PARTON SHOWER + NLO:
A POSITIVE-WEIGHT HARDEST EMISSION GENERATOR

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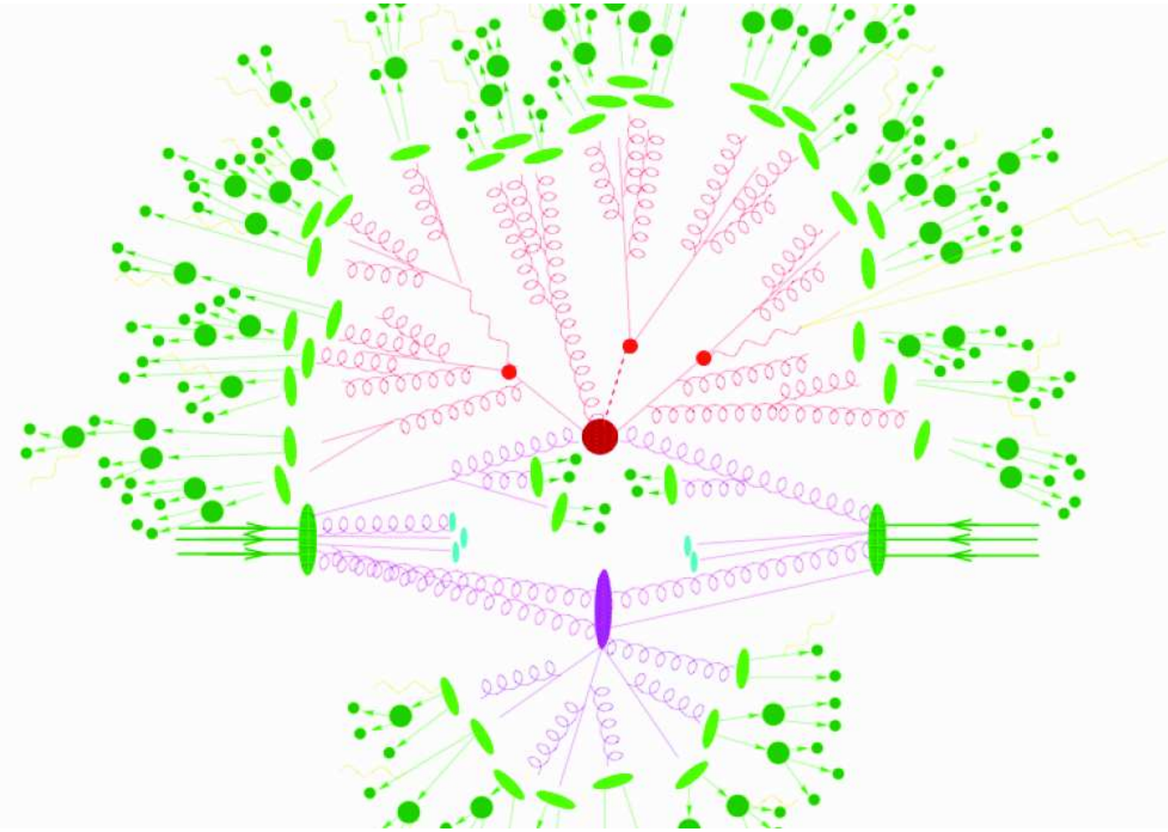
- Basics of Shower Monte Carlo programs
- The POWHEG formalism
- Conclusions



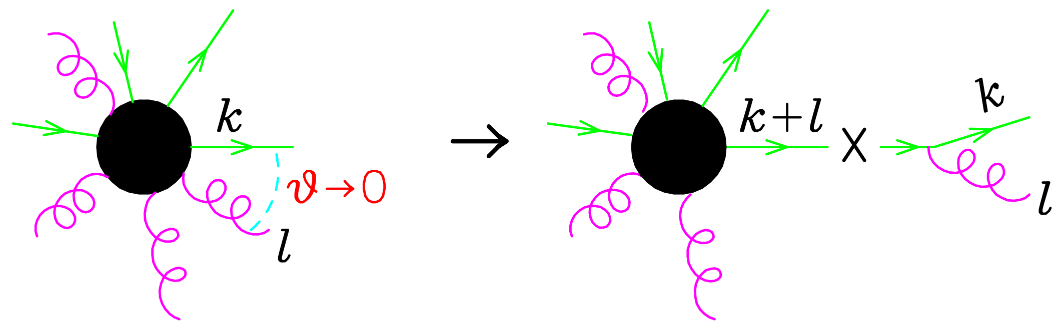
Motivations

- In high-energy collider physics not many questions can be answered without a Shower Monte Carlo (SMC).
- Even if QCD were solved exactly, it is unlikely that complex, high-energy phenomena will be described better than in SMC models.
- After LEP, QCD testing is less important. With LHC, QCD modeling is a primary issue.
- SMC models have long been neglected in theoretical physics: time to look back at them
- Thinking in terms of shower algorithms gives us an easy to grasp, intuitive understanding of complex QCD phenomena (and a practical way to verify our ideas).
- Shower algorithms summarize most of our knowledge in PQCD: infrared cancellations, Altarelli-Parisi equations, soft coherence, Sudakov form factors. All have a simple interpretation in terms of shower algorithms

Shower basics: collinear factorization

QCD emissions are enhanced near the collinear limit

Cross sections factorize near collinear limit



$$|M_{n+1}|^2 d\Phi_{n+1} \Longrightarrow |M_n|^2 \, d\Phi_n \, \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z) \, dz \, \frac{d\phi}{2\pi}$$

$$d\Phi_{n+1} = d\Phi_n \, d\Phi_r \qquad d\Phi_r \div dt \, dz \, d\phi$$

t : $(k+l)^2, \; p_T^2, \; E^2\theta^2 \ldots$

$z = k^0/(k^0+l^0)$: energy (or $p_\parallel$ or p^+)fraction of quark

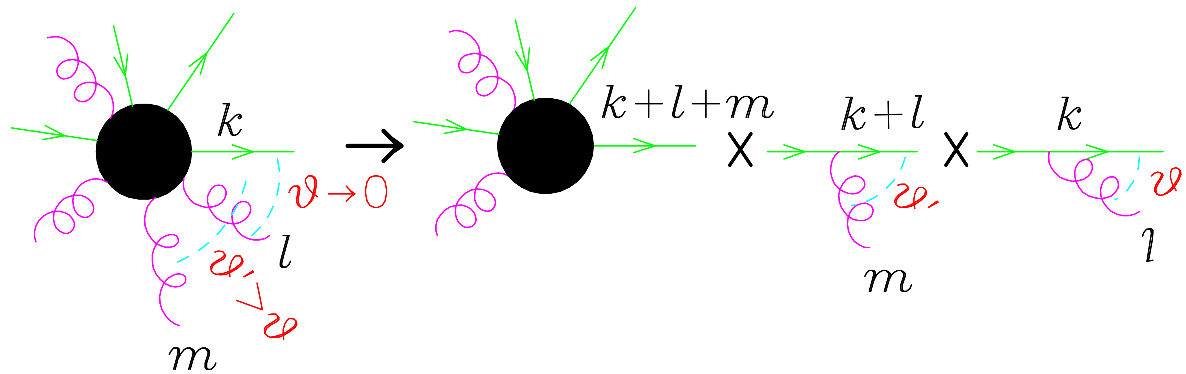
$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$: Altarelli-Parisi splitting function

(ignore $z \to 1$ IR divergence for now)

Shower basics: collinear factorization

If another gluon becomes collinear, iterate the previous formula:

$\theta', \theta \to 0$ with $\theta' > \theta$



$$|M_{n+1}|^2 d\Phi_{n+1} \implies |M_{n-1}|^2 d\Phi_{n-1} \times \frac{\alpha_s}{2\pi} \frac{dt'}{t'} P_{q,qg}(z')\, dz' \frac{d\phi'}{2\pi}$$
$$\times \frac{\alpha_s}{2\pi} \frac{dt}{t} P_{q,qg}(z)\, dz \frac{d\phi}{2\pi} \theta(t'-t)$$

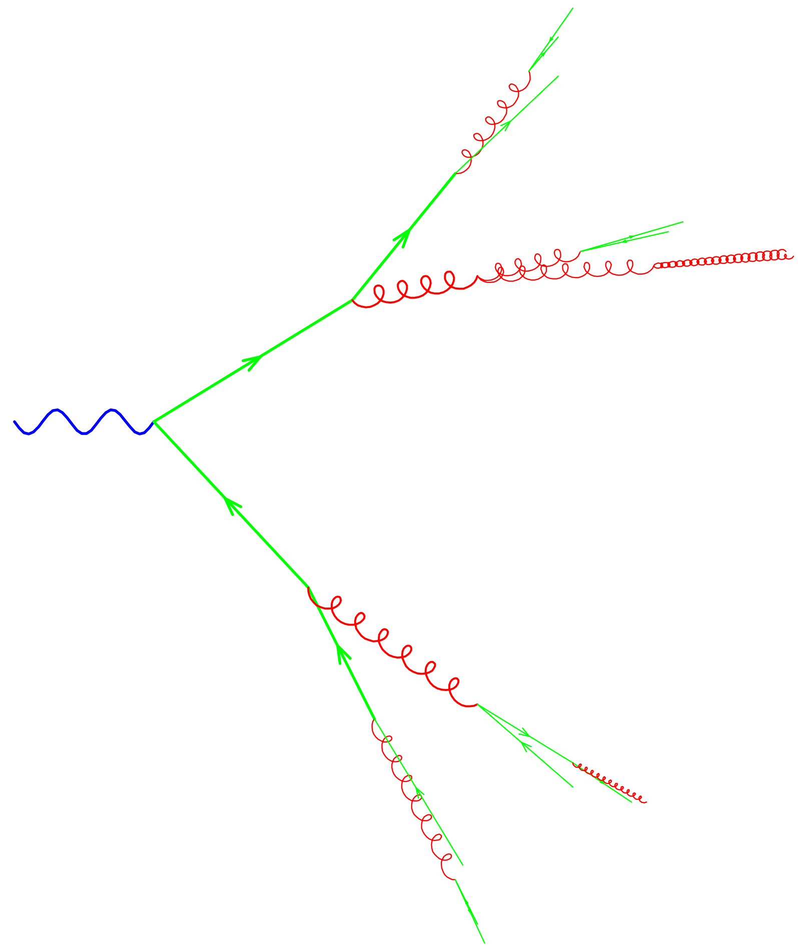
Collinear partons can be described by a factorized integral ordered in t.
For m collinear emissions:

$$\int_{t_{\min}}^{t^{\max}} \frac{dt_1}{t_1} \int_{t_{\min}}^{t_1} \frac{dt_2}{t_2} \dots \int_{t_{\min}}^{t_{m-1}} \frac{dt_m}{t_m} \propto \frac{\log^m \frac{t_{\max}}{t_{\min}}}{m!} \approx \frac{\log^m \frac{Q^2}{\Lambda^2}}{m!}, \qquad \Lambda \approx \Lambda_{\mathrm{QCD}}$$

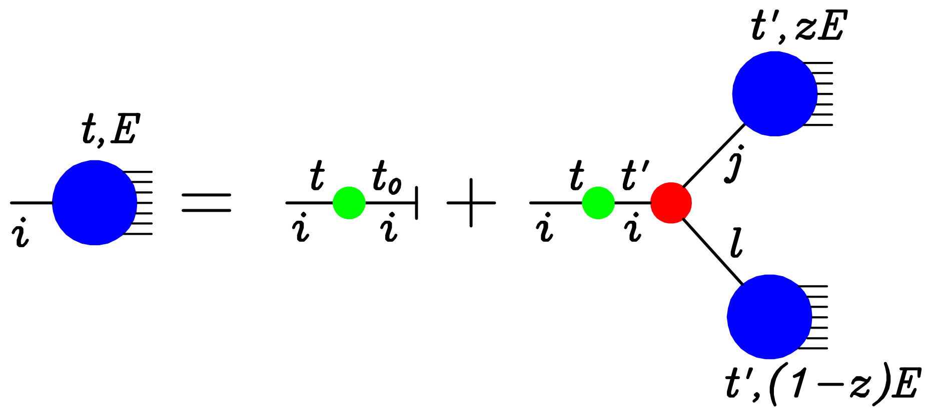
Typical dominant configuration at very high Q^2

$\gamma^* \longrightarrow$ hadrons

- Besides $q \rightarrow qg$, also $g \rightarrow gg$, $g \rightarrow q\bar{q}$ come into play.
- In the typical configurations, intermediate angles are of order of geometric average of upstream and downstream angles.
- Each angle is $\mathcal{O}(\alpha_s)$ smaller than its upstream angle, and $\mathcal{O}(\alpha_s)$ bigger than its downstream angle.
- As relative momenta become smaller α_s becomes bigger, and this picture breaks down.



Final Recipe

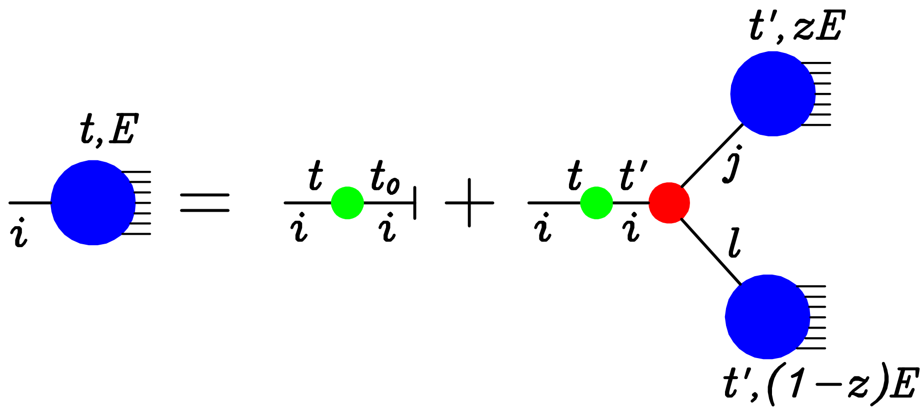


$$\mathcal{S}_i(t,E) = \Delta_i(t,t_0)\langle\mathbb{I}| + \sum_{(jl)} \int_{t_0}^{t} \frac{\alpha_S(t')}{2\pi}\frac{dt'}{t'} \int dz \int \frac{d\phi}{2\pi} \Delta_i(t,t')\ \mathcal{S}_j(t',zE)\ \mathcal{S}_l(t',(1-z)E)$$

- consider all tree graphs.
- assign values to the radiation variables Φ_r (t, z and ϕ) to each vertex.
- at each vertex, $i \to jk$, include a factor

$$\frac{dt}{t}\,dz\,\frac{\alpha_s(t)}{2\pi}\,P_{i,jk}(z)\,\frac{d\phi}{2\pi}$$

Final Recipe



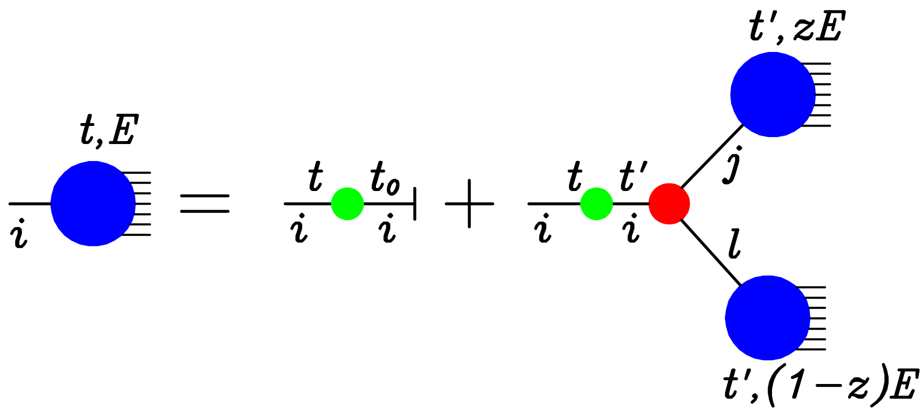
- include a factor $\Delta_i(t_1, t_2)$ to each internal parton i, from hardness t_1 to hardness t_2.

$$\Delta_i(t_1, t_2) = \exp\left[-\sum_{(jl)} \int_{t_2}^{t_1} \frac{dt}{t} \frac{\alpha_S(t)}{2\pi} \int dz\, P_{i,jl}(z) \int \frac{d\phi}{2\pi}\right]$$

The weights $\Delta_i(t_1, t_2)$ are called Sudakov form factors. They resum all the dominant virtual corrections to the tree graph (in the collinear approximation).
Notice that $0 < \Delta < 1$. So Δ can be interpreted as the non-emission probability.

- include a factor $\Delta_i(t, t_0)$ on final lines (t_0 = IR cutoff)

First branching



The probability of the first branching is independent of subsequent branchings
It is given by

$$\frac{\alpha_S(t')}{2\pi}\,\frac{dt'}{t'}\,P_{i,jk}(z)\,dz\,\frac{d\phi}{2\pi}\,\Delta_i(t,t')$$

Integrating in $dz, d\phi$, summing over jl, the t' distribution is

$$\Delta_i(t,t')\,\frac{\alpha_S(t')}{2\pi}\,\frac{dt'}{t'}\int\sum_{(jk)}P_{i,jk}(z)\,dz\frac{d\phi}{2\pi}=-d\Delta_i(t,t')$$

i.e. the distribution is uniform in the Sudakov form factor!

Shower algorithm

- generate a uniform random number $0 < r < 1$
- solve the equation $\Delta_i(t, t') = r$ for t'
- if $t' < t_0$ stop here (final state line)
- generate z, jk with probability $P_{i,jk}(z)$, and $0 < \phi < 2\pi$ uniformly
- restart from each of the two branches, with hardness parameter t'.

Accuracy: soft divergences and double-log regions

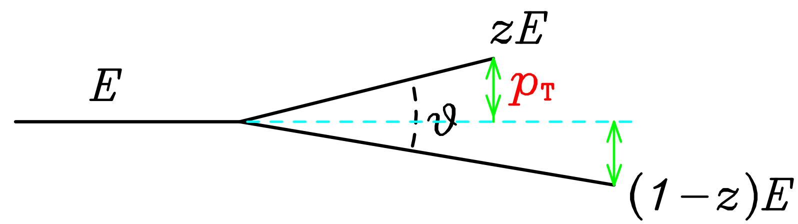
$z \to 1$ ($z \to 0$) region problematic. In fact, for $z \to 1$, $P_{qq}, P_{gg} \propto 1/(1-z)$

Choice of hardness variable t makes a difference

$$\text{virtuality:} \quad t \equiv E^2 z(1-z) \overbrace{\theta^2}^{2(1-\cos\theta)}$$

$$p_T^2: \quad t \equiv E^2 z^2 (1-z)^2 \theta^2$$

$$\text{angle:} \quad t \equiv E^2 \theta^2$$



$$\text{virtuality} : z(1-z) > t/E^2 \implies \int \frac{dt}{t} \int_{\sqrt{t}/E}^{1-\sqrt{t}/E} \frac{dz}{1-z} \approx \frac{\log^2 \frac{t}{E^2}}{4}$$

$$p_T^2 : z^2(1-z)^2 > t/E \implies \int \frac{dt}{t} \int_{t/E}^{1-t/E} \frac{dz}{1-z} \approx \frac{\log^2 \frac{t}{E^2}}{2}$$

$$\text{angle} : \implies \int \frac{dt}{t} \int_0^1 \frac{dz}{1-z} \approx \log t \log \Lambda$$

Sizable difference in double-log structure!

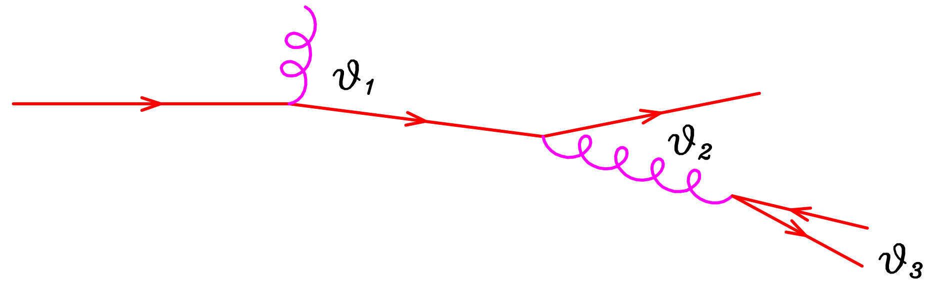
Angular ordering

Mueller (1981) showed that angular ordering is the correct choice

$$\frac{d\theta}{\theta}\,\frac{\alpha_s\left(p_T^2\right)}{2\pi}\,P(z)\,dz$$

$$\theta_1 > \theta_2 > \theta_3 \ldots$$

$$p_T^2 = E^2 z^2 (1-z)^2 \theta^2$$



$\alpha_s(p_T^2)$ for a correct treatment of charge renormalization in soft region.

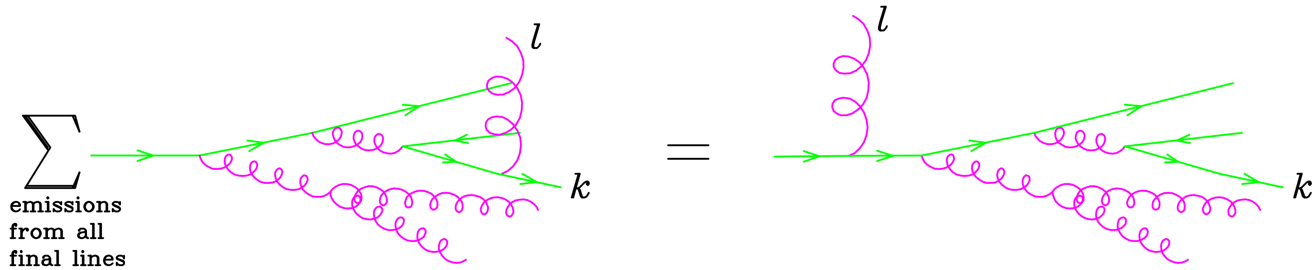
$$\Delta_i(t,t') = \exp\left[-\int_{t'}^{t}\frac{dt}{t}\int_{\sqrt{\frac{t_0}{t}}}^{1-\sqrt{\frac{t_0}{t}}} dz\,\frac{\alpha_s(p_T^2)}{2\pi}\sum_{(jk)} P_{i,jk}(z)\right]$$

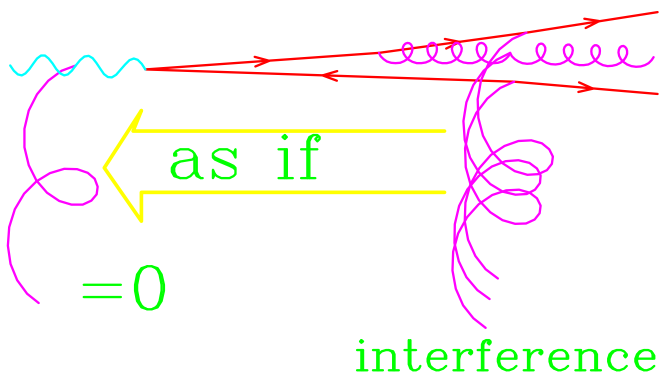
$$\approx \exp\left\{-\frac{c_i}{4\pi b_0}\left[\log\frac{t}{\Lambda^2}\log\frac{\log\frac{t}{\Lambda^2}}{\log\frac{t_0}{\Lambda^2}} - \log\frac{t}{t_0}\right]_{t'}^{t}\right\} \qquad (c_q = C_F, c_g = 2C_A)$$

Sudakov dumping stronger than any power of t.

Color coherence

Soft gluons emitted at large angles from final-state partons add coherently





- angular ordering accounts for soft gluon interference.
- intensity for photon jets $= 0$
- intensity for gluon jets $= C_A$ instead of $2C_F + C_A$

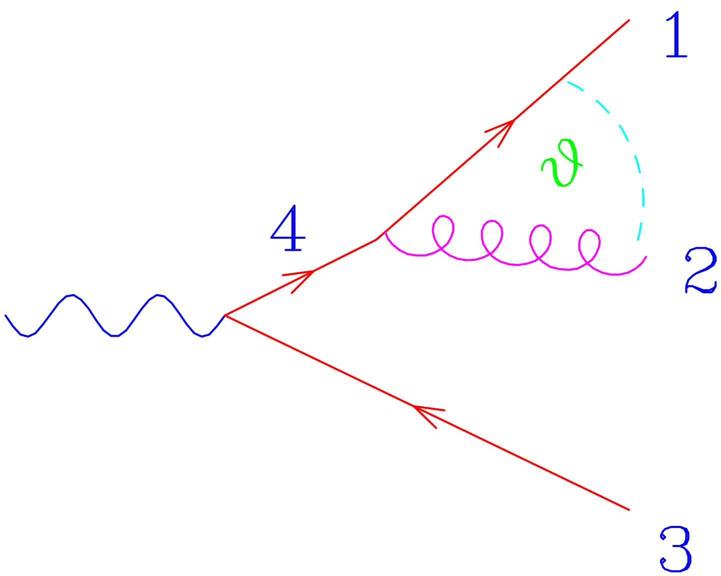
In angular ordered shower Monte Carlo, large-angle soft emission is generated first.
Hardest emission (i.e. highest p_T) happens later.

POWHEG

The POWHEG (POsitive-Weight Hardest Emission Generator) method [Nason, hep-ph/0409146] deals with two main issues:

1. transform an angular ordered shower into a shower where the hardest emission happens first
   - generate first event with hardest emission
   - generate all subsequent emissions with a p_T veto equal to the hardest emission p_T
   - pair up the partons that are nearest in p_T
   - generate an angular ordered shower associated with the paired parton, stopping at the angle of the paired partons (truncated shower)
   - generate all subsequent vetoed showers
2. include exact NLO cross section

Example of truncated shower: e^+e^-



- nearby partons: 1 and 2
- truncated shower: 1 and 2 pair, from θ up to a maximum angle. The truncated shower reintroduces coherent soft radiation from 1 and 2 at angles larger than θ (angular-ordered shower Monte Carlo programs generate those earlier).
- 1 and 2 shower from θ to cutoff
- 3 showers from maximum to cutoff

Truncated showers not yet implemented.

No evidence of effects from their absence up to now [Nason and Ridolfi, hep-ph/0606275], [Latunde-Dada, Gieseke and Webber, hep-ph/0612281]

Reaching the NLO accuracy

$$\Phi_{n+1} = \Phi_{n+1}(\Phi_n, \Phi_r) \qquad d\Phi_{n+1} = d\Phi_n \, d\Phi_r \qquad d\Phi_r = dt\, dz \frac{d\phi}{2\pi}$$

$$d\sigma^{\rm LO} = B(\Phi_n)\, d\Phi_n \left[\Delta^{\rm LO}(t_{\rm min}) + \int d\Phi_r \, \Delta^{\rm LO}(t) \, \frac{\alpha_s}{2\pi} \, P(z) \, \frac{1}{t}\right]$$

$$d\sigma^{\rm NLO} = \bar{B}(\Phi_n)\, d\Phi_n \left[\Delta^{\rm NLO}_{t_{\rm min}} + \int d\Phi_r \, \Delta^{\rm NLO}_t \, \frac{R(\Phi_{n+1})}{B(\Phi_n)}\right]$$

$$\bar{B}(\Phi_n) = B(\Phi_n) + \underbrace{\overbrace{V(\Phi_n)}^{\text{infinite}} + \overbrace{\int d\Phi_r \, R(\Phi_{n+1})}^{\text{infinite}}}_{\text{FINITE!}}$$

$$\Delta^{\rm LO}(t) = \exp\left[-\int_t d\Phi'_r \, \frac{\alpha_s}{2\pi} \, P(z') \, \frac{1}{t'}\right] \qquad \Delta^{\rm NLO}_t = \exp\left[\underbrace{-\int d\Phi'_r \, \frac{R(\Phi_n, \Phi'_r)}{B(\Phi_n)} \, \theta(t'-t)}_{\text{FINITE because of } \Theta \text{ function}}\right]$$

with $t' = p_T(\Phi_n, \Phi'_r)$ = transverse momentum of the emitted parton.
If we expand the shower expression in α_s, we obtain a result that is accurate at the NLO level.

POSITIVE if $\bar{B}$ is positive!

POsitive-Weight Hardest Emission Generator

POWHEG is a method (NOT only a program)

- ✓ it is independent from parton-shower programs
- ✓ it can use existing NLO results
- ✓ it generates events with positive weights
- ✓ NLO accuracy for integrated quantities
- ✓ LL-collinear + double-log (soft-collinear) + large-N_c-soft-log accurate on first radiation (for 3 external colored partons, it is exact at NLL accuracy). It should be easy to extend to more colored partons.

It has already been successfully used by

- [Nason and Ridolfi hep-ph/0606275] in ZZ production
- [Latunde-Dada, Gieseke and Webber, hep-ph/0612281] in e^+e^- to hadrons
- [Frixione, Ridolfi and Nason] in heavy-quark production. To appear soon.

Work in progress

POWHEG is fully general and can be applied to any subtraction framework.

We [Frixione, Nason and C.O.] are now working on providing any user with all the formulae and ingredients to implement an existing NLO calculation in the POWHEG formalism.

We have looked in details at POWHEG in the two subtraction schemes:

- the FKS (Frixione, Kunszt, Signer)
- the CS (Catani, Seymour).

The paper will appear very soon.

Strategy

- ✓ Shower Monte Carlo programs to do the final shower already exist
- ✓ Most of them implement a p_T veto
- ✓ Most of them comply with a standard interface to hard processes, the so called Les Houches Interface (LHI)

SO...

- construct a POWHEG for a NLO process. Output on LHI
- if needed, construct a generator capable to add truncated showers to events from the LHI. Output again on LHI
- use standard Shower Monte Carlo to perform the p_T vetoed final shower from the event on LHI.