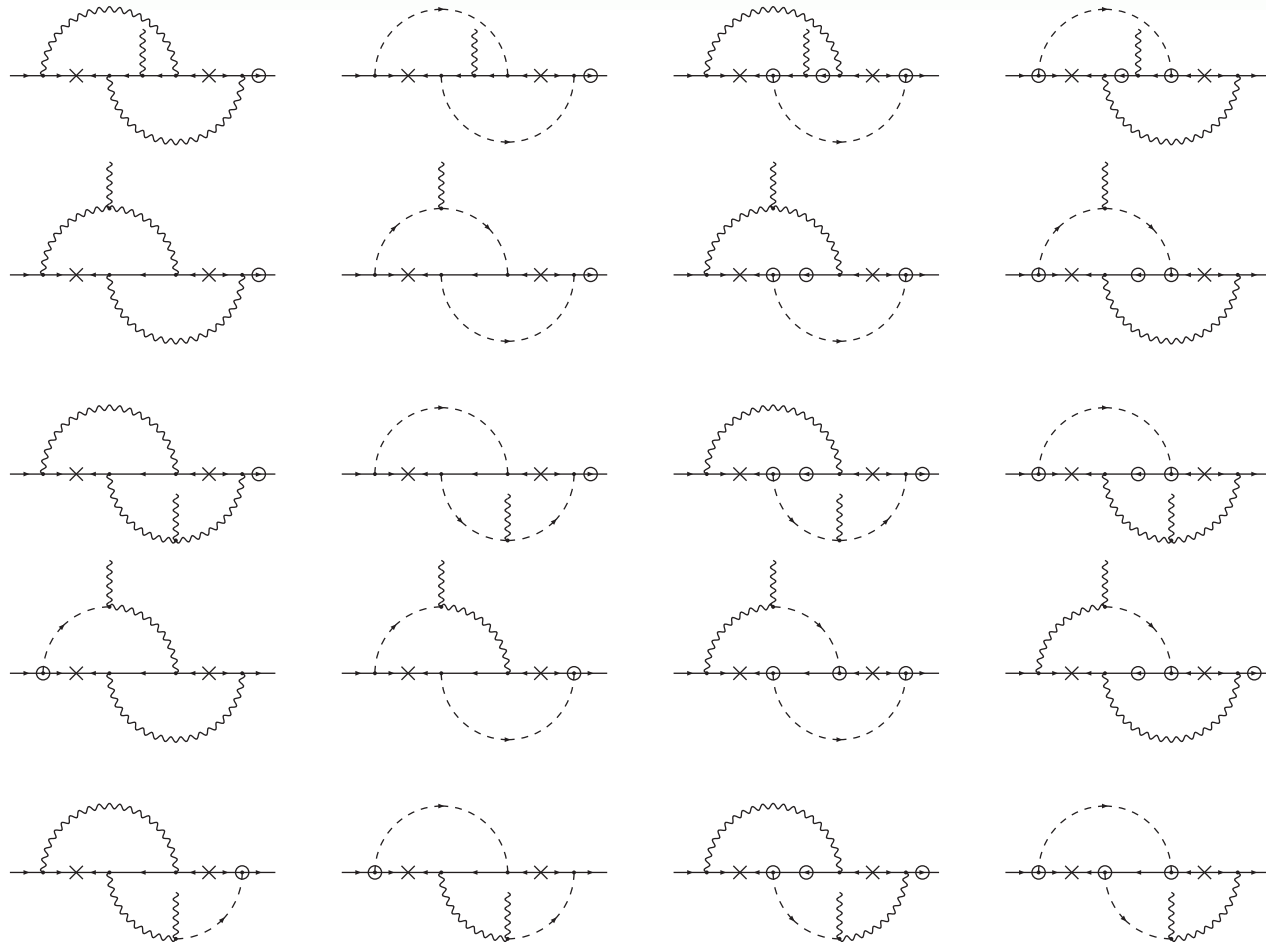
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 - light masses automatically zero.
 - mixings and masses are independent.
 - lepton EDM's can be **large**!?

20 Diagrams



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Give to graduate student!

Results by Saifuddin Rayyan 1



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⑥ Use:

M_W , neutrino Dirac mass \gg charged lepton masses

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⑥ Final result:

$d =$ see next page

Results by Saifuddin Rayyan 2

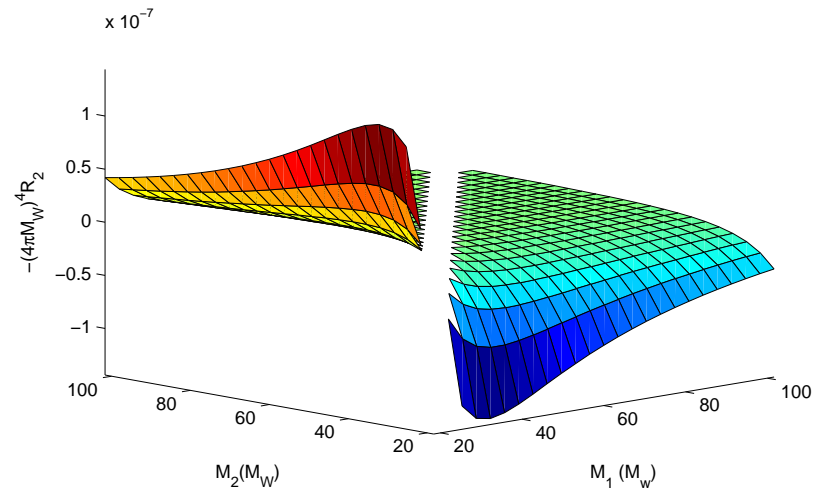
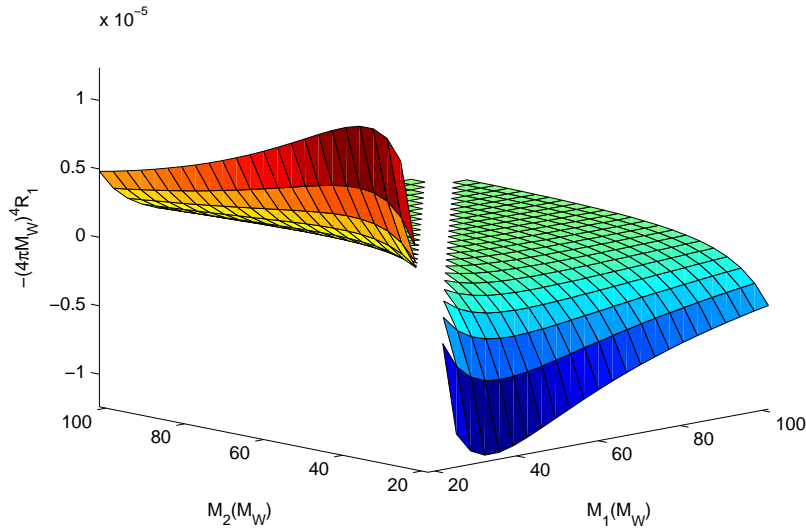
$$\begin{aligned} & \frac{eg^4}{2} m \sum_{\beta} \sum_{i>j} M_i M_j \Im \left(\tilde{V}_{\alpha i} \tilde{V}_{\beta i} \tilde{V}_{\beta j}^* \tilde{V}_{\alpha j}^* \right) R_1(M_i, M_j) \\ & + 2em \sum_{\beta} \sum_{i>j} M_i M_j \Im \left(\tilde{\Lambda}_{\alpha i} \tilde{\Lambda}_{\beta i} \tilde{\Lambda}_{\beta j}^* \tilde{\Lambda}_{\alpha j}^* \right) R_2(M_i, M_j) \\ & + \frac{eg^3}{\sqrt{2}} M_W \sum_{\beta} \sum_{i>j} M_i M_j \Im \left(\tilde{\lambda}_{\alpha i} \tilde{V}_{\beta i} \tilde{V}_{\beta j}^* \tilde{V}_{\alpha j}^* \right) R_3(M_i, M_j) \\ & + \sqrt{2} eg M_W \sum_{\beta} \sum_{i>j} M_i M_j \Im \left(\tilde{\Lambda}_{\alpha i} \tilde{\Lambda}_{\beta i} \tilde{V}_{\beta j}^* \tilde{\lambda}_{\alpha j}^* \right) R_4(M_i, M_j) \end{aligned}$$

Results by Saifuddin Rayyan 2

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Expressions for the R -functions fill many pages.

Results by Saifuddin Rayyan 3



Dependence of R_1 and R_2 on the two Majorana masses.

Numerical Results

Current Experimental Limits:

$$d_e = (6.9 \pm 7.4) \times 10^{-28} \text{ e} \cdot \text{cm}$$
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Preliminary Result for Okamura Model:

d_e can be as large as or larger than $O(10^{-28}) \text{ e} \cdot \text{cm}$

(Actual number depends on choice of parameters.)

Conclusions



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- ⑥ Next generation of experiments could either discover lepton EDM or place strong constraints on TeV-scale seesaw models.